IMF WORKING PAPER

WP/93/60

INTERNATIONAL MONETARY FUND

Fiscal Affairs Department

Collection Lags and the Optimal Inflation Tax: A Reconsideration

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July 1993

Abstract

The observation that collection lags combine with inflation to erode fiscal revenues has long been a strong argument against seigniorage (Tanzi (1978)). However, with the exception of Dixit (1991), who used a general equilibrium model to reject this argument, the optimal tax literature has not analyzed how collection lags affect desired tax structures. In this paper, this issue is re-examined using an overlapping generations version of Dixit's model. It is shown that depending on the specification of the collection cost function and the size of government spending in GDP, collection lags may increase, leave unchanged, or reduce the desired rate of inflation.

JEL Classification Nos: H21, E31, E60

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Contents

Summary

It has long been argued that the case for inflationary finance is greatly weakened when allowance is made for lags in the payment of taxes that erode fiscal revenues (Tanzi (1978)). Recently, however, Dixit (1991) has rejected this argument on the basis of a general equilibrium optimal tax analysis. Specifically, he employed a version of Végh's (1989) "shopping time" monetary model with costly income taxation to show that introducing collection lags and allowing the government to recalculate its optimal tax mix may result in unchanged or even higher rates of inflation.

This paper reconsiders the effects of collection lags on the optimal tax menu in a version of Samuelson's (1958) consumption loans model. A Ramsey formula is derived that demonstrates that optimal inflation is (a) proportional to the marginal cost of income tax collections; and (b) inversely proportional to the marginal propensity to consume and the interest elasticity of real money demand. It is also shown that, depending on the specification of the collection cost function and the size of government spending in GDP, collection lags may result in higher, unchanged, or lower rates of desired inflation. Specifically, if real collection costs are a function of real revenues realized, there is a threshold value of the size of government spending in GDP such that the optimal rate of inflation is lower (higher) when lags are present (absent). However, if real collection costs are a function of real revenues accrued, the optimal tax menu does not change in the presence of collection lags.

I. Introduction

One of the basic issues in both public finance and monetary economics is the desirability of using inflationary finance as a means of generating government revenue. In the monetary literature, the orthodox position is that associated with Friedman's (1969) optimum quantity of money rule which argues that the nominal rate of interest should be zero. 1/ The public finance literature has been dominated by Phelps (1973), who used a money-in-the-utility function model to show that seigniorage can be part of a second-best tax system. Phelps' starting point was the observation that to implement Friedman's rule, lump sum transfers must be feasible. If they are not, taxation of all commodities--including consumption and liquidity--may be required to raise government revenue. 2/ Later contributions to the public finance literature criticized Phelps' treatment of liquidity as a separate commodity, focusing on fiscal inefficiencies to rationalize resort to seigniorage. 3/ With the exception of Kimbrough (1986), who salvages Friedman's rule in a second-best setting, in this literature, the fiscal inefficiencies assumed (such as positive foreign nominal rates of interest, collection costs or a large underground economy) introduce distortions of their own for ordinary taxes. Resort to seigniorage, therefore, helps reduce these inefficiencies and allows it to coexist with consumption (or income) taxes in optimal tax menus.

With the exception of Dixit (1991), however, the literature on optimal inflation has not considered the implications of collection lags for this argument. As has long been emphasized by Tanzi (1978), the case for inflationary finance is substantially weakened if high rates of inflation combine with substantial collection lags to erode the real value of ordinary

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1/ Friedman argued that to allocate resources efficiently in a monetary economy, the social marginal benefit of money must be brought in line with its social marginal cost. Since fiat money is (almost) costless to produce, the nominal rate of interest should be set to zero, for example, by contracting the money supply at a rate equal to the real rate of interest. See Woodford (1990) for an extensive discussion of the optimum quantity of money rule.

2/ Kimbrough (1986), however, has argued that Friedman's rule may be optimal even in second best environments. His reasoning is based on the view (formalized in the "shopping time" model) that fiat money is not a final good but rather an intermediate input in the transactions technology. A theorem of Diamond and Mirrlees (1971) on second-best taxation is then applicable to the effect that if the production function exhibits constant returns to scale and all final goods are taxable, then intermediate inputs ought to not be taxed. For a discussion of the applicability of Friedman's rule in the shopping time monetary model, see Végh and Guidotti (1992).

3/ See Frenkel (1987), Mourmouras (1991) or Végh (1991).

taxes. 1/ This argument has been challenged recently by Dixit who provides a welfare analysis of inflation in a version of Végh's (1989) model incorporating collection costs and collection lags. Dixit observes that rational governments will, in a general equilibrium environment, react to the presence of lags by adjusting all taxes, not just the rate of inflation. Since this will change prices and the real cost of collections, there is a richer menu of possibilities to consider. Two examples are provided that reverse the traditional argument. In the first, the length of collection lags is irrelevant for the optimal choice of inflation, as if full interest were charged to compensate for the delay in tax payments. More interesting is the second case in which the presence of lags has the effect of raising the excess burden of income taxes, thereby warranting greater reliance on seigniorage than in economies with no lags.

While relying on the public finance approach to inflation, this paper reconsiders the optimum mix of inflation and costly income taxes for alternative specifications of the monetary model and the collection cost technology. In particular, the optimal tax analysis is performed in the context of Samuelson's (1959) consumption loans model in which, unlike the shopping time model, the major distortion caused by money-financed deficits is that on intertemporal consumption allocations. Several interesting results emerge. First, it is established that regardless of the length of the collection lag, optimal inflation is proportional to the marginal cost of collecting income taxes, implying that price stability ought to be pursued whenever tax collections are costless at the margin. Second, and in accordance with the Ramsey "inverse elasticity rule," it is shown that the optimal rate of inflation is inversely related to both the marginal propensity to consume and the interest elasticity of real money balances. Third, the desired rate of inflation when a one period lag is present is lower (higher) than the rate of inflation warranted in the no-lag economy provided that g, the share of government spending in GDP, is below (above) a certain threshold value. This establishes that Professor Dixit's unconditional rejection of the traditional presumption--that collection lags ought to reduce the optimal rate of inflation--on theoretical grounds alone

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1/ Admittedly, the applicability of abstract theories of inflation (especially of the steady state variety) is limited. For instance, the collection period (which is exogenous in both Dixit's and our analysis) is in reality an endogenous variable which depends, among others, on the prevailing rate of inflation. It is interesting that whereas theorists allow income taxes and inflation to be changed costlessly, they treat the collection interval as fixed. In practice, of course, the frequency of collections is less "sticky" than income tax rates. Whereas the former can be changed by having the tax administration issue new implementing orders, the latter must (in democratic systems) go through the legislative process. In periods of high inflation, rational governments attempt to protect the real value of revenues by shortening the collection period. See Tanzi (1991) on this point and on the other limitations of normative tax theory.

is unwarranted. As a practical matter, the threshold value of g, which is crucial for the comparison, turns out to be a function of the marginal cost of collection (see Table 1 and Figure 1.)

These results are due to a combination of factors. First, in the present model the efficiency trade-off is between money-financed deficits that lower real interest rates and distort intertemporal choice and income taxes that require real resources for collection. This leads naturally to Ramsey formulae incorporating marginal collection costs and interest and income elasticities of currency demand. 1/ By contrast, in the shopping time model used by Végh (1989) and Dixit (1991), a version of Irving Fisher's theory of interest is maintained according to which the inflationary process does not impact on real interest rates, the whole profile of which is taken to be exogenous (Dixit, p. 645). Second, while for a given positive rate of inflation, the introduction of payment lags for taxes reduces their real yield, it also raises desired real currency balances, leading to complicated changes in the tax bases and the optimal tax menu. Finally, unlike Dixit who maintains (p. 648) that "the nominal collection cost technology exactly keeps pace with inflation," this paper allows real collection costs to either remain constant or decline in the face of higher inflation. This feature proves to be important in optimal tax calculations as well.

The rest of the paper is organized as follows: after describing the model in section II, section III presents the public finance analysis and derives the Ramsey formula for an economy in which there are no (significant) lags in income tax collections. In section IV, the optimal rate of inflation is calculated for two parametric examples corresponding, respectively, to Dixit's benchmark specification of constant marginal collection cost and the more realistic increasing marginal cost case. In section V, the impact of a one-period collection lag on the optimal tax structure is analyzed and the optimal rates of inflation are compared for the lag and no-lag environments. Section VI then examines the sensitivity of these results to changes in the specification of the collection cost function and proves a version of Dixit's neutrality proposition for a plausible alternative functional form. Some concluding comments are presented in section VII.

