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Global Equilibrium Exchange Rates: Euro, Dollar, "Ins," "Outs," and Other Major Currencies in a Panel Cointegration Framework

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Abstract

This paper presents a methodology for calculating bilateral equilibrium exchange rates for a panel of currencies in a way that guarantees global consistency. The methodology has three parts: a theoretical model that encompasses the balance of payments and the Balassa-Samuelson approaches to real exchange rate determination; an unobserved components decomposition in a cointegration framework that identifies a time-varying equilibrium real exchange rate; and an algebraic transformation that extracts bilateral equilibrium nominal rates. The results uncover that, by the start of Stage III of the European Economic and Monetary Union (EMU), the euro was significantly undervalued against the dollar and the pound, but overvalued against the yen. The paper also shows that the four major EMU currencies locked their parities with the euro at a rate close to equilibrium.

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Contents Page

I. INTRODUCTION

The advent of Stage III of the European Economic and Monetary Union (EMU) raised several issues regarding the equilibrium exchange rate of the euro against other major currencies. First, since EMU implies the irrevocable fix of the members' currencies (the *in* countries) to the euro (€), the adequacy of the chosen parities will be crucial to understand future relative price developments. Second, although the euro has just replaced the ECU in the foreign exchange markets, this conversion and its recent evolution have opened the debate on the "right" dollar/euro parity. Finally, the existence of four European Union (EU) countries outside EMU (the *out* countries), which may join in the future, raises the issue of their appropriate definitive euro parity.

In order to address these types of issues, this paper present a methodology for the calculation of equilibrium bilateral exchange rates in a way that guarantees consistency at the global level, and assesses the degree of misalignment of some major currencies—the euro among them—as well as those of *in* and *out* countries.

We start by defining the concept of multilateral equilibrium real exchange rate in a simple theoretical model. From the definition of the real exchange rate, two components can be distinguished, which relate to the external and internal balance of the economy: (i) the concept of external balance, based on the asset market models developed by Frenkel and Mussa (1985); and (ii) the concept of internal balance, based on the productivity hypothesis advanced by Balassa (1964) and Samuelson (1964). The theoretical model used in this paper takes advantage of this decomposition to derive an equilibrium real exchange rate that is consistent with both approaches to real exchange rate determination.

From an empirical point of view, we use cointegration techniques to map the equilibrium conditions derived from the theoretical model into the available data. In this regard, using a vector of currencies for the period 1980–98 allows for the possibility of testing for cointegration in a panel context. After showing that a cointegration relationship between the real exchange rate and the fundamentals of its external and internal components exists for the panel of currencies under study, we use an orthogonal decomposition of the cointegration matrix into a permanent and a transitory component. The time varying permanent component, for which confidence bands are also computed, is identified as the equilibrium multilateral real exchange rate for each currency.

At this stage, the divergence between the equilibrium and the actual value of the multilateral rate provides an estimate of the misalignment of each currency relative to its trading partners. The next step is to derive the bilateral rates of the currencies: since the panel of currencies covers most of the trade among developed countries, the link between multilateral and bilateral rates at a global level can be exploited to derive consistent estimates of the equilibrium bilateral rates, in both nominal and real terms.

With these elements, we obtain a complete picture of the estimated misalignment of the bilateral exchange rates for each country at the inception of Stage III of EMU (end-1998).

Taking the euro a as reference, as we do in the empirical analysis, the results can be divided into three different groups of countries:

- Major currencies: The euro was about 7.5 percent undervalued against the U.S. dollar, which implies an equilibrium nominal rate of 1.26 dollars per euro. It was also slightly undervalued against the Canadian dollar (2.8 percent), but overvalued against the yen (6.25 percent).

- *Out* currencies: The pound sterling was overvalued against the euro (15.5 percent), implying an equilibrium rate of about 0.8 pounds per euro. The Danish krone was slightly overvalued (1.5 percent), the Swedish krona was somewhat undervalued (3.8 percent), and the Greek drachma was in equilibrium.

- *In* currencies: Of the four major EMU currencies, the Deutsche mark displayed a significant overvaluation at entry time (3 percent), the Italian lira was moderately undervalued (about 4 percent), and the French franc and the Spanish peseta entry rates were in equilibrium.

The issue of equilibrium exchange rates has received considerable attention in the literature (see, among many others, Faruqee (1995), Isard and Faruquee (1998), and the papers in MacDonald and Stein (1999)). We consider our effort to be a valuable undertaking for five reasons. First, we devise a theoretical model that encompasses both the external and the internal equilibrium approaches to exchange rate determination. Second, we take advantage of recently developed panel integration and cointegration techniques that overcome the low power of standard tests. Third, by using an unobserved components approach to the extraction of the equilibrium rate, we exploit all the available information contained in the multivariate cointegration relationship. Fourth, we go beyond the calculation of multilateral misalignments and compute bilateral equilibrium rates that are directly comparable with market rates. Finally, we provide an assessment of currencies at a critical historical moment, namely the locking of parities of the euro.

The rest of the paper is organised as follows. Section 2 presents the theoretical framework that lays out the basis for the empirical exercise. After decomposing the real exchange rate into an external and an internal component, we briefly present the theoretical model used to derive the equilibrium rate exchange rate and its determinants. Section 3 introduces the empirical approach to computing equilibrium real exchange rates and Section 4 describes the data. Section 5 presents the results for multilateral and bilateral rates, and the final section draws some conclusions.

II. A STYLIZED MODEL OF THE EQUILIBRIUM EXCHANGE RATE

The concept of long-run or equilibrium exchange rate has been addressed in the literature with different approaches, starting from the simple and popular concept of purchasing power parity (PPP), implying a constant equilibrium real exchange rate (see,

among many others, Dornbusch (1987) for a survey of PPP). The empirical failure of PPP, documented recently in Breuer (1994), opened the door to two main lines of research on determination of the real exchange rates, which emphasized the underlying net foreign asset position and sectoral (tradable-nontradable) balance of a country, respectively. On the one hand, the balance of payments approach, which builds on the identity between the capital and the current account, was initiated by Nurske (1945), and is based on the adequacy of the current account to sustain notional or equilibrium capital flows and keep in check saving-investment balances. Frenkel and Mussa (1985) adopted this model to derive the equilibrium real exchange rate; more recently, Gagnon (1996) found that (accumulated) current account balances explain the behaviour of the real exchange rate. Properly refined and extended, this approach is also the basis of FEER computations by Williamson (1994) and the IMF's macroeconomic balance methodology (IMF 1998). On the other hand, the work of Balassa (1964) and Samuelson (1964), pointed to differences in productivity growth between countries and sectors as the main determinants of the long-run behaviour of the real exchange rate; recent contributions by de Gregorio et al. (1994), for instance, underlined the importance of sectoral demand. This hypothesis has been shown to explain to some extent the behaviour of exchange rates in the long run (Canzoneri et al. (1999)).

To provide a rationale for our empirical exercise, we present an illustrative model that essentially encompasses both perspectives on exchange rate determination. The starting point is the decomposition of the exchange rate into two different relative prices: (i) the price of domestic relative to foreign tradables and (ii) the relative price of non-tradables relative to tradables within each country. Each component is related to one of the theories mentioned above. The first component captures the competitiveness of the economy and determines the evolution of the foreign asset position, while the second plays a central role in adjusting excess demand across sectors in the economy. We build on this decomposition and derive an extended version of the stock-flow analysis presented in Faruquee (1995), explicitly accounting for the role of sectoral evolutions, along the lines of Broner et al. (1998). The long-run solution to the model determines an equilibrium value for the real exchange rate consistent with the internal and the external balance in the economy.

A. Real Exchange Rate Decomposition

There are two countries in the world, each producing two goods: one tradable (subscript T, in what follows) and one non-tradable (N). The real exchange rate (q) is defined as the relative price of domestic to foreign goods in the consumption basket, p and p^*, respectively,[2] expressed in domestic currency.

$$q = p - (s + p^*) \qquad (2.1)$$

[2] An asterisk denotes foreign variables.

where s is the (log) nominal exchange rate, defined as the price of foreign currency in terms of domestic currency. Thus, an increase q represents an appreciation of the real exchange rate.

The consumer price index (CPI) for each country is a weighted average of the tradable, non-tradable, and imported (tradable) prices, all expressed in their home currency:

$$\begin{aligned} p &= (1-\alpha_N-\alpha_{T^*})p_T + \alpha_N p_N + \alpha_T (s+p^*_T) \\ p^* &= (1-\alpha^*_N-\alpha^*_T)p^*_T + \alpha^*_N p^*_N + \alpha^*_T (p_T - s) \end{aligned} \qquad (2.2)$$

where the αs are the weights of the respective goods. Substituting these expressions in *(2.1)*, assuming that $\alpha_N = \alpha^*_N$, and rearranging terms we obtain[3]

$$q = (1-\alpha_{T^*}-\alpha^*_T)q_X + \alpha_N q_I \qquad (2.3)$$

where $q_X = \left[p^T - (s+p_T{}^*)\right]$ is the relative price of domestic to foreign tradables and $q_I = \left[(p_N - p_T) - (p_N{}^* - p_T{}^*)\right]$ is the price of non-tradables relative to tradables across countries.

B. The Model

Following this decomposition, the model distinguishes between an external and an internal dimension of equilibrium. Each relative price adjusts to achieve equilibrium in one of the markets, and hence we will denote q_X and q_I as the internal and the external relative prices, respectively. The equilibrium exchange rate ($\bar{q}$, where the bar denotes equilibrium values) will require simultaneous equilibrium in both markets, and thus will be a combination of the equilibrium internal and external relative prices.

The **external balance** clears the tradable goods market, and it is characterised by the achievement of a desired stock of net foreign assets. Adjustment to equilibrium is reflected in the evolution of the current account balance, which in turn leads to an accumulation of net foreign assets (f). By definition, the current account balance (ca) is the sum of the trade balance (xn) and the net income that residents receive (or pay) on their foreign asset holdings, all expressed in real terms. The current account position of the foreign country is the same but with the opposite sign:

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[3] Alberola and Tyrväinen (1998) compute the shares of non-tradables in the CPI for EMU countries and the results are clustered in a small range (between 62 percent and 72 percent). The shares of imported tradables, however, depend on the openness degree and vary widely among countries.

$$ca = -ca^* = xn + i^* f \qquad (2.4)$$

where i^* is the international real interest rate. A positive stock of net foreign assets ($f>0$) reflects a creditor position for the country.

The trade balance depends on the evolution of the external relative price: an increase in the relative price of domestic tradables (q^X) shifts consumption toward foreign tradables and worsens the trade balance, when the Marshall-Lerner condition holds. Hence,

$$ca = -\gamma q_X + i^* f \qquad (2.5)$$

To close the model we define the relationship between the current and the capital accounts. A sustainable balance of payments position is one that reflects a current account balance financed by a sustainable accumulation of capital flows, which in turn depends on the underlying determinants of the net foreign asset position. We follow Frenkel and Mussa (1985) who model the rate of accumulation of foreign assets as depending not only on the adjustment to its desired level (F) but also on the differences between short and long-run real rates ($i - \bar{i}$) on financial assets, since a positive wedge biases the allocation of saving toward the present:

$$\text{ca} = \eta(\text{F} - \text{f}) + \mu(i - \bar{i}) \qquad (2.6)$$

Assuming that the long rate equals the world rate, $i^* = \bar{i}$, and that the uncovered parity holds, the divergence between domestic and foreign real interest rates reflects expected real exchange rate changes:

$$i - i^* = -E(\dot{q}) \qquad (2.7)$$

The **internal balance** is characterised in terms of excess demand functions in the non-tradable sector for each country, d_N and d^*_N:

$$\begin{aligned} d_N &= -\alpha_N xn - \theta[(p_N - p_T) - (k + z)] \\ d^*_N &= \alpha_N xn - \theta[(p^*_N - p^*_T) - (k^* + z^*)] \end{aligned} \qquad (2.8)$$

The first term in the right hand side of each equation states that excess demand is proportional to the excess of aggregate domestic spending over domestic production measured in terms of the foreign tradable, which in turn is equal to the trade balance with a negative sign; α_N, the share of non-tradables in total expenditure, is the proportionality factor. The second term conveys, in the first place, the Balassa-Samuelson productivity hypothesis: k and k^* are variables representing sectoral productivity differentials (an increase in k amounts to an increase in the relative productivity of the tradable sector); the assumption of complete labor mobility within countries, or of centralised wage bargaining at the national level, ensure nominal wage homogeneity across sectors. Since the non-tradable market clears

domestically, the prices of non-tradables must increase relative to those of tradables $k > 0 \Rightarrow p_N > p_T$, otherwise production of non-tradables would shrink and an excess demand for non-tradables would arise. Sectoral demand shocks may also be behind the excess demand for non-tradables, as de Gregorio et al. (1994), among others, have emphasised. In this spirit, z and z^* account for positive relative demand shocks in the non-tradable sectors, such as public expenditure or tariffs shocks, which have the same effect as productivity shocks on relative sectoral prices. Finally, θ is the price elasticity of excess demand and is assumed to be equal in both countries.

