IMF WORKING PAPER

WP/96/21

INTERNATIONAL MONETARY FUND

Research Department

Jumps, Martingales, and Foreign Exchange Futures Prices

Prepared by Zu-Liu Hu 1/

Authorized for distribution by Peter Wickham

February 1996

Abstract

A common specification about the behavior of foreign exchange spot and futures prices is that they follow continuous diffusion processes. The empirical regularities uncovered from daily and weekly currency futures data, however, cast doubts on the validity of this model. First, contrary to the suggestions in the literature, changes in foreign currency futures prices are serially correlated; variance ratio tests and other related tests overwhelmingly reject Samuelson's martingale hypothesis. Second, foreign exchange futures prices do not appear to have continuous sample path; the evidence suggests the presence of a jump component, which may lead to pricing bias when applying the standard Black-Scholes option pricing formula to foreign exchange markets.

JEL Classification Numbers:
F31, G13

1/ I would like to thank Carlos Asilis, Robert Flood, Kenneth Froot, Robert Merton, Jeffrey Sachs, and Peter Wickham for helpful comments. The author also thanks, Maria Orihuela for assistance in text preparation. All views expressed in this paper are those of the author's and do not necessarily represent those of the International Monetary Fund.

Contents

Summary

Do currency futures prices follow Samuelson's martingale hypothesis that they display zero-risk primia? While equilibrium asset pricing theories show that asset prices possess the martingale property only under certain very restrictive assumptions, there is interest in testing the martingale hypothesis because of its important implications for theoretical models of exchange rates and for trading practices in foreign exchange markets.

The empirical results to date are inconclusive, however. The unbiasedness hypothesis using daily data for currency futures prices has been rejected. When the uncorrelatedness of increments in futures prices is examined, evidence is found against the null hypothesis only for daily data, while results from weekly data support the martingale hypothesis.

This paper re-examines the serial-correlation structure of daily and weekly currency futures prices, using a different methodology called variance ratio tests. The results for daily data confirm the previous finding against the martingale hypothesis. In contrast to suggestions in the literature, however, results obtained from weekly data also reject the martingale hypothesis. The evidence indicates that daily and weekly changes of foreign exchange futures prices are serially correlated.

The paper also examines alternative specifications that may account for the departure from the martingale model. A mixed diffusion-jump process is found to give a good representation of the short-run dynamics for currency futures prices. The result suggests that applying the standard Black-Scholes option pricing formula to foreign exchange market may produce nontrivial pricing bias. A modified options pricing model that allows for discontinuity in the price process of the underlying asset is more appropriate.

I. Introduction

An important question regarding the behavior of currency futures prices is whether they are martingales, as first suggested by Samuelson (1965). Although traditionally the martingale property has been thought as an essential implication of the market efficiency hypothesis, equilibrium asset pricing theories rarely suggest that asset prices need be martingales unless certain restrictive assumptions (such as investors' risk neutrality) are satisfied. Nevertheless, there continues to be considerable interest in testing the martingale hypothesis. One reason is that in practice, the lognormal diffusion model is commonly postulated for asset prices including foreign exchange spot and futures prices, so that the Black-Scholes formula can be directly applied to pricing of foreign currency spot and futures options contracts. The lognormal diffusion price process implies that futures price is a Brownian martingale under a change of probability measure.

Two aspects of the martingale hypothesis for foreign exchange futures prices have been subject to scrutiny in the literature. One is the unbiasedness of futures price as predictor of spot exchange rate. Hodrick and Srivastava (1987) rejected the unbiasedness hypothesis using daily data for currency futures prices. The other aspect, the uncorrelatedness of increments in futures prices, was examined by McCurdy and Morgan (1987, 1988), who found evidence against the null hypothesis for daily data but their results from weekly data supported the martingale hypothesis. The apparently different messages delivered by these studies using currency futures data led Hodrick (1987) to remark that "reconciliation of these results (McCurdy and Morgan (1987, 1988)) with the results of Hodrick and Srivastava (1987) indicating serial correlation in risk premiums appears to be an outstanding item on the research agenda."

This article re-examines the serial correlation structure of daily and weekly currency futures prices. Variance ratio tests for daily data confirm the previous finding against the martingale hypothesis. In contrast to the suggestions in the literature, however, our results obtained from weekly data also reject the martingale hypothesis. The evidence indicates that daily and weekly changes of foreign exchange futures prices are serially correlated. The rest of the paper is organized as follows. Section II briefly describes the methodology. Section III presents results of variance ratio tests of the martingale hypothesis for currency futures prices. Section IV examines alternative specifications that may account for the departure from the martingale model. A mixed diffusion-jump process is found to give good representation of the short-run dynamics for currency futures prices.

II. Methodology

Let F_t denote the foreign exchange futures price at time t and define $X_t = \ln F_t$ as the log price process. A commonly postulated model is that X_t satisfies the following stochastic integral equation under some probability measure P:

$$X_t = X_o + \int_0^t \mu \, ds + \int_0^t \sigma \, dB_s , \qquad t \geq 0 \tag{1}$$

where μ and σ are the drift and diffusion parameters and B_s is a standard Brownian motion at time s under the probability measure P. Since the first integral in (1) is almost always a function of bounded variation, and the second integral is a continuous time martingale, then X_t is a semimartingale. Note that, following the Girsanov's Theorem, X_t may in fact be a martingale under an equivalent probability measure Q. 1/ Thus the maintained hypothesis is given by the following relation: 2/

$$E^Q[X_t \mid \mathcal{F}_s] = X_s \qquad \forall \quad s \leq t \tag{2}$$

An important property of X_t satisfying (2) is that its increments, $X_1 - X_0$, $X_2 - X_1$, ..., are uncorrelated, so that the variance of any increment, $X_t - X_s$, $\forall\ t \geq s$, is proportional to the length of the interval, t-s. Suppose that we sample the continuous time process in (1) at equal intervals and obtain a sequence of discrete time observations, X_0, X_1, X_2, ..., X_{Tk+1}, then, we can construct the following intertemporal variance ratio:

