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Wage Contracts, Capital Mobility, and Macroeconomic Policy

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Abstract

This paper examines the long-run effects of macroeconomic policy shocks on the behavior of output, inflation, real wages and the real exchange rate in a small open economy. The analysis is based on a two-sector, three-good optimizing model with imperfect capital mobility, nominal wage contracts with backward- or forward-looking price expectations, and endogenous mark-up pricing in the nontraded goods sector. The effects of a cut in government spending on nontraded goods are shown to be independent of the expectational mechanism embedded in wage contracts. A reduction in the nominal devaluation rate lowers steady-state output in the tradable sector under backward-looking contracts, but exerts an expansionary effect under forward-looking contracts.

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### Summary

Nominal wage rigidity and imperfect capital mobility are two important macroeconomic features of many developing countries. This paper integrates both features in an analysis of the effects of macroeconomic policy shocks on output, real wages, the real exchange rate, and the current account. The analysis is based on a two-sector, three-good optimizing model with endogenous mark-up pricing in the nontraded goods sector. Both backward- and forward-looking wage contracts are considered. Because of the highly complex dynamic structure of the model (which is shown to depend crucially on the nature of the expectational mechanism embedded in wage contracts), the analysis focuses on the steady-state effects of policy shocks.

The paper shows that the long-run macroeconomic effects of a reduction in spending on nontraded goods is shown to be independent of the mechanism through which contracts are formed. It lowers the marginal value of wealth (and thus stimulates private consumption), reduces the product wage (thereby raising output in the export sector), and leads to a depreciation of the real exchange rate. Real holdings of foreign bonds (measured in domestic currency terms) rise, whereas net foreign assets held by the central bank may rise or fall. A cut in public spending has no effect on either inflation and output of nontraded goods, or on nominal and real interest rates. By contrast, the steady-state effects of a reduction in the devaluation rate on several macroeconomic variables is shown to depend crucially on whether wage contracts are backward or forward looking. In the case of backward-looking contracts, the real exchange rate depreciates while the product wage rises, leading to a reduction in output in the export sector. With forward-looking contracts, the real exchange rate appreciates while the real product wage falls, leading to an expansion in output of exportables. In both cases, however, inflation, the rate of growth of nominal wages, and the nominal interest rate fall in the same proportion as the devaluation rate--with no net effect on the real interest rate.

The thrust of the analysis in the paper is that the extent to which the nature of wage contracts alters the long-run effects of macroeconomic policies--particularly on the real sector of the economy--depends on the type of shocks considered. An understanding of the mechanisms through which wage contracts are formed is thus important for the choice of policy instruments in stabilization programs.



## I. Introduction

An important area of investigation in development macroeconomics has been the determination of the degree of wage flexibility and the speed of international capital mobility. Recent findings suggest that the labor market in developing countries is often characterized by nominal wage rigidity--in contrast with the common, and often implicit, assumption of real wage resistance in the early development literature--and, on the financial side, by imperfect capital mobility. Although the literature remains somewhat unsettled on issues such as the exact nature of the expectational mechanism that best characterizes the formation of inflationary expectations embedded in wage contracts, it has been argued that nominal wage inertia--which is often viewed as one of the principal causes of the persistence of inflation in semi-industrialized countries--represents one of the main features of the labor market in a large number of Asian and Latin American countries. Similarly, there exists a growing consensus that the assumption of perfectly mobile capital may not be a sensible approximation for developing countries (in contrast to industrial countries), although the degree of capital mobility appears to vary substantially between upper middle-income developing nations and low-income countries. <sup>1/</sup>

Wage rigidity and imperfect capital mobility have important implications for macroeconomic management, many of which have long been recognized by economists. The literature on contractionary devaluation, for instance, has emphasized the potentially perverse effect that wage formation may play in the transmission process and the ultimate effects of exchange rate changes (Agénor and Montiel, 1994, chapter 7). Similarly, the speed of capital mobility has been shown to play an important role in explaining the behavior of interest rates and the dynamics of portfolio adjustment induced by domestic monetary and fiscal policy shocks. However, both features have not so far been integrated simultaneously in a setup deemed appropriate for macroeconomic policy analysis in developing countries.

This paper attempts to provide such a framework. It examines the implications of both backward- and forward-looking contracts for the long-run effects of macroeconomic policy shocks, using a specification of wage contracts first proposed by Willman (1988). Section II develops the analytical framework, which consists of a two-sector, three-good optimizing model with endogenous mark-up pricing in the nontraded goods sector. Section III studies how alternative assumptions regarding contract formation affect the dynamic structure

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<sup>1/</sup> See, for instance, Labán and Larraín (1993) for evidence on capital mobility in Chile. Agénor and Montiel (1994) and Montiel (1994) provide a comprehensive review of the recent literature on capital mobility and wage formation in developing countries. Obstfeld (1994) examines the available evidence for industrial countries.

and the stability conditions of the model. Section IV examines the steady-state effects of fiscal and exchange rate policies. Section V summarizes the major implications of the analysis.

## II. The Analytical Framework

Consider a small open economy in which perfect foresight prevails and four types of agents operate: households, producers, the government, and the central bank. The exchange rate is depreciated by the central bank at a constant rate, and there are no restrictions on access to foreign exchange. The economy produces two goods, a home good which is used only for final domestic consumption, and an exportable good, whose output is entirely sold abroad. The capital stock in each sector is fixed, while labor is homogeneous and perfectly mobile across sectors.

Households consume both home and imported goods, supply labor inelastically and hold three categories of imperfectly substitutable financial assets in their portfolios: domestic money, foreign bonds and domestic government bonds. Domestic money bears no interest, but the transactions technology is such that holding money reduces transactions costs. As a result of the small country assumption, the real rate of return on foreign bonds is determined on world capital markets. The government consumes home and imported goods, collects income and lump-sum taxes, and pays interest on its domestic debt. It finances its budget deficit either by issuing nontraded bonds, borrowing from the central bank, or by varying taxes levied on households. Nominal wages are set through contracts negotiated between households and producers.

### 1. Households

The imported good consumed by households is not produced domestically and is imperfectly substitutable to the nontraded good. Consumption decisions are assumed to follow a two-step process: households first determine the optimal level of total consumption, and then allocate that amount between consumption of the two goods. 1/

The representative household's discounted lifetime utility is given by 2/

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1/ Conditions under which a two-stage budgeting process of this type is optimal are discussed by Deaton and Muellbauer (1980).

2/ Except otherwise indicated, partial derivatives are denoted by corresponding subscripts, while the total derivative of a function of a single argument is denoted by a prime. A sign over a variable refers to the sign of the corresponding partial derivative. We also define  $\dot{z}_t = dz_t/dt$ .

$$\int_0^{\infty} u(c_t) e^{-\alpha t} dt, \quad u(c_t) = \ln c_t, \quad (1)$$

where  $\alpha$  denotes the rate of time preference (assumed constant), and  $c_t$  total consumption. For convenience, the instantaneous utility function is assumed to be logarithmic.