Nonzero excess demand for non-tradables signals disequilibrium in the internal allocation of resources, which is adjusted by movements in the relative price of non-tradables. We assume sluggishness in the adjustment of the demand for non-tradables, owing to stickiness in prices, and the speed of adjustment (ρ>0) is set to be the same in both countries $p_N - p_T = \rho d_N, p^*_N - p^*_T = \rho d^*_N$, so that:

$$q_I = \rho(d_N - d^*_N) \tag{2.9}$$

Once we have described the structure of the model, the next step is to characterize the global equilibrium. Assuming rational expectations and operating on the previous expressions, the model reduces to a system of three differential equations with one predetermined variable (the stock of foreign assets, f), and two non-predetermined variables (the internal and external relative prices, q_I and q_X,), with forcing variables k, z and F:

$$\begin{aligned} \dot{q}_X &= \frac{(1-2\alpha_N{}^2\rho)v q_X + \alpha_N\rho\left[2\theta q_I - (r^*+\mu)f + \mu F - \theta(k-k)^* - \theta(z-z^*)\right]}{\gamma(1-\alpha_T-\alpha^*_T)} \\ \dot{q}_I &= \rho\left[2\alpha_N v q_X - 2\theta q_I + \theta(k-k)^* + \theta(z-z^*)\right] \\ \dot{f} &= -v q_X + r^* f \end{aligned} \tag{2.10}$$

The long-run solution of the model implies that the dynamics of the dependent variables are driven only by the forcing variables and the stability of the system requires the existence of as many unstable roots in the solution as the number of non-predetermined variables (see Buiter (1989)). For illustrative purposes we can assume that the levels of the forcing variables are fixed in the long run,[4] so that the steady state equilibrium of the model is obtained by setting $\dot{q}_X = \dot{q}_I = \dot{f} = 0$:

[4] The forcing variables have long-run dynamics, which explain the variability of the equilibrium real exchange rates derived in the empirical part.

$$\bar{q}_X = \frac{r^* F}{v}$$

$$\bar{q}_I = \alpha_N r^* F + \frac{(k-k)^* + (z-z^*)}{2} \qquad (2.11)$$

$$\bar{f} = F$$

The interpretation of this solution is straightforward: equilibrium in the net foreign asset position is attained when the actual stock equals the desired stock. Determinants of the desired stock of net foreign assets are diverse, and arise from structural features of the economy ranging from demographic trends to savings behavior or investment opportunities. The equilibrium external relative price $\bar{q}_X$ is a positive function of F. Note that this relaxes the assumption of PPP in the tradable goods sector, a common feature of real exchange rate models.[5] Thus, a higher F implies larger interest receipts, which can finance the larger trade balance deficit arising from a more appreciated currency in equilibrium. Finally, the evolution of $\bar{q}_I$ is a positive function not only of sectoral productivity differentials (across countries) but also of the desired stock of net foreign assets; this latter effect stems from the fact that a higher F implies higher domestic expenditure, which leads to an excess demand for tradables that increases their price.

Since the variable under study is the real exchange rate, it is convenient to derive its equilibrium level, $\bar{q}$, which is attained when both the external and the internal relative prices are in equilibrium. From (2.3) and (2.11), it immediately follows that:

$$\bar{q} = (1 - \alpha_{T*} - \alpha^*_T)\frac{r^* F}{v} + \alpha_N \left[\alpha_N r^* F + \frac{(k-k)^* + (z-z^*)}{2} \right] \qquad (2.12)$$

III. THE EMPIRICAL METHODOLOGY

A. The Empirical Model

The theoretical model has identified three fundamentals for the evolution of the real exchange rate: the level of net foreign assets (f), a measure of relative sectoral productivity (k-k^*), and exogenous demand factors (z-z^*) that may affect sectoral allocation. However, we encounter a problem at this stage, namely that these fundamentals are not easy to identify in practice.

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[5] To allow for deviations in the law of one price in the tradable sector, it suffices that domestic and foreign tradables be imperfect substitutes, as Broner et al. (1998) show.

Regarding the level of net foreign assets, the problem is easily overcome. Although this is not a standard item in national income accounts, it can be traced to the evolution of the current account.

The problems related to sectoral productivity and demand shocks are more severe. Since demand shocks also drive sectoral productivity, the latter could be an adequate variable to consider. However, measures of sectoral productivity are quite controversial (see Bernard and Jones (1995)) and, more importantly, data are not available on a timely basis and are not homogeneous across countries.[6] Therefore, it is necessary to use a proxy for sectoral productivity, which is readily available. We take advantage of the already robust evidence of a long-run relation between sectoral productivity and sectoral prices (see, among others, de Gregorio et al. (1994), Canzoneri et al. (1999), Alberola and Tyrväinen (1998)) to use an index of relative sectoral prices as a proxy for sectoral productivities.

More precisely, we use the comparative index of the relative price of non-tradable versus tradable goods devised by Kakkar and Ogaki (1999). Their comparative index, denoted by *n*, consists of the domestic ratio of the consumer price index *CPI* to the wholesale price index *WPI* relative to the foreign ratio:

$$n = CPI/WPI \Big/ CPI^*/WPI^* \qquad (3.1)$$

The *CPI* contains a large share of non-tradables (mainly services), whereas the wholesale index contains mainly tradables. Thus, the ratio of *CPI* to *WPI* is an increasing function of the relative price of non-tradable goods. The variable *log (CPI)* corresponds to p in *(2.2)*, *log (WPI)* is the proxy for p^T, and the denominator corresponds to foreign country variables. Operating in the expression, it immediately follows that the relative sectoral price differential index (n) equals the product of the internal real exchange rate and the weight of non-tradables in the consumption basket, $\alpha_N q_1$.

Hence, a suitable empirical model to estimate under these assumptions would be

$$q_t = \beta_0 + \beta_1 f_t + \beta_2 n_t + u_t \qquad (3.2)$$

whereby we would explain the evolution of the real exchange rate as a function of its fundamentals.[7]

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[6] In fact, data on sectoral productivity exist normally at annual frequency and are made available with two to three years lag to the databases.

[7] Notice that n cannot be identified with the internal equilibrium exchange rate because, as equation *(2.12)* shows, the latter depends on both the level of net foreign assets and the

(continued...)

At this stage, one could think that finding a long-run cointegration relationship in *(3.2)* between the real exchange rate and its fundamentals would yield an estimate of its equilibrium rate. However, this result does not hold: for this to be true, we must first observe the ***equilibrium*** levels of the fundamentals, and then apply *(3.2)* to them. Unfortunately, we can observe only the actual values of the variables, and therefore some further econometric manipulation is needed to estimate the equilibrium real exchange rate.

Intuitively, the observed exchange rate could be decomposed into two components: the first one, when the fundamentals are at their steady state levels, would be the equilibrium exchange rate

$$\bar{q}_t = \beta_0 + \beta_1 \bar{f}_t + \beta_2 \bar{n}_t \qquad (3.3)$$

where, operating on *(2.11)* and *(2.12)*, the parameters are expected to take the values $\beta_1 = \frac{(1-\alpha_T-\alpha^*_T+\alpha^2 v)r^*}{v}$ and $\beta_2 \cong 1$; the second component, when the fundamentals are away from their respective steady states, would correspond to the deviations of the exchange rate from its equilibrium level.

$$\hat{q}_t = \beta_0 + \beta_1 \hat{f}_t + \beta_2 \hat{n}_t + u_t \qquad (3.4)$$

where $\hat{f}_t$ and $\hat{n}_t$, refer to deviations of fundamentals from their equilibrium values.

Thus, a strategy toward the estimation of the equilibrium real exchange rate could be based on the econometric decomposition of the observed real exchange rate into a transitory and a permanent component. The estimated equilibrium exchange rate is taken to be the permanent component, while the transitory component reflects deviations with respect to equilibrium. In what follows, we first relate the concept of equilibrium exchange rate with the concept of cointegration, and then we show how cointegration allows for the extraction of the two unobserved components from the observed exchange rate and fundamental series.

B. Cointegration and Orthogonal Decomposition

In order to understand the link between equilibrium and cointegration, it is useful to depart from the theory of purchasing power parity (PPP), which implies a constant value for the equilibrium real exchange rate $\bar{q}$. In econometric terms, PPP implies a stationary process for the real exchange rate or, in other words, that q_t is integrated of order zero (*I(0)*). On the contrary, if the real exchange rate contains a unit root (i.e., it is an *I(1)* variable), no constant equilibrium can be defined for q_t and the PPP hypothesis is rejected.

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determinants of sectoral allocation. Thus, β_1 concentrates the effect of net foreign assets on both the external and the internal equilibrium exchange rates.

However, failure of PPP to hold does not necessarily imply that no equilibrium exists. Rather, the equilibrium may be time varying. In our case, if q_t, f_t, and n_t are *cointegrated,* then u_t in *(3.1)* will be I(0), and an equilibrium real exchange rate will exist. In other words, q_t will fluctuate around a time-varying equilibrium characterized by the long-run cointegration relationship $[1 \ -\beta_1 \ -\beta_2]$.

Thus, the presence of cointegration allows for the existence of a time-varying equilibrium. However, as observed above, the time-varying equilibrium exchange rate cannot be inferred by simply imposing the cointegration vector on the observed values of the explanatory variables. In this regard, cointegration among a set of variables presents a very desirable property: it allows for the decomposition of the relationship among the variables into two components. A permanent or secular component, which would be I(1), describes the long-run properties of the relationship among the variables, and can be identified with a time-varying equilibrium path; and a transitory component, which would be I(0), corresponds to deviations over time from the permanent component, and would represent departures of the fundamentals from their steady state values.[8]

The decomposition of the observed series into the permanent and transitory components will require the identification of the basic properties of these unobserved components (see Maravall (1993) for a theoretical discussion of the identification of permanent and transitory components). There are several procedures in the literature to address this issue, including Quah (1992), Kasa (1992), and Gonzalo and Granger (1995). In principle, we can characterize a transitory component as having limited memory; in other words, the effects of a shock to the component die out over time. However, it is perfectly possible that a shock to a transitory component has permanent effects on the aggregated series. For example, it would be enough to assume that the transitory component Granger-causes the permanent component to obtain this effect. In such a case, the economic interpretation of the components may be misleading, for whether a shock is temporary or permanent would depend on whether the researcher is observing the component or the aggregated series.

The decompositions advanced by Quah (1992) and Kasa (1992) present this undesirable property. In order to overcome this problem, Gonzalo and Granger (1995) derive a decomposition where the transitory component does not Granger-cause the permanent component in the long run, and where the permanent component is a linear combination of contemporaneous observed variables. In other words, the first restriction implies that a change in the transitory component today will not have an effect on the long-run values of

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[8] Another solution to this problem would be to simply calculate the equilibrium paths of the fundamentals by fitting them a trend or a smoothing filter (see, for example, Clarida and Gali (1994), Baxter (1994), and Faruquee (1995)). This approach, however, would discard all the information contained in the multivariate cointegration relationship.

the variables.[9] The second restriction makes the permanent component observable and assumes that the contemporaneous observations contain all the necessary information to extract the permanent component.

Analytically, consider the 3x1 vector $x_t=[\, q_t, f_t\, n_t]'$ which under the null of one cointegration vector admits the following representation:

$$\Delta x_t = D_1 \Delta x_{t-1} + \ldots + D_{p-1} \Delta x_{t-p+1} + \Pi x_{t-p} + e_t \qquad (3.5)$$

where e_t is a vector white noise process with zero mean and variance Σ and Π is 3 x 3 matrix with rank 1. Given that Π is not full rank, it can be written as the product of two rectangular matrices α and β of order 3 x 1 such that $\Pi=\alpha\beta'$. The vector β is the cointegration vector and the vector α is the factor-loading vector. Next, we can define the orthogonal complements $\alpha_\perp$ and $\beta_\perp$ as the eigenvectors associated with the unit eigenvalues of the matrices $(I- \alpha (\alpha' \alpha)^{-1} \alpha')$ and $(I- \beta (\beta' \beta)^{-1} \beta')$, respectively. Notice that $\alpha'_\perp \alpha = 0$ and $\beta'_\perp \beta = 0$. With this notation it is possible to write

$$x_t = \beta_\perp (\alpha'_\perp \beta_\perp)^{-1} \alpha_\perp x_t + \alpha(\beta'\alpha)^{-1} \beta' x_t \qquad (3.6)$$

where $\beta_\perp (\alpha'_\perp \beta_\perp)^{-1} \alpha_\perp x_t$ would capture the permanent component and $\alpha (\beta' \alpha)^{-1} \beta' x_t$ the transitory component. Gonzalo and Granger (1995) show that the transitory components defined in this way will not have any effect on the long-run value of the variables captured by the permanent components.