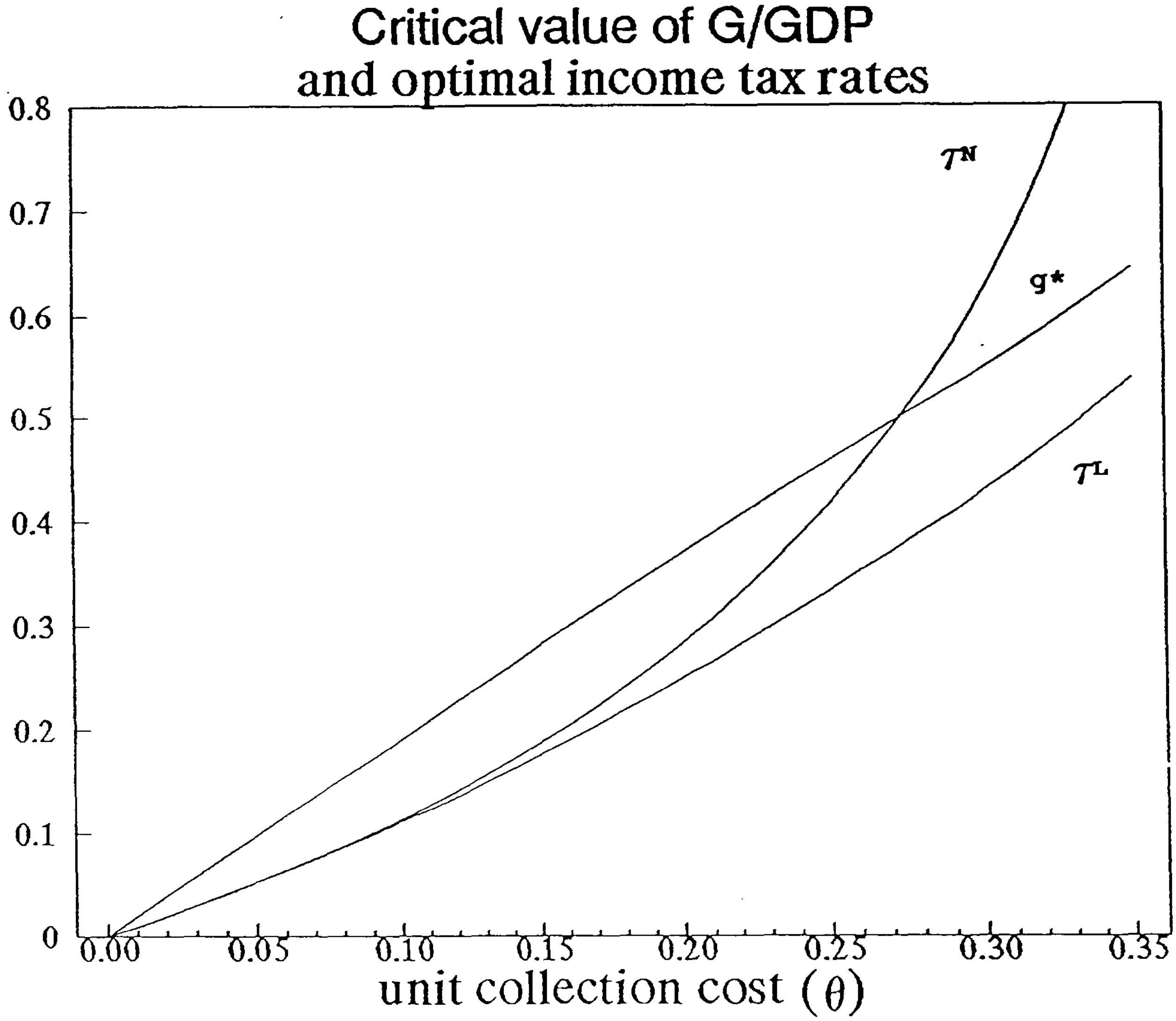
1/ This distortion, which is not made explicit in welfare analyses of the costs of inflation, seems to be important in practice--particularly in developing countries. In these countries, the ability of (especially small) savers to index asset returns through financial markets is hindered by prohibitive transactions costs and the primitive state of development of these markets. See Wallace (1980) on this point.

Table 1. The Critical Value of g, and Optimal (τ,π) as Functions of the Unit Cost of Collection (θ)

θ	π^N	π^L	τ^L	τ^N	g*
0.00	0.0000	0.0000	0.0000	0.0000	0.0000
0.01	0.0204	0.0204	0.0101	0.0101	0.0199
0.02	0.0417	0.0417	0.0204	0.0204	0.0396
0.03	0.0638	0.0638	0.0309	0.0310	0.0591
0.04	0.0870	0.0870	0.0417	0.0418	0.0785
0.05	0.1111	0.1111	0.0526	0.0529	0.0976
0.06	0.1364	0.1364	0.0638	0.0644	0.1166
0.07	0.1628	0.1628	0.0753	0.0761	0.1355
0.08	0.1905	0.1905	0.0870	0.0883	0.1542
0.09	0.2195	0.2195	0.0989	0.1009	0.1727
0.10	0.2500	0.2500	0.1111	0.1139	0.1911
0.11	0.2821	0.2821	0.1236	0.1274	0.2094
0.12	0.3158	0.3158	0.1364	0.1415	0.2276
0.13	0.3514	0.3514	0.1494	0.1563	0.2456
0.14	0.3889	0.3889	0.1628	0.1717	0.2636
0.15	0.4286	0.4286	0.1765	0.1878	0.2815
0.16	0.4706	0.4706	0.1905	0.2048	0.2993
0.17	0.5152	0.5152	0.2048	0.2228	0.3170
0.18	0.5625	0.5625	0.2195	0.2417	0.3347
0.19	0.6129	0.6129	0.2346	0.2619	0.3524
0.20	0.6667	0.6667	0.2500	0.2833	0.3700
0.21	0.7241	0.7241	0.2658	0.3062	0.3876
0.22	0.7857	0.7857	0.2821	0.3308	0.4053
0.23	0.8519	0.8519	0.2987	0.3572	0.4229
0.24	0.9231	0.9231	0.3158	0.3857	0.4406
0.25	1.0000	1.0000	0.3333	0.4167	0.4583
0.26	1.0833	1.0833	0.3514	0.4503	0.4762
0.27	1.1739	1.1739	0.3699	0.4871	0.4941
0.28	1.2727	1.2727	0.3889	0.5275	0.5121
0.29	1.3810	1.3810	0.4085	0.5720	0.5303
0.30	1.5000	1.5000	0.4286	0.6214	0.5486
0.31	1.6316	1.6316	0.4493	0.6765	0.5671
0.32	1.7778	1.7778	0.4706	0.7383	0.5858
0.33	1.9412	1.9412	0.4925	0.8081	0.6047
0.34	2.1250	2.1250	0.5152	0.8873	0.6240
0.35	2.3333	2.3333	0.5385	0.9782	0.6435

π is in absolute figures over a twenty year period
θ, τ, and g* are expressed in absolute figures (ratios)

Figure 1



g^* = critical value of government spending to GDP ratio as function of unit cost of income tax collections

τ^N = optimal income tax rate in absence of collection lags

τ^L = optimal income tax rate in presence of collection lags

II. The Model

Consider the following simple version of the consumption loans model. The economy consists of an infinite sequence of two period-lived overlapping generations, each of equal size that for simplicity is normalized to one. At each date, $t = 1,2,...$ the representative young agent is endowed with $\bar{n}$ units of labor and a technology $f(n)$ which allows her to produce a single perishable consumption good. The production function $f(\cdot)$ satisfies $f(0) = 0$, $f' \geq 0$, $f'' \leq 0$. A fiat currency issued by the government is the only asset. At the initial period $t = 1$, the initial old agent (belonging to generation 0) owns some quantity of fiat currency $M(0)$. Agents are retired in the second period of life and must rely on accumulated currency balances to purchase the consumption good. Letting $w = f(\bar{n})$ denote real income and $\tau(t)$ the income tax rate, after-tax real income is $A(t) \equiv (1-\tau(t))w(t)$. In addition, let $c_t(k)$ denote consumption in period k by agent t and $m(t)$ her nominal currency holdings at the end of t. The price level at t is denoted $p(t)$, the rate of inflation between t and t+1 $\pi(t) \equiv [p(t+1)-p(t)]/p(t)$, the gross return on real currency holdings $R(t) \equiv 1/[1+\pi(t)]$, and the end-of-period economy-wide stock of fiat currency $M(t)$. Agents are endowed with perfect foresight throughout. Given the current and expected future price levels, $p(t)$ and $p(t+1)$ respectively, and $\tau(t)$, agent t selects a nonnegative vector $(c_t(t),c_t(t+1),m(t))$ to maximize lifetime utility

$$u(c_t(t),c_t(t+1)) \tag{1}$$

subject to

$$c_t(t) + \frac{m(t)}{p(t)} \leq (1-\tau(t))w \quad \text{and} \quad c_t(t+1) \leq \frac{m(t)}{p(t+1)} \tag{2}$$

