IMF WORKING PAPER

WP/93/48

INTERNATIONAL MONETARY FUND

Research Department

DEER Hunting: Misalignment, Debt Accumulation and Desired Equilibrium Exchange Rates

Prepared by Michael J. Artis* and Mark P. Taylor**

Authorized for Distribution by Peter B. Clark

June 1993

Abstract

This paper deals with hysteresis in the desired equilibrium exchange rate (DEER) arising from misalignment. When the actual real exchange rate departs from its DEER value, current account realizations--and consequently, debt service obligations--will differ from those assumed in the initial DEER calculation, necessitating its recomputation. The paper derives a formal expression for this hysteresis effect in the DEER, studies the convergence properties of a system in which the evolution of actual exchange rates depends on the DEER and provides illustrative calculations of its historical significance. Finally, the paper derives and applies rules of thumb for computing the hysteresis effect when considering the rate of approach of an exchange rate to its DEER value.

JEL Classification Numbers: F31

* University of Manchester and Centre for Economic Policy Research.
** International Monetary Fund and Centre for Economic Policy Research.
The Research reported in this paper was begun while Michael Artis was a Visiting Scholar in the Research Department of the Fund.

Contents

		Page
Summary		
I.	Introduction	1
II.	Debt Accumulation and Misalignment	3
III.	Catching a Moving Bus: The Dynamics of Adjustment	7
IV.	How Important Has Hysteresis Been	9
V.	Converging on the DEER: Rules of Thumb	11
VI.	Adjusting DEER Trajectories: How Good are the Rules of Thumb?	14
VII.	Conclusions	16
References		17

List of Tables

Table 1.	Percentage Deviation of Illustrative G-5 DEER Trajectories from Constant DEERs 1/	12
Table 2.	Movements in the DEER: Five-Year Convergence	14
Table 3.	Movements in the DEER: Ten-Year Convergency	15
Table 4.	Hypothetical Five-Year DEER Trajectories for the G5	17
Table 5.	Hypothetical Ten-Year DEER Trajectories for the G5	18

List of Figures

Figure 1.	Internal and External Balance	4a
Figure 2.	Hysteresis Effects	4b
Figure 3.	The Phase Portrait for Given Initial Misalignment	8a
Figure 4.	Illustrative DEER Trajectories for the G5	12a

Summary

This paper tackles the issue of hysteresis in the desired equilibrium exchange rate (DEER) arising from misalignment and changes in debt stocks. The DEER is a "steady state" concept, calculated as that (real) rate of exchange that will ensure external balance when the economy is operating at a full rate of utilization (internal balance). When the actual exchange rate is different--a so-called misalignment--the external balance realization will, in general, be different from that implicit in the DEER. This implies that the economy's net foreign asset stock will also be different from that implicit in the DEER, as will debt-service obligations. But different debt-service obligations require a different DEER, which must be recomputed. The dependence of the desired equilibrium exchange rate on the actual rate of exchange indicates that it is subject to hysteresis.

The first task of the paper is to formalize the presence of hysteresis effects in the DEER. This process is straightforward and indicates that a misalignment has an effect of opposite sign on the DEER. That is, if the actual exchange rate is depreciated relative to the DEER, the DEER value appreciates. This result suggests that the stability properties of a system in which the actual rate may respond to the desired equilibrium exchange rate should be examined. Such and examination yields some quite intuitive results with respect to the adjustment speed.

The presence of hysteresis effects in the DEER is not in doubt, but particular interest attaches to their empirical significance. Three relevant sets of calculations are performed in this respect. First, the analytical formalization derived below is used to examine the possible significance of hysteresis effects for the Group of Five during 1975-90. For the United States, particularly, a prolonged misalignment over this period indicates a sizable hysteresis effect. Second, rules of thumb are derived for computing the amount by which the desired equilibrium exchange rate will shift for a given initial misalignment and the length of the period of adjustment envisaged for the actual rate to converge on the desired rate. Finally, a 1990 set of DEERs for the Group of Five is used to compute by how much the DEERs would need to be adjusted if convergence were to take place over five or ten years, with account being taken at the same time of the adjustment of utilization rates over the same horizons. The results broadly confirm the rules of thumb and suggest that the extent to which the desired rate shifts, relative to the initial amount of misalignment, is not negligible.

I. Introduction

Our objective in this paper is to investigate the effects of hysteresis--operating through shifting international net asset stocks during periods of misalignment--on the desired equilibrium exchange rate. Although our discussion relies in large part on analytical arguments, our aim is to provide practical guidelines which are of use in assessing situations of exchange rate misalignment.

The concept of the equilibrium exchange rate is not unique. As noted by Frenkel and Goldstein (1986), there are at least three approaches to determining the equilibrium exchange rate--corresponding to structural exchange rate models such as the monetary model or the portfolio balance model of exchange rate determination, the purchasing power parity approach, and the "underlying balance" approach. 1/ In this paper, we are concerned with the underlying balance approach. According to this approach, the equilibrium exchange rate is defined as the real effective exchange rate that is consistent with medium-term internal and external macroeconomic balance. This definition is discussed further below.

The former of these two conditions can be identified with equilibrium employment and output, which would be typically taken to be the non-accelerating inflation rate unemployment level or NAIRU), the latter with current account balance or, in the presence of sustainable medium-term capital inflow or outflow, a corresponding deficit or surplus. In any case, the current account should incorporate the flow of debt service payments or receipts (and other investment income) corresponding to the underlying net foreign asset position of the economy. These definitions are explored further below.

The underlying balance approach to the equilibrium exchange rate was developed by Fund staff during the early 1970s (see International Monetary Fund (1984)). 2/ More recently, the equilibrium rate associated with underlying balance has been labeled the "fundamental equilibrium exchange rate" (FEER) (Williamson (1985)). The concept of "fundamental" equilibrium would be more applicable, however, to a long-term situation where all underlying economic forces had worked themselves out. Moreover, we wish to stress that the concept of the equilibrium real exchange rate consistent with underlying macroeconomic balance is essentially normative, as it is contingent upon a set of desired macroeconomic objectives. In this paper, therefore, we shall use the term "desired equilibrium exchange rate" (DEER) to refer to this concept. The DEER has been used as an analytical device by a number of authors to assess exchange rate misalignment (Williamson (1985, 1990), Barrell and Wren-Lewis (1984)), as well as in the context of discussions of "blueprints" for international policy coordination

---

1/ See MacDonald and Taylor (1992) for a recent survey relating to the first two of these approaches. Frenkel and Goldstein (1986) discuss the relative merits of the three approaches.

2/ See also Nurske (1945) and International Monetary Fund (1970) for precursors of this approach.

(Williamson and Miller (1987)), Frenkel and Goldstein (1986), Currie and Wren-Lewis (1989)), and in discussions of the "appropriate level" at which to join a pegged exchange rate system such as the European Monetary System (EMS) (Wren-Lewis, et al. (1991)).

This paper addresses an issue concerning DEER computations which, although it has not been entirely overlooked in the literature, has been given relatively little attention, namely: How sensitive is the DEER to the path chosen for convergence towards it? A given DEER trajectory has associated with it a particular path for the current account and debt service flows; a deviation of the actual rate from the DEER immediately implies a different current account and correspondingly a change in debt service flows compared to the original. The level of the real exchange rate consistent with medium-term external balance must therefore change. Thus, the final DEER arrived at will not be independent of the path chosen towards it.

Where equilibrium values turn out to be dependent on the dynamic path of adjustment, the situation is generally termed one of "hysteresis" (Cross (1992)). Contemporary economic analysis is turning up a number of instances in which it appears that hysteresis effects may be important, for example, in the determination of the NAIRU (Lindbeck and Snower (1986)), and in the analysis of trade responses to exchange rate variation, taking account of the sunk costs of setting up in overseas markets (Baldwin and Krugman (1986)). For economists trained in the neoclassical tradition, hysteresis is a troublesome feature since its presence renders inapplicable the method of comparative statics upon which much of modern economic analysis is founded. 1/ The implicit use of something at least approximating the comparative static method is common in analyses involving the DEER. For example, enumerating the attractions of the DEER apparatus, Wren-Lewis (1991), notes: "First and foremost, it provides a fixed point for a variable that is notoriously difficult to forecast and analyze empirically." 2/

In the remainder of this paper we evaluate the importance of hysteresis effects on the DEER as they arise through the debt service consequences of misalignment. 3/ That there are hysteresis effects of this kind is not in dispute; our aim is to provide a sense of their empirical significance. In particular, we provide broad rules of thumb by which to judge the importance

---

1/ The locus classicus on the method of comparative statics is Samuelson (1947). Cuthbertson and Taylor (1987, chapter 1) provide a textbook discussion.

