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Stabilization Dynamics and Backward-Looking Contracts 1/

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Abstract

Exchange rate-based stabilizations often result in an initial output expansion. One explanation for this phenomenon has been that, in the presence of inflation inertia, a reduction in the nominal interest rate causes the domestic real interest rate to fall, thus increasing aggregate demand. This paper reexamines this issue in the context of an intertemporal optimizing model. In contrast to previous results, the analysis shows that, if the intertemporal elasticity of substitution is smaller than the elasticity of substitution between traded and home goods, a permanent reduction in the rate of devaluation leads to a fall in aggregate demand.

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### Summary

Exchange rate-based stabilizations often result in an initial output expansion and real appreciation. One explanation is that, in the presence of inflation inertia, a reduction in the nominal interest rate causes the domestic real interest rate to fall, thereby increasing aggregate demand. Inflation inertia also causes a sustained real appreciation of the domestic currency. This paper re-examines this phenomenon in the context of an intertemporal optimizing model.

The paper's central finding is that a credible, once-and-for-all reduction in the rate of devaluation gives rise to appreciation of the real exchange rate and an initial expansion in aggregate demand only if the intertemporal elasticity of substitution exceeds the static elasticity of substitution between traded and nontraded (home) goods. Otherwise, aggregate demand does not increase during the first stages of the program. Specifically, when the intertemporal elasticity of substitution is lower than the elasticity of substitution between traded and home goods, aggregate demand for home goods falls following a permanent reduction in the devaluation rate. The paper also confirms the findings of earlier, reduced-form models (under all parameter configurations) that real interest rates initially fall on impact, and the domestic currency appreciates in real terms during the first stages of the program.



## I. Introduction

During the second half of the 1970s, the Southern-Cone countries of Latin America (Argentina, Chile, and Uruguay) implemented stabilization plans that relied on unilaterally setting the path of the exchange rate against the U.S. dollar. Inflation was expected to converge rapidly to the international inflation rate plus the pre-set rate of devaluation. However, contrary to policymakers' expectations, inflation exhibited considerable inertia, which resulted in a sizable real appreciation of the domestic currency. The apparent failure of the Southern-Cone programs to put a firm rein on inflation has attracted considerable professional attention, starting with the pioneering paper by Diaz-Alejandro (1981). 1/

Perhaps the biggest surprise of the Southern-Cone experiments was that aggregate demand seemed to respond in a perverse manner: instead of subsiding in view of an appreciated real exchange rate, aggregate demand increased at the beginning of the programs. The contractionary costs associated with inflation stabilization appeared only later in the programs. This boom-recession cycle has also been observed in other exchange rate-based stabilization programs, as emphasized by Kiguel and Liviatan (1992). 2/

Two polar explanations of the intriguing phenomena observed in the Southern-Cone programs have been offered in the literature. The first explanation relies on the presence of backward-looking price and wage behavior. In particular, Rodriguez (1982)—inspired by the December 1978 Argentine tablita—was the first to offer a coherent explanation of the simultaneous occurrence of real appreciation and high aggregate demand. The key feature of Rodriguez's (1982) model is the assumption of backward-looking (i.e., adaptive) expectations. Under perfect capital mobility, a reduction in the rate of devaluation leads to lower nominal interest and, since inflationary expectations are sticky, to lower real interest rates. As a result, aggregate demand expands, which raises inflation even further and causes the domestic currency to appreciate in real terms. The cumulative effects of the real appreciation eventually lead the economy into a recession. Since (ex-post) real interest rates fell and/or were negative at the beginning of the Southern-Cone programs, the mechanism emphasized by Rodriguez (1982) is clearly applicable. 3/

In a similar vein, Dornbusch (1982) assumes that the inflation rate is sticky (i.e., at any point in time, the rate of inflation is a predetermined variable, although it may vary over time). An analytical innovation of Dornbusch (1982), however, is the assumption of rational expectations. As in Rodriguez (1982), a stabilization plan that relies on decreasing the rate

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1/ See, among many others, Rodriguez (1982), Dornbusch (1982), Corbo, de Melo, and Tybout (1986), Edwards (1991), Kiguel and Liviatan (1992), and Végh (1992).

2/ In contrast, money-based stabilizations usually result in an initial output contraction (see Calvo and Végh (1990) and Kiguel and Liviatan (1992)).

3/ In contrast, in the heterodox programs of the mid-1980's in Argentina, Brazil, Israel, and Mexico, (ex-post) real interest rates increased (see, for instance, Végh (1992)).

of devaluation results, under perfect capital mobility, in lower nominal interest rates. Since the inflation rate is predetermined, the domestic real interest rate declines, thus stimulating aggregate demand. The real appreciation that results from the presence of inflation inertia eventually throws the economy into a recession. 1/ 2/

At the other end of the spectrum, we find models in which prices are set in a forward-looking manner, but policy is not fully credible. Calvo and Végh (1991), for example, study the implications of announcing a permanent reduction in the rate of devaluation which is not fully credible in the sense that the public believes that the policy will be abandoned sometime in the future. The model assumes a cash-in-advance constraint and rational staggered-price setting. In a cash-in-advance economy, the effective price of consumption equals its market price plus the opportunity cost of the associated monetary holdings (i.e., the nominal interest rate). The expectation of the future abandonment of the present stabilization policy induces individuals to expect higher nominal rates in the future. Hence, in the public's mind, the effective price of future consumption is higher than that of present consumption. Thus, present consumption (or aggregate demand) expands. Since the expansion is likely to persist during the first stages of the stabilization program, forward-looking price-setters start raising prices, even though prices are sticky in the short run. Consequently, the expansion in aggregate demand implies that inflation falls by less than the rate of devaluation, which results in a sustained real appreciation of the domestic currency. Hence, in Calvo and Végh's (1991) model, inflation inertia stems from lack of credibility rather than backward-looking behavior. Price stickiness, in turn, implies that the initial expansion in aggregate demand reduces excess capacity and lowers the rate of unemployment. The initial boom is followed by a later recession, as the real appreciation reduces the demand for home goods. 3/

Thus, both backward- and forward-looking models seem to offer a reasonable description of the phenomena associated with the Southern-Cone stabilizations. In our view, however, the analytical relevance of backward-looking behavior remains to be fully established since the earlier literature relied on ad-hoc specifications (i.e., Rodríguez (1982) and Dornbusch (1982)) and, in the case of Rodríguez (1982), on non-rational behavior. The purpose of this paper is therefore to explore the analytical implications of backward-looking behavior in a utility-maximizing, rational-expectations framework. An advantage of this analytical strategy is that the "demand side" is derived from standard

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1/ It should be noted that Dornbusch (1982) does not focus on the initial boom but rather on the later recession.

2/ In the specific case of the Chilean stabilization, Edwards (1991) argues that backward-looking wage indexation was the primary cause of inflation inertia. When inflation is falling over time, backward-looking indexation leads to higher real wages which may result in a consumer-led boom.

3/ See Végh (1992) for a review of the stylized facts associated with exchange-rate based stabilization in terms of Calvo and Végh's (1991) model.

optimization assumptions, which helps to better isolate the effects of backward-looking price and wage behavior. More importantly, however, we show that starting from "first principles" is much more than an academic exercise since it has important implications for the predictions of the model. Specifically, a fully-credible reduction in the devaluation rate may result in a contraction of aggregate demand, thus turning standard results on their heads.

We assume that, as a result of backward-looking contracts, the rate of inflation at time  $t$  is a weighted average of past inflation plus a term reflecting the state of aggregate demand at time  $t$ . Thus, our backward-looking assumption is formally very close to that to Rodriguez (1982), except that the backward-looking term reflects wage-setting behavior, not expectations (which, as indicated before, are assumed to be rational). In addition, we assume a representative consumer à la Ramsey-Lucas who is subject to a cash-in-advance constraint, in addition to the usual intertemporal budget constraint.

The central result of the paper is that a credible, once-and-for-all reduction in the rate of devaluation gives rise to the stylized facts discussed above—appreciation of the real exchange rate and an initial expansion in aggregate demand—only if the intertemporal elasticity of substitution exceeds the static elasticity of substitution between traded and non-traded (or home) goods. Otherwise, aggregate demand does not increase during the first stages of the program. Specifically, when the intertemporal elasticity of substitution is lower than the elasticity of substitution between traded and home goods, aggregate demand for home goods falls following a permanent reduction in the devaluation rate. We also show that, in line with Dornbusch (1982) and Rodriguez (1982), initially (and under all parameter configurations) real interest rates fall on impact and the domestic currency appreciates in real terms during the first stages of the program.

