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Is the Parallel Market Premium a Reliable Indicator of
Real Exchange Rate Misalignment in Developing Countries?

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Abstract

It is often argued that the parallel market premium is a useful indicator of real exchange rate misalignment in developing countries. The empirical evidence does not, however, suggest the existence of a robust correlation between these two endogenous variables that is independent of the nature of economic shocks and various structural relationships in the economy. This paper presents an analytical investigation of the reliability of the parallel market premium as an indicator of real exchange rate misalignment in the context of a fully optimizing model of a developing country. The analysis suggests that one should exercise caution in drawing inferences about the sign and magnitude of real exchange rate misalignment from the parallel market premium.

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I. Introduction

Real exchange rate misalignment--the occurrence of a sustained departure of the actual real exchange rate from its equilibrium value--has been a recurring policy problem in many developing countries. Overvaluation of the real exchange rate has had undesirable effects on net exports and growth in some countries, while undervaluation has created problems for monetary control and inflation in others. Getting the real exchange rate "right" in a world in which the fundamental structural determinants of the equilibrium real exchange rate are constantly changing remains one of the most important goals of economic policy in developing countries.

The problems associated with real exchange rate misalignment have led policymakers in developing countries to try to reduce the degree of overvaluation or undervaluation of their currencies by appropriate policies for the nominal exchange rate. The idea behind such policies is that the transition from, for example, a situation of overvaluation to one of real exchange rate equilibrium can be long and drawn out, particularly if institutional factors--such as wage- and price-setting behavior--are not very favorable. Under these circumstances, a change in the nominal exchange rate (a devaluation) can help to reestablish equilibrium more quickly and thereby mitigate some of the costs associated with the transition.

A well-known problem associated with the implementation of such policies is that the extent of real exchange rate misalignment--and in some cases even its sign--may be difficult to gauge. This is because information about the extent of misalignment requires knowledge of the level of the equilibrium real exchange rate, which depends on both structural factors (including trade and industrial policies, the degree of capital mobility, and the terms of trade) and on macroeconomic factors, such as the level and composition of government spending and taxation as well as the international macroeconomic environment. The problem of real exchange rate misalignment thus lies at the heart of both macroeconomic and structural policy in developing countries.

In practice, the problem of estimating the extent of real exchange rate misalignment has been addressed in a variety of ways. One approach has been to determine some base period in which the actual and equilibrium real exchange rates were equal, and then to attempt to determine the extent by which the equilibrium real rate has changed as a result of changes in its fundamental structural determinants, so that a comparison can be made with the path of the actual real exchange rate. ^{1/} This approach, however, has not been used as frequently as one might think, probably because the effects of exogenous and policy-induced shocks on the equilibrium real exchange rate depend on structural relationships in the economy about which policymakers are likely to have insufficient information.

^{1/} For an attempt at such an exercise, see Khan and Ostry (1992).

An alternative approach, which is simpler and more direct, involves using information from the parallel market to gauge the extent of real exchange rate misalignment. 1/ The existence of a premium on foreign exchange in the free market is taken as indicative of an excess demand for foreign exchange at the official exchange rate, which in turn is interpreted as arising from an overvaluation of the domestic currency at the prevailing official exchange rate.

This intuition is buttressed by models in the literature in which the premium and the degree of misalignment both respond endogenously to some shock, such as an increase in the stock of credit. For example, models in the currency-substitution tradition (Calvo and Rodriguez (1977)) have been used by Edwards (1989) and Kamin (1993) to analyze the effects of unsustainable financial policies on the parallel market premium and the divergence of the real exchange rate from its long-run equilibrium value. The robust finding from these models is that, along the adjustment path, overvalued real exchange rates are associated with high premia. One interpretation of this relationship views the equilibrium exchange rate as a weighted average of the free rate and the official rate. Consequently in many developing countries, exchange rate policy is designed to reduce the gap between the two rates by depreciating the official exchange rate (see, for example, Aghevli *et. al.* (1991)). In effect, the misalignment of the official rate is being contained by targeting the premium at a reduced level. 2/

This view of the relationship between the premium and real exchange rate misalignment has some empirical support. For example, studies of a number of devaluation episodes (see Edwards (1989) and Kamin (1993)) find that the parallel market premium often rises very rapidly in the period immediately preceding a major devaluation, and then falls off just after the devaluation. A positive correlation between the premium and the extent of underlying real exchange rate overvaluation would seem to be suggested in such cases.

Nonetheless, from an analytical standpoint, the case for treating the size of the parallel market premium as an indicator of the magnitude of real exchange rate misalignment seems far from obvious. Both the premium on foreign exchange in the free market and the real exchange rate in the official market are endogenous variables with complex macroeconomic roles and, as such, the correlation between them should depend in general on the sources of shocks impinging on the economy. Moreover, the parallel market premium is an asset price, which can be expected to exhibit much greater volatility than the official real exchange rate, in particular by responding to transitory shocks that leave the equilibrium real exchange rate unaffected. The very different time-series properties of these two

1/ On the use of the premium as an indicator of real exchange rate misalignment in developing countries, see Quirk *et. al.* (1987).

2/ For an analysis of the Bolivian case, see Kharas and Pinto (1989).

variables raises some doubt about the reliability of the premium as an indicator of real exchange rate misalignment.

Empirically, parallel market premia tend to be a good deal more variable than real exchange rates in developing countries, and can easily reach levels of several hundred percent at times without discernible corresponding changes in the underlying extent of real exchange rate misalignment. Furthermore, the sign of the correlation between the parallel market premium and the official real exchange rate seems to vary from country to country and time period to time period. ^{1/} For example, in the study cited previously, Kamin (1993) found that in about a third of his sample of devaluation episodes, the premium actually fell prior to an exchange rate correction. This certainly calls into question the presumption that there is a robust correlation between real exchange rate misalignment and the parallel market premium that is independent of the nature of the underlying shocks and the structure of the economy.