In an interior solution (to be assumed throughout) the marginal rate of substitution between second and first-period consumption u_2/u_1 must be equal to the ratio of relative prices $p(t+1)/p(t) \equiv R(t)$. Using the condition $u_2 = Ru_1$ and the budget constraints (2) at equality, consumer demand schedules and a demand schedule for real currency balances may be written, respectively, as $c_t(t) = c_1(A(t),R(t))$, $c_t(t+1) = c_2(A(t),R(t))$ and $m(t)/p(t) = s(A(t),R(t))$. Let $\epsilon(s,A) \equiv \partial\ln(s)/\partial\ln(R)$ denote the interest elasticity of real currency demand and $s_A \equiv \partial s/\partial A$ the marginal propensity to save out of current income.

Government spending on public goods and services (net of collection costs) is financed via a flat-rate income tax and seigniorage. Public goods enter individual utility functions in a separable manner that does not affect rankings of private goods. The sequence of government expenditures on public goods $\{G(t)\}$ may therefore be treated as exogenous. Income taxes require real resources for collection. Specifically, if τw is gross tax revenue in real terms, then net revenue is $\tau w - \phi(\tau w)$ where $\phi(\cdot)$ is a monotonic increasing function describing the resource cost of income tax collections to the government. The function ϕ satisfies $\phi(0) = 0$, $\phi' \geq 0$, $\phi'' \geq 0$. The government cash-flow constraint for period t may be written

$$G(t) = \tau(t)w - \phi(\tau(t)w) + \frac{M(t)-M(t-1)}{p(t)} \qquad t = 1,2,... \tag{3}$$

Given Π and the initial condition M(0) a <u>perfect foresight competitive equilibrium</u> is a set of sequences $\{c_t(t), c_t(t+1), s(t), M(t), p(t), \pi(t), R(t), G(t), \tau(t)\}$ that for all t = 1,2,... satisfy the conditions of individual optimization and are consistent with market clearing and the sequence of government budget constraints. A <u>stationary equilibrium</u> is a set of scalars $(c_1, c_2, s, G, \tau, \pi, R, p(1), c_0(1))$ and geometrically growing levels of p(t), M(t) for all t = 1,2,... satisfying the following:

$$u_2(A-s,Rs) = Ru_1(A-s,Rs) \tag{3a}$$

$$G = \tau w - \phi(\tau w) + (1-R)s(A,R) \tag{3b}$$

$$G(1) = \tau(1)w(1) - \phi(\tau(1)w(1)) + \frac{M(1)-M(0)}{p(1)} \tag{3c}$$

$$M(1)/p(1) = s(A,R) \tag{3d}$$

$$\frac{M(t+1)}{M(t)} = \frac{p(t+1)}{p(t)} = \frac{1}{R} \tag{3e}$$

Note that in equation (3b) the term $1-R \equiv \pi/(1+\pi)$ is the effective inflation tax rate on real currency balances.

III. <u>Optimal Taxation</u>

In this section, the model developed above is used to illustrate conditions under which inflationary finance can be part of a second-best tax menu. In particular, a Ramsey formula is derived which makes explicit the dependence of the desired rate of inflation on the specification of the collection cost function and such variables as the marginal propensity to consume and the income and interest elasticities of money demand. Basically, the assumption that income taxes require real resources for collection introduces the distortion necessary to allow seigniorage to be a part of the optimal government finance strategy. Thus, while higher rates of inflation reduce the real rate of interest and distort intertemporal choice, higher rates of income taxation entail direct collection costs. In the optimal position, marginal excess burdens of the two taxes are equalized.

Formally, let the social welfare criterion be the steady state utility $V(A,R) = u(A-s,Rs)$ of each generation $t = 1,2,\ldots$ The authorities set the pair (R,τ) to maximize V subject to (3b) and the functional form of the real money demand function $s(A,R)$ dictated by private optimization. Letting μ denote the Lagrange multiplier associated with (3b) and assuming an interior solution, the first order necessary conditions (FONC) of this problem are (3b) and

(4a) $\quad u_1 = \mu[1-\phi'-(1-R)s_A]$ $\qquad$ and $\qquad$ (4b) $\quad u_2 = \mu[1-(1-R)s_R/s]$

Dividing equation (4a) by equation (4b) and using the identity $u_1 = Ru_2$ yields

$$R = \frac{1-\phi'-(1-R)s_A}{1-(1-R)s_R/s} \tag{5}$$

Equation (5) may be solved for the <u>optimal inflation tax</u> 1-R as a function of the marginal cost of collection ϕ' and the income and interest elasticities of real money demand. Cross-multiplying and rearranging terms leads to the following simple formula:

$$1-R = \frac{\phi'}{1 - s_A + \epsilon(s,R)} \tag{6a}$$

The Ramsey formula (6a) suggests, first, that <u>the desired inflation tax is proportional to the marginal cost of collection ϕ'</u>. Thus, regardless of the values of the income and interest elasticities of money demand, price stability ($R=1$ or $\pi=0$) should be strived for if tax collections are costless at the margin. The reason for this result is that in the present model, agents neither value leisure directly nor do they have opportunities to engage in untaxed home production. With agents' entire labor endowments inelastically supplied to the taxed activity, costless flat-rate income taxes are equivalent to lump-sum taxes and ought to be used to raise 100 percent of revenue. 1/

Equation (6a) also suggests that the optimal inflation tax is inversely related to the marginal propensity to consume $1-s_A$ and the interest elasticity of real money demand $\epsilon(s,R)$. A higher marginal propensity to consume lowers one-for-one the amount of desired real currency balances carried forward from the current period. As such, it reduces the base of the inflation tax and raises the excess burden of a given rate of inflation. For entirely analogous reasons, the optimal inflation rate is lower the more real currency demand is interest-elastic.

IV. Examples

To gain additional insight into the nature of optimal taxes, two special cases are considered below which lead to closed form solutions of the optimal (τ,π) pair. The first case corresponds to Végh's and Dixit's benchmark specification of log utility and constant collection costs, while the second corresponds to the more realistic case of increasing marginal collection costs.

Suppose first that consumer preferences are given by $u(c_1,c_2) = \log(c_1) + \beta\log(c_2)$, where $\beta > 0$ is the subjective time discount factor. Consumer optimization leads to a constant saving rate ($s = \beta/(1+\beta)$), and interest-inelastic real currency demand ($\epsilon(s,R) = 0$). The Ramsey formula (6a) then becomes $1-R = (1+\beta)\phi'$. Assuming in addition that the marginal cost of collection is constant, say $\phi' = \theta_1 > 0$, establishes that optimal inflation is (a) constant and independent of the level of government spending; and (b) inversely related to the subjective rate of time preference $1/\beta-1$.

Given the desired value of 1-R, the optimal income tax rate is computed from (3b). Letting $g = G/w$ denote the share of government spending in national income, the optimal value of τ is:

$$\tau = \frac{g-\beta\theta_1}{1-(1+\beta)\theta_1} \tag{6b}$$

1/ The presence of untaxed activities would alter this result. If workers value leisure or have employment opportunities in the underground economy, proportional income taxes would distort some margin of choice by consumers--the labor-leisure choice in the first case, and the regular-underground employment in the second. In either case, efficient taxation would entail positive income and inflation taxes even in the absence of collection costs. See Mourmouras (1991) for an explicit analysis.