2/ It should be noted, however, that in the same paper, Wren-Lewis critically analyzes many of the assumptions of the DEER approach and explicitly mentions the hysteresis effects which are the topic of this paper (page 83).

3/ Note that we specifically do not address hysteresis effects that might arise in other ways, e.g., through shifts in the NAIRU or on account of the presence of set-up costs in international trade.

of such effects for any given degree of perceived exchange rate misalignment and desired period of monotonic adjustment towards the DEER.

We begin in Section II by discussing the issue of hysteresis in the DEER in more detail and by showing analytically that the DEER is not independent of the path chosen to achieve it. In Section III we derive the full set of dynamic solutions to the problem when the real exchange rate adjusts by a proportion of its deviation from the DEER. In particular, we derive restrictions--which turn out to be quite intuitive--on the partial adjustment process, which are necessary in order to ensure eventual convergence on the DEER. In Section IV we attempt to gauge how important hysteresis effects have been by examining the historical path of real exchange rates for the G-5 countries over an eleven-year period. In Section V we derive our rules of thumb for assessing the importance of hysteresis effects for a given path of adjustment and set of initial conditions. In Section VI we take--for purely illustrative purposes--estimates of the DEER for each of the G-5 countries in 1990 provided by Williamson (1990), and analyze the importance of hysteresis effects assuming that the implied degree of misalignment is reduced to zero over either a five- or a ten-year period. A final section concludes. 1/

II. Debt Accumulation and Misalignment

The DEER is defined as that value of the real exchange rate that will reconcile internal and external equilibrium "in the medium term." As already explained, external equilibrium is defined in terms of a desired value of the current account balance; this value may be non-zero if there is a presumption about a "normal" rate of capital inflow or outflow. Otherwise a zero value seems a plausible objective, corresponding as it does to a constant level of net foreign assets. Internal balance is usually defined as potential full employment output; in most computations (e.g., those of Williamson (1985, 1990)) this appears to be assumed to be independently computed and to be independent of the real exchange rate itself and we shall follow that tradition here. This is illustrated in Figure 1, the NAIRU schedule is drawn as a vertical line in real exchange rate (S) and utilization (u) space. In a very open economy, the NAIRU may be modelled as a function of the real exchange rate. An appreciation of the real exchange rate allows a higher real wage (markup of wages over prices) for a given rate of utilization, since holding the current account constant in volume terms, the improvement in the terms of trade implies an increase in real revenues which may be passed on to workers. This view, which is represented analytically in the Layard-Nickell (1985) "Battle of the Markups" view of

---

1/ This paper is not concerned with estimating desired equilibrium exchange rates, but rather with illustrating how such estimates must be adjusted in order to allow for movements in net asset stocks arising from misalignment. Any actual numbers assigned to DEERS in the paper are for purely illustrative purposes, and should therefore be treated as strictly hypothetical.

inflation is reflected in the econometric modelling work of Wren-Lewis et al (1991). In terms of the diagram, the NAIRU schedule would be positively sloped from left to right. The current account (CA) schedule is drawn for a given level of the current account balance and slopes down from left to right for well-known (net import propensity) reasons: as utilization increases, (net) imports tend to rise, requiring a devaluation of the real exchange rate as an offset. The solution for S*, u* gives the DEER and the internal balance or utilization rate. 1/

For given assumed values of the target current account and internal balance, then, the corresponding "fundamental equilibrium exchange rate" (DEER) can be computed. It should be clear that the DEER needs to be computed as a trajectory if there is evolution in the values of the current account and internal balance targets, (or predictable change in the structure of the economy for given values of those targets), even if we set on one side all issues of hysteresis. (An example of a step change in an otherwise flat DEER trajectory is introduced below in Section IV.)

It is easy to see from considering Figure 1 how hysteresis in the DEER can arise. Suppose that the actual exchange rate happens to correspond initially to its DEER value and that internal balance is at the optimal level--in other words, that there is no problem of the starting point or transition period. In terms of the figure, we are u*, S*. Now suppose that in the next period the actual real exchange rate departs from its DEER value--specifically that it appreciates--whilst utilization remains at u*. The appreciation causes the current account to deteriorate relative to the initial equilibrium target positions, which is assumed to be zero. Then the DEER calculation must be performed afresh. The deficit increases net foreign indebtedness and creates an obligation to service debt interest. Even disregarding any desire to rectify the increase in indebtedness, the obligation to service more debt must cause the CA schedule to move in to the left: the real exchange rate which would have been consistent with the current account target in the absence of the increased debt service will now produce a deficit due to the increased debt-service obligation. More precisely, the trade account target has changed to provide a surplus sufficient to cover the increased debt service obligation. The current account target remains the same since debt service is incorporated in it, but the "structure" of the economy has changed, forcing CA to shift inward.

The departure of the actual exchange rate from its DEER value (trajectory) thus forces a revision of the DEER. A "hysteresis loop" (Cross (1992)) would ensue if the previous DEER were to be re-established. The exchange rate would need to "overdepreciate" in order to reinstate the previous schedule (Figure 2). A displacement of the actual real rate of

---

1/ In some computations, a desired value of the fiscal deficit is also involved, but the assumption here is that there is no additional effect of fiscal policy to be allowed for on top of its effect on u--i.e., that changes in fiscal policy will not shift the schedules. To a good first approximation, this seems a reasonable assumption.