The results of our model reflect the tension between intertemporal substitution effects and relative price effects stemming from the real appreciation of the domestic currency. Specifically, the real appreciation has two effects. First, by lowering the consumption-based real rate of interest, the real appreciation tends to raise present consumption of both goods relative to the future, which increases excess demand for home goods. The extent of this effect is governed by the intertemporal elasticity of substitution. Second, by making home goods more expensive over time, the real appreciation induces a rising time path of consumption of traded goods. Since traded-goods wealth does not change, consumption of traded goods must fall on impact to accommodate the future increase. Given that the relative price of traded goods in terms of home goods (i.e., the real exchange rate) is a predetermined variable, aggregate demand for home goods falls on impact as well. The magnitude of this effect is governed by the elasticity of substitution between traded and home goods.

In sum, the real appreciation generates expansionary effects through intertemporal consumption substitution and recessionary effects through substitution between traded and home goods. In this light, it is not

surprising that a recession ensues whenever the intertemporal elasticity of substitution is lower than the elasticity of substitution between traded and home goods. Available econometric evidence (see Ostry and Reinhart (1992)) indicates that, for developing countries, the intertemporal elasticity of substitution is lower than the elasticity of substitution between traded and home goods. On this basis, therefore, we would expect aggregate demand to fall as a result of a fully-credible exchange rate-based stabilization program. Our results thus call into question the notion that the initial expansion in an exchange rate-based stabilization may be due to backward-looking wage and price behavior. Therefore, in an indirect manner, this paper can be taken as suggesting that credibility considerations appear to be essential for understanding such phenomena. 1/

The paper is organized as follows: Section II presents the basic model; Section III discusses the intuition behind the main results of the paper (formal proofs are relegated to an appendix); and Section IV concludes. The appendix also develops a reduced-form version of the model to show that the difference between the results of our model and those of Rodriguez (1982) is due to the latter's lack of optimizing behavior rather than to the absence of rational expectations.

## II. The Model

This section deals first with the consumer's problem, and then derives equilibrium conditions. The supply side is considered next. Finally, the dynamic system to be used in the next section is derived.

### 1. Consumer's problem

Consider a small open economy that operates under predetermined exchange rates. Perfect capital mobility prevails. The economy is inhabited by a large number of identical individuals who are blessed with perfect foresight. The representative consumer derives utility from the consumption of a traded good (whose price in terms of foreign currency is given and constant) and a non-traded (or home) good. His or her lifetime utility is given by:

$$\int_0^{\infty} u(c_t^*, c_t) \exp(-\beta t) dt, \quad (1)$$

where  $c^*$  and  $c$  denote consumption of traded and home goods, respectively, and  $\beta(>0)$  is the subjective discount rate. The instantaneous utility

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1/ For more direct evidence about the relevance of credibility considerations, see Reinhart and Végh (1992). An alternative explanation for the boom-recession cycle, which relies on the timing of purchases of durable goods, can be found in De Gregorio, Guidotti, and Végh (1992).



function,  $u(\cdot)$ , is assumed to exhibit constant relative risk aversion:

$$u(c_t^*, c_t) = \frac{z_t^{\frac{1-\frac{1}{\rho}}{\rho}} - 1}{1 - \frac{1}{\rho}}, \quad (2)$$

where  $\rho(>0)$  is the intertemporal elasticity of substitution, and  $z$ , an index of total consumption (hereafter referred to as "aggregate consumption"), takes the constant-elasticity-of-substitution (CES) functional form:

$$z_t = \left[ q c_t^{*\frac{\sigma-1}{\sigma}} + (1-q) c_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (3)$$

where  $\sigma(>0)$  denotes the elasticity of substitution between traded and home goods and  $q$ , which satisfies  $0 < q < 1$ , is a constant. 1/

Consumers may hold two financial assets: domestic money, which bears no interest, and an internationally-traded bond, which yields a constant real return  $r$  (in terms of traded goods). Real financial wealth, denoted by  $a$ , is thus

$$a_t = m_t + b_t, \quad (4)$$

where  $m$  and  $b$  denote real domestic money balances and the real stock of bonds in the hands of the public, respectively. 2/ The lifetime budget constraint of the representative consumer is given by:

$$a_0 + \int_0^\infty (y_t^* + y_t/e_t + \tau_t) \exp(-rt) dt - \int_0^\infty (c_t^* + c_t/e_t + i_t m_t) \exp(-rt) dt, \quad (5)$$

where  $a_0$  is initial real financial wealth;  $y^*$  and  $y$  denote output of traded and home goods, respectively;  $e$  is the real exchange rate (i.e., the relative price of traded goods in terms of home goods);  $\tau$  are real lump-sum transfers from the government; and  $i$  denotes the instantaneous nominal interest rate in terms of domestic currency. 3/ Equation (5) states that

1/ Note that when  $\sigma=1$ , aggregate consumption, given by equation (3), takes a Cobb-Douglas form, where  $q$  denotes the (constant) share of traded goods in total consumption.

2/ With the exception of the real exchange rate (see below), all "real" variables denote nominal variables deflated by the price of traded goods.

3/ The real exchange rate is defined as  $e=EP^*/P$ , where  $E$  is the nominal exchange rate (in units of domestic currency per unit of foreign currency),  $P^*$  is the (constant) foreign currency price of traded goods, and  $P$  is the nominal price of home goods.

lifetime expenditure cannot exceed, and at an optimum will be equal to, lifetime income.

The consumer must use money to purchase goods. Assuming a positive nominal interest rate, the cash-in-advance constraint takes the form:

$$\alpha(c_t^* + c_t/e_t) = m_t, \quad (6)$$

where  $\alpha$  is a positive constant. 1/ Using equation (6), the consumer's lifetime budget constraint, equation (5), can be rewritten as:

$$a_0 + \int_0^\infty (y_t^* + y_t/e_t + \tau_t) \exp(-rt) dt = \int_0^\infty (c_t^* + c_t/e_t)(1 + \alpha i_t) \exp(-rt) dt. \quad (7)$$

The consumer chooses paths of  $c^*$  and  $c$  to maximize lifetime utility, given by (1), subject to the intertemporal budget constraint, equation (7), given initial real financial wealth,  $a_0$ , and expected (equal to actual, given the assumption of perfect foresight) paths of  $y$ ,  $y^*$ ,  $r$ ,  $\tau$ ,  $e$ , and  $i$ . The first-order conditions are 2/

$$u_{c^*}(c_t^*, c_t) = \lambda(1 + \alpha i_t), \quad (8)$$

$$\frac{u_{c^*}(c_t^*, c_t)}{u_c(c_t^*, c_t)} = e_t, \quad (9)$$

where  $\lambda$  is the Lagrange multiplier associated with budget constraint (7), which can be interpreted as the marginal utility of wealth. Equation (8) is the familiar condition whereby the consumer equates the marginal utility of consumption of traded goods to the marginal utility of wealth,  $\lambda$ , times the effective price of consumption,  $1 + \alpha i$ . The effective price of consumption includes the opportunity cost of holding the real money balances needed to purchase goods. First-order condition (9) equates the marginal rate of substitution between traded and home goods to their relative price,  $e$ . Taking into account the particular functional form of the utility function,

1/ Since only permanent policy changes will be considered, the results would not change if money was an argument in the utility function, as long as it entered additively or multiplicatively. This ensures that first-order condition (9) (see below) is independent of real money balances.

2/ To ensure the existence of a steady-state, it is assumed that  $\beta = r$ .

given by equations (2) and (3), equation (9) can be rewritten as

$$\frac{c_t}{c_t^*} = \eta^\sigma e_t^\sigma, \quad (10)$$

where  $\eta \equiv (1-q)/q$ . Equation (10) indicates that the larger is the elasticity of substitution between traded and home goods,  $\sigma$ , the greater is the increase in the ratio of consumption of home to traded goods that results from a given increase in the real exchange rate (i.e., an increase in the relative price of traded goods).

Let us now derive the laws of motion for aggregate consumption, and consumption of traded and home goods. Totally differentiating first-order conditions (8) and (10), taking into account (2) and (3), yields the law of motion for aggregate consumption:

$$\frac{\dot{z}_t}{z_t} = \left( \frac{\rho}{1+\Gamma_t} \right) \frac{\dot{e}_t}{e_t}, \quad (11)$$

where

$$\Gamma_t \equiv \eta^{-\sigma} e_t^{1-\sigma} > 0. \quad (12)$$

As discussed below, equation (11) indicates that the growth of aggregate consumption over time is dictated by the difference between the real interest rate relevant for aggregate consumption—referred to as the "consumption-based" real interest rate—and the subjective rate of time preference, due to standard intertemporal considerations (see Dornbusch (1983)).