Nominal wealth of the representative household  $A_t$  is given by

$$A_t = M_t + B_t + E_t B_t^*, \quad (2)$$

where  $M_t$  denotes the nominal money stock,  $B_t$  the stock of government bonds, and  $E_t B_t^*$  the domestic-currency value of the stock of foreign bonds (with  $E_t$  denoting the nominal exchange rate and  $B_t^*$  the foreign-currency value of foreign assets). Letting  $P_t$  denote the domestic price level (defined below),  $m_t = M_t/P_t$  real money balances,  $b_t = B_t/P_t$  the real stock of government bonds, and  $b_t^* = E_t B_t^*/P_t$  the real stock of foreign bonds, real wealth  $a_t$  can be defined as

$$a_t = m_t + b_t + b_t^*. \quad (2')$$

The flow budget constraint is given by

$$\begin{aligned} \dot{a}_t = & (1-\iota)(q_t + i_t b_t + i_t^* b_t^*) - [1+v(\frac{m_t}{c_t})]c_t \\ & - \tau_t + \epsilon_t b_t^* - \pi_t a_t - \gamma(b_t^*)^2/2, \end{aligned} \quad (3)$$

where  $q_t$  denotes net factor income (derived below),  $\tau_t$  the real value of lump-sum taxes,  $0 < \iota < 1$  the tax rate on income,  $i_t$  the domestic nominal interest rate,  $i_t^*$  the interest rate on foreign bonds,  $\epsilon_t = \dot{E}_t/E_t$  the predetermined rate of depreciation of the exchange rate, and  $\pi_t = \dot{P}_t/P_t$  the overall inflation rate. For simplicity, income taxes are assumed to be levied at the same rate on factor income and interest payments on holdings of domestic and foreign bonds. The function  $v()$  characterizes transactions costs associated with consumption. Holding money is assumed to reduce transactions

costs--so that  $v' < 0$ --but entails diminishing returns ( $v'' > 0$ ). 1/ The term  $\pi_t a_t$  accounts for capital losses on total wealth resulting from inflation, while the term  $b_t^* \epsilon_t$  represents capital gains on the stock of foreign bonds resulting from exchange rate depreciation. The last term in equation (3) (where  $\gamma > 0$ ) represents an additional tax on foreign bonds, which rises at an increasing rate with actual holdings. Such a tax may be imposed by the government as a result of distributional considerations. 2/ As shown below, it implies that domestic and foreign bonds are imperfect substitutes, with  $\gamma$  being interpreted as a measure of the degree of capital mobility.

Using (2'), equation (3) can be written as

$$\begin{aligned} \dot{a}_t = & \rho_t a_t + (1-\iota)q_t - [1+v(\frac{m_t}{c_t})]c_t - (1-\iota)i_t m_t, \\ & + [(1-\iota)(i_t^* - i_t) + \epsilon_t]b_t^* - \tau_t - \gamma b_t^{*2}/2, \end{aligned} \quad (3')$$

where  $\rho_t = (1-\iota)i_t - \pi_t$  denotes the after-tax domestic real rate of interest.

In the first stage of the consumption decision process, households treat  $\pi_t$ ,  $\epsilon_t$ ,  $q_t$ ,  $i_t$ ,  $i_t^*$  and  $\tau_t$  as given and maximize (1) subject to (3') by choosing a sequence  $\{c_t, m_t, b_t, b_t^*\}_{t=0}^{\infty}$ . 3/ The required optimality conditions are given by:

$$1/c_t = \lambda_t \left\{ [1+v(\frac{m_t}{c_t})] - m_t c_t^{-1} v'(\frac{m_t}{c_t}) \right\}, \quad (4a)$$

$$- v'(\frac{m_t}{c_t}) = (1-\iota)i_t, \quad (4b)$$

1/ Put differently, the representative household must purchase the gross amount  $c_t[1+v(m_t/c_t)]$  in order to consume the net amount  $c_t$ . This formulation has been used by, among others, Kimbrough (1992). An alternative formulation would be to introduce money directly in the utility function. Feenstra (1986) examines the conditions under which this approach is equivalent to the explicit consideration of transactions costs in the household's budget constraint.

2/ This specification was proposed by Turnovsky (1985) in a different context. Agénor (1994) also uses a similar formulation in a one-good optimizing framework and discusses an alternative rationale (the existence of a confiscation risk).

3/ Recall that the supply of labor is inelastic.



$$b_t^* = [(1-\iota)(i_t^* - i_t) + \epsilon_t]/\gamma, \quad (4c)$$

$$\dot{\lambda}_t = (\alpha - \rho_t)\lambda_t, \quad (4d)$$

and the transversality condition  $\lim_{t \rightarrow \infty} (a_t e^{-\alpha t}) = 0$ .  $\lambda_t$ , the costate variable associated with the flow budget constraint, measures the shadow value of the representative household's wealth.

Equation (4a) shows that total consumption is inversely related to the marginal utility of wealth, whose dynamics are described by equation (4d) as a function of the difference between the discount rate and the real after-tax interest rate. Equation (4b) equates at the margin the benefit from holding an additional unit of money (the associated reduction in transactions costs) to its opportunity cost, given by the after-tax domestic nominal interest rate. It determines implicitly the demand for real money balances. Equation (4c) indicates that holdings of foreign bonds depend positively on the difference between the after-tax rates of return on foreign and domestic assets. When the tax parameter  $\gamma \rightarrow 0$ , equation (4c) yields the uncovered interest parity condition  $(1-\iota)i_t = (1-\iota)i_t^* + \epsilon_t$ . By contrast, when the tax parameter  $\gamma$  is large, holdings of foreign bonds tend to zero. In such conditions, the domestic interest rate becomes independent of the world interest rate. 1/

Equation (4b) can be written as

$$m_t = m[(1-\iota)i_t]c_t, \quad m' = -1/v'' < 0 \quad (5)$$

which shows that the demand for money depends positively on the level of transactions--as measured by total consumption expenditure--and negatively on the after-tax nominal interest rate. Substituting equations (4b) and (5) in (4a) yields

$$1/c_t = \lambda_t \left\{ 1 + v[m((1-\iota)i_t)] + (1-\iota)i_t m'[(1-\iota)i_t] \right\} = \lambda_t P^+[(1-\iota)i_t],$$

where  $P()$  denotes the effective price of total consumption in period  $t$ . This price is equal to the direct price of the consumption bundle plus the transactions costs associated with purchasing goods, augmented by the opportunity cost (net of taxes) of holding domestic

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1/ Turnovsky (1985) discusses the restrictions that must be imposed on the rate of time preference  $\alpha$  to ensure that steady-state consumption is stationary under perfect capital mobility, and that the domestic interest rate is independent of the foreign interest rate under zero capital mobility.