Figure 1: Internal and External Balance

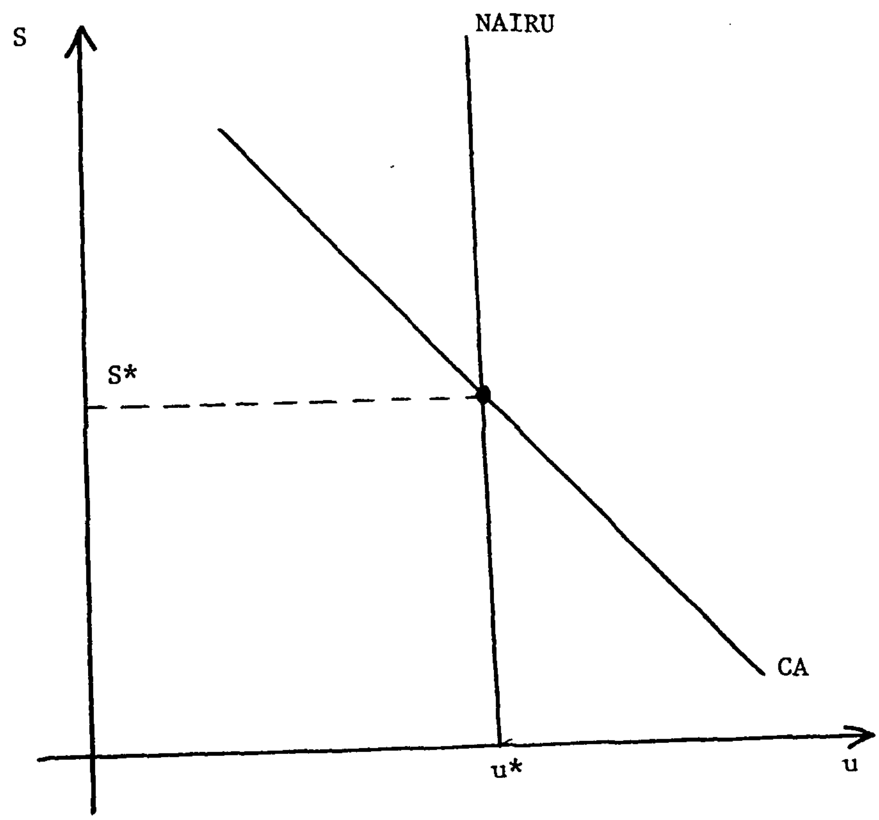
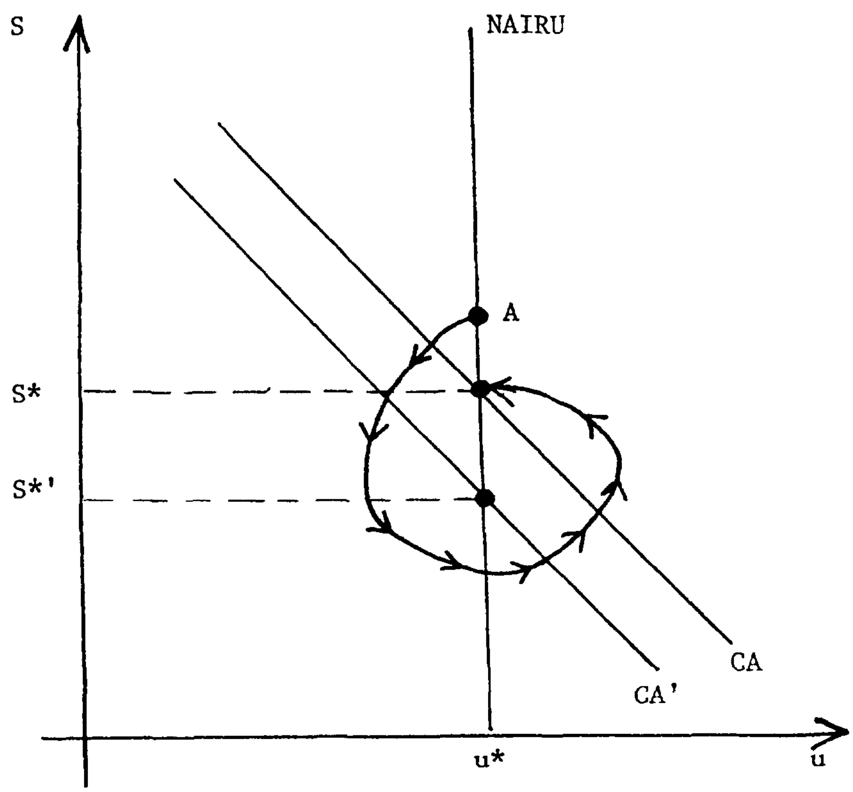


Figure 2: Hysteresis Effects



exchange from its DEER value--say, to point A--involves a real appreciation and a current account deficit (relative to the current account balance underlying CA). This requires a re-evaluation of the CA schedule, to CA′, and a devaluation of the DEER from S* to S*′. For the DEER to be re-established at S*, an overdepreciation would be needed to reduce the stock of debt to the original level, resulting in the "hysteresis loop" shown.

How large a revision of the DEER is required as a result of misalignment? To derive an answer to this question, imagine again that starting from a favorable position, (i.e., where the current real exchange rate is at its DEER value and internal balance is realized), the current exchange rate departs from its DEER value by, say, x percent. For concreteness, suppose this is an appreciation. Then a deficit in the current account will appear of $x(\mu+\tau)X$ where μ and τ are, respectively, the import and export elasticities and X is the volume of exports (or imports--we suppose the two to be approximately equal). If this is a one-off deviation of "one-year" duration, then the DEER will have to be devalued to the extent necessary to service the additional debt incurred. It is convenient to assume that the DEER adjustment depends on the same elasticities, 1/ but the adjustment need only be large enough to service the cost of the debt incurred; then it is easy to see that, if the interest rate is r, the adjustment required is $\delta = rx$. (The DEER devaluation, δ, must yield $rx(\mu+\tau)X$ to cover the debt service, or $\delta(\mu+\tau)X = rx(\mu+\tau)X$.) If the deviation is sustained for two "years," the total adjustment required will be twice as large. Thus, each initial x percent deviation of the actual from the fundamental equilibrium exchange rate will require a DEER adjustment of rx percent in the opposite direction if DEERs are adjusted annually.

Notice that this makes no allowance for what might logically be seen as the need to reverse the increment in debt acquired in this example through the appreciation of the current exchange rate over its DEER value. The reason this might seem logical is that the DEER external balance criterion is typically for a zero net foreign asset accumulation (or for a particular baseline growth in net foreign assets). Thus, an event that causes a departure from this initial desired condition should lead to an action calculated to offset it. In this case every x percent appreciation of the actual rate over its DEER value should lead to a reduction big enough to pay back the debt over a defined period of time. The size of hysteresis effects will clearly be potentially much larger if the DEER adjustment is required to be big enough to repay the debt incurred rather than simply to service the additional interest obligation. In what follows, however, we calculate DEER adjustments on the more modest of the two possible criteria, looking therefore for a DEER adjustment sufficient only to cover the cost of the additional interest burdens arising from misalignment. This case is clearly the most conservative one to take and sets a natural "lower band" to the

---

1/ It might be objected that the short-run elasticities differ from the medium-run elasticities used in constructing the DEER. A further adjustment could be made for any such differences.

size of the hysteresis problem. Had we chosen to assume that debt repayment objectives were involved, we should also have been obliged to specify--quite arbitrarily--the speed with which debt repayments were to occur. We now proceed to derive a formula for the DEER adjustment process. The result expresses formally the hysteresis effect--the dependence of the DEER on the path of the actual exchange rate.

Suppose that initially, in year 0, the actual real exchange rate is at its DEER value, which has a flat (stationary) trajectory at that point in time. Internal balance is assumed to be maintained at its optimal level throughout. Then any deviation of the actual rate from the DEER value implies a deviation from current account balance and requires a recomputation of the DEER on the lines indicated above.

Approximately, then:

$$F_n = F_{n-1} - r(S_{n-1} - F_{n-1}) \qquad (1)$$

or

$$F_n = (1 + r)F_{n-1} - rS_{n-1} \qquad (2)$$

where F_n is the logarithmic value of the DEER in year n, S_n is the logarithm of the actual exchange rate in year n, and r is the rate of interest.

Equation (2) implies that

$$F_{n-1} = (1 + r)F_{n-2} - Rs_{n-2} \qquad (3)$$

$$F_{n-2} = (1 + r)F_{n-3} - rS_{n-3} \qquad (4)$$

Recursive substitution (of (4) and (3) into (2) etc.,) yields

$$F_n = (1+r)^n F_0 - r\sum_{i=1}^{n} (1 + r)^{i-1} S_{n-i} \qquad (5)$$

Equation (5) shows how the initial stationary trajectory for F, F_0, will require updating in the light of the evolution of the actual real exchange rate. Thus, the DEER is not independent of the history of exchange rate movements. In particular, if the authorities wished to move the current exchange rate to the DEER, at the end of n periods, they would need to choose a path for the real exchange rate (S_1, S_2 ... S_n) such that $S_n = F_n$ with F_n as defined in (5). Thus, given a deviation of the actual rate from the DEER and a desire ultimately to equate the two, the DEER arrived at when the actual rate again coincides with it will not be independent of the path taken by the exchange rate towards this goal. Indeed, as equation (1) makes clear, if S deviates from F, F will actually move <u>away</u> from S at speed r per period. Intuitively, therefore, we should expect eventual convergence of F and S to require a movement of S towards F at a speed greater than r in the following period. This intuition turns out to be correct when we analyze the dynamics of adjustment more rigorously.

III. Catching a Moving Bus: The Dynamics of Adjustment

In Section II we derived a dynamic equation for the DEER (equation (1)). We now examine the problem of adjustment to the DEER, in the framework of a general partial adjustment equation:

$$S_n = S_{n-1} - \theta(S_{n-1} - F_{n-1}), \tag{6}$$

with $\theta > 0$.