1/ The probability measure Q has Radon-Nikodym derivative

$$\frac{dQ}{dP} = \xi_T$$

where,

$$\xi_t = \exp\left[\int_0^t \theta_s \, dB_s - \frac{1}{2}\int_0^t \theta_s \cdot \theta_s \, ds\right] , \qquad t \in [0, T]$$

and θ satisfies

$$E\left[\exp\left(\frac{1}{2}\int_0^T \theta_s \cdot \theta_s \, ds\right)\right] < \infty .$$

2/ A distinction should be made between the behavior of the level of futures price and that of its log. The martingale property of the log price process does not imply that the level is a martingale. In this paper we follow the literature to examine the martingale hypothesis for the log price process of foreign exchange futures.

$$VR(k) = \frac{Var(X_{t+k} - X_t)}{k\,Var(X_{t+1} - X_t)} \tag{3}$$

where Var is the variance operator. With homoskedasticity (i.e., constant diffusion parameter), the null hypothesis (2) implies that the variance ratio VR(k) equals unity. Fama and French (1988), Lo and MacKinlay (1988), and Poterba and Summers (1988) have all used variance ratios to test the random walk property of stock returns.

Since there is mounting evidence that volatilities of foreign exchange rates are time varying, see Baillie and Bollerslev (1989), Hsieh (1988, 1989), McCurdy and Morgan (1987, 1988), among others, statistical tests of serial correlation should be robust to heteroskedasticity in the data. While the Monte Carlo evidence provided by Poterba and Summers (1988) suggested that heteroskedasticity may not significantly affect the power of variance ratio tests, Lo and Mackinlay (1988) developed a formal sampling theory for the variance ratio statistic for both the homoskedastic and the possible heteroskedastic increments.

The benchmark discrete time model corresponding to (1) is given by:

$$\Delta X_t = \mu + \varepsilon_t , \qquad \varepsilon_t \mid \Omega_{t-1} \sim N(0, \sigma^2). \tag{4}$$

where Δ denotes the first difference operator.

With i.i.d. Gaussian disturbances ε_t, it can be shown that the variance test statistic has asymptotically standard normal distribution: 1/

$$V_z = \sqrt{Tk}\,(VR(k) - 1)(2(2k - 1)(k - 1)/3k)^{-1/2} \sim N(0,1) \tag{5}$$

where,

$$VR(k) \equiv \sigma_k^2 / \sigma_1^2 \tag{6}$$

is the variance ratio and

$$\mu = \frac{1}{Tk} \sum_{t=1}^{Tk} (X_t - X_{t-1}) \tag{7}$$

1/ For details of the proof, see Lo and MacKinlay (1988).

$$\sigma_1^2 = \frac{1}{Tk-1} \sum_{t=1}^{Tk} (X_t - X_{t-1} - \mu)^2 \tag{8}$$

$$\sigma_k^2 = \frac{1}{n} \sum_{t=k}^{Tk} (X_t - X_{t-k} - k\mu)^2, \qquad n = k(Tk - k + 1)(1 - \frac{k}{Tk}) \tag{9}$$

are the unbiased mean and variance estimators respectively. Moreover, in the presence of general heteroskedasticity such as the GARCH process (Engle (1982), Bollerslev (1986)), an adjusted test statistic that still has asymptotically standard normal distribution can be written as:

$$V_z^* = \sqrt{Tk}\,(VR(k) - 1)/\sqrt{\theta}, \qquad \theta(k) = \sum_{i=1}^{k-1} [\frac{2(k-i)}{k}]^2 \delta(i) \tag{10}$$

where

$$\delta(i) = \frac{\sum_{t=i+1}^{Tk} (X_t - X_{t-1} - \mu)^2 (X_{t-i} - X_{t-i-1} - \mu)^2}{[\sum_{t=1}^{Tk} (X_t - X_{t-1} - \mu)^2]^2} \tag{11}$$

is a heteroskedasticity-consistent estimator of the asymptotic variance of the i^{th}-order autocorrelation coefficient of ΔX_t.

In the following sections we apply the variance ratio-based statistics V and V^* outlined above to test the martingale hypothesis for foreign exchange futures prices. We also explore plausible alternatives to the benchmark model equation (4) to explain the behavior of currency futures prices.

III. Variance Ratio Tests for Daily Rates of Change in Futures Prices

We use price data for currency futures contracts traded at the International Monetary Market of the Chicago Mercantile Exchange. We focus on five most heavily traded currency futures contracts--the British Pound (BP), the Canadian dollar (CD), the Deutsche mark (DM), the Japanese yen (JY) and the Swiss franc (SF). The daily closing prices are taken from Reuters. All the series, except for the British pound futures price, started on January 1974 and ended on January 1993, with a sample size of 4816. The British pound futures price started on January 1975 and has 4362 observations.

Our initial analysis indicates strong GARCH effects for daily changes in futures price. Likelihood ratio tests reject the benchmark model equation (4), that is, the discrete time version of the constant parameter lognormal diffusion model, in favor of a GARCH (1,1) specification for all five currency futures prices. We therefore use only the adjusted variance ratio test statistic V^* for daily data.