money balances to facilitate transactions. An increase in the after-tax nominal interest rate, given the marginal value of wealth, raises transactions costs as well as the opportunity cost of the existing stock of real money holdings, therefore raising the marginal cost of consumption. The effective price is thus positively related to the opportunity cost of holding real money balances. The above equation can also be written as

$$c_t = c[\lambda_t, (1-i_t)i_t]. \quad (6)$$

In the second stage of the consumption decision process, the representative household maximizes a sub-utility function  $v[c_N(t), c_I(t)]$ , which is assumed to be homogeneous of degree one, subject to the static budget constraint:

$$P_N(t)c_N(t) + E_t P_I^* c_I(t) = P_t [1 + v(\frac{m_t}{c_t})] c_t, \quad (7)$$

where  $P_N(t)$  denotes the price of the home good,  $P_I^*$  the world price of imports--which is constant and set to unity in what follows-- $c_N(t)$  purchases of nontraded goods, and  $c_I(t)$  expenditure on imported goods. The solution to this program yields the familiar result according to which the representative household sets the marginal rate of substitution between home and foreign goods equal to their relative price  $z_t \equiv E_t/P_N(t)$ , that is, the real exchange rate. Using this condition, together with equation (7), and assuming that the sub-utility function is Cobb-Douglas--so that expenditure shares are constant--yields the appropriate definition of the domestic consumer price index,  $P_t$ :

$$P_t = P_N(t)^\delta E_t^{1-\delta} = E_t z_t^{-\delta}, \quad 0 < \delta < 1 \quad (8)$$

where  $\delta$  denotes the share of total spending falling on home goods. Consequently, using equations (7) and (8) yields

$$c_N(t) = \delta z_t^{1-\delta} [1 + v()] c_t, \quad (9a)$$

$$c_I(t) = (1-\delta) z_t^{-\delta} [1 + v()] c_t, \quad (9b)$$

$$\pi_t = \epsilon_t - \delta \dot{z}_t / z_t. \quad (9c)$$

## 2. Production, income, and wage formation

Firms in the home goods sector set prices  $P_N(t)$  as a mark-up  $\theta_t$  over nominal wages  $w_t$ , so that

$$\pi_N(t) = \dot{\theta}_t / \theta_t + \dot{w}_t / w_t, \quad (10)$$

where  $\pi_N(t)$  denotes the rate of inflation in home goods prices. 1/

Output of the home goods sector is determined by the demand side of the market. Using equation (9a), the equilibrium condition of the home goods market is given by

$$q_N(t) = g_N + \delta z_t^{1-\delta} [1+v()]c_t, \quad (11)$$

where  $g_N$  denotes the exogenous level of government spending on home goods. The mark-up rate varies procyclically over time, and depends positively on excess demand for home goods:

$$\dot{\theta}_t / \theta_t = \xi [q_N(t) - \tilde{q}_N], \quad \xi > 0 \quad (12)$$

where  $\tilde{q}_N$  denotes the exogenous level of capacity output. 2/

Technology for the production of the export good is characterized by decreasing returns to labor:

$$q_E(t) = F[n_E(t)], \quad F' > 0, \quad F'' < 0 \quad (13)$$

where  $q_E$  denotes output of exportables, and  $n_E$  the quantity of labor employed in the export sector. Firms maximize real profits given by

1/ For a discussion of the optimality of mark-up pricing, see Naish (1993). The assumption of mark-up pricing represents a key analytical feature of New Structuralist macroeconomic models, as discussed for instance by Taylor (1983). Note that in the presence of imported intermediate inputs, prices of nontraded goods would also depend on the exchange rate.

2/ Appendix I provides a more explicit rationale for an equation similar to (12). Evidence on the cyclicity of mark-up rates in developing countries is mixed. Some aggregate econometric studies appear to support the "pure" mark-up hypothesis-- which is consistent with profit maximization if the production technology is characterized by constant returns to scale--with a limited role attached to excess demand (see, for instance, Parkin, 1991). In what follows we will in any case assume that  $\xi$ , although positive, is small.

$q_E(t) = \omega_t n_E(t)$ , where  $\omega_t \equiv w_t/E_t$  denotes the product wage in the export sector, in which it is assumed that the world price of exports is constant and set to unity--implying that the domestic price is equal to the nominal exchange rate. From the first-order condition for profit maximization, the supply function of exports can be derived as

$$q_E^S(t) = q_E(\omega_t), \quad q_E' < 0 \quad (14)$$

which indicates that output of exportable goods is inversely related to the product wage.

Using equations (8) and (14), real factor income  $q_t$ , measured in terms of the price of the consumption basket, is given by

$$q_t = z_t^{\delta-1} [q_N(t) + z_t q_E(\omega_t)]. \quad (15)$$

Nominal wages are set under two alternative contract mechanisms. Under the first scheme, wages are backward-looking and depend only on past levels of prices:

$$w_t = \sigma \int_{-\infty}^t e^{-\sigma(t-k)} P_k dk, \quad \sigma > 0$$

where  $\sigma$  is a discount factor. Differentiating this equation with respect to time yields:

$$\dot{w}_t = -\sigma(w_t - P_t). \quad (16)$$

Under the second scheme, wage contracts are assumed to be forward-looking and to depend solely on future prices:

$$w_t = \sigma \int_t^{\infty} e^{\sigma(t-k)} P_k dk,$$

so that

$$\dot{w}_t = \sigma(w_t - P_t). \quad (17)$$

Equations (16) and (17) show therefore that changes in the nominal wage under backward- and forward-looking contracts respond in opposite directions to movements in the wage-price differential. 1/

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1/ Equations (16) and (17) are adapted from Willman (1988). The forward-looking case is conceptually close to Calvo's (1983) overlapping contract formulation; see also Miller (cont'd on page 9)

Dividing equations (16) and (17) by  $w_t$ , using equation (8) and noting the definition of the product wage in the export sector yields

$$\dot{w}_t/w_t = -\sigma(1 - z_t^{-\delta}/\omega_t), \quad (16')$$

$$\dot{w}_t/w_t = \sigma(1 - z_t^{-\delta}/\omega_t). \quad (17')$$

### 3. Government and the central bank

There are no commercial banks in the economy, and the central bank only lends to the government. The nominal money stock is therefore equal to

$$M_t = D_t + E_t R_t, \quad (18)$$

where  $D_t$  denotes the stock of domestic credit allocated by the central bank to the government, and  $R_t$  the stock of net foreign assets, measured in foreign currency terms. The central bank fixes the rate of growth of the nominal credit stock at  $\mu_t \geq 0$ , so that

$$\dot{d}_t = (\mu_t - \pi_t)d_t, \quad (19)$$

where  $d_t$  denotes the real credit stock.