Income taxes are positive as long as the unit cost of collection is not too large (in the sense of satisfying $g > \beta\theta_1$ and $1 > (1+\beta)\theta_1$.) As expected, in this range the optimal income tax rate varies inversely with the marginal cost of collection. For example, if $\beta = 1$ and $\theta_1 = 0.10$, $\pi = 0.25$; that is, with no discounting of the future and a marginal collection cost of 10 percent, the optimal inflation rate is 25 percent regardless of the size of the government budget. 1/ If in addition $g = 0.40$, then $\tau = 0.375$. In other words, if government spending is 40 percent of GDP and costs of collections are 10 percent of budgetary revenues, gross budget revenues are 37.5 percent of GDP, collection costs are 3.75 percent of GDP, and the optimal money-financed deficit is 6.25 percent of GDP. If $\theta_1 = 0.05$ and $g = 0.40$ $\pi = 0.111$ and $\tau = 0.389$. In Figures 2 and 3, the optimal (τ,π) pair is drawn against g and θ_1, respectively. Table 2 shows how the optimal (τ,π) pairs vary for selected values of θ_1.

The conclusion that inflation is part of an <u>interior</u> optimal tax package is in sharp contrast with the results obtained by Frenkel (1987). This author studied a log-linear cash in advance model of labor-leisure choice and showed that if the marginal cost of income tax collections is constant, then the optimal policy involves financing government expenditures via inflation taxes <u>in its entirety</u>. This result stems directly from Frenkel's specification of money demand and his assumption that inflation is costless while income taxes require resource costs. In particular, Frenkel assumes demand for real currency balances to be proportional to income. Since the bases for the income and inflation taxes are then proportional, the costless inflation tax will always be preferred over the costly income tax. By contrast, in the overlapping generations model, the two tax bases are separate but related. On the one hand, income taxes are levied on current income at the source; on the other hand, seigniorage subjects to tax that part of current income which is <u>not</u> consumed in the present period, creating a distortion of intertemporal choice. This distinction creates a nontrivial trade-off between the two types of taxes which is optimally exploited by the policymaker and is reflected in equation (6a).

<u>Quadratic collection costs</u>

In this section, the more realistic case of increasing marginal collection costs is taken up. Following Végh (1989) and Dixit (1991), suppose that the function ϕ is quadratic:

$$\phi(\tau w) = \theta_1(\tau w) + \frac{\theta_2}{2}(\tau w)^2, \qquad \theta_1 > 0 \text{ and } \theta_2 > 0 \tag{7}$$

1/ Assuming the length of the period to be approximately 20 years, this translates to an annual rate of inflation of 1.1 percent.

Table 2. Selected Values of Optimal (τ,π) as Functions of g

g	θ	π^N	π^L	τ^N	τ^L
30	0.3	4.69	2.93	0.00	13.16
31	0.3	4.69	2.99	2.50	14.34
32	0.3	4.69	3.04	5.00	15.52
33	0.3	4.69	3.10	7.50	16.70
34	0.3	4.69	3.16	10.00	17.88
35	0.3	4.69	3.21	12.50	19.07
36	0.3	4.69	3.27	15.00	20.25
37	0.3	4.69	3.34	17.50	21.43
38	0.3	4.69	3.40	20.00	22.62
39	0.3	4.69	3.46	22.50	23.81
40	0.3	4.69	3.53	25.00	25.00
41	0.3	4.69	3.59	27.50	26.19
42	0.3	4.69	3.66	30.00	27.38
43	0.3	4.69	3.73	32.50	28.58
44	0.3	4.69	3.80	35.00	29.77
45	0.3	4.69	3.87	37.50	30.97
46	0.3	4.69	3.95	40.00	32.17
47	0.3	4.69	4.02	42.50	33.37
48	0.3	4.69	4.10	45.00	34.57
49	0.3	4.69	4.18	47.50	35.78
50	0.3	4.69	4.26	50.00	36.98
51	0.3	4.69	4.35	52.50	38.19
52	0.3	4.69	4.43	55.00	39.40
53	0.3	4.69	4.52	57.50	40.61
54	0.3	4.69	4.61	60.00	41.82
55	0.3	4.69	4.70	62.50	43.03
56	0.3	4.69	4.80	65.00	44.25
57	0.3	4.69	4.89	67.50	45.46
58	0.3	4.69	4.99	70.00	46.68
59	0.3	4.69	5.10	72.50	47.90
60	0.3	4.69	5.20	75.00	49.13