We compare the 2-day, 4-day, 8-day and 16-day variance of futures price changes with the variance of daily price changes. The estimated variance ratios are reported in Table 1. Under the random walk martingale hypothesis, the value of these ratios is 1. The variance ratios for BP, CD, DM and SF all have values above unity, indicating positive autocorrelation for the 2-day, 4-day, 8-day and 16-day returns. 1/ The variance ratio for JY suggests negative serial correlation for short-period returns on the Japanese Yen futures. The heteroskedasticity-robust test statistics V^* all have zero probability values, indicating that the corresponding variance ratios are statistically different from 1. The martingale hypothesis for foreign exchange futures prices is therefore decisively rejected with daily data.

The results of variance ratio tests using daily sampled foreign exchange futures price data confirm the previous finding (Hodrick and Srivastava (1987), McCurdy and Morgan (1987,1988)) against the martingale hypothesis. The rejection of the null hypothesis could be due to the day-of-the-week effect and holiday effect, which McCurdy and Morgan (1987,1988), and Hertzel, Kendall and Kretzmer (1990) found significant in daily currency

---

1/ It can be shown that

$$VR(k) \simeq 1+\frac{2(k-1)}{k}\rho(1)+\frac{2(k-2)}{k}\rho(2)+\ldots+\frac{2}{k}\rho(k-1), \qquad \forall \quad k\geq 2.$$

where $\rho(k)$ denotes the k^{th}-order autocorrelation coefficient estimator of ΔX_t. The above relation indicates that the variance ratio VR(k) is approximately a linear combinations of the first k-1 autocorrelation coefficient estimators of the first differences with arithmetically declining weights. The variance ratio VR(k) is also closely related to the Ljung-Box (1978) Q statistic of order k-1,

$$Q = T(T+2)\sum_{j=1}^{k-1}\rho_j^2$$

which is asymptotically distributed as χ^2 with k-1 degrees of freedom.

Table 1. Variance Ratio Tests of the Martingale Hypothesis for Foreign Exchange Futures Prices: Daily Data 1/

Time Period	Number Tk of Base Observations	Number k of Base Observations Aggregated to Form Variance Ratio: 2	4	8	16
British pound					
Oct 20, 1975-Jan 29, 1993	4362	1.028	1.025	1.031	1.044
		(89.739)	(45.266)	(35.876)	(35.073)
Canadian dollar					
Jan 2, 1974-Jan 29, 1993	4816	1.053	1.031	1.007	0.978
		(193.232)	(61.488)	(9.238)	(-14.023)
Deutsche mark					
Jan 2, 1974-Jan 29, 1993	4816	1.008	1.027	1.038	1.094
Japanese yen					
Jan 2, 1974-Jan 29, 1993	4816	0.967	0.943	0.954	1.051
		(-60.272)	(-65.518)	(-40.427)	(35.264)
Swiss franc					
Jan 2, 1974-Jan 29, 1993	4816	1.021	1.039	1.043	1.075
		(87.637)	(85.279)	(60.243)	(71.214)

1/ The variance ratios VR(k) are given in the main rows, and the heteroskedasticity-robust test statistics V_z^* are in parenthesis.

futures data. 1/ Strictly speaking, the presence of a strong trading day effect per se is inconsistent with the martingale hypothesis. Since our results using daily data are in agreement with the literature, below we will analyze foreign exchange futures price data sampled at weekly intervals in greater detail.

IV. Testing the Martingale Hypothesis for Foreign Exchange Futures Prices Using Weekly Data

The weekly currency futures price data are derived from the Reuters daily price file. We pick Wednesday's closing price for each week and if Wednesday is a holiday, then Thursday's closing price is used. We obtain a sample of 1048 weekly observations of futures prices for CD, DM, JY and SF from January 3, 1973 up to January 27, 1993. The number of observation for BP futures price is 902, covering the period from October 22, 1975 to January 27, 1993. By using weekly sampled data, the trading day effect and the impact of limit moves for daily prices can be mitigated (McCurdy and Morgan (1987, 1988)).

Table 2 gives the estimated autocorrelations of the first difference in weekly futures price up to lag 15. Note that these coefficients are significant at some lags for all currencies. For example, the weekly changes in DM futures price have negative autocorrelation coefficient at lags 4, 5, and 8, and positive autocorrelation coefficient at lags 9 and 13. These serially correlated increments are inconsistent with the martingale hypothesis.

Table 3 reports the results of variance ratio tests for weekly data under the assumption of homoskedasticity. Returns on BP futures from the two week interval to the 16-week interval display positive autocorrelation, and returns on DM futures for these horizons display negative autocorrelations while for the other three currency futures, returns exhibit the pattern of mean reversion--they first show positive then negative serial correlations. There is strong evidence against the null hypothesis. The V_Z statistics marked with asterisks indicate that the corresponding variance

---

1/ Another possible factor is the presence of limit moves in the futures prices imposed by the Chicago Mercantile Exchange, which may induce some positive serial correlation in price changes. However, there are several reasons to believe that limit moves may not affect the overall conclusions here. First, limit moves were relatively concentrated on the early period of the sample while the martingale hypothesis is rejected for all sub-periods. Second, McCurdy and Morgan (1987, 1988) conducted analysis after removing observations affected by daily price limits and found that the results remained unchanged. Third, institutionally imposed limit moves should have less impact on the serial correlation structure of weekly logarithmic price differences. Nevertheless, the conclusions drawn from analysis of weekly data (see below) are essentially the same.

