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Openness, Human Development, and Fiscal Policies:  
Effects on Economic Growth and Speed of Adjustment

Prepared by Delano Villanueva\*

Authorized for Distribution by Donald Mathieson

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Abstract

The model developed here postulates that learning through experience plays a critical role in raising labor productivity over time, with three major consequences. First, the steady-state growth rate (of output) becomes endogenous and is influenced by government policies. Second, the speed of adjustment to steady-state growth is faster, and enhanced learning further reduces adjustment time. Third, both steady-state growth and the optimal net rate of return to capital are higher than the sum of exogenous rates of technical change and population growth. Simulation results confirm the model's faster speed of adjustment, while regression analysis explains a large part of divergent growth patterns across countries in terms of the extent of openness and human development and of the quality of fiscal policies.

JEL Classification Numbers:  
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## I. Introduction

The basic neoclassical growth model developed by Solow (1956) and Swan (1956) has been the workhorse of growth theorists during the last three decades and a half. Its simple structure--consisting of a well-behaved neoclassical production function, investment-saving relation, and a labor growth function--is an elegant solution to the "knife-edge" problem posed by Harrod (1939) and Domar (1946). By allowing smooth factor substitution and wage-price flexibility, the capital-output ratio is made a monotonic function of the capital-labor ratio. The growth rate of the capital stock (the warranted rate) adjusts to the exogenously given growth rate of the labor force (the natural rate) to maintain full-employment real output.

The Solow-Swan (henceforth, SS) model, however, have certain equilibrium properties that bother many growth theorists: An increase in the saving rate, while raising the level of per capita real income, has no effect on the growth rate of output. The surprising result on growth neutrality has a simple explanation: Although a higher saving rate raises the growth rate of output by increasing the investment rate, the increase in economic growth occurs only during the transition toward the next equilibrium; sooner or later, the labor input would be a bottleneck, limiting further output expansion. The growth rate of output would eventually fall back to the constant natural rate of growth.

The time it takes the economy to reach this balanced growth path is also of considerable interest--particularly to policymakers. In the context of the SS model, if the objective of economic policy is to raise the equilibrium level of per capita real income (for example, by raising the government saving rate), a fast adjustment would be desirable.

Using a Cobb-Douglas production with constant returns to scale and Harrod-neutral technical progress, Sato (1963) has shown that the time required for the SS model to reach equilibrium was of the order of a hundred years! <sup>1/</sup> Moreover, the lower the rate of depreciation or the higher the share of capital, the slower the adjustment. An intuitive explanation for these results is that a slower rate of depreciation or a larger share of capital would enable firms to substitute capital for labor and thus postpone for a longer period the bottleneck posed by a fixed growth rate of labor.

The SS model's prediction that the rates of saving, depreciation, and population growth, and government policies cannot affect the equilibrium growth rate of per capita real income, which is fixed by an exogenously determined rate of labor-augmenting technological progress, appears to be counterfactual. It seems reasonable to conjecture that, over the long haul, countries with policies that promote saving and investment, reduce the

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<sup>1/</sup> Such a slow adjustment would render somewhat irrelevant the equilibrium behavior of the model because of the likelihood that the other parameters of the system would have changed in the interim.

depreciation of the capital stock, and create more open trading systems tend to grow faster, while those with rapid population increase, sluggish expansion in expenditures on human development and basic needs, and high ratios of government deficits to GDP tend to grow slower.

The relatively slow adjustment of the SS model toward its steady state is partly due to the (assumed) inability of the natural rate to adjust to changes in capital intensity as the economy moves from one equilibrium position to another in response to an exogenous shock. It would seem plausible to consider that a partly endogenous natural rate, via learning through experience, would contribute to a faster speed of adjustment (see Section IV). In this case, the limiting behavior of the SS model would assume much more relevance to policymakers.

This study is both theoretical and empirical. It belongs to the class of new "endogenous growth" (henceforth, EG) models. 1/ It is a variant of Conlisk's (1967) endogenous-technical-change model and of Arrow's (1962) "learning by doing" model, wherein experience (measured in terms of either cumulative past investment or output) plays a critical role in raising labor productivity over time. The presence of learning through experience has three major theoretical consequences. First, equilibrium growth becomes endogenous and is influenced by government policies. 2/ Second, the speed of adjustment to growth equilibrium is faster, and enhanced learning further reduces adjustment time. Third, both equilibrium economic growth and the optimal net rate of return to capital are higher than the sum of the exogenous rates of technical change and population growth.

The EG model's equilibrium behavior is found to be consistent with the substantial diversity in per capita growth patterns actually observed across countries. Such diverse growth experiences, which are predicted by the model, can be explained by differences in saving rates, ratios of government deficits to GDP, population growth rates, and certain parameters that influence the learning coefficient, such as changes in openness to world trade and growth in government outlays on education and health.

The rest of the paper is organized as follows. Section II presents the EG model, analyzes its stability, and derives its short-run and long-run properties. Section III derives and discusses some Golden Rule results from the model. Section IV discusses the model's speed of adjustment toward equilibrium, using analytic, simulation, and empirical methods. Finally, Section V summarizes the findings and draws some policy implications.

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1/ See, among others, Romer (1986), Lucas (1988), Becker et al. (1990), Grossman and Helpman (1990), and Rivera-Batiz and Romer (1991).

2/ Equilibrium growth in Arrow's (1962) learning-by-doing model, although a function of the "learning coefficient," nevertheless remains independent of the saving rate and the depreciation rate. See footnote 1 on page 5 for details.

## II. The EG Model

The model is summarized by the following system of 4 behavioral and 2 definitional relationships.

$$Y = F(K, N) = Nf(k) \quad (1)$$

$$dK/dt = s(\theta, .)Y - \delta(\mu)K \quad (2)$$

$$dL/dt = nL \quad (3)$$

$$dT/dt = \alpha(\chi, \xi, \omega, .)K/L + \lambda T \quad (4)$$

$$N = TL \quad (5)$$

$$k = K/N \quad (6)$$

### Variables

Y	:	real GDP
K	:	capital stock
N	:	labor, manhours in efficiency units
L	:	population, manhours
T	:	labor productivity or technical-change multiplier, index number
k	:	ratio of K to N
s	:	ratio of real saving-investment to Y
$\delta$	:	depreciation of capital
$\alpha$	:	learning coefficient

### Parameters

n	:	population growth rate
$\chi$	:	change in ratio to GDP of foreign trade (sum of exports and imports)
$\xi$	:	growth rate of real government expenditures on education and health
$\omega$	:	growth rate of real government expenditures for social security and housing
$\mu$	:	growth rate of real government expenditures on operations and maintenance
$\theta$	:	ratio of government deficits to GDP
$\lambda$	:	rate of exogenous labor-augmenting technical change
d(.) / dt:		time derivative

Equation (1) is a standard neoclassical production function satisfying the Inada (1963) conditions. <sup>1/</sup> Equation (2) is the expression for capital accumulation: the increment in the capital stock is equal to gross domestic saving less depreciation. The proportion  $s$  of GDP saved and invested is assumed to be sensitive to government policies, in particular to the ratio of the fiscal deficit to GDP,  $\theta$ . High values of  $\theta$  directly lower  $s$ , as the public sector dissaves. There are indirect effects as well. High levels of  $\theta$  indicate large government borrowings from financial markets. Either through high interest rates or lower credit availability, private sector capital accumulation is adversely affected. Thus, it is assumed that  $s'(\theta) < 0$ . There are other (unspecified) factors affecting  $s$ . For example, interest rate liberalization may increase the private saving rate, which would tend to pull aggregate  $s$  up, but may also entail increases in the rate of government dissaving in the presence of a large stock of public debt, which would drag total  $s$  both directly and indirectly (via negative effects on the private saving-investment rate, as mentioned above). It is also assumed that  $\delta'(\mu) < 0$  -- the rate of depreciation  $\delta$  is a negative function of the real growth of expenditures on operations and maintenance  $\mu$ , that is, a higher  $\mu$  lowers the rate of depreciation of existing capital stock  $K$ . The population grows at an exogenously constant rate  $n$  in equation (3).

The key relationship in the model is equation (4). It postulates that technical change  $dT/dt$  improves with the aggregate capital stock per capita  $K/L$ . Output per capita  $Y/L$  can be used instead. For example, manhours in the production of an airframe during the 1930s tended to decline with the number of airframes produced. A more current example is the introduction of both high-speed and personal computers, which has improved the productivity of engineers and scientists (including economists). Since  $(dT/dt)/T$  is a function of  $Y/TL = Y/N = f(k)$ , using  $K/L$  is equivalent to using  $Y/L$  as the forcing variable behind improvements in labor productivity. The parameter  $\alpha$  is the learning coefficient. If  $\alpha = 0$ ,  $T$  grows exogenously at a constant rate  $\lambda$  and the EG model collapses into the SS model. The restrictions  $\alpha \geq 0$  are assumed and empirically tested in Section IV. Since the assumption that  $\alpha > 0$  is crucial to the arguments and propositions in this paper, an extended discussion of its rationale is useful.

