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FEERs and Uncertainty: Confidence Intervals for the Fundamental Equilibrium Exchange Rate of the Canadian Dollar

Prepared by Charles Kramer 1/

Authorized for Distribution by Steven V. Dunaway

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Abstract

Models of Fundamental Equilibrium Exchange Rates (FEERs) impose internal and external balance, and so appeal to fundamental notions of equilibrium from a macroeconomic perspective. However, the need to estimate internal and external imbalances creates uncertainty in the approach. Parameters must be estimated, and equilibrium balances must be gauged using judgement. Hence it makes sense to consider the FEER as a statistical estimate rather than a fixed number, and to calculate confidence intervals for the FEER. This paper calculates such confidence intervals with data for Canada, under a variety of assumptions. The estimated confidence intervals are quite wide, principally because of uncertainty about price elasticities in the underlying trade equations.

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1/ Many colleagues provided very helpful advice and comments. I thank especially (though without implication) Tamim Bayoumi, Hamid Faruqee, Thomas Helbling, Jorge Márquez-Ruarte, Guy Meredith, Carmen Reinhart, and John Williamson for their advice and comments. Any errors, omissions, or conclusions are mine, however.

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Summary

Apparent misalignments in exchange rates have been the focus of much recent interest. This interest has been spurred by wide swings in exchange rates in the face of small swings in fundamentals and by the desire to calculate parities for the exchange rate mechanism of the European Monetary System. One way to gauge the degree of misalignment is by calculating the fundamental equilibrium exchange rate (FEER)--the exchange rate compatible with full employment and external balance. Recent studies have constructed FEERs using models ranging from simple partial-equilibrium trade models to full-blown macroeconomic models.

Estimated FEERs depend on estimated parameters and, like other estimates based on data, are subject to sampling uncertainty. This means that the estimates should not be taken too literally. As Clark and others (1994) state, "it is clearly more realistic to think of ranges rather than of point estimates in the assessment of exchange rates" (p. 20). A confidence interval would be useful because it would quantify the estimation error and permit the user to decide whether the current exchange rate was a significant distance from equilibrium.

This paper calculates sampling distributions for the FEER of the Canadian dollar on the basis of a simple model of trade. The analysis focuses on the uncertainty introduced by employing estimated trade elasticities. Elasticities are drawn at random from ranges typical of estimates found in the literature and also from empirical (bootstrapped) distributions. The estimated FEER shows a substantial range of variation that primarily reflects such uncertainty. The results suggest that FEER estimates should be treated with caution because they offer a fairly imprecise measure of misalignment.

I. Introduction

"Misalignments" in exchange rates have been the focus of much recent interest. 1/ This interest has been spurred by wide swings in exchange rates in the face of small swings in fundamentals and by the desire to calculate parities for the Exchange Rate Mechanism of the European Monetary System. 2/ One way to gauge the degree of misalignment is by calculating the Fundamental Equilibrium Exchange Rate (FEER)--the exchange rate compatible with full employment and external balance. 3/ Recent studies have constructed FEERs using models ranging from simple partial-equilibrium trade models to full-blown macroeconomic models. 4/

Estimated FEERs depend on estimated parameters, and, like other estimates based on data, are subject to sampling uncertainty. This means that the estimates should not be taken too literally. As Clark et al. (1994) state, "it is clearly more realistic to think of ranges rather than of point estimates in the assessment of exchange rates" (p. 20). A confidence interval would be useful, as it would quantify the estimation error and permit the user to decide whether the current exchange rate is a significant distance from equilibrium.

This paper presents calculated sampling distributions for the FEER of the Canadian dollar, based on a simple model of trade. 5/ The analysis focuses on the uncertainty introduced by employing estimated trade elasticities. The estimated FEER shows a substantial range of variation that primarily reflects such uncertainty, in particular the possibility that the trade balance may be insensitive to movements in the real exchange rate. 6/ The results suggest that FEER estimates should be treated with caution, as they may offer a fairly imprecise measure of misalignment.

1/ The notion that market-determined exchange rates can be "misaligned" is controversial. I assume in this paper that it makes sense to treat a fitted value from an exchange-rate model as an estimate of the equilibrium exchange rate.

2/ See for example Clark et al. (1994).

3/ Sometimes also referred to as a Desired Equilibrium Exchange Rate (DEER) so as to emphasize its normative nature.

4/ Examples include Goldman Sachs (1995), Chandler and Laidler (1995), Church (1992), and the papers in Williamson (1994). Other approaches to assessing movements in real exchange rates, including purchasing power parity, terms of trade, and productivity trends (Balassa-Samuelson), are discussed in Amano and van Norden (1995), Ronald MacDonald (1995), Asea and Corden (1994), and Isard and Symansky (1995).

5/ Though the data employed are for Canada, nothing about the appropriateness of the exchange rate for the Canadian dollar is implied by or should be inferred from what follows.