Table 2. Autocorrelations of Weekly Foreign Exchange Futures Prices Changes: January 3, 1973-January 27, 1993 1/

Lags	British Pound 2/	Canadian Dollar	Deutsche Mark	Japanese Yen	Swiss Franc
1	0.0246	-0.0006	-0.0023	0.0023	-0.0009
	(0.0333)	(0.0309)	(0.0309)	(0.0309)	(0.0309)
2	-0.0088	-0.0002	-0.0115	-0.0025	-0.0082
	(0.0323)	(0.0139)	(0.0301)	(0.0309)	(0.0231)
3	0.0608	-0.0005	0.0002	0.0012	-0.0029
	(0.0333)	(0.0309)	(0.0023)	(0.0309)	(0.0121)
4	0.0521	-0.3330	-0.2010	-0.0015	-0.2020
	(0.0335)	(0.0309)	(0.0309)	(0.0309)	(0.0309)
5	-0.0124	-0.3340	-0.3970	-0.3280	-0.3910
	(0.0336)	(0.0342)	(0.0321)	(0.0309)	(0.0321)
6	0.0290	0.0003	0.0098	-0.0025	0.0090
	(0.0336)	(0.0371)	(0.0365)	(0.0341)	(0.0364)
7	0.0439	0.0003	0.0133	-0.0028	0.0073
	(0.0826)	(0.0371)	(0.0271)	(0.0341)	(0.0364)
8	0.0290	-0.0014	-0.1940	-0.332	-0.1890
	(0.0336)	(0.0371)	(0.0365)	(0.0341)	(0.0364)
9	-0.0217	-0.3340	0.1950	0.0026	0.1930
	(0.0322)	(0.0371)	(0.0375)	(0.0370)	(0.0373)
10	-0.0108	-0.0537	0.0467	0.0690	-0.0253
	(0.0337)	(0.0399)	(0.0384)	(0.0370)	(0.0383)
11	-0.0019	-0.0002	-0.0088	-0.0008	-0.0075
	(0.0337)	(0.0399)	(0.0384)	(0.0370)	(0.0383)
12	0.0050	0.0003	-0.0011	0.0079	-0.0015
	(0.0337)	(0.0399)	(0.0328)	(0.0370)	(0.0383)
13	0.0205	0.0011	0.398	0.3310	0.3980
	(0.0310)	(0.0399)	(0.0384)	(0.0370)	(0.0383)
14	0.0047	-0.0005	-0.008	0.0005	0.0030
	(0.0203)	(0.0399)	(0.0422)	(0.0398)	(0.0420)
15	0.0077	0.0391	0.0280	0.0066	0.0909
	(0.0337)	(0.0399)	(0.0421)	(0.0398)	(0.0420)

1/ Standard errors in parenthesis.
2/ October 22, 1975-January 27, 1993 for the British Pound futures.

Table 3. Variance Ratio Tests of the Martingale Hypothesis for Foreign Exchange Futures Prices: Weekly Data

Time Period	Number Tk of Base Observations	Number k of Base Observations Aggregated to Form Variance Ratio 2	4	8	16
British pound futures					
Oct 22, 1975-Jan 27, 1993	902	1.026	1.035	1.150	1.288
		(0.783)	(0.566)	(1.522)	(1.965)*
Oct 22, 1975-Jun 6, 1984	451	1.019	1.035	1.208	1.378
		(0.404)	(0.394)	(1.493)	(1.820)
Jan 13, 1984-Jan 27, 1993	451	1.031	1.038	1.124	1.241
		(0.652)	(0.427)	(0.892)	(1.164)
Oct 22, 1975-Dec 28, 1983	428	1.023	1.036	1.211	1.425
		(0.465)	(0.400)	(1.477)	(1.990)*
Canadian dollar futures					
Jan 3, 1973-Jan 27, 1993	1048	1.001	1.005	0.421	0.340
		(0.043)	(0.080)	(-6.336)*	(-4.853)*
Jan 3, 1973-Jan 12, 1983	524	1.003	1.010	0.424	0.345
		(0.068)	(0.121)	(-4.454)*	(-3.405)*
Jan 19, 1983-Jan 27, 1993	524	1.009	0.850	0.745	0.794
		(0.208)	(-1.834)*	(-1.972)*	(-1.068)
Jan 5, 1977-Dec 28, 1993	365	1.038	1.155	1.110	0.828
		(0.729)	(1.585)	(0.707)	(-0.746)
Deutsche mark futures					
Jan 3, 1973-Jan 27, 1993	1048	0.999	0.991	0.496	0.288
		(-0.014)	(-0.160)	(-5.518)*	(-5.233)*
Jan 3, 1973-Jan 12, 1983	524	1.001	0.996	0.498	0.289
		(0.029)	(-0.052)	(-3.884)*	(-3.696)*
Jan 19, 1983-Jan 27, 1993	524	1.009	1.021	1.070	1.220
		(0.212)	(0.259)	(0.544)	(1.142)
Jan 2, 1974-Dec 28, 1983	522	1.108	1.228	1.277	1.414
		(2.425)*	(2.778)*	(2.140)*	(2.148)*
Japanese yen futures					
Jan 3, 1973-Jan 27, 1993	1048	1.004	1.007	0.762	0.351
		(0.136)	(0.127)	(-2.601)*	(-4.774)*
Jan 3, 1973-Jan 12, 1983	524	1.006	1.011	0.768	0.351
		(0.129)	(0.140)	(-1.796)	(-3.374)
Jan 19, 1983-Jan 27, 1983	524	1.043	1.158	1.270	1.399
		(0.982)	(1.936)*	(2.086)*	(2.075)*
Jan 5, 1977-Dec 28, 1983	365	1.084	1.260	1.433	1.485
		(1.600)	(2.655)*	(2.794)*	(2.103)*

Table 3 (Concluded). Variance Ratio Tests of the Martingale Hypothesis for Foreign Exchange Futures Prices: Weekly Data