The SS model's characterizing assumption  $\alpha = 0$  may be true in a world devoid of technical change, as labor supply may be measured by the size of population. In this case, it may be plausible to assume that labor has no endogenous growth component, since population in many countries appears to grow independently of the economic system. But the real world is one of continuous technical change. While a portion of this may be exogenous, some technical change is clearly endogenous and partly labor-augmenting. Workers learn through experience, and their productivity is likely to be enhanced by the arrival of new and advanced capital goods. That is, the EG model's

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<sup>1/</sup>  $\lim_{K \rightarrow 0} \partial F / \partial K = \infty$  as  $K \rightarrow 0$ ;  $\lim_{K \rightarrow \infty} \partial F / \partial K = 0$  as  $K \rightarrow \infty$ ;  $f(0) \geq 0$ ;  $f'(k) > 0$ ; and  $f''(k) < 0$ .

assumption that  $\alpha > 0$  seems more plausible than the SS model's assumption that  $\alpha = 0$ . 1/

In the restriction  $\alpha > 0$ , the learning coefficient  $\alpha$  is allowed to vary positively with changes in the ratio of foreign trade to GDP  $\chi$ , real growth of outlays on education and health  $\xi$ , social security, housing, and recreation  $\omega$ , and other unspecified factors. The role of a rapid growth of foreign trade in stimulating a higher learning coefficient is two-fold. 2/ First, the import-export sector serves as a vehicle for technology transfer through the importation of advanced capital goods, as elucidated by Bardhan and Lewis (1970), Chen (1979), and Khang (1987), and as a channel for positive intersectoral externalities through the development of efficient and internationally competitive management, training of skilled workers, and the spillover consequences of scale expansion (Keesing (1967), and Feder (1983)). Second, rising exports relieve the foreign-exchange constraint. The importation of technologically-superior capital goods is enlarged by growing export receipts and higher flows of foreign credits and direct investment, which take into account the country's ability to repay out of export earnings. 3/

It is also reasonable to posit that an acceleration in the growth of real outlays on education and health would be associated with a higher value of the labor's learning potential, as would growth in real expenditures on social security, housing, and recreation. Finally, equations (5) and (6) are standard definitional relations involving  $N$  and  $k$ .

#### 1. Reduced model

The growth rate of the capital stock is derived by dividing equation (2) by  $K$ , using (1) and (6):

$$(dK/dt)/K = s(\theta, \cdot)f(k)/k - \delta(\mu) \quad (7)$$

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1/ Arrow's (1962) learning-by-doing model has a steady-state solution for the growth rate of output equal to  $(\lambda+n)/(1-\alpha)$ , wherein the technical change function is  $(dT/dt)/T = \alpha(dK/dt)/K + \lambda$ ,  $0 < \alpha < 1$ . Although steady-state growth is thus a multiple of  $\lambda+n$ , growth remains independent of  $s$  and  $\delta$ ; besides, this model has the property that  $\partial(g^*-n)/\partial n = \alpha/(1-\alpha) > 0$ , that is, an increase in population growth **raises** equilibrium rate of **per capita** growth! This proposition is rejected by the empirical finding reported in Section IV that an increase in the rate of population growth **depresses** the average growth rate of per capita output during 1975-86 in a sample of 36 developing countries from five regions.

2/ See the discussion on the production linkage summarized in Khan and Villanueva (1991). Edwards (1992) and Knight et al. (forthcoming) present evidence on the relationship between trade openness and economic growth.

3/ The transfer of efficient technologies and the availability of foreign exchange have featured prominently in recent experiences of rapid economic growth (Thirlwall (1979)).

$$(dK/dt)/K = s(\theta, .)f(k)/k - \delta(\mu) \quad (7)$$

The growth rate of efficient labor is derived by differentiating (5) with respect to time, using (1) and (3)-(6):

$$(dN/dt)/N = \alpha(\chi, \xi, \omega, .)k + n + \lambda \quad (8)$$

Differentiating (6) with respect to time and substituting (7) and (8), the growth rate of the capital-labor ratio  $k$  is thus equal to:

$$\begin{aligned} (dk/dt)/k &= (dK/dt)/K - (dN/dt)/N \\ &= s(\theta, .)f(k)/k - \alpha(\chi, \xi, \omega, .)k - [n + \lambda + \delta(\mu)] \end{aligned} \quad (9)$$

The reduced model, equation (9), is a single differential equation involving the variables  $(dk/dt)/k$  and  $k$  alone.

Per capita income,  $Y/L$ , grows according to:

$$(dY/dt)/Y - n = \alpha(\chi, \xi, \omega, .)k + \pi(k)(dk/dt)/k + \lambda \quad (10)$$

which is also a single-valued function of  $k$ . Here,  $\pi$  is the share of income going to capital; this share is in general a function of  $k$ . <sup>1/</sup> The equilibrium capital intensity,  $k^*$ , is the root of (9) equated to zero,

$$s(\theta, .)f(k^*)/k^* - \alpha(\chi, \xi, \omega, .)k^* - [n + \lambda + \delta(\mu)] = 0. \quad (11)$$

And the equilibrium growth rate of per capita income is given by:

$$[(dY/dt)/Y]^* - n = [(dK/dt)/K]^* - n = s(\theta, .)f(k^*)/k^* - [n + \delta(\mu)] \quad (12a)$$

$$= [(dN/dt)/N]^* - n = \alpha(\chi, \xi, \omega, .)k^* + \lambda \quad (12b)$$

## 2. Stability

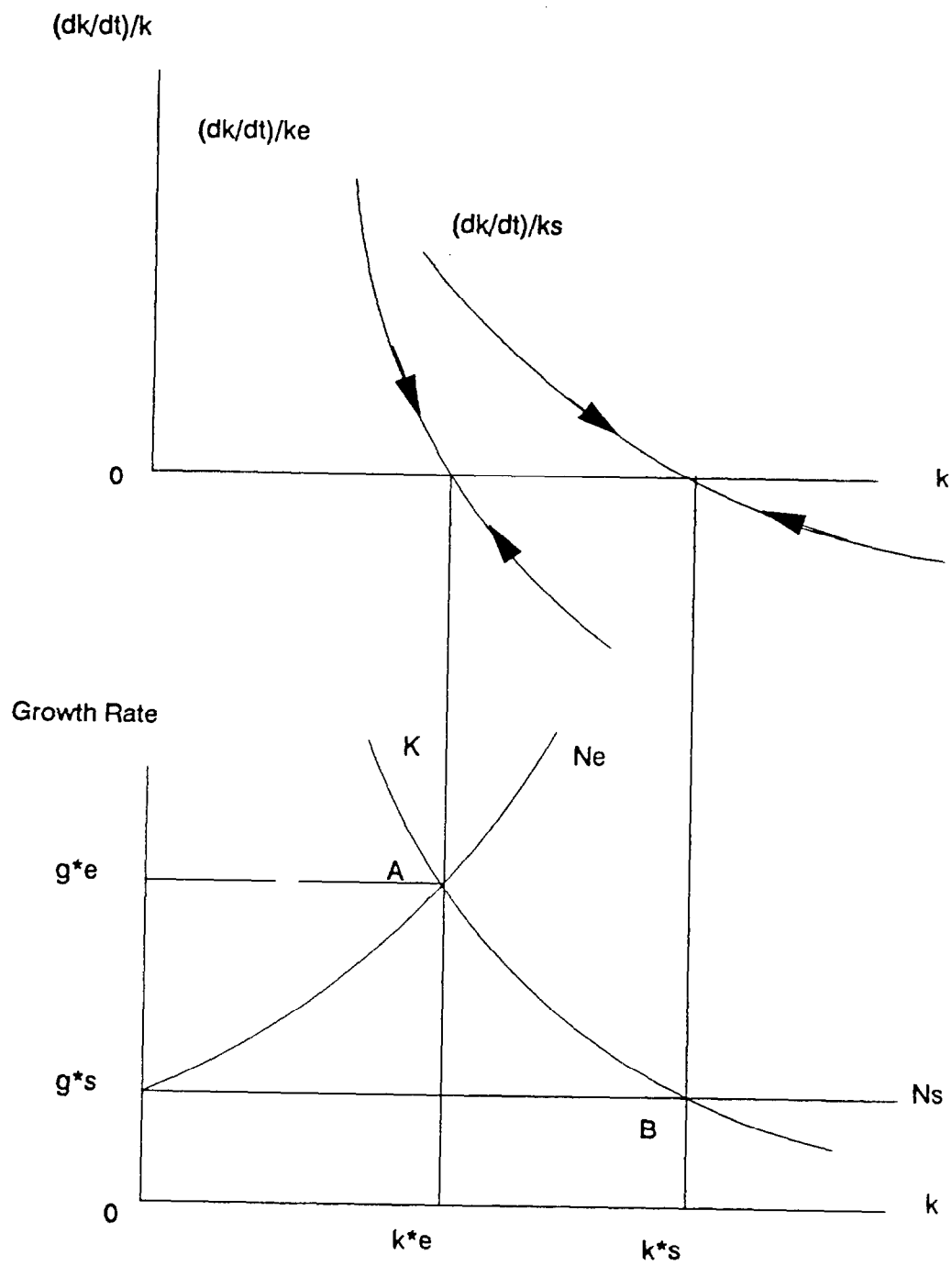
Given the Inada conditions on the production function, equations (7)-(9) graph as in Figure 1. The upper panel graphs equation (9), while the lower panel graphs equations (7) and (8). The downward slopes of the curves representing equations (7) and (9) and the upward slope of the curve representing equation (8) follow from the assumption of a positive but