6/ Such insensitivity is consistent with empirical estimates in the literature. For example, estimated price elasticities below unity in absolute value are hardly unusual in trade equations fitted to data for the United States, Japan, and Canada (see Marquez (1995)), and indeed for many developing countries (see Reinhart (1995)).

Some earlier work has performed sensitivity analysis on FEERs, but sensitivity analysis is different in spirit from what is done in this paper. 1/ For example, a sensitivity analysis might calculate three FEER estimates, using an elasticity of 1.0, an elasticity of 1.5, and an elasticity of 2.0. This would be useful for gauging how sensitive the FEER estimate is to the assumption that the elasticity is 1.5, for example. However, unless the estimator for the elasticity has a distribution that takes on the values 1.0, 1.5, and 2.0, each with probability 1/3, the resulting "distribution" for the FEER will not resemble the actual sampling distribution of the FEER, and hence will not be useful for testing that an estimated FEER implies a statistically significant misalignment. In the experiments presented in this paper, elasticities are drawn from their estimated sampling distributions and used to calculate the sampling distribution of the FEER. This sampling distribution can then be used for hypothesis testing, as it accurately reflects the dependence of the FEER on randomness in estimated elasticities.

The paper is laid out as follows. Section 2 describes the FEER model used in this paper. Section 3 discusses the notion of the FEER as a statistical estimate. Section 4 describes how the distribution of this statistical estimate is simulated. Section 5 presents the results of these simulations, and Section 6 concludes the paper. An appendix provides details of how the empirical (bootstrapped) distributions for elasticities were constructed.

II. The Fundamental Equilibrium Exchange Rate (FEER)

This paper follows the methodology outlined by Clark et al. (1994) and used by, *inter alia*, Chandler and Laidler (1995) in studying the equilibrium exchange rate for Canada. The three steps in calculating the FEER are sketched below.

The first step is to calculate the underlying current account, CA_u. This is done by adjusting the actual current account, CA_a, for foreign and domestic output gaps and the effects of changes in lagged real exchange rates, i.e.

$$\frac{CA_u}{Y}=\frac{CA_a}{Y}+\alpha_x\frac{X}{Y}YGAP^f-\alpha_m\frac{M}{Y}YGAP-[\beta_x\frac{X}{Y}+(\beta_m-1)\frac{M}{Y}]*(R-RLAG) \tag{1}$$

where

1/ Some studies of FEERs that contain sensitivity analysis are discussed in Clark et al. (1994).

$$RLAG=\frac{\theta_0 R+\theta_1 R_{-1}+\theta_2 R_{-2}}{\theta_0+\theta_1+\theta_2}\,;\theta_0+\theta_1+\theta_2=1.$$

α_x and α_m denote the elasticities of real exports and real imports with respect to foreign and domestic income; β_x and β_m denote the elasticities of real exports and real imports with respect to their relative prices; X, M, and Y denote nominal exports, imports and output; $YGAP^f$ denotes the export-weighted foreign output gap and YGAP denotes the domestic output gap (both measured in differences from potential in logarithms, where a positive gap indicates output below potential); R denotes the logarithm of the real exchange rate (in this methodology, the multilateral real effective exchange rate), defined so that an increase in R represents an appreciation of the home currency; and RLAG is a weighted average of current and past exchange rates. In keeping with the respective literatures on FEERs and empirical models of trade, β_x and β_m above are both positive numbers, while in the empirical section below, both take their estimated (negative) sign. 1/

Once the underlying current account has been calculated, the second step is to specify the equilibrium or desired current account, CA_e. For example, CA_e could be the current account consistent with a desirable ratio of net foreign assets to GDP, or the current account consistent with a particular model of saving and investment.

The third and last step is the adjustment from the underlying current account to the equilibrium current account. This implies that the real exchange rate adjusts to its equilibrium value, R_e: 2/

$$\frac{CA_u}{Y}-\frac{CA_e}{Y}=[\beta_x\frac{X}{Y}+(\beta_m-1)\,\frac{M}{Y}]\,(R_e-R),$$

1/ That is, in this part of the paper, β_x is the parameter in the model for exports

$$X = -\beta_x\ P_x + \alpha_x\ y^*$$

while in the empirical part, β_x is the slope parameter in the regression

$$X = \beta_x\ P_x + \alpha_x\ y^*.$$

2/ For the sake of simplicity, this expression, like the previous one, is a comparative statics definition that ignores hysteresis effects (see Bayoumi et al. (1994)).

or

$$R_e - R = \frac{\frac{CA_u}{Y} - \frac{CA_e}{Y}}{[\beta_x \frac{X}{Y} + (\beta_m - 1)\frac{M}{Y}]} . \qquad (2)$$

$(R_e - R)$ is the estimated degree of misalignment. For example, $(R_e - R) > 0$ indicates that the home currency is undervalued relative to its FEER value. $(R_e - R)$ is the focus of this study.