The purpose of this paper is to explore these issues in the context of a fully optimizing model of a developing country that simultaneously determines the degree of misalignment of the real exchange rate and the premium in the parallel market. The paper's objective is to assess the reliability of the premium as an indicator of exchange rate misalignment by answering the following questions: Letting e and \bar{e} denote respectively the actual and equilibrium real exchange rates, and letting b denote (one plus) the premium in the free exchange market, can we identify a structural relationship $b-1 = f(e-\bar{e})$ that would provide *quantitative* information about the degree of real exchange rate overvaluation from observations of the premium? If not, can we at least establish that the sign of $(b-1)$ is the same as the sign of $(e-\bar{e})$, so that the premium serves as a *qualitative* indicator of misalignment? If neither of these can be established in a standard model, then the former cannot be taken as a reliable indicator of the latter.

This paper is organized as follows. We set up the model in Section II, beginning by describing the consumer's optimization problem. Section III discusses the equilibrium and solution of the model. Section IV contains the central point of the paper, showing that adjustment in an economy (modeled along fairly standard lines) to even a simple shock (consisting in this case of a permanent productivity shock) can be rather complex. We will show in particular that while the adjustment of the real exchange rate to its new long-run equilibrium is monotonic, that of the parallel market premium is not, implying that neither the sign nor the magnitude of the premium is informative about the extent of misalignment in this case. Section V summarizes the main conclusions.

^{1/} See for example the discussion in Edwards (1989), chapter 4.

II. The Household's Problem

Consider a small open economy in which agents derive utility from the consumption of traded and nontraded goods, denoted respectively by c_T and c_N . The instantaneous utility function u is thus given by:

$$(1) \quad u(t) = u[c_T(t), c_N(t)].$$

Consumption is subject to a cash-in-advance constraint of the form:

$$(2) \quad m(t) = \alpha c(t)$$

where $m = M/P$ is the stock of money deflated by the true consumption-based price index, P ; c is the real value of aggregate consumption, which is related to consumption of traded and nontraded goods by $c = e^\beta(c_T + c_N/e)$, where e is the price of tradables relative to nontradables (the real exchange rate), $0 < \beta < 1$ is a parameter whose economic interpretation will be given below; and $\alpha > 0$. Apart from domestic money, household financial wealth includes foreign-currency denominated securities, which are denoted by F .

A "dual" exchange rate regime is assumed to be in place, involving a fixed "official" exchange rate, s , and a floating "unofficial" or "parallel" market exchange rate, v . All commercial transactions are assumed to take place at the official rate s , while all other (financial) transactions take place at the market-determined rate v . We denote by b the ratio of the financial to the commercial exchange rate, so that $b = v/s$ denotes one plus the premium in the parallel market. Thus, the real value (in terms of traded goods) of household financial wealth, denoted by a , may be written as:

$$(3) \quad a = me^{-\beta} + bF,$$

It is assumed that agents can engage in cross-transactions between official and unofficial markets. These leakages, denoted by L , carry a cost $T(L)$, where $T(0)=0$, $T'(0)=0$, and $T''(0)>0$. ^{1/} Thus, the increase in resources available to consumers by engaging in "fraudulent" activity (leakages) must be traded off against the costs associated with such activity, which are captured by the $T(L)$ function. Taking into account these leakages, the instantaneous budget constraint of the representative household may be written as:

$$(4) \quad \dot{a} = y + (r^* + \hat{b})bF - c_T - c_N/e - T(L) + (b-1)L,$$

^{1/} The first optimizing model of dual rates with leakages is due to Bhandari and Végh (1990). The main differences between their paper and ours relate to the goods structure and the economy's access to world capital markets.

where a dot above a variable denotes a time derivative, while a circumflex denotes a proportional rate of change, that is $\hat{x} = \dot{x}/x$; y denotes the real value (in terms of traded goods) of output of both traded and nontraded goods $\underline{1}$; r^* is the world interest rate; and the last two terms in (4) denote, respectively, the costs associated with fraudulent transactions, and the benefits associated with such activity. Thus, the change in real household wealth (per unit of time) is simply equal to income (which consists of income from production (y), plus interest earnings on foreign securities (r^*bF) plus capital gains ($\hat{b}bF$)) less consumption, plus the net benefits associated with fraudulent activity.

The problem of the representative household is to choose values of consumption of traded and nontraded goods, c_T and c_N , of foreign securities F , and of the amount of leakage into the free market L , to maximize the discounted sum of utility given by:

$$(5) \quad \int_0^{\infty} u(c_T(t), c_N(t)) \exp(-\delta t) dt$$

subject to the budget constraint given by equation (4) and a transversality condition which requires that the present value of household net worth converge to a nonnegative number. In addition to the budget constraint (4) and the transversality condition, the optimality conditions for this problem are given by:

$$(6) \quad u_1[c_T(t), c_N(t)] = \lambda[1 + \alpha(r^* + \hat{b})],$$

$$(7) \quad u_2[c_T(t), c_N(t)] = \lambda[1 + \alpha(r^* + \hat{b})]/e,$$

$$(8) \quad T'(L) = b - 1,$$

$$(9) \quad \dot{\lambda} = \lambda[\delta - (r^* + \hat{b})].$$

The interpretation of these conditions is standard. The first two (equations (6) and (7)) require that the marginal utility of consumption of each of the two goods be equated with the product of the marginal utility of wealth, λ , and the price of the good. The price itself involves two components. First, there is the market price (unity in the case of traded goods and the reciprocal of the real exchange rate e in the case of home goods); second, because of the cash-in-advance constraint, there is the opportunity cost of holding money, which is α (the factor of proportionality between consumption and money) times the real interest rate, $r^* + \hat{b}$. Clearly, the domestic real interest rate is equal to the world rate plus a term (the rate of change in the parallel market premium) that captures the capital gains or losses from holding foreign-currency denominated assets.

1/ The production side of the economy is described in Section III.

The third optimality condition (equation (8)) relates the amount of leakage to the level of the parallel market premium. Given that $T(L)$ is a convex function, the condition states that the higher is the premium, $b-1$, the greater the amount of leakage between the official and parallel markets. Finally, equation (9) is the standard condition relating the path of the costate variable to the difference between the subjective rate of time preference and the domestic real interest rate.

