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Portfolio Performance of the SDR and Reserve
Currencies: Tests Using the ARCH Methodology

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Abstract

In managing their foreign exchange exposure, international investors, including central banks, often compare actual portfolios with hypothetical portfolios that have been calculated using certain assumptions regarding the statistical properties of interest rates and exchange rates. One of these assumptions is that the variability of returns on various currency assets is time invariant. This assumption is tested in this paper using autoregressive conditional heteroskedastic (ARCH) models. Using weekly data for the period February 1982 to December 1991 for major reserve currencies, including the SDR, we find evidence that the variances of returns do vary over time (i.e., they do not exhibit stationarity) and that ARCH models that specify changing variances are superior to models that assume constant variance. By incorrectly assuming constant variability of returns, currency-asset allocations are not necessarily optimal and the measures of riskiness of a fixed-income portfolio may not be accurate. Furthermore, the error introduced by incorrectly assuming stationarity is smaller with the SDR than with any other national currency in the portfolio to be managed.

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Summary

This paper tests the common assumption used in portfolio theory that returns on various currency fixed-income investments do not vary over time. This assumption underlies most optimal portfolio models investors use in managing their financial assets. If this assumption does not hold, the resulting allocation of assets is not necessarily optimal, and the obtained measure of the riskiness of a portfolio may not be accurate. Suboptimal allocations may have significant implications for the hedging approaches necessary to control the implied interest and exchange rate risk and may thus expose investors to greater risk of loss than they are willing to accept. If estimates of changing variances can be made with reasonable accuracy, appropriate portfolio shifts can be made over time. Such a strategy can help central banks manage their foreign exchange reserves.

The tests employ the autoregressive conditional heteroscedasticity (ARCH) methodology. The existence of heteroscedasticity makes it difficult to base inferences and predictions on least-squares estimation. When the conditional variances of returns are not constant through time, they can be calculated with the estimation procedure used in ARCH models. Estimation is possible because the autoregressive (AR) representation of a time series with ARCH disturbances allows for the changing variances of investment returns to be forecast, even though the level of returns cannot be predicted. For this study, the standard univariate ARCH model was estimated using weekly data on short-term investment returns on various currencies, including composites such as the ECU and the SDR, for February 1982 to December 1991.

The empirical results indicate that the returns on nine currency investments exhibit statistically significant and persistent changes over time. Specifically, the estimated constant term in an autoregressive specification for returns is positive for all currencies, implying an upward trend in total returns, although the process is generally not explosive. The estimated ARCH parameter for SDR returns is the smallest among all currency returns exhibiting one lag of ARCH effects, indicating that SDR investments provide greater insulation from unexpected shocks. Finally, the statistical tests presented in the paper indicate that the conventional assumption of a standard normal distribution for the errors of the underlying portfolio model is not consistent with the data for the 1982-91 period.

I. Introduction

In the calculation of optimal fixed-income portfolios based on historical means and variances, which are often used by investors to determine the "best" allocation of their wealth in terms of fixed-income investment instruments and by some central banks in the management of their external reserve assets, the standard assumption is that historical volatilities of returns from investments in various currency-denominated government securities remain constant over time. The same assumption is also often made by the capital asset pricing model and the modern portfolio theory in the calculation of diversification benefits from investing in multiple assets and by the Black-Scholes model in the calculation of option-pricing formulas. The implications of incorrectly assuming constant variability of returns over time are loss of efficiency in terms of portfolio allocations and inappropriate inferences in terms of hedging the associated risk. Recent empirical work (see Section II) has shown that large changes in equity returns and exchange rates, with high sampling frequency, tend to be followed by large changes of either sign, that is, the variance of these returns and exchange rates is not constant and, therefore, this traditional assumption of many standard models of portfolio allocation does not hold. However, very few, if any, studies have dealt with returns from a multicurrency fixed-income portfolio.

A suitable model for dealing with non-constant volatilities of returns is the ARCH model. ^{1/} This paper attempts to model total global fixed-income investment returns for a U.S. dollar-based investor as a univariate autoregressive conditional heteroskedastic (ARCH) process as an alternative to the more common portfolio model that assumes that the variance of returns from investing in various currency-denominated assets is stationary or time invariant. The ARCH model expresses the current conditional variance of a time series of total returns as a linear function of their past-squared residuals. ARCH models permit the prediction of changes in the conditional variance of asset returns on different currencies, but not the changes in the unconditional mean value of asset returns. Knowledge of the statistical properties of the process that generates the observed rates of return on various investments, and in particular of how the process depends upon developments in the financial markets, is important for characterizing asset-return volatility and for modelling optimal portfolios of assets denominated in various currencies.

Traditional models assume a constant one-period forecast variance and covariance matrix for the returns on different investments. This assumption can be generalized to introduce a new class of stochastic processes called ARCH processes. These processes imply serially uncorrelated errors which have zero mean and nonconstant variances that are conditional on past returns. To test whether the error terms follow an ARCH process, the autocorrelation of the squared ordinary-least-squares (OLS) residuals has been calculated. After the detection of ARCH effects based on the

^{1/} See Engle and Rothschild (1992).

autocorrelation statistics, a factor-ARCH model 1/ can be used to reduce the number of parameters to be estimated.

Our empirical results show that the variability of returns from investing in various currencies and currency composites is affected by lagged residuals, while the number of lags differs among currencies. The estimated constant terms in the ARCH specification, a_0 , for all currencies are significant and non-negative, indicating a positive autonomous conditional volatility for currencies. Further, the estimated parameters of the lagged squared residuals in the ARCH process, a_1 , exhibit strong statistical significance for all currency asset returns, indicating that the stochastic processes generating these currency returns are covariance-nonstationary. However, investment in SDRs provides greater insulation to investors than investing in any other currency with one lag of ARCH effects, as SDR returns demonstrate the lowest transmission of unexpected weekly shocks on their volatility. 2/ In these circumstances and as already noted, an ARCH specification is a more efficient approach for modelling all considered asset returns than is the OLS approach. Finally, the Lagrange multiplier (LM) statistic for first-order ARCH specification is significant for all currencies, which also indicates that the conditional variances are not constant through time.