π is in percent per annum
g is in percent of GDP
τ is in percent

Figure 2a

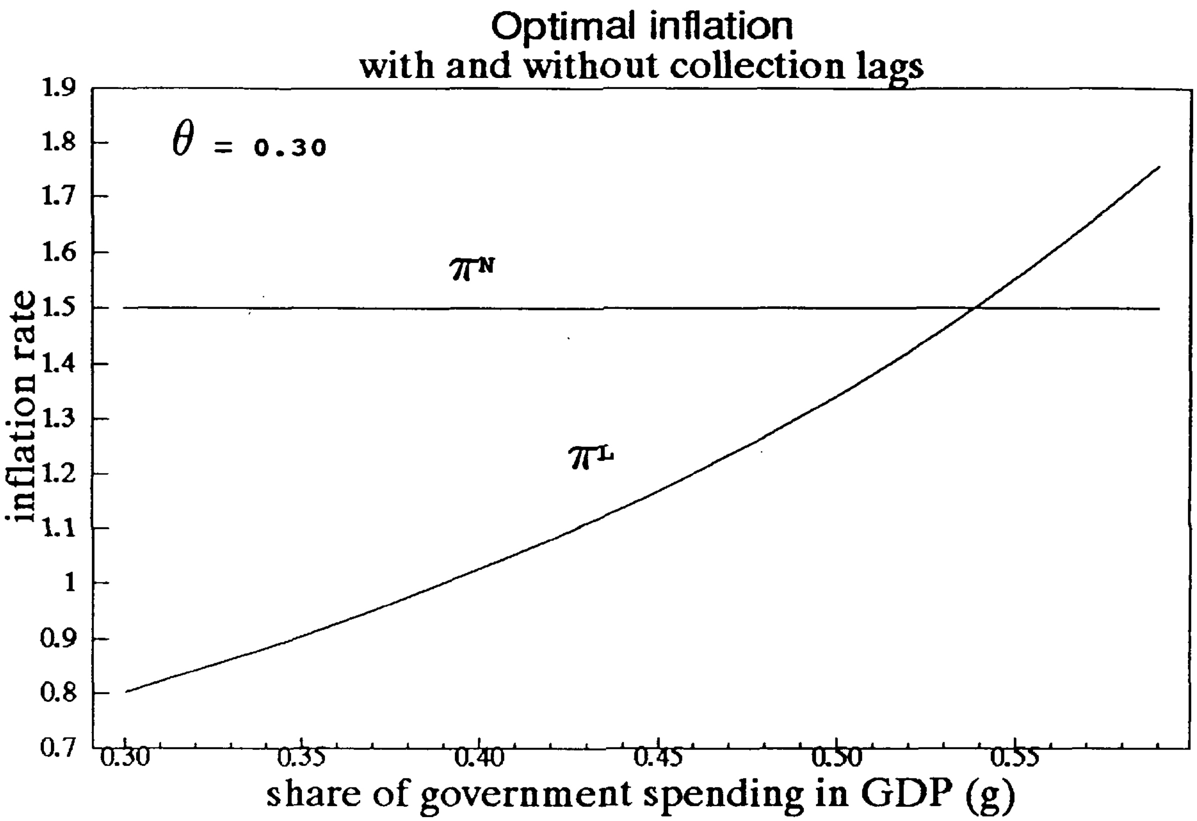


Figure 2b

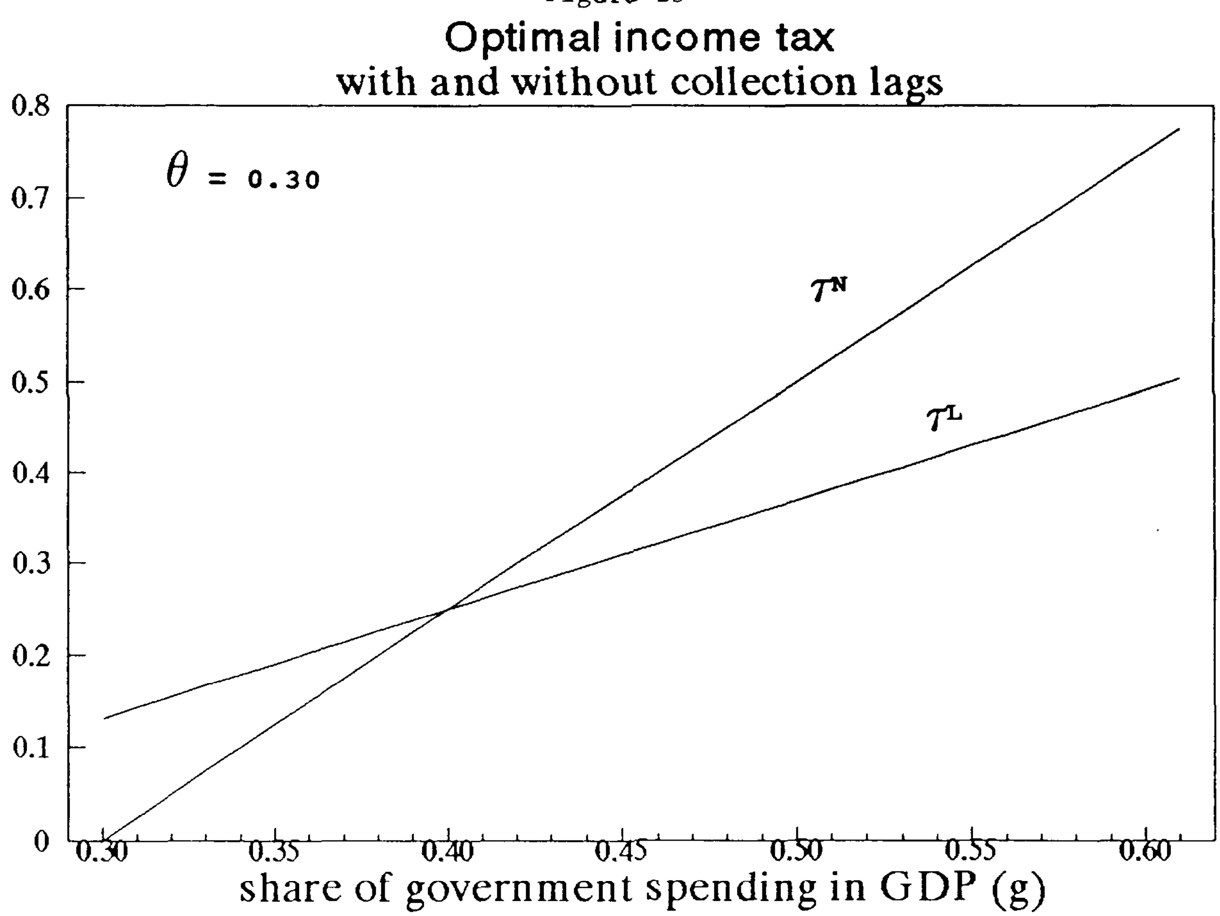


Figure 3a

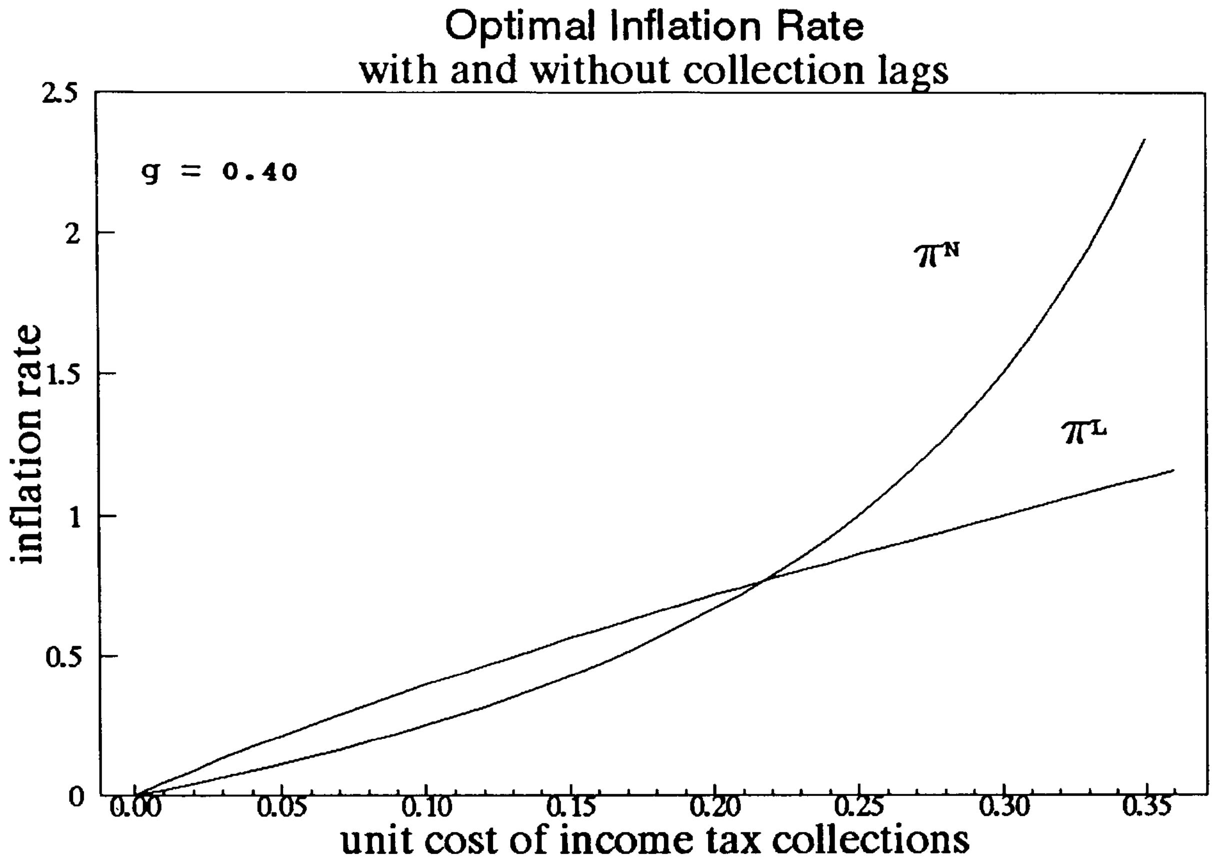
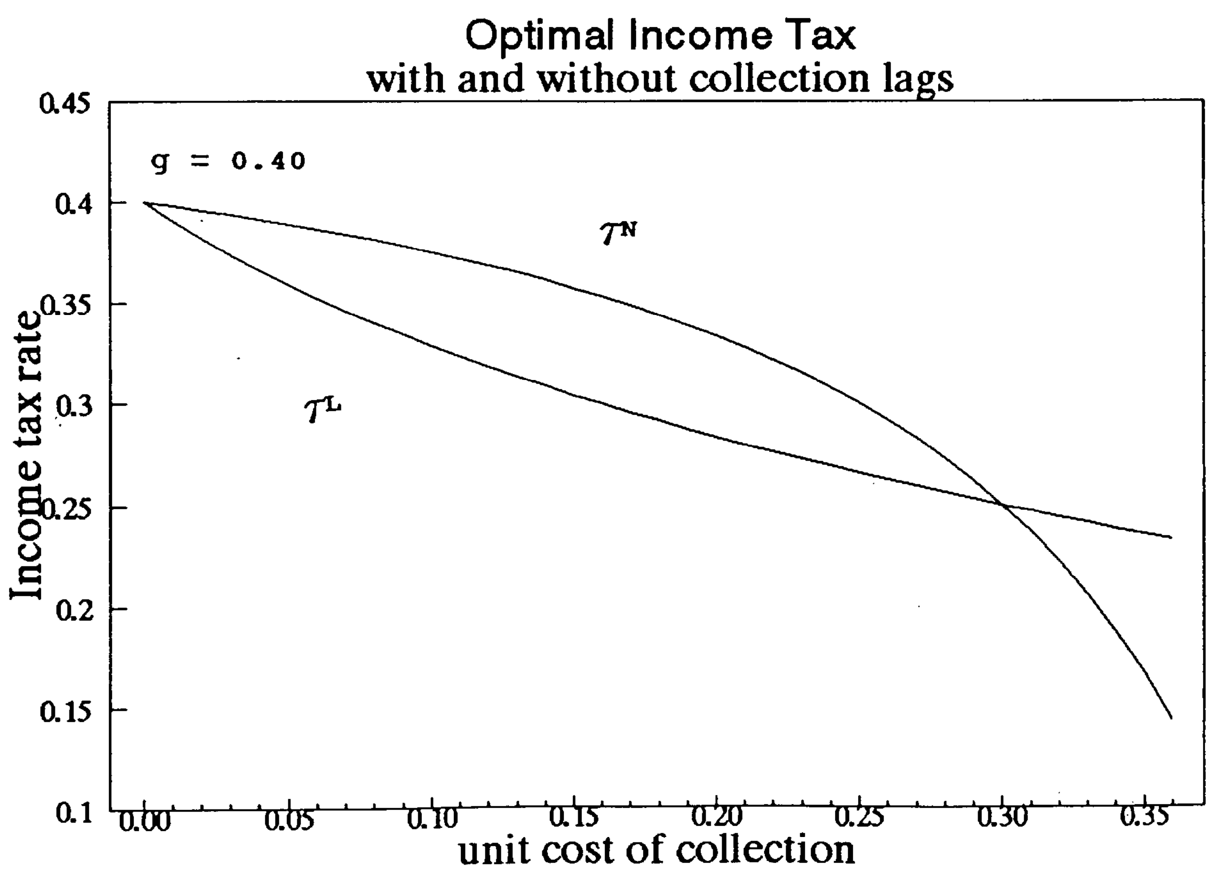