III. The FEER as a Statistical Estimate

It is obvious that the estimated degree of misalignment is a function of unknown quantities, in particular of trade elasticities and the equilibrium current account. In practice, the equilibrium current account is replaced by an estimate based on judgement, and trade elasticities are replaced by statistical estimates. Thus the FEER is also a statistical estimate, and like other statistical estimates has a sampling distribution. If known, that sampling distribution could be used for formal hypothesis testing. For example, it could be used to construct a test that the estimated degree of misalignment is not significantly different from zero.

A closed-form distribution for the FEER that could be applied in a sample of any size would be ideal. At best, however, one might be able to derive an analytic approximation to the distribution of the FEER that is accurate in large samples. For example, given that the elasticities are normally distributed in large samples, the FEER estimate of misalignment will be normally distributed in large samples as well. 1/ However, it is hard to say when a sample is large enough for the approximation to be a good one. For the data used in this paper, some tests (not shown) imply that the approximation is poor--that the estimated FEER is not normally distributed. 2/

Since an analytic approximation to the distribution of the FEER is likely to be poor, the study proceeded by a different route, namely, by simulating repeated samples. That is, elasticities were drawn at random from specified distributions, and then used to calculate the FEER, just as would be done in an actual random sample. This process was repeated 10,000 times, generating elasticities at random 10,000 times and calculating 10,000 FEERs, under various assumptions about the equilibrium current account and other aspects of the model. The 10,000 FEER estimates for each case then

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1/ This follows from the so-called "delta method" of approximation. See Serfling (1980).

2/ Normality tests (see Epps and Pulley (1983)) rejected the null of normality at the one percent level for the simulated values of the FEERs presented later. Test results are available on request.

yielded a picture of the sampling distribution of the FEER. More details on how this was done are in the following section.

IV. Simulating the sampling distribution of the FEER

Calculating the FEER misalignment estimate required trade elasticities, a value for the equilibrium current account, and miscellaneous data such as imports, exports, and GDP. The source for each of these is described in turn.

Elasticities were generated at random by two methods. In the first method, the elasticities reflected the range of estimates typically encountered in the literature. Elasticities were drawn at random from uniform distributions, where the minimum and maximum of each distribution were fixed at the minimum and maximum elasticities for Canada shown in the survey of empirical trade models by Goldstein and Khan (1985). These ranges are shown in the tabulation below. The estimated elasticities from the recent study of the FEER for Canada by Chandler and Laidler (1995) fall in these ranges. 1/

Ranges for Trade Elasticities 2/

	Parameter			
	Export Income Elasticity α_x	Export Price Elasticity β_x	Import Income Elasticity α_m	Import Price Elasticity β_m
Minimum	0.69	-1.10	0.90	-2.50
Maximum	1.97	-0.23	1.87	-0.20

As a second method, a bootstrap methodology was used: elasticities were drawn at random from an empirically-determined distribution of estimates. 3/ This empirical distribution was created by simulating how trade elasticities would vary as they were re-estimated in repeated samples. The resulting FEER estimates reflect the true sampling variability in estimated elasticities.

1/ As stated earlier, while the expressions in the previous section treat all elasticities as having positive signs, the elasticities in this section take their estimated sign (which is negative for price elasticities).

2/ See Goldstein and Khan (1985), Tables 4.1 and 4.3.

3/ The Appendix briefly discusses the bootstrap methodology and explains its application to the problem at hand. See Davidson and MacKinnon (1993), Chapter 21, for a more detailed discussion of bootstrapping and simulation in econometrics.

Three values of the equilibrium current account were specified: a surplus of one percent of GDP, balance (no deficit or surplus), and a deficit of one percent of GDP. The trade balance for 1994 that stabilizes the ratio of net international liabilities to GDP is estimated at about 1.5 percent of GDP. 1/ Hence, a range of (-1, +1) for the desired current account seems reasonable, and includes levels of the current account that would put external debt on a declining path as a percentage of GDP, which is one of the objectives of the Canadian authorities. 2/

Exports, imports, GDP, the current account, and the domestic output gap (all for 1995) were taken from the World Economic Outlook database. 3/ Estimates made by the Fund's Research Department were used for the foreign output gap and the weights on lagged exchange rates (θ_0, θ_1, and θ_2).

The simulations account for variability in elasticities and judgement about the equilibrium current account. However, no attempt was made to account for uncertainty in the measure of the output gap, nor for likely developments in fiscal policy or demographics. As such, the estimates presented below understate the uncertainty in FEER estimates.

V. Results

This section presents the results of the simulations. Four sets of simulations are presented below. The first set of simulations proceeded as described above. The second set of simulations imposed the Marshall-Lerner condition on the simulated elasticities. The third set of simulations added randomness in the equilibrium current account, and the final set of simulations examined the sampling distribution in the underlying current account.

1. The sampling distribution of the FEER

Chart 1 displays the simulated joint distribution of the estimated degree of misalignment (R_e-R) and the estimated underlying current account

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1/ See Sheila MacDonald (1995).

2/ See The Economic and Fiscal Update, Department of Finance, Canada, December 6, 1995.