The questions we pose are the following: given (1) and (6), what restrictions must be placed on θ to ensure eventual convergence and what does θ imply about the nature of convergence? These are interesting questions since, at the normative level, policy makers may have a plan for converging on the rate they calculate as the DEER. In this case answers to the question posed are directly relevant to the choice of speed of adjustment towards the DEER (of course, this assumes that the authorities have the means to influence S in the right direction). Thus, although (6) may be a simplification in the sense that we make no attempt to reconcile it with any theoretical model of exchange rate movements, it is useful because it allows us to derive sharp analytic conclusions concerning the speed of adjustment towards the DEER which are likely to apply, at least qualitatively, within a more elaborate framework.

We define the absolute deviation of the actual real exchange rate from the DEER in period n as:

$$d_n \equiv |S_n - F_n| \tag{7}$$

Then convergence on the DEER may be formally defined as:

$$\lim_{n\to\infty} d_n = 0 \tag{8}$$

Now a sufficient condition for (8) is that the series of absolute deviations be convergent (Widder (1961)):

$$\lim_{n\to\infty} \sum_{i=1}^{n} d_n = L \tag{9}$$

where L is finite. Thus, we can apply results on the convergence of positive series to answer our questions. In particular, D'Alembert's ratio test (Widder (1961)) implies that (8) is satisfied if

$$d_n/d_{n-1} < 1 \tag{10}$$

which is merely the requirement that successive deviations from the DEER should decrease in absolute magnitude. Using (1) and (6) and the definition of d_n, equation (7), inequality (10) becomes:

$$|1 - \theta + r| < 1 \tag{11}$$

From inspection of (11), and for given r, it is possible to deduce information about the behavior of the system for any value of θ on the real line:

(a) $\theta < r$: monotonic divergence: F_n and S_n move steadily away from one another.

(b) $\theta = r$: bounded monotonic divergence. F_n and S_n move on parallel trajectories.

(c) $r < \theta < 1 + r$: monotonic convergence. F_n and S_n converge monotonically over more than one period.

(d) $\theta = 1 + r$: one-period convergence. F_n and S_n are equal after exactly one period.

(e) $1 + r < \theta < 2 + r$: oscillating convergence. S_n moves alternately above and below F_n as the two converge.

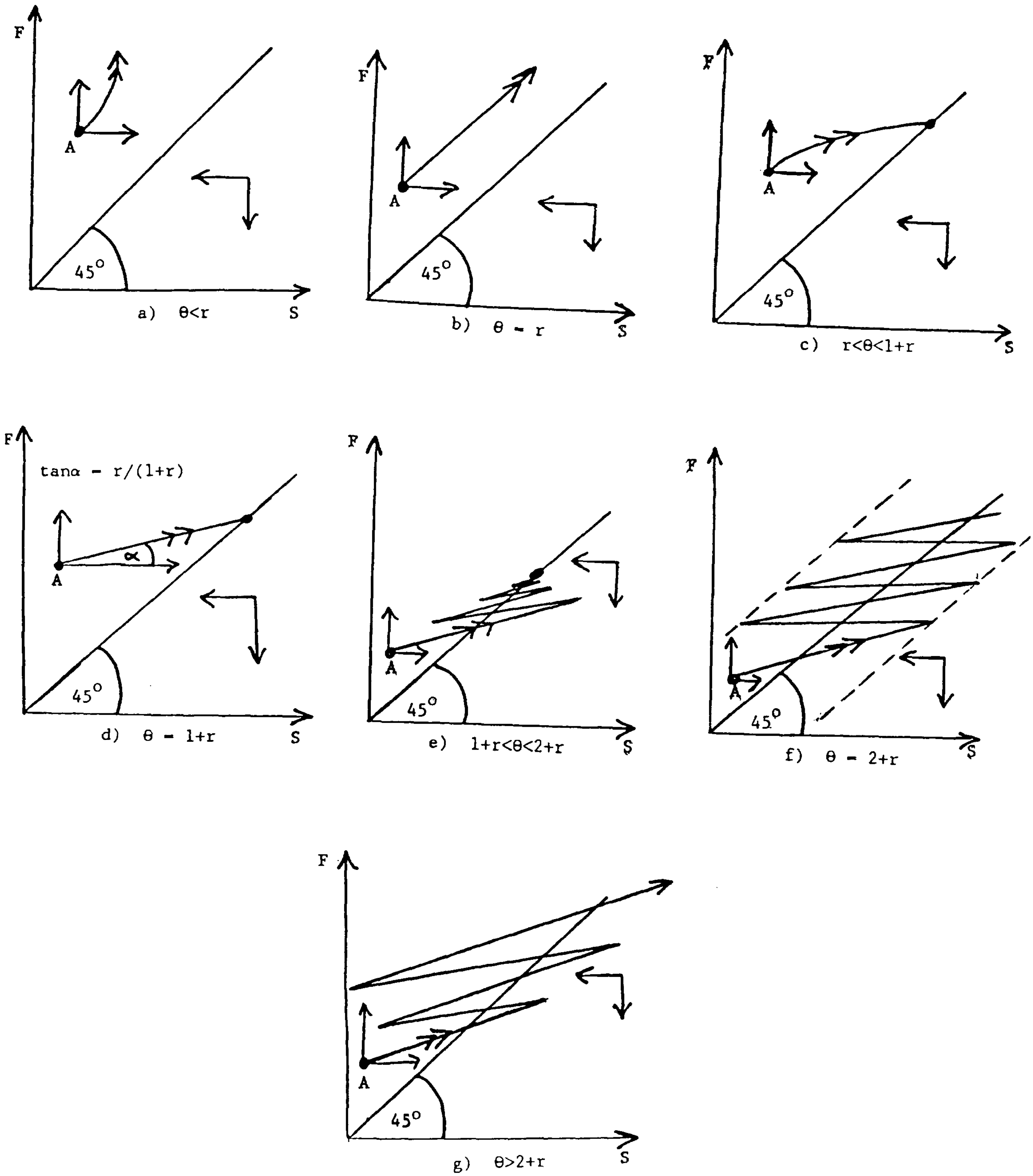
(f) $\theta = 2 + r$: bounded oscillating divergence. S_n moves alternately above and below F_n but the absolute deviation, d_n, remains constant.

(g) $\theta > 2 + r$: oscillating divergence. S_n moves alternately above and below F_n and d_n grows over time.

The phase portrait of the system in (S_n, F_n)-space, for cases (a)-(g), is given in Figure 3.

For values of θ in the open interval (r, 1+r), the speed of adjustment of S_n towards the DEER is sufficiently greater than the speed at which the

Figure 3. The Phase Portrait for Given Initial Misalignment



DEER is moving away for the two to eventually coincide--a little like running for a moving bus. 1/

If, on the other hand, θ lies in the closed-open interval [1+r, 2+r), the exchange rate continually overshoots the DEER but by smaller and smaller absolute amounts, so that convergence is eventually achieved but the adjustment path is not monotonic.

In the case where $\theta = 1+r$, convergence is achieved in one period with the DEER revised upward by r times the initial divergence and S higher by (1+r) times the initial divergence.

Given the range of values of θ for which the system is convergent--the open interval (r, 2+r)--it would seem, prima facie, that eventual convergence on the DEER is quite likely. Note, however, that equation (6) is in effect an assumption that S will tend to move towards F. In fact, it is not immediately obvious that this will be the case, since the DEER is a normative rather than a positive concept. 2/

Given, then, that issue of convergence is largely empirical, we need to turn to the data to examine whether the hysteresis effects are likely to be important. This is the subject of the next section.

IV. How Important Has Hysteresis Been?

Our approach to the question "How important has hysteresis been?" involves applying the measures developed in Section II to a historical data set. The approach is illustrative and, given its purpose, allows us to make simplifying assumptions. In this spirit we shall assume--purely for illustrative purposes and without any presumption that this was actually the case--that at the starting point, DEERs coincide with actual real exchange rates for all the countries in the sample; and, with an exception to be discussed below, that the initial DEER trajectories are flat throughout the period under examination. However, we are obliged to take account of a feature of real world experience from which we abstracted in the earlier sections. There, deviations of the actual exchange rate from the DEER were treated as if they were deviations at a constant level of utilization, hence implying a deviation from the current account target, with attendant

---

1/ The authors have in mind a London bus of traditional type, which allows passengers to leap on at the rear while the bus is moving. A North American alternative would be a San Francisco cable car.