The central bank receives interest on its holdings of foreign assets and its loans to the government. <sup>1/</sup> Real profits of the central bank  $\Omega_t$  are therefore determined by

(cont'd from page 8) and Sutherland (1993).  $w_t$  in equation (17) can be interpreted as the nominal wage stipulated in new contracts as well as those renewed at time  $t$ . Assuming that wage contracts are negotiated directly between producers and individual employees,  $w_t$  can be viewed as measuring the marginal cost of labor and is thus the relevant price that producers incorporate in their price-setting decisions. However, in contrast to Calvo's formulation in which the domestic price level is a weighted average of past contract wages (and is thus predetermined at any moment in time), in the present formulation home goods prices--and therefore the real exchange rate--can jump on impact.

<sup>1/</sup> The interest rate paid by the government on central bank loans is assumed equal to the market rate of interest on domestic bonds. Since only the consolidated budget of the public sector is considered in what follows, this assumption carries little importance.

$$P_t \Omega_t = i_t D_t + E_t(i_t^* + \epsilon_t) R_t, \quad (20)$$

where  $\dot{E}_t R_t$  measures capital gains on reserves.

The government's revenue sources consist of income taxes and lump-sum taxes on households, transfers from the central bank, and revenue from the taxation of foreign assets held by households. It consumes both home and imported goods, and pays interest on its domestic debt. It finances its budget deficit by borrowing from the central bank and/or issuing nontraded bonds. In nominal terms, the flow budget constraint of the government can be written as:

$$\begin{aligned} \dot{B}_t + \dot{D}_t = & P_N(t) g_N + E_t g_I + (1-\iota) i_t B_t - \iota (P_t q_t + i_t^* E_t B_t^*), \\ & + i_t D_t - P_t (\tau_t + \Omega_t + \gamma b_t^{*2}/2), \end{aligned} \quad (21)$$

where  $g_I$  denotes government spending on imports. Using (19), equation (21) can be written in real terms as

$$\begin{aligned} \pi_t m_t + \dot{b}_t + \dot{d}_t = & z_t^{\delta-1} (g_N + z_t g_I) + \rho_t b_t - \iota (q_t + i_t^* b_t^*) \\ & - \tau_t - \gamma b_t^{*2}/2 - i_t^* z_t^{\delta} R_t. \end{aligned} \quad (22)$$

Equation (22) indicates that government spending on domestic and foreign goods plus net interest payments on the domestic debt, minus lump-sum and income taxes, proceeds from the taxation of private foreign assets, and interest income on reserves, must be financed by seignorage revenue, issuance of bonds or an increase in central bank credit. 1/

To close the model requires specifying the equilibrium condition for the money market. Equations (5) and (18) imply

$$d_t + z_t^{\delta} R_t = m[(1-\iota) i_t] c_t, \quad (23)$$

which can be solved for the market-clearing domestic interest rate.

1/ Using equation (22) and setting  $\dot{d}_t = 0$  (see below), the government's intertemporal budget constraint can be written in a form that restricts the present value of government purchases of goods and services to be no greater than revenue from initial holdings of reserves plus the present value of total tax revenue, subject to an appropriate transversality condition on  $b_t$ .

### III. Contract Formation and Dynamic Structure

The analytical framework developed in the previous section can be used to study the effects of a variety of macroeconomic policy shocks. Before doing so, however, it is convenient to re-write the model in a more compact form in order to characterize its dynamic structure.

The definitional equation for  $z_t$  yields

$$\dot{z}_t/z_t = \epsilon_t - \pi_N(t), \quad (24)$$

or, using equations (10), (11) and (12):

$$\dot{z}_t/z_t = \epsilon_t - \dot{w}_t/w_t - \xi \left\{ g_N + \delta z_t^{1-\delta} [1+v()] c_t - \tilde{q}_N \right\}.$$

Substituting equations (16') and (17') in the above expression implies, with backward-looking wage contracts:

$$\dot{z}_t/z_t = \epsilon_t + \sigma(1-z_t^{-\delta}/\omega_t) - \xi \left\{ g_N + \delta z_t^{1-\delta} [1+v()] c_t - \tilde{q}_N \right\}, \quad (25a)$$

and with forward-looking contracts:

$$\dot{z}_t/z_t = \epsilon_t - \sigma(1-z_t^{-\delta}/\omega_t) - \xi \left\{ g_N + \delta z_t^{1-\delta} [1+v()] c_t - \tilde{q}_N \right\}. \quad (25b)$$

Suppose now that the central bank sets the rate of growth of nominal credit so as to compensate the government for the loss in value of the real outstanding stock of credit due to inflation

( $\mu_t = \pi_t$ ). Under this rule,  $\dot{d}_t = 0$ . Assume also that the government foregoes the issuance of bonds to finance its deficit and instead varies lump-sum taxes so as to balance the budget. Equation (22) yields therefore

$$\tau_t = z_t^{\delta-1} (g_N + z_t g_I) - \iota(q_t + i_t^* b_t^*) + \rho_t \tilde{b} - (i_t^* + \epsilon_t) z_t^{\delta} R_t - \pi_t \tilde{d} - \gamma b_t^{*2}/2 \quad (26)$$

where  $\tilde{b}$  and  $\tilde{d}$  denote the constant levels of domestic bonds and credit, which are normalized to zero in what follows. Substituting equations (7), (11), (15) and (26) in (3') yields 1/

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1/ Note that the quadratic term appearing in the private sector budget constraint (3') does not appear in equation (27), since it also appears in equation (26).

$$\dot{R}_t + \dot{B}_t^* = q_E(\omega_t) - c_t - g_I + i_t^*(R_t + B_t^*), \quad (27)$$

which represents the consolidated flow budget constraint of the economy. 1/ From the equilibrium condition of the money market (equation 23), the domestic interest rate is given by

$$i_t = i(m_t, c_t)/(1-\iota). \quad (28)$$

Substituting this result in equation (6) yields

$$c_t = c(\lambda_t, m_t). \quad (29)$$

An increase in the real money stock raises aggregate consumption because the associated expansion in the money supply reduces the domestic interest rate, which stimulates private spending. A rise in the shadow value of wealth lowers directly private expenditure, for a given level of the domestic interest rate.

Substituting equation (29) back in (28) yields

$$i_t = \Phi(\lambda_t, m_t)/(1-\iota), \quad (28')$$

where  $\Phi() = i[m_t, c(\lambda_t, m_t)]$ . In general, the net effect of a change in real money balances on the nominal interest rate is ambiguous. On the one hand, an increase in the real money stock requires a fall in domestic interest rates to maintain equilibrium in the money market. On the other, however, the fall in interest rates tends to reduce transactions costs and the opportunity cost of holding money, thus increasing consumption spending and partly offsetting the initial downward movement. The net effect is here assumed to be negative.

Using (5) and (29), gross private expenditure can be written as

$$[1+v()]c_t = c(\lambda_t, m_t). \quad (29')$$

Substituting equations (8b) and (29') in (27) yields

$$\dot{R}_t + \dot{B}_t^* = q_E(\omega_t) - (1-\delta)z_t^{-\delta}c(\lambda_t, m_t) - g_I + i_t^*(R_t + B_t^*). \quad (30)$$

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1/ Integrating equation (27) yields the economy's intertemporal budget constraint, which requires that the current level of foreign assets be equal to the discounted stream of the excess of domestic absorption of imported goods over future production of exports.