In addition, the results of several diagnostic checks on the distributional properties of total returns indicate that they deviate from the normal distribution. In particular, the kurtosis statistics of the probability distribution of returns on all currency investments are substantially larger than those from a standard normal distribution, indicating that these returns are leptokurtic (fat-tailed). This result is further supported by Bera-Jarque tests indicating not only a highly significant non-normality of the probability function for returns on these currencies but also a significant likelihood that the variance of these returns is not constant over time. The Bera-Jarque (BJ) test for normality 3/ is highly significant for the deutsche mark, the pound sterling, the French franc, the Japanese yen, the ECU, the SDR, and gold with ARCH(1,1), as well as for the U.S. dollar with ARCH(6,2). Only the BJ test for the average composite index with ARCH(1,1) is not statistically significant.

The organization of this paper is as follows: Section II provides a brief review of the literature. Section III outlines the testing and estimation procedures for ARCH effects. Section IV presents an application of the standard univariate ARCH model on weekly returns from short-term investments in nine different currencies and analyzes the main results. Section V concludes by summarizing the evidence from modeling short-term fixed income returns as ARCH processes and discusses the practical implications for the management of reserves.

1/ See Appendix I, pp. 15-20.

2/ Although the estimated coefficient a_1 for the average composite index (ACI) is lower than that for the SDR, the sum of the estimated coefficients a_1 , $i=1, \dots, 5$, for the ACI process is greater than the estimated coefficient a_1 for the SDR.

3/ For the use of this test, see Table 2.

II. A Brief Literature Review

The uncertainty of speculative prices has been observed to be changing through time (Mandelbrot (1963) and Fama (1965)). The tendency of large (small) price changes in the high frequency financial data to be followed by other large (small) price changes is often called "volatility clustering." One of the specifications that has emerged for characterizing such changing variances is the ARCH model (Engle (1982)) and its various extensions. In his seminal paper, Engle suggests that one possible parametrization for variances is to express them as a linear function of past-squared values of the errors of the model. With financial data, the ARCH model captures the tendency for volatility clustering, and numerous empirical applications of the ARCH methodology in characterizing asset return variances and covariances have already appeared in the literature. ARCH effects have generally been found to be highly significant in equity markets; for example, highly significant test statistics for ARCH models have been reported for various individual returns on stock market investments (Engle and Mustafa (1989)).

In a series of papers, the ARCH model has been analyzed, generalized, extended to the multivariate context, and used to test for time-varying risk premia in financial markets. These papers include Engle (1983) and Engle and Kraft (1983). The ARCH model was extended to the multivariate framework in Kraft and Engle (1982). Diebold and Nerlove (1989) use a multivariate approach exploiting factor structure, which facilitates tractable estimation via a substantial reduction in the number of parameters to be estimated. The factor structure captures commonality in volatility movements in financial time series. Engle, Ng, and Rothschild (1990) suggest using the factor-ARCH model as a structure for the conditional covariance matrix of excess asset returns. One- and two-factor ARCH models have been applied to pricing of treasury bills, with the results showing reasonable stability over time.

The importance of ARCH models in finance comes partly from the direct association of variance and risk, and the fundamental trade-off relationship between risk and return. The ARCH-M (in mean) model developed by Engle, Lilien, and Robins (1987) expresses the conditional mean as a function of the conditional variance of the ARCH process. It thus provided a tool for estimation of the (possibly) linear relationship between the return and variances of a given investment portfolio.

Another interesting observation in financial data is that the unconditional price and return distributions tend to have fatter tails than the normal distribution (i.e., are leptokurtic) (Mandelbrot (1963) and Fama (1965)). Ignoring the fat tails and the time-varying variances could lead to erroneous detection of abnormal returns (De Jong, Kemna, and Kloeck (1990)). Stock returns tend to exhibit non-normal unconditional sampling distributions, both in the form of skewness and excess kurtosis (Fama (1965)). For many financial time series, the leptokurtosis cannot be fully explained. The conditional normality assumption in ARCH generates some

degree of unconditional excess kurtosis, but this is typically less than adequate to fully account for the fat-tailed properties of the data. One solution to the kurtosis problem is the adoption of conditional distributions with fatter tails than the normal distribution. Skewness and kurtosis are shown to be important in characterizing the conditional density function of returns on stocks (Engel and Gonzales-Rivera (1989)).

In addition to the leptokurtic distribution of stock return data, Black (1976) noted a negative correlation between current returns and future volatility. The linear generalized ARCH(p,q) model, where the variance only depends on the magnitude and not on the sign of the estimated residuals, is not able to capture this negative relationship since the conditional variance is only linked to past conditional variances and squared innovations, and hence the sign of returns plays no role in affecting the volatilities. This limitation of the standard ARCH formulation is one of the primary motivations for the exponential-GARCH(p,q) model by Nelson (1990). In these types of generalized ARCH models, the volatility depends not only on the magnitude of the past surprises but also on their corresponding signs.

Bollerslev, Engle, and Wooldridge (1988) first estimated a multivariate GARCH(p,q) process for returns on bills, bonds, and stocks where the expected return was assumed to be proportional to the conditional covariance of each return with that of a fully diversified or market portfolio. They found that conditional covariances are quite variable over time and are a significant determinant of the time-varying risk premia. Bollerslev (1990) applied also a multivariate GARCH model to short-run nominal exchange rates. Hall, Miles, and Taylor (1988), similarly applied a multivariate GARCH-M estimation of the capital asset pricing model. Pagan and Schwert (1990) compared several statistical models for monthly stock return volatility and showed the importance of nonlinearities in stock return behavior that are not captured by conventional ARCH or GARCH models. They also showed the nonstationarity of the volatility of stock returns. Vries (1991) compared stable and GARCH processes for modeling financial data and showed that the unconditional distribution of variables generated by a GARCH-like process, which models the clustering of volatility and exhibits the fat-tail property as well, can be stable. Further, Bollerslev (1987) had earlier introduced an extension of ARCH and GARCH models, called an integrated GARCH model, which tried to explain the persistence of variance over time. A detailed explanation of the major specifications and estimation procedures of the ARCH-family models is given in Appendix I.