3/ Complete data for 1995 did not become available until after the simulations were performed, so some of the data employed were based on staff estimates. Some of the experiments were later repeated with complete data for 1995, and the results were qualitatively the same as the ones shown here--the FEER estimates showed a wide range of uncertainty.

(CA_u), while Table 1 shows some statistics on the distribution of (R_e-R). 1/ All simulations show an overvalued real effective exchange rate on average. The median is higher than the mean in every case, implying positive skewness, and thus measuring significance in the usual way (a point estimate two standard deviations from zero) may not yield accurate inferences. Another measure of uncertainty, one that does not rely on the symmetry of the underlying distribution, is the percentile range. For example, the range between the 95th and 5th percentiles is a 90 percent confidence interval for the estimate.

The bounds of the 90 percent confidence intervals (denoted by q_{05} and q_{95}) are shown in the last two columns of Table 1. The confidence intervals imply that even when using the uniform distribution for elasticities (cases A-C), the uncertainty in the estimated degree of misalignment is not negligible. When using the empirical distribution for elasticities (cases D-F), it is quite large indeed. For the case with an equilibrium current account deficit of 1 percent of GDP, with 90 percent confidence, the degree of required depreciation is between 18 percent and about 1 percent (using uniform elasticities) or between 67 percent and -45 percent (using empirically-distributed elasticities). 2/ In either case, discerning the degree of misalignment is difficult, and the confidence intervals are generally quite a bit wider than Williamson's (1994) rule of thumb of plus or minus 10 percent. 3/

2. Sampling distribution of the FEER under the Marshall-Lerner condition

The experiments described in the previous section did not impose the Marshall-Lerner condition that a real appreciation worsens the current account. This was because nothing in theory (or practice) guarantees that the condition must hold. 4/ However, it is sufficiently well-entrenched as a stylized fact of empirical trade models that many readers may be curious about its implications. Also, some would prefer that the

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1/ In some random draws, the denominator of the expression for the FEER (equation (2)) was very small. The subsequent large outliers in the results unduly influenced the descriptive statistics. To remedy this problem, 2.5% of the distribution of the FEER was trimmed from each tail (trimming the distributions of the elasticities did not reduce the number of outliers). This trimming means that the statistics will understate the true degree of imprecision in the FEER estimator, however.

2/ Since the Marshall-Lerner condition is not imposed at this stage, some of the random draws might imply that an appreciation is needed to reduce the current account deficit to its equilibrium level. The implications of imposing the Marshall-Lerner condition are explored in the next section.

3/ See Williamson (1994), p. 9.

4/ In fact, in a dynamic model the Marshall-Lerner condition may not have much meaning. For example, Backus, Kehoe and Kydland (1992) show that the Marshall-Lerner condition may have no relevance for the sign of the correlation between the exchange rate and the trade balance in a dynamic model.

counterintuitive condition that a real appreciation improves the trade balance be ruled out. Moreover, it is clear from expression (2) that when the Marshall-Lerner condition is close to failing, the denominator of R_e-R is close to zero, and so R_e-R is very large. Hence, the experiments above were repeated, but using combinations of β_x and β_m for which β_x and β_m satisfied the Marshall-Lerner condition.

Before proceeding, though, it is worth emphasizing that for two reasons, the Marshall-Lerner condition is not an innocuous constraint in the context of this work. First, imposing the Marshall-Lerner condition here is not the same as assuming that it holds in a particular model of trade: it is the same as assuming that it could never fail to hold in an estimated model. This is a statement not about estimates observed in the literature, or about the outcome of any test involving those estimates, but about the joint distribution for estimated price elasticities. Imposing Marshall-Lerner amounts to throwing away some (in practice, a lot) of that joint distribution.

Second, imposing the Marshall-Lerner condition means throwing away an important part of the joint distribution, as it biases the results towards finding 'significant' overvaluation or undervaluation. The reason is that if, for example, current-account surplus is too small, a depreciation is the only cure when Marshall-Lerner is imposed. From expression (2), it is clear that (ignoring variation in the gap between the underlying and equilibrium current accounts, which is small), experiments that impose the Marshall-Lerner condition will find 'significant' misalignments, because all the FEER estimates will fall on one side of zero. Therefore, estimated fractiles could never bracket zero. For this reason, not too much should be drawn from the fact that the confidence intervals for the estimated degree of misalignment do not bracket zero when Marshall-Lerner is imposed, and the estimates in this section are not empirical estimates of the uncertainty in FEERs.

Table 2 displays the results of the simulations with the Marshall-Lerner condition imposed (for ease of comparison with the previous experiments, the corresponding results for the unrestricted case are reproduced from Table 1). Imposing the Marshall-Lerner condition shifts the distribution of the estimated degree of misalignment to the left, as evidenced by the smaller (more negative) means and medians for the distributions. This is not surprising, as the Marshall-Lerner condition means that only depreciation (not appreciation) can close the gap between the underlying and equilibrium current accounts. Accordingly, the confidence intervals for the bootstrapped cases no longer bracket zero--indeed, unless the estimated underlying current account is quite variable, they cannot bracket zero. Finally, while imposing the Marshall-Lerner condition makes the confidence intervals smaller, it does not make them small. The narrowest confidence interval brackets 20 percent overvaluation to about 2 percent overvaluation, while the widest confidence interval brackets 230 percent overvaluation to 17 percent overvaluation.