The consumption-based real interest rate is a weighted average of the real interest rates relevant for the consumption of traded and home goods. To see this, note that the real interest rate relevant for consumption of traded goods is  $r$  (the world real interest rate), while the real interest rate relevant for consumption of home goods is  $r + \dot{e}/e$  (referred to as "the domestic real interest rate," and denoted by  $r^d$ ). 1/ To fix ideas, consider first the Cobb-Douglas case (i.e.,  $\sigma=1$ ), where  $q$  and  $1-q$  denote the shares of traded and home goods, respectively. 2/ The average of the two real interest rates, weighted by consumption shares in total consumption, is thus  $qr + (1-q)(r + \dot{e}/e)$ , which equals  $r + (1-q)\dot{e}/e$ , the consumption-based real

1/ Note that, under perfect capital mobility,  $r^d = i - \pi$  (where  $\pi$  is the inflation rate of home goods), which is the definition usually found in macro-models.

2/ By share of home goods, we mean the share of  $c/e$  (i.e., consumption of home goods in terms of traded goods) in  $c/e + c^*$  (i.e., total consumption in terms of traded goods).

interest rate. Since it can be shown that, when  $\sigma=1$ ,  $1/(1+\Gamma)$  equals  $1-q$ , the term that multiplies  $\rho$  on the RHS of equation (11) is simply  $(1-q)(\dot{e}/e)$ , which is indeed the difference between the consumption-based real interest rate and the subjective rate of time preference,  $\beta$  (recall that, by assumption,  $\beta=r$ ).

In the CES case, the share of home goods in total consumption is no longer constant. Indeed, it is easy to check that, for the CES case, the share of home goods in total consumption is  $1/(1+\Gamma)$ , while that of traded goods is  $\Gamma/(1+\Gamma)$ . <sup>1/</sup> As in the Cobb-Douglas case, the consumption-based real interest rate is a weighted average of the real interest rates of traded and home goods. Specifically, the weighted average, given by  $[\Gamma/(1+\Gamma)]r + [1/(1+\Gamma)](r + \dot{e}/e)$ , equals  $r + [1/(1+\Gamma)]\dot{e}/e$ , the consumption-based real interest rate. Hence, the term that multiplies  $\rho$  on the RHS of equation (11) can be viewed as the difference between the consumption-based real interest rate and the rate of time preference.

When the relative price of traded goods is rising over time (i.e.,  $\dot{e} > 0$ ), the consumption-based real interest rate exceeds the world real interest rate because a unit of traded goods borrowed today will be more costly to repay tomorrow in terms of the consumption-basket. As a result, today's aggregate consumption is lower than in the future, with the magnitude being dictated by the intertemporal elasticity of substitution,  $\rho$ . When  $\rho$  is large, for instance, small deviations of the consumption-based real interest rate from  $\beta$  will cause large intertemporal consumption substitution.

The laws of motion for consumption of traded and home goods, which also follow from totally differentiating (8) and (10) and using (2) and (3), are given by

$$\frac{\dot{c}_t}{c_t} = \frac{\dot{z}_t}{z_t} + \sigma \left( \frac{\Gamma_t}{1+\Gamma_t} \right) \frac{\dot{e}_t}{e_t}, \quad (13)$$

$$\frac{\dot{c}_t^*}{c_t^*} = \frac{\dot{z}_t}{z_t} - \sigma \left( \frac{1}{1+\Gamma_t} \right) \frac{\dot{e}_t}{e_t}. \quad (14)$$

Equations (13) and (14) show that consumption of traded and home goods responds to two forces. The first force is aggregate consumption—first term on the right-hand side (RHS) of equations (13) and (14)—which captures

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<sup>1/</sup> As one would expect, the share of home goods in total consumption is increasing in  $1-q$ . The effect of an increase in the real exchange rate on the share of home goods depends on whether  $\sigma$  is greater or smaller than unity. If  $\sigma < 1$ , a rise in  $e$  decreases the share of home goods in total consumption expenditure, while the opposite is true if  $\sigma > 1$ .

intertemporal consumption substitution. If aggregate consumption is, say, rising over time, reflecting the considerations discussed above, consumption of both traded and home goods rises over time as well. Note that if  $\sigma$  is set equal to 0 in equations (13) and (14) (i.e., traded and home goods are consumed in fixed proportions), the evolution of consumption of either good over time is identical to that of aggregate consumption. The second effect derives from substitution between the two goods, which is captured by the second term on the RHS of equations (13) and (14). If the real exchange rate is, say, increasing over time, traded goods will be more expensive relative to home goods tomorrow than today, which induces substitution away from traded goods—second term on the RHS of equation (14)—and towards home goods—second term on the RHS of equation (13). The substitution between goods is higher the larger is  $\sigma$ .

Consider a situation in which the real exchange rate is increasing over time (i.e., the domestic currency is depreciating in real terms). Aggregate consumption is also rising over time (equation (11)) since the consumption-based real interest rate is above the subjective rate of time preference. This implies that, on this account, both  $c^*$  and  $c$  are increasing over time as well. At the same time, the real depreciation makes traded goods more expensive, thus inducing substitution away from traded goods and towards home goods. In the case of home goods, both effects reinforce each other. In the case of traded goods, however, the two effects go in opposite direction. Formally, using equation (11), equations (13) and (14) can be rewritten as:

$$\frac{\dot{c}_t}{c_t} = \left( \frac{\rho + \sigma \Gamma_t}{1 + \Gamma_t} \right) \frac{\dot{e}_t}{e_t}, \quad (15)$$

$$\frac{\dot{c}_t^*}{c_t^*} = \left( \frac{\rho - \sigma}{1 + \Gamma_t} \right) \frac{\dot{e}_t}{e_t} \quad (16)$$

Equation (15) shows how both effects reinforce each other so that an increasing real exchange rate path implies a rising consumption path of home goods. Equation (16) indicates that, as suggested above, there is tension between intertemporal consumption substitution (as measured by  $\rho$  in equation (16)) and substitution between goods (as measured by  $\sigma$  in equation (16)). Specifically, when  $\rho = \sigma$ , both effects cancel each other and the time path of traded goods is flat. When  $\rho > \sigma$ , the intertemporal channel prevails over the substitution channel and thus an increasing real exchange rate results in a rising path of  $c^*$ . When  $\rho < \sigma$ , substitution between goods dominates so that  $c^*$  falls as the real exchange rate rises.

## 2. Equilibrium conditions

To derive the resource constraint for the economy as a whole, consider first the government's budget constraint:

$$\int_0^{\infty} \tau_t \exp(-rt) dt = h_0 + \int_0^{\infty} (\dot{m}_t + \epsilon_t m_t) \exp(-rt) dt, \quad (17)$$

where  $\epsilon (= \dot{E}/E)$  denotes the rate of devaluation, and  $h_0$  is the government's initial stock of bonds. Equation (17) states that the present discounted value of transfers to the private sector, given by the LHS of equation (17), equals the present discounted value of revenues from money creation, given by the RHS of equation (17), plus the initial net assets of the government.

The assumption of perfect capital mobility implies that (recall that  $P^*$  is constant)

$$i_t = r + \epsilon_t. \quad (18)$$

Equilibrium in the home goods market requires that

$$c_t = y_t. \quad (19)$$

The economy's resource constraint follows from combining equations (7), (17), (18), and (19):

$$k_0 + \int_0^{\infty} y_t^* \exp(-rt) dt = \int_0^{\infty} c_t^* \exp(-rt) dt, \quad (20)$$

where  $k_0 = b_0 + h_0$  denotes the economy's initial stock of bonds. Equation (20) states that the initial net stock of bonds plus the present discounted value of traded resources, given by the LHS, must equal the present discounted value of consumption of traded goods, given by the RHS.

The flow constraint corresponding to equation (20) is (see Calvo and Végh (1991))

$$\dot{k}_t = y_t^* + rk_t - c_t^*, \quad (21)$$

which indicates that the current account balance,  $\dot{k}$ , is the difference between income of traded resources and consumption of traded goods.

## 3. Supply side

For simplicity, it will be assumed that the supply of traded goods is exogenously given at the (constant) level  $y^*$ , while the home-goods sector operates under sticky prices (i.e., the nominal price of home goods is given at each point in time), and output is demand determined. Prices (or wages)

are assumed to be set according to a backward-looking rule. Specifically, we assume that inflation of home goods,  $\pi$ , is governed by the following backward-looking mechanism:

$$\pi_t = \omega_t + \theta(c_t - \bar{y}), \quad (22)$$

where  $\bar{y}$  denotes full-employment output of home goods and  $\theta(>0)$  measures the effect of excess aggregate demand on inflation of home goods. The variable  $\omega$ , which may be interpreted as the rate of increase of nominal wages, is inherited from the past and evolves over time according to

$$\dot{\omega}_t = \gamma(\pi_t - \omega_t), \quad (23)$$

where  $\gamma$  is a positive parameter. Equation (23), which captures backward-looking indexation in the simplest possible way, states that the rate of growth of nominal wages is adjusted according to the difference between inflation of home goods and the current rate of growth of nominal wages. 1/

To illustrate the implications of this set-up, integrate backwards (23) and substitute the resulting expression for  $\omega$  into (22), to obtain

$$\pi_t = \gamma \int_{-\infty}^t \pi_s \exp[-\gamma(t-s)] ds + \theta(c_t - \bar{y}). \quad (24)$$

Equation (24) indicates that current inflation depends on a weighted average of past inflation rates—with inflation rates in the recent past receiving the greatest weight—and current excess aggregate demand, which is what the notion of "inflation inertia" is usually taken to mean. (see, for instance, Dornbusch and Simonsen (1987)).