Figure 3b



With the collection cost function as specified in (7) and log utility, it turns out that <u>the optimal income and inflation tax rates are linearly related</u>: substituting $\phi' = \theta_1+\theta_2(\tau w)$, $\epsilon(s,R) = 0$ and $1-s_A = 1/(1+\beta)$ into equation (6a) yields

$$(8) \qquad 1-R \equiv \pi/(1+\pi) = (1+\beta)[\theta_1+\theta_2\tau w]$$

To compute the reduced form of τ and π, the right-hand side of (8) may be substituted into the government budget constraint (3b). After some simple algebra the optimal value of τ is seen to be a solution to the following quadratic equation:

$$(9) \qquad (\tfrac{1}{2}+\beta)\theta_2 w\tau^2 - [1-(1+\beta)\theta_1+\beta\theta_2 w]\tau + g-\beta\theta_1 = 0$$

The roots of equation 9 are real if $\Delta = [1-(1+\beta)\theta_1+\beta\theta_2 w]^2-4(\tfrac{1}{2}+\beta)\theta_2 w(g-\beta\theta_1)$ is positive, a condition that will always hold if g is not "too large." The reduced form of τ is then given by the smaller of the two roots, namely:

$$(10) \qquad \tau = \frac{1-(1+\beta)\theta_1+\beta\theta_2 w - \Delta^{\frac{1}{2}}}{2(\tfrac{1}{2}+\beta)\theta_2 w}$$

There are two differences between (8)-(10) and (6a-b). First, in (8) the greater fiscal inefficiency raises π (note that the inflation tax in (8) is uniformly greater in $\tau\epsilon[0,1]$ than $(1+\beta)\theta_1$). Secondly, whereas equation (8) shows that higher values of τ (and g) raise π, it will be recalled that in the benchmark case inflation is constant and independent of g. (To show that $\partial\tau/\partial g > 0$ it suffices to differentiate (9) with respect to g. From equation 8, it then follows that $\partial(1-R)/\partial g = 2\theta_2 w\cdot\partial\tau/\partial g > 0$ so that both tax instruments are normal.)

V. <u>Collection Lags and Optimal Inflation</u>

In order to assess the impact of collection lags on the optimal tax menu, in this section the basic model is extended to incorporate a one-period lag in income tax collections. As before, a stream of government spending $\{G(t)\}$ is financed by income taxes $\{\tau(t)\}$ and currency issues $\{M(t) - M(t-1)\}$ for $t = 1,2,\ldots$ Let the nominal income tax liability accrued at t, $\tau(t)p(t)w(t)$, become due at t+1. The payment lag allows workers (who also own the firms in the economy) both to use these balances for an additional period and (assuming positive inflation) to reduce their tax liability in real terms. To the initial conditions Π and M(0) must now be added the nominal tax liability due in the initial period, say $\tau(0)p(0)w(0)$.

The lag in tax payments means the time at which costs of collections are incurred must be elaborated as well. Given the payment lag, nominal collections at t are $p(t-1)\tau(t-1)w(t1)$. The real value of these collections in period t is $p(t-1)\tau(t-1)w(t-1)/p(t) = R(t-1)\tau(t-1)w(t-1)$ and declines with inflation. Two plausible formulations are considered. In this section it is first assumed that real collection costs at t, $\phi(t)$, are a function of the real value of revenues collected at t. In other words, $\phi(t) = \phi(R(t-1)\tau(t-1)w(t-1))$. This implies that the real costs of collecting a given nominal liability are lowered by inflation. While this is a plausible formulation, it must be emphasized that it is entirely ad hoc. 1/ An alternative adopted in section VI, is to write the collection cost function as $\phi(t) = \phi(\tau(t-1)w(t-1))$, implying that costs of collection are completely indexed for inflation. 2/

Formally, the private optimization problem is now to maximize $u(c_t(t),c_t(t+1))$ subject to:

$$p(t)c_t(t) + m(t) \leq p(t)w(t) \tag{11}$$

$$p(t+1)c_t(t+1) \leq m(t) - \tau(t)p(t)w(t) \tag{12}$$

Using the notation developed in section II, equations (11)-(12) may be written

$$c_t(t) + s(t) \leq w(t) \tag{11'}$$

$$c_t(t+1) \leq R(t)s(t) - R(t)\tau(t)w(t) \tag{12'}$$

1/ Note that this formulation is consistent with the view that nominal collection costs at t, say $\Phi(t)$, are proportional to the nominal value of the liability to be collected $\tau(t-1)p(t-1)w(t-1)$. Dividing through $\Phi(\tau(t-1)p(t-1)w(t-1)) = \theta_1\tau(t-1)p(t-1)w(t-1)$ by p(t) shows that real collection costs $\phi(t) = \Phi(t)/p(t)$ is equal to $\theta_1\tau(t-1)p(t-1)w(t-1)/p(t)$, or that $\phi(t) = \theta_1 R(t-1)\tau(t-1)w(t-1)$. This specification, of course, corresponds to Végh's and Dixit's benchmark case of constant real marginal collection costs.

2/ The third alternative, namely where real collection costs are raised by higher inflation, is left as an exercise for the interested reader.

In equation (11′), s(t) is an agent's <u>gross</u> real currency balances at the end of t, while in (12′) $R(t)\tau(t)w(t)$ is the agent's effective real tax liability. Combining (11′) and (12′), the agent's consumption set is

$$(13) \qquad c_t(t) + c_t(t+1)/R(t) \leq A(t) \equiv (1-\tau(t))w(t)$$

As before, the solution to this problem is a pair of consumer demand schedules $c_1(A,R)$ and $c_2(A,R)$ and a real currency demand schedule $s = w - c_1$. Given R(t) and $\tau(t)$, individual consumption sets are not affected by the lags as agents react to the change in the timing of taxes by altering their financial decisions. Note in particular that in anticipation of the tax liability to be incurred one period ahead, agents accumulate higher levels of real currency balances than in a regime in which taxes were paid with no lag. This alters the real currency demand schedule--the base of the inflation tax. This change in turn affects the government's choice set and the outcome of the optimal tax calculation (see below).

Since payment of taxes accrued at t-1 is not made until t, the government's cash flow constraint may be written as follows:

$$(14) \qquad p(t)G(t) = p(t-1)\tau(t-1)w(t-1) - p(t)\phi\left(\frac{p(t-1)\tau(t-1)w(t-1)}{p(t)}\right) + M(t) - M(t-1)$$

The initial conditions of the model include the nominal quantity of taxes due at t = 1, namely $p(0)\tau(0)w(0)$. (The real value of this nominal revenue is endogenous, as it depends on p(1), the date 1 price level.) Dividing (14) by p(t) yields

$$(15) \qquad G(t) = R(t-1)\tau(t-1)w(t-1) - \phi(R(t-1)\tau(t-1)w(t-1)) + \frac{M(t)-M(t-1)}{p(t)}$$

In equation (15), the Tanzi effect is reflected in the term $R\tau w = \tau w/(1+\pi)$ in the right-hand side where given τ, the real value of income tax collections is lowered by higher inflation. Notice that if ϕ' is less than one, the real value of income tax collections <u>net</u> of these costs, $R\tau w - \phi(R\tau w)$, is raised by lowering the rate of inflation. This can be seen by differentiating $R\tau w - \phi(R\tau w)$ with respect to R holding w and τ constant:

(16) $$\frac{\partial}{\partial R}[R\tau w - \phi(R\tau w)] = \tau w(1-\phi') > 0 \quad \text{if} \quad \phi' < 1$$

A stationary equilibrium is a pair (R,τ) satisfying

(17) $$G = R\tau w - \phi(R\tau w) + (1-R)s(A,R)$$

Given a feasible choice of (R,τ), the equilibrium sequences for the price level and money supply each grow at the gross rate of inflation R^{-1}. These price sequences are completely known once the initial price level and currency issue, $p(1)$ and $M(1)$ respectively, are determined. Given values for G and the initial conditions $p(0)$, $\tau(0)$, $w(0)$, $M(0)$, the equilibrium values of $p(1)$ and $M(1)$ may be calculated using the government budget constraint for the initial period $t = 1$, equation (18) below,

(18) $$G = p(0)\tau(0)w(0)/p(1) + M(1)/p(1) - M(0)/p(1),$$

and the stationary value of the real stock of currency $M(1)/p(1) = s(A,R)$.