We proceed under the simplifying assumption that the instantaneous utility function in (1) is logarithmic:

$$(1') \quad u[c_T(t), c_N(t)] = (1-\beta)\ln c_T(t) + \beta\ln c_N(t).$$

Denoting aggregate consumption in terms of traded goods by $Z = c_T + c_N/e$, we have from (1') and the first two optimality conditions that consumption of traded and nontraded goods are given by:

$$(10) \quad c_T(t) = (1-\beta)Z(t), \quad c_N(t) = \beta e(t)Z(t).$$

Thus, consumption of the two goods depends on the level of total expenditure Z and on the the real exchange rate e . 1/ We can also solve for the marginal utility of wealth in terms of aggregate expenditure and the domestic interest rate:

$$(11) \quad \lambda = 1/(Z[1+\alpha(r^*+\hat{b})]).$$

III. Equilibrium and Solution of the Model

We begin by discussing some issues that affect the dynamics of the aggregate economy, but which are exogenous from the point of view of the individual household and hence were ignored in the previous section. First, it should be noted that from the point of view of the economy as a whole, the legal stock of foreign securities is fixed and the actual stock can therefore only be altered through leakages. Thus, recalling that interest earnings on foreign securities are repatriated through the parallel market (since the official market is by assumption only for commercial transactions), we have:

$$(12) \quad \dot{F} = L + r^*F,$$

where the second term in (12) represents interest earnings on the existing stock of foreign securities.

Second, the country will be assumed to face an upward-sloping supply of foreign loans--i.e., the cost of funds to domestic residents depends on the actual stock of foreign securities held by the country as a whole. Specifically, it is assumed that $r^* = r^*(F)$, with $r^*(F)$ being a decreasing

1/ Clearly, β is the expenditure share of nontraded goods.

function that satisfies $r^*(0) = \delta$, where δ is the rate of time preference. Thus, when the country is neither a net creditor nor a net debtor ($F = 0$), the rate of interest its residents face is simply equal to the rate of time preference δ , but the rate of interest faced by the domestic residents rises with $-F$, that is with the extent of the country's net debtor position. This makes r^* a decreasing function of F . 1/

Third, in the previous section, the household took the production side of the economy as given. To close the model, however, the conditions of production need to be specified. We adopt a particularly simple approach here, wherein there is no investment or other dynamic problem that firms need to solve so that production decisions maximize period by period profits. It is also assumed that all prices are flexible so that the economy operates continually at full employment.

A convenient way of modelling the supply side is to assume that the economy possesses a concave transformation frontier between traded and nontraded goods. 2/ It operates at the point of tangency between that frontier and a straight line with slope equal to (minus) the reciprocal of the real exchange rate. This implies supply functions of the form:

$$(13) \quad y_T = y_T(e; \theta), \quad \partial y_T / \partial e > 0, \quad \partial y_T / \partial \theta > 0; \quad y_N = y_N(e), \quad \partial y_N / \partial e < 0,$$

where θ is a productivity parameter in the traded goods sector to be discussed below. 3/ Using the definition of output in terms of traded goods y together with the envelope condition also implies:

$$(14) \quad y(e; \theta) = y_T(e; \theta) + y_N(e)/e, \quad y_1 \equiv \partial y / \partial e > 0, \quad y_2 \equiv \partial y / \partial \theta > 0.$$

Finally, we can use this simple specification of production together with the demand functions given in equation (10) to solve for the equilibrium real exchange rate in terms of the level of total expenditure, Z . Specifically, since in equilibrium the nontraded goods market must clear, we have:

$$(15) \quad y_N(e) = \beta e Z.$$

Differentiating (15) we can solve for the equilibrium real exchange rate in terms of total expenditure:

1/ Essentially, the role of this assumption is to pin down the stock of foreign securities in the steady state.

2/ For a similar modelling of the supply side, see Lizondo and Montiel (1991).

3/ To simplify the analysis, it is assumed that the output (productivity) shock affects only the tradables sector. This implies that the productivity shock does not enter into equation (16) below which relates the equilibrium real exchange rate to the level of total expenditure.

$$(16) \quad e = e(Z), \quad e'(Z) < 0.$$

Clearly, an increase in total expenditure creates excess demand for home goods, thereby requiring an appreciation of the real exchange rate (a decrease in e) to restore equilibrium in the home goods market.

With these three issues in hand, the specification of the model is complete. Notice that the analytical setup is a fairly standard, general one based on familiar components, so the comovements between the premium and the real exchange rate in response to shocks in such a model should be of particular interest. We shall derive four such equations, two of which will refer to the "jumping" variables b and the domestic real interest rate $r^* + \hat{b}$, which we will denote as R , and two of which involve the predetermined variables Z and F . We begin by deriving the dynamics of private consumption expenditure Z . Equation (11) links Z to the marginal utility of wealth. Proceeding in standard fashion, replace $r^* + \hat{b}$ by R in (11), differentiate with respect to time, and substitute from (9). The result is:

$$(17) \quad \dot{Z} = (R - \delta - [\alpha R / (1 + \alpha R)] \hat{R}) Z,$$

which is the familiar condition relating the slope of the time path of consumption to the difference between the consumption real interest rate and the discount rate. 1/

Substituting the cash-in-advance constraint into the expression for household wealth (recalling that $c = e^{\beta} Z$) gives the following alternative expression for wealth:

$$(18) \quad a = \alpha Z + bF.$$

Differentiating (18) with respect to time, equating the resulting expression for \dot{a} to the one given in equation (4), and using equation (12) to eliminate \dot{F} , gives the following alternative expression for \dot{Z} :

$$(19) \quad \dot{Z} = (1/\alpha)(y - Z - T(L) + L).$$

Inverting the optimality condition in (8), we can write

$$(20) \quad L = H(b-1),$$

1/ In any period, the effective price of consumption is equal to $1 + \alpha R$, which is the sum of its market price (unity) and the opportunity cost of holding the α units of money needed to purchase a unit of the good (αR). The consumption rate of interest, which is the true intertemporal price of consumption, is equal to the interest rate R less the "rate of inflation" defined as the rate of change of the effective price of consumption, $[\alpha R / (1 + \alpha R)] R$.