The identification of the permanent component with equilibrium implies that

$$x_t = \beta_\perp (\alpha'_\perp \beta_\perp)^{-1} \alpha_\perp x_t$$

and

$$\hat{x}_t = \alpha(\beta'\alpha)^{-1} \beta' x_t$$

from where the estimation of the equilibrium exchange rate and its deviation directly follows.

C. Panel Cointegration

We rely on panel integration and cointegration techniques to infer the long-run properties of our series. It is well known the notorious low power of standard unit root and cointegration techniques when applied to the individual time series available for the length of the post war period, especially in the case of series that are stationary but have highly persistent dynamics. Papers by Shiller and Perron (1985) and Pierce and Snell (1995)

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[9] In essence, this decomposition rules out hysteresis effects in exchange rates.

confirm that it is the time span, and not the frequency of the data, that matters for the power of these tests. Given the short sample of available data, a practical alternative to increase the power of the tests is to add the cross-sectional dimension to the exercise. For the sake of completeness, we will also present the results of time series unit root (ADF) and cointegration (Johansen) tests, although our judgement will be based on the results of the more powerful panel tests.

In this regard, recent research by Quah (1994), Levin and Lin (1994), Im, Pesaran, and Shin (1997), and Pedroni (1998) has developed panel unit root and cointegration statistics that, under fairly general conditions, have more power than the standard time series tests. Moreover, the tests by Im, Pesaran, and Shin (IPS) (1997) and Pedroni (1998) allow for heterogeneity in the dynamics of each of the cross section units in the panel. That is, under the null hypothesis of a unit root in either the series of interest or the residuals of a cointegration regression, the dynamics of each cross section unit are allowed to differ. Under the alternative hypothesis of no unit root, there is no homogeneity restriction. This flexibility makes it appropriate to use these tests in this framework, where the parameters controlling the long-run equilibrium and the short-run dynamics are likely to differ across countries.[10]

Since standard time series techniques, such as the Augmented Dickey-Fuller test for the unit root hypothesis and Johansen tests for cointegration, are widely used in the empirical literature, we now turn to the discussion of how to construct and implement the panel unit root and cointegration tests. In all cases, the tests are computed on the basis of well-known statistics calculated for each cross section unit. The general expression of the tests for a panel spanning T years for N cross section units is

$$t_{NT} = N^{1/2}(\bar{t}_T - E(t_T))/(\text{var}(t_T))^{1/2}$$

where $\bar{t}_T = \Sigma^N_{i=n} t_{iT}$ and t_{iT} is a statistic computed on each cross section unit.

Im, Pesaran and Shin (1997) propose a test statistic *(t-IPS)* to test for the null hypothesis of a unit root in a panel. Their test is based on the average of the standard ADF t statistics obtained from individual tests and hence, as noted above, it does not require any kind of homogeneity restriction. Thus, it retains the flexibility of the individual unit root tests by allowing for heterogeneous autoregressive roots, while increasing the power. The finite common moments $E(t_T)$ and $var(t_T)$ are obtained by Monte Carlo methods and are tabulated in Im, Pesaran, and Shin (1997). Their study shows that under the null hypothesis of a unit root, the panel unit root statistic is distributed as a standard normal.

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[10] See, among others, Canzoneri, Cumby, and Diba (1996), Chinn and Johnson (1996) or Bayoumi and MacDonald (1999) for applications of panel unit root and cointegration techniques to exchange rates.

For the case of cointegration, Pedroni (1998) proposes several panel cointegration tests. In this paper we will use two of them that may be constructed using as a basis well known univariate unit root tests, the *Group PP* (*GPP*) and the *Group t* (*Gt*). *GPP* is computed on the basis of the individual Phillips-Perron statistics applied on the residuals of each cointegration regression. Likewise, *Gt* is computed on the basis of the individual ADF *t*-statistics applied on the same residuals. Notice that both statistics allow for full heterogeneity across cross-section units. Pedroni (1998) tabulates, also by Monte Carlo methods, the finite moments $E(t_T)$ and $var(t_T)$ for each test, which in this case depend also on the number of regressors in the cointegration regression. In both cases, the panel cointegration tests are asymptotically normal.

IV. THE DATA

Our paper considers twelve currencies (eleven countries plus the euro composite) and covers the period 1980 Q1-1998 Q4 that ends with the creation of EMU. The sample can be divided into three groups: the euro plus some other major currencies (United States, Japan and Canada); EU countries outside EMU, the *out* countries (Denmark, Sweden, Greece, and United Kingdom); and the four largest EMU economies, the *in* countries (Germany, France, Italy, and Spain). The relevant variables are the real effective exchange rate (q_t), the stock of net foreign assets (f_t), and an index of relative sectoral prices (n_t).[11] It is important to note that the proposed model can only be tested in a multi-country context, since the data on the external position are always defined with respect to the rest of the world.

For the real effective exchange rate (q_t) we use the CPI-based index of the real effective exchange rate constructed by the IMF for all the considered currencies except the euro. In the construction of the series, the weight of each currency w_i, (where i indicates the trading partners) in the computation of each real exchange rate depends on the share of trade of the corresponding country.[12] Following common practice, we use the natural logarithm of the series. For the euro, we use a series of real effective exchange rate constructed by the BIS, based on the exchange rates of the eleven euro area countries, weighted by manufacturing exports.[13]

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[11] An extensive description of the data and their sources can be found in Appendix II.

[12] The group of trade partners is wider than the currencies considered. The additional countries are Australia, Hong-Kong, New Zealand, Norway, Switzerland, Taiwan and the rest of EU countries not considered in the study (Austria, Belgium, Netherlands, Finland, Ireland and Portugal). Luxembourg has been excluded.

[13] A brief explanation of the BIS methodology can be found in the Data Appendix.

The construction of the index of relative sectoral prices (n_t) has been introduced above. The ratio of *CPI* to *WPI* has to be considered relative to the rest of the countries, whose weights are given by w_i.

$$n_{it} = \alpha_N q_{Iit} = \log\left(\frac{(CPI_t/WPI_t)}{\prod_i (CPI_t/WPI_t)^{w_{it}}}\right) \qquad (4.1)$$

For the euro, $n_{\epsilon t}$ was computed by dividing the relative sectoral prices of the euro area by the geometric mean of relative sectoral prices in the rest of the world:

$$n_{\epsilon t} = \log\left(\frac{\prod_j (CPI_{jt}/WPI_{jt})^{w_{\epsilon j}}}{\prod_i (CPI_{it}/WPI_{it})^{w_{i\epsilon}}}\right) \qquad (4.2)$$

where $w_{\epsilon j}$ is the share of each euro-area country in internal trade, and $w_{i\epsilon}$ is the share in euro area trade of each country outside the euro area. Here, we also use the natural logarithms in the estimation process.

Finally, the computation of the stock of net foreign assets (f_t) requires an estimate of the initial stock. Data on the stock of net foreign assets were obtained from the OECD. The evolution of the net foreign asset position for each country is then obtained by adding up the current account balances ca_t.

$$f_{it} = f_{i0} + \sum_{j=1}^{t} ca_{ij} \qquad (4.3)$$

and, in order to adjust for the size of the country, net foreign assets were normalised by GDP.[14]

In the case of the euro, we first aggregate the stocks of net foreign assets of the eleven member countries and then we compound them with current account data for the euro area. With this approach, the stock of net foreign assets of euro-area members held by the rest of EMU countries is netted out.

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[14] A detailed compilation of the stock of net foreign assets is beyond the scope of this paper. The series we obtain, however, are broadly similar to the more carefully calculated by Milesi-Ferreti and Lane (1999).

V. THE COMPUTATION OF MULTILATERAL AND BILATERAL EQUILIBRIUM RATES

A. Cointegration Vectors

In this section we present the results of the unit root and cointegration tests that serve as the basis for the computation of the equilibrium real exchange rates. As mentioned above, we use panel integration and cointegration techniques to infer the long-run properties of our series.

The results of the unit root tests appear in Table 1. In its upper part it shows the results of the panel unit root tests (*t-IPS*) which, at standard significance levels, do not reject the null of all the series being I(1). For completeness, the results for the individual ADF tests are also displayed, with similar results. The null of a unit root is rejected only for the French *q* and the Swedish *n*. Thus, the evidence points overwhelmingly to the presence of unit roots in all three variables.

Table 1. Unit Root Tests

	q	*f*	*n*
Panel (t-IPS)	-1.68	1.72	-0.16
Individual series analysis			
Euro	-1.87	-0.90	-1.64
U.S.	-1.49	-1.46	-1.37
Japan	-1.72	-0.53	-2.63
Canada	-0.23	-0.27	-0.79
U.K.	-2.58	-0.80	-1.89
Sweden	-1.98	-2.26	-3.01*
Denmark	-1.53	-0.15	-1.60
Greece	-1.14	-0.89	-2.04
Germany	-1.10	-2.48	-2.28
France	-4.28 *	0.43	-1.97
Italy	-2.86	-2.39	-1.95
Spain	-1.52	-1.89	-2.16

Note: 95 percent critical values: ADF: -2.9.
t-IPS:-1.69. An asterisk indicates the rejection of a unit room at the 5 percent significance level.

The next step is testing for cointegration and, if the null of no cointegration is rejected, estimating the cointegration relationships. As above, we provide the results of both panel and single equation tests. The latter are performed following Johansen (1988). Table 2 shows some disparity in the results, with the U.K., Canada and the euro failing to reject the

null of no cointegration. The more powerful panel cointegration tests, however, both strongly reject the null of no cointegration at the 5 percent significance level.

Table 2. Cointegration Tests		
	Panel Cointegration Tests	
	GPP	GT
	-1.95*	-3.1
Individual series	Trace	Lambda
Euro	26.40	16.34
U.S.	45.16*	28.71*
Japan	37.34*	25.24*
Canada	23.49	16.86
U.K.	12.66	8.18
Sweden	28.21	19.92**
Denmark	24.18	24.77*
Greece	10.36	9.14
Germany	36.37*	19.21**
France	32.44*	17.69
Italy	27.37	22.23*
Spain	29.30	20.81**

Note:* denotes significant at 5 percent;
** denotes significant at 10 percent.
Critical value of panel test at 5 percent: -1.69.
The panel cointegration tests are: Pedroni Group PP (GPP) and Pedroni Group t (Gt).

Table 3 displays the cointegration vectors for the countries under study. Note that all of them display the right negative signs, and that the value of the parameter associated with n is systematically very close to one, as expected.

Table 3. Cointegration Vectors			
	q	*f*	*n*
Euro	1	-0.30	-1.03
U.S.	1	-0.85	-0.94
Japan	1	-0.66	-1.03
Canada	1	-0.51	-1.01
U.K.	1	-0.16	-1.06
Sweden	1	-0.08	-1.01
Denmark	1	-0.15	-1.02
Greece	1	-0.01	-0.91
Germany	1	-0.67	-1.02
France	1	-0.01	-1.01
Italy	1	-0.61	-1.02
Spain	1	-0.48	-1.02

Using these cointegrating vectors and the loading factors of the cointegration relationships (α's), the real exchange rate series are decomposed into a permanent and a transitory component, following the Granger and Gonzalo (1995) methodology described in Section IIIB. The permanent and transitory components represent in our empirical model the real equilibrium exchange rate and the deviations from equilibrium, respectively.