Optimal policy

The presence of payment lags introduces two mutually opposing forces that in principle have an ambiguous net effect on total "revenue" (seigniorage plus income taxes). On the one hand, inflation lowers the real value of revenue from a given rate of income tax due to the Tanzi effect. On the other hand, the increase in real money demand associated with the presence of collection lags makes a given rate of inflation more "productive." As suggested by Dixit, to ascertain how the payment lags affect the desired rate of inflation in the face of these forces, the optimal tax problem must be recalculated in its entirety.

Formally, given the private decision rules $c_1(A,R)$, $c_2(A,R)$ and $s = w-c_1$, the government selects (τ,R) to maximize $u(w-s,Rs-\tau w)$ subject to

(19) $$G = R\tau w + (1-R)s - \phi(R\tau w)$$

In the Appendix it is shown that the optimal rate of inflation is given by

$$\pi = \frac{\phi'}{\epsilon(s,R) + 1 - s_A + [\epsilon(s,A)\tau/(1-\tau) - 1]} \tag{20}$$

Equation (20) retains the basic characteristics of equation (6a), the optimal inflation formula in the absence of any lags. First, desired inflation continues to be proportional to the marginal cost of collection so that if $\phi' = 0$ price stability ought to still be pursued regardless. Second, the optimal inflation rate is inversely related to the marginal propensity to consume and the interest elasticity of money demand. To ascertain the relative magnitude of desired inflation in the lag and no-lag cases, the right-hand sides of equation (6a) and (20) may be compared directly. These equations differ only by the term in brackets in the denominator of (20), $[\epsilon(s,A)\tau/(1-\tau) - 1]$. It follows that <u>if the term $\epsilon(s,A)\tau/(1-\tau) - 1$ is positive (negative) the desired rate of inflation is lowered (raised) by the presence of payments lags</u>. Note that this term, which is increasing in τ and $\epsilon(s,A)$, is positive if $\epsilon(s,A)\geq(1-\tau)/\tau$. Thus, for a given value of $\epsilon(s,A)\epsilon(o,\infty)$ the presence of lags will lead to lower desired inflation whenever τ exceeds some critical value. However, since τ is endogenous and the terms in the right-hand sides of (6a) and (20) are all functions, a simpler comparison criterion cannot, in general, be established.

Sharper results are possible for the benchmark case considered by Dixit and Végh in which the functions in (6a) and (20) are constants. As stated in the Introduction, when this comparison is undertaken it turns out that a theoretical case can be made for the Tanzi position, at least for levels of government spending below a certain threshold. 1/ Assuming $u(c_1,c_2) = \log(c_1) + \log(c_2)$, and $\phi' = \theta > 0$ optimal inflation π^L (L for lag) is easily calculated to be

$$\pi^L = \frac{1+\tau}{1-\tau}2\theta \tag{21}$$

The reduced forms of τ and π^L can be computed analytically as follows: first substitute the real money demand function $s = w(1+\tau)/2$ in the steady state form of the government budget constraint equation (17). After some rearranging, this reduces to

$$\tau = \frac{2g-(1-R)}{2R(1-\theta)+(1-R)} \tag{22}$$

1/ Analysis of the more realistic case of <u>quadratic</u> collection costs yields similar results. The proof of this assertion for the quadratic case involves some tedious algebra--finding the roots of a fourth order polynomial equation--and is available upon request.

Equation (22) may be written in the following form which is convenient for subsequent substitutions:

$$(22') \qquad \frac{1+\tau}{1-\tau} = \frac{1-\theta+(1+\pi)g}{1-\theta+\pi-(1+\pi)g}$$

Substituting the right-hand side of (22') into the Ramsey formula (21) yields the following equation in π:

$$(23) \qquad \pi = 2\theta\frac{1-\theta+(1+\pi)g}{1-\theta+\pi-(1+\pi)g}$$

Simplifying (23) yields the following quadratic equation in π:

$$(24) \qquad (1-g)\pi^2 - (\theta+g-1+2\theta g)\pi - 2\theta(1-\theta+g) = 0$$

The roots of (24) are real and of opposite sign as $\Delta = (\theta+g-1+2\theta g)^2 + 8(1-g)\theta(1-\theta+g)$ is positive, in which case the reduced form of π is:

$$(25) \qquad \pi^L = \frac{(\theta+g-1+2\theta g)^2 + \Delta^{1/2}}{2(1-g)}$$

The pairs (τ^L,π^L) satisfying (25) and (21) are drawn against g in Figures 2a and 2b, and against θ in Figures 3a and 3b. It may be observed that even in the benchmark case the introduction of a payment lag makes desired inflation rise with g. This outcome may be contrasted with the results of section III where the optimal rate of inflation in the benchmark case was shown to be independent of g. 1/ Given the definition $R = 1/(1+\pi)$ equation (6a) describing the desired rate of inflation π^N (for no-lag) may be written as follows:

$$(26) \qquad \pi^N = 2\theta/(1-2\theta)$$

In Figure 2a, equation (26) is shown as the horizontal line while equation (25) is shown as the upward sloping curve. For a given value of θ, the

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1/ If marginal collection costs are increasing, inflation continues to be an increasing function of g.

relative size of inflation in the two environments depends on the magnitude of g. In particular, there is a threshold $g^* > 0$ solving $\pi^L(g^*) = \pi^N(g^*)$ such that (i) $\pi^L(g) < \pi^N(g)$ whenever $0 < g < g^*$ and $\pi^L(g) > \pi^N(g)$ whenever $g > g^*$. That is, <u>the desired rate of inflation is lower in the presence of collection lags for all values of g not exceeding the threshold</u>. The threshold value of g can be calculated analytically: setting $\pi^L = \pi^N = 2\theta/(1-2\theta)$ in equation (25) yields after a few steps of algebra the following closed form for g^*:

$$g^* = \frac{\theta}{1-\theta}[2\theta^2-3\theta+2] \tag{27}$$

Figure 1 and Table 1 show the critical value of g and the associated (τ,π) pairs as a function of θ. Notice that g^* is an increasing function of θ with $g^*(0) = 0$ and $g^*(0.5) = 1$. It can be shown that for values of θ greater than approximately 0.27, this function is strictly convex, implying that <u>as the degree of fiscal inefficiency grows, the critical value of g grows at an increasing rate and approaches unity as the unit cost of collection θ approaches 0.5 from below</u>.

VI. An Invariance Proposition

In this section, we explore the implications of altering the specification of the collection cost technology in the manner suggested in section V. This is important because, as emphasized by Dixit (1991, p. 648), no deep theory exists of the nature of these costs. It will be recalled that in Section V real collection costs were assumed to be a stable and increasing function of real <u>realized</u> tax revenues, $\phi = \phi(R\tau w)$, implying that for given statutory tax rates and nominal collections, real collection costs are lowered by higher inflation. In this section, the function ϕ is written $\phi = \phi(\tau w)$ so that collection costs are assumed to be a function of <u>accrued</u> real revenues, or that nominal costs of collection rise in proportion with prices. The main result is a neutrality proposition for the benchmark specification of constant marginal collection costs and log utility establishing that <u>the optimal (τ,R) pair is independent of the collection lag</u>.