III. Estimation and Testing for ARCH Effects

Autoregressive conditional heteroskedasticity (ARCH) models account for the empirical observation that large changes in asset returns tend to be followed by further large changes in these returns (though of unpredictable sign), and small changes in asset returns tend to be followed by small

changes, thus leading to periods of persistently high or low volatility. 1/ We utilize the ARCH model by assuming that the conditional mean, $E(y_t|y_{t-1})$, of each asset return, y_t , is linearly unpredictable, while its conditional variance, h_t , is predictable by an ARCH model. It is well known that the unconditional distribution of ARCH processes have thicker tails, even though their conditional distributions are normal. 2/

The first order autoregressive, AR(1), model for y_t , combined with ARCH(1,1) errors, can be written as 3/

$$y_t = \phi_0 + \phi_1 y_{t-1} + \epsilon_t \quad (1)$$

where $E(\epsilon_t|y_{t-1}) = 0$

$$\text{var}(\epsilon_t|y_{t-1}) = E(\epsilon_t^2|y_{t-1}) = h_t$$

and

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 \quad (2)$$

where $|\phi_1|$ is assumed to be less than one for the regression to be stable. To ensure that h_t is positive and that the process is stationary, we must have $\alpha_0 \geq 0$ and $1 \geq \alpha_1 \geq 0$. Intuitively, α_0 can be interpreted as a "normal" stationary element in the series on asset returns, and α_1 as the heteroskedastic coefficient, which shows the correlation between changes in returns and their corresponding volatilities in the previous period. For a U.S. dollar-based investor, for example, y_t denotes asset returns in U.S. dollars obtained by investing in various currency assets and is defined as the product of the three-month treasury bill rate for each asset multiplied by the percentage change of the corresponding U.S. dollar exchange rate, all expressed on a weekly basis. 4/

1/ This stylized fact is often referred in the relevant literature as "volatility clustering."

2/ See Engle (1982).

3/ That is, an AR(1) model means that y_t depends on y_{t-1} , but not on earlier (y_{t-2} , y_{t-3} , ...) observations. An ARCH(1, 1) model means that y_t depends on y_{t-1} , but not on earlier (y_{t-2} , y_{t-3} , ...) observations, and that h_t depends on ϵ_{t-1}^2 , but not on earlier (ϵ_{t-2}^2 , ϵ_{t-3}^2 , ...) squared errors.

4/ In calculating returns, we have omitted capital gains or losses resulting from changes in the prices of bills on the assumption that such short-term securities generally have relatively small price variations over the course of a week.

The generalization of the ARCH(p,q) model for asset return y_t is as follows:

$$y_t = \phi_0 + \phi_1 y_{t-1} + \dots + \phi_p y_{t-p} + \varepsilon_t \quad (3)$$

where $\phi_p(L) = \sum_{i=1}^p \phi_i L^i$ is the polynomial autoregressive lag operator.

Note that

$$y_t \sim N(\phi_p(L)y_t, h_t) \quad (4)$$

$$\text{with } h_t = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 \quad (5)$$

where ε_t follows a white noise process defined by $E(\varepsilon_t) = 0$ for all t , $E(\varepsilon_t \varepsilon_s) = 0$ for $t \neq s$, and $E(\varepsilon_t^2) = \sigma^2$ for all t . The conditional variance of ε_t , h_t , is allowed to vary over time and is a linear function of past squared residuals.

Testing for ARCH effects requires to test the null hypothesis that the conditional variance h_t is constant over time, or

$$H_0: \alpha_1 = \alpha_2 = \dots = \alpha_q = 0 \quad (\text{no ARCH effects})$$

$$H_1: \alpha_1 \neq \alpha_2 \neq \dots \neq \alpha_q \neq 0 \quad (\text{ARCH effects exist}).$$

To test the null hypothesis H_0 , we have utilized the following procedure: first, the residuals $\hat{\varepsilon}_t$ are estimated from equation (3) by using ordinary least squares (OLS); second, the number of lags of $\hat{\varepsilon}_t$ is determined by utilizing all available information, through maximization of its log-likelihood function; and third, estimates of $\alpha_0, \dots, \alpha_q$ are obtained by applying OLS to

$$h_t = \alpha_0 + \sum_{i=1}^q \alpha_i \hat{\varepsilon}_{t-i}^2.$$

In order to test for an ARCH process of order q , we form the regression

$$E(\hat{\varepsilon}_t^2) = h_t = \alpha_0 + \sum_{i=1}^q \alpha_i \hat{\varepsilon}_{t-i}^2 \quad (6)$$

where $\hat{\varepsilon}_t^2$ is the square of the OLS residuals obtained from equation (3). 1/ To test for higher order ARCH specifications and the joint significance of the coefficients attached to equation (6), the Lagrange multiplier (LM) statistic, is calculated as $T \cdot R^2$ (where R^2 is

1/ See Diebold (1988), p. 19; Greene (1990), p. 417.

the coefficient of determination and T is the sample size) which has an asymptotic $\chi^2(q)$ distribution. Significantly large values of the statistic will lead to the rejection of the null hypothesis H_0 of homoskedasticity in favor of an ARCH process of order q .

Finally, if ARCH effects are detected, maximum likelihood estimation rather than OLS should be applied ^{1/} and the additional assumption of (conditional) normality of returns should be employed in the third step of the above procedure. A detailed explanation of the various specifications and estimation procedures of the ARCH-type models is given in Appendix I.

IV. Empirical Results

In the univariate version of ARCH models, as described above, the conditional variance of a currency return is modelled as an autoregressive process of its lagged-squared residuals so that during periods of high variability of a return, its estimated variance will increase or remain high, and during periods of relative stability its estimated variance will decrease. In this section, the univariate ARCH model is estimated for a U.S. dollar-based investor using weekly data. The data set contains returns from investing in short-term securities in the U.S. dollar, the deutsche mark, the Japanese yen, the pound sterling, the French franc, Special Drawing Rights (SDR), European currency unit (ECU), an average composite index (ACI), and gold. The ACI represents the average share of currencies in official holdings of foreign exchange reserves of IMF member central banks for the period 1981-87. Our sample spans the period February 1982 to December 1991. The data set is described in Appendix II.