where $H'(b-1) = 1/T''(L) > 0$. More intuitively, equation (20) states that the amount of leakage is an increasing function of the parallel market premium, $b-1$. Substituting (20) into (19) then gives:

$$(21) \quad \dot{Z} = (1/\alpha)\{y - Z - T[H(b-1)] - H(b-1)\}.$$

Equating the righthand sides of equations (17) and (21) provides a dynamic equation for the domestic real interest rate:

$$(22) \quad \dot{R} = -[(1+\alpha R)/(\alpha^2 Z)]\{y - Z - T[H(b-1)] - H(b-1)\} + (R-\delta)(1+\alpha R)/\alpha.$$

The dynamic system that characterizes the economy consists of the following four equations:

$$(23) \quad \dot{b} = [R - r^*(F)]b,$$

$$(24) \quad \dot{Z} = (1/\alpha)\{y - Z - T[H(b-1)] - H(b-1)\},$$

$$(25) \quad \dot{R} = -[(1+\alpha R)/(\alpha^2 Z)]\{y - Z - T[H(b-1)] - H(b-1)\} + (R-\delta)(1+\alpha R)/\alpha,$$

$$(26) \quad \dot{F} = H(b-1) + r^*(F)F.$$

Equation (23) simply follows from the definition of the real interest rate; equations (24) and (25) were given previously as equations (21) and (22) and are rewritten here for convenience; equation (26) incorporates the leakage function (20) and the dependence of the external interest rate facing domestic residents on the stock of foreign assets into the equation for \dot{F} given previously as equation (12). The state variables are Z and F , while the non-predetermined variables are b and R .

Imposing the steady-state conditions $\dot{b} = \dot{Z} = \dot{R} = \dot{F} = 0$, we can solve for the steady-state values (where an overbar denotes a steady state):

$$(27) \quad \bar{b} = 1,$$

$$(28) \quad \bar{Z} = y[e(\bar{Z}); \theta],$$

$$(29) \quad \bar{R} = \delta,$$

$$(30) \quad \bar{F} = 0.$$

The logic of the steady state is seen by observing that if the net stock of foreign securities reaches a constant value which differs from zero, this implies that the constant value reached by the premium ($b-1$) must also differ from zero (equation 26)), and that the value of the world interest rate must differ from the rate of time preference (by definition of the relationship between F and $r^*(F)$). This would make the last term in equation (25) nonzero, which in turn implies that the term in curly brackets in that equation must also be nonzero. Since this term also appears on the right hand side of (24), it would not be possible for expenditure Z to reach

a constant value, which is in contradiction with the definition of the steady state. Therefore, the steady-state values must be those given by equations (27)-(30).

Our interest is in analyzing the comovements of the premium and the real exchange rate in this economy when the economy is hit by an exogenous shock. For that purpose, we will consider a particularly simple shock, in the form of a permanent improvement in productivity. In the remainder of this section, we provide a formal solution for the response dynamics, which we then analyze in more detail in the section that follows. To analyze the dynamics in the vicinity of the new steady state, we linearize the system of equations (23)-(26) around the steady state to obtain:

$$(31) \quad \begin{bmatrix} \dot{b} \\ \dot{R} \\ \dot{Z} \\ \dot{F} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & b_4 \\ R_1 & R_2 & R_3 & 0 \\ Z_1 & 0 & Z_3 & 0 \\ F_1 & 0 & 0 & F_4 \end{bmatrix} \begin{bmatrix} db \\ dR \\ dZ \\ dF \end{bmatrix} + \begin{bmatrix} 0 \\ R_5 \\ Z_5 \\ 0 \end{bmatrix} d\theta.$$

The partial derivatives in (31) are given by:

$$\begin{aligned} b_4 &= -r^{*'}(F) > 0, \\ R_1 &= [(1+\alpha R)/(\alpha^2 Z)]H' > 0, \\ R_2 &= (1+\alpha R)/\alpha > 0, \\ R_3 &= [(1+\alpha R)/(\alpha^2 Z)](1-y_1 e') > 0, \\ R_5 &= -[(1+\alpha R)/(\alpha^2 Z)]y_2 < 0, \\ Z_1 &= -H'/\alpha < 0, \\ Z_3 &= -(1-y_1 e')/\alpha < 0, \\ Z_5 &= y_2/\alpha > 0, \\ F_1 &= H' > 0, \\ F_4 &= \delta > 0. \end{aligned}$$

The trace of the matrix on the righthand side of (31) is given by:

$$\text{Trace} = 2\delta + (y_1 e')/\alpha > 0,$$

while the determinant is given by:

$$\text{Determinant} = -r^*H'(1+\alpha\delta)(1-y_1e')/\alpha^2 > 0.$$

The fact that the trace is positive implies that not all roots of the system can be negative, while the fact that the determinant is positive implies that the number of positive roots is either two or four. Since there are two predetermined variables and two non-predetermined variables, the former case (two positive roots) implies saddle-point stability of the equilibrium, whereas the case of four positive roots implies global instability.

To determine which of these two cases is relevant, we examine the characteristic polynomial in more detail. Defining the latter by $P(\lambda)$, we have by definition:

$$(32) \quad P(\lambda) = \det \begin{pmatrix} -\lambda & 1 & 0 & b_4 \\ R_1 & R_2-\lambda & R_3 & 0 \\ Z_1 & 0 & Z_3-\lambda & 0 \\ F_1 & 0 & 0 & F_4-\lambda \end{pmatrix}.$$

Expanding the expression for the polynomial, one obtains:

$$(33) \quad P(\lambda) = \gamma_0 + \gamma_1\lambda + \gamma_2\lambda^2 + \gamma_3\lambda^3 + \gamma_4\lambda^4,$$

where, upon substitution for the coefficients as defined below equation (31), one obtains:

$$\gamma_0 = -r^*H'(1+\alpha\delta)(1-y_1e')/\alpha^2 > 0,$$

$$\gamma_1 = r^*H'[\delta+(y_1e')/\alpha] - (\delta/\alpha)(1+\alpha\delta)[(1-y_1e')/\alpha + H'/(aZ)] < 0,$$

$$\gamma_2 = \delta[\delta+(y_1e')/\alpha] - [(1+\alpha\delta)/(\alpha^2Z)][H'+(y_1e'-1)^2/\alpha] + r^*H' \leq 0,$$

$$\gamma_3 = 2\delta + (y_1e')/\alpha > 0,$$

$$\gamma_4 = 1.$$

By definition, γ_0 is equal to the product of all the roots (the determinant), γ_1 is equal to the sum of all products of three of the roots, γ_2 is equal to the sum of all products of two of the roots, and γ_3 is equal to the sum of all products of one of the roots (or the trace). Because γ_1 is negative, it must be the case that at least one of the roots of the system is negative. This rules out the case in which all roots are positive, and implies that the system is saddlepoint stable, possessing exactly two positive roots and two negative roots.