2/ Indeed, given the widespread use of nonfundamental or "chartist" analysis in foreign exchange markets (Taylor and Allen (1992)), and that a high proportion of the chartist techniques which are used are in some way extrapolative (ibid.), one might expect there to be substantial divergences between actual exchange rates and DEERs over time since the use of such extrapolative advice by the market would tend to push the nominal and hence the real exchange rate in the opposite direction to the DEER.

consequences for debt service obligations. A glance at Figure 1, however, serves as a reminder that deviations of actual exchange rates from DEER values need not necessarily imply movement off the CA schedule. An appreciation of the exchange rate and a simultaneous fall in utilization, for example, could imply that the economy has moved left along the CA schedule, with no consequences for debt service or DEER revision. When using historical data therefore it is necessary to purge movements in actual real exchange rates of that part which can be held to reflect changes in utilization. In essence, this involves estimating the slope of the CA schedule. Empirically, one can use medium-term elasticities derived from an econometric macro model to do this. In this paper, we employ elasticities derived from the IMF's multi-country macroeconometric model, MULTIMOD (Masson, Symansky, and Meredith (1990)).

Given values for changes in the rate of capacity utilization, output elasticities yield the implied change in the current account. The real exchange rate elasticities then yield the change in the real exchange rate that would eliminate this. That is, if ϵ_y is the output elasticity of net imports and Δu the percentage change in utilization, the corresponding real exchange rate change, ΔS, can be found from

$$\epsilon_y \, \Delta u \, X = X(\mu + \tau) \, \Delta S \qquad (12)$$

or

$$\Delta S = \frac{\epsilon_y \, \Delta u}{\mu + \tau} \qquad (13)$$

where $\mu + \tau$ is the sum of import and export elasticities with respect to the real exchange rate and X is the level of exports (or imports).

The exercise described was conducted on data for the G-5 countries over the period 1979-90. Our simplifying assumptions thus equate the actual real exchange rates (effective rates) with DEERs for those countries at the outset (1979) of the period and project a flat trajectory for the DEERs throughout the period with the exception of a small modification for the effect of OPEC II. The steep increase in oil prices at the end of 1979 is assumed to have raised the U.K. DEER by 10 percent in 1980 and later years, with corresponding adjustments to the DEERs for other G-5 member countries. 1/ An annual interest rate of 5 percent was assumed. As noted above, we used medium-term elasticity estimates derived from MULTIMOD. We

---

1/ Adjustment to other exchange rates was based on sterling's weight in the relevant index, using MERM weights.

also used IMF data on the estimated level of capacity utilization, which is derived using the Hodrick-Prescott filter. 1/

Assuming, then, misalignment of zero for each of the G-5 countries in 1979, 2/ Figure 4 depicts actual movements in the real exchange rate, 3/ movements in the DEER and a "constant" DEER trajectory adjusted only for the OPEC II shock as discussed above.

As Figure 4 shows, the DEER trajectory is distorted away from the constant path in each case. It appears most significantly, however, in the cases where there are large sustained deviations of the real exchange rate from the DEER in one direction. In the case of the U.S., most strikingly, the chronic overvaluation of the dollar leads to an increasing deviation of the DEER trajectory away from the constant path reaching some 14 percent by 1990 and averaging 7 1/2 percent per year (Table 1). In cases where the exchange rate is alternately over- and undervalued--most notably France--however, the DEER trajectory is kept approximately flat, so that the loss to assuming a constant DEER trajectory may be slight (Table 1). As we might expect, then, we see that exchange rate volatility (defined as relatively high-frequency oscillations in the exchange rate) is less damaging than sustained misalignment (defined as sustained deviation from the DEER).

Overall, therefore, the results of this section demonstrate that hysteresis effects on the DEER, operating through the effects of debt accumulation or decumulation, may have been significant historically, particularly for countries whose real exchange rate has moved predominantly in one direction over a sustained period of time--such as the U.S. dollar during the 1980s.

V. Converging on the DEER: Rules of Thumb

In this section, we seek to answer the following question: given an initial misalignment of the exchange rate, and a desire to correct the misalignment over a certain finite period, by how much does the required movement in the exchange rate differ from the initial misalignment? For example, suppose that the exchange rate is undervalued according to its value relative to the DEER, and that it is desired to correct this over a period of, say, five years. Assuming that the desired adjustment is monotonic (e.g., a constant percentage annual appreciation), then, by our arguments, the DEER at the end of the fifth year, when the misalignment is zero, will be higher than the initial calculated DEER. Thus, the total

---

1/ See Prescott (1986) or King and Rebelo (1989) for a discussion of the Hodrick-Prescott filter.

2/ Note that this is for purely illustrative purposes, with no presumption that any of these real exchange rates actually did coincide with the FEER in 1979.

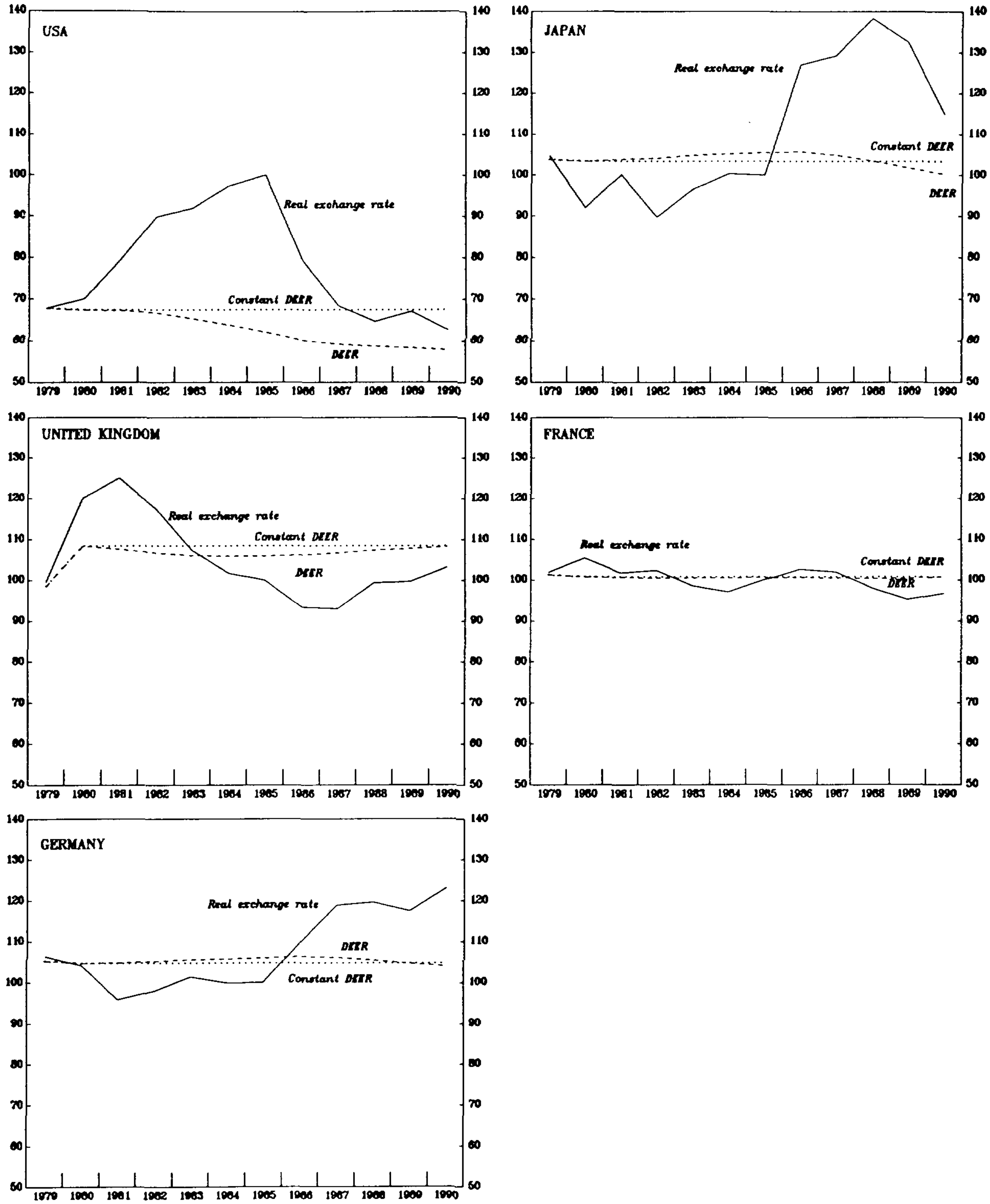
3/ From the IMF's International Financial Statistics (IFS) data tape; 1985=100.