Time Period	Number Tk of Base Observations	Number k of Base Observations Aggregated to Form Variance Ratio 2	4	8	16
Swiss franc futures					
Jan 3, 1973-Jan 27, 1993	1048	1.001	0.995	0.500	0.289
		(0.033)	(-0.092)	(-5.469)*	(-5.227)*
Jan 3, 1973-Jan 12, 1983	524	1.003	0.999	0.501	0.287
		(0.061)	(-0.008)	(-3.859)*	(-3.703)*
Jan 19, 1983-Jan 27, 1993	524	1.005	1.040	1.123	1.291
		(0.115)	(0.492)	(0.954)	(0.511)
Jan 2, 1974-Dec 28, 1983	522	1.068	1.121	1.199	1.330
		(1.549)	(1.475)	(1.535)	(1.710)*

Note: The variance ratios VR(K) are given in the main rows, and the unadjusted test statistics VZ are in parenthesis. Test statistics marked with asterisks (*) indicate that the corresponding variance ratios are statistically different from 1 percent at the 5 percent level of significance. The time period in the fourth row corresponds to the sample period previously examined in McCurdy and Morgan (1987), except for the British pound futures, for which their sample period is from January 2, 1974 to December 18, 1983.

ratios are statistically different from 1 at the 5 percent level of significance.

The V^* statistics are reported in Table 4. These statistics are robust to the general heteroskedasticity such as the GARCH process (Engle (1982), Bollerslev (1986)). The first row displays the variance ratios and the corresponding test statistics for the whole 1048-week (902-week for the British pound) sample period, the next two rows present the results for the two 524-week (451-week for the British pound) subperiods, and the last row gives the results for the sample period previously examined by McCurdy and Morgan (1987, 1988). All the test statistics have zero probability values, overwhelmingly rejecting the martingale hypothesis for all five currency futures and for all periods. Moreover the rejection of the null is unlikely to be due to the time-varying volatility since the V^* statistics are robust to the conditional heteroskedasticity present in the foreign exchange data.

The results of variance ratio tests using weekly data differ from those obtained by McCurdy and Morgan (1987, 1988). They used the regression tests, i.e., regressing the weekly futures return on lagged returns and testing that the coefficients are zero. They found that with weekly data the martingale hypothesis is retained for all five currency futures prices after allowing for conditional heteroskedasticity. They concluded that the rejection of the null hypothesis with daily data is primarily due to the complicated trading day effects in the daily data. Poterba and Summers (1988), however, suggested that the regression tests may have less power against the null hypothesis than the variance ratio tests.

For comparison we also performed alternative tests--the Dickey-Fuller (1979, 1981) unit root tests and the Ljung-Box (1978) portmanteau tests for the k^{th}-order serial correlation. The Dickey-Fuller test is related to our interests here because, if the null of random walk martingale is true, then a unit root is present in the futures price series. The relation between the Ljung-Box Q statistic and the variance ratio tests is given in footnote 1, page 5. The results are presented in Table 5. The Dickey-Fuller t test and the Augmented Dickey-Fuller test strongly reject the unit-root hypothesis for weekly foreign exchange futures prices. The Ljung-Box Q tests also decisively reject the null of uncorrelated price changes for weekly futures data for all currencies except for the British pound. The p-values for Q(15), which assumes homoskedasticity, and for $Q^*(15)$, which is adjusted for heteroskedasticity, are all zero for prices other than that of the British pound futures. These results support those obtained from the variance ratio tests and further strengthen the case against the martingale hypothesis.

Table 4. Variance Ratio Tests of the Martingale Hypothesis for Foreign Exchange Futures Prices: Weekly Data 1/

Time Period	Number Tk of Base Observations	Number k of Base Observations Aggregated to Form Variance Ratio 2	4	8	16
British pound futures					
Oct 22, 1975-Jan 27, 1993	902	1.026	1.035	1.150	1.288
		(18.820)	(13.706)	(36.909)	(49.104)
Oct 22, 1975-Jun 6, 1984	451	1.014	1.035	1.208	1.378
		(7.241)	(7.107)	(27.608)	(34.264)
Jun 13, 1984-Jan 27, 1993	451	1.031	1.038	1.124	1.241
		(11.690)	(7.725)	(16.104)	(21.726)
Oct 22, 1975-Dec 28, 1983*	428	1.023	1.036	1.211	1.425
		(8.087)	(7.000)	(26.500)	(36.436)
Canadian dollar futures					
Jan 3, 1973-Jan 27,1993	1048	1.001	1.005	0.421	0.340
		(48.719)	(90.368)	(-45.104)	(-29.009)
Jan 3, 1973-Jan 12, 1983	524	1.003	1.010	0.424	0.345
		(46.040	(80.825)	(-31.649)	(-20.323)
Jan 19, 1983-Jan 27, 1993	524	1.009	0.850	0.745	0.794
		(4.876)	(-26.820)	(-29.285)	(-17.597)
Jan 5, 1977-Dec 28, 1983*	365	1.038	1.155	1.110	0.828
		(13.054)	(27.991)	(12.275)	(-13.097)
Deutsche mark futures					
Jan 3, 1973-Jan 27, 1993	1048	0.999	0.991	0.496	0.288
		(-5.647)	(-37.526)	(-56.396)	(-41.015)
Jan 3, 1973-Jan 12, 1983	524	1.001	0.996	0.498	0.289
		(11.138)	(-11.999)	(-39.455)	(-28.700)
Jan 19, 1983-Jan 27, 1993	524	1.009	1.021	1.070	1.220
		(4.511)	(5.240)	(10.994)	(23.344)
Jan 2, 1974-Dec 28, 1983*	522	1.108	1.228	1.277	1.414
		(53.991)	(57.759)	(43.326)	(42.954)
Japanese yen futures					
Jan 3, 1973-Jan 27, 1993	1048	1.004	1.007	0.762	0.351
		(63.763)	(56.910)	(-31.158)	(-36.717)
Jan 3, 1973-Jan 12, 1983	524	1.006	1.011	0.768	0.351
		(49.686)	(51.346)	(-21.365)	(-25.760)
Jan 19, 1983-Jan 27, 1993	524	1.043	1.158	1.270	1.399
		(16.664)	(35.248)	(40.327)	(42.02)
Jan 5, 1977-Dec 28, 1983*	365	1.084	1.260	1.433	1.485
		(27.479)	(46.211)	(48.962)	(36.884)