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<sup>1/</sup> For a degree  $\beta$  homogeneous production function  $Y = F(K, N)$ ,  $\pi(k) = kf'(k)/\beta f(k)$ . The sign of  $\pi'(k)$  follows the sign of  $\epsilon(k) - 1$ , where  $\epsilon(k) = f'(k)[\beta f(k) - kf'(k)]/k[(\beta - 1)f'(k)^2 - \beta f(k)f''(k)]$  is the elasticity of substitution. If  $F$  is Cobb-Douglas,  $\pi(k) = \alpha$ , where  $\alpha$  is the constant exponent of  $K$ , and  $\epsilon(k) = 1$ . If  $F$  is CES,  $\pi(k) = 1/[1 + (1 - \alpha)(1/\alpha)k^{-\sigma}]$  and  $\epsilon(k) = 1/(1 - \sigma)$ . Notice that if  $\sigma = 0$ , CES reduces to Cobb-Douglas.



Figure 1. The EG and SS Models





diminishing marginal product of capital. The reasons why the  $(dk/dt)/k$  curve lies partly in the first quadrant and partly in the fourth quadrant in Figure 1 are given by the other Inada conditions, that is, for some initial values of the capital-labor ratio, it is possible for capital to grow either faster or slower than labor. It is obvious by inspection that, at any point on the  $(dk/dt)/k_e$  curve, the economy would move in the direction indicated by the arrows. Thus,  $k$  tends to settle at an equilibrium value  $k_e^*$ , which is globally stable. Points off  $k_e^*$  along the curve imply nonzero rates of change in  $k$ , and  $k$  will change toward  $k_e^*$ . For example, in Figure 1, points to the left of  $k_e^*$  imply positive values of  $(dk/dt)/k$ . This means that  $K$  is growing faster than  $N$ , and the ratio  $K/N$  will rise. The increase in  $k$  lowers the income-capital ratio and, hence, the saving- and investment-capital ratios. The growth of  $K$  slows. Meanwhile, a higher  $k$  induces an increase in labor-augmenting technical change through enhanced learning and experience. The growth of  $N$  is stimulated. This process would continue until the growth rates of  $K$  and  $N$  converge at the stationary value  $k_e^*$ . <sup>1/</sup> At this equilibrium point,  $K$  and  $N$  would grow at the same rate  $g_e^*$  and, by the constant returns assumption, output  $Y$  also would grow at this rate, given by equations (12a) and (12b).

### 3. Equilibrium capital intensity and growth in the EG and SS models

The SS and EG models are graphically portrayed in Figure 1. In the lower panel, the natural rate schedule,  $N_e$ , is upward sloping in the EG model, owing to the presence of learning-by-doing and the assumption of a positive marginal product of capital. The natural rate schedule in the SS model is shown as the horizontal line  $N_s$  with vertical height equal to a constant  $g_s^*$  ( $= \lambda + n$ ). The warranted rate schedule  $K$  is assumed to be identical in the two models.

In the upper panel, reflecting the different natural rate schedules, the capital accumulation schedules assume the shape and intersection (with the  $k$ -axis) indicated by the two curves, with  $(dk/dt)/k_s$  flatter and to the right of  $(dk/dt)/k_e$ . The equilibrium positions of the EG and SS models are indicated by the points A and B, respectively, in the lower panel. The growth rate of output is higher in the EG model, by the magnitude  $\alpha(.)f(k^*)$ , that is,  $g_e^* > g_s^*$ . The capital-labor ratio, however, is lower in the EG model ( $k_e^* < k_s^*$ ). The growth rate is higher because of induced learning-by-doing in the EG model. The EG model's capital intensity level is lower because of a higher level of the effective labor input.

### 4. Comparative dynamics

Table 1 summarizes the qualitative effects of changes in the structural parameters on the equilibrium capital intensity  $k^*$  and on the equilibrium per capita growth rate of income  $g^* \cdot n$ . Algebraically, the partial

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<sup>1/</sup> The opposite sequence of events is true for points to the right of  $k_e^*$ , implying negative values of  $(dk/dt)/k$ .

Table 1. The Effects of Changes in the Structural Parameters on the Equilibrium Values of Capital Intensity ( $k^*$ ) and the Per Capita Growth Rate ( $g^*-n$ ) in the EG and SS Models

An Increase In	EG		SS	
	$k^*$	$g^*-n$	$k^*$	$g^*-n$
Saving rate ( $s$ )	+	+	+	0
Ratio of foreign trade to GDP ( $\chi$ )	+	+	na	na
Growth in real spending on education & health ( $\xi$ )	-	+	na	na
Growth in real spending on social security, etc. ( $\omega$ )	-	+	na	na
Growth in real spending on operations & maintenance ( $\mu$ )	+	+	na	na
Ratio of fiscal deficits to GDP ( $\theta$ )	-	-	na	na
Population growth ( $n$ )	-	-	-	0
Exogenous technical change ( $\lambda$ )	-	+	-	+

Notes: + = increase; - = decrease; 0 = no change; na = not applicable.

Source: For EG model, equations (11), (12a)-(12b) of text. For SS model, same set of equations with  $\alpha$  set equal to zero.

derivatives of  $k^*$  and  $g^*$  with respect to any structural parameter may be obtained by differentiation of equations (11) and (12a)-(12b).

a. The short-run and long-run effects of an increase in the saving rate on economic growth in the EG and SS models 1/

The effects of an increase in the saving rate  $s$  on the transitional and equilibrium growth rate of output in the EG and SS models can be analyzed in detail with the aid of Figure 2, in which the initial equilibrium positions in the EG and SS models are indicated by points A and B, respectively. An increase in the saving rate shifts the warranted rate curve to  $K'$  in either model. The new equilibrium positions are indicated by points D in the EG model and C in the SS model. In both EG and SS models, the capital-labor ratio goes up, albeit the new ratio remains lower in the EG model (in relation to the new ratio in the SS model), owing to positive learning-by-doing. However, the new equilibrium growth rate increases in the EG model, but remains unchanged in the SS model. The discussion below traces the adjustment dynamics to the new growth equilibrium in the two models, as a result of an increase in the saving rate. The transitional dynamics of the SS model is taken up first, followed by that of the EG model.

During the transition between equilibrium points B and C, the rate of growth of output in the SS model is momentarily higher--by  $EB$ --than the natural rate  $g_s^*$  because of a higher warranted rate occasioned by a higher ratio of saving to income. 2/ The capital-labor ratio begins to rise, which slows the warranted rate. Since the natural rate is completely independent of the capital-labor ratio, only the warranted rate adjusts (downward) along the segment  $EC$ . Over time, labor becomes a bottleneck, and the growth rate slows (converges) to the constant natural rate  $g_s^* (= n + \lambda)$  at C. At this point, the capital-labor ratio stops rising and settles at a new and higher level  $k_s^*$ . The effect of an increase in the saving rate is thus to raise the equilibrium capital-labor ratio (from  $k_s^*$  to  $k_s^{*'}), 3/$

1/ The effects of a reduction in the rate of depreciation--exogenously in the SS model and endogenously in the EG model via a higher growth rate of real expenditures on operations and maintenance--are similar.

2/ The transitional growth rate of output,  $(dY/dt)/Y$ , is equal to  $\lambda + n + \pi(k)k/k$ , where  $\pi(k) = kf'(k)/f(k)$ . Now, both  $\pi(k)$  and  $k/k$  are positive anywhere between  $k_s^*$  and  $k_s^{*'}$ . It follows that  $(dY/dt)/Y > \lambda + n$  during the transition from B to C. At either B or C,  $\pi > 0$  and  $k/k = 0$ , so that  $(dY/dt)/Y = \lambda + n$  at either equilibrium point. The convergence property of neoclassical growth models, including both SS and EG models, can be demonstrated with the aid of Figure 2. As the initial capital intensity (or initial income per worker) moves farther to the left of  $k_s^{*'}$  (or  $k_e^{*'}$ ), i.e., gets smaller, the average growth rate of per capita income rises, i.e., the length of the line increases between C (or D) and any point on the  $K'$  curve corresponding to the initial level of capital intensity.

3/ And thus the equilibrium level of real income per efficient worker.

owing to a permanent upward shift of the warranted rate curve associated with an increase in the saving rate.