For given values of past-realized investment returns, α_0 and α_i , $i=1, \dots, 5$, were estimated for each total return on the above mentioned nine currency-denominated assets (see Table 1). All estimated coefficients, α_0 and α_i , are non-negative, thus ensuring that a positive variance is obtained. Furthermore, the summation of α_i 's ($i \geq 1$) is always less than one, indicating that the estimated variance is finite. The optimal lag length, q , is determined as the value that maximizes the log-likelihood function in a grid search over lags 1 to 6. To maximize the likelihood function, an iterative procedure based upon the method of BHHH ^{2/} was used. As can be seen from Table 1, which presents the estimation results for the optimal lag lengths, the optimal lag length is one for the deutsche mark, the pound sterling, the French franc, the Japanese yen, ECU, SDR, and gold; two for the U.S. dollar; and five for the average composite index. The difference in the optimal lag length between the U.S. dollar and the rest of the employed currencies, except the average composite index, may be attributed to the fact that U.S. dollar returns comprise only interest rates on the respective assets, without taking into account any counteracting

^{1/} See Engle (1982), pp. 996-999.

^{2/} The Berndt, Hall, Hall, and Hausman (1974) iterative algorithm.

Table 1. ARCH (p,q) Estimation Results for Total Returns (Weekly, 1982:5-1991:50)

$$y_t = \phi_0 + \phi_1 y_{t-1} + \dots + \phi_p y_{t-p} + \varepsilon_t \quad \text{and} \quad h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \dots + \alpha_q \varepsilon_{t-q}^2$$

	U.S. dollar	Deutsche mark	Japanese yen	Pound sterling	French franc	SDR	ECU	Average composite index	Gold
$\hat{\phi}_0$	0.001 (2.889)*	0.126 (2.262)*	0.135 (2.406)*	0.118 (2.038)*	0.136 (2.458)*	0.082 (2.883)*	0.095 (1.714)*	0.016 (1.939)*	-0.059 (-0.062)
$\hat{\phi}_1$	1.191 (27.026)*	0.331 (8.087)*	0.347 (7.394)*	0.311 (7.679)*	0.332 (8.119)*	0.330 (7.500)*	0.302 (7.134)*	0.362 (7.829)*	0.248 (5.454)*
$\hat{\phi}_2$	-0.291 (-4.597)*							-0.105 (-2.094)*	
$\hat{\phi}_3$	0.212 (4.184)*							0.102 (1.999)*	
$\hat{\phi}_4$	0.009 (0.211)							-0.039 (-0.816)	
$\hat{\phi}_5$	0.055 (1.565)								
$\hat{\phi}_6$	-0.187 (-8.734)*								
$\hat{\alpha}_0$	0.000 (11.199)*	1.355 (12.767)*	1.25 (15.332)*	1.374 (14.310)*	1.270 (14.431)*	0.369 (13.072)*	1.163 (10.963)*	0.018 (5.368)*	3.329 (22.932)*
$\hat{\alpha}_1$	0.538 (9.665)*	0.144 (2.473)*	0.17 (4.742)*	0.181 (3.870)*	0.197 (3.107)*	0.114 (2.002)*	0.203 (3.065)*	0.106 (1.726)*	0.194 (4.369)*
$\hat{\alpha}_2$	0.162 (2.944)*							0.074 (1.467)**	
$\hat{\alpha}_3$								0.108 (1.452)**	
$\hat{\alpha}_4$								0.130 (2.179)*	
$\hat{\alpha}_5$								0.037 (0.708)	
Log like- lihood	2234.24	-845.77	-829.02	-857.23	-841.03	-503.10	-755.69	158.17	-1081.92
Lagrange multiplier ^{1/}	112.72*	10.99*	6.09*	8.18*	19.15*	4.45*	10.21*	16.69*	0.60

* and ** denote statistically significant t-statistics at 0.05 and 0.1 levels, respectively.

^{1/} The Lagrange multiplier (LM) test is a test for the existence of ARCH effects in a series. The LM statistic is the product of the sample size, T, times the coefficient of determination, R^2 . Its distribution is a χ^2 with degrees of freedom equal to the number of lags.

movements in the exchange rate as is the case for the other currencies. The ARCH coefficients for almost all currency returns are significantly greater than zero, as indicated by the asymptotic t-statistics given in parentheses.

The specification test for qth-order ARCH disturbances is based on the qth-order autocorrelation of the squared residuals. The LM test statistics for the presence of ARCH is asymptotically distributed as $\chi^2(q)$ under the null hypothesis of no qth-order ARCH effects. The critical values of the $\chi^2(1)$, $\chi^2(2)$, and $\chi^2(5)$ distributions at the 5 percent significance level are 3.84, 5.99, and 11.07, respectively. The presence of first-order ARCH effects is accepted at the 5 percent level for the deutsche mark, the Japanese yen, the pound sterling, the French franc, SDR, and ECU returns. The null hypothesis of no second-order ARCH effects is rejected for returns on the U.S. dollar, and that of no fifth-order ARCH effects is rejected for average composite investment returns. These results appear to provide statistical evidence that the variances of the respective currency returns are not independent over time, as they are related through the square of past residuals. As regards to gold returns, the evidence is inconclusive as to whether they follow an ARCH process since the t-statistic of the corresponding ARCH coefficient is statistically significant, while the LM statistic suggests the absence of ARCH effects.