One might object that a "critical" part of the joint distribution of price elasticities is still included in these simulations, as the sum of import and export price elasticities might be close to 1. In this case, the denominator of (2) could still be arbitrarily small, so that R_e-R could be arbitrarily large. Forcing the sum of the price elasticities to be at least 1.5, say, might reduce the variability of R_e-R substantially. This is true, but irrelevant for the purpose pursued here--the purpose of assessing the sampling distribution of the FEER. It is hardly surprising that throwing away sample information on elasticities reduces the variability of the FEER, and it means that we no longer have an accurate picture of the sampling distribution of the FEER.

Also, some recent estimates of price elasticities do not support restricting elasticities to be large. Two recent papers that employ state-of-the-art econometric methods find trade price elasticities for the United States, Japan, and Canada (Marquez (1995)) and for a number of developing countries (Reinhart (1995)) that are well below unity in absolute value. Moreover, to focus unduly on point estimates is to miss the point of the paper. This paper is not about point estimates, but about uncertainty in point estimates. That is, the question is not whether price elasticities are large, but whether the data say that small elasticities are statistically possible. The recent evidence--here, in Marquez, and in Reinhart--suggests that small elasticities are very much possible.

3. Sampling distribution of the FEER when CA_e is random

As noted, these experiments understate the true degree of uncertainty, by ignoring the uncertainty in estimates of output gaps, for example. There is also judgmental uncertainty about the desired current account. It would be useful to quantify this uncertainty by treating the desired current account as a random variable rather than a parameter. Hence, this uncertainty was simulated by drawing equilibrium current accounts at random from a uniform distribution with bounds equal to the maximum and minimum of the values used above (-1 and +1). The Marshall-Lerner condition is imposed for these simulations.

The simulations using a random equilibrium current account are presented in Table 3. Since the average equilibrium current account is zero in these simulations, they correspond to Cases B and E in Table 2 where the equilibrium current account is fixed at zero, and so are labeled Case B' and Case E' in the table. The means, medians and confidence intervals are scarcely affected by randomness in the equilibrium current account. The effect of uncertainty in the judgmental estimate of the equilibrium current account seems small relative to the effect of uncertainty in the elasticity estimates.

4. Sampling distribution of the underlying current account

It is also useful to isolate the degree of uncertainty in the underlying current account. Table 4 presents descriptive statistics for the distribution of the underlying current account (since CA_e is not used in estimating the underlying current account, there are only two cases, one for each type of distribution for the elasticities). Without the Marshall-Lerner condition imposed, the 90 percent confidence interval for the estimated underlying current account is about (-2.9, -1.7) for the case with uniformly-distributed elasticities and about (-2.4, -1.9) for the case with bootstrapped elasticities, in both cases well on the deficit side. In each case, there is a significant distance between the estimated underlying current account and the equilibrium current accounts specified above; for example, a deficit of one percent of GDP lies above these confidence intervals. This means that the estimated misalignment is imprecise not because the estimated current-account adjustment is imprecise, but because of the way that the FEER depends on estimated price elasticities.

VI. Conclusions

Since the FEER is a function of both sample data and judgement, the point estimate that emerges from the procedure should be treated with caution. The results described above suggest that the uncertainty about the point estimate is very large, even when the degree of implied adjustment in the current account is significant. Most of this uncertainty comes from the use of estimated trade elasticities. Accounting for judgmental uncertainty in the specification of the equilibrium current account only slightly magnifies the variability of the FEER estimates.

The variability in the FEER estimates for Canada is large enough that it is often impossible to tell whether the real exchange rate is overvalued or undervalued with any precision. As a result, the usefulness of misalignment indicators based on such trade models seems questionable. In fact, for the estimates that most plausibly reflect true sampling uncertainty (those using bootstrapped elasticities and without the Marshall-Lerner condition imposed), the confidence intervals indicate an insignificant degree of misalignment. Even given the relative simplicity of the model relative to some others used to produce FEERs, large apparent misalignments can be small relative to sampling uncertainty.

The nature of the problem suggests that more complicated trade models may not yield more precise estimates. The fundamental problem lies in the use of trade elasticities in the denominator of the expression for the FEER. When these trade elasticities are close to failing the Marshall-Lerner condition, the FEER estimate becomes very large. 1/ Any trade model that imputes a real exchange rate adjustment to equilibrate the current account will have the same problem, regardless of how the equilibrium current is estimated. Moreover, the estimated degree of misalignment is relatively insensitive to randomness in the equilibrium current account, as the simulations in section 5.c show. It thus seems that more precise estimates

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1/ As mentioned above, my view is that the literature (see Marquez (1995) and Reinhart (1995)) and the bootstrap experiments clearly imply that estimated price elasticities can be small indeed.

of FEERs will require much more precise estimates of trade elasticities than we presently have, or a model that does not depend on trade elasticities.