Equations (18), (29') and (30) indicate that the net effect of a real exchange rate depreciation (a rise in  $z_t$ ) on consumption of imported goods is, in general, ambiguous. On the one hand, it reduces spending on imports directly since their relative price rises. On the other, it raises the real money stock (since the domestic-currency value of the stock of foreign reserves increases), thereby reducing the domestic interest rate and stimulating total spending. In what follows, we will assume that the direct effect dominates, so that a depreciation of the real exchange rate reduces purchases of imported goods.

Finally, from the definition of the product wage in the export sector, we have

$$\dot{\omega}_t/\omega_t = \dot{w}_t/w_t - \epsilon_t = \mp \sigma(1-z_t^{-\delta}/\omega_t) - \epsilon_t, \quad (31)$$

where the sign preceding the term in brackets on the right-hand side of the equation is negative if contracts are backward-looking, and positive if contracts are forward-looking.

Equations (4c), (4d), (9c), (18), (25a) or (25b), (28'), (30) and (31) describe the evolution of the economy along any perfect foresight equilibrium path. 1/ In a more compact form, these equations can be written as, assuming a constant foreign interest rate and a constant devaluation rate equal to  $\epsilon$ :

$$z_t^\delta B_t^* = [(1-\iota)i^* - \Phi(\lambda_t, m_t) + \epsilon]/\gamma, \quad (32a)$$

$$\dot{\lambda}_t = [\alpha - \Phi(\lambda_t, m_t) + \pi_t]\lambda_t, \quad (32b)$$

$$\dot{\omega}_t/\omega_t = \mp \sigma(1-z_t^{-\delta}/\omega_t) - \epsilon, \quad (32c)$$

$$\dot{z}_t/z_t = \epsilon \pm \sigma(1-z_t^{-\delta}/\omega_t) - \xi[g_N + \delta z_t^{-\delta} c(\lambda_t, m_t) - \bar{q}_N], \quad (32d)$$

$$\dot{R}_t + \dot{B}_t^* = q_E(\omega_t) - c_I(\lambda_t, m_t, z_t) - g_I + i^*(R_t + B_t^*), \quad (32e)$$

$$\pi_t = \epsilon - \delta \dot{z}_t/z_t, \quad (32f)$$

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1/ Note that in this model, if the mark-up remains constant over time (so that  $\xi = 0$ ), the nominal wage becomes equal to the price of nontraded goods (setting  $\theta = 1$ ) and the product wage in the export sector equal to the inverse of the real exchange rate. One of the dynamic equations driving these variables becomes therefore redundant. In addition, the system becomes recursive: equations (25) could be solved separately for the real exchange rate, which would thus depend only on changes in the nominal devaluation rate.

$$m_t = z_t^\delta R_t, \quad (32g)$$

where  $c_I() \equiv (1-\delta)z_t^{-\delta}c(\lambda_t, m_t)$ , and with equation (26) determining residually lump-sum taxes.

Equations (32) can be further condensed into a dynamic system in four variables: the marginal utility of wealth, the real exchange rate, the product wage in the export sector, and total holdings of foreign bonds measured in foreign currency terms,  $F_t = R_t + b_t^*$  (see Appendix II). With forward-looking contracts, the model has three jump variables ( $\lambda_t$ ,  $z_t$  and  $\omega_t$ ) and one backward-looking variable ( $F_t$ ), hence requiring three unstable roots. Given this condition, and since total holdings of foreign assets evolve continuously from their initial level  $F_0$ , the appropriate initial jump in the real exchange rate and the product wage following any exogenous shock will take place through a jump in the price of nontraded goods and nominal wages respectively, whereas the jump in the marginal value of wealth will occur through an adjustment in private expenditure. With backward-looking contracts, the model has one forward-looking variable,  $\lambda_t$ , and three backward-looking variables,  $z_t$ ,  $F_t$  and  $\omega_t$ , hence requiring one unstable (positive) root and three stable (negative) eigenvalues for the saddlepoint property to be satisfied. Given that holdings of foreign assets, the real exchange rate and the product wage evolve continuously from their initial levels  $F_0$ ,  $z_0$  and  $\omega_0$ , the appropriate initial jump in the marginal value of wealth following an exogenous shock to the economy will also be obtained by a discrete adjustment in private consumption. Appendix II establishes the conditions under which the model is saddlepoint stable under alternative specifications of wage contracts. 1/

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1/ An important characteristic of the present model is that regardless of the nature of wage contracts, both official reserves and the private stock of foreign bonds are allowed to jump in response to policy-induced shifts in domestic interest rates (see Agénor, 1994). An instantaneous shift in private holdings of foreign bonds is accompanied by an offsetting movement (at the given official exchange rate) in the stock of foreign reserves held by the central bank, which leaves the overall stock of foreign assets  $F_t$  constant. Over time, of course, both components of  $F_t$  would in general display transitional dynamics.

#### IV. Steady-State Effects of Policy Shocks

In what follows the model developed in the previous sections is used to examine the macroeconomic effects of two policy measures that have figured prominently in stabilization programs implemented in developing countries, namely a reduction in government spending on home goods and a reduction in the nominal devaluation rate. Because of the highly complex nature of the short-run dynamics of the model described in the the previous sections under both backward- and forward-looking contracts, only the steady-state effects of these shocks will be examined. 1/

In the steady state,  $\dot{\lambda}_t = \dot{z}_t = \dot{\omega}_t = \dot{R}_t = \dot{B}_t^* = 0$ . Equation (32b) therefore implies that the after-tax real interest rate is equal to the rate of time preference:

$$(1-\iota)\tilde{i} - \tilde{\pi} = \alpha, \quad (33a)$$

where a '~' is used to denote steady-state values. The market for nontraded goods must be in equilibrium in the steady state, implying from equations (11) and (32g) that

$$\delta \tilde{z}^{1-\delta} c(\tilde{\lambda}, \tilde{z}^\delta \tilde{R}) = \tilde{q}_N - g_N. \quad (33b)$$

This condition implies, using equation (32d) with  $\dot{z}_t = 0$ :

$$\tilde{z}^{-\delta} / \tilde{\omega} = 1 \pm \epsilon / \sigma. \quad (33c)$$

From equations (24) and (32f), the rate of increase in home goods prices and the overall domestic inflation rate must be equal to the devaluation rate ( $\tilde{\pi}_N = \tilde{\pi} = \epsilon$ ). From equation (10) or (32c), since equilibrium in the nontraded goods market implies that the mark-up rate is constant, the rate of growth of nominal wages must also be equal to the devaluation rate. Substituting equation (33a) and  $\tilde{\pi} = \epsilon$  in (32a) yields

$$\tilde{z}^\delta \tilde{B}^* = [ (1-\iota)\tilde{i}^* - \alpha ] / \gamma, \quad (33d)$$

from which it can be inferred that the steady-state real stock of foreign bonds, measured in foreign currency terms, is invariant to domestic policy shocks other than a change in the tax rate on income

1/ As shown in Appendix II, strong simplifying assumptions may be needed to allow an explicit short-run dynamic analysis; the results would thus depend on the precise assumptions made. A numerical approach would only partly alleviate the problem, since it would remain sensitive to the choice of parameter values for which limited empirical evidence is available.