The significance tests for the α_i parameters and the LM statistics are corroborated by several additional diagnostic tests on the distributional properties of the total returns as given in Table 2. These statistics indicate that most currency returns, except for the ECU and the SDR, display negative skewness. This characterizes a distribution of returns where negative changes are less than positive ones. A small negative skewness statistic indicates that large positive changes are outnumbered by negative ones of smaller magnitude. Moreover, all of the currency returns considered here exhibit high leptokurtosis, which commonly appears in high-frequency exchange rate data. 1/ As mentioned above, when the conditional distribution is normal with ARCH disturbances, its unconditional distribution is known to be leptokurtic, 2/ implying that an ARCH model could partly eliminate the associated leptokurtosis. For distributions which are symmetrical and unimodal, kurtosis is a measure of flatness or peakedness of the distribution; leptokurtic curves tend to be sharply peaked curves relative to the normal distribution. The kurtosis statistics for all currency returns are significantly larger than three, which would not be consistent with a standard normal distribution of the total returns in these assets. 3/

1/ See Diebold (1988), p. 5.

2/ See Engle (1982); Diebold (1988), pp. 8-11; Kroner and Claessens (1991), p. 136.

3/ Standard normal distributions have a kurtosis equal to three. Distributions with a kurtosis higher than three are leptokurtic in that they have more observations in the tails, away from the mean, when compared with a normal distribution.

Table 2. Preliminary Data Analysis on Total Returns 1/

Statistic	U.S. dollar	Deutsche mark	Japanese yen	Pound sterling	French franc	SDR	ECU	Average composite index	Gold
Skewness <u>2/</u>	0.998	0.281	0.899	0.443	0.074	-0.226	-0.209	0.070	0.651
Kurtosis <u>3/</u>	10.506	3.133	3.190	3.218	3.264	3.081	3.282	3.245	3.2546
Bera-Jarque <u>4/</u>	4817.583*	9.197*	220.964*	65.608*	14.496*	4.519*	10.524*	2.689	1158.28*
Augmented Dickey-Fuller <u>5/</u>	-20.929*	-17.363*	-22.918*	-12.918*	-17.542*	-12.293*	-16.671*	-12.381*	-22.367*

* Indicates statistically significant statistics at 5 percent level.

1/ The number of observation is 496.

2/ Skewness indicates how asymmetric the distribution is. The skewness statistic is zero in the population of a normal distribution.

3/ A high value of the kurtosis coefficient indicates the presence of a significant number of observations at the tails of the distribution. The kurtosis statistic is three in the population of a normal distribution. If the kurtosis coefficient is higher than 3, this indicates a higher number of observations at the tails than suggested under the normal distribution. For the calculation of the Kurtosis statistic, see Diebold (1988) pp. 8-11.

4/ The Bera-Jarque test for normality is a joint test for skewness and kurtosis.

5/ The augmented Dickey-Fuller statistic tests for unit roots in an autoregressive series and is robust to ARCH specification. If the augmented Dickey-Fuller statistic is significant, the existence of a characteristic root outside the unit circle cannot be rejected and therefore the respective series is considered non-stationary.

In addition, the Bera-Jarque (BJ) test statistic 1/ for the deutsche mark, the pound sterling, the French franc, the Japanese yen, ECU, SDR, and gold with ARCH(1,1) is highly significant. Furthermore, the BJ test for the U.S. dollar with ARCH(6,2) and for the average composite index with ARCH(4,5) are also significant. The resulting high negative values for the augmented Dickey-Fuller tests implies that the characteristic roots of the Taylor expansion of the unconditional variance of ε_t^2 lie outside the unit circle for all currency returns. 2/

The finding that the LM statistic for testing for ARCH effects is significant for all currency returns (except gold), suggests that investment in these currencies will result in increased estimated variances, and therefore increased risk, during periods of large unexpected shocks to returns, and diminished variances during periods of relative stability. This result indicates that it might be desirable to split the sample into periods of high and low variability of returns and examine whether ARCH effects persist for these currencies. The values for the parameter α_0 range from zero for U.S. dollar returns to 3.329 for gold and are all statistically significant. The parameter α_i ($i=1, \dots, 5$) represents the relative contribution of the respective previous period's squared error of the return equation to the conditional variance in the current period. The parameter values of α_i range from 0.106 for the average composite index to 0.538 for U.S. dollar returns and are all statistically significant at the 5 percent level. In the case of the U.S. dollar, this means that about 54 percent of a week's disturbances is carried over into the following week. Among currency returns displaying one lag of ARCH effects, returns on the SDR demonstrate the lowest transmission of the weekly deviations of the SDR returns from their mean on the volatility of SDR returns, while the ECU returns had the highest. This suggests that shorter term investment periods are more likely to prove "successful" for the SDR, relative to the ECU and to the rest of the currencies, in terms of achieving a predicted risk/return tradeoff. The result for the ECU may be attributed to the fact that it contains currencies with larger volatility than those included in the SDR basket.

The constant term, ϕ_0 , takes into account a possible non-stationary element (drift), like a time trend, in total returns. Under the null hypothesis, currency returns would allow an autoregressive structure with constant variance residuals. Under the alternative, an ARCH structure on the residuals is imposed. During the sample period, the constants in all currency returns, except that of gold, were positive and only gold returns were not significantly different from zero. These terms may be interpreted as mean weekly returns, thus the intercept for deutsche mark returns

1/ The Bera-Jarque test statistic is approximately distributed as a central χ^2 (2) under the null hypothesis of normality in the underlying distribution of returns (see Hendry (1989), pp. 32-33).

2/ The augmented Dickey-Fuller test is a stationarity test, where the coefficient of the lagged dependent variable is tested to see whether it is equal or greater than unity (unit root test). A coefficient that is less than one indicates a stationary time series.

indicates that the annualized average weekly returns on deutsche mark investments were 10.28 percent. ^{1/} Further, since ϕ_1 for all currency returns is less than one, it would seem that all returns are stable. Note that U.S. dollar returns exhibit a different pattern of generating process than other currency returns owing to the fact that returns on U.S. assets consist only of the respective interest rates. U.S. dollar returns at time t are found to be affected by a six-lag structure, although the coefficients of the fourth and fifth lags are not statistically significant. However, the coefficient of the sixth lag on the U.S. dollar-returns process is highly significant, indicating an apparent cyclicity of U.S. interest rates of similar periodicity. Since the sum of the six coefficients of the lag structure is 0.989, U.S. dollar returns appear to be also stable.