The general solution of the model may therefore be written as:

$$(33) \quad \begin{bmatrix} b \\ R \\ Z \\ F \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{bmatrix} \begin{bmatrix} \omega_1 \exp(\lambda_1 t) \\ \omega_2 \exp(\lambda_2 t) \\ \omega_3 \exp(\lambda_3 t) \\ \omega_4 \exp(\lambda_4 t) \end{bmatrix},$$

where the elements of each eigenvector may be solved from:

$$(34) \quad 0 = \begin{bmatrix} -\lambda_i & 1 & 0 & b_4 \\ R_1 & R_2 - \lambda_i & R_3 & 0 \\ Z_1 & 0 & Z_3 - \lambda_i & 0 \\ F_1 & 0 & 0 & F_4 - \lambda_i \end{bmatrix} \begin{bmatrix} h_{1i} \\ h_{2i} \\ h_{3i} \\ h_{4i} \end{bmatrix}$$

for each of the four eigenvalues λ_i , $i = 1-4$. If we let λ_3 and λ_4 be the two positive eigenvalues, then convergence of the dynamic system requires $\omega_3 = \omega_4 = 0$. To solve for the remaining two scalars, we use the initial conditions, which imply:

$$(35) \quad \omega_1 = h_{42}/(h_{31}h_{42} - h_{32}h_{41})(Z_0 - \bar{Z}) - h_{32}/(h_{31}h_{42} - h_{32}h_{41})(F_0 - \bar{F}),$$

$$(36) \quad \omega_2 = h_{31}/(h_{31}h_{42} - h_{32}h_{41})(Z_0 - \bar{Z}) - h_{41}/(h_{31}h_{42} - h_{32}h_{41})(F_0 - \bar{F}).$$

We can now write the solution of the system as:

$$(37) \quad b - \bar{b} = \Delta^{-1} \{ [h_{11}h_{42}\exp(\lambda_1 t) + h_{12}h_{31}\exp(\lambda_2 t)](Z_0 - \bar{Z}) \\ - [h_{11}h_{32}\exp(\lambda_1 t) + h_{12}h_{41}\exp(\lambda_2 t)](F_0 - \bar{F}) \},$$

$$(38) \quad R - \bar{R} = \Delta^{-1} \{ [h_{21}h_{42}\exp(\lambda_1 t) + h_{22}h_{31}\exp(\lambda_2 t)](Z_0 - \bar{Z}) \\ - [h_{21}h_{32}\exp(\lambda_1 t) + h_{22}h_{41}\exp(\lambda_2 t)](F_0 - \bar{F}) \},$$

$$(39) \quad Z - \bar{Z} = \Delta^{-1} \{ [h_{31}h_{42}\exp(\lambda_1 t) + h_{32}h_{31}\exp(\lambda_2 t)](Z_0 - \bar{Z}) \\ - [h_{31}h_{32}\exp(\lambda_1 t) + h_{32}h_{41}\exp(\lambda_2 t)](F_0 - \bar{F}) \},$$

$$(40) \quad F - \bar{F} = \Delta^{-1} \{ [h_{41}h_{42}\exp(\lambda_1 t) + h_{42}h_{31}\exp(\lambda_2 t)](Z_0 - \bar{Z}) \\ - [h_{41}h_{32}\exp(\lambda_1 t) + h_{42}h_{41}\exp(\lambda_2 t)](F_0 - \bar{F}) \},$$

where $\Delta = h_{31}h_{42} - h_{32}h_{41}$.

IV. Adjustment to a Productivity Shock

In this section, we analyze the properties of this solution in detail, focusing specifically on the behavior of the premium and the real exchange rate. To begin with, it is worth noting that the discussion that follows would not be altered in any important way if the source of the shock were instead an improvement in the (exogenous) terms of trade faced by the country $\underline{1}$ /, or simply some productivity-enhancing policy reform. Alternatively, one may prefer to substitute an endowment structure for the simple transformation function described in the previous section. In this case, the shock would simply be an increase in the endowment of the tradable good. Finally, as mentioned previously, we confine the shock to the tradables sector mainly to simplify the analysis of its effect on the equilibrium real exchange rate (equation (16)). It may also be noted that the model can be used to analyze a variety of other disturbances, but this is not necessary for the purpose at hand, which is simply to illustrate the potential problems that may arise in using the parallel market premium to draw inferences about real exchange rate misalignment.

Consider then the effect of a permanent favorable productivity shock. As far as the steady state is concerned, all variables (\bar{b} , \bar{R} , and \bar{F}) remain constant at their original steady-state values (given by equations (27), (29), and (30), respectively), except for expenditure, where equation (28) implies that a positive (negative) shock causes the steady-state level of expenditure to rise (fall). From equation (16), therefore, the long-run value of the equilibrium real exchange rate decreases for a positive shock, i.e., there is an equilibrium real appreciation in the long run. Notice that the steady-state premium does not change in this model. This feature permits policymakers to feel confident about the equilibrium value of the premium after a shock, and thus about whether the observed premium is above or below its long-run value, without the necessity of solving the model. The issue, then, is whether this conveys information about the position of the real exchange rate relative to its long-run value.