Construction of Empirical Distribution for Trade Elasticities

In constructing a simulated distribution for the FEER, trade elasticities were "bootstrapped" or resampled from empirical distributions for $(\hat{\alpha}_x, \hat{\beta}_x)$ and $(\hat{\alpha}_m, \hat{\beta}_m)$. The bootstrap technique constructs an estimate of the distribution of some statistic of interest by repeatedly sampling the data--in effect, treating the sample as if it were the population. The estimated distribution can then be used to perform inference. Bootstrapping is valuable when little is known about the distribution of a statistic in small samples. 1/

A simple example can illustrate the principle. Suppose that one had 10 observations on income, and wanted to test the (two-sided) hypothesis that average income is equal to $500 at the five percent significance level. Basic statistics suggests that 10 may be too small a sample size to use a normal distribution for the hypothesis test. A hypothesis test could then be constructed as follows.

Step 1. Repeat these two steps 1000 times:

1.a Draw a random sample of size 10 from the data set with replacement, where each observation has the same probability of being chosen. Call this a "synthetic sample".

1.b Calculate the sample mean ($\bar{x}$), the standard deviation of the sample mean (s), and the test statistic ($\bar{x}$-500)/s for each synthetic sample.

Step 2. Sort the 1,000 test statistics from smallest to largest.

Under the null hypothesis that the mean is 500, the 25th statistic is the 2.5 percent probability cutoff, and the 975th statistic is the 97.5 percent probability cutoff.

Step 3. Calculate ($\bar{x}$-500)/s for the original 10 observations. Compare it to the two cutoff values above. If it is less than the 2.5 percent probability cutoff or greater than the 97.5 percent cutoff, reject the null hypothesis that average income is equal to $500 at the five percent significance level.

The useful feature of this procedure is that since the distribution of the test statistic is constructed from the data, no explicit assumptions

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1/ See Davidson and MacKinnon (1993), Chapter 21, for a discussion of bootstrapping and related topics.

about the distribution of the data are required. 1/ Of course, the bootstrap procedure might give misleading results if the sample were not representative of the population, but this is equally true of the usual textbook inference procedure.

The rest of this section describes how the bootstrap procedure was used to construct an empirical distribution for elasticities. The first step was to estimate representative elasticities. This required data on exports and imports at constant prices, the relative prices of imports and exports, and foreign and domestic income at constant prices. The relative price of exports was measured by the ratio of the Canadian export price deflator (converted to U.S. dollar terms) to export-weighted partner-country export prices in U.S. dollar terms. The relative price of imports was measured by the ratio of the import price deflator to the consumer price index. Domestic income was measured by real GDP, and foreign income was measured by an index of export-weighted partner-country real GDP. All data were quarterly, and the sample covered 1971:I to 1995:II.

Since the series showed evidence of unit-root nonstationarity and cointegration, the Stock and Watson (1993) estimator was used to estimate the elasticities. Starting with 8 leads and 8 lags in the short-run adjustment mechanism, insignificant short-run coefficients were trimmed to yield a model that passed tests for autoregressive conditional heteroskedasticity, a structural break at mid-sample, and tests for autocorrelation, after a correction for first-order autoregressive (AR(1)) disturbances.

The rest of the procedure followed Li (1994). The sample of import elasticities was generated as follows (the export elasticities were generated in an analogous fashion). Denote by M the dependent variable (imports in constant-dollar terms), and denote by Z a vector containing the independent variables (the relative price of imports and domestic income at constant prices). 2/ Residuals (e_t), estimated coefficients (δ,π), and an estimated AR(1) coefficient ρ were calculated by estimating the model

$$M_t = \mu + \sum_{j=p}^{P} \delta_j \Delta Z_{t+j} + \pi Z_t + e_t, \; e_t = \rho e_{t-1} + u_t, \; u_t \text{ i.i.d.} \qquad \text{(A1)}$$

with $P > 0 > p$ (P leads and p lags). The series $s_t \equiv \Delta Z_t$ was calculated as well.

Next, drawing at random with replacement from the estimated disturbances $\{\hat{u}_t\}_{t=1}^{T}$ and from the shocks $\{s_t\}_{t=1}^{T}$, synthetic series $e_t^* \equiv \hat{\rho}\, e_{t-1}^* + \hat{u}_t$ and $Z_t^* \equiv Z_{t-1}^* + s_t$ were constructed. These were used in turn to

1/ There are implicit assumptions, however. In particular, drawing at random implicitly assumes that incomes are independently and identically distributed. Since the same assumption would generally be made in the textbook procedure, this is not too strong an assumption.