For most currency returns, the lagged squared errors provide a large contribution to the conditional variance, as indicated by the significant α_1 coefficients, which imply that currency returns are generated by processes with non-constant unconditional variance. Because the principal assumption of constant variances of the error terms in an OLS estimation is violated, the latter methodology should not be used as an estimation procedure for currency returns. Furthermore, the coefficient structure of the ARCH model for U.S. dollar returns is monotonically decreasing, indicating that a squared error from the previous week has a greater effect on current conditional variance than a squared error from two weeks ago.

Another important observation on the statistical properties of the stochastic process followed by SDR returns is that the conditional variance of the SDR returns displays an optimal lag length of one, as only the coefficient of the first period (squared) residual is statistically significant. Thus, an ARCH process of order one is indicated by the data. The estimated SDR returns have finite variance since the coefficient of the lagged residuals is less than one. The resulting ARCH effects for the SDR returns are further supported by their distributional properties of high leptokurtosis and the highly significant Bera-Jarque tests indicating a non-normal distribution for these SDR returns. These empirical findings indicate that the conditional variance of the SDR returns is not constant over time and that optimization of a portfolio which includes the SDR as an

^{1/} The implied annualized average weekly returns are calculated as

$$\left[\left(1 + \frac{\bar{y}}{100} \right)^{52} - 1 \right] * 100, \text{ where } \bar{y} = \phi_0 / \sum_{i=1}^6 \phi_i$$

Accordingly, the implied annualized average weekly returns for U.S. dollar investments are 4.84 percent, the Japanese yen investments 11.34 percent, the pound sterling investments 9.30 percent, the French franc investments 11.15 percent, SDR investments 6.56 percent, ECU investments 7.33 percent, the Average Composite index investments 1.23 percent, and gold 0.96 percent, during the period February 1982 to December 1991.

V. Conclusions

This paper provides empirical evidence that would permit investors, including central banks, to make more accurate calculations of optimal strategies in the management of their reserves. The gains in accuracy depend directly on how the variance or risk measures on various currency investment returns change over time. To test whether the variability of returns is constant we employ the ARCH model, which was further utilized to provide estimates of the variability of the various currency investment returns.

On the basis of weekly data for investment returns on eight currencies and gold for the period February 1982 to December 1991, the empirical tests show that the variances of total returns on all currencies and gold change significantly over time. Evidence of positive and highly significant autocorrelation for weekly returns on those assets and their predictable level of changing variability is not consistent with the assumption of constant (time-invariant) risk that is often used in asset allocation models. The empirical results presented in this paper generally suggest that for a U.S. dollar-based investor, an ARCH model which takes into account the non-stationarity in return variability provides more accurate estimates of the variability of total returns on various currency-denominated investments than ordinary least-squared models.

In particular, the sign of the intercept in the ARCH specifications is non-negative for all currencies, indicating that the variability of asset returns would increase in the absence of any influences from previous period returns. A further interesting empirical result is that the volatility of returns from investing in the deutsche mark, the pound sterling, the French franc, the Japanese yen, ECU, SDR, and gold assets is affected by the square of one lag of their past residuals, while the variability of returns on U.S. dollar assets is affected by the square of two lags of their past residuals and that of a currency composite by the square of five lags of its past residuals. This result can be interpreted to mean that for individual currencies the effect of changes in variability on future variability lasts for only a short period and diminishes over time. The variability of returns on U.S. investments may be attributed to the fact that they comprise only interest rates on the respective assets. Interest rates, as opposed to exchange rates, tend to exhibit longer persistence, i.e., they are influenced by longer lags. For an average currency composite investment, however, the effect lasts for a significantly longer period.

The returns from investment in the SDR and ECU follow an ARCH process of order one and their probability distribution appears to be highly leptokurtic. These results are broadly similar to those for individual currencies. In particular, the coefficient of the past-period error introduced in the variance of SDR returns is smaller than in that of any other currency returns owing to a large extent to the built-in diversification of currencies in the SDR basket. In contrast, ECU returns exhibited the highest transmission of unexpected weekly shocks on their respective volatility during the sample period. The difference between the

statistical results for the SDR and those for the ECU could be attributed to the fact that the ECU contains currencies of lesser importance and larger volatility in foreign exchange markets than those included in the SDR basket. Further study of the characteristics of the SDR and the ECU might shed some light on more fundamental comparisons between the performance of these two currency composites as a reserve asset. In addition, application of Monte-Carlo simulations with a traditional portfolio model and one using the ARCH specification may further quantify the deviations from optimal allocations and losses incurred in terms of risk-adjusted returns.

Specification and Estimation of the ARCH-Type Models

1. The ARCH model

Estimation of the ARCH process depends upon the specification of the conditional mean. To determine an adequate representation for the mean, we first compute the sample autocorrelation and partial autocorrelation functions for the logarithm of the levels and log differences of each asset return. If the results are consistent with the existence of a unit root (the null hypothesis) using the Dickey-Fuller method, which is robust to ARCH, then we should use the following ARCH model:

$$y_{it} = y_{it-1} + \varepsilon_{it} \quad (8)$$

where

$$\varepsilon_{it} | (y_{it-1}, y_{it-2}, \dots), \sim N(0, h_{it}) \quad (9)$$

and

$$h_{it} = \alpha_{i0} + (\alpha_i/m) [q \cdot \varepsilon_{it-1}^2 + (q-1) \cdot \varepsilon_{it-2}^2 + \dots + \varepsilon_{it-q}^2] \quad (10)$$

where y_{it} is the logarithm of the return on currency i -denominated asset and $m = q(q+1)/2$. For a given value of q , α_{i0} and α_i are estimated for each currency asset individually. In equation (8), we impose some restrictions on y_{it-1} such as that the first-order autoregressive coefficient is one and all other coefficients are zero. To be able to test the significance of the first-order autoregressive coefficient, we can estimate equation (8) and (10) using the residuals from a different order autoregression on ε_{it} . If the results are close, we can say that the first-order autoregression coefficient is appropriate to construct the model. The objective is to obtain efficient estimates of α_{i0} and α_i from equation (10) for a given value of q .