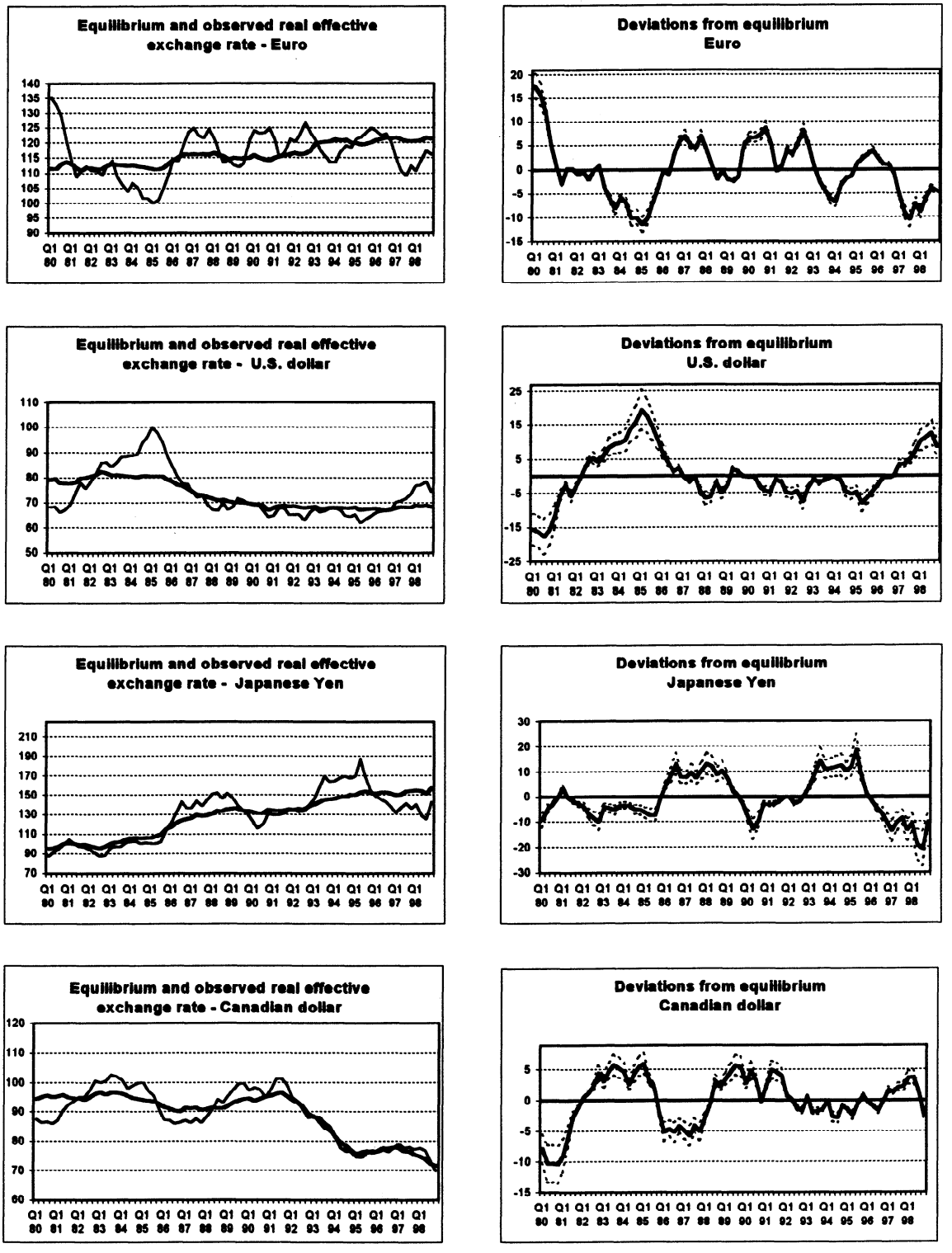
Figure 1 presents the results. The left column displays the actual and equilibrium multilateral exchange rates, and the right column presents deviations from equilibrium (the difference between actual and estimated equilibrium rates), with computed 95 percent standard error bands;[15] values above zero imply an overvaluation of the multilateral rate. Table 4 shows the misalignment of the multilateral exchange rate, $\hat{q}$, as of the fourth quarter of 1998.

Table 4. Multilateral Misalignments (as of end-1998, in percent)							
EURO	U.S.	JAPAN	CANADA	U.K.	SWEDEN	DENMARK	GREECE
-4.48	8.31	-10.24	-2.25	15.73	-5.15*	1.07	-0.71
(-0.33)	(1.22)	(-1.68)	(-0.33)	(2.92)	(3.37)	(0.23)	(0.2)
		FRANCE	GERMANY	SPAIN	ITALY		
		-2.21	2.66	-2.32	-8.23*		
		(-0.86)	(0.51)	(-1.11)	(5.07)		

Note: Standard errors in parentheses. An asterisk means nonsignificant at the 90 percent level.

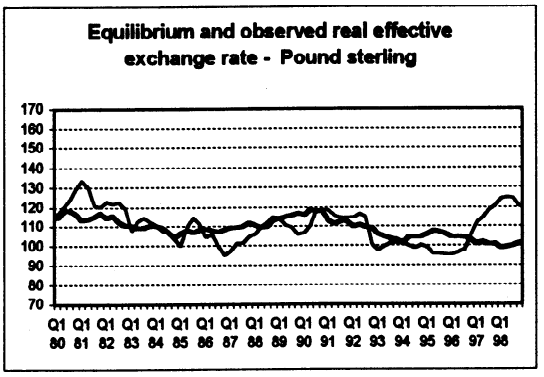
[15] See Alberola and Lopez (1999) and Appendix I for an explanation of how these bands are computed.

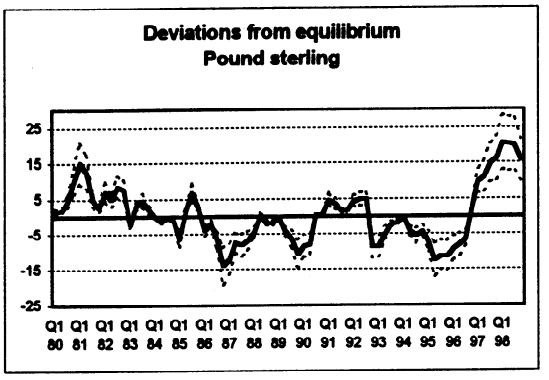
Figure 1. Multilateral Equilibrium and Misalignment[16]
Euro, U.S. Dollar, CAN Dollar and JP Yen

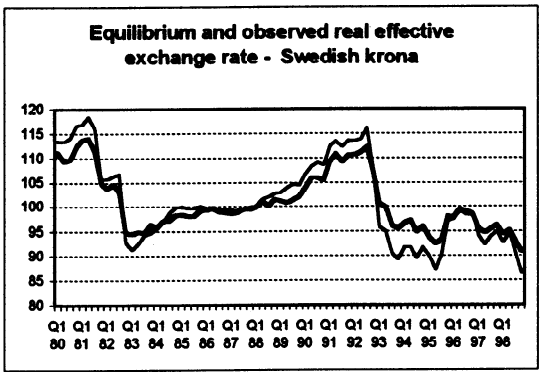


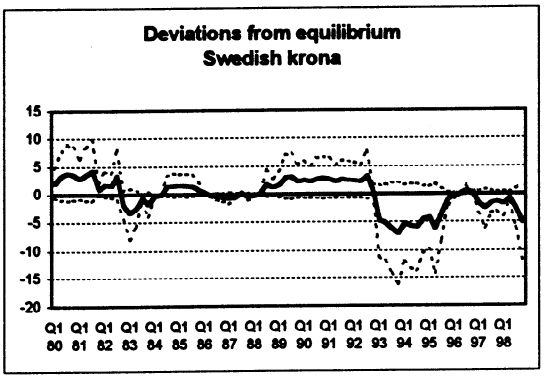
[16] In the left panel, equilibrium effective exchange rates are represented with a thicker line. In the right panel, deviations from equilibrium are expressed in percentage points. Standard error bands are represented with a dotted line (- -)

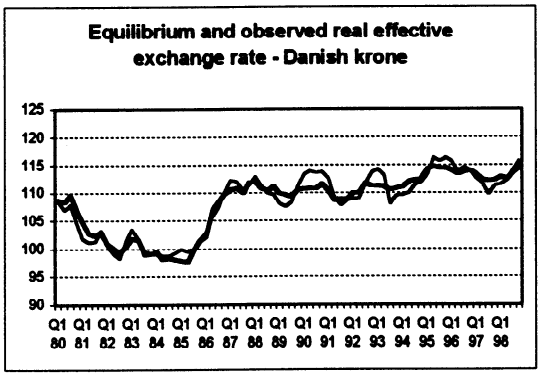
Figure 1. Sterling, Swedish Krona, Danish Krona and Greek Drachma (Continued)

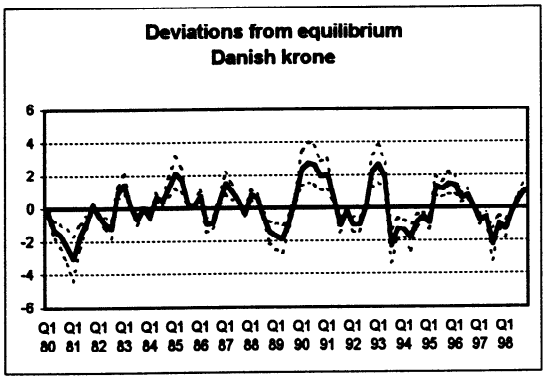


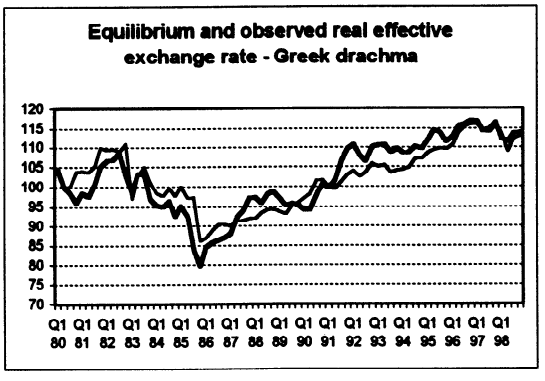












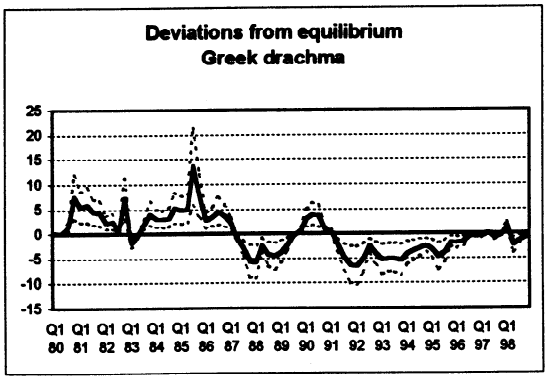
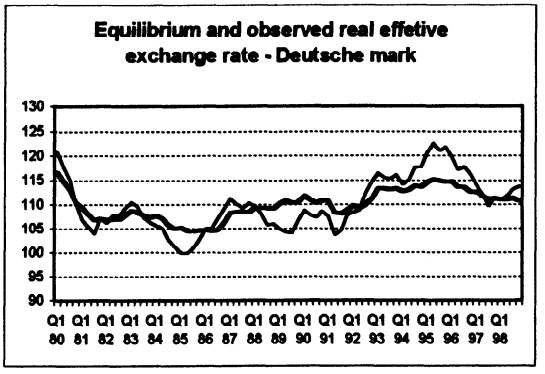
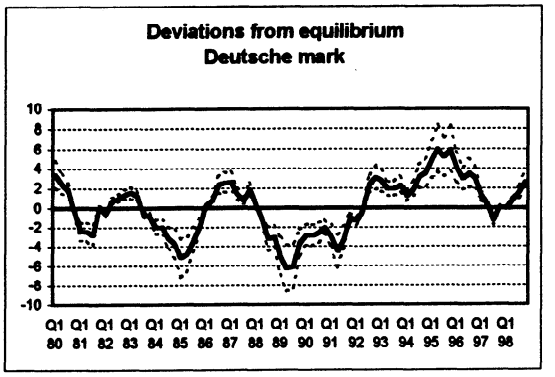
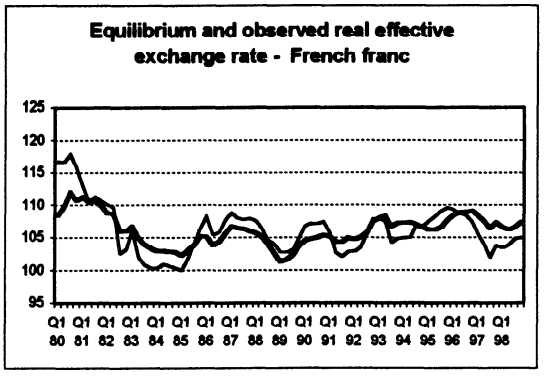
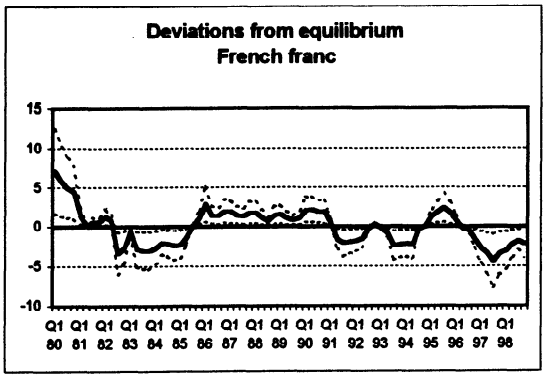


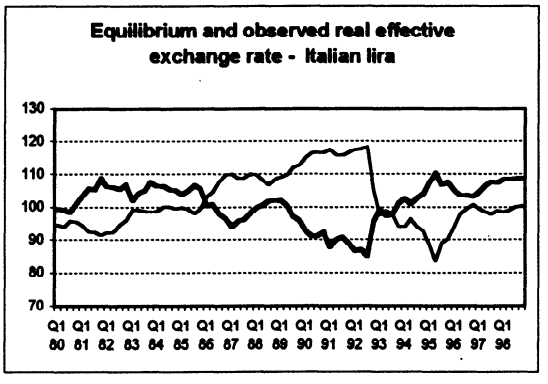
Figure 1. Deutsche Mark, French Franc, Italian Lira and Spanish Peseta (Concluded)