To determine the out-of-steady-state dynamics, it is useful to turn to Figure 1, which plots the level of expenditure Z on the horizontal axis and the level of (one plus) the parallel market premium b on the vertical axis. Imposing the steady-state condition $\dot{Z} = 0$ in equation (31) yields a negatively-sloped locus in the b - Z space, with slope given by $-(1-y_1e')/H'$. Since the steady-state premium must be zero, the intersection of this locus with the horizontal line at $b = 1$ determines the steady-state value of expenditure. Z_0 denotes the initial steady-state level of expenditure while \bar{Z} denotes the higher post-productivity shock level. The initial steady

$\underline{1}/$ We could think in this case of the model as consisting of an exportables and a nontradables sector on the supply side, and importables and nontradables on the demand side. Also, it should be noted that the analysis of a *negative* shock would be completely symmetric to that which is presented below for a positive shock.

state is located at point A while the new steady state is located at point B, since the parallel market premium is zero, i.e., $b = 1$, in both steady states, while the position of the $\dot{Z} = 0$ locus is shifted to the right as higher spending is induced by the higher (post-shock) level of productivity. The arrows of motion relative to the new $\dot{Z} = 0$ locus in the figure follow from the fact that $Z_3 < 0$, as established below equation (31).

Consider now the dynamics of adjustment of b and Z from A to B. Since expenditure is linked to the stock of money by the cash-in-advance constraint, and since the money stock can only change over time through the process of hoarding, Z cannot jump on impact. This means that the instantaneous equilibrium of the economy must be somewhere along the perpendicular rising from point Z_0 . To determine the paths followed by b and Z from an initial point on this perpendicular to the new steady state at B, we proceed in several steps. First, we derive slopes of the dominant and nondominant eigenvectors (i.e., the eigenvectors associated with the smallest and largest of the negative eigenvalues, λ_1 and λ_2 , respectively). These loci have the property that the paths of b and Z cannot cross them, since if the economy is ever on one of these vectors, it can be shown to remain on it as it converges to the new steady state. From (37) and (39), the slope of the dominant eigenvector is equal to h_{11}/h_{31} , while the slope of the nondominant eigenvector is equal to h_{12}/h_{32} . To determine the signs of the slopes, we use equation (34) which implies:

$$(41) \quad \text{slope of dominant eigenvector} = (\lambda_1 - Z_3)/Z_1 > 0,$$

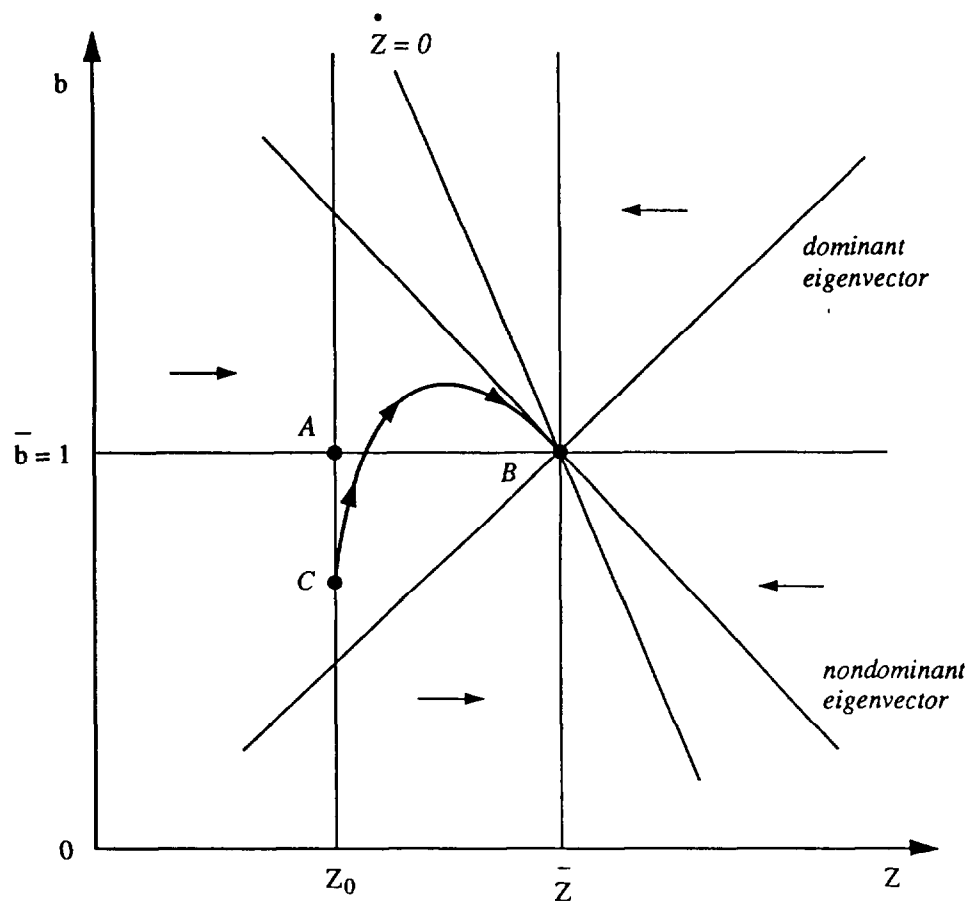
$$(42) \quad \text{slope of nondominant eigenvector} = (\lambda_2 - Z_3)/Z_1 < 0,$$

where the signs in (41) and (42) follow from evaluating the polynomial in (32) $P(\lambda)$ at $\lambda = Z_3$. ^{1/} Unless the initial values of the state variables Z and F are such as to place the economy precisely on the dominant eigenvector, the economy will converge to the steady state at B along the nondominant eigenvector drawn in Figure 1, since the influence of the largest negative root must gradually vanish over time.

It remains only to determine the sign of the jump in b resulting from the productivity disturbance. An appendix at the end of the paper establishes that in fact, the premium and therefore b must make a discrete jump downwards on impact, to a point such as C in Figure 1. This means that the adjustment path must follow the dark arrowed path in the figure, with expenditure rising throughout the adjustment, but with the premium falling on impact, then rising temporarily before falling back towards zero in the final stages of the adjustment.

^{1/} Specifically, from (32), it can be shown that $\lim_{\lambda \rightarrow -\infty} P(\lambda) > 0$ and $P(Z_3) < 0$. From these two facts, it follows that $\lambda_1 < Z_3 < \lambda_2 < 0$. The signs of the slopes in (41) and (42) follow directly.