2. An ARCH-M model

The ARCH-M model is of the form

$$y_t = a + \psi h_t + \varepsilon_t \quad (11)$$

where $\mu_t = a + \psi h_t$ and $\mu_t = x_t' \beta$, with

$$h_t = \alpha_0 + \sum_{k=1}^q \alpha_k \varepsilon_{t-k}^2 \quad (12)$$

and, where $x_t' \beta$ denotes the conditional mean of y_t . ε_t is an error term with zero mean and conditional variance $E(\varepsilon_t^2 | F_t) = h_t$, where $F_t = (\varepsilon_{t-1}, \varepsilon_{t-2}, \dots, \varepsilon_{t-q})$. The ARCH model is extended to allow the conditional variance as a determinant of the mean and is called ARCH-M. In this way, changing conditional variances directly affect the expected return on a currency. In this model, an increase in the conditional variance will be associated with an increase or a decrease in the conditional mean of y_t depending on the sign of the partial derivative of y_t with respect to h_t in equation (11).

Equations (11) and (12) are estimated by the maximum likelihood (ML) estimation method, and t-statistics of coefficients are interpreted on the basis of a significance level. If the coefficients ψ and α are significant, then we say there is not only an ARCH effect ($\alpha \neq 0$), but also a time-varying expected return of an asset of interest ($\psi \neq 0$). Many theories in finance involve an explicit trade-off between the risk and the expected return. The ARCH-M model is ideally suited to handling such questions in a time series context where the conditional variance may be time varying.

To perform a diagnostic test for the linear relationship assumed between y_t and h_t , we can use $\log(h_t)$ instead of h_t and compare the log-likelihood values of models with h_t and $\log(h_t)$. The model with a larger log-likelihood value will be the final preferred model.

3. A multivariate factor-ARCH model

The ARCH model cannot account for the fact that substantial commonality appears to exist in asset-return volatility movements. A factor-analytic approach enables us to simultaneously address the problems of: (i) a large number of parameters to be estimated, and (ii) commonality in asset-return volatility movements. What a factor structure tries to capture is that the covariance matrix of asset returns can be approximated by a simpler, lower dimensional structure. We will say that there is a strict K-factor

structure if the return on the i -th currency is generated by

$$X_i = \mu_i + \beta_{i1}F_1 + \dots + \beta_{ik}F_k + \nu_i \quad (13)$$

where the factors F_k are uncorrelated with idiosyncratic disturbances, ν_i , which are in turn correlated with each other.

The factor-ARCH model is of the form

$$y_t = \lambda F_t + e_t \quad (14)$$

where $E(F_t) = E(e_{jt}) = 0$ for all j and t ,
 $E(F_t F_s) = 0$ for $t \neq s$
 $E(F_t e_{js}) = 0$ for all t, s, j
 $E(e_{jt} e_{ks}) = \gamma_j$ if $j=k, t=s$
 $E(e_{jt} e_{ks}) = 0$ if $j \neq k, t \neq s$
 $F_t | F_{t-1}, F_{t-2}, \dots, F_{t-n} \sim N(0, \sigma_t^2)$

$$\sigma_t^2 = \alpha_0 + \theta \sum_{i=1}^n (n+1-i) F_{t-i}. \quad (15)$$

Note that y, λ , and e are all vectors. The common factor F_t represents general influences that tend to affect all asset returns. The impact of the common factor on the return of asset j is reflected in the value λ_j . y_t is an $m \times 1$ vector of returns on different currency assets, and m denotes the number of assets.

a. Diagnostics for a factor-ARCH model

To be able to support the claim that there is a commonality of high volatility of asset returns, we employ the following methods: (i) plotting the asset returns, the squares of the asset returns, and the products of the asset returns with the return of dollar, and also trying to detect a common volatility in these variables; and (ii) estimating common factors (F_t) and checking that the trend of (F_t) is similar to that of any variable included in the derivation of common factors. If we find a similarity between the trend of (F_t) and that of a variable in the common factor, we conclude that this particular variable has a central role in the behavior of the common factor.

b. The estimation technique

First, the number of common factors, F_t , is determined. Second, a time series of factor values is constructed so that equation (14) becomes applicable. Namely, we determine the eigenvalues of the correlation matrix of asset returns on different currencies in order to figure out the statistically significant factors (where K denotes the number of statistically significant factors) representing all the variables according to their shares in the total variance. Finally, we test and model ARCH effects in the extracted factors, \hat{F}_t , by estimating equation (15).

The Lagrangian multiplier (LM) test statistic is used to test for a white noise process against an ARCH(n) alternative. Equation (15) represents an ARCH(n) model since there are n lag values of estimated factors \hat{F}_t . In model (14), we assume that e_t is white noise defined as $E(e_t) = 0$ for all t , $E(e_t^2) = \sigma^2$ for all t , and $E(e_t e_s) = 0$ if $t \neq s$. The LM statistic is developed as follows:

$$\text{Multivariate likelihood} = \sum_{t=1}^T \ln(L_t) \quad (16)$$

where $\ln(L_t) = -(N/2)\ln(2\pi) + (1/2)\ln|H_t^{-1}| - (1/2)e_t' H_t^{-1} e_t$, with H_t defined as the error variance-covariance matrix and N being the number of asset returns on different currencies.

$$\text{Univariate likelihood} = \sum_{T=1}^T \sum_{j=1}^N \ln(L_{tj}) \quad (17)$$

where

$$\ln(L_{tj}) = -(1/2)\ln(2\pi) + (1/2)\ln|(\sigma_{tj}^2)^{-1}| - (1/2)e_{tj}'(\sigma_{tj}^2)^{-1}e_{tj}$$

and j = U.S. dollar/deutsche mark, U.S. dollar/French franc, ..., U.S. dollar/Japanese yen. If the value of the multivariate likelihood, equation (16), is greater than that of the univariate likelihood, equation (17), then we conclude that the multivariate factor approach is better than the univariate one. That is, we accept there is a common factor displaying ARCH effects.