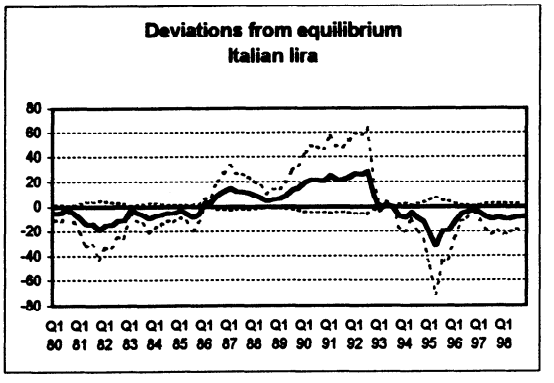


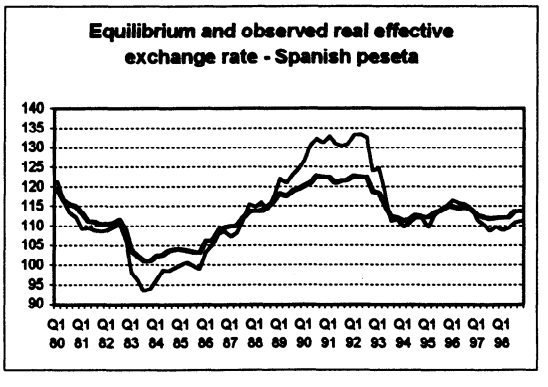


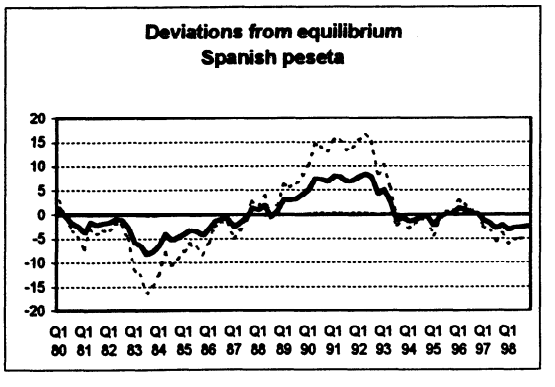












Starting with the euro, a slight appreciation trend can be observed in the long run. Deviations from multilateral equilibrium, apart from the initial period of overvaluation—which coincided with the weakness of the dollar at the beginning of the eighties—have been moderate. By the start of EMU, the euro is estimated to be slightly undervalued, between 3.8 percent and 5.1 percent, with a point estimate of 4.5 percent.

The dollar, on the contrary, displays a depreciation trend in its multilateral equilibrium rate, and deviations from trend have tended to be larger. During the 1980s, overvaluation peaked at more than 15 percent, and the recent surge of the dollar has resulted in an overvaluation above 10 percent. The overvaluation by the end of 1998 is estimated at between 5.8 and 10.7 percent, with a point estimate of 7.5 percent.

The Japanese yen displays a strong appreciation trend over the period, although the current crises have placed it well below its long-run estimated equilibrium value (between 13.6 percent and 6.8 percent undervaluation). The behaviour of the Canadian dollar has been less volatile and the current undervaluation is estimated to be small (between 1.5 percent and 3 percent).

Moving to the *out* countries, we observe that the current overvaluation of the pound sterling is exceptional, at least relative to the historical series, and is estimated to range between 10 percent and 21.6 percent, with a point estimate of 15.7 percent. The Swedish krona displays some problems because it is estimated with a low degree of precision: although the point estimate shows a 5 percent undervaluation, it is not significantly different from zero. The Danish krone has displayed remarkable stability along its appreciating trend, and the overvaluation by end-1998 was estimated to be between 0.5 percent and 1.5 percent. Finally, the Greek drachma is slightly undervalued with respect to its equilibrium level.

Finally, the past behaviour of the major EMU currencies, the *in* countries, presents a remarkable stability with respect to the equilibrium values, with the exception of the lira, which in any case displays extremely wide standard error bands. By the fixing of the euro parities, the Deutsche mark was somewhat overvalued, between 1.6 percent and 3.7 percent, and the French franc and the Spanish peseta were slightly undervalued, between -0.5 percent and -4 percent and -0.1 percent and -4.5 percent, respectively. The estimate of the Italian lira points to an important undervaluation, about 8.2 percent, which however turns out not to be statistically significant.

B. Bilateral Equilibrium Rates

The results for multilateral equilibrium exchange rates, although interesting in themselves, are uninformative as regards the equilibrium position between pairs of currencies. Moreover, with the current trend toward a world with few major currencies, the relevant questions usually revolve around equilibrium bilateral rates: what is the equilibrium dollar/euro rate? Is the yen undervalued against the dollar? What would be the *right* entry rate in EMU for the sterling pound?

A simple algebraic operation allows us to answer these questions. Note that the (log) real multilateral exchange rate for country *i* (q_i) is the trade-weighted average of the (log) bilateral real exchange rates of the trade partners vis-à-vis country *i* (e_{ij}):

$$q_i = \sum_j w_{ij}\, e_{ij} \qquad (5.1)$$

where w_{ij} are the trade weights, which add up to one $\sum w_{ij} = 1$. Alternatively, the bilateral rate can be expressed in terms of an arbitrary numeraire currency, say *n*, making use of the cross-rates equivalence in logarithmic terms: $e_{ij} = e_{jn} - e_{in}$. Therefore, it is possible to express the ($n \times 1$) column vector of multilateral exchange rates, denoted by Q, with the numeraire currency being the last element, in terms of the exchange rate vector E, whose elements are the bilateral exchange rates against the numeraire currency, as follows:

$$Q = (W-I)\, E \qquad (5.2)$$

W is the ($n \times n$) trade matrix with zeros in the diagonal and I is the identity matrix of order *n*. Matrix $(W-I)$ must be singular because E contains only $n-1$ independent exchange rates. This property imposes a linear constraint across the real exchange rates, which allows for the calculation of globally consistent bilateral rates,[17] since one of the multilateral rates in Q is redundant. Thus, by eliminating this redundant exchange rate and solving for the reduced system, consistent bilateral exchange rates can be derived.

To do so, the row and column corresponding to the numeraire currency are discarded, and the remaining $n-1$ multilateral rates are expressed relative to the numeraire currency, $Q_{-} - 1q_n$. The subscript $(.)$ denotes that the *n*th currency has been deleted and 1 is a conformable $(n-1)$ vector of ones. From (5.2) it follows that:

$$Q_{-} - 1q_n = (W-I)_{-} E - 1q_n \qquad (5.3)$$

Since q_n is the trade-weighted average of the $n-1$ bilateral rates for the numeraire currency, from (5.1) we can write

$$Q_{-} - 1q_n = CE_{-} \qquad (5.4)$$

where C is the following ($n-1 \times n-1$) matrix

$$C = [(W-I)_{-}\, 1(w_{n1}, w_{n2,\ldots}, w_{nn-1})]$$

---

[17] See Isard and Faruqee (1998), Chapter 7, for more details on the algebraic foundation of the linear constraint.

From here, the derivation of bilateral equilibrium exchange rates is obtained by pre-multiplying both sides of *(5.4)* by the inverse of *C*. Since we have derived the deviations of multilateral rates from equilibrium ($\hat{q}$), the problem can be re-specified to compute the bilateral equilibrium exchange rates deviations from equilibrium, denoted by $\hat{e}$. Thus, we have

$$\hat{E}_{.} = C^{-1}[\hat{Q} - 1\hat{q}_{n}] \qquad (5.5)$$

where $\hat{E}_{.}$ is the *n-1* vector of bilateral equilibrium deviations with respect to the numeraire currency.

This method can be applied to transform our vector of deviations of multilateral rates into a matrix of deviations of bilateral rates. It is important to note that this transformation requires that such vector encompass all of the world, with two consequences. First, as long as the euro enters the global analysis, euro countries cannot be considered; computation of bilateral rates for them will require a different approach, as we will see in Section V.C. Second, a completeness problem arises, since the countries under study cover most, but not all, of the world. Thus, the *rest of the world (RoW)* must be included in the analysis, and this can be done by expanding the *W* matrix by one column and one row. The column accounts for the weight of the rest of the world in each country's trade, and the row contains the share of trade of the rest of the world trade with each of the countries considered. Moreover, an assumption is required for the equilibrium real exchange rate deviations for the rest of the world ($\hat{q}_{RoW}$): we will assume that this deviation is zero. Given that the weight of *RoW* in the trade matrix is small (see below), changes in this assumption are not expected to have important consequences on the results.

The trade matrix (*W*) appears in the data appendix, where the euro, which has been considered as numeraire, is placed in the last row. The vector of deviations from the multilateral equilibrium ($\hat{Q}$) can be found in Table 4. The bilateral deviations with respect to the euro are derived by substituting these two variables into *(5.5)*, and appear in the first left column of Table 5. The remaining bilateral rate deviations have been obtained by computing the cross-rates of each currency with the euro.[18] It is important to stress that this methodology guarantees that all of these bilateral rates are globally consistent with the multilateral equilibrium estimation.

---

[18] In order to calculate the standard errors of the bilateral deviations we would need the theoretical expression of their covariance matrix. As a first approximation, a measure of standard error could be derived by taking the maximum range of deviations of each bilateral rate, given by the confidence bands of the respective multilateral rates and the point estimates of the other currencies. This calculation would yield non significant deviations with respect to the euro only for Sweden and Greece.

Table 5. Bilateral Equilibrium Deviations ($\hat{e}_i$) of									
	EURO	USA	JAP	CAN	UK	SWE	DK	GRE	RoW
Relative to EURO	0.00								
U.S.	-7.41	0.00							
JAPAN	6.23	13.64	0.00						
CANADA	-2.78	4.63	-9.01	0.00					
U.K.	-15.47	-8.06	-21.70	-12.69	0.00				
SWEDEN	3.75	11.16	-2.48	6.53	19.22	0.00			
DENMARK	-1.54	5.87	-7.77	1.24	13.92	-5.29	0.00		
GREECE	0.40	7.81	-5.83	3.18	15.87	-3.35	1.94	0.00	
RoW	-0.83	6.58	-7.06	1.95	14.64	-4.58	0.72	-1.23	0.00
A (-) sign implies undervaluation with respect to the reference country.									

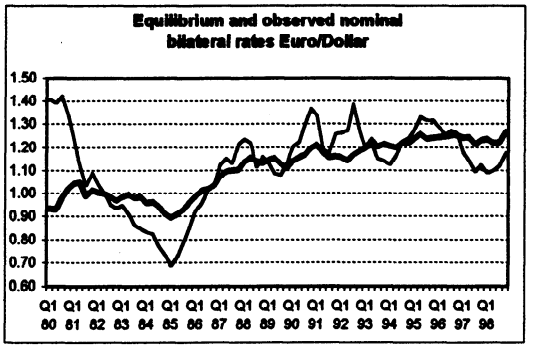
The computation of the matrix of bilateral rates has been done for the final period, to avoid an excessive bulk of information, but it is of course possible to compute the series of historical bilateral rates in real terms and also in nominal terms. We have performed this exercise for the three major world currencies: the euro, dollar and yen. The left column of Figure 2 shows the deviations of the exchange rate from equilibrium, whereas the right column of Figure 2 shows the implied equilibrium bilateral exchange rate together with the observed nominal bilateral rates.

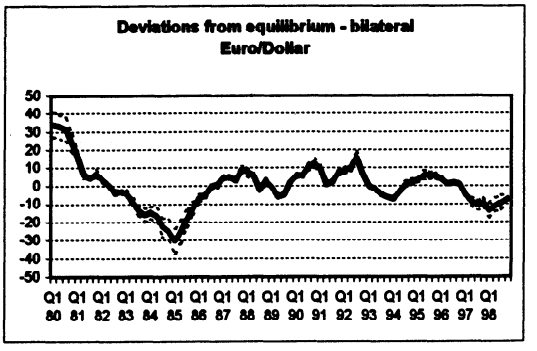
The estimation of the bilateral equilibrium exchange rates and its comparison with current values allows us to answer most of the relevant questions posed above. It shows that, by end-1998:

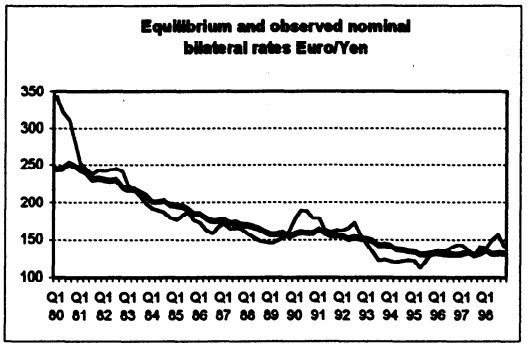
- The euro was significantly undervalued against the dollar (7.5 percent) implying by the start of EMU a nominal equilibrium parity of about 1.26 dollar per euro. The ensuing euro depreciation has widened the disequilibrium to about 20 percent by end-1999. On the contrary, the euro/yen rate was 6.23 percent overvalued by the end of 1998.