Figure 1
Dynamics in $b - Z$ space



Intuitively, the favorable productivity shock increases income, which is consistent with a permanent increase in household expenditure. However, the adjustment in spending cannot be instantaneous, because an increase in spending requires an increase in the money stock, and the latter can only occur gradually over time through the process of hoarding. Initially, then, spending remains below permanent income and the associated saving results in the gradual accumulation of money, permitting spending to rise over time. This accounts for the monotonic adjustment of Z . At the same time, the path of b cannot be monotonic because if it were, the stock of foreign exchange could not remain unchanged from one steady state to the next, as it is required to be by equation (30). With monotonic adjustment in b , foreign exchange balances would either be drawn down or accumulated continuously.

The contrast between the behavior of b and Z along the adjustment path has important implications for the use of the premium as an indicator of real exchange rate misalignment. Figure 2 plots the time paths of the various variables resulting from the adjustment of the economy portrayed in the first figure. As can be seen, the monotonic adjustment of expenditure implies that the adjustment of the real exchange rate is also monotonic by equation (16), while the path of the parallel market premium necessarily overshoots. This means that, throughout the adjustment, the currency is undervalued in real terms (obviously if the shock had instead been negative, the problem would have instead been real exchange rate overvaluation). However, as can be seen, the parallel market premium is initially negative (foreign currency sells at a discount in the unofficial market), then rises to a positive value, before falling again towards zero. Thus, only the first part of the adjustment process coincides with the conventional view that undervaluation (overvaluation) should be associated with parallel market discounts (premiums). Clearly, the parallel market premium cannot by itself even provide reliable information about the sign of real exchange rate misalignment at a given point in time, let alone its magnitude. ^{1/}

V. Conclusion

Because most developing countries maintain a managed official exchange rate, policymakers in such countries commonly have to make judgments about the extent of real exchange rate misalignment. The degree of misalignment is very difficult to measure, since the equilibrium real exchange rate is unobservable and depends on a range of structural and macroeconomic factors. Thus, economists looking at this issue have frequently used the parallel market premium, about whose equilibrium value they have more confidence, to draw inferences regarding the extent of real exchange rate misalignment.

^{1/} An alternative to looking at the premium at a point in time is to look at its average behavior over some period of time. Whether this yields a more reliable indicator depends on how long the economy spends in each phase of the adjustment path and on how long after the shock an assessment is made.

The conventional wisdom is that there is a robust correlation between the extent of overvaluation or undervaluation of a currency and the parallel market premium, a view which appeals to intuition, as well as having both some analytical and empirical support. Against this view, however, one must set the observation that, since the degree of misalignment and the parallel market premium are both endogenous real variables, both the nature of shocks and the economic structure are likely to play a role in determining the sign and magnitude of comovements between these variables. Moreover, available empirical evidence suggests that the premium is far more variable than any likely deviations of actual from equilibrium real exchange rates, reaching at times several hundred percent with no ex post evidence of comparable magnitudes of real exchange rate misalignment.

This paper investigated, within the context of a fully optimizing model of a small open developing country, how the parallel market premium and the real exchange rate jointly respond to an illustrative shock, taken here to be a permanent productivity disturbance. The analysis suggested that the informational content of the premium may be limited, since in response to a shock, the premium was at various times along the adjustment path positive and negative, while the degree of overvaluation of the currency was always positive for a negative shock, or negative for a positive shock.

The obvious policy implication of the analysis is that by itself the premium is an unreliable indicator of the sign and magnitude of real exchange rate misalignment. The premium indeed may provide useful information about the relationship of the real exchange rate to its equilibrium level. For example, the magnitude of the initial jump in the premium will be correlated with that of the initial extent of real exchange rate misalignment in our model. However, it is dangerous to draw inferences about deviations of the actual from the equilibrium real exchange rate based on observations of the premium at a given moment in time. In the model of this paper, other variables, such as the behavior of the trade balance, would provide more reliable information on the extent of real exchange rate misalignment.