Table 4 (Concluded). Variance Ratio Tests of the Martingale Hypothesis for Foreign Exchange Futures Prices: Weekly Data

Time Period	Number Tk of Base Observations	Number k of Base Observations Aggregated to Form Variance Ratio			
		2	4	8	16
Swiss franc futures					
Jan 3, 1973-Jan 27, 1993	1248	1.001	0.995	0.500	0.289
		(10.336)	(-24.951)	(-56.053)	(-41.095)
Jan 3, 1973-Jan 12, 1983	524	1.003	0.999	0.501	0.287
		(17.601)	(-1.989)	(-39.278)	(-28.919)
Jan 19, 1983-Jan 27, 1993	524	1.005	1.040	1.123	1.291
		(2.454)	(10.155)	(19.582)	(31.742)
Jan 2, 1974-Dec 28, 1983*	522	1.068	1.121	1.199	1.330
		(32.877)	(30.356)	(30.873)	(33.401)

1/ The variance ratios VR(k) are given in the main rows, and the heteroskedasticity-robust test statistics V_z^* are in parenthesis. The time period marked with asterisks (*) corresponds to the sample period previously examined in McCurdy and Morgan (1987), except the British pound futures for which their sample period is from January 2, 1974 to December 28, 1983.

Table 5. Alternative Tests of Autocorrelation in Weekly Changes of Foreign Exchange Futures Prices 1/

Futures Contracts	Sample Period	DF	ADF	$Q^*(15)$	Q(15)
British pound	10/22/75-01/27/93	-1.672	-29.114	19.692	15.200
		[0.091]	[0.000]	[0.184]	[0.418]
Canadian dollar	01/03/74-01/27/93	-7.924	-31.448	349.695	352.176*
		[0.000]	[0.000]	[0.000]	[0.000]
Deutsche mark	01/03/74-01/27/93	-7.599	-31.368	287.839	456.232*
		[0.000]	[0.000]	[0.000]	[0.000]
Japanese yen	01/03/74-01/27/93	-7.294	-31.433	343.610	347.114*
		[0.000]	[0.000]	[0.000]	[0.000]
Swiss franc	01/03/78-01/27/93	-7.377	-31.419	286.054	450.238*
		[0.000]	[0.000]	[0.000]	[0.000]

1/ DF is Dickey-Fuller (1979, 1981) unit root test, ADF is the augmented Dickey-Fuller test, Q(15) is the Ljung-Box portmanteau test for autocorrelations to the order of 15 and $Q^*(15)$ is the heteroskedasticity-adjusted Ljung-Box Q statistics. The corresponding p-values are in brackets.

V. Jump Processes in the Foreign Exchange Futures Market

The above analysis on the serial correlation structure of the data suggests that the benchmark diffusion model with constant parameters--the lognormal diffusion process , or its discrete time version in equation (4)--the random-walk model with drift, is inappropriate for describing the behavior of foreign exchange futures prices. Hsieh (1988, 1989), McCurdy and Morgan (1987, 1988), Baillie and Bollerslev (1989, 1990), and many others have found that allowing for time-dependent conditional heteroskedasticity such as a GARCH model may lead to a better representation of the exchange rate movements. An alternative model is a mixed diffusion-jump process. Jorion (1988) showed that the jump process is likely more important in the foreign exchange market than in other financial markets such as the stock market. A specification incorporating a jump component may account for the leptokurtosis present in the residual of a pure diffusion model.

The mixed diffusion-jump model for asset price process, as proposed by Merton (1976), can be written as

$$\frac{dF_t}{F_t} = (\mu + \frac{\sigma^2}{2})dt + \sigma dB_t + dq_t \tag{12}$$

where B_t is a standard Brownian motion and dq_t is the Poisson process with a mean number of jumps per unit time λ and a jump size Y, which is assumed to have a lognormal distribution, that is, $\ln Y \sim N(\nu, \xi^2)$. Thus, in discrete time, the logarithmic difference in futures prices, ΔX_t, takes the form of

$$\Delta X_t = \mu + \sigma B_t + \sum_{j=1}^{J_t} \ln Y_t \tag{13}$$

where J_t is the actual number of jumps during the interval.

To capture the time-dependent conditional heteroskedasticity, the mixed lognormal diffusion-jump model can be further generalized to allow for the presence of the GARCH process:

$$\begin{aligned} \Delta X_t &= \mu + \epsilon_t + \sum_{j=1}^{J_t} \ln Y_t, \qquad \epsilon_t \mid \Omega_{t-1} \sim N(0, h_t) \\ h_t &= \sigma^2 + \sum_{j=1}^{q} \alpha_j \epsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \end{aligned} \tag{14}$$

The estimates for model equation (4), the discrete time counterpart of the lognormal diffusion model, and the corresponding diagnostic tests are presented in Table 6. The Jargue-Bera (1987) tests strongly reject the assumption of normality for all futures prices. There are significant skewness and kurtosis in all of the residuals. Not surprisingly, the Q(15) tests indicate significant serial correlation of the residuals for most currencies. The ARCH tests, however, do not support an ARCH formulation for CD, DM, JY and SF. Only for the British pound futures is the ARCH effect found to be significant. In fact the GARCH (1,1) model fits the data well for the British pound futures. The estimates of the GARCH parameters are shown in Table 7. The likelihood ratio test indicates that the GARCH model dominates the lognormal diffusion process.