Table 1. Percentage Deviation of Illustrative G-5 DEER Trajectories from Constant DEERs 1/

Year	United States	Japan	United Kingdom	France	Germany
1980	0.00	0.00	0.00	0.00	0.00
1981	0.27	0.57	0.61	0.21	0.07
1982	1.23	0.76	1.55	0.29	0.50
1983	3.20	1.46	2.19	0.38	0.79
1984	5.38	1.84	2.32	0.31	0.96
1985	7.94	2.07	2.23	0.19	1.24
1986	10.82	2.35	2.05	0.22	1.53
1987	12.35	1.44	1.53	0.35	1.36
1988	13.13	0.22	0.93	0.47	0.71
1989	13.61	1.45	0.54	0.36	0.01
1990	14.30	2.94	0.15	0.08	0.61
Sum	82.22	15.11	14.09	2.87	7.78
Mean	7.47	1.37	1.28	0.26	0.71

1/ The "Constant DEER" paths incorporates an adjustment for OPEC II--see text.

Figure 4. Illustrative DEER Trajectories for the G5



required movement in the exchange rate will be greater than the initial difference between the exchange rate and the DEER. The additional movement in the exchange rate over the five-year period (over the initial misalignment measure) is thus, in some sense, a measure of the importance of hysteresis effects arising during misalignment.

If the rate of capacity utilization is held constant at 100 percent, then the overall movement in the DEER will be equal to the cumulative misalignment each period multiplied by the interest rate.

Tables 2 and 3 give some illustrative examples of movements in the DEER, assuming an interest rate of five percent, initial undervaluation or overvaluation of 10 or 20 percent, and a constant annual percentage change in the exchange rate. These trajectories are essentially found by fine-tuning the annual percentage change in the exchange rate to find $S_n = F_n$ ($n = 5, 10$) with F_n as given in (5), for given F_0 and S_0. 1/

Consider first the results assuming five-year convergence (Table 2). These show that the effect of a changing net asset stock (due to misalignment) is to increase the total amount of required exchange rate adjustment by some 1.5 percentage points for an initial 10 percent misalignment, and by 2.5 to 3 percentage points for an initial 20 percent misalignment. Note that the revision due to debt accumulation or decumulation is greater in the case of undervaluation, since the initial exchange rate on which the percentage is calculated is at a depressed level.

If the desired period of convergence is increased to ten years (Table 3), then the revisions increase to approximately 3 percentage points for an initial 10 percent misalignment, to some 5 percentage points for an initial 20 percent overvaluation and to a sizeable 7 percentage points for an initial 20 percent undervaluation.

As a very rough rule of thumb, therefore, for economies operating near full capacity utilization, the results of this section suggest the following: The initial measure of misalignment should be increased by about 1.5 percentage points for each 10 percent of misalignment if the desired period of convergence is five years, and by some 3 percentage points for each 10 percent of misalignment if convergence over ten years is desired. Thus, an initial misalignment of 15 percent would suggest a required exchange rate movement of about 17 1/4 percent over five years.

1/ In fact, the precise calculations were slightly different because we used percentage differences rather than logarithmic differences, as in equation (1). A spread sheet program was used to perform the calculations.

Table 2. Movements in the DEER: Five-Year Convergence 1/

a) Initial 10 percent overvaluation

Year	0	1	2	3	4	5
Exchange Rate	100.00	97.61	95.27	92.99	90.70	88.60
DEER	90.00	89.55	89.18	88.90	88.70	88.60
Misalignment (percent)	+10.00	+8.26	+6.40	+4.41	+2.28	+0.00

Annual exchange rate movement = -2.39 percent

Overall movement in the DEER = -1.56 percent

Overall movement in the exchange rate = -11.40 percent

Difference between initial misalignment and overall exchange rate movement = 1.4 percentage points

b) Initial 10 percent undervaluation

Year	0	1	2	3	4	5
Exchange rate	100.00	102.24	104.52	106.86	109.25	111.70
DEER	110.00	110.55	111.00	111.34	11.58	111.70
Misalignment (percent)	-10.00	-8.13	-6.20	-4.19	-2.13	0.00

Annual exchange rate movement = +2.24 percent

Overall movement in the DEER = +1.54 percent

Overall movement in the exchange rate = +11.70 percent

Difference between initial misalignment and overall exchange rate movement = 1.7 percentage points

c) Initial 20 percent overvaluation

Year	0	1	2	3	4	5
Exchange rate	100.00	95.03	90.30	85.81	81.55	77.49
DEER	80.00	79.20	78.54	78.03	77.67	77.49
Misalignment (percent)	+20.00	+16.66	+13.03	+9.07	+4.75	0.00

Annual exchange rate movement = -4.97 percent

Overall movement in the DEER = -3.14 percent

Overall movement in the exchange rate = -22.51 percent

Difference between initial misalignment and overall exchange rate movement = 2.51 percentage points

d) Initial 20 percent undervaluation

Year	0	1	2	3	4	5
Exchange rate	100.00	104.34	108.87	113.60	118.54	123.68
DEER	120.00	121.20	122.18	122.93	123.43	123.68
Misalignment (percent)	-20.00	-16.16	-12.22	-8.21	-4.13	0.00

Annual exchange rate movement = +4.34 percent

Overall movement in the DEER = +3.07 percent

Overall movement in the exchange rate = +23.68 percent

Difference between initial misalignment and overall exchange rate movement = 3.68 percentage points

1/ We assume a constant, 100 percent rate of capacity utilization, and an interest rate of 5 percent per annum.

Table 3. Movements in the DEER: Ten-Year Convergence 1/

a) Initial 10 percent overvaluation

Year	0	1	2	3	4	5	6	7	8	9	10
Exchange rate	100.00	98.06	97.33	96.02	94.74	93.46	92.21	90.97	89.75	88.54	87.35
DEER	90.00	89.55	89.14	88.76	88.43	88.13	87.88	87.67	87.51	87.41	87.35
Misalignment (percent)	10.00	9.23	8.42	7.56	6.66	5.70	4.69	3.62	2.49	1.28	0.00

Annual exchange rate movement = 1.34 percent

Overall movement in the DEER = -2.94 percent

Overall movement in the exchange rate = -12.65 percent

Difference between initial misalignment and overall exchange rate movement = 2.65 percentage points

b) Initial 10 percent undervaluation

Year	0	1	2	3	4	5	6	7	8	9	10
Exchange rate	100.00	101.25	102.52	103.81	105.11	106.42	107.75	109.10	110.47	111.86	113.26
DEER	110.00	110.55	111.06	111.52	111.93	112.30	112.61	112.86	113.06	113.19	113.26
Misalignment	-10.00	-9.18	-8.33	-7.43	-6.50	-5.52	-4.50	-3.44	-2.34	-1.19	0.00

Annual exchange rate movement = + 1.25 percent

Overall movement in the DEER = + 2.96 percent

Overall movement in the exchange rate = 13.26 percent

Difference between initial misalignment and overall exchange rate movement = 3.26 percentage points

c) Initial 20 percent overvaluation

Year	0	1	2	3	4	5	6	7	8	9	10
Exchange rate	100.00	97.20	94.48	91.84	89.27	86.77	84.34	81.98	79.69	77.46	75.29
DEER	80.00	79.20	78.47	77.80	77.21	76.69	76.24	75.87	75.59	75.40	75.29
Misalignment	+20.00	+18.52	+16.95	+15.28	+13.51	+11.62	+9.61	+7.45	+5.14	+2.66	0.00

Annual exchange rate movement = -2.80 percent

Overall movement in the DEER = -5.8 percent

Overall movement in the exchange rate = -24.71

Difference between initial misalignment and the overall exchange rate movement = 4.71 percentage points

d) Initial 20 percent undervaluation

Year	0	1	2	3	4	5	6	7	8	9	10
Exchange rate	100.00	102.43	104.92	107.47	110.08	112.75	115.49	118.30	121.17	124.12	127.13
DEER	120.00	121.20	122.31	123.32	124.23	125.03	125.71	126.27	126.70	126.98	127.13
Misalignment	-20.00	-18.33	-16.58	-14.76	-12.86	-10.89	-8.85	-6.74	-4.56	-2.31	0.00

Annual exchange rate movement = +2.43 percent

Overall movement in the DEER = +5.94 percent

Overall movement in the exchange rate = +27.13 percent

Difference between initial misalignment and the overall exchange rate movement = 7.13 percentage points

1/ We assume a constant, 100 percent rate of capacity utilization, and an interest rate of 5 percent per annum.