#### 4. Dynamic system

The dynamics of the system can be studied by considering a two-equation system in  $c$ , consumption of home goods, and  $\omega$ , the rate of growth of nominal wages. By definition,  $e = EP^*/P$ . Hence (recalling that  $P^*$  is assumed constant) 2/

1/ Alternatively, one could assume that inflation is a predetermined variable and that the change in the rate of inflation is given by  $\dot{\pi}_t = \kappa(\epsilon_t - \pi_t) + \xi(c_t - \bar{y})$ , where  $\kappa$  and  $\xi$  are positive parameters, along the lines of Dornbusch (1982). The same solution methods discussed in Appendix I can be used to solve the model under this alternative specification. It can be verified that the same key results discussed below would follow.

2/ Since the rate of devaluation will be assumed constant over time, the time subscript on  $\epsilon$  will be dropped.

$$\frac{\dot{e}_t}{e_t} = \epsilon - \pi_t. \quad (25)$$

Substituting equations (25) and (22) into (15) yields:

$$\frac{\dot{c}_t}{c_t} = A_t[\epsilon - \omega_t - \theta(c_t - \bar{y})], \quad (26)$$

where  $A_t = (\rho + \sigma\Gamma_t)/(1 + \Gamma_t) > 0$ .

Substituting equation (22) into (23) yields the second dynamic equation:

$$\dot{\omega}_t = \gamma\theta(c_t - \bar{y}) \quad (27)$$

When linearized around the steady-state, equations (26) and (27) constitute a dynamic system in  $c$  and  $\omega$ . 1/ The system is stable (i.e., both roots have negative real parts). 2/ Furthermore, the adjustment can be cyclical (i.e., the roots may be complex). 3/ Hence, under perfect foresight—and for a given set of parameters—all continuous equilibrium paths (the only ones considered in this paper) converge to the steady-state. Figure 1 illustrates the phase diagram for the system (26) and (27). The directional arrows correspond to point E. Whatever the initial condition (points A, B, or C), the system converges to the steady-state, point E. 4/5/ To understand the dynamics of the adjustment process, suppose that, initially, the system is at point B. At point B, there is no excess aggregate demand because  $c = \bar{y}$  so that inflation equals the rate of growth of nominal wages (see equation (22)). Inflation, however, is above the devaluation rate (given by  $\epsilon^L$  in Figure 1), so that the real exchange rate is decreasing (i.e., the relative price of home goods is increasing), which reduces consumption of home goods (equation (15)). As consumption of home goods falls below full-employment output, the resulting negative excess aggregate demand reduces the rate of inflation (equation (22)). The fall in

1/ The steady-state values of  $\omega$ ,  $c$ , and  $e$  are given below in equations (28), (31), and (33), respectively. Note that although  $e$  enters the system through  $\Gamma$  (recall equation (12)), the linearized system does not depend on  $e$ .

2/ The trace of the matrix associated with the linear approximation is  $-A\theta\bar{y}(<0)$ , while the determinant is  $A\gamma\theta\bar{y}(>0)$ .

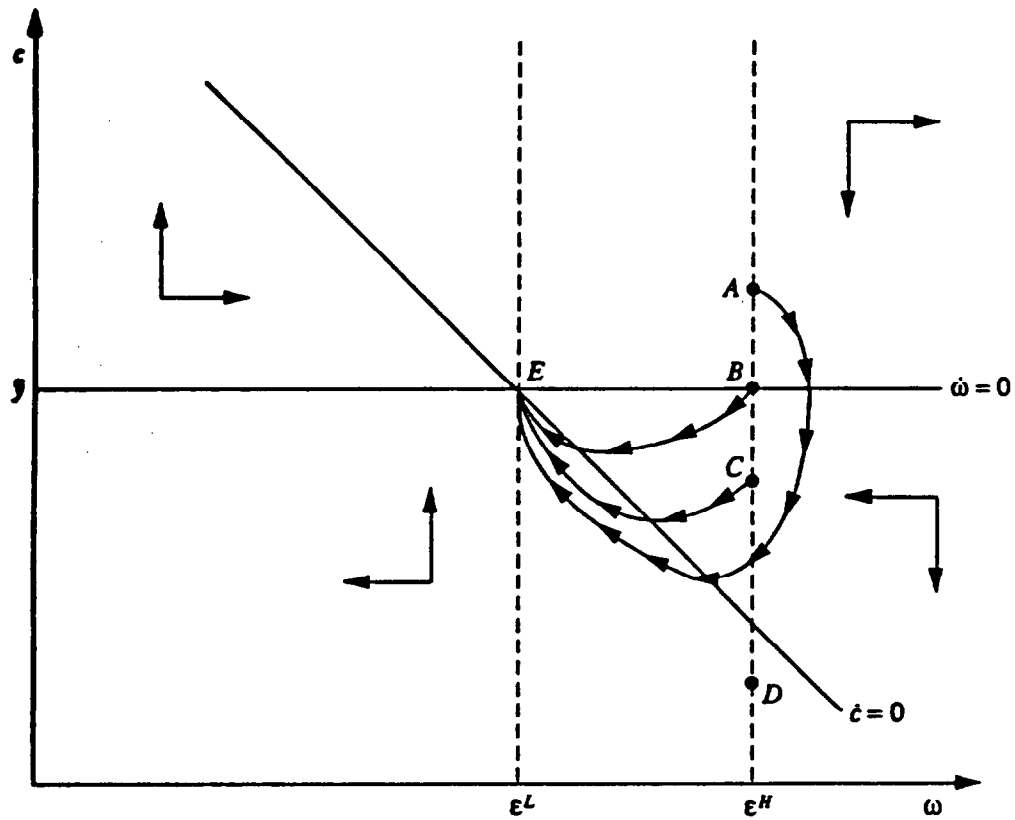
3/ For the sake of exposition, we will concentrate on the case in which roots are real, and indicate how the results extend to the complex-roots case in Appendix I.

4/ As explained below, the initial condition will be determined by the initial jump in  $c^*$  (recall (10)), which ensures that the equilibrium path is unique.

5/ Appendix I shows that point D is not a feasible initial condition when there is a permanent reduction in the rate of devaluation.



Figure 1  
Dynamic System





inflation induces a reduction in the rate of growth of nominal wages (equation (23)), which further lowers inflation (by equation (22)). At some point, inflation falls below the rate of devaluation (after the dynamic path in Figure 1 crosses the  $\dot{c}=0$ -schedule), so that inflation undershoots its long-run value. 1/ The real exchange rate begins to rise (i.e., the relative price of home goods begins to fall), which increases consumption of home goods. The rise in consumption of home goods tends to increase the inflation rate, which eventually returns to the level  $\epsilon^L$ .

### III. Permanent Reduction in the Devaluation Rate

This section examines the effects of a permanent reduction in the devaluation rate. We will show that the initial change in consumption of home goods depends on the relationship between the intertemporal elasticity of substitution,  $\rho$ , and the elasticity of substitution between home and traded goods,  $\sigma$ .

Suppose that, initially (i.e., for  $t < 0$ ), the devaluation rate is  $\epsilon^H$ , and is expected to remain at that level forever. The economy is thus at a steady-state (which corresponds to point B in Figure 1) characterized by (as follows from equations (3), (10), (20), (23), (25), and (27)):

$$\omega_{ss} = \epsilon^H \quad (28)$$

$$\pi_{ss} = \epsilon^H, \quad (29)$$

$$c_{ss}^* = y^* + rk_0 \quad (30)$$

$$c_{ss} = \bar{y}, \quad (31)$$

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1/ Note that when  $\dot{c}=0$ ,  $\dot{\epsilon}=0$  (equation (15)). Hence, at that point,  $\pi=\epsilon^L$  (equation (25)).

$$z_{ss} = \left[ q c_{ss}^* \frac{\sigma-1}{\sigma} + (1-q) \bar{y} \frac{\sigma-1}{\sigma} \right] \frac{\sigma}{\sigma-1}, \quad (32)$$

$$e_{ss} = \frac{1}{\eta} \left( \frac{\bar{y}}{c_{ss}^*} \right)^{\frac{1}{\sigma}} \quad (33)$$

$$r_{ss}^d = r, \quad (34)$$

At a steady-state, nominal variables are growing at the rate  $\epsilon^H$ , as indicated by equations (28) and (29). Consumption of traded goods equals its permanent income level (equation (30)), while consumption of home goods equals its full-employment level (equation (31)). Aggregate consumption varies directly with consumption of traded goods (equation (32)). The steady-state real exchange rate varies inversely with consumption of traded goods (equation (33)). The steady-state domestic real interest rate equals the world real interest rate (recall that  $r^d = r + \dot{e}/e$ ), as indicated by equation (34)).