4. A GARCH(p,q) model

The general form of the GARCH(p,q) model is

$$y_t = \phi_1 y_{t-1} + \dots + \phi_p y_{t-p} + \varepsilon_t \quad (18)$$

where

$$\phi_p(L)y_t = \sum_{i=1}^p \phi_i L^i y_t$$

Note that

$$y_t \sim N(\phi_p(L)y_t, h_t)$$

with

$$h_t = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^k \beta_j h_{t-j} \quad (19)$$

where $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_i \geq 0$ for all i , y_t denotes the asset return on each of the currencies, and $\phi(L)$ is a polynomial lag operator in L containing the unknown parameters (ϕ_1, \dots, ϕ_p) to be estimated. Here ε_t denotes a discrete-time real-valued stochastic process with a normal conditional distribution given by

$$\varepsilon_t | \Omega_{t-1} \sim N(0, h_t)$$

where Ω_{t-1} is the information set available at time $t-1$. The information set Ω_t includes past realizations of y_t and lagged values of h_t . A GARCH model can be used to analyze the composition of asset returns in different currencies. The unique feature of ARCH and GARCH processes is that, unlike standard time-series models, the conditional variance is allowed to change over time. Analogous to ARCH models, GARCH models can be used to characterize the conditional mean and variance of expected asset returns. A GARCH model is an extension to the ARCH specification for modeling asset returns in different currencies. It differs from standard time-series models in that the conditional variance of the underlying stochastic process is specified as a function of past realizations of the random variable of interest. To summarize, if the data suggest that heteroskedasticity of the form implied by (19) is present, then proxy variables for the subjective expectations of y_t could be generated in a manner consistent with the GARCH(p,q) process.

5. A GARCH-M model

A time-varying expected asset return in different currencies should be built around a structure with a time-varying conditional covariance. A model ideally suited for this purpose is the GARCH in mean (GARCH-M) model. For y_t , the GARCH(p,q)-M model takes the general form

$$y_t = c + \psi h_t + \varepsilon_t \quad (20)$$

where the conditional mean of y_t is $\mu_t = X_t' \beta = c + \psi h_t$. Denote

$$\phi_p(L) = \sum_{i=1}^p \phi_i L^i$$

where $y_t \sim N(\phi_p(L)y_t, h_t)$

$$\text{with } h_t = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^k \beta_j h_{t-j}. \quad (21)$$

Calculation of Asset Returns

1. Calculation of asset returns in different currencies

For the sake of convenience, since exchange rates are expressed bilaterally against the U.S. dollar, the analysis is dollar-based. The same procedure can be repeated for the mark-, yen-, or franc-based investors. We define dollar-based returns as follows:

$$y_{t,k} = [(1 + i_{t-1,k}) * (1 + e_{t,k}) - 1] * 100$$

where

$$e_{t,k} = (E_{t,k} - E_{t-1,k}) / E_{t-1,k}$$

- and $y_{t,k}$ = The U.S. dollar-based returns of investment on currency k at time t.
 $i_{t-1,k}$ = Three-month treasury bill rate on a weekly basis for the country corresponding to currency k, at time t-1.
 $e_{t,k}$ = Rate of appreciation/depreciation of currency k against the U.S. dollar, at time t.
 $E_{t,k}$ = Spot rate defined as U.S. dollar per unit of local currency k, at time t.
 k = the U.S. dollar, the deutsche mark, the pound sterling, the French franc, the Japanese yen, ECU, SDR, and an average composite index.

The source for exchange rates and interest rates for the various countries employed is the IMF Treasurer's Department database.

2. Computation of gold returns in different currencies

The formula used in the calculation of returns on gold is:

$$g_t = [(G_t - G_{t-1}) / G_{t-1}] * 100$$

where g_t and G_t denote, respectively, the weekly percentage change in the U.S. dollar price of gold and the level of the dollar price of gold, at time t. Further, we define

$$GR_{t,k} = g_t + e_{t,k} * 100$$

where $GR_{t,k}$ denotes the weekly gold returns in U.S. dollars by investors of currency k at time t. The source for gold prices is the IMF Treasurer's Department database.

3. Calculation of the average composite exchange and interest rates

The hypothetical basket consists of seven currencies: the U.S. dollar (US), the pound sterling (PS), the deutsche mark (DM), the French franc (FF), the Swiss franc (SF), the Netherlands guilder (NG), and the Japanese yen (JY). A prespecified amount 1/ of currencies for the basket and an average of exchange rates (London, noon quotations in terms of U.S. dollar) for the period 1981-87 are used to calculate the weight of each currency in formulating the average composite exchange and interest rates. These figures are as follows:

Calculation of Weights for the Composite Exchange and Interest Rates

	Amount of currencies in the basket (1)	Exchange rate (London, noon quotation in U.S. dollar) (2)	Weights = (1) ÷ (2) (3)
U.S. dollar	0.7217	US/US = 1.000	w1 = 0.721654
Pound sterling	0.0118	PS/US = 0.420	w2 = 0.028120
Deutsche mark	0.2775	DM/US = 1.974	w3 = 0.140601
French franc	0.0576	FF/US = 4.560	w4 = 0.012631
Swiss franc	0.0423	SF/US = 1.781	w5 = 0.023759
Netherlands guilder	0.0225	NG/US = 2.142	w6 = 0.010526
Japanese yen	12.7213	JY/US = 202.870	w7 = 0.062706

The formulas by which average composite exchange rates (ACE) and average composite interest rates (ACI) are computed are as follows:

$$ACE_t = [w1*(US/US)_t + \dots + w7*(JY/US)_t]$$

$$ACI_t = [w1*(USI)_t + w2*(PSI)_t + \dots + w7*(JYI)_t]$$

where (USI), (PSI), ..., (JYI) denote three-month treasury bill interest rates in the United States, United Kingdom, ..., Japan, respectively.

1/ It represents the average share of currencies in total official holdings of foreign exchange reserves for the period 1981-87.

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