Formally, $R \geq 0$, and τ are selected to maximize social welfare

$$U = \log(c_1) + \log(c_2) \tag{28}$$

subject to its resource constraint (29) and private sector demand and supply schedules (30)-(33):

$$G = (1-R)s + R\tau w - \phi(\tau w) \tag{29}$$

$$c_1 = \tfrac{1}{2}w(1-\tau) \tag{30}$$

$$c_2 = \tfrac{1}{2}w(1-\tau)R \tag{31}$$

$$s = \tfrac{1}{2}w(1+\tau) \tag{32}$$

This problem is equivalent to selecting $R \geq 0$ and τ to maximize

$$U = 2\log(1-\tau) + \log(R) \tag{33}$$

subject to

$$G = (1-R)\tfrac{1}{2}w(1+\tau) + R\tau w - \phi(\tau w) \tag{34}$$

If $\phi(\tau w) = \theta\tau w$ for some $\theta > 0$, we can solve the problem by substitution. Use (34) to write τ as a function of R and θ:

$$\tau = \frac{2g+R-1}{R+1-2\theta} \tag{35}$$

This implies that

$$1-\tau = 2\frac{1-\theta-g}{R+1-2\theta} \tag{36}$$

The objective function (33) can then be written

$$U = 2\log\left(2\frac{1-\theta-g}{R+1-2\theta}\right) + \log(R) \tag{37}$$

Maximizing U with respect to R ≥ 0 is equivalent to maximizing

$$\Omega(R) = 2\log\left(\frac{1-\theta-g}{R+1-2\theta}\right) + \log(R) \tag{38}$$

The first and second derivatives of Ω are

$$\Omega' = \frac{-2}{(R+1-2\theta)} + \frac{1}{R} < 0 \tag{39}$$

and

$$\Omega'' = \frac{2}{(R+1-2\theta)^2} + \frac{-1}{R^2} < 0 \tag{40}$$

The optimal value of R is $R = 1 - 2\theta$, implying that the optimal inflation rate is unchanged from the value obtained earlier in the absence of lags.

Returning to the general specification of the collection cost function, the problem of maximizing (28) subject to (29)-(32) may be solved by forming the Lagrangian

$$L = 2\log(1-\tau) + \log(R) + \lambda\left[(1-R)\tfrac{1}{2}w(1+\tau) + R\tau w - \phi(\tau w) - G\right] \tag{41}$$

The first order necessary conditions (FONC) for this problem are

$$\frac{1}{R} = \lambda w\left[w(1+\tau)/2 - \tau w\right] \tag{42}$$

and

$$\frac{2}{1-\tau} = \lambda w\left[\frac{1-R}{2} + R - \phi'\right] \tag{43}$$

Dividing (42) by (43) and simplifying yields $1-R = 2\phi'$ which is identical to the formula for the inflation tax rate derived in section III under the assumption that there were no collection lags (simply set $\beta = 1$, $s_A = 0.5$ and $\epsilon(s,R) = 0$ in equation 6a). In order to prove that collection lags are neutral, it remains to show that the reduced form for τ is unchanged as well. Below this is first shown for the linear marginal collection cost case. That is, it is established that substituting equation (32) and the formula $1-R = 2[\theta_1+\theta_2\tau w]$ in equation (29) and solving for τ yields the same formula as equation (9) on page 9. Upon substitution, the optimal value of τ must solve

$$(44) \qquad g = (\theta_1+\theta_2\tau w)(1+\tau) + [1-2\theta_1-2\theta_2\tau w]\tau - \theta_1\tau - \frac{\theta_2 w\tau^2}{2}$$

After some steps of algebra this is seen to be equivalent to

$$(45) \qquad 0 = (3/2)\theta_2 w\tau^2 + [1-2\theta_1+\theta_2 w]\tau - (g-\theta_1)$$

Comparison of equation (45) with equation (9) on page 8 for $\beta=1$ shows that the two are equivalent. Finally, to establish the claim for the linear marginal cost case, simply note that equation (45) with $\theta_2=0$ implies equation (6b) on page 6. This proves that the presence of collection lags does not change the optimal (R,τ) pair in both the case of linear and quadratic collection costs.

A note of caution is in order in interpreting this invariance proposition. While this proposition does suggest that the optimal inflation tax rate 1-R is invariant to the lag, it does not suggest that inflation tax revenue (defined as the product (1-R)s(A,R) of the inflation tax rate and the real currency stock) is invariant to collection lags. Analogously, this proposition does not suggest invariance of income tax revenue. Clearly, with inelastic labor, positive inflation, and an unchanged rate of labor income taxation, the presence of collection lags lowers the effective yield of labor taxes. According to the proposition, all revenue losses are made up by higher real seigniorage earnings. In the presence of an unchanged rate of inflation, the additional seigniorage earnings are possible by the higher stock of real balances accumulated in order to meet future income tax liabilities.

VII. Conclusions

This paper analyzed how the presence of collection lags affects the magnitude of the optimal inflation rate in an overlapping generations version of the Frenkel-Végh model of costly income taxation. Ramsey pairs (τ,π) were derived from first principles and the traditional argument, that inflation ought to reduce the optimal rate of inflation, was confirmed for cases where the level of government spending in GDP was below a threshold level. Professor Dixit's conclusions are confirmed for tax rates above the threshold. This demonstration casts some doubt on the claim that, as a general principle, the Tanzi hypothesis is not consistent with optimal tax theory.

Appendix

The Appendix derives the optimal tax formula applicable in Section V. The following notation is used:

w	= real pre-tax income
τ	= income tax rate
A	= $(1-\tau)w$ = after-tax income
c_1	= consumption in first period of life
c_2	= consumption in second period of life
s	= saving
π	= rate of inflation
G	= rate of real government spending on goods and services
$\phi(\cdot)$	= collection cost function
$\phi'(\cdot)$	= marginal cost of income tax collections
R	= $1/(1+\pi)$
$1-R$	= rate of inflation tax

Formally, given the private decision rules $c_1(A,R)$, $c_2(A,R)$ and $s = w-c_1$, the pair (τ,R) is selected to maximize indirect utility $u(w-s,Rs-\tau w)$ subject to

$$G = \tau w + (1-R)s - \phi(\tau wR) \tag{A1}$$

The Lagrangian expression for this problem is

$$L = u(w-s,Rs-\tau w) + \lambda\Big[\tau w + (1-R)s - \phi(\tau w) - G\Big] \tag{A2}$$

The first order necessary conditions (FONC) for an interior solution are

$$\tau: \quad u_1(-s_A A_\tau) + u_2(Rs_A A_\tau - w) + \lambda\Big[w(1-\phi) + (1-R)s_A A_\tau\Big] = 0 \tag{A3}$$

$$R: \quad u_1(-s_R) + u_2(s + RS_R - \tau w) + \lambda\Big[\tau w(1-\phi') - s + (1-R)s_R\Big] = 0 \tag{A4}$$

Note that $A = (1-\tau)w$ implies $A_\tau = -w$. Also, from private optimization, $u_1 = Ru_2$. Substituting these in (A3)-(A4) and simplifying yields

$$u_2 wR = \lambda\Big[w(1-\phi') - s + (1-R)s_R\Big] \tag{A5}$$

and

$$\text{(A6)} \qquad u_2(s-\tau w) = -\lambda\left[\tau w(1-\phi') - s + (1-R)s_R\right]$$

These imply that

$$\text{(A7)} \qquad \frac{R}{s - \tau w} = \frac{R(1-\phi') - (1-R)s_A}{\tau w(1-\phi') - s + (1-R)s_R}$$

Cross-multiplication yields

$$\text{(A8)} \qquad -R\tau w(1-\phi') + Rs - R(1-R)s_R = sR(1-\phi') - s(1-R)s_A - \tau w(1-\phi') + \tau w(1-R)s_A$$

After some simplification this can be written

$$\text{(A9)} \qquad -R(1-R)s_R = -sR\phi' - s(1-R)s_A + \tau w(1-R)s_A \qquad \Rightarrow$$

$$\text{(A10)} \qquad (1-R)Rs_R/s = R\phi' + (1-R)s_A - (1-R)s_A\tau w/s.$$

The expression Rs_R/s on the left-hand side is the interest elasticity of real currency demand $\epsilon(s,R)$, while the term $s_A\tau w/s$ equals $\epsilon(s,A)\tau/(1-\tau)$. Thus (A10) may be rewritten as follows:

$$\text{(A11)} \qquad (1-R)\epsilon(s,R) = -sR\phi' - s(1-R)s_A + \tau w(1-R)s_A \qquad \Rightarrow$$

$$\text{(A12)} \qquad (1-R)\left[\epsilon(s,R) - s_A + \epsilon(s,A)\frac{\tau}{1-\tau}\right] = R\phi'$$

Since the factor $(1-R)/R$ equals the rate of inflation π, equation (A12) can be written

$$\text{(A13)} \qquad \pi = \frac{\phi'}{\epsilon(s,R) - s_A + \epsilon(s,A)\frac{\tau}{1-\tau}}$$

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