In the EG model, following an increase in the saving rate, equilibrium shifts from A to D. At the starting position A, capital would grow faster than labor (by FA), and the capital-labor ratio would rise (from  $k_e^*$  toward  $k_e^{*'}).$  As this happens, the marginal and average product of capital would fall, thus lowering the level of saving per unit of capital and slowing the warranted rate (downward along FD). On the other hand, the natural rate, instead of remaining constant as in the SS model, would accelerate (from A to D, along the  $N_e$  curve) because of a higher rate of labor-augmenting technical change associated with a rising capital-labor ratio. This process would continue until the warranted and natural rates are equalized--through a continuous increase in the capital-labor ratio--at the new equilibrium value  $k_e^{*'} at D, at which point the warranted rate would have fallen to the new and higher value of the natural rate, equal to the new and higher growth rate of output  $g_e^{*'} (> g_e^*).$$

b. The effects of changes in openness, expenditures on human development, and exogenous technical change

The effects of these factors can be analyzed with the help of Figure 3, which is similar in construction to Figure 2. Since many of these parameters are absent from the SS model, 1/ the illustrations refer only to the EG model. Changes in the ratio to GDP of foreign trade (sum of exports and imports) and growth in real outlays on education, health, and social security, housing, and recreation are reflected in changes in the learning coefficient  $\alpha$ , while changes in the exogenous rate of technical change  $\lambda$  enter the natural rate schedule directly.

An increase in any of these parameters shifts the capital accumulation schedule in the southwest direction (upper panel) and the natural rate schedule in the northwest direction (lower panel). With reference to Figure 3, the adjustment dynamics are the following. After the parametric increase, the rate of change in  $k$  is negative at the old equilibrium value  $k_0^*$ . This means that the natural rate is above the warranted rate, as shown in the lower panel. Thus, the level of capital intensity begins to fall toward  $k_1^*$ . As  $k$  falls, income per unit of capital rises, stimulating saving and investment, and the warranted rate goes up. At the same time, a lower stock of capital reduces the rate at which technological progress is taking place, depressing the natural rate. This process continues until the two rates meet at  $k_1^*$ , where the rate of change in  $k$  is, again, zero. The new equilibrium position is characterized by a lower level of capital intensity and a higher growth rate of per capita output and income.

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1/ Except for the exogenous rate of technical change  $\lambda$ , whose effects on capital intensity and per capita growth are similar in the two models.

Figure 2. Effects of an Increase in Saving Rate

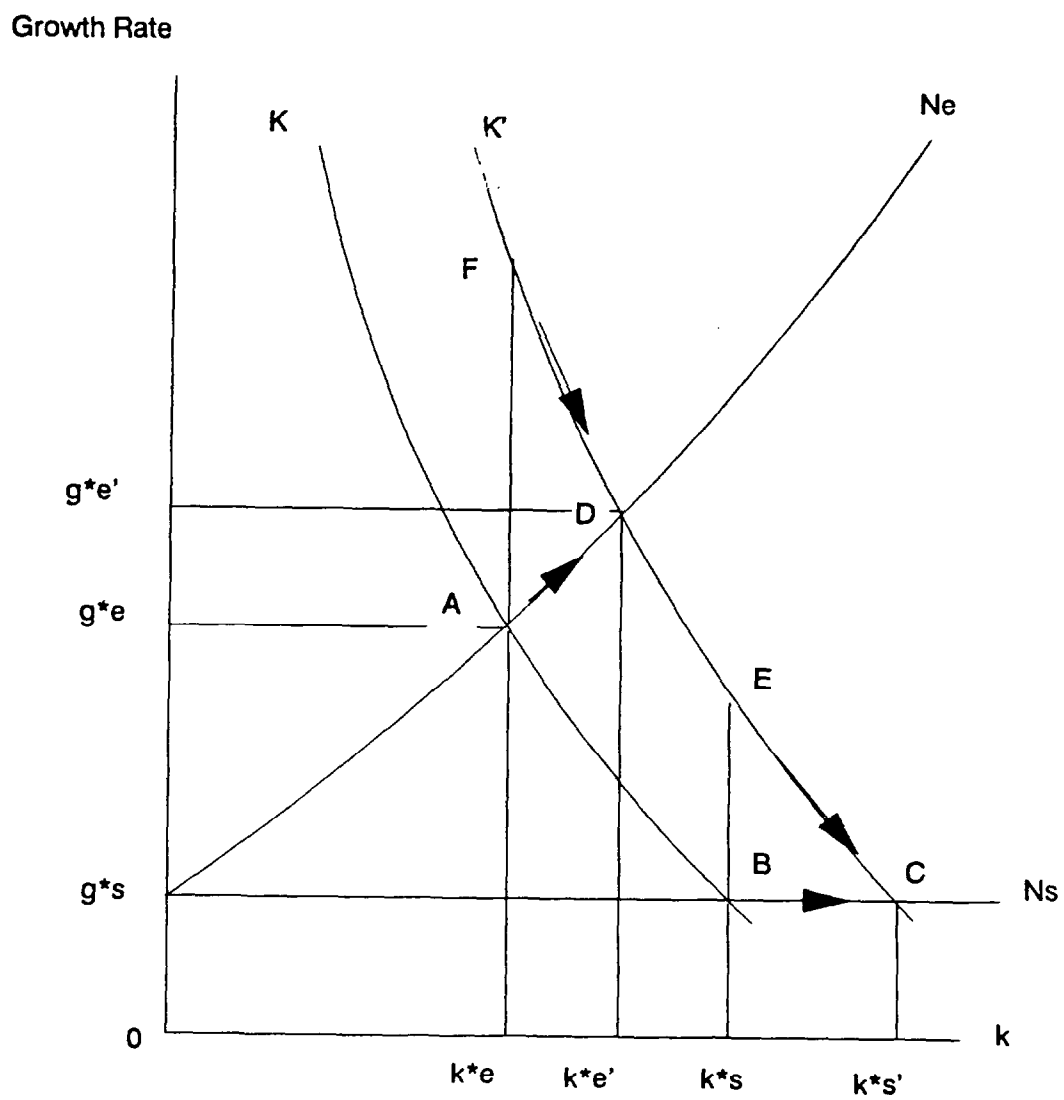
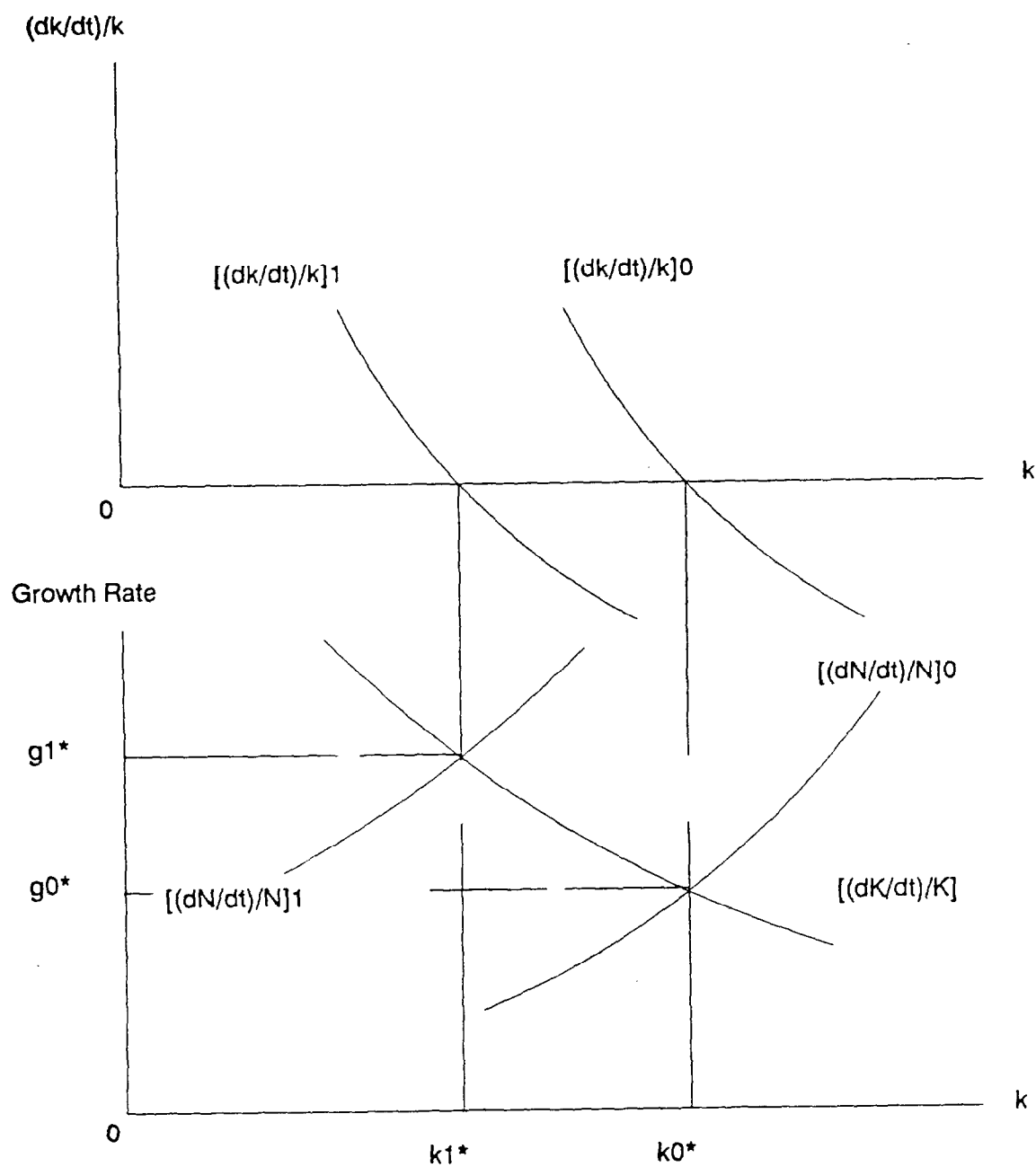