(see Agénor, 1994).

Equations (32e) and (32g) imply

$$q_E(\tilde{\omega}) - c_I(\tilde{\lambda}, \tilde{z}^\delta \tilde{R}, \tilde{z}) - g_I + i^*(\tilde{R} + \tilde{B}^*) = 0. \quad (33e)$$

Equations (33a)--with  $\tilde{\pi} = \epsilon$ , and  $(1-\iota)\tilde{l}$  replaced by  $\Phi(\tilde{\lambda}, \tilde{z}^\delta \tilde{R})$ --and (33b)-(33e) can be solved simultaneously for the steady-state values of  $\tilde{\lambda}$ ,  $\tilde{R}$ ,  $\tilde{B}^*$ ,  $\tilde{z}$  and  $\tilde{\omega}$ , from which the long-run effects of a reduction in the devaluation rate or in government spending on nontraded goods can be assessed. Tedious but straightforward calculations indicate that, with backward-looking contracts:

$$\begin{array}{ccc} \begin{array}{c} - + \\ \tilde{\lambda} = \lambda(\epsilon, g_N), \end{array} & \begin{array}{c} \pm \pm \\ \tilde{R} = R(\epsilon, g_N), \end{array} & \begin{array}{c} - - \\ \tilde{z} = z(\epsilon, g_N), \end{array} \\ \\ \begin{array}{c} - + \\ \tilde{\omega} = \omega(\epsilon, g_N), \end{array} & \begin{array}{c} + + \\ \tilde{B}^* = B^*(\epsilon, g_N), \end{array} & \begin{array}{c} + 0 \\ \tilde{\pi} = \pi(\epsilon, g_N), \end{array} \end{array}$$

while with forward-looking contracts:

$$\begin{array}{ccc} \begin{array}{c} \pm + \\ \tilde{\lambda} = \lambda(\epsilon, g_N), \end{array} & \begin{array}{c} - \pm \\ \tilde{R} = R(\epsilon, g_N), \end{array} & \begin{array}{c} + - \\ \tilde{z} = z(\epsilon, g_N), \end{array} \\ \\ \begin{array}{c} + + \\ \tilde{\omega} = \omega(\epsilon, g_N), \end{array} & \begin{array}{c} - + \\ \tilde{B}^* = B^*(\epsilon, g_N), \end{array} & \begin{array}{c} + 0 \\ \tilde{\pi} = \pi(\epsilon, g_N). \end{array} \end{array}$$

Consider first a reduction in public spending on home goods. The steady-state effects of this policy shock are the same, regardless of the nature of the expectational mechanism embedded in wage contracts: it lowers the marginal value of wealth (high stimulates private consumption), reduces the product wage and raises output in the export sector while leading to a depreciation of the real exchange rate to maintain equilibrium in the nontraded goods market. The desired stock of foreign bonds (whose foreign currency value is independent of the policy shocks considered here, as indicated earlier) rises as a result of the fall in the relative price of domestic goods. In equilibrium, output of nontraded goods is equal to its capacity level, and is thus independent of changes in government expenditure. The movements in net foreign assets of the central bank is generally ambiguous. On the one hand, higher exports and the real exchange rate depreciation (which dampens demand for imported goods) have a positive effect on reserves. On the other hand, the reduction in the marginal value of wealth raises total consumption expenditure and thus the demand for imports. A sufficient condition for the net effect on reserves to be positive is that the net effect on consumption of imported goods be negative. Since, from equation (33a), the real after-tax interest

rate remains constant (and equal to the rate of time preference), the nominal interest rate must rise in the same proportion as the inflation rate. But since the domestic inflation rate is not affected by the reduction in government spending (it depends only on the devaluation rate), the nominal interest rate must also remain unchanged ( $d\tilde{i}/dg_N = 0$ ). <sup>1/</sup> Thus, the "effective" price of consumption does not change, and private expenditure expands only as a result of the fall in the marginal utility of wealth. Finally, it can be noted that from equations (8) and (33c):

$$\tilde{P}/\tilde{w} = 1 \pm \epsilon/\sigma \Rightarrow d(\tilde{w}/\tilde{P})/dg_N = 0, \quad (34)$$

which indicates that  $\tilde{w}/\tilde{P}$ , the real wage measured in terms of the consumption basket, is not altered by the government spending shock.

Consider now a reduction in the devaluation rate. Regardless of the wage contract mechanism considered, this policy shock leads to a proportional reduction in overall inflation, the rate of inflation in nontradable prices, and nominal wage growth. Since the shock has no effect on the real after-tax interest rate--which remains equal to the rate of time preference--the nominal interest rate (as shown by equation 33a) must fall in the same proportion as the devaluation rate. For all other variables, however, the mechanism through which wage contracts are formed does affect the long-run outcome. With backward-looking contracts, the marginal value of wealth rises--thus dampening consumption and requiring a real exchange rate depreciation (a fall in the relative price of home goods) to maintain equilibrium in the market for nontraded goods. The product wage rises, leading to a fall in output of exportables, and the domestic-currency value of the stock of foreign bonds falls. The net effect on central bank reserves is ambiguous, since both exports and imports (as a result of the increase in the marginal value of wealth and the real depreciation) fall. By contrast, with forward-looking contracts, the net effect of a reduction in the devaluation rate on the marginal value of wealth--and thus total consumption and the demand for imports--is ambiguous. The real exchange rate appreciates--thus raising the real value in domestic currency terms of holdings of foreign bonds--and the

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<sup>1/</sup> Note that using this result and equation (28') yields

$$d\tilde{m}/dg_N = - (\Phi_\lambda/\Phi_m) d\tilde{\lambda}/dg_N,$$

which implies that, since the stock of domestic credit is constant:

$$d\tilde{R}/dg_N = - \tilde{R}d\tilde{z}/dg_N - (\Phi_\lambda/\Phi_m) d\tilde{\lambda}/dg_N \stackrel{>}{<} 0,$$

where  $\tilde{z} = 1$ . This result confirms the ambiguous nature of the effect of a reduction in government spending on reserves.

product wage falls, thus raising output of exportable goods. Here central bank holdings of foreign exchange unambiguously increase. The real consumption wage rises ( $d(\tilde{w}/\tilde{P})/d\epsilon > 0$ ) with backward-looking contracts and falls ( $d(\tilde{w}/\tilde{P})/d\epsilon < 0$ ) under forward-looking contracts.