2/ M and Z were expressed in logarithms.

construct M_t^* by replacing (δ,π) with $(\hat{\delta},\hat{\pi})$, e_t with $\hat{e}_t$, and ρ with $\hat{\rho}$ in equation (A1). The parameters (δ,π) were then re-estimated from the synthetic data M^* and Z^*, yielding estimates (δ^*,π^*) from fitting

$$M_t^* = \mu^* + \sum_{j=-p}^{p} \delta_j^* \Delta Z_{t+j}^* + \pi^* Z_t^* + e_t^*,$$

again with an AR(1) correction. This process was repeated 1000 times for each equation, yielding 1000 simulated estimates of α_x, β_x, α_m, and β_m.

These estimates were sampled at random for each of the 10,000 trials where R_e - R was calculated. The samples were drawn in matched pairs: that is, if the 100th estimated α_x and the 200th estimated α_m were chosen for a particular estimate of R_e - R, then the 100th estimated β_x and the 200th estimated β_m were chosen as well. This scheme preserved the dependence between $\hat{\alpha}_x$ and $\hat{\beta}_x$ and the dependence between $\hat{\alpha}_m$ and $\hat{\beta}_m$. Chart 2 summarizes the results of the bootstrapping experiment, showing the distributions of the estimated import and export elasticities.

Table 1. Estimates of the Degree of Real Misalignment

	Mean	Median	90 Percent Confidence Interval
<u>Case A</u> Elasticities: Drawn from uniform distribution CA_e: Fixed at -1	-4.35	-3.17	(-17.35, -0.72)
<u>Case B</u> Elasticities: Drawn from uniform distribution CA_e: Fixed at 0	-7.73	-5.60	(-30.39, -2.00)
<u>Case C</u> Elasticities: Drawn from uniform distribution CA_e: Fixed at 1	-11.11	-8.02	(-43.76, -3.25)
<u>Case D</u> Elasticities: Drawn from bootstrap distribution CA_e: Fixed at -1	-0.96	6.32	(-66.55, 45.23)
<u>Case E</u> Elasticities: Drawn from bootstrap distribution CA_e: Fixed at 0	-2.32	11.76	(-130.70, 84.17)
<u>Case F</u> Elasticities: Drawn from bootstrap distribution CA_e: Fixed at 1	-3.69	17.11	(-190.09, 127.89)

Table 2. Sensitivity of Estimates of the Degree of Real Misalignment to Imposition of the Marshall-Lerner Condition 1/

	Mean	Median	90 Percent Confidence Interval
Case A Elasticities: Drawn from uniform distribution CA_e: Fixed at -1			
Without Marshall-Lerner	-4.35	-3.17	(-17.35, -0.72)
With Marshall-Lerner	-6.39	-3.65	(-20.46, -1.73)
Case B Elasticities: Drawn from uniform distribution CA_e: Fixed at 0			
Without Marshall-Lerner	-7.73	-5.60	(-30.39, -2.00)
With Marshall-Lerner	-11.54	-6.55	(-37.03, -3.33)
Case C Elasticities: Drawn from uniform distribution CA_e: Fixed at 1			
Without Marshall-Lerner	-11.11	-8.02	(-43.76, -3.25)
With Marshall-Lerner	-16.68	-9.46	(-53.55, -4.86)
Case D Elasticities: Drawn from bootstrap distribution CA_e: Fixed at -1			
Without Marshall-Lerner	-0.96	6.32	(-66.55, 45.23)
With Marshall-Lerner	-30.44	-19.88	(-89.31, -7.29)

1/ The corresponding cases from Table 1 are reproduced here for ease of comparison.

Table 2. Sensitivity of Estimates of the Degree of Real Misalignment to Imposition of the Marshall-Lerner Condition (Concluded) 1/

	Mean	Median	90 Percent Confidence Interval
Case E Elasticities: Drawn from bootstrap distribution CA_e: Fixed at 0			
Without Marshall-Lerner	-2.32	11.76	(-130.70, 84.17)
With Marshall-Lerner	-51.83	-33.00	(-157.22, -12.20)
Case F Elasticities: Drawn from bootstrap distribution CA_e: Fixed at 1			
Without Marshall-Lerner	-3.69	17.11	(-190.09, 127.89)
With Marshall-Lerner	-73.22	-46.79	(-228.09, -17.15)

1/ The corresponding cases from Table 1 are reproduced here for ease of comparison.

Table 3. Sensitivity of Estimates of the Degree of Real Misalignment to Uncertainty about Equilibrium Current Account 1/

	Mean	Median	90 Percent Confidence Interval
Case B Elasticities: Drawn from uniform distribution CA_e: Fixed at 0	-11.54	-6.55	(-37.03, -3.33)
Case B' Elasticities: Drawn from uniform distribution CA_e: Drawn from uniform (-1,1) distribution	-11.71	-6.56	(-37.12, -2.69)
Case E Elasticities: Drawn from bootstrap distribution CA_e: Fixed at 0	-51.83	-33.00	(-157.22, -12.20)
Case E' Elasticities: Drawn from bootstrap distribution CA_e: Drawn from uniform (-1,1) distribution	-51.72	-32.94	(-162.01, -10.66)

1/ For all cases, the Marshall-Lerner condition is imposed. The corresponding cases (B and E) from Table 2 are reproduced here for ease of comparison.