Suppose now that at  $t=0$ , the authorities announce a permanent reduction in the devaluation rate from  $\epsilon^H$  to  $\epsilon^L$ . The announcement is assumed to be fully credible so that the public acts on the expectation that the devaluation rate will remain at  $\epsilon^L$  for the indefinite future. In terms of Figure 1, the steady-state becomes point E. Since  $\omega$  cannot jump, the key issue is the initial jump in  $c$ . We will argue that depending on whether  $\rho$  is equal to, greater than, or smaller than  $\sigma$ , consumption of home goods remains constant, increases, or decreases, respectively.

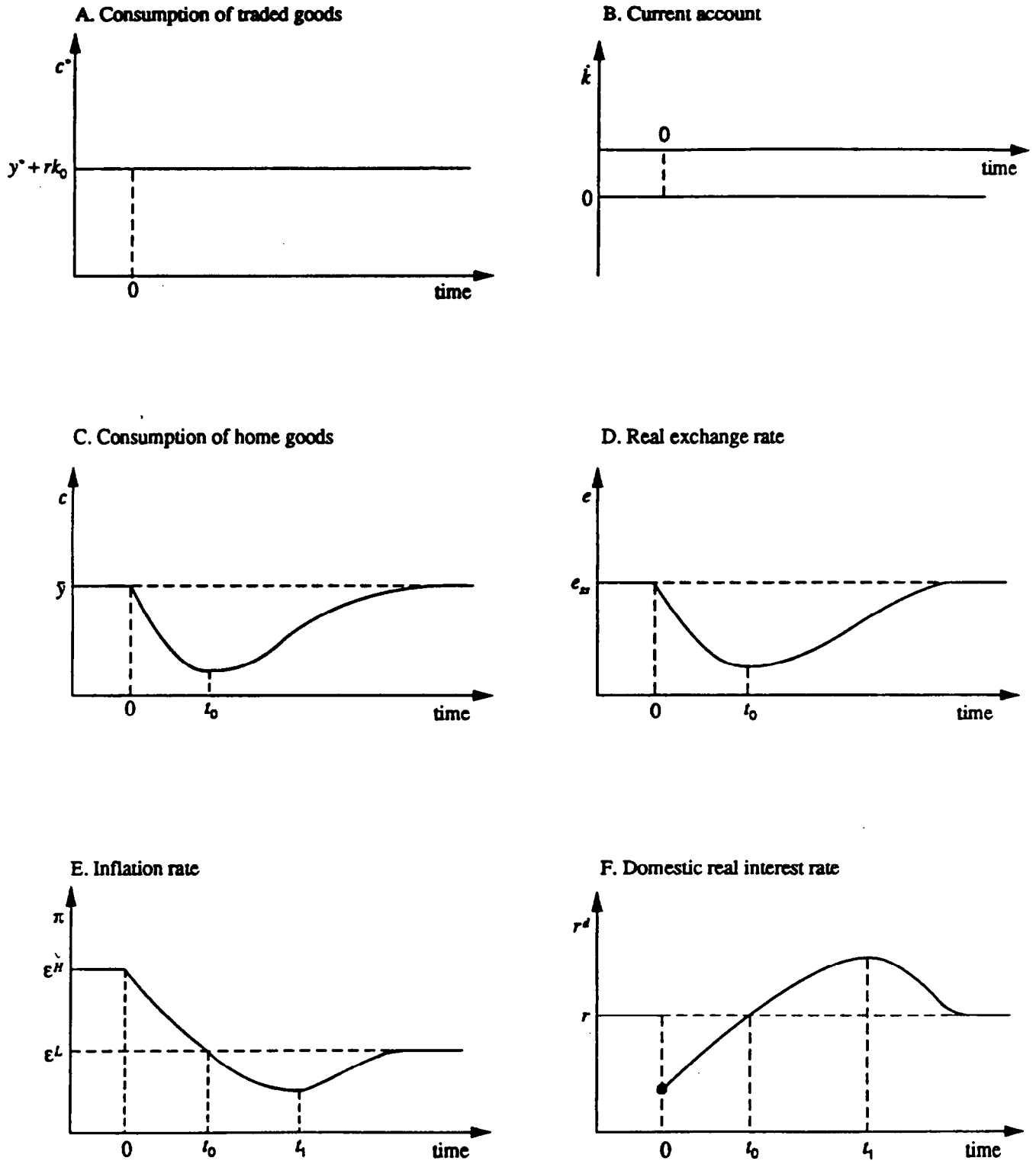
#### Case 1: $\rho = \sigma$

This is the simplest case from an analytical point of view, and constitutes the benchmark case. The reason is that, as indicated by equation (16), the path of  $c^*$  is flat over time because, whatever the path of the real exchange rate, the intertemporal effects are exactly offset by the substitution effects between the two goods, as explained above. This implies that  $c^*$  cannot jump at  $t=0$  because, if it did, the resource constraint (20) would be violated. Figure 2, Panel A, illustrates the corresponding path of  $c^*$ . <sup>1/</sup> Naturally, the current account remains balanced (Figure 2, Panel B).

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<sup>1/</sup> To facilitate the comparison between the three cases, the path of the same six variables (in the real-roots case) will be drawn in each case.

Figure 2  
Permanent Reduction in Devaluation Rate:  $\rho = \sigma$





Since neither  $c^*$  nor  $e$  jump at  $t=0$ , equation (10) indicates that  $c$  also remains constant on impact. Hence, the dynamic system stays at point B in Figure 1 at time 0 and follows the arrowed path over time. The path of  $c$  is illustrated in Figure 2, Panel C. Recalling equation (15), it follows that this pattern of  $c$  can only be consistent with real appreciation in the first phase of the program and depreciation later, as illustrated in Figure 2, Panel D. <sup>1/</sup> The path of aggregate consumption (not drawn),  $z$ , mimics the path of  $c$ , given that  $c^*$  remains at its initial level.

The path of inflation of home goods, which follows from equation (22) and Figure 1, is illustrated in Figure 2, Panel E. At  $t=0$ , inflation does not change because the rate of growth of nominal wages is given and there is no excess aggregate demand. As the real exchange rate appreciates, however, consumption falls below its full-employment level, which reduces inflation through two channels. First, negative excess aggregate demand directly reduces inflation (see equation (22)). Second, because inflation is now below the rate of growth of nominal wages, the latter begins to be adjusted downward, as follows from equation (23), which feeds back into lower inflation through equation (22). At  $t=t_0$ , inflation falls below its new steady-state value,  $\epsilon^L$ , reaches a minimum at  $t=t_1$  and then increases back to  $\epsilon^L$  as the effects of higher consumption of home goods prevail over the continuing fall of the rate of growth of nominal wages (see equation (22)).

The time path of the domestic real interest rate is illustrated in Figure 2, Panel F. Recall that  $r^d = i - \pi$ . On impact,  $\pi$  remains constant but  $i$  falls one to one with the fall in  $\epsilon$  due to the assumption of perfect capital mobility. Hence,  $r^d$  falls on impact. After the initial fall, the path of  $r^d$  mirrors that of inflation since  $\dot{r}^d = -\dot{\pi}$ . Thus,  $r^d$  reaches a maximum at  $t=t_1$  and then falls back to its unchanged steady-state value.

In summary, in the benchmark case in which  $\rho = \sigma$ , there is no expansion in the home-goods sector in spite of the fact that the domestic real interest rate falls. This is because, in the presence of optimizing consumers, a fall in the real interest rate relevant for home-goods consumption dictates the slope but not the level of consumption of home goods. This model clearly illustrates this point since, as we will see, the domestic real interest rate falls for any values of  $\rho$  and  $\sigma$ . However, consumption of home goods may increase, decrease, or stay constant on impact. Hence, there is no relation between the level of consumption of home goods on impact and the behavior of the domestic real interest rate.