Figure 3. Effects of Increased Openness  
and Expenditures on Human Development





c. The effects of changes in the ratio of fiscal deficits to GDP and in the population growth rate

Finally, Figure 4 illustrates the effects of increases in the fiscal deficit ratio and in the rate of population growth on equilibrium capital intensity and on the growth rate of per capita output in the EG model. An increase in population growth or in the rate of government dissaving  $\frac{1}{\theta}$  (by lowering the investment rate) shifts the capital accumulation schedule in the southwest direction in both panels. At the old equilibrium capital intensity, the rate of change in  $k$  turns negative (upper panel), implying that the warranted rate falls short of the natural rate (lower panel). As  $k$  falls, income per unit of capital increases, raising saving and investment and, hence, the warranted rate. At the same time, the natural rate decreases, because a lower  $k$  induces a lower rate of learning. This process would continue until the economy settles at a new equilibrium position, characterized by a convergence of the warranted and natural rates, a lower level of capital intensity, and a slower growth rate of per capita income.

III. Optimal Long-Run Growth

Output per unit of effective labor in the long run is  $y^* = f(k^*)$ . If the level of  $y^*$  is considered a measure of the standard of living, and since  $f'(k^*) > 0$ , it is possible to raise living standards by increasing  $k^*$ . This can be done by adjusting the saving rate,  $s$ , via, for example, lowering the fiscal deficit parameter  $\theta$ . If consumption per unit of effective labor (or any monotonically increasing function of it) is taken as a measure of the social welfare of the society, the saving rate that will maximize social welfare by maximizing long-run consumption can be determined. Phelps (1966) refers to this path as the "Golden Rule of Accumulation."

Consumption per unit of effective labor is  $c = C/N = Y/N - S/N$ .  $Y/N$  is  $f(k)$  and  $S = I = dK/dt + \delta(\mu)K$ . Thus,  $c = f(k) - [(dK/dt + \delta(\mu)K)]/N = f(k) - k[(dK/dt)/K] - \delta(\mu)k$ . On the balanced growth path,  $(dK/dt)/K = \alpha(.)k^* + \lambda + n$ , where  $\alpha(.) = \alpha(\chi, \xi, \omega, .)$ . Thus:

$$c^* = f(k^*) - [\alpha(.)k^* + \lambda + n + \delta(\mu)]k^* \quad (13)$$

Maximizing  $c^*$  with respect to  $s$ ,

$$\partial c^*/\partial s = [f'(k^*) - 2\alpha(.)k^* - (\lambda + n + \delta(\mu))]\partial k^*/\partial s = 0 \quad (14)$$

Since  $\partial k^*/\partial s > 0$ , the Golden Rule condition is,

$$f'(k^*) - \delta(\mu) = g^*(k^*) + \alpha(.)k^* \quad (15)$$

---

$\frac{1}{\theta}$  As noted earlier, as the public sector dissaves less resources will be available to accumulate capital. Moreover, the ensuing large government borrowings from financial markets would tend to raise interest rates or lower available credit, adversely affecting private capital accumulation.

where  $g^*(k^*) = \alpha(.)k^* + \lambda + n$  is the equilibrium growth rate of output. The second-order condition for a maximum is satisfied, since

$$\partial^2 c^* / \partial s^2 = [f''(k^*) - 2\alpha(.)] \partial k^* / \partial s < 0. \quad (16)$$

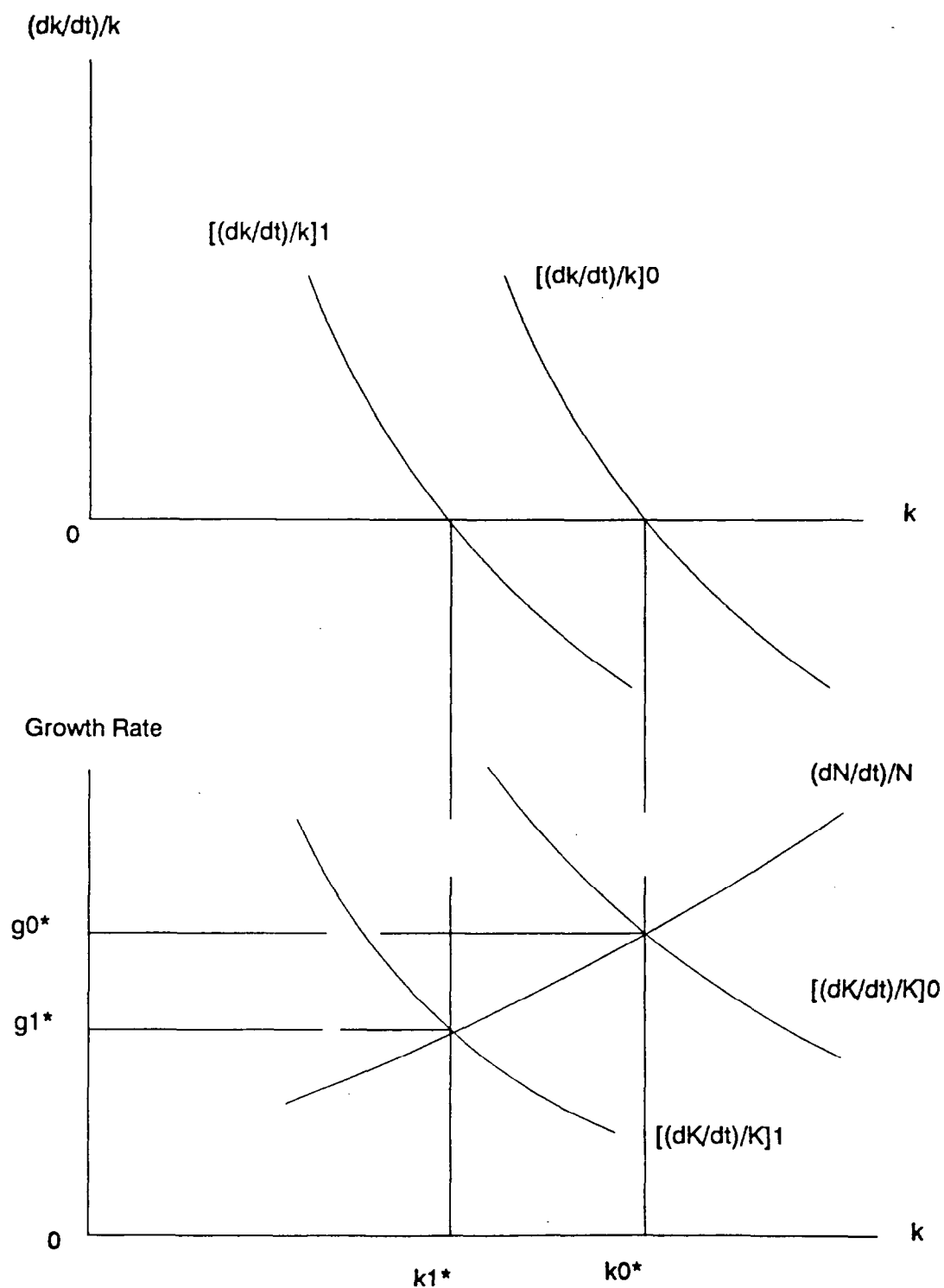
Equation (15) says that, for social welfare to be maximized, the saving-investment ratio should be raised up to a point where the net rate of return to capital (which is equal to capital's marginal product less depreciation) equals the long-run growth rate of output plus the product of the learning coefficient and the equilibrium capital intensity. The second term is nothing more than the endogenous component of labor-augmenting technical change--the component of  $(dT/dt)/T$  induced by learning and experience associated with a higher level of capital intensity, which, in turn, is caused by a higher saving rate. If there is no learning ( $\alpha = 0$ ), equation (15) reduces to  $f'(k^*) - \delta = \lambda + n$ , which is the familiar Golden Rule result from standard neoclassical growth theory. It is evident that the optimal net rate of return to capital should be higher than  $\lambda + n$  when  $\alpha > 0$ --when there is learning by doing--because of two factors. First, when the saving rate  $s$  is raised, the equilibrium growth  $g^*$  will be higher than  $\lambda + n$ , by the amount  $\alpha(.)\partial k^* / \partial s$ . Second, capital should be compensated for the effect on equilibrium output growth through the induced learning term  $\alpha(.)k^*$ .