VI. Adjusting DEER Trajectories: How Good are the Rules of Thumb?

Our final exercise involves investigating what adjustments should be made to some actual DEER estimates, due to Williamson (1990), when a certain convergence path is assumed and hysteresis effects are taken into account. This allows an assessment of the rules of thumb derived in the previous section.

Williamson (1990) presents a range of estimates of DEERs for the G-7 countries in 1990. 1/ To illustrate the effects of hysteresis, we consider his base-case estimates obtained using the Global Econometric Model (GEM) developed by the U.K. National Institute for Economic and Social Research, since GEM appears to be Williamson's preferred model in this context (1990, p. 70). These estimates (Williamson (1990, Tables 6-7)) suggest that, at the end of 1989, the U.S. dollar was overvalued relative to the DEER by some 9 percent, the yen was undervalued by 15 percent, sterling was overvalued by 10 percent, the franc was overvalued by 8 percent, and the mark was undervalued by about 18 percent. 2/ These figures were applied to real exchange rates for the G-5 in the fourth quarter of 1989 to obtain an estimate of the DEER in 1990 which could be compared with the average real exchange rate for that year to gauge the initial degree of misalignment. 3/ Given an initial estimate of capacity utilization in 1990, we then assume that, over a five- or ten-year period, utilization adjusts linearly to full capacity and the exchange rate adjusts by a constant percentage annual change in order to achieve convergence on the DEER at the end of the period. Essentially, this involves setting the trajectory for capacity utilization exogenously, and fine-tuning the annual percentage change in the real exchange rate so that $S_n = F_n$ as defined in (5) but with the exchange rate purged of changes due to movements in capacity utilization according to (13). 4/

The results are reported in Tables 4 and 5. They tend to confirm the usefulness of the rules of thumb derived in the previous section. For example, with an initial misalignment of 9.33 percent, five-year convergence on the DEER for the U.S. dollar entails an extra 1.34 percentage point movement in the real exchange rate (over the initial 9.33 percent), which corresponds closely to the "1.5 percentage points per 10 percent of misalignment corrected over five years" rule. For convergence over ten years, an extra 2.5 percentage points of adjustment in the dollar is required, compared to the "3 percentage points per 10 percent of misalignment corrected over ten years" rule. Similar results are obtained for the remainder of the G-5 countries.

1/ More exactly, end-1989.

2/ These estimates are used for purely illustrative purposes. They are not endorsed either by the present authors or by the International Monetary Fund.

3/ Real exchange rate data were taken from the IFS data tape.

4/ Again, the actual computations were slightly more complex since actual percentage changes, rather than log-linear approximations, were used.

Table 4. Hypothetical Five-Year DEER Trajectories for the G5 1/

	Year	Exchange Rate	DEER	Utilization Rate
a) United States	1990	62.53	56.69	100.29
	1991	61.13	56.43	100.24
	1992	59.77	56.21	100.18
	1993	58.43	56.04	100.12
	1994	57.13	55.92	100.06
	1995	55.85	55.85	100.00

Initial misalignment = +9.33 percent
Annual exchange rate movement = 2.23 percent
Overall movement in the DEER = -1.48 percent
Overall movement in the exchange rate = -10.68 percent
Difference between initial misalignment and overall exchange movement = 1.34 percentage points

	Year	Exchange Rate	DEER	Utilization Rate
b) Japan	1990	115.00	141.11	100.96
	1991	120.61	142.71	100.77
	1992	126.50	144.00	100.58
	1993	132.67	144.98	100.38
	1994	139.15	145.63	100.19
	1995	145.94	145.94	100.00

Initial misalignment = -22.70 percent
Annual exchange rate movement = +4.88 percent
Overall movement in the DEER = +3.43 percent
Overall movement in the exchange rate = +26.91 percent
Difference between initial misalignment and overall exchange movement = 4.21 percentage points

	Year	Exchange Rate	DEER	Utilization Rate
c) United Kingdom	1990	103.07	84.87	101.09
	1991	98.57	84.12	100.87
	1992	94.26	83.50	100.65
	1993	90.15	83.00	100.43
	1994	86.12	82.65	100.22
	1995	82.44	82.44	100.00

Initial misalignment = +17.66 percent
Annual exchange rate movement = -4.37 percent
Overall movement in the DEER = -2.86 percent
Overall movement in the exchange rate = -20.01 percent
Difference between initial misalignment and overall exchange movement = 2.35 percentage points

Table 4. Hypothetical Five-Year DEER Trajectories for the G5 1/

	Year	Exchange Rate	DEER	Utilization Rate
d) France	1990	96.72	86.30	101.16
	1991	94.20	85.83	100.93
	1992	91.75	85.44	100.70
	1993	89.37	85.13	100.46
	1994	87.04	84.91	100.23
	1995	84.78	84.78	100.00

Initial misalignment = +10.78 percent
Annual exchange rate movement = -2.60 percent
Overall movement in the DEER = -1.76 percent
Overall movement in the exchange rate = -12.35 percent
Difference between initial misalignment and overall exchange movement = 1.57 percentage points

	Year	Exchange Rate	DEER	Utilization Rate
e) Germany	1990	122.96	145.49	101.24
	1991	127.86	146.83	100.99
	1992	132.96	147.91	100.74
	1993	138.26	148.72	100.50
	1994	143.77	149.25	100.25
	1995	149.50	149.50	100.00

Initial misalignment = -18.33 percent
Annual exchange rate movement = +3.99 percent
Overall movement in the DEER = +2.76 percent
Overall movement in the exchange rate = +21.59 percent
Difference between initial misalignment and overall exchange movement = 3.26 percentage points.

1/ We assume an interest rate of 5 percent per annum.

Table 5. Hypothetical Ten-Year DEER Trajectories for the G-5 1/

Country	Year	Exchange Rate	DEER	Utilization Rate
a) United States	1990	62.53	56.69	100.29
	1991	61.74	56.43	100.26
	1992	60.97	56.18	100.24
	1993	60.20	55.96	100.21
	1994	59.45	55.76	100.18
	1995	58.70	55.59	100.15
	1996	57.96	55.44	100.12
	1997	57.23	55.31	100.09
	1998	56.52	55.21	100.06
	1999	55.81	55.14	100.03
	2000	55.10	55.10	100.00

Initial misalignment = +9.33 percent
Annual exchange rate movement = -1.26 percent
Overall movement in DEER = -2.80 percent
Overall movement in exchange rate = -11.88 percent
Difference between initial misalignment and overall exchange rate movement = 2.5 percentage points

Country	Year	Exchange Rate	DEER	Utilization Rate
b) Japan	1990	115.00	141.11	100.96
	1991	118.13	142.71	100.86
	1992	121.35	144.19	100.77
	1993	124.66	145.53	100.67
	1994	128.06	146.74	100.58
	1995	131.55	147.79	100.48
	1996	135.13	148.68	100.38
	1997	138.81	149.40	100.29
	1998	142.60	149.95	100.19
	1999	146.48	250.30	100.10
	2000	150.47	150.47	100.00