More generally, we can nest alternative models for foreign exchange data and test these different model specifications. Table 8 reports results of fitting a mixed diffusion-jump model for foreign exchange futures prices. The log-likelihood function for this model can be written as

$$L = -T\lambda - \frac{T}{2}\ln(2\pi) + \sum_{t=1}^{T} \ln\left[\sum_{j=0}^{\infty} \frac{\lambda^j}{j!} \frac{1}{\sqrt{h_t + \xi^2 j}} \exp\left(\frac{-(x_t - \mu - \nu j)^2}{2(h_t + \xi^2 j)}\right)\right] \quad (15)$$

This likelihood function can be used to construct a generalized likelihood ratio:

$$\Lambda = -2(\ln L(x;\varphi_0) - \ln L(x;\varphi^*)) \quad (16)$$

where φ^* is the maximum likelihood estimate under the general specification of mixed diffusion-Poisson jump process, and φ_0 is the parameter vector estimate that maximizes the likelihood function under the null hypothesis that a jump component is absent. The test statistic Λ is asymptotically distributed as χ^2 with degrees of freedom equal to the difference of the number of the parameters between the two nested models.

It can be seen that a specification that captures the jump component leads to much better empirical fit for the data. The parameters of the Poisson jump process are found significant for the British pound futures price even after allowing for the time-dependent conditional heteroskedasticity. Similarly the futures price data display significant discontinuity for other currencies. Moreover the jump process can account for substantial leptokurtosis in the unconditional distribution of the changes in the currency futures prices. The skewness and kurtosis in the residuals are substantially reduced compared to those in the lognormal model in Table 6. The likelihood ratio test statistics strongly reject both the lognormal-diffusion model and the GARCH model in favor of a mixed diffusion-jump process.

Table 6. Diffusion Models with Constant Parameters for Foreign Exchange Futures Prices: Weekly Data 1/

	British Pound	Canadian Dollar	Deutsche Mark	Japanese Yen	Swiss Franc
μ	-0.354×10^{-3}	0.021	-0.024	0.021	-0.024
	(1.107)	(2.060)	(2.644)	(2.069)	(2.546)
σ^2	0.268×10^{-3}	0.038	0.047	0.039	0.048
	(38.821)	(112.777)	(122.026)	(112.141)	(121.986)
Log-likelihood	2428.11	532.22	345.53	528.27	343.87
ARCH	5.905	0.009	0.025	0.009	0.025
	[0.015]	[0.926]	[0.875]	[0.924]	[0.875]
Q(15)	15.467	351.784	456.442	346.89	450.04
	[0.418]	[0.000]	[0.000]	[0.000]	[0.000]
Skewness	-0.417	6.187	2.906	6.150	2.860
Kurtosis	4.964	345.657	203.406	337.721	202.350
JB	938.593	0.517×10^{7}	0.178×10^{7}	0.493×10^{7}	0.177×10^{7}
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]

1/ Asymptotic t-statistics are in parenthesis. ARCH is the Engle test for ARCH(1) residuals, Q(15) is the Ljung-Box Q statistic for autocorrelation of order 15. JB is the Jarque-Bera (LM) normality test. The p-values are shown in brackets.

Table 7. A Diffusion Model with Time-Varying Second Moments--GARCH (1,1) for the British Pound Futures Prices: Weekly Data

$\mu(\mathrm{x}10^3)$	$\sigma^2(\mathrm{x}10^3)$	α_1	β_1	Log-likelihood	Q(15)	Skewness	Kurtosis	JB	LR
0.549	0.0147	0.075	0.874	2461.21	10.807	-0.401	3.998	39.250	66.2
(1.107)	(4.932)	(4.352)	(34.324)		[0.523]			[0.000]	[0.000]

Note: Asymptotic t-statistics are shown in parenthesis and asymptotic p-values are in brackets. Q(15) is Ljung-Box Q statistic for autocorrelation of order 15. JB is Jarque-Bera (LM) normality test and LR is the likelihood ratio test for the hypothesis that the GARCH parameters α_1, and β_1, are jointly zero.

Table 8. Mixed Diffusion-Jump Models for Foreign Exchange Futures Prices: Weekly Data

	British Pound	Canadian Dollar	Deutsche Mark	Japanese Yen	Swiss Franc
$\mu(x10^{-2})$	-0.026 (-1.023)	0.197 (0.627)	0.287 (0.569)	0.304 (0.749)	0.310 (0.615)
σ^2	$0.125x10^{-3}$ (4.811)	0.015 (223.586)	0.025 (226.290)	0.015 (280.134)	0.026 (225.823)
α_1	0.109 (4.677)				
β_1	0.881 (5.212)				
λ	1.032 (1.720)	1.504 (1.981)	0.788 (1.302)	1.205 (2.039)	1.115 (1.876)
$\nu(x10^{-2})$	-0.011 (-0.972)	0.310 (1.172)	-0.155 (-0.834)	0.213 (0.951)	0.188 (1.235)
ξ^2	$0.142x10^{-2}$ (7.142)	0.035 (5.210)	0.023 (3.651)	0.021 (4.105)	0.029 (25.138)
Log-likelihood	2475.59	554.88	394.79	554.57	381.69
Q(15)	8.317 [0.910]	43.587 [0.000]	35.864 [0.002]	29.229 [0.015]	32.403 [0.006]
Skewness	-0.219	0.318	0.445	0.231	0.502
Kurtosis	3.998	3.289	4.701	3.653	3.873
JB	9.236 [0.009]	8.756 [0.013]	11.012 [0.004]	9.594 [0.008]	8.703 [0.013]
LR	28.76 [0.000]	45.32 [0.0]	98.52 [0.0]	52.60 [0.0]	75.63 [0.0]