Figure 2
Adjustment to a Favorable Productivity Shock

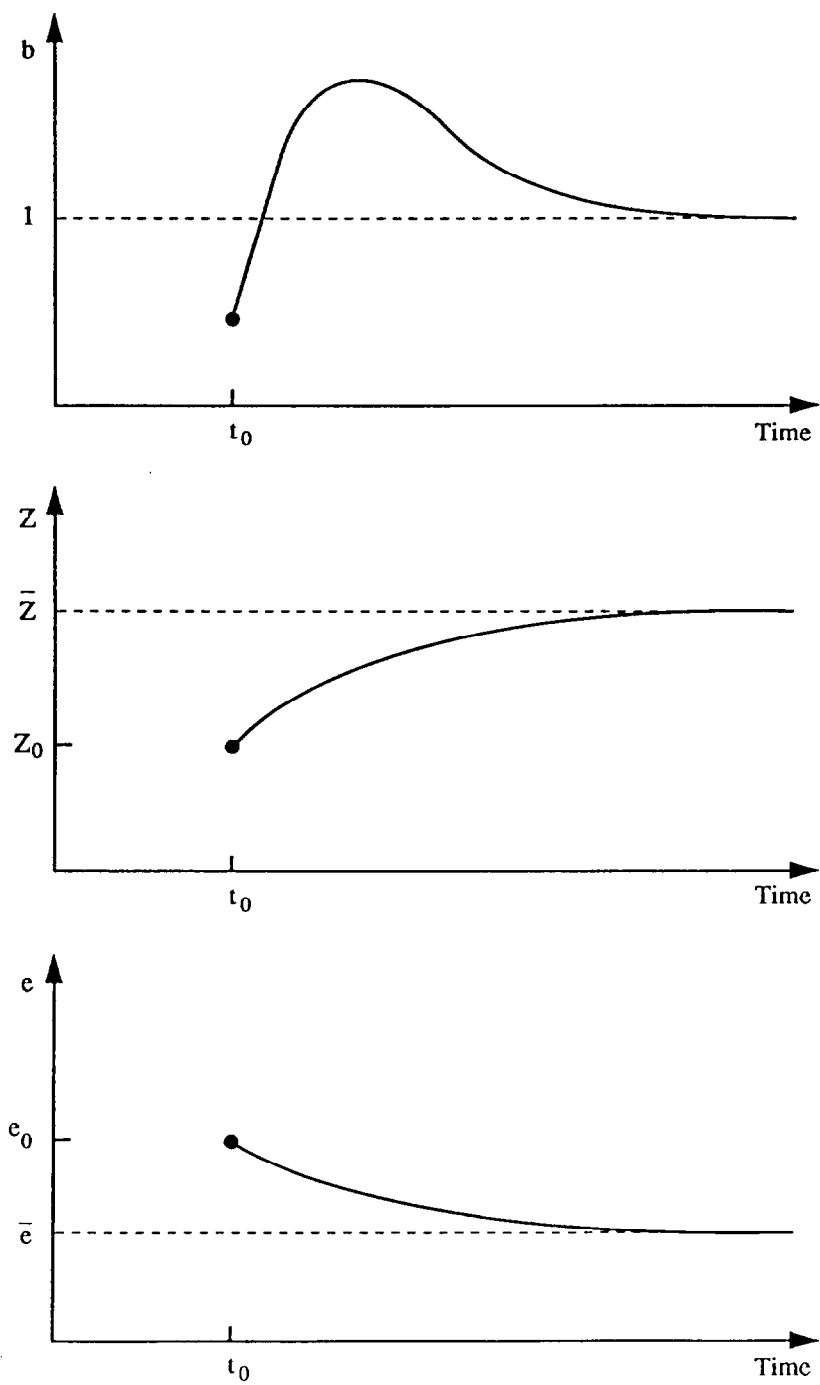
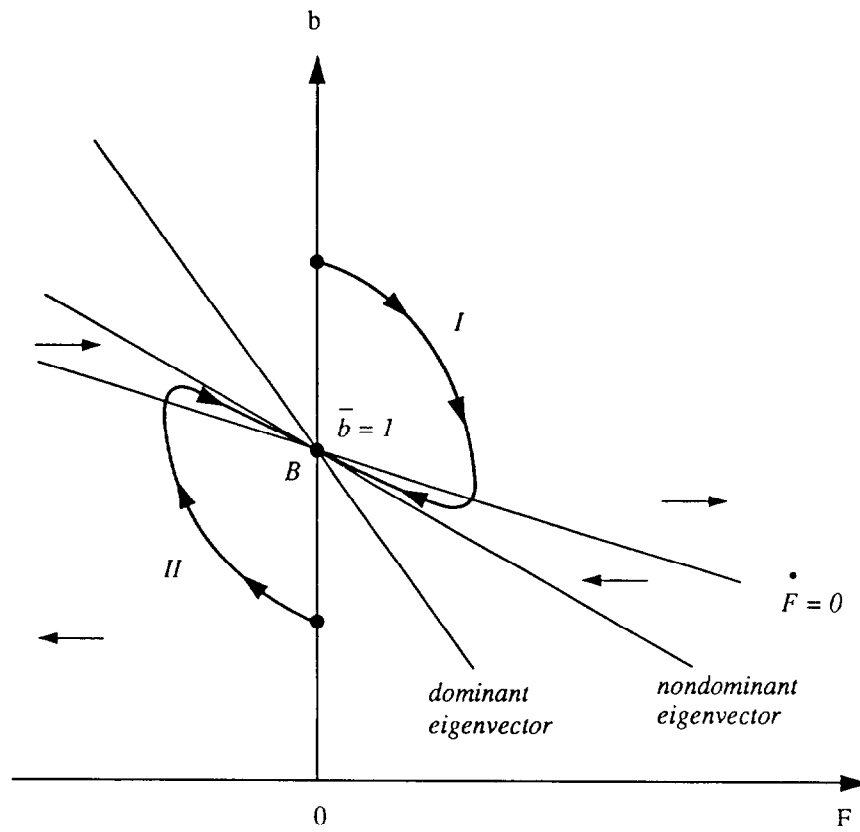


Figure 3
Dynamics in b - F space



Appendix

The purpose of this appendix is to show that in response to a positive productivity shock, the premium must jump down on impact and, therefore, that the adjustment of the economy to such a shock must be as presented in Figure 1. Figure 3 plots on the horizontal axis the stock of foreign securities F , and on the vertical axis, the parallel market premium (plus unity), b . The slope of the $\dot{F} = 0$ locus (which is drawn relative to the new, post-shock steady state equilibrium at B) is equal to $-F_4/F_1$ which is negative (equation (31)). ^{1/} From equations (37) and (40), we can derive the slopes of the dominant and nondominant eigenvectors:

$$(A1) \quad \text{slope of dominant eigenvector} = (\lambda_1 - F_4)/F_1 < 0,$$

$$(A2) \quad \text{slope of nondominant eigenvector} = (\lambda_2 - F_4)/F_1 < 0,$$

where, by the fact that $\lambda_1 < \lambda_2 < 0$, the dominant eigenvector is more steeply sloped than the nondominant eigenvector, and both are more steeply sloped than the $\dot{F} = 0$ locus, as drawn in Figure 3. As mentioned previously, unless the initial steady state values of the two state variables are such as to place the economy on the dominant eigenvector, the path followed by the economy in Figure 3 eventually converges to the slope of the nondominant eigenvector.

The question, as in Figure 1, is where the economy jumps on impact along the perpendicular to the point $F = 0$, which is the new (and old) steady state value of F . There are two possibilities, labelled respectively I and II in Figure 3. Under path I, the premium jumps up on impact and, given the arrows of motion drawn relative to the $\dot{F} = 0$ locus, the premium must approach its steady state level from below. Under path II, the premium jumps down on impact and, given the arrows of motion, must approach the steady state at B from above.

It may be recalled from Figure 1 that, whether b jumps up or down on impact, it must approach the steady state from above. But this rules out path I in Figure 3 since, in this case, b approaches the steady state from below. Therefore, b must jump down on impact, and the economy must follow the path indicated in Figure 1 (with the dynamics of the individual variables as drawn in Figure 2), and likewise path II in Figure 3.

^{1/} Because the new and old steady-state values of b and F are identical, the new and old steady states in Figure 3 coincide, and are given by point B.

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