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Intertemporal Substitution in Consumption Revisited

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Abstract

Some of the highly controversial questions in macroeconomics critically hinge on the value of a single parameter of consumer preference--the elasticity of intertemporal substitution. This paper provides new estimates of this parameter for individual G-7 and a panel of twenty OECD countries. We find that single equation GMM estimates are typically small and imprecise, consistent with Hall's (1988) finding from the U.S. data. Estimation of a system of equations that takes into account the cross-equation restrictions implied by theory, however, generally gives larger and better determined values for the parameter. The panel procedure also yields relatively large estimates. Overall our multi-country results contradict the hypothesis of zero intertemporal substitution.

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E2, E6, H3

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Summary

The elasticity of intertemporal substitution is an important determinant of the response of saving and consumption to the real interest rate. Summers (1984) argued that the intertemporal substitution effect was strong, whereas Hall (1988), drawing evidence from U.S. data, concluded that its value is close to zero. Hall maintained that the previous higher estimates obtained by Summers (1982) and others are due to inappropriate treatment of the time aggregation bias and can therefore be dismissed.

This paper, which addresses the specification issues raised by Hall, extends the earlier research to an international context by examining data from twenty OECD countries. The Kreps-Porteus nonexpected utility preference is adopted, and distributional restrictions are imposed to derive a simple relation that governs the covariation of consumption growth and asset returns, which allows unambiguous identification of the intertemporal substitution parameter.

The single-equation generalized method of moments estimates for each of the seven major industrial countries are typically small and imprecise, corroborating Hall's earlier finding from the U.S. data. The full information maximum likelihood estimation, however, gives larger and more precise values for the parameter, possibly because of the efficiency gain of system estimation. The panel procedure also yields relatively large estimates. Overall, the multicountry evidence seems to contradict the hypothesis of zero intertemporal substitution.

The results presented in this paper imply, among other things, that a shift toward expenditure taxation would probably lead to increases in private savings.

I. Introduction

Some of the highly controversial questions in macroeconomics critically hinges on the value of a single parameter of consumer preference--the elasticity of intertemporal substitution. This parameter measures the responsiveness of consumption and savings to movements in the real interest rates. A large value of this parameter means that consumers will either postpone or increase current consumption whenever there are expected changes in the real interest rates. If the elasticity of intertemporal substitution is substantial, then proposals about using tax policy to stimulate private savings are justified, deadweight loss of capital income taxation can be important, while the burden of a pay-as-you-go social security system or the national debt may not be as onerous as previously believed, and real interest shocks should have strong impact on current account dynamics because domestic consumption growth and investment move in the opposite directions. As Robert Hall (1988) points out, "the magnitude of this intertemporal substitution effect is one of the central questions of macroeconomics."

Early studies, often based on traditional aggregate consumption function, typically concluded that the interest elasticity of savings is low, with the notable exception of Boskin (1978), who obtained a value of 0.4. Boskin's result was significantly strengthened by the subsequent work of Summers (1982, 1984), and others. In summary of his findings, Summers (1984) stated: "available evidence tends to suggest that savings are likely to be interest elastic. I find in the more reliable estimates in my working paper (Summers 1982) values of the intertemporal elasticity of substitution which cluster at the high end of the range Evans (1983) and I considered (above one)." The view of substantial intertemporal substitution in consumption, however, has been challenged by Hall (1988). He estimated that the value of the elasticity is close to zero, using several U.S. data sets of varying sample periods and observation intervals. He modeled the relation between consumption and real rates of return in a framework in which agents' intertemporal substitution and risk aversion parameters can be explicitly differentiated. He argued that the previous higher estimates are almost all due to inadequate treatment of time aggregation and other modelling problems and therefore can be dismissed. Hall's finding contrasts sharply with the previously accumulated evidence. Because of its important implications for policy making, it seems worthwhile to carefully check the robustness of his result.

This paper extends that research to an international context by examining data from a panel of twenty OECD countries and from individual G-7 (Group of Seven) economies. In this study, we take care of the estimation issues raised by Hall. Imposing distributional restrictions, we derive a simple linear relation between consumption growth and asset returns, which permits unambiguous identification of the intertemporal substitution parameter. We explore the possible efficiency gain of system estimation by taking into account the cross-equation restrictions implied by theory. Our multi-country results suggest that values of the elasticity of intertemporal substitution tend to cluster around one, contradicting the hypothesis of zero intertemporal substitution. The remainder of the paper is organized as follows. Section II presents the general model and in particular derives a linear relation between consumption growth and asset returns. Section III briefly discusses data, and Section IV presents empirical results and evaluates alternative estimation strategies. The final section concludes the paper and offers some suggestions for future research.