Note: Asymptotic t-statistics are shown in parenthesis. Q(15) is Ljung-Box Q statistics for auto correlation of the order 15, JB is Jarque-Bera (LM) normality test, LR is likelihood ratio test for the diffusion model (possibly with time-varying parameters) against a mixed diffusion-jump model, with the test statistics being $\chi^2(3)$. The p values are in brackets.

These results have important implications for the pricing of options on currency futures. They suggest that applying the standard Black-Scholes options pricing formula to foreign exchange market may produce nontrivial pricing bias. A modified options pricing model, along the lines of Merton (1976), that allows for discontinuity in the price process of the underlying asset, are more appropriate.

VI. Conclusion

The characterization of exchange rate movements, including second-order dynamics, have important implications for many issues in international finance, such as the effect of exchange rate on international trade, international portfolio management and pricing of currency options. One of the best established facts about exchange rate behavior is that the unconditional distributions are leptokutic, with significant ARCH effects. At the same time, the traditional random-walk model with drift and Gaussian errors is found to well represent the serial correlation structure of exchange rate data, supporting the lognormal diffusion specification. Baillie and Bollerslev (1989), for example, found a unit root in daily spot exchange rate series. Using data on foreign exchange futures prices, McCurdy and Morgan (1987, 1988) could not reject the Martingale hypothesis for weekly data.

By contrast, variance ratio tests used in the current study strongly reject the martingale hypothesis for both daily and weekly currency futures returns, indicating that changes in currency futures prices are serially correlated. These rejections are unlikely caused by the presence of heteroskedasticity. Furthermore, a significant jump component is detected from the data, indicating discontinuity in the sample path of currency futures prices. These results suggest that the popular diffusion model, even after allowing for time-varying volatility, is inappropriate for describing the empirical behavior of foreign exchange rate data. The challenge facing theoretical models of exchange rate determination is that they must explain the discontinuity in the sample path as well as the pattern of serial correlation and conditional heteroskedasticity.

References

Baillie, R.T., and T. Bollerslev, (1989), "The Message in Daily Exchange Rates: a Conditional-Variance Tale," Journal of Business and Economic Statistics, Vol. 7, No.3, 297-305.

Bollerslev, T., (1986), "Generalized Autoregressive Conditional Heteroskedasticity," Journal of Econometrics, Vol. 31, pp. 307-27.

Dickey D.A. and W.A. Fuller, (1979), "Distributions of the Estimators for Autoregressive Time Series with a Unit Root," Journal of American Statistical Association, Vol. 74, pp. 427-31.

———, (1981), "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root," Econometrica, Vol. 49, pp. 1057-72.

Engle, R.F., (1982), "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation," Econometrica, Vol. 55, pp. 391-407.

Fama, E., and K. French, (1988), "Permanent and Temporary Components of Stock Prices," Journal of Political Economy, Vol. 96, pp. 246-73.

Hertzel M.C., C.S. Kendall and P.K. Kretzmer, (1990), "The Volatility of Asset Returns During Trading and Nontrading Hours: Some Evidence from the Foreign Exchange Markets," Journal of International Money and Finance, Vol. 9, pp. 335-43.

Hodrick, R.J., (1987), The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets, Harwood Academic Publishers.

———, and S. Srivastava, (1987), "Foreign Currency Futures," Journal of International Economics, Vol. 22, pp. 1-24.

Hsieh, D.H.A., (1988), "The Statistical Properties of Daily Foreign Exchange Rates:1974-1983," Journal of International Economics, Vol. 24, pp. 129-45.

———, (1989), "Modeling Heteroskedasticity in Daily Foreign Exchange Rates," Journal of Business and Economic Statistics, Vol. 7, pp. 307-17.

Jarque, C.M. and A.K. Bera, (1987), "A Test for Normality of Observations and Regression Residuals," International Statistical Review, Vol. 55, pp. 163-72.

Ljung, G.M., and G.E.P. Box, (1978), "On a Measure of Lack of Fit in Time Series Models," Biometrika, Vol. 65, pp. 297-303.

Lo, A. and A. C. MacKinlay, (1988), "Stock market Prices Do Not Follow Random Walks: Evidence from a Simple Specification Test," Review of Financial Studies, Vol. 1, pp. 41-66.

McCurdy, T., and I.G. Morgan, (1987), "Tests of the Martingale Hypothesis for Foreign Currency Futures with Time Varying Volatility," International Journal of Forecasting, Vol. 3, pp. 131-48.

———,(1988), "Testing the Martingale Hypothesis in Deutsche Mark Futures with Models Specifying the Form of the Heteroskedasticity," Journal of Applied Econometrics, Vol.3, 187-202.

Merton, R.C., (1976), "Option Pricing when Underlying Stock Returns are Discontinuous," Journal of Financial Economics, Vol. 3, pp. 125-44.

Poterba, J. and L. Summers, (1988), "Mean Reversion in Stock Returns: Evidence and Implications," Journal of Financial Economics, Vol. 22, pp. 27-59.

Samuelson, P., (1965), "Proof that Properly Anticipated Prices Fluctuates Randomly," Industrial Management Review, Vol. 6, pp. 41-50.