An alternative interpretation of the above Golden Rule can be given. A standard neoclassical result is that the optimal saving rate  $s$  should be set equal to the income share of capital  $\pi$ . With endogenous learning-by-doing, the optimal saving rate should be set at a fraction of  $\pi$ , the fraction being equal to  $(g^* + \delta) / [g^* + \delta + \alpha(.)k^*]$ . <sup>1/</sup> Here,  $g^* + \delta + \alpha(.)k^* = f'(k^*)$ --given by equation (15)--is the (gross) social marginal product of capital, inclusive of the positive externalities via learning experience associated with capital accumulation in the EG model. Equivalently put,

---

<sup>1/</sup> Equations (11), (15) and the definition  $\pi = k^*f'(k^*)/f(k^*)$  are used to derive this result. When  $\alpha = 0$ , the proportionality factor assumes a value of unity, and the standard neoclassical result holds. In terms of the parametric values assumed in the simulations reported in Table 2 below (Section IV), when the learning coefficient  $\alpha$  is greater than zero, the optimal saving rate should be set at about three quarters of the assumed income share of capital  $\pi$ , or at 0.3 when  $\pi = 0.4$ . The simulations also show that the higher the learning coefficient, the lower the optimal saving rate as a proportion of capital's income share. According to the standard model, the optimal saving rate should always be set equal to  $\pi$ , which is at 0.4 in the numerical examples. The higher saving rate implied by the standard model owes to its neglect of endogenous growth and positive externalities through learning-by-doing associated with saving and capital accumulation. By contrast, in the EG model the economy benefits from such endogenous growth and positive externalities, so that a smaller saving-investment rate is all that is required (relative to the rate required by the standard model).

Figure 4. Effects of Increases in Ratio of Fiscal Deficits to GDP and in Population Growth





income going to capital as a share of total output should be a multiple of the amount saved and invested to compensate capital for the additional output generated by endogenous growth and induced learning. A value of  $\pi$  equal to  $s$ , implicit in the standard model, would undercompensate capital and thus would be suboptimal from society's point of view.

#### IV. The Speed of Adjustment Toward Equilibrium

The equilibrium results derived in the preceding section would not be relevant to the real world if the time period for the model to reach its equilibrium were unduly long. There are three approaches to the analysis of adjustment dynamics in the speed-of-approach literature:

- (i) Analytical approach, with less explicit results but without resorting to a full-scale numerical simulation;
- (ii) Simulation, such as the work of Sato (1963), where a specific functional form for the production function and representative values of the structural parameters are used, and adjustment paths from hypothetical disequilibria are calculated to obtain estimates of the time (in years) needed to reach equilibrium; and
- (iii) Empirical approach, where the model's equilibrium predictions are examined whether they accord with observed growth patterns of real economies over reasonably long periods.

##### 1. Analytical approach

The (negative) slope of the  $(dk/dt)/k_e$  curve (see Figure 1) at the equilibrium capital intensity  $k_e^*$  is a measure of the local adjustment speed. The steeper the slope, the faster the steady state  $k_e^*$  is reached. The absolute value (a.v.) of the slope of the above curve at  $k_e^*$  may be obtained by differentiating equation (9) with respect to  $k$  and evaluating at  $k_e^*$ :

$$V = \text{a.v. } [(d/dk)((dk/dt)/k)]^* = (n+\lambda+\delta+\alpha k_e^*)[(1-\pi(k_e^*))]/k_e^* + \alpha \quad (17)$$

The key feature of the EG model that distinguishes it from the SS model is the assumed presence of learning-by-doing, represented by a positive learning coefficient  $\alpha$ . In the absence of learning through experience ( $\alpha = 0$ ), (17) reduces to the SS expression. It is obvious by inspection of (17) that, with  $\alpha > 0$ , the slope of the  $(dk/dt)/k_e$  curve is steeper than the slope of the  $(dk/dt)/k_s$  curve (when  $\alpha = 0$ ); see Figure 1. Thus, the EG model takes lesser time to reach equilibrium, compared with the SS model. Moreover, it can be shown that enhanced learning--represented by an increase in  $\alpha$ --would reduce further the adjustment time, provided that the elasticity of substitution is not less than one, such as when the production function is CES. This can be seen by differentiating (17) with respect to  $\alpha$ , which yields:

$$((1-k_e^*)\alpha - (n+\lambda+\delta))(1-\pi(k_e^*)) - \pi'(k_e^*)(n+\lambda+\delta+\alpha k_e^*)(1/k_e^*)(\partial k_e^*/\partial \alpha),$$

which is positive, if the production function is CES (in which case,  $\pi'(k_e^*) \geq 0$ ), and if  $k_e^* \geq 1$  (in words, the equilibrium capital per effective worker is not less than a unit of the currency). It has been shown earlier that  $\partial k_e^*/\partial \alpha < 0$ . The simulations using a Cobb-Douglas production function (a special case of CES) reported in Table 2 below show the same results.

The above results can be given an intuitive interpretation. It has been shown that the equilibrium growth rate of output is  $((dN/dt)/N)^* = ((dK/dt)/K)^*$ . Both the natural and warranted rates adjust endogenously to changes in capital intensity. With the brunt of adjustment toward equilibrium being shared by changes in the natural rate, the time needed to reach equilibrium is much less in the EG model (see earlier discussion of Figure 1). In sharp contrast, the time required to reach equilibrium is much longer in the SS model because the adjustment burden is borne entirely by changes in the warranted rate.

## 2. Simulation

The reduced model, equation (9) is:

$$(dk/dt)/k = sf(k)/k - \alpha k - (n+\lambda+\delta)$$

Assuming a Cobb-Douglas form for  $f(k) = k^a$ , where  $0 < a < 1$  is the exponent of the capital stock (in this particular case, also equal to capital's share in income  $\pi$ , which is constant and independent of  $k$ ), the reduced model becomes:

$$dk/dt = sk^a - \alpha k^2 - (n+\lambda+\delta)k = g(k). \quad (18)$$

The solution to this differential equation is complicated because it is a nonlinear function. However, a linear approximation is possible in the neighborhood of the steady-state constant value  $k^*$ : 1/

$$\begin{aligned} dk/dt &= g(k^*) + g'(k^*)(k - k^*) \\ &= [ask^{*a-1} - 2\alpha k^* - (n+\lambda+\delta)](k - k^*), \end{aligned}$$

since  $g(k^*) = 0$ .

---

1/ The constant  $k^*$  is the unique root of (18) equated to zero:  $sk^{*a} - \alpha k^{*2} - (n+\lambda+\delta)k^* = 0$ . Given  $s = 0.2$ ,  $a = 0.4$ ,  $\alpha = 0.01$ ,  $n = 0.025$ ,  $\lambda = 0.005$ , and  $\delta = 0.04$ ,  $k^*$  assumes the value of 3.00, and the balanced growth path is equal to an annual rate of 0.06. If  $\alpha = 0$ , as in the SS model, and assuming the other parameters unchanged,  $k^*$  solves to a higher level at 5.75, and balanced growth to a lower rate of 0.03 per annum.



Table 2. Adjustment Time Estimates (in Years)  
as  $y_t$  Approaches a Limit of  $y_\infty$  1/  
( $\alpha = 0$ )

$P_t$	$y_0 - y_\infty > 0$		$y_0 - y_\infty < 0$	
	$y_0 = 0.045$	$y_0 = 0.035$	$y_0 = 0.015$	$y_0 = 0.025$
0.25	33.8	9.3	3.9	5.4
0.50	55.7	21.1	10.1	13.6
0.75	80.3	39.6	22.6	28.5
0.90	105.9	62.6	41.7	49.3

1/ With  $a = 0.4$ ,  $\delta = 0.04$ ,  $\lambda = 0.005$ ,  $n = 0.025$ ,  $s = 0.2$ . With these parametric values,  $k^* = 5.75$  and  $y_\infty = 0.03$ .

Adjustment Time Estimates (in Years)  
as  $y_t$  Approaches a Limit of  $y_\infty$  1/  
( $\alpha = 0.01$ )

$P_t$	$y_0 - y_\infty > 0$		$y_0 - y_\infty < 0$	
	$y_0 = 0.08$	$y_0 = 0.07$	$y_0 = 0.01$	$y_0 = 0.05$
0.25	6.4	4.3	1.5	2.6
0.50	13.5	9.8	3.9	6.5
0.75	23.3	18.6	9.1	13.6
0.90	34.7	29.4	17.5	23.5

1/ With  $a = 0.4$ ,  $\delta = 0.04$ ,  $\lambda = 0.005$ ,  $n = 0.025$ ,  $s = 0.2$ . With these parametric values,  $k^* = 3.00$  and  $y_\infty = 0.06$ .