Intuitively, the reason why the long-run effects of a reduction in government spending on home goods does not depend on the structure of wage contracts is because the shock has no effect on steady-state inflation, which depends only on the devaluation rate. More generally, as can be inferred from equation (34), any shock that leaves the real wage measured in terms of the consumption basket unchanged implies that the structure of wage contracts has no bearing on the direction of the associated effects. By the same token, the opposite effects induced by a reduction on the devaluation rate on the product wage and the real exchange under alternative contract specifications is entirely driven by the direction of the long-run effect on the real consumption wage.

## V. Summary and Conclusions

Imperfect capital mobility and nominal wage rigidity are two important macroeconomic features of many developing countries. The purpose of this paper has been to integrate simultaneously both features in an analysis of the effects of macroeconomic policy shocks on output, real wages, the real exchange rate and the current account in an intertemporal framework. The analysis was based on a two-sector, three-good optimizing model with endogenous mark-up pricing in the nontraded goods sector. Our discussion considered both backward- and forward-looking wage contracts. Because of the highly complex dynamic structure of the model (which was shown to depend crucially on the type of wage contracts considered), the analysis focused only on the steady-state effects of policy shocks.

The long-run macroeconomic effects of a reduction in spending on nontraded goods was shown to be independent of the mechanism through which contracts are formed. It lowers the marginal value of wealth (and thus stimulates consumption), reduces the product wage in the export sector (thereby raising output in that sector), and leads to a depreciation of the real exchange rate. Real holdings of foreign bonds--measured in domestic currency terms--rise, while net foreign assets of the central bank may rise or fall. A cut in public spending has no effect on either inflation and output of nontraded goods, or on nominal and real interest rates. By contrast, the steady-state effects of a reduction in the devaluation rate on some of the variables was shown to depend crucially on whether wage contracts are backward or forward looking. In the case of backward-looking contracts, the real exchange rate depreciates whereas the product wage rises, leading to a reduction in output in the export sector. With forward-looking contracts, the real exchange rate appreciates whereas the real product wage falls, leading to an expansion in output of

exportables. In both cases, however, inflation and the rate of growth of nominal wages fall, while the nominal interest rate falls in the same proportion as the devaluation rate--with no net effect on the real interest rate, which remains tied to the rate of time preference. The divergent nature of these results was shown to depend critically on the behavior of the real wage measured in terms of the consumption basket.

The thrust of the analysis is therefore that whether the nature of wage contracts has implications for the long-run effects of macroeconomic policies--particularly on the real sector of the economy--depends on the type of shocks considered. The real effects associated with changes in some macroeconomic policy instruments may be largely independent of the mechanism through which contracts are formed, whereas other types of stabilization policies may have a substantial impact on the behavior of output, real wages, and the real exchange rate. Understanding the mechanisms through which wage contracts are formed is thus important for the choice of policy instruments in stabilization programs.

Alternative Derivation of the Mark-up Rule

This Appendix provides a more explicit derivation of the mark-up pricing decision, but with a resulting mark-up rule that shares the same properties as equation (12). 1/ Suppose that production in the nontraded goods sector takes place under decreasing returns to labor, so that  $q_N = q_N(L_N)$ , with  $q'_N > 0$  and  $q''_N < 0$ . The condition that firms will hire labor up to the point where the value of the marginal product of labor equals the product wage is given by

$$q'_N[L_N^d(t)] = w_t/P_N(t) = \omega_t z_t, \quad (A1)$$

where all variables are as defined in the text. This marginal productivity condition does not necessarily hold given our assumptions that both prices and the nominal wage are predetermined, and that output is demand determined: the effective level of demand for nontraded goods may either exceed or fall short of the level of output at which, given  $\omega_t$  and  $z_t$ , the value of the marginal product equals the real wage.

Suppose that firms will produce the effective demand level of output even if they incur a short-run loss at the margin by meeting that demand, that is, even if  $q'_N[\cdot] < \omega_t z_t$ . Let  $\mu_t$  denote the excess of the product wage over the value of the marginal product of labor in the nontraded goods sector, that is

$$\mu_t = \omega_t z_t - q'_N[L_N^d(t)]. \quad (A2)$$

$\mu_t$  is a measure of disequilibrium in the nontraded goods market which determines the rate at which producers should adjust their prices. When  $\mu_t < 0$  for instance, firms would, at existing output prices, like to hire more labor and produce more output, if only they could sell the additional output at the going market price. When firms are thus quantity-constrained in the market for nontraded goods, they will reduce the rate of increase of their prices. Analogously,  $\mu_t > 0$  will act as a signal for firms to raise the rate at which they increase the price of their output. Thus, the rate of change of the mark-up rate can be written as a linear function of  $\mu_t$ :

$$\dot{\theta}_t/\theta_t = \xi \mu_t, \quad \xi > 0 \quad (A3)$$

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1/ The derivation below is based in part on Buiter (1978).



APPENDIX I

From the equilibrium condition of the nontraded goods market (equation 11) and the production function defined above, we have 1/

$$L_N^d = q_N^{-1} [\delta z_t^{1-\delta} c(\lambda_t, m_t) + g_N] = L_N^d(z_t, \lambda_t, m_t; g_N), \quad (A4)$$

which implies that, using (A2) and (A3) and noting that  $q_N' < 0$ :

$$\dot{\theta}_t / \theta_t = \xi \mu(z_t, \lambda_t, m_t, \omega_t; g_N). \quad (A5)$$

Finally, equations (11), (12) and (29') in the text yield

$$\dot{\theta}_t / \theta_t = \xi [\delta z_t^{1-\delta} c(\lambda_t, m_t) + g_N - \tilde{q}_N], \quad (A6)$$

which can be shown to possess the same properties as (A5) with respect to changes in all variables--except for  $\omega_t$  which does not appear in (A6). But since  $\xi$  is taken to be relatively small here, the presence of  $\omega_t$  in (A6) would not alter the dynamic equation of the the real exchange rate, as implied by equation (A3) in Appendix II.

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1/ The effective supply of labor in the nontraded goods sector is assumed to be not less than the amount of labor required to produce the effective demand level of output. Put differently, firms are assumed to face no constraint on the labor market.