II. Theory of the Consumer Under Uncertain Real Rates of Return

The advance in modern asset pricing theory has provided a boost to the study of consumer's intertemporal decisions under uncertainty. The consumption-based asset pricing theories, such as the discrete-time model of Lucas (1978) and the continuous-time model of Breeden (1979), impose a restriction on the joint stochastic process of aggregate consumption and asset returns. If we specify the return process and make predictions about consumption behavior, then the restriction can be interpreted as a theory of consumption, as in the permanent income hypotheses. Under the assumptions of representative agent and time-additive Von Neumann-Morgenstein (VNM) expected utility function, the first-order condition for optimal consumption is given by the stochastic Euler equation,

$$U'(C_t) = \beta E_t[U'(c_{t+1}) R_{it+1}]; \forall i \in [1, 2, \dots, N] \tag{1}$$

where U(.) is the consumer's period utility function, β the subjective discount factor and R_{it+1} is the return on the i^{th} asset expressed in units of consumption good. If U(.) is of the CRRA form,

$$U(c_t) = \frac{C_t^{1+\gamma}}{1+\gamma}; \qquad \gamma < 0, \tag{2}$$

then (1) becomes

$$E_t[\beta (\frac{C_{t+1}}{c_t})^\gamma R_{it+1}] = 1; \qquad \forall i \tag{3}$$

Hansen and Singleton (1982, 1983), Brown and Gibbons(1985), among others, have examined the time series relationship given by (3) 1/. They all interpret the parameter γ as the Arrow-Pratt coefficient of relative risk aversion (RRA). With the utility specification of (2), the reciprocal of this parameter, $1/\gamma$, is also the elasticity of intertemporal substitution (EIS).

This exact, quantitative relation between RRA and EIS imposed a prior by the time-additive, VNM expected utility framework is overly restrictive, because intertemporal substitution and risk aversion are two distinctive attributes of consumer preferences. While one concerns attitudes towards variation in consumption across time in a deterministic environment the other concerns the attitude towards variation across states of nature at a given point of time. Although we want to have a clear understanding of their respective role in determining consumer behavior, the conventional time-additive, Von Neumann-Morgenstein expected utility framework does not offer a clean separation of the two. The nonseparability of EIS and RRA has caused much confusion in the empirical identification of consumer's preference parameters. A close-to-zero value of the elasticity of intertemporal substitution, as that estimated by Hall (1988), for example, implies an almost infinite degree of risk aversion, which contradicts the observed consumer attitudes towards risk. This implausible inference led Hall to conclude that it is the intertemporal substitution alone that controls the relation between mean consumption growth and asset returns and that the Euler equation characterization given by (3) reveals just EIS without saying anything about RRA. We show what assumptions are needed in order for Hall's statement to remain valid.

The desired separation of EIS and RRA can be achieved in a nonexpected utility framework proposed by Kreps and Porteus (1978). The Kreps-Porteus (KP) preferences generalize the two-period "ordinal certainty equivalent" of Selden (1978) to a multiperiod stochastic setting. 2/ The KP preferences preserve both the stationarity (in the sense of Koopmans [1960]) and time-consistency (in the sense of Johnsen and Donaldson [1985]). A central idea is that consumers exhibit nonindifference toward the timing of resolution of uncertainty. The KP intertemporal preferences have the following recursive structure,

$$V_t = U(c_t, \mu[V_{t+1} | F_t]) \tag{4}$$

1/ Mankiw, Rotemberg and Summers (1985) extended this framework to include leisure as a decision variable, and Dunn and Singleton (1986) considered the multi-good case.

2/ Selden's two-period "Ordinal Certainty Equivalent" framework was adopted by Hall (1988).

where $\mu[.]$ is the certainty equivalent of stochastic future utility, U_t is an aggregator function that combines the deterministic current period consumption c_t with the certainty equivalent to obtain present period lifetime utility, and F_t is the filtration of underlying state variables in the economy or the information set as is commonly called in economics literature. In this setup, the aggregator U_t alone defines preference for intertemporal substitution over deterministic consumption paths while the certainty equivalent $\mu[.]$ reflects consumer's risk preference.

Epstein and Zin (1989) and Weil (1990) have independently given very similar parameterization to the KP preferences. We adopt a representation by Esptein and Zin (1989) in this paper. They proposed the following functional form for the recursive intertemporal utility,

$$V_t = [(1-\beta) c_t^{\frac{\sigma-1}{\sigma}} + \beta (E_t V_{t+1}^{\alpha})^{\frac{\sigma-1}{\alpha\sigma}}]^{\frac{\sigma}{\sigma-1}}, \tag{5}$$

where E_t is the conditional expectation operator given F_t.

With this specification, the role of utility parameters are easily interpreted. The parameter β reflects the rate of time preference, α reveals risk aversion, while σ is the elasticity of intertemporal substitution. Therefore (5) permits disentangling risk aversion parameter from the intertemporal substitution parameter.

Consumers endowed with KP preferences of (5) prefer early (late) resolution of uncertainty when $\alpha < (>) (\sigma-1)/\sigma$. As noted by Epstein and Zin (1989), and Weil (1990), the parametric form (5) nests the traditional time-additive, VNM expected utility function when $\alpha = (\sigma-1)/\sigma$, in which case there is indifference to the timing of resolution.

The optimal conditions analogous to (3) for the representative consumer with KP preferences of the form (5) are given by

$$E_t [\beta (\frac{c_{t+1}}{c_t})^{-1/\sigma} R_{M,t+1}]^{\frac{\alpha\sigma}{\sigma-1}} = 1 \tag{6}$$

and

$$E_t [\beta^{\frac{\alpha\sigma}{\sigma-1}} (\frac{C_{t+1}}{c_t})^{\frac{\alpha}{1-\sigma}} R_{M,t+1}^{\frac{\sigma(\alpha-1)+1}{\sigma-1}} R_{i,t+1}] = 1 \tag{7}$$

where $R_{M,t+1}$ is the return on the market portfolio and $R_{i,t+1}$ is the return on the i^{th} asset.