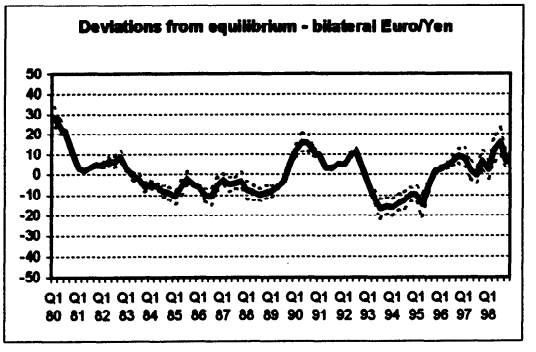
- The dollar was strongly overvalued against the yen (13.64 percent) and to a lesser extent against the Canadian dollar (4.63 percent).

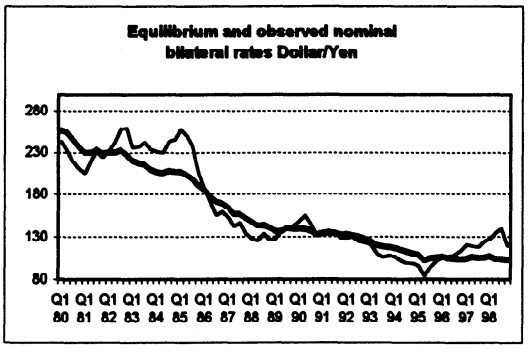
Figure 2. Bilateral Equilibrium and Misalignment[19]
Euro/Dollar, Euro/Yen and Dollar/Yen

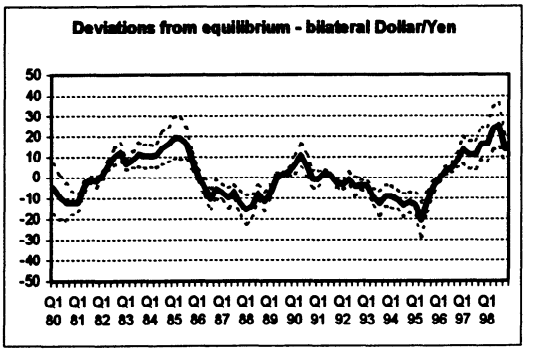












[19] In the left panel, equilibrium effective exchange rates are represented with a thicker line. In the right panel, deviations from equilibrium are expressed in percentage points. Standard error bands calculated as explained in footnote 18 appear with a dotted line.

- The pound was strongly overvalued against all other currencies (more than 15 percent against the euro, more than 21 percent against the yen). The equilibrium entry rate for the pound by the end of 1998 is estimated to be 0.81 pounds per euro.

- The Swedish krona was strongly undervalued against the dollar and the pound, and somewhat undervalued against the euro (3.7 percent). The Danish krone was slightly overvalued against the euro, and the Greek drachma was essentially in equilibrium against the euro.

In order to check the robustness of these results with respect to the equilibrium assumption for the rest of the world, we have performed a sensitivity analysis to account for different deviations of $\hat{q}_{RoW}$. Only for very large assumed deviations (more than 30 percent) do some of the qualitative results start to change, confirming the robustness of our estimations.

C. Equilibrium Rates of EMU Currencies Against the Euro

Because the euro-area currencies are part of a bigger aggregate, the previous procedure to obtain bilateral rates cannot be applied to them. Nevertheless, given the multilateral equilibrium of both the euro and each of the major countries in the euro area, we can compute the bilateral deviation from equilibrium of each EMU currency relative to the euro, denoted by $\hat{e}_{i€}$.

Following the methodology of the previous section, the multilateral exchange rate can also be expressed as a weighted average of other multilateral rates comprising groups of countries. In the case of EMU currencies, it is convenient to distinguish two components in the (deviation) of multilateral rates ($\hat{q}_i$): the multilateral rate relative to the currencies outside EMU ($\hat{q}_{i,ex€}$) and relative to the rest of EMU members ($\hat{e}_{i€}$). From *(5.1)*, we obtain:

$$\hat{q}_i = \sum_j w_{ij}\, \hat{q}_{ij} = (1 - w_i)\hat{e}_{i€\_} + w_i \hat{q}_{i,ex€} \qquad (5.6)$$

where *(1-w)* is the relative weight of the euro area in the country's trade. Regarding $\hat{e}_{i€\_}$ two points are worth stressing. First, although it is taken as a bilateral rate, the exchange rate against the euro for any country has actually been, until its launching in January 1999, a multilateral rate, in the sense that the euro is a trade-weighted basket of currencies. In spite of this, we will express it as a bilateral exchange rate, owing to the fact that it is conventionally considered as such. Second, the euro is usually defined as a basket of all EMU currencies, and therefore the bilateral exchange rate with respect to the euro for any EMU country is different from $\hat{e}_{i€\_}$ since this definition does not contain the currency *i*. We will follow the

standard definition that implies the following correction for the euro rate: $\hat{e}_{i\epsilon} = (1-b_i)\hat{e}_{i\epsilon\_}$, where b_i is the weight of currency i in the standard computation of the euro.[20]

Therefore, deviations of the multilateral exchange rate for each country can be expressed as:

$$\hat{q}_i = \frac{(1-w_i)}{1-b_i}\hat{e}_{i\epsilon} + w_i\hat{q}_{i,ex\epsilon} \qquad (5.7)$$

Note that $\hat{e}_{i\epsilon}$ could also be derived taking advantage of cross rates:[21]

$$\hat{e}_{i\epsilon} = \hat{q}_{i,ex\epsilon} - \hat{q}_\epsilon \qquad (5.8)$$

where $\hat{q}_\epsilon$ is the deviation of the euro against the ex -EMU countries, that is, the multilateral equilibrium rate of euro computed in the previous section.

Solving for the unknown $\hat{q}_{i,ex\epsilon}$ in the previous expression and substituting it into *(5.7)*, the exchange rate for each country with respect to the euro is given by:

$$\hat{e}_{i\epsilon} = \Phi_i(\hat{q}_i - w_i\hat{q}_\epsilon) \qquad (5.9)$$

where $\Phi_i = \frac{1-w_i}{1-b_i w_i}$. Substituting the estimated deviations of the multilateral rate equilibrium for each country and the euro, the equilibrium deviations of each EMU country with respect to the euro are obtained, and appear in Table 6.

Now we are in the position of assessing the nominal entry rates of the four major EMU countries:

- Germany's exchange rate relative to the euro was slightly overvalued at entry, about 3 percent.
- France and Spain entered EMU at basically their equilibrium rates against the euro.
- Italy's exchange rate was moderately undervalued at entry (almost four percent).[22]

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[20] In our approach, which follows the BIS methodology, *b* is given by the share of each country in external EMU trade.

[21] For example, the (log) dollar-yen can be derived as the difference between the (log) dollar-pound and (log) yen-pound rates.

[22] This undervaluation would not be significant if standard errors were calculated.

Table 6. Deviations from Equilibrium of the Exchange Rates of EMU Countries with Respect to the Euro (end-1998, in percent)	
	$\hat{e}_{i\epsilon}$
France	-0.13
Germany	3.00
Spain	-0.42
Italy	-3.76
A (-) sign implies undervaluation with respect to reference	

VI. CONCLUSIONS

The behaviour of exchange rates has always raised the question of misalignments from equilibrium. In this context, the birth of a new large currency, the euro, raises three main questions: (i) what is its "right" value against the other two major currencies, the dollar and the yen?; (ii) was the final locking of parities among EMU members at an appropriate value, and what are the implications for future developments in relative prices?; and (iii) what is the appropriate entry rate for the aspiring euro members?

In this connection, this paper has proposed a methodology for the analysis of equilibrium exchange rates that allows us to answer this type of questions. From a theoretical point of view, we have outlined a model that encompasses two well-known theories of real exchange rate determination. From an empirical point of view, we have exploited the advantages of panel cointegration and unobserved component decomposition to estimate multilateral equilibrium values. Finally, a simple algebraic transformation has allowed us to shift from multilateral to bilateral rates, which are directly comparable to market rates. This methodology has been applied to all the major currencies (the euro, dollar, yen, and Canadian dollar) plus the *in* countries (those already in EMU), and the *out* countries (those awaiting entry).

The results have shown that, by end-1998, the pound and, to a lesser extent, the dollar, were both overvalued against the euro, and that the recent weakness of the latter has widened this misalignment. Regarding prospective EMU members, the results indicate that the pound should depreciate considerably before entering EMU, while for Sweden, Denmark, and Greece deviations from equilibrium are currently small. Finally, and despite the large volatility of EMU currencies in the period after the ERM crises, the final parities of the four major EMU currencies with respect to the euro seem to be quite close to equilibrium.

Overall, the theoretical appeal of the model, the robustness of the econometric results, the long-run perspective of the methodology—implying parameter stability—and its computational simplicity make this approach to the estimation of equilibrium exchange rates a suitable tool for exchange rate monitoring. Further research will be directed toward assessing the forecasting capability of this methodology.

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Derivation of the Asymptotic Distribution of Deviations from the Multilateral Equilibrium

The deviation from the multilateral equilibrium is defined in *(3.6)* as

$$\hat{C}_t = \hat{\alpha}(\hat{\beta}'\hat{\alpha})^{-1}\hat{\beta}' x_t$$

Notice that, conditional on x_t, the only source of variation on $\hat{C}_t$ could arise from $\hat{\alpha}$ and $\hat{\beta}$. The first order expansion of $\hat{C}_t$ around α and β yields

$$\hat{C}_t - C_t = C_t/\alpha'(\hat{\alpha}-\alpha) + C_t/\beta'(\hat{\beta}-\beta) + O_p(T^{-1})$$

and

$$T^{1/2}(\hat{C}_t - C_t) = C_t/\alpha' T^{1/2}(\hat{\alpha}-\alpha) + C_t/\beta' T^{1/2}(\hat{\beta}-\beta) + O_p(T^{1/2})$$

Since $\hat{\beta}$ is T consistent,

$$T^{1/2}(\hat{\beta}-\beta) \xrightarrow{p} 0$$

and therefore we can write

$$T^{1/2}(\hat{C}_t - C_t) = C_t/\alpha' T^{1/2}(\hat{\alpha}-\alpha) + o_p(1)$$

Thus, all the variation in $\hat{C}_t$ arises from $\hat{\alpha}$. Tedious but straightforward matricial algebra yields

$$C_t/\alpha' = -C_t(\beta'\alpha)^{-1}\beta' + (\Lambda' C_t \otimes I_N) = Z$$

where $\Lambda = \alpha(\alpha'\alpha)^{-1}$, $\otimes$ is the Kroneker product, and I_N is an identity matrix of order N. We can therefore write

$$T^{1/2}(\hat{C}_t - C_t) = ZT^{1/2}(\hat{\alpha}-\alpha) + o_p(1)$$

or

$$T^{1/2}(\hat{C}_t - C_t) = ZZ_1 T^{1/2}(\hat{\Pi}-\Pi) + o_p(1)$$

where $Z_1 = (B' \otimes I_N)$ and $B = \beta(\beta'\beta)^{-1}$. The asymptotic distribution of $T^{1/2}(\hat{\Pi}-\Pi)$ is normal with variance Σ_Π (see Lutkepohl (1993) for a proof and for the form of Σ_Π). This implies that $\hat{C}_t$ will also be asymptotically normal

$$T^{1/2}(\hat{C}_t - C_t) \sim N(0, ZZ_1\Sigma_\Pi Z_1'Z')$$

Data Sources

We have tried to achieve the highest feasible consistency subject to data availability. When possible, sources are homogeneous across countries. Note that, while this paper was written, European countries were transforming their statistics according to the ESA95, making it impossible to obtain all national account data with the same standard. Thus, some of the series used here will be soon replaced by their ESA counterparts. Nevertheless, series and sources have been carefully chosen to guarantee consistency both within and across countries.

- Consumer Price Index (CPI). All CPI series were obtained from the IMF with the exception of Hong Kong, Ireland and Taiwan.

- Wholesale Price Index or Production Price Index (WPI-PP). Most WPI's data were obtained from the IMF with the exception of Australia, Norway and Taiwan. A table with the corresponding Datastream codes for the two price indexes follows.