Table 4. Estimates of the Underlying Current Account

	Mean	Median	90 Percent Confidence Interval
<u>Case A</u> Elasticities: Drawn from uniform distribution			
Without Marshall-Lerner	-2.30	-2.31	(-2.90, -1.70)
With Marshall-Lerner	-2.24	-2.24	(-2.57, -1.92)
<u>Case B</u> Elasticities: Drawn from bootstrap distribution			
Without Marshall-Lerner	-2.10	-2.09	(-2.36, -1.85)
With Marshall-Lerner	-2.46	-2.47	(-2.69, -2.25)

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CHART 1

JOINT DISTRIBUTION OF ESTIMATED DEGREE OF MISALIGNMENT AND ESTIMATED UNDERLYING CURRENT ACCOUNT

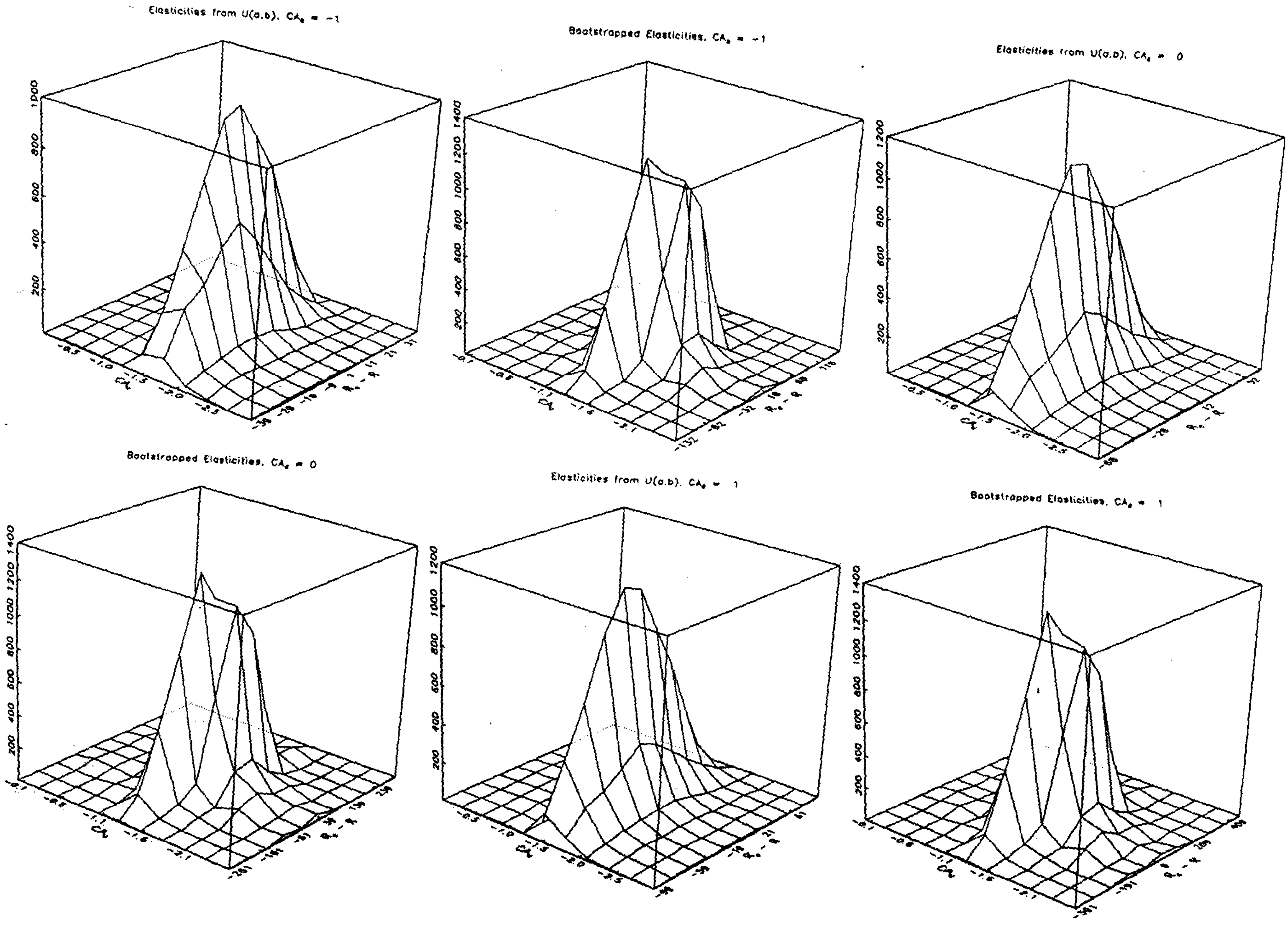

CHART 2

DISTRIBUTION OF SIMULATED IMPORT AND EXPORT ELASTICITIES