Initial misalignment = -22.70 percent
Annual exchange rate movement = +2.73 percent
Overall movement in DEER = +6.64 percent
Overall movement in exchange rate = +30.85 percent
Difference between initial misalignment and overall exchange rate movement = 8.15 percentage points

Country	Year	Exchange Rate	DEER	Utilization Rate
c) United Kingdom	1990	103.07	84.87	101.09
	1991	100.53	84.12	100.98
	1992	98.05	83.43	100.87
	1993	95.63	82.80	100.76
	1994	93.27	82.24	100.65
	1995	90.97	81.73	100.54
	1996	88.73	81.30	100.43
	1997	86.54	80.93	100.33
	1998	84.40	80.64	100.22
	1999	82.32	80.43	100.11
	2000	80.29	80.29	100.00

Initial misalignment = +17.66 percent
Annual exchange rate movement = -2.47 percent
Overall movement in DEER = -5.39 percent
Overall movement in exchange rate = -22.10 percent
Difference between initial misalignment and overall exchange rate movement = 4.44 percentage points

Table 5 (continued). Ten-Year DEER Trajectories for the G-5

d) France	1990	96.72	86.30	101.16
	1991	95.30	85.83	101.04
	1992	93.30	85.40	100.93
	1993	92.52	85.01	100.81
	1994	91.16	84.65	100.70
	1995	89.82	84.34	100.58
	1996	88.50	84.06	100.46
	1997	87.20	83.83	100.35
	1998	85.91	83.64	100.23
	1999	84.65	83.50	100.12
	2000	83.41	83.41	100.00

Initial misalignment = +10.78 percent
Annual exchange rate movement = -1.47 percent
Overall movement in DEER = -3.35 percent
Overall movement in exchange rate = -13.76 percent
Difference between initial misalignment and overall exchange rate movement = 2.98 percentage points

e) Germany	1990	122.96	145.49	101.24
	1991	125.70	146.83	101.12
	1992	128.49	148.06	100.99
	1993	131.35	149.17	100.87
	1994	134.27	150.17	100.74
	1995	137.26	151.04	100.62
	1996	140.31	151.77	100.50
	1997	143.44	152.37	100.37
	1998	146.63	152.81	100.25
	1999	149.89	153.10	100.12
	2000	153.22	153.22	100.00

Initial misalignment = -18.33 percent
Annual exchange rate movement = +2.22 percent
Overall movement in DEER = +5.31 percent
Overall movement in exchange rate = +24.61 percent
Difference between initial misalignment and overall exchange rate movement = 6.29 percentage points

1/ We assume an interest rate of 5 percent per annum.

VII. Conclusions

The efficient conduct of policy on matters relating to the international monetary system requires some basis for the evaluation of market-determined exchange rates. One such basis can be found in the concept of the fundamental equilibrium exchange rate (DEER) to which Williamson has appealed in his advocacy of exchange rate target zones. The fundamental rate appeals to the notion that in the medium term it is desirable to obtain both internal balance and external balance. The DEER is simply that rate of exchange which fulfills this condition. It is reasonably straightforward, given estimates of the relevant elasticities, to compute values for the fundamental rate on "medium-term" assumptions, that is to say, ignoring the dynamics of adjustment.

However, it appears that the true value of the DEER must depend on the path taken towards it. The reason is that, while the actual exchange rate deviates from its DEER value, so in general will the current account realizations deviate from those implicit in the initial calculation of the DEER trajectory. For this reason debt service obligations will differ from those assumed in the initial computation of the trajectory. The DEER will change and a recomputation is called for. The DEER is thus not independent of the path taken towards it, and suffers from hysteresis.

In this paper we were able to show that this problem could be formalized in a fairly straightforward manner, enabling us to obtain a formal representation of the hysteresis effect on the DEER. Using this formalization we were then able to demonstrate the properties required for convergence of the actual exchange rate on the DEER. Since the existence of hysteresis operating through misalignment and debt stocks is not in doubt the important question is its empirical significance. The paper attempted to address this in three different ways, using the analytical formalization derived earlier. First, we asked whether over a historical time period, misalignment would have contributed significantly to recomputations of the DEER; it appeared that it could. Then we sought rules of thumb for the updating of the DEER for a given degree of initial misalignment and for particular horizons and monotonic adjustment of the actual rate towards the DEER. Finally, we calculated by how much DEERs would change if values calculated in 1990 by Williamson were achieved by 1995 or by the year 2000. The results, which took account of the need to adjust capacity utilization levels at the same time, largely confirmed the rules of thumb suggested in the earlier exercise; just as important, the revisions emerged as significant in size when compared with the initial amount of misalignment.

References

Baldwin, R., and P. Krugman, "Persistent Trade Effects of Large Exchange Rate Shocks", Working Paper No. 2167, National Bureau of Economic Research, Cambridge, Ma. (1986).

Barrell, R., and S. Wren-Lewis, "Equilibrium Exchange Rates for the G7," Discussion Paper No. 323, Centre for Economic Policy Research, London (1989).

Cross, R., "On the Foundations of Hysteresis in Economic Systems," International Centre for Macroeconomic Modelling, University of Strathclyde, Discussion Paper No. 4 (1992).

Currie, D., and S. Wren-Lewis, "Evaluating Blueprints for the Conduct of International Macropolicy," American Economic Review, Vol. 79 (1989), pp. 264-69.

Cuthbertson, K., and M.P. Taylor, Macroeconomic Systems (Oxford: Blackwell, 1987).

Frenkel, J.A., and M. Goldstein, "A Guide to Target Zones," International Monetary Fund (Washington), Staff Papers, Vol. 33 (1986), pp. 633-73.

International Monetary Fund, The Role of Exchange Rates in the Adjustment of International Payments: A Report by the Executive Directors (Washington: International Monetary Fund, 1970).

_____, Issues in the Assessment of the Exchange Rates of Industrial Countries, Occasional Paper No. 29 (Washington: International Monetary Fund, 1984).

King, R., and S. Rebelo, "Low-Frequency Filtering and Real Business Cycles," Working Paper No. 205, Rochester Center for Economic Research (1989).

Layard, R., and S. Nickell, "The Causes of British Unemployment," National Institute Economic Review, No. 111 (1985), pp. 62-85.

Lindbeck, A., and D. Snower, "Wage Setting, Unemployment and Insider-Outsider Relations," American Economic Review, Vol. 76 (1986), pp. 235-39.

MacDonald, R., and M.P. Taylor, "Exchange Rate Economics: A Survey," International Monetary Fund (Washington), Staff Papers, Vol. 39 (1992), pp. 1-57.

Masson, P., S. Symansky, and G. Meredith, MULTIMOD Mark II: A Revised and Extended Model, Occasional Paper No. 71 (Washington: International Monetary Fund, 1990).

Nurske, R., Conditions of International Monetary Equilibrium, Essays in International Finance, No. 4 (Princeton, N.J.: Princeton University Press, 1945).

Prescott, E.C., "Theory Ahead of Business Cycle Measurement," Federal Reserve Bank of Minneapolis Quarterly Review (Fall 1986), pp. 9-22.

Samuelson, P.A., Foundations of Economic Analysis (Cambridge, Massachusetts: Harvard University Press, 1947).

Taylor, M.P., and H.L. Allen, "The Use of Technical Analysis in the Foreign Exchange Market," Journal of International Money and Finance, Vol. 11 (1992), pp. 304-14.

Widder, D., Advanced Calculus (Englewood Cliffs, N.J.: Prentice-Hall, 1961).

Williamson, J., The Exchange Rate System, Institute for International Economics (Washington, 1985).

_____, "Equilibrium Exchange Rates: An Update," unpublished paper, Institute for International Economics (Washington, October 1990).

_____, and M. Miller, Target Zones and Indicators: A Blueprint for the International Coordination of Economic Policy (Washington, D.C.: Institute for International Economics, 1987).

Wren-Lewis, S., P. Westaway, S. Soteri, and R. Barrell, "Evaluating the United Kingdom's Choice of Entry Rate Into the ERM," Manchester School, Vol. 59, Supplement (1991), pp. 1-22.