Table 7. Price Indices

Country	CPI	Source	WPI-PP	Source
Australia	AUI64...F	IMF	AUOCPPMFF	OECD
Austria	OEI64...F	IMF	OEI63...F	IMF
Belgium	BGI64...F	IMF	BGI63...F	IMF
Canada	CNI64...F	IMF	CNI63...F	IMF
Denmark	DKI64...F	IMF	DKI63...F	IMF
Finland	FNI64...F	IMF	FNI63...F	IMF
France	FRI64...F	IMF	FRI63...F	IMF, Bloomberg
Germany	BDI64...F	IMF	BDI63...F	IMF
Greece	GRI64...F	IMF	GRI63...F	IMF
Hong Kong	HKCPALLAF	HK, MDS[23]	HKI63...F	IMF
Ireland	IRCP....F	CSO, Ireland[24]	IRI63...F	IMF, Bloomberg
Italy	ITI64...F	IMF	ITI63...F	IMF
Japan	JPI64...F	IMF	JPI63...F	IMF
Netherlands	NLI64...F	IMF	NLI63...F	IMF
New Zealand	NZI64...F	IMF	NZI63...F	IMF
Norway	NWI64...F	IMF	NWOCPTOTF	OECD
Portugal	PTI64...F	IMF	PTI63...F	IMF
Spain	ESI64...F	IMF	ESI63...F	IMF
Sweden	SDI64...F	IMF	SDI63...F	IMF
Switzerland	SWI64...F	IMF	SWI63...F	IMF
Taiwan	TWCP....F	DGBAS	TWWHLPRCF	National Statistics
U.K.	UKI64...F	IMF	UKI63...F	IMF
U.S.	USI64...F	IMF	USI63...F	IMF
Yugoslavia	YGI64...F	IMF	YGI63...F	IMF

[23] MDS stands for Monthly Digest of Statistics.

[24] CSO stands for Central Statistic Office

- **Real effective exchange rate.** All data on real effective exchange rates for non-Euro currencies were obtained from the IMF. The data are deflated with the CPI and seasonally adjusted. We obtained the real effective exchange rate for the euro from the BIS. They consider the trade matrix in manufacturing for each country and proceed by

  a) Deducting intra euro area trade and re-computing the respective trade matrices;
  b) Calculating an extra euro area real effective exchange rate –taken into account only extra-euro area trade-;
  c) Weighting each real effective exchange rate by each share of extra-€ area trade. The following table presents the corresponding IMF codes.

Table 8. Real Effective Exchange Rates		
Country	REER	Source
Sweden	SDI..RECE	IMF
Denmark	DKI..RECE	IMF
Norway	NWI..RECE	IMF
UK	UKI..RECE	IMF
Germany	DEI..RECE	IMF
Greece		Eurostat
U.S.	USI..RECE	IMF
Canada	CNI..RECE	IMF
Japan	JPI..RECE	IMF
Euro		BIS

- **Trade weights** Data on trade weights are taken from the IMF trade statistics and are consistent with the construction of multilateral exchange rates. For the period 1980–99, four sets of trade weights are used (1977,81,88,95). The euro aggregate has been constructed by aggregation of extra-euro trade, to be consistent with the BIS methodology for the construction of the euro. The **trade matrix** used in the estimation of the bilateral rates requires a rest of world aggregate, which is obtained as residual. The trade matrix corresponding to 1995 is:

1995	U.S.	Japan	Canada	U.K.	Sweden	Denmark	Greece	RoW	Euro
U.S.	0.00	0.30	0.25	0.09	0.02	0.00	0.00	0.06	0.30
JAPAN	0.53	0.00	0.03	0.07	0.01	0.01	0.00	0.07	0.30
CANADA	0.82	0.06	0.00	0.02	0.01	0.00	0.00	0.02	0.08
U.K.	0.16	0.07	0.01	0.00	0.03	0.01	0.01	0.12	0.63
SWEDEN	0.12	0.05	0.01	0.12	0.00	0.06	0.00	0.04	0.61
DENMARK	0.07	0.04	0.01	0.11	0.13	0.00	0.01	0.13	0.51
GREECE	0.06	0.05	0.00	0.08	0.02	0.01	0.00	0.06	0.74
RoW	0.28	0.14	0.02	0.05	0.00	0.00	0.00	0.00	0.53
EURO	0.23	0.13	0.02	0.28	0.01	0.01	0.01	0.34	0.00

- **Nominal GDP.** All GDP data, but Greece, are annualized, quarterly and seasonally adjusted, expressed at market prices. For Greece, only annual data are available, and we allocated growth equally on every quarter. We use GDP to normalise our data on f.

- **Current Account.** All CA data are seasonally adjusted apart from France and Italy. Those series did not present a strong seasonal component so we decided to work with the original data instead of treating them.

- **Stock of net foreign assets.** For all countries but Greece we obtained f_0 from the OECD Economic Outlook, December 1996 (Annex Table 53). For Greece we used the estimation of Bloomberg. We cumulate the stock of f from 1994 Q4 OECD data in U.S. dollars, and the CA and GDP series were converted into U.S. dollars using the end of period bilateral exchange rate.

Table 9. Gross Domestic Product and Current Account

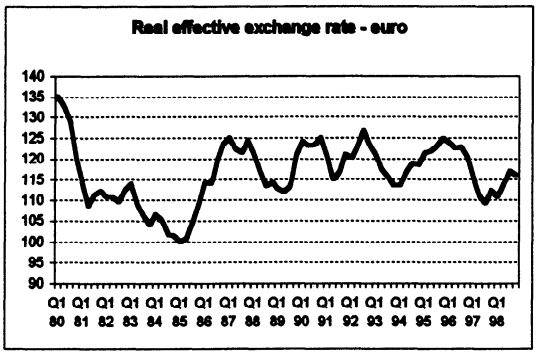
Country	GDP	Source	CA	Source
Sweden	SDGDP...A	Nat. Statistics,	SDCURACBA	Nat .Statistics,
Denmark	DKGDPCR.	OECD Main Indicators	DKCURACBA	Nat. Statistics,
Norway	NWGDP...A	Nat. Statistics,	NWCURBALA	Nat. Statistics,
United Kingdom	UKYBHA..	Nat. Statistics, ESA	UKCURBALA	Nat. Statistics ESA
Germany [25]	QSSGDPNBBK	Nat. Statistics	Q134SSLEISAL	Nat. Statistics
Greece	GRGDPCR.	OECD Main indicators[26]	GROCCBALA	OECD Main indicators
United States	USI99B.CB	IMF	USCURACBB	US$ Bill
Italy	FRGDP...B	Nat. Statistics,	FRCURA.QA[27]	Nat. Statistics, Eurostat
France	ITGDP...B	Nat. Statistics,	ITCURBALA	Nat. Statistics, Eurostat
Canada	CNI99B.CB	IMF	CNCURACBB	Nat. Statistics
Japan	JPI99B.CB	IMF	JPCUR...B	Nat. Statistics
Euro area	Aggregating Euro-11 GDP's			European Central Bank

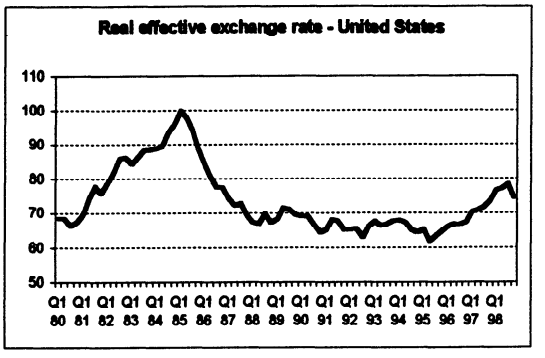
[25] Data on GDP for Western Germany becomes Pan-Germany in 1992 Q2 while current account data becomes Pan-Germany in 1991 Q1. Difference between the ratio CA/GDP from 1991 Q1 for West Germany and Pan-Germany GDP is negligible.

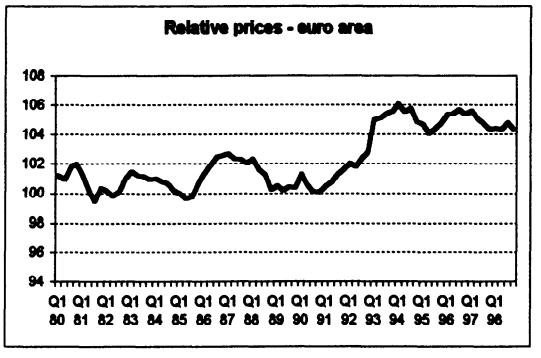
[26] Data were interpolated from annual figures.

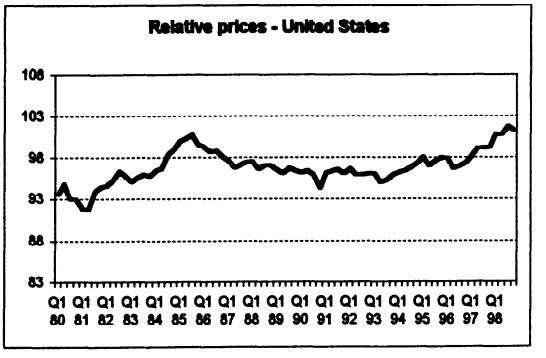
[27] This series was transformed from euros into national currency. The same applies to the Italian current account data.

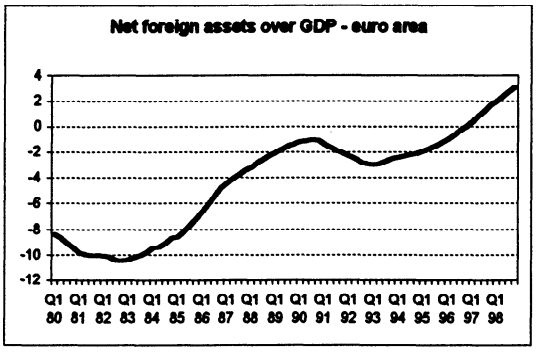
Figure 3. Euro Area and United States











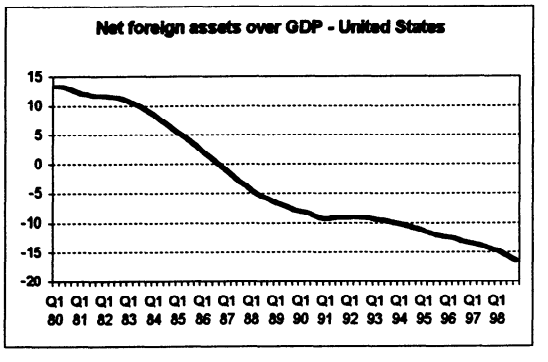
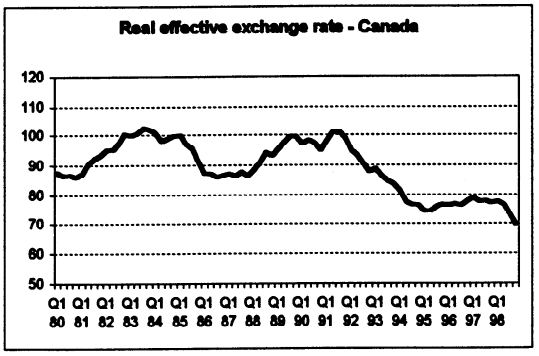
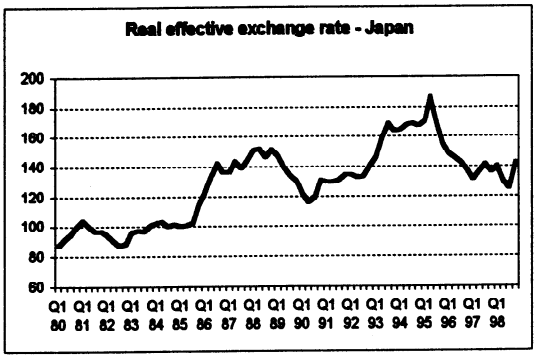
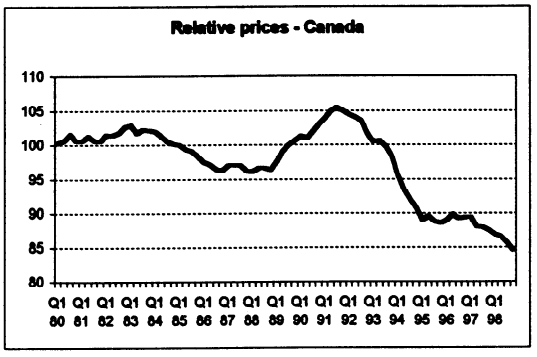
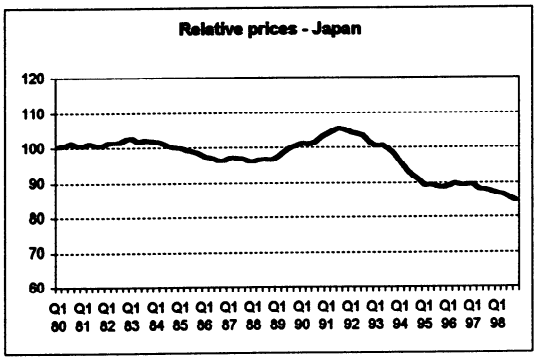


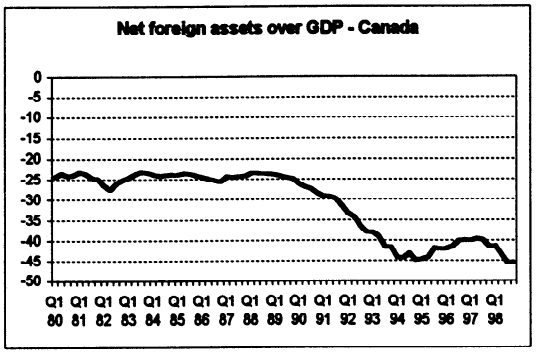
Figure 3. Canada and Japan (Continued)