Table 2 (Concluded). Adjustment Time Estimates (in Years)  
as  $y_t$  Approaches a Limit of  $y_\infty$  1/  
( $\alpha = 0.02$ )

$P_t$	$y_0 - y_\infty > 0$		$y_0 - y_\infty < 0$	
	$y_0 = 0.08$	$y_0 = 0.07$	$y_0 = 0.01$	$y_0 = 0.05$
0.25	3.8	3.0	1.3	2.1
0.50	8.4	6.9	3.3	5.1
0.75	15.3	13.2	7.6	10.5
0.90	23.6	21.2	14.2	18.0

1/ With  $a = 0.4$ ,  $\delta = 0.04$ ,  $\lambda = 0.005$ ,  $n = 0.025$ ,  $s = 0.2$ . With these parametric values,  $k^* = 2.40$  and  $y_\infty = 0.078$ .

Or,

$$dk/dt = A(k - k^*), \quad (19)$$

where  $A = ask^{a-1} - 2\alpha k^* - (n+\lambda+\delta) < 0$ . 1/

Equation (19) is of a "variables separable" form, which can be separated as:

$$[1/(k-k^*)]dx = A dt. \quad (20)$$

Integrating both sides,

$$\int [1/(k-k^*)]dx = At + \text{constant},$$

$$\log(k-k^*) = At + \text{constant},$$

$$k - k^* = \text{constant } e^{At},$$

$$k = k^* + Ce^{At}, \quad (21)$$

where C is a constant of integration. 2/

Substituting (21) into (19)--

$$(dk/dt)/k = A[1-(k^*/(k^*+Ce^{At}))] \quad (22)$$

Now, from (10), the growth rate of output is given by:

$$(dY/dt)/Y = y_t = a(dk/dt)/k + y_\infty, \quad (23)$$

where:

$$y_\infty = \alpha k^* + \lambda + n. \quad (24)$$

Substituting (22) and (24) into (23)--

$$y_t = aA[1-(k^*/(k^*+Ce^{At}))] + y_\infty, \quad (25)$$

Setting  $y_t = y_0$  and  $t = 0$  in (25),

$$y_0 = aA[1-(k^*/(k^*+C))] + y_\infty, \quad (26)$$

1/ As mentioned in the preceding footnote, for values of the parameters and of  $k^*$  assumed therein, a particular value for A equal to -0.0886 is obtained for  $\alpha = 0.01$ .

2/ Note that as t goes to infinity, the second term on the right-hand side of (21) goes to zero (since  $A < 0$ ), and thus k approaches  $k^*$ .

which can be solved for the constant C,

$$C = (y_0 - y_\infty)k^*/(y_\infty - y_0 + aA) \quad (27)$$

Substituting (27) into (25),

$$y_t = aA[1 - (k^*/(k^* + ((y_0 - y_\infty)k^*/(y_\infty - y_0 + aA))e^{At}))] + y_\infty, \quad (28)$$

Next, define the adjustment ratio  $p_t$  as:

$$p_t = (y_t - y_0)/(y_\infty - y_0) \quad (29)$$

Substituting (28) into (29) solves for the time  $t$  (in years) required to get a fraction  $p_t$  of the way from  $y_0$  to  $y_\infty$ , from which Table 2 is computed:

$$t = (1/A) \ln[(1 - p_t)(y_\infty - y_0 + aA)/((1 - p_t)(y_\infty - y_0) + aA)] \quad (30)$$

where  $\ln$  is the natural logarithm operator.

Table 2 reveals that the adjustment times in an endogenous growth model are generally only about a quarter or a third of those in an exogenous growth model, depending on the value of the learning coefficient  $\alpha$ . For example, whereas an exogenous growth model ( $\alpha = 0$ ) takes from 42 to 106 years for equilibrium growth to be nearly reached, an endogenous growth model ( $\alpha > 0$ ) takes anywhere from 14 to 35 years to achieve 90 percent adjustment to the steady state growth path, depending on the learning coefficient  $\alpha$  (Table 2 alternately uses values of 0, 0.01, and 0.02 for  $\alpha$ ).

Table 2 also illustrates the effects of an increase in the learning coefficient from 0.01 to 0.02: the equilibrium capital intensity falls from 3.00 to 2.40 and equilibrium growth rises from 6 to 7.8 percent annually; moreover, adjustment times are reduced by 30-50 percent. 1/

### 3. Empirical approach

The model's predictions about the per capita output growth and capital stock trends, which have been summarized in Table 1, are reproduced below, where the directional impact is given by the sign above each argument inside the two functions.

$$g^{*-n} = \psi(s, \theta, \chi, \xi, \omega, \mu, n, \lambda) \quad (31)$$

$$k^* = \phi(s, \theta, \chi, \xi, \omega, \mu, n, \lambda) \quad (32)$$

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1/ These simulation results are confirmed by the qualitative analysis of the EG model in Section II, summarized in Table 1.

Equations (31) and (32) are in general nonlinear functions. Without the fiscal deficit variable  $\theta$ , a linear approximation to (31) and (32) can produce coefficient estimates of arbitrary magnitude and significance. For example, suppose that growth rates initially rise and then fall as the growth of government expenditures continuously increases, with attendant heavy financing burdens, measured by rising values of  $\theta$ . In this case, positive coefficients of government expenditures will be obtained for linear regressions using data with low  $\theta$ , negative coefficients for those that rely on high  $\theta$ , and coefficients biased toward zero for linear regressions using both low and high  $\theta$ . The EG model developed in Section II and the linear regression results reported below thus include the ratio  $\theta$  of government deficits to GDP.

No data for  $k^*$  exist in developing countries, so that equation (32) cannot be estimated. However, since there are data on  $g^{*-n}$ , equation (31) can be tested. In general, the average per capita growth rate,  $g^{*-n}$ , is inversely related to the starting value of per capita real income,  $y_0$ --the familiar convergence property of neoclassical growth models (including the present one). <sup>1/</sup> Thus, for empirical testing, the following linear specification can be considered:

$$g^{*-n} = a_0 + a_1s + a_2\chi + a_3\xi + a_4\omega + a_5\theta + a_6n + a_7y_0 + a_8\lambda + a_9\mu \quad (33)$$

Of the nine explanatory variables in equation (33), data on only the last two are unavailable. Recall that  $\mu$  is the real growth of expenditures on operations and maintenance of capital assets, while  $\lambda$  is the exogenous rate of labor-augmenting technological progress. The parameter  $\lambda$  can be interpreted as capturing all the unobserved country-specific factors that raise labor productivity--cultural, social, ethnic, political, religious. Regional dummy variables will be included to reflect such factors. The unobserved series  $\mu$  is assumed to enter the error term in a well-behaved manner. For present purposes, the following multiple regression can be estimated:

$$g^{*-n} = a_0 + a_1s + a_2\chi + a_3\xi + a_4\omega + a_5\theta + a_6n + a_7y_0 + a_8\text{dummy} \quad (34)$$

The EG model's equilibrium predictions (where the learning coefficient  $\alpha > 0$ ) are that  $a_1, a_2, a_3, a_4 > 0$ , and  $a_5, a_6, a_7 < 0$ . The SS model (where  $\alpha = 0$ ) predicts that  $a_1 = a_2 = a_3 = a_4 = a_5 = a_6 = 0$ , and  $a_7 < 0$ . The data set consists of annual averages of observations over the period 1975-86 for 36 developing countries from 5 regions (see the Appendix for the data, sources and definitions, and list of countries in the sample).

The regression results are reported below, where the insignificant coefficients of the regional dummy variables are suppressed (t-values are in parentheses):

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<sup>1/</sup> See footnote 2 on page 9.

$$\begin{aligned}
 g^*-n &= 0.01 + 0.183s + 0.038\chi + 0.093\xi + 0.063\omega - 0.189\theta \\
 &\quad (0.50) \quad (3.06) \quad (2.43) \quad (1.91) \quad (1.54) \quad (2.39) \\
 &\quad - 0.665n - 0.000015y_0 \\
 &\quad (1.90) \quad (2.59)
 \end{aligned} \tag{35}$$

$$R^2 = 0.7952; \text{ SEE} = 0.0144$$

An  $R^2$  of close to 0.8 is relatively high for a cross-country regression. <sup>1/</sup> All the regression coefficients have the expected signs. The coefficients for the saving rate, ratio of foreign trade to GDP, the ratio of fiscal deficits to GDP, and the initial level of per capita income are statistically significant at the 5 percent level or better. The coefficients for the growth of real expenditures on education and health and for the rate of population growth are statistically significant at the 10 percent level or better. The coefficient for the growth of real expenditures on social security, housing, and recreation is marginally significant.

Since  $\theta$  (government dissaving) is a part of total  $s$ , a discussion of the coefficients of  $s$  and  $\theta$  in the above regression would be useful. The EG model divides the total long-run impact of changes in  $s$  on  $g^*-n$  into two components: (i) an element arising from changes in the private saving rate induced by changes in its determinants other than changes in  $\theta$ ; and (ii) a composite factor stemming from changes in  $s$  directly as a result of changes in  $\theta$  and indirectly via induced changes in the private saving rate. Component (i) is measured by the coefficient of  $s$  in the above regression equation, while component (ii) is captured by the coefficient of  $\theta$  in the same regression. Since the estimates of these two coefficients are nearly identical (with opposite signs), the results suggest a symmetric response of  $g^*-n$ , in opposite direction, either to a change in the private saving rate or to a change in the rate of government dissaving.