#### Case 2: $\rho < \sigma$

In this case, which seems to be the relevant one empirically (see Ostry and Reinhart (1992)), consumption of both home and traded goods falls on

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<sup>1/</sup> Note that, since  $c_{ss}^*$  does not change,  $e_{ss}$  remains the same (see equation (33)).

impact. 1/ First, note that since  $\rho < \sigma$ , equation (16) indicates that consumption of traded goods and the real exchange rate move in opposite directions, because the substitution effects between goods (which call for  $c^*$  and  $e$  to move in opposite directions) more than offset the intertemporal substitution effects (which call for  $c^*$  and  $e$  to move in the same direction). Furthermore, whatever the initial position of the dynamic system in Figure 1 (i.e., points A, B, or C), we know that  $c$  follows a U-shape path. 2/ This implies that the real exchange rate path also follows a U-shaped time path (recall equation (15)), as illustrated in Figure 3, Panel D. 3/

Let us now consider the initial jump in consumption. Appendix I shows that both  $c^*$  and  $c$  fall on impact, and hence follow the paths illustrated in Panels A and C in Figure 3, respectively. Intuitively, in the early stages of the program, the real appreciation (i.e., the fall in the relative price of traded goods) induces a rising path of consumption of traded goods as the consumer substitutes away from home goods and towards traded goods. Since traded-goods wealth, given by the LHS of equation (20), is not affected by changes in the devaluation rate, the present discounted value of consumption of traded goods must remain unchanged. Given this wealth constraint, consumption of traded goods must fall on impact to accommodate the future increase. If it did not fall, the wealth constraint would be violated. On impact, the real exchange rate is given, so that, by equation (10), a fall in consumption of traded goods must be accompanied by a fall in consumption of home goods.

The path of the current account is illustrated in Figure 3, Panel B. On impact, the current account jumps into surplus as a result of the fall in  $c^*$ . Eventually, the current account turns into deficit as consumption of traded goods increases above its initial level. 4/

The inflation rate of home goods,  $\pi$ , falls on impact reflecting the reduction in excess aggregate demand, as illustrated in Figure 3, Panel E. Afterwards, the path of inflation is similar to that which occurs in the previous case. The behavior of the domestic real interest rate is also qualitatively the same. Quantitatively, the initial fall in the domestic real interest rate is smaller than in the previous case because, in the present case, inflation of home goods initially falls.

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1/ Technically, this case and the next one are somewhat involved. Hence, formal proofs will be relegated to Appendix I.

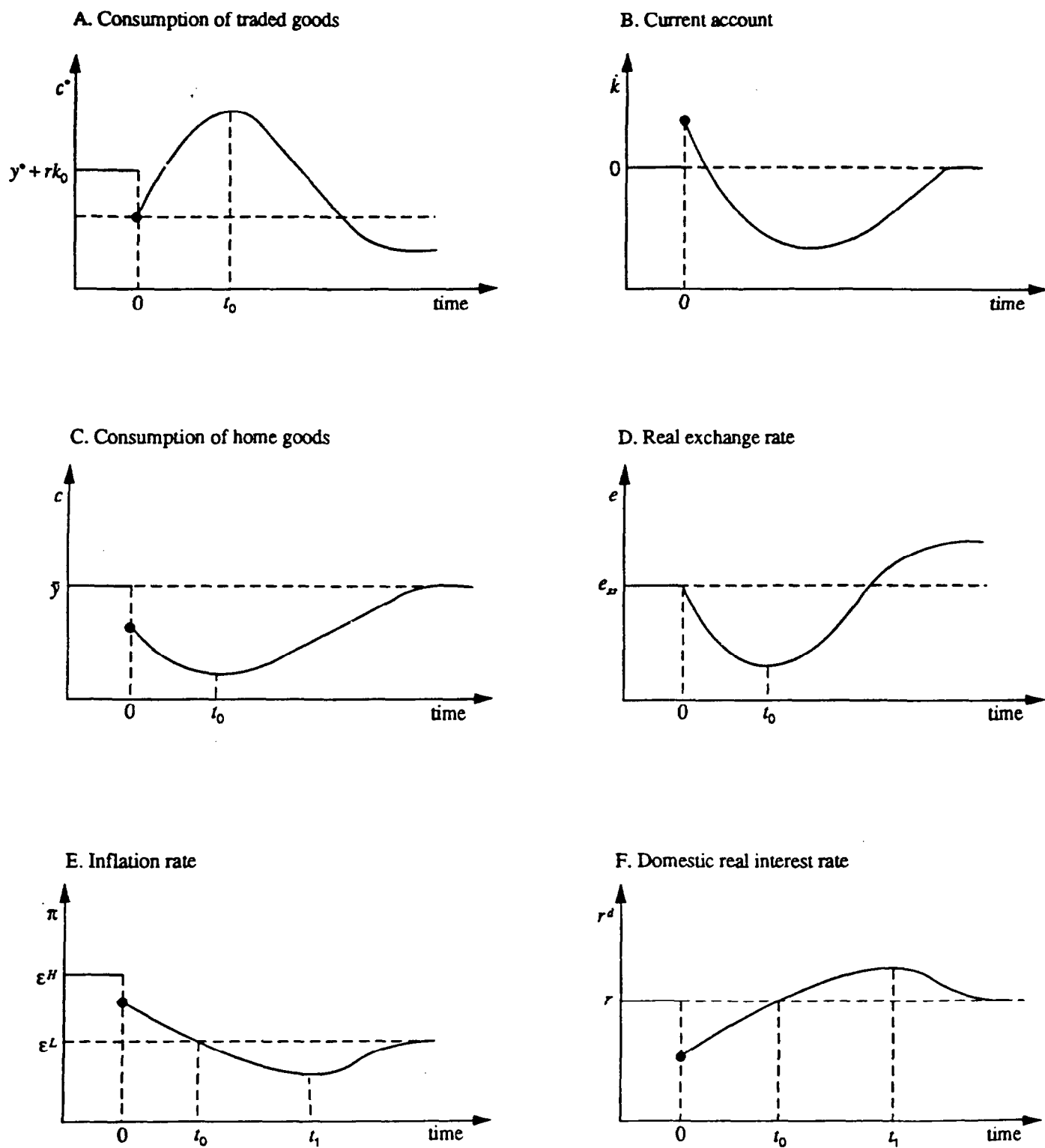
2/ Appendix I shows that, for the real-roots case, point D cannot be the initial condition.

3/ Note that since, as shown below, steady-state consumption of traded goods decreases, the steady-state real exchange rate increases.

4/ Formally, we know that a current account deficit will eventually emerge because the steady-state value of  $c^*$  decreases, which implies that the steady-state level of net foreign assets,  $k$ , also decreases. Hence, at some point in time, there must be a deficit in the current account which lowers the stock of net foreign assets.



Figure 3  
Permanent Reduction in Devaluation Rate:  $\rho < \sigma$





In sum, we have shown that when  $\rho < \sigma$ , an exchange rate-based stabilization results in an initial recession. Since empirical evidence for developing countries suggests that  $\rho$  is indeed smaller than  $\sigma$  (see Ostry and Reinhart (1992)), we conclude that backward-looking indexation by itself may not be capable of explaining the initial boom observed in exchange rate-based stabilizations. 1/

Case 3:  $\rho > \sigma$

In this case, there is an expansion (i.e., consumption of home goods increases on impact). 2/ The starting point of the analysis is, as before, equations (15) and (16). Since  $\rho > \sigma$ , consumption of traded goods and the real exchange rate move in the same direction because intertemporal considerations more than offset the substitution effects between traded and home goods. It follows from Figure 1 that, whatever the initial condition (points A, B, or C), consumption of home goods falls at first and then rises. 3/ Consumers will be induced to choose such a path if the real exchange appreciates at first and depreciates later (recall equation (15)), as illustrated in Figure 4, Panel D. Since intertemporal substitution considerations now dominate over substitution effects, consumption of traded goods decreases at first and increases later.

Appendix I shows that, in the real-roots case, both  $c^*$  and  $c$  increase on impact, and thus follow the paths illustrated in Panels A and C in Figure 4, respectively. Intuitively, in the early stages of the program, the real appreciation induces a decreasing path of  $c^*$  because the fall in the consumption-based real interest rate makes today's consumption cheaper than future consumption. Given that traded-goods wealth remains unchanged, consumption of traded goods must increase on impact to accommodate the declining path that will follow. If it did not, the resource constraint (20) would be violated. The initial rise in consumption of traded goods implies that consumption of home goods also increases on impact (see equation (10)).

The initial upward jump in consumption of traded goods is reflected in an initial current account deficit (see Figure 4, Panel B). Eventually, a current account surplus must emerge because, at the steady-state, consumption of traded goods and, therefore, the stock of net foreign assets are higher than in the initial steady-state.

The initial expansion in the home-goods sector implies that inflation of home goods increases on impact. Inflation then follows the path that it does in the previous two cases, undershooting its (lower) steady-state

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1/ As shown in Appendix I, this result also holds when roots are complex.

2/ This result is proved in Appendix I for the real-roots case. We conjecture that this will also be the case when roots are complex (see the discussion in Appendix I), but a formal proof remains an open issue.

3/ Appendix I shows that a point such as D in Figure 1 cannot be the initial condition.

value. 1/ The path of the domestic real interest rate is qualitatively the same as before. 2/ On impact, however, the domestic real interest rate falls by more than it does in the previous two cases due to the additional effect of higher inflation.

In sum, we have shown that when the intertemporal elasticity of substitution is higher than the elasticity of substitution between traded and home goods, there is an expansion in the home-goods sector. Interestingly, under this parameter configuration, the same results as in Rodriguez (1982) obtain: aggregate demand expands, the real interest falls, and inflation increases. The final section discusses the relationship between this model and Rodriguez's (1982).

#### IV. Final Remarks

This paper has analyzed the effects of backward-looking contracts on exchange rate-based stabilization in a utility-maximizing model. The model shows that a permanent reduction in the rate of devaluation may lead to an initial consumption boom only if the elasticity of substitution between traded and home goods is smaller than the intertemporal elasticity of consumption substitution. In the opposite case, a recession ensues because the expansionary effects generated by intertemporal substitution are more than offset by the recessionary effects that result from substitution between home and traded goods. Thus, the model casts some doubts on the extreme view that backward-looking indexation is the central factor accounting for the kind of phenomena observed in the Southern Cone, since available econometric evidence suggests that the intertemporal elasticity of substitution is the smaller of the two (see Ostry and Reinhart (1992)).

It should be noted that in all cases our model predicts a fall in the domestic real interest rate. However, contrary to ad-hoc models (for instance, Rodriguez (1982)), our model does not necessarily imply an expansion of aggregate demand. In micro-founded demand models, the real rate of interest determines only the rate of growth of aggregate demand, and not its level. Hence, the larger the real rate of interest, the larger will be the rate of growth of consumption. This underlines the risks of relying on ad-hoc demand models, and the relevance of starting from "first principles"—especially when one is dealing with "transitional" dynamics.

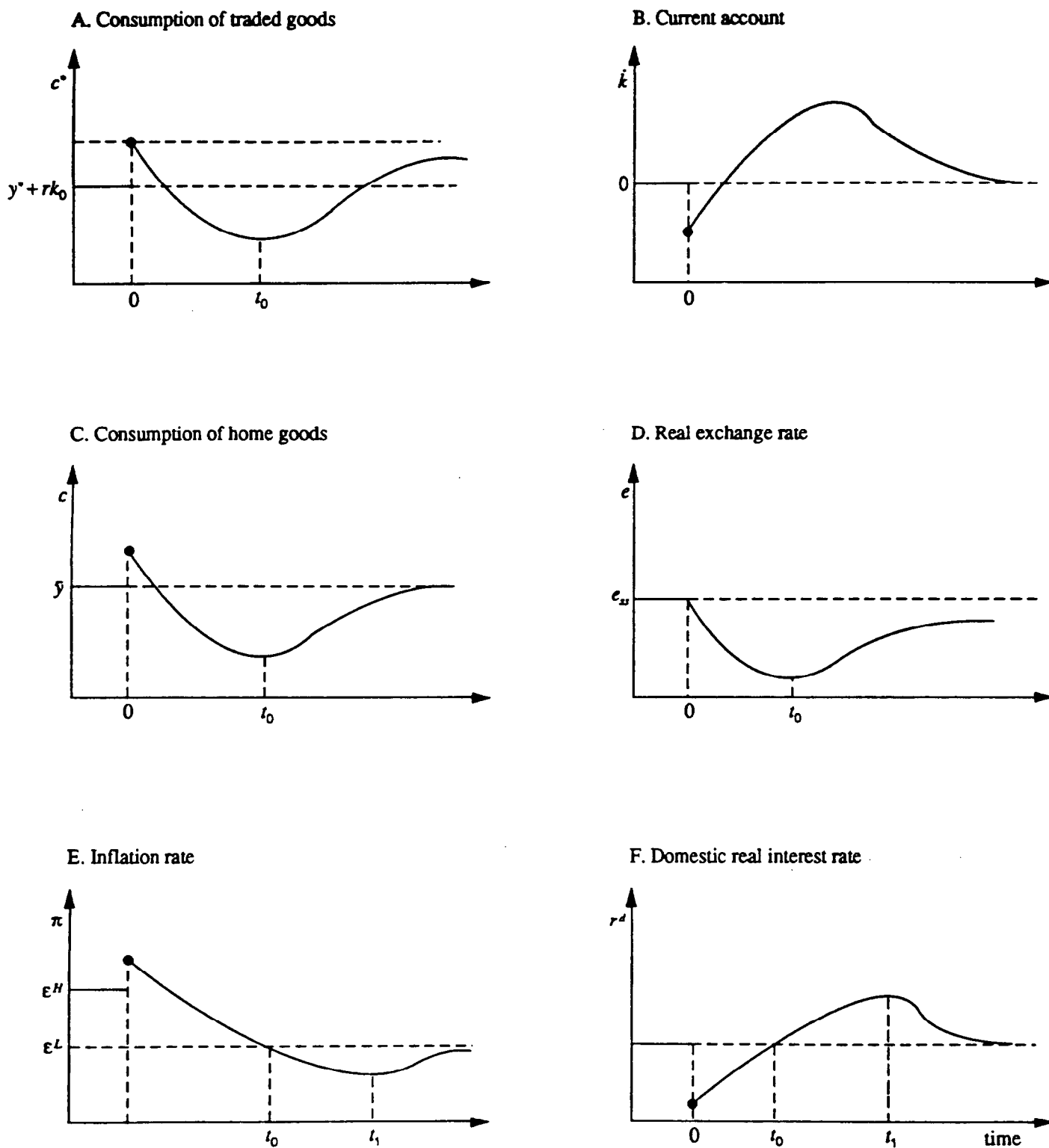
Appendix II makes the above point clear by presenting a reduced-form version of the model (keeping the assumption of rational expectations), in which aggregate demand is assumed to depend negatively on the domestic real interest rate and positively on the real exchange rate. We show that, following a permanent reduction in the rate of devaluation, aggregate demand

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1/ It cannot be ruled out, however, that after increasing on impact, inflation rises further (i.e.,  $\dot{\pi}_0 > 0$ ) before beginning to fall.

2/ If  $\dot{\pi}_0 > 0$ , however, then  $\dot{r}_0^d < 0$ ; that is, the domestic real interest would fall further before beginning to increase.

Figure 4  
Permanent Reduction in Devaluation Rate:  $\rho > \sigma$





for home goods always increases on impact, as in Rodriguez's (1982) model. Hence, the key to the different results that may arise in our model compared to Rodriguez's is the latter's lack of optimizing behavior and not the lack of rational expectations. It should also be noted that ad-hoc models do not take into account resource constraints which, as the above discussion suggests, play a critical role in optimizing models.

The presence of capital mobility is critical for the existence of an initial overheating (or contraction, for that matter) of the economy. In effect, if (fully effective) capital controls were in place, then the real money supply would not change on impact which, through the cash-in-advance constraint (6), ensures that consumption expenditure remains constant on impact. Since the real exchange rate cannot jump on impact, constant real expenditure implies that consumption of either good cannot change on impact--since if one did, the other would as well (recall equation (10)), and hence total expenditure would change. Hence, the fact that, when capital controls are in place, the money supply becomes an additional nominal anchor prevents aggregate demand from changing on impact.

In practice, there are several ways in which policymakers may try to interfere with capital mobility. Policies range from the imposition of taxes on capital inflows, to increasing banks' reserve requirements in order to make domestic deposits less attractive. Sterilization of capital inflows is occasionally tried but--as experience and theory show (see Calvo, Leiderman, and Reinhart (1992) for a discussion)--such a policy may turn out to be counterproductive. Sterilization keeps domestic interest rates at a premium compared to international ones. Thus, a growing quasi-fiscal deficit (i.e., central bank deficit) is likely to develop.

This appendix proves the results discussed in Section III.

Proposition 1. If  $\rho < \sigma$ ,  $c$  falls on impact following a permanent reduction in the devaluation rate.

Proof. We first focus on the real-roots case. Two pieces of information are key to the proof. The first piece of information provides a relationship between the changes in  $c$  and  $c^*$  between time 0 and  $\infty$ . Using (2) and (3), it follows from equations (8) and (9) that (where  $\Delta c$  and  $\Delta c^*$  denote the changes in those variables between  $t=0$  and  $\infty$ ) 1/

$$\frac{\Delta c^*}{c_0^*} = \left( \frac{\rho - \sigma}{\rho + \sigma \Gamma_0} \right) \frac{\Delta c}{c_0} \quad (35)$$

Equation (35), which is the first piece of information, states that, since  $\rho < \sigma$ ,  $\text{sign}(\Delta c^*) = -\text{sign}(\Delta c)$ . The second piece of information is the resource constraint (20), which provides an integral constraint on the path of  $c^*$ .

We will now contradict the hypothesis that  $c$  either stays constant or increases on impact. Suppose that  $c$  does not jump at  $t=0$ , which implies, by equation (10), that  $c^*$  does not jump at  $t=0$  either. Then,  $\Delta c=0$ , since  $c_\infty = \bar{y}$ . Hence, by (35),  $\Delta c^*=0$ ; that is,  $c_\infty^* = c_0^*$ . From Figure 1 and equations (15) and (16), it then follows that after remaining constant on impact,  $c^*$  follows an inverted-U path and returns to its initial steady-state value. Such a path violates the resource constraint (20) because  $c^*$  never falls below the initial permanent income level.

Suppose that  $c$ , and hence, by equation (10),  $c^*$ , jump upwards at  $t=0$ . Then,  $\Delta c < 0$ , because  $c_\infty = \bar{y}$ . Hence, by (35),  $\Delta c^* > 0$ ; that is,  $c_\infty^* > c_0^*$ . Hence, after increasing on impact,  $c^*$  follows an inverted-U shape, and never falls below  $c_0^*$ . Such a path violates the resource constraint (20). We thus conclude that  $c$ , and hence  $c^*$ , must fall on impact. 2/

Consider now the complex-roots case. Suppose that  $c$  remains unchanged on impact. Then, by an analogous reasoning to the real-roots case just examined, it follows that  $c^*$  remains unchanged on impact, follows an

1/ Naturally, we are still treating  $\Delta$  as an infinitesimal change.

2/ Note, however, that a point such as D in Figure 1 cannot be the initial condition (i.e., the initial condition must lie above the  $\dot{c}=0$ -schedule). Indeed, it can be verified that for points which lie below the  $\dot{c}=0$ -schedule but above or on the non-dominant ray eigenvector,  $c^*$  would always fall over time, which would violate the resource constraint (20). (It can be shown that the non-dominant ray eigenvector is steeper (in absolute value) than the dominant-ray eigenvector which, in turn, is steeper (in absolute value) than the  $\dot{c}=0$ -schedule.) For points that lie below the non-dominant ray eigenvector, the path of  $c^*$  follows a U-pattern and either violates the resource constraint (20) or the condition that  $\Delta c^* < 0$ .



inverted-U path at first, then falls below its initial value, and rises again completing the first cycle. 1/ The time path thus displays dampened oscillations around its unchanged steady-state value. Since the real part is negative, the amplitude of the path gets smaller as time goes by, which implies that the area covered in each cycle is greater than initial permanent income. This violates the budget constraint (20).

Suppose now that  $c$  jumps upwards on impact, so that  $c^*$  also increases on impact and displays dampened oscillations around the new steady-state value, which is higher than the value that  $c^*$  takes on impact. Analogous considerations lead us to conclude that this path violates the budget constraint. We thus conclude that, when  $\rho < \sigma$ , consumption of home goods falls on impact. 2/

The following lemma will be needed to prove Proposition 2.

Lemma 1. If  $\rho > \sigma$ ,  $c$  cannot remain constant on impact.

Proof. As before, the proof proceeds by contradiction. Since  $\rho > \sigma$ , equation (35) indicates that the changes in  $c$  and  $c^*$  are now positively related. Suppose that  $c$  does not jump on impact, which implies that  $\Delta c = 0$ . Then, by (35),  $\Delta c^* = 0$ . If roots are real,  $c^*$  does not change on impact, follows a U-path, and returns to its initial steady-state value. This path violates the budget constraint (20). If roots are complex,  $c^*$  remains unchanged on impact, follows a U-path at first, then rises over its initial value and falls again completing the first cycle. The time path then displays dampened oscillations around its unchanged steady-state value. Since the amplitude of the cycles gets smaller as time goes by, the area covered by each cycle is below initial permanent income, which implies that the resource constraint (20) is violated.

Proposition 2. Let roots be real for the case  $\rho = \sigma$ . Then, if  $\rho > \sigma$  and roots are real,  $c$  increases on impact following a permanent reduction in the devaluation rate. 3/

Proof. From Lemma 1, we know that  $c$  cannot remain constant on impact. Now suppose that  $c$  jumps downward on impact. Three cases must be considered. First, suppose that  $c$  falls to a point which lies above the  $\dot{c} = 0$ -schedule in Figure 1. Then,  $\Delta c > 0$ , which implies, by (35), that  $\Delta c^* > 0$ ; that is,  $c_0^* < c_\infty^*$ . Hence, after falling on impact,  $c^*$  follows a U-path and reaches a steady-

1/ It should be clear from (the linear approximation of) the dynamic system (26) and (27) and equations (15), (16), and (25), that if  $c$  exhibits periodic fluctuations, so does  $c^*$  (although the amplitude will differ).

2/ As in the real-roots case, it can be checked that the initial condition cannot lie below the  $\dot{c} = 0$ -schedule because the present value of consumption of traded goods would fall short of traded-good resources.

3/ In the complex-roots case, we have not been able to rule out the possibility that consumption may fall on impact (see the discussion below).

state level which is below initial permanent income, thus violating the resource constraint (20). 1/ Second, suppose that  $c$  falls to a point which lies below the  $\dot{c}=0$ -schedule but above or on the non-dominant ray eigenvector. Then,  $c^*$  falls on impact, and increases over time back to a level below initial permanent income, which violates the resource constraint (20). Third, suppose that  $c$  falls to a point below the non-dominant ray eigenvector. We will resort to a continuity argument to rule out this case. Suppose that the change in  $c$  at  $t=0$  is a continuous function of  $\rho$ . Recall from Section III that  $c$  does not change on impact when  $\rho=\sigma$ . If  $\rho$  is increased by a small amount, it follows by the continuity assumption that, if  $c$  falls, it would lie above the  $\dot{c}=0$ -schedule. But, from the preceding discussion, we know that, if roots are real, the resulting path of  $c^*$  violates the resource constraint (20). Hence, if  $\rho$  is larger than  $\sigma$  by a small amount and roots are real,  $c$  must rise on impact. Now, if  $c$  were to fall for some higher value of  $\rho$ , the continuity assumption implies that there should exist a value of  $\rho(>\sigma)$ , for which  $c$  would not change on impact. But such a value of  $\rho$  does not exist, as Lemma 1 shows. We then conclude that  $c$ , and thus  $c^*$ , increase on impact. 2/

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1/ The conclusion that the steady-state value of  $c^*$  falls is reached as follows. From the first order conditions (8) and (9), it follows, using (2) and (3), that  $\Delta e$  has the same sign as  $\Delta c$  (which is positive in this case). Hence,  $e_{ss}$  increases, which implies, by equation (33), that  $c_{ss}^*$  falls.

2/ In the complex-roots case, Lemma 1 shows that if  $c$ , and hence  $c^*$ , do not change on impact, the present value of consumption of traded goods falls short of traded-goods wealth. We would conjecture that for a lower initial value of  $c$  (i.e., if  $c$  falls at  $t=0$ ), the area will be smaller thus violating the budget constraint as well. A formal proof, however, remains an open issue.

We now show that, under a reduced-form specification in which excess aggregate demand for home goods depends negatively on the domestic real interest rate and positively on the real exchange rate, the present model yields the same key result as the model of Rodriguez (1982): a permanent reduction in the devaluation rate leads to an increase in aggregate demand for home goods on impact.

Formally, write equation (22) as:

$$\pi_t = \omega_t + \theta[\phi(r_t^d, e_t) - \bar{y}], \quad \phi_{r^d}(\bullet) < 0, \phi_e(\bullet) > 0. \quad (36)$$

where  $\phi(\bullet)$  denotes excess aggregate demand for home goods. Recall that the domestic real interest rate,  $r_t^d$ , is defined as:

$$r_t^d = 1 - \pi_t. \quad (37)$$

Equation (23), which describes the evolution of  $\omega$ , remains valid. The model is closed with the assumption of perfect capital mobility, equation (18), the law of motion of the real exchange rate, equation (25), and the assumption of perfect foresight. This five-equation model, given by equations (18), (23), (25), (36), and (37), can be reduced to a two-equation dynamic system that consists of equation (23) and

$$\dot{\pi}_t = \frac{1}{1 + \theta\phi_{r^d}(\bullet)} [\dot{\omega}_t + \theta e_t \phi_e(\bullet)(\epsilon - \pi_t)]. \quad (38)$$

When linearized around the steady-state, equations (23) and (38) constitute a dynamic system in  $\omega$  and  $\pi$  that is locally stable. 1/ Given the policy parameter  $\epsilon$ —and recalling that  $r^d = r + \epsilon - \pi$ —equation (36) determines the initial condition for  $\pi$ .

It can be shown that if the rate of devaluation is permanently reduced, the domestic real interest rate falls—because the nominal interest rate falls and the inflation rate of home goods increases—leading to an increase in aggregate demand for home goods since, by assumption, aggregate demand depends negatively on the domestic real interest rate and the real exchange rate is a non-jumping variable.

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1/ It is assumed that  $1 + \theta\phi_{r^d}(\bullet) > 0$  and  $\gamma\phi_{r^d}(\bullet) + e_{ss}\phi_e(\bullet) > 0$ .

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