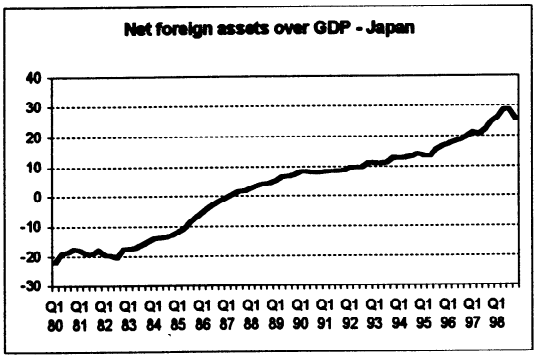
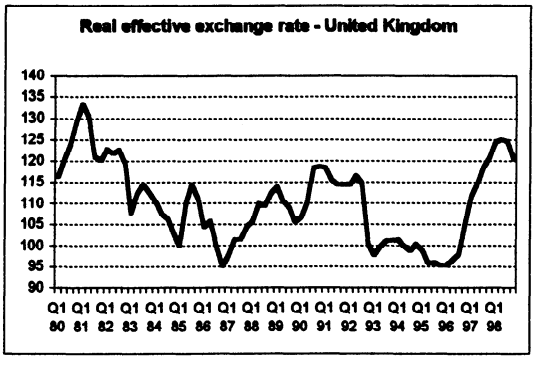
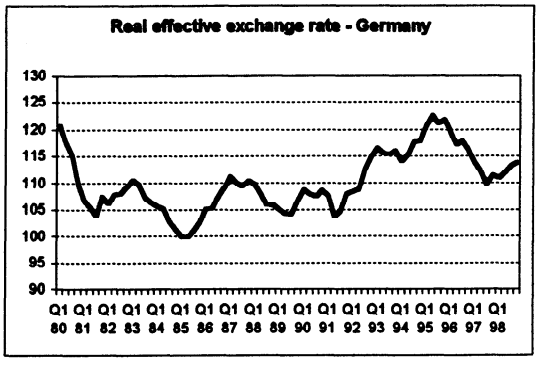
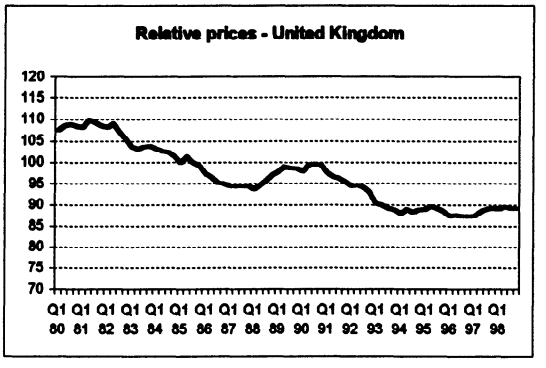
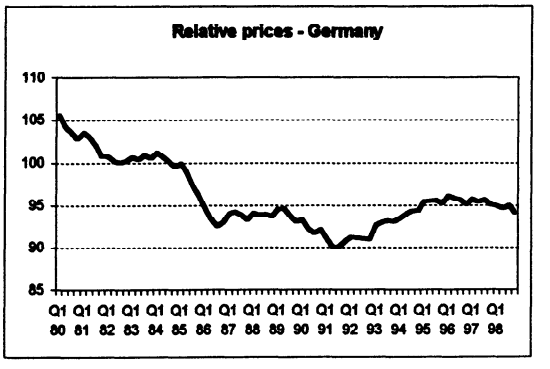


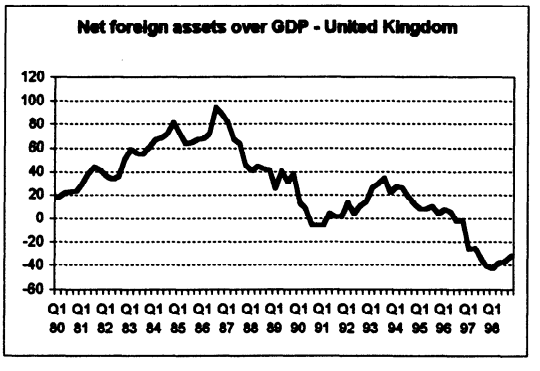
Figure 3. United Kingdom and Germany (Continued)











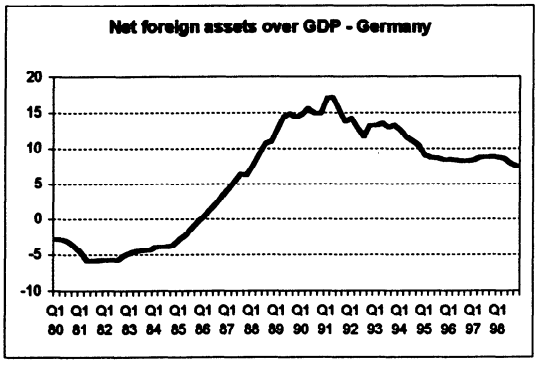


Figure 3. France and Italy (Continued)

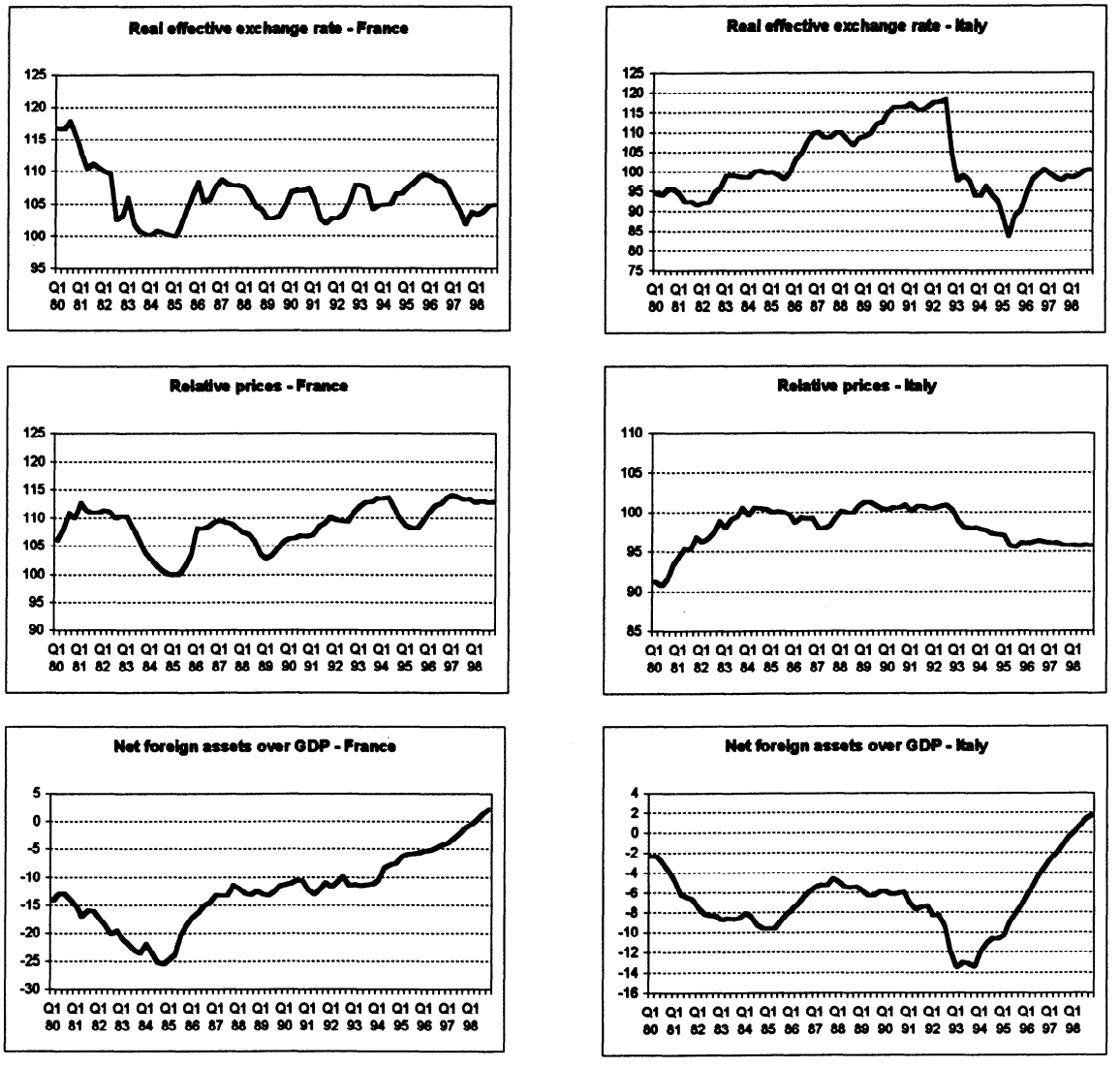
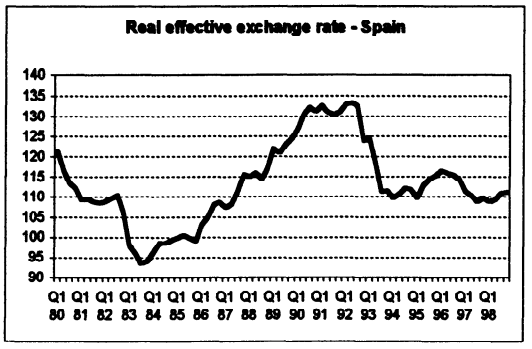
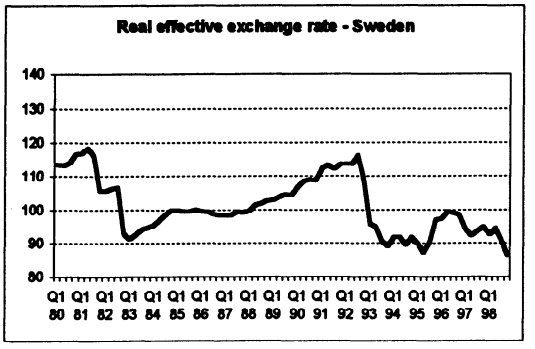
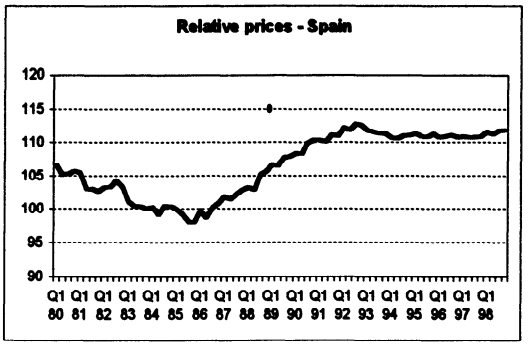
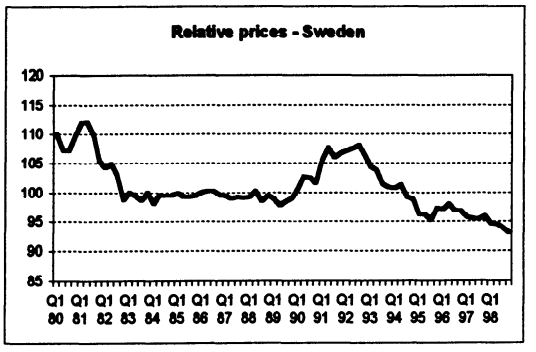


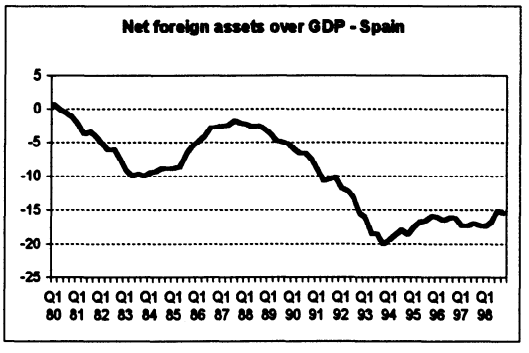
Figure 3. Spain and Sweden (Continued)











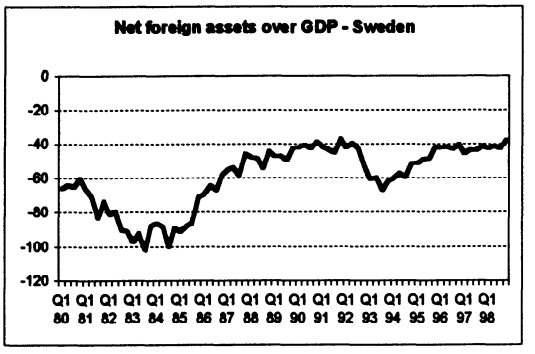


Figure 3. Denmark and Greece (Concluded)