Condensed Dynamic Form and Stability Conditions

To determine the condensed dynamic form of the model, we begin by eliminating the stock of foreign bonds, given by equation (32a), using the stock budget constraint. Since the constant stocks of credit and holdings of domestic bonds are set equal to zero, the budget constraint of the representative household (equation 2') together with equations (8) and (18) imply that  $m_t = z_t^\delta R_t = a_t - z_t^\delta B_t^*$ . Substituting this result in (32a) and taking a linear approximation to  $\Phi()$  yields

$$z_t^\delta B_t^* = [(1-\iota)i^* + \epsilon - \Phi_m a_t - \Phi_\lambda \lambda_t]/(\gamma - \Phi_m) = H(\lambda_t, a_t; \epsilon), \quad (A1)$$

where  $H_a = -\Phi_m/(\gamma - \Phi_m) < 1$ . This result implies that

$$m_t = a_t - z_t^\delta B_t^* = a_t - H(\lambda_t, a_t; \epsilon) = h(\lambda_t, a_t; \epsilon), \quad (A2)$$

where  $h_a = 1 - H_a < 1$ . Consider for the moment the case where contracts are forward-looking. Substituting equation (A2) in (32d) yields

$$\dot{z}_t/z_t = \epsilon - \sigma(1 - z_t^{-\delta}/\omega_t) - \xi \left\{ g_N + \delta z_t^{-\delta} c[\lambda_t, h(\lambda_t, a_t; \epsilon)] - \tilde{q}_N \right\},$$

or equivalently

$$\dot{z}_t/z_t = \kappa(a_t, z_t, \lambda_t, \omega_t; \epsilon, g_N). \quad (A3)$$

The sign of the partial derivative of  $\kappa()$  with respect to  $z_t$  is in general ambiguous. Assuming that  $\xi$  (the speed of adjustment of the mark-up rate to excess demand for home goods) is relatively small ensures that  $\kappa_z < 0$ .

Since  $a_t = z_t^\delta (R_t + B_t^*)$  we have, using equation (9c):

$$\dot{a}_t = z_t^\delta (\dot{R}_t + \dot{B}_t^*) + (\epsilon - \pi_t) a_t.$$

If  $z_t$  is a jump variable,  $a_t$  will also be a jump variable. To avoid this complication, let  $F_t = R_t + B_t^*$  denote the real value (in foreign currency terms) of private financial wealth. Consequently,

APPENDIX II

$a_t = z_t^\delta F_t$  and using equations (32e) and (34):

$$\dot{F}_t = i^* F_t + q_E(\omega_t) - C_I(\lambda_t, F_t, z_t; \epsilon) - g_I, \quad (A4)$$

where  $C_I() = c_I[\lambda_t, h(\lambda_t, z_t^\delta F_t; \epsilon), z_t]$ .

Finally, substituting equation (A3) in equation (32b) yields, together with equations (32c), (A3) and (A4), a dynamic system in the marginal utility of wealth  $\lambda_t$ , the product wage in the export sector  $\omega_t$ , the real exchange rate  $z_t$ , and private wealth measured in foreign currency terms  $F_t$ , which can be linearized around the initial steady state to give

$$\begin{bmatrix} \dot{\lambda}_t \\ \dot{z}_t \\ \dot{F}_t \\ \dot{\omega}_t \end{bmatrix} = \begin{bmatrix} \Lambda_\lambda & \Lambda_z & \Lambda_F & 0 \\ \kappa_\lambda & \kappa'_z & \kappa_a & \kappa_\omega \\ F_\lambda & F_z & F_F & q'_E \\ 0 & w_z & 0 & w_\omega \end{bmatrix} \begin{bmatrix} \lambda_t - \bar{\lambda} \\ z_t - \bar{z} \\ F_t - \bar{F} \\ \omega_t - \bar{\omega} \end{bmatrix}, \quad (A5)$$

where, assuming that the initial steady-state values of the real exchange rate and the product wage are normalized to unity:

$$\begin{aligned} \Lambda_\lambda &= -\bar{\lambda}(\Phi_\lambda + \Phi_m h_\lambda) > 0, & \Lambda_z &= -\bar{\lambda} \delta \Phi_m h_a \bar{F} > 0, & \Lambda_F &= -\bar{\lambda} \Phi_m h_a > 0, \\ \kappa'_z &= \delta \kappa_a \bar{F} + \kappa_z < 0, & F_\lambda &= -C_{I\lambda} > 0, & F_z &= -\delta C_{Ia} \bar{F} - C_{Iz} \stackrel{?}{<} 0, \\ F_F &= i^* - C_{Ia} \stackrel{?}{>} 0, & w_z &= \sigma \delta > 0, & w_\omega &= -\kappa_\omega = \sigma > 0, \end{aligned}$$

with  $w_z + \kappa'_z < 0$ . Assuming that the world interest rate is small implies  $F_F < 0$ . As argued earlier, the direct relative price effect on the demand for imported goods will be assumed to dominate the indirect interest rate effect, so that  $F_z > 0$ .

With forward-looking contracts, the model has three jump variables ( $\lambda_t$ ,  $z_t$  and  $\omega_t$ ) and one backward-looking variable ( $F_t$ ), hence requiring three unstable (positive) eigenvalues and one stable (negative) eigenvalue for the saddlepoint property to be satisfied. Conditions under which the model is saddlepoint stable are given below.

APPENDIX II

The general solution of the system can be written as

$$\begin{bmatrix} \lambda_t - \tilde{\lambda} \\ z_t - \tilde{z} \\ F_t - \tilde{F} \\ \omega_t - \tilde{\omega} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{bmatrix} \begin{bmatrix} C_1 \exp(\mu_1 t) \\ C_2 \exp(\mu_2 t) \\ C_3 \exp(\mu_3 t) \\ C_4 \exp(\mu_4 t) \end{bmatrix}, \quad (A6)$$

where  $\mu_k$ ,  $k=1,2,3,4$  denote the eigenvalues of the system. The elements  $h_{jk}$  of each eigenvector can be solved from the system of linearly dependent equations

$$\begin{bmatrix} \Lambda_\lambda - \mu_k & \Lambda_z & \Lambda_F & 0 \\ \kappa_\lambda & \kappa'_z - \mu_k & \kappa_a & \kappa_\omega \\ F_\lambda & F_z & F_F - \mu_k & q'_E \\ 0 & w_z & 0 & w_\omega - \mu_k \end{bmatrix} \begin{bmatrix} h_{1k} \\ h_{2k} \\ h_{3k} \\ h_{4k} \end{bmatrix} = 0, \quad (A7)$$

for each of the four eigenvalues. 1/ Let  $\mu_2$ ,  $\mu_3$  and  $\mu_4$  denote the positive eigenvalues. To ensure saddlepath stability requires setting  $C_2 = C_3 = C_4 = 0$  in equations (A6).

Under backward-looking contracts, the dynamic system can be written in a way similar to (A5), with now  $w_z < 0$ ,  $w_\omega < 0$ ,  $\kappa_\omega > 0$ ,  $\Lambda_z < 0$ , and--assuming  $\xi$  is small-- $\kappa'_z > 0$ . The model has one forward-looking variable,  $\lambda_t$ , and three backward-looking variables,  $z_t$ ,  $F_t$  and  $\omega_t$ , hence requiring one unstable (positive) root and three stable (negative) eigenvalues for the saddlepoint property to be satisfied.

To determine the conditions under which the model is saddlepoint stable, consider first the case of forward-looking contracts, that is, system (A5). The trace of the coefficient matrix  $M$  is equal to

1/ Note that the components of each eigenvector are all different from zero since, by definition, an eigenvector