The empirical results clearly show that the following factors promote per capita economic growth: steady increases in saving-investment rates, in the ratio of foreign trade (exports plus imports) to GDP, and in the growth of real expenditures on education and health. On the other hand, rapid population growth rates and high ratios of fiscal deficits to GDP are followed by slow average growth of per capita output. There is also empirical support for the convergence property of the EG and SS models--the significant negative relationship between the initial level of per capita real income and subsequent average growth.

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<sup>1/</sup> Ramanathan (1982) notes that typical values of  $R^2$  for equations estimating the growth performance in developing countries using cross-country data fall in the range 0.3-0.4.

## V. Conclusions

This paper has presented a simple neoclassical growth model with endogenous technical change (EG model) and contrasted its equilibrium properties with those of the more standard SS model. It is found that, contrary to the predictions of the SS model, the equilibrium growth rate of per capita output is influenced in a systematic way by changes in the private rates of saving and depreciation, population growth, and in public policies with regard to opening up of the economy (trade liberalization), fiscal deficits, spending for human resource development (the growth of real expenditures on education and health), and net investment (public capital formation and real expenditures on operations and maintenance of existing capital assets).

In the absence of learning by doing, the model's optimal net rate of return to capital is equal to the sum of the population growth  $n$  and the exogenous rate of labor-augmenting technical change  $\lambda$ , or that the optimal saving rate should be set equal to the share of capital in aggregate output --these are familiar Golden Rule theorems from standard optimal growth theory. With learning-by-doing, these standard Golden Rule results are revised: The optimal net rate of return to capital is higher than  $n + \lambda$ , or alternatively, the optimal saving rate should be set at only a fraction of capital's income share, because of endogenous growth and the induced learning associated with increases in the capital stock.

The analytic and simulation results appear to favor the EG over the SS model. Simulations show that the speed of adjustment toward equilibrium is substantially faster in a model of endogenous growth. Moreover, an increase in learning-by-doing further reduces the adjustment time. The empirical results also validate the EG model, particularly those relating to the positive per capita growth effects of public policies for greater openness of the trading system, high saving rates, and rapid growth in expenditures on human development, and those relating to the negative per capita growth effects of rapid population growth and high ratios of fiscal deficits to GDP. Finally, the convergence property of the EG model has been confirmed (as has the convergence of the SS model). However, the result on the saving rate-growth relationship is tenuous, in view of the short time interval (12 years) of the sample. Since the realized growth dynamics in the SS model over this relatively short period would also show a positive relationship, the empirical results would hardly invalidate the SS approach, pending additional research. Efforts are currently underway to use the very long time series (from 1950 to 1985) from Summers and Heston (1988) in testing the equilibrium relationships among the growth rates of per capita real income, saving rates, population growth rates, and the growth and size of government. The 36 years spanned by this data set would meet the adjustment time estimates of 14 to 35 years for equilibrium growth to be reached (but not the adjustment time estimates of 42 to 106 years in a model without endogenous learning).

The policy implications are straightforward. Public policies that raise the capital-labor ratio have magnified effects on the growth rate of per capita income, owing to induced learning-by-doing associated with a rising capital stock. Policies that enhance the learning process also accelerate the speed of adjustment toward the balanced growth path. Examples of such policies include measures to raise saving and investment, permit the steady expansion of the tradable sector, and accelerate the growth of real expenditures on education and health. On the other hand, there are clear limits to the size of government in relation to GDP, because of the increasingly heavy costs of burgeoning deficits.



Data Used in the Study

APPENDIX

A. Sources and Definitions

The data, except for foreign trade flows, are drawn from Orsmond (1990), which are based on the IMF's Government Financial Statistics and International Financial Statistics. Foreign trade flows are taken from the World Economic Outlook database. The sample consists of observations averaged over the period 1975 through 1986 for 36 developing countries.

PYG : Real per capita GDP growth rate, annual average;  
KY : Gross investment divided by nominal GDP, annual average;  
XC : Change in ratio of sum of nominal exports and imports to nominal GDP between 1975 and 1986;  
EG : Growth rate of government expenditures on education and health, annual average, deflated by GDP deflator for budget year;  
SG : Growth rate of government expenditures on social security, housing, and recreation, annual average, deflated by GDP deflator for budget year;  
DY : Nominal fiscal deficits divided by nominal GDP, annual average;  
PG : Population growth rate, annual average;  
GDP75 : Per capita income level in 1975 US dollars;  
DUM(i) : Dummy variable that assumes the value of 1 for region i, zero otherwise, i = AFRICA, ASIA, MIDDLE EAST, WESTERN HEMISPHERE.

B. List of Countries

The countries in the sample are:

Botswana	Mexico
Burkina Faso	Morocco
Cameroon	Myanmar
Chile	Nepal
Costa Rica	Pakistan
Dominican Republic	Panama
Egypt	Singapore
El Salvador	Sri Lanka
Ethiopia	Tanzania
Fiji	Thailand
Guatemala	Togo
Indonesia	Tunisia
Iran	Turkey
Kenya	Uruguay
Korea	Yemen Arab Republic
Liberia	Zambia
Mauritius	Zimbabwe

C. The Data

	PYG	KY	XC	EG	SG	DY	PG	GDP75
Botswana	7.4	29.8	3.5	17.5	38.7	-4.2	4.7	350.0
Korea	7.1	28.8	10.6	10.2	12.9	1.7	1.4	580.0
Singapore	5.7	40.3	35.6	12.8	12.8	-1.7	1.2	2540.0
Yemen Arab Rep.	3.7	27.9	-9.4	32.1	2.0	10.8	2.8	140.0
Pakistan	3.4	17.0	1.1	10.1	32.2	7.5	3.1	140.0
Cameroon	3.1	22.4	-7.6	0.0	13.3	0.5	3.0	310.0
Mali	3.9	24.1	11.1	9.0	6.1	4.1	2.1	360.0
Indonesia	4.0	24.4	-4.0	9.3	2.0	2.2	2.0	210.0
Paraguay	2.6	23.7	11.6	1.2	9.2	0.3	3.2	550.0
Myanmar	3.2	15.9	-0.3	7.0	10.0	-0.1	2.4	150.0
Sri Lanka	3.6	23.3	13.8	6.7	0.8	10.5	1.6	220.0
Tunisia	2.5	29.4	2.5	5.0	9.5	4.7	2.6	710.0
Kenya	0.8	20.9	-15.8	5.8	5.1	5.7	4.2	230.0
Panama	1.9	23.4	-26.7	4.1	9.1	7.6	2.6	1030.0
Mauritius	3.1	24.4	-3.1	7.7	-0.5	8.5	1.3	300.0
Burkina Faso	2.7	22.9	4.6	6.7	18.7	0.3	1.6	100.0
Egypt	1.8	25.8	25.2	9.4	0.3	13.1	2.5	310.0
Turkey	2.1	20.2	15.4	0.7	2.7	4.3	2.0	830.0
Chile	2.2	14.7	8.0	3.4	7.3	-0.3	1.7	860.0
Morocco	1.3	25.2	-4.3	5.3	6.5	10.8	2.5	500.0
Nepal	0.9	17.7	10.7	11.3	14.3	4.6	2.8	110.0
Mexico	0.9	21.1	5.2	4.0	0.0	6.6	2.6	1360.0
Ethiopia	-1.2	10.3	8.0	6.3	10.8	6.0	4.6	90.0
Dominican Republic	0.4	21.2	-8.9	2.0	1.6	2.1	2.8	670.0
Costa Rica	0.0	21.8	-38.9	1.5	9.5	3.3	3.1	950.0
Zimbabwe	-0.5	17.8	-29.8	10.9	5.0	7.9	2.9	570.0
Fiji	0.2	20.6	-14.3	4.8	13.0	3.8	2.0	1030.0
Guatemala	-0.4	15.3	-16.5	13.9	-1.2	2.7	2.5	570.0
Togo	-0.9	29.1	0.4	5.9	6.6	11.3	2.9	260.0
Tanzania	-1.8	18.5	-16.0	-5.6	3.8	8.8	3.6	160.0
Venezuela	-1.4	26.4	-17.4	2.7	2.6	0.7	3.1	2380.0
Uruguay	0.8	13.3	5.8	-1.8	3.8	2.6	0.6	1370.0
Iran, Islamic								
Republic of	-2.6	22.7	-58.9	3.5	10.1	5.8	3.8	1449.7
Zambia	-3.2	18.6	9.0	-3.4	-0.5	13.7	3.5	550.0
Liberia	-3.3	22.4	-29.2	6.1	13.7	7.8	3.3	410.0
El Salvador	-2.1	15.9	-21.1	-3.4	0.3	2.8	1.9	440.0

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