The Euler equations (6) and (7) imply a highly nonlinear relation between consumption growth and asset returns. Since both risk aversion and intertemporal substitution parameters play a role in determining this relation, Hall's (Hall [1988]) argument that only EIS matters is no longer valid in general when consumers are endowed with KP preferences.

Although in principle we can identify separately all the preference parameters from (6) and (7), in practice these two equations pose formidable challenge to empirical implementation. First, typical nonlinear estimation procedures involve the minimization of an appropriately specified criterion function over the parameters. The iteration algorithm requires good starting values for the parameters in order to reach convergence. For a highly nonlinear regression model like (6) and (7), however, it is not an easy matter to find such "good" starting values. Second, The Euler equations (6) and (7) involve the return on the optimal market portfolio, $R_{M,t+1}$, which is nonobservable. To facilitate empirical analysis it is useful to find some form of simplification.

We assume that the joint distribution of consumption and returns is lognormal. 1/ Let

$$x_t = \frac{c_t}{c_{t-1}} \tag{8}$$

$$U_{mt} = \left[\left(\frac{C_{t+1}}{C_t} \right)^{-1/\sigma} R_{M,t+1} \right]^{\frac{\alpha\sigma}{\sigma-1}} \tag{9}$$

and

$$U_{imt} = \left[\left(\frac{C_{t+1}}{C_t} \right)^{\frac{\alpha}{1-\sigma}} R_{M,t+1}^{\frac{\sigma(\alpha-1)+1}{\sigma-1}} R_{i,t+1} \right] \tag{10}$$

$i = 1, \ldots, N$. Then the Euler equations (6) and (7) can be rewritten as

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1/ Hansen and Singleton (1983) exploited the lognormality assumption to estimate utility parameters from the Euler equation of (3) in the conventional expected intertemporal utility framework.

$$E_{t-1}(U_{mt}) = \beta^{\frac{\alpha\sigma}{1-\sigma}} \tag{11}$$

and

$$E_{t-1}(U_{imt}) = \beta^{\frac{\alpha\sigma}{1-\sigma}} \qquad i = 1, \ldots, N. \tag{12}$$

Next let $x_t = \log X_t$, $r_{Mt} = \log R_{Mt}$, $r_{it} = \log R_{it}$, $\Psi_t = (x_t, r_{Mt}, r_{1t}, \ldots, r_{Nt})'$, $u_{mt} = \log U_{mt}$, $u_{imt} = \log U_{imt}$ $(i = 1, \ldots, N)$, and ψ_{t-1} denote the information set $\{\Psi_{t-s}: s \geq 1\}$. We further assume that $\{\Psi_t\}$ is a stationary Gaussian process.

This assumption implies that the distributions of both u_{mt} and u_{imt} conditional on ψ_{t-1} are normal with means μ_{mt-1}, μ_{imt-1}, and constant variances Σ_m^2, Σ^2_{im}, respectively. Hence, we have

$$E(U_{mt}) = \exp(\mu_{mt-1} + \frac{1}{2}\Sigma_m^2) \tag{13}$$

and

$$E(U_{imt}) = \exp(\mu_{imt-1} + \frac{1}{2}\Sigma_{im}^2) \qquad i = 1, \ldots, N. \tag{14}$$

Since $\psi_{t-1} \subset F_{t-1}$, we can take expectations of both sides of (11) and (12) respectively, conditional on ψ_{t-1} to obtain

$$E(U_{mt} \mid \psi_{t-1}) = \beta^{\frac{\alpha\sigma}{1-\sigma}} \tag{15}$$

and

$$E(U_{imt} \mid \psi_{t-1}) = \beta^{\frac{\alpha\sigma}{1-\sigma}} \qquad i = 1, \ldots, N. \tag{16}$$

Now equating the right-hand sides of equations (13) and (15) yields

$$\mu_{m,t-1} = \frac{\alpha\sigma}{1-\sigma}\log\beta - \frac{1}{2}\Sigma_m^2 \tag{17}$$

Define

$$\Gamma_{mt} = u_{mt} - \mu_{m,t-1}$$

$$= \frac{\alpha\sigma}{\sigma-1}\log\beta - \frac{\alpha\sigma}{\sigma-1}x_t + \frac{\alpha\sigma}{\sigma-1}r_{Mt} + \frac{1}{2}\Sigma_m^2 \tag{18}$$

Then, $E(\Gamma_{mt} \mid \psi_{t-1}) = 0$ and

$$E(r_{Mt}|\psi_{t-1}) = -\frac{1}{2}\frac{(\sigma-1)\Sigma_m^2}{\alpha\sigma} - \log\beta + \frac{1}{\sigma}E(x_t|\psi_{t-1}) \tag{19}$$

Similarly from equations (14) and (16) we have

$$\mu_{im,t-1} = \frac{\alpha\sigma}{1-\sigma}\log\beta - \frac{1}{2}\Sigma_{im}^2 \tag{20}$$

and if we define

$$\Gamma_{imt} = u_{imt} - \mu_{im,t-1}$$

$$= \frac{\alpha\sigma}{\sigma-1}\log\beta - \frac{\alpha}{\sigma-1}x_t + \frac{\sigma(\alpha-1)+1}{\sigma-1}r_{Mt} + r_{it} + \frac{1}{2}\Sigma_{im}^2 \tag{21}$$

Then, $E(\Gamma_{imt} \mid \psi_{t-1}) = 0$ and

$$\frac{\sigma(\alpha-1)+1}{\sigma-1}E(r_{Mt}|\psi_{t-1}) + \frac{1}{2}\Sigma_{im}^2 + \frac{\alpha\sigma}{\sigma-1}\log\beta - \frac{\alpha}{\sigma-1}E(x_t|\psi_{t-1}) + E(r_{it}|\psi_{t-1}) = 0 \tag{22}$$

Substituting (19) into (22) and rearrange the terms we obtain

$$E(\,x_t \mid \psi_{t-1}\,) \;=\; A_i \;+\; \sigma\, E(\,r_{it} \mid \psi_{t-1}\,) \qquad i = 1, \;\ldots.\; N. \tag{23}$$

where A_i is a term involving the parameters and the constant conditional second moments of x_t, r_{Mt}, and r_{it}.

Equation (23) implies that the movements in the conditional distributions of consumption growth and asset returns are completely summarized by movements in the conditional means. For empirical analysis we can rewrite (23) as

$$x_t \;=\; A_i \;+\; \sigma\, r_{it} \;+\; \varepsilon_{it} \tag{24}$$

where

$$E(\varepsilon_{it} \mid \psi_{t-1}) \;=\; 0$$

and

$$Var(\varepsilon_{it} \mid \psi_{t-1}) \;=\; \sigma_x^2 \;-\; 2\,Cov(x_t,\; r_{it}) \;+\; \sigma^2\sigma_{ri}^2$$

Equation (24) gives a convenient representation of the time series relations between consumption growth and asset returns. The log normality assumption reduces the complex Euler equations (6) and (7) to a simple linear relation similar to that employed by Hall (1988). From equation (24) we can unambiguously identify the parameter of intertemporal substitution, σ.

In the empirical section of this paper, we apply equation (24) to two assets, the aggregate stock market portfolio, and short-term Treasury bills or typical savings accounts if data for the Treasury bill yields are not available. In addition, we select the unrestricted representation for consumption growth from the following vector autoregressive model

$$Y_t \;=\; \sum_{s=1}^{p} B_s\, Y_{t-s} \;+\; \epsilon_t \tag{25}$$

to augment equation (24) for the two asset returns. Note that Y_t in (25) is a vector of consumption growth and asset returns. To summarize, we estimate the parameter σ for individual G-7 countries from three alternative models. Model I estimates equation (24) for a single asset return only, Model II consists of (24) for two asset returns and Model III is a system of two asset return equations of the form (24) and an unrestricted autoregressive representation for consumption growth. We finally estimate this parameter from the panel data for twenty OECD countries. This way we can compare the results from alternative estimation strategies.

III. Data

In empirical work examining the intertemporal relations between aggregate consumption and asset returns one has to decide which measure of consumption to be used. The results are typically found very sensitive to the measurement of consumption series. Most existing studies implicitly assumed that different components enter the utility function separately and employed either nondurables or nondurables plus services. An overwhelming conclusion is that consumer nondurables and services appear too smooth relative to the movements in asset returns. It seems to us inappropriate to exclude durables because expenditure on durables is the component of consumption that most likely responds to changes in the real interest rates. Very few households would curtail their current food consumption or other necessities in order to speculate in the stock market. It is much more plausible to think consumers might postpone purchasing a new piece of furniture, a new car or a new home if real interest rates are expected to decline tomorrow relative to today. 1/ Further, the definitions of nondurables and durables are often arbitrary in most official statistics. Shoes and clothes, for example, are typically classified as nondurables. However, most of these goods can last at least a year, considerably longer than the assumed decision horizons of consumers when monthly or quarterly data are used. The benefits from certain services such as consumer spending on preventive health care and fitness programs are not limited to the current period. These issues may affect empirical results appreciably. Therefore, we use the comprehensive measure of private consumption that includes consumer durables as well as nondurables and services. It seems that though attention is directed to consumer expenditure in the press and policy debates, academic economists insist that what enters utility function is consumption rather than consumer expenditure. By employing the total consumption expenditure, we thus implicitly assume that expenditure is a good measure of total consumption. In doing so, we are aware of the potential problem arising from treating durables as stocks rather than service flows. We therefore for comparison also check our results against those obtained when only nondurables and services are used.

As in Hall's study, we consider stock returns, Treasury bills yield and the deposit rate on savings accounts, three types of asset returns that determine consumers' real rate of interest rates. For Canada, the

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1/ Mankiw (1985) found that expenditure on consumer durables is far more sensitive to changes in the interest rate than is expenditure on nondurables and services.

United Kingdom, and the United States, we always use return on Treasury bills. We do not have data on short-term Treasury bill yields for Germany and Japan while the series for three-month Treasury bill rates for France and Italy are too short to be useful. For these countries we always use deposit rates on savings accounts. The nominal consumption, stock prices, consumer price index (CPI) are taken from IMF's International Financial Statistics. The dividend yields, Treasury bill yields, and deposit rates on savings accounts are from OECD Financial Statistics. Real series are obtained by deflating the corresponding nominal series with CPI. Detailed information on the quarterly asset return data is given in the appendix.

Table 1 gives summary statistics for real consumption growth, real stock market return and real Treasury bill yields and/or deposit rates on savings accounts in G-7 countries. Asset returns are adjusted for capital gains and dividend tax. For some countries detailed information on capital gains and dividend taxation is not available, we simply assume that the investment income is subject to ordinary income tax. A quick glance at this table gives one the impression that consumption growth is sluggish relative to changes in real asset returns. It can be seen that real rates of return consistently have much more variability than consumption growth for all countries. The standard deviations for quarterly consumption growth range from 0.9 percent to 1.5 percent while the standard deviations for ex-post return on the stock market is typically more than 6 percent. Also note that the mean ex-post real return on the stock market is substantially higher than that on Treasury bills or on savings accounts. However the stock market return also has considerably more variability than the Treasury bill yields or deposit rates.

Table 2, Part a, presents cross-country correlations for G-7 (Group of Seven) countries. It appears that there is quite a bit cross-country covariation in consumption growth. Asset returns also tend to move together across these countries, though the correlations are not perfect since the financial markets are far from being completely integrated during the sample period. Part b of the table shows that the cross-country covariation of annual consumption growth and asset returns are somewhat stronger than the corresponding quarterly series.

IV. Empirical Results

The representation of time series relations between consumption growth and asset returns set up in Section II is suitable for using a variety of estimation techniques. As a first step, we apply the Generalized Method of Moments (GMM) to equation (24), using only one of the asset returns each time. We call this Model I. This single equation instrumental variable approach was that adopted by Hall (1988). We then apply equation (24) simultaneously to both asset returns (Model II). The theory imposes the same restriction to both assets, and Model II takes this joint restriction into account. Finally, we propose an unrestricted equation for consumption growth and estimate the system of the unrestricted equation for consumption

Table 1. Summary Statistics for Consumption Growth and Real Asset Returns in G-7 Countries: Quarterly Data

Country	Sample Period	Number of Observ.	Mean	Std. Dev.	Autocorrelation					
					$\rho1$	$\rho2$	$\rho3$	$\rho4$	$\rho8$	$\rho12$
Canada										
Consumption Growth	60:1-91:4	128	0.0088	0.0106	0.140	0.292	0.179	0.147	-0.023	-0.061
Stock Returns	60:1-91:4	128	0.0377	0.0706	0.241	0.037	-0.071	-0.099	0.032	0.060
T-bill Yields	60:1-91:4	128	0.0054	0.0080	0.323	0.278	0.212	0.149	0.155	0.198
France										
Consumption Growth	65:1-91:3	107	0.0072	0.0098	-0.056	-0.041	0.168	0.109	0.008	0.131
Stock Returns	65:1-91:3	107	0.0436	0.0994	0.028	-0.010	0.110	0.087	0.046	0.001
Savings Deposit Rate	65:1-91:3	107	-0.0027	0.0053	0.509	0.153	0.048	0.273	0.189	0.219
Germany										
Consumption Growth	60:1-91:4	128	0.0078	0.0119	0.071	0.88	0.170	0.174	0.037	0.035
Stocks Returns	60:1-91:4	128	0.0359	0.0821	0.294	-0.021	0.055	-0.035	-0.040	0.076
Savings Deposit Rate	60:1-91:4	128	0.0034	0.0063	0.212	0.130	0.052	0.071	-0.041	-0.017
Italy										
Consumption Growth	70:1-91:3	87	0.0094	0.0086	0.506	0.157	0.057	0.105	-0.068	0.093
Stock Returns	70:1-91:3	87	0.0163	0.1158	0.355	0.169	0.117	0.136	0.019	-0.009
Savings Deposit Rate	70:1-91:3	87	-0.0032	0.032	0.689	0.577	0.580	0.518	0.210	0.214
Japan										
Consumption Growth	60:1-91:4	128	0.0145	0.0147	-0.209	0.025	0.198	-0.042	0.122	0.123
Stock Returns	60:1-91:4	128	0.0337	0.0779	0.253	0.101	0.093	0.072	0.046	0.205
Savings Deposit Rate	60:1-91:4	128	0.004	0.0122	0.359	0.384	0.333	0.128	0.135	0.053
United Kingdom										
Consumption Growth	60:1-91:4	128	0.0056	0.0151	-0.117	0.111	0.048	-0.006	-0.065	0.061
Stocks Returns	60:1-91:4	128	0.0514	0.0862	0.271	-0.020	0.053	-0.073	0.073	0.051
T-bill Yield	60:1-91:4	128	0.0025	0.0138	0.427	0.388	0.299	0.245	0.244	0.191
United States										
Consumption Growth	60:1-92:1	129	0.0075	0.0091	0.313	0.259	0.266	0.196	0.110	-0.051
Stocks Returns	60:1-92:1	129	0.0423	0.0647	0.291	-0.028	-0.083	-0.085	0.098	0.008
T-bill Yield	60:1-92:1	129	0.0034	0.0064	0.732	0.581	0.666	0.529	0.350	0.272

Table 2a. Cross-Country Correlations of Consumption Growth and Asset Returns: Quarterly Data

	United States	United Kingdom	France	Germany	Italy	Canada	Japan
(I) Consumption Growth: 70:1 - 91:3							
United States	1.0000						
United Kingdom	0.4772	1.0000					
France	0.2651	0.2562	1.0000				
Germany	0.2819	0.3805	0.3480	1.0000			
Italy	0.2034	0.2967	0.579	0.1296	1.0000		
Canada	0.5119	0.2555	0.2773	0.2051	0.1196	1.0000	
Japan	0.2864	0.3521	0.1862	0.1916	0.1716	0.2101	1.0000
(II) Stock Return: 70:1 - 91:3							
United States	1.0000						
United Kingdom	0.6844	1.000					
France	0.4416	0.3907	1.0000				
Germany	0.5023	0.4845	0.3588	1.0000			
Italy	0.5128	0.3645	0.4465	0.5149	1.0000		
Canada	0.8124	0.5763	0.3961	0.3828	0.4012	1.0000	
Japan	0.4470	0.4952	0.2699	0.3094	0.4335	0.4208	1.0000
(III) T-bill: 70:1 - 91:3							
United States	1.0000						
United Kingdom	0.5539						
France	0.3934	1.0000					
Germany	0.3626	0.5213	0.2569	1.0000			
Italy	0.3875	0.4198	0.5939	0.4497			
Canada	0.5719	0.4421	0.3146	0.4132	0.2730	1.0000	
Japan	0.5317	0.6310	0.3860	0.4422	0.4457	0.2614	1.0000

Table 2b. Cross-Country Correlations of Consumption Growth and Asset Returns: Annual Data

	United States	United Kingdom	France	Germany	Italy	Canada	Japan
(I) Consumption Growth: 1950 - 1991							
United States	1.0000						
United Kingdom	0.4865	1.0000					
France	0.2468	0.1718	1.0000				
Germany	0.2214	0.1225	0.2019	1.0000			
Italy	0.1474	0.2521	0.3829	0.3707	1.0000		
Canada	0.5742	0.2691	0.441	0.2143	0.2779	1.0000	
Japan	0.4499	0.3794	0.3938	0.5976	0.4901	0.2360	1.0000
(II) Stock Returns: 1950 - 1991							
United States	1.0000						
United Kingdom	0.7115	1.0000					
France	0.4923	0.5205	1.0000				
Germany	0.4823	0.3879	0.5559	1.0000			
Italy	0.3295	0.3039	0.6281	0.4299	1.0000		
Canada	0.7400	0.4265	0.5300	0.1949	0.3567	1.0000	
Japan	0.4621	0.6483	0.6321	0.3635	0.6033	0.4083	1.0000

growth and equation (24) for two asset returns. The advantage of this model is that it accommodates the inclusion of endogenous variables on the right hand side as well as contemporaneous correlation of the disturbances. There is potential efficiency gain to treat the equations as a system of simultaneous equations with cross-equation constraints explicitly imposed on them as opposed to run regression separately on each equation.

We assume that consumers make their intertemporal consumption and portfolio decisions based on their knowledge about current and past consumption and returns. We therefore limit the set of instrumental variables to include only (endogenous) consumption and asset returns. Throughout the analysis we make alternate use of three sets of instrumental variables, denoted by INS1, INS2, and INS3, respectively. The first set, INS1, consists of a constant, the first lag of consumption growth, and the first lag of asset returns. The second set INS2 is INS1 lagged once, and the third, INS3, is INS1 lagged twice. The estimated VAR model (25) suggests that values of consumption growth and asset returns beyond their second lags are not very useful in forecasting consumption growth. Therefore, we do not experiment with instruments beyond INS3.

Hall (1988) called special attention to the time aggregation problem when aggregate consumption data are used to estimate consumers' utility parameters. He particularly pointed out the importance of selecting appropriate instruments. If time aggregation bias is important, the first set of instrumental variables, INS1, are inappropriate. Since we used three sets of instruments with different lags, we could detect the possible bias in the results. Our initial estimation used the raw data. To make our estimates robust to the potential time aggregation bias, we also performed appropriate autoregressive transformation to the data following the same procedure used by Hall (1988) and re-estimated all models using the transformed data and the corresponding instruments. Finally, we conducted formal tests for serial correlation for all equations to evaluate the importance of time aggregation bias.

The single equation instrumental variable estimates of the elasticity of intertemporal substitution (EIS) for G-7 countries are presented in Tables 3 and 4. Table 3 reports results when stock return is used while Table 4 give results when Treasury bill yields or deposit rates are used. The point estimates of EIS from this model are typically imprecise, with few of them twice larger the corresponding standard errors. The largest estimated value of EIS is 0.95 for Japan. However its standard error is too large to consider the estimate reliable. There are no notable differences between the results when the potentially inappropriate instruments INS1 were used and those obtained by using INS2 and INS3. These single equation instrumental variable estimates are consistent with Hall's (1988) earlier finding from the U.S. data.

Table 3. Model I: Single Equation GMM Estimates of EIS
Using Stock Returns 1/

Country	INS1		INS2		INS3	
Canada	0.4146	(0.3068)	-0.2182	(0.2093)	-0.1560	(0.1301)
France	0.2801	(0.2527)	0.6065	(0.5105)	-0.1999	(0.3044)
Germany	0.2974	(0.4342)	0.5152	(0.2333)	-0.6034	(0.4159)
Italy	0.4135	(0.2504)	0.4947	(0.3201)	-0.2014	(0.1631)
Japan	0.9114	(0.5812)	0.7993	(0.4722)	0.7214	(0.4013)
United Kingdom	0.6597	(0.4980)	-0.3603	(0.2438)	0.1092	(0.0918)
United States	0.7039	(0.4145)	0.3988	(0.2061)	0.0468	(0.0365)

1/ Standard errors (in parenthesis) are robust to serial correlation and conditional heteroscedasticity.

Table 4. Model I: Single Equation GMM Estimates of EIS
Using Treasury-Bill Returns 1/

(Deposit rates)

Country	INS1		INS2		INS3	
Canada	0.6070	(0.5061)	0.5069	(0.4354)	-0.6401	(0.4141)
France	0.2105	(0.1465)	0.1083	(0.1809)	0.0678	(0.0760)
Germany	0.7089	(0.4033)	0.2601	(0.1840)	0.2080	(0.1730)
Italy	0.2385	(0.1324)	-0.0767	(0.1308)	0.1571	(0.1173)
Japan	0.2584	(0.2048)	0.6897	(0.4560)	0.9526	(0.6658)
United Kingdom	0.3333	(0.1715)	-0.2705	(0.1540)	0.3460	(0.1649)
United States	0.5074	(0.3219)	-0.6178	(0.4023)	0.2280	(0.1085)

1/ Standard errors (in parenthesis) are robust to serial correlation and conditional heteroscedasticity.

Applying GMM to Model II with multiple returns also gives small point estimates, as can be seen in Table 5. And these estimates are also typically imprecise. There was considerable improvement in the results given by Table 6 and Table 7, however, when Model III, the system of simultaneous equations was estimated. The Full Information Maximum Likelihood (FIML) estimates are all above unity, and are well determined. The corresponding GMM estimates in Table 7 are somewhat smaller. Although both GMM and FIML estimators are efficient under our Gaussian distribution assumption, it seems that the FIML approach yields more precise estimates.

We rerun all three models using transformed consumption series and corresponding instruments. The results were indistinguishable from those using the raw data. The point estimates of EIS are again small and imprecise when single equation instrumental variable technique was used. The results from the system estimation are also unaffected. This suggests that time aggregation may not be a serious problem in the raw data. Formal statistical tests (Lagrange Multiplier Test) indeed cannot reject the null of no serial correlation at five percent significance level. When we checked our results against those obtained by using nondurable goods and services, we found that the results from single equation GMM estimation were not much different. Since the series of nondurable goods and services is very smooth relative to changes in real interest rates, the estimated values of σ from FIML are consequently reduced somewhat, with all except the two estimates (for Japan and Italy) falling below unity.

Finally, we constructed a panel data set for twenty OECD countries. We used annual consumption growth and stock return series because annual data are less subject to measurement error. Under the assumption of common slope coefficient, that is, the elasticity of intertemporal substitution is the same across these countries, the panel procedure likely provides good estimates for the parameter since it exploits the cross-country correlations which contain useful information about the covariation of consumption growth and asset returns. The panel estimates, as shown in Table 8, support the case for substantial intertemporal substitution. Both the within (fixed effect) and the variance component (random effect) estimates exceed unity.

In summary, the results suggest that values of the elasticity of intertemporal substitution are positive and may well be above unity. The multicountry evidence presented in this paper is inconsistent with the extreme view that there is zero intertemporal substitution in consumption.

V. Conclusions

We have derived from a simple model with Kreps-Portues nonexpected preferences a linear relation between consumption growth and asset returns. This model allows unambiguous identification of the intertemporal substitution parameter. Applying the model to aggregate consumption data and asset returns in individual G-7 countries and a panel of twenty OECD countries, we have obtained estimates of the elasticity of intertemporal

Table 5. Model II: GMM Estimates of EIS Via Two Asset Returns 1/

Country	INS1		INS2		INS3	
Canada	0.6130	(0.4677)	0.8241	(0.5212)	-0.2125	(0.3172)
France	0.2886	(0.1205)	0.5302	(0.2218)	0.2191	(0.0781)
Germany	0.7908	(0.2505)	0.7222	(0.3902)	0.5036	(0.2732)
Italy	0.4601	(0.3511)	0.3982	(0.1822)	0.5301	(0.3999)
Japan	0.9124	(0.4912)	0.8913	(0.4716)	0.7105	(0.4095)
United Kingdom	0.6013	(0.4212)	0.7293	(0.3743)	0.4204	(0.2376)
United States	0.7761	(0.4108)	0.8847	(0.5136)	0.8058	(0.5019)

1/ Standard errors (in parenthesis) are robust to serial correlation and conditional heteroscedasticity.

Table 6. Model III: Full Information Maximum Likelihood Estimates of EIS 1/

Country	EIS Estimates	
Canada	1.2662	(0.4039)
France	1.0183	(0.4041)
Germany	1.5079	(0.4014)
Italy	1.3341	(0.7106)
Japan	1.9634	(0.7952)
United Kingdom	1.4145	(0.6191)
United States	1.1815	(0.3799)

1/ The standard errors (in parenthesis) are computed from the matrix of sums of squares of the outer products of the gradient of the likelihood function with respect to both the structural parameters and the unique elements of the inverse residual covariance matrix. See Berndt, E.K., B.H. Hall, R.E. Hall, and J.A. Hausman (1974).

Table 7. Model II: GMM Estimates of EIS Via The System of Three Equations 1/

Country	INS1		INS2		INS3	
Canada	0.8030	(0.4977)	0.9259	(0.4830)	0.7125	(0.3972)
France	0.9152	(0.3947)	1.1053	(0.5118)	0.8065	(0.4131)
Germany	0.9508	(0.4541)	0.8925	(0.4056)	0.8036	(0.4232)
Italy	0.7143	(0.3701)	0.5700	(0.5014)	0.4881	(0.2059)
Japan	1.9415	(0.4201)	0.9712	(0.4560)	0.7818	(0.4095)
United Kingdom	0.4332	(0.3916)	0.5293	(0.2043)	0.5204	(0.2076)
United States	0.6893	(0.2018)	0.5047	(0.3022)	0.6558	(0.3119)

1/ Standard errors (in parenthesis) are robust to serial correlation and conditional heteroscedasticity.

Table 8. Panel Estimation of the Elasticity of Intertemporal Substitution

(i) Within (fixed effects) estimates

σ = 1.0152, standard error = 0.4421

F (18.571) = 20.083 ρ value = 0.0000

(ii) Variance components (Brandom effects) estimates

σ = 1.0422, standard error = 0.4510

$\chi^2(1)$ = 0.0161 ρ-value = 0.0002 substitution. The single equation GMM estimates are typically small, corroborating Hall's earlier finding from the U.S. data. However the estimates from the system estimation and the panel procedure tend to cluster around one, contradicting the hypothesis of zero intertemporal substitution.

If we put faith in the results of this paper, then we ought to feel reassured about some of established presumptions about consumer behavior. The multicountry evidence documented here, together with the earlier finding of Boskin (1978), Summers (1982), and others, seem to suggest that aggregate consumption growth is responsive to real after-tax interest rates. A shift towards consumption taxation that raises real net return to capital assets, for example, may stimulate private savings rate.

As we mentioned, although total consumption or its nondurable component is very smooth, the durable goods spending has more variability. There are notable co-movements in durable goods spending and the levels of interest rates. Our use of total consumption expenditure can potentially be a source of misspecification, because it may distort the autocorrelation properties of consumption growth as well as the mean and variance of consumption. It remains a topic for future research to treat durable goods appropriately in the framework of KP preferences.

More fundamentally, the representative agent, complete market model adopted in this paper may not capture the true consumer behavior under uncertainty. If we introduce a model with more realistic features such as liquidity constraints, transaction costs, rules of thumb behavior, or nontradable human capital, we may obtain much more accurate estimates for the magnitude of elasticity of intertemporal substitution.

Asset Returns Data

Canada: Stock prices are industrial share prices on closing quotations at the end of the quarter on the Toronto Stock Exchange for a composite of 300 shares. Stock returns are the dividend yields over the quarter plus capital gains at end of the quarter. Treasury bills yields are the weighted average of the yields on successful bids for three-month bills. Deposit rate data relate to chartered banks' rates on 90-day Canadian dollar deposits. Quarterly data are averages of data for the last Wednesday in each month.

France: Stock price index is based on a sample of 180 stocks on the Paris Exchange. Stock returns are dividend yields over the quarter plus capital gains at end of the quarter. Treasury bill yields are weighted average rates on 13-week Treasury bills. Deposit rate is end of the third month rate on savings deposits, bank accounts on pass book.

Germany: Stock price index is the average of daily quotations covering 95 percent of common shares of German industrial companies. Stock return is derived from dividend yields over the quarter and capital gains at end of the quarter. Yields on short-term Treasury bills are not available. Deposit rate is the rate on three-month deposits under 1 million deutsche marks.

Italy: The stock price series is an average of daily spot closing quotations of common shares of 40 major companies on the Milan Exchange. Stock return is derived from dividend yields over the quarter and capital gains at end of the quarter. Treasury bill yields are the weighted average yields, before tax, on newly-issued three-month Treasury bills. Deposit rate is the weighted average rate prevailing on savings deposits with commercial and savings bank during the month ending the quarter.

Japan: Stock price index is the average of daily closing prices for all stocks listed on the Tokyo Exchange. Stock returns are dividend yields over the quarter plus capital gains at end of the quarter. Yields on short-term Treasury bills not available. Deposit rate is the three-month time deposit rate.

United Kingdom: Stock price data refer to the average of daily quotations of 500 industrial ordinary shares. Stock returns are dividend yields over the quarter plus capital gains at end of the quarter. Treasury bill rate is the tender rate at which 91-day bills are allotted. Deposit rates are the rates paid on seven day notice accounts of London clearing banks.

United States: Stock prices are the Standard and Poor's 500 price index. Stock returns are dividend yields over the quarter plus capital gains at end of the quarter. Treasury bill rate is the discount on new issues of three-month bills. Deposit rate is the certificates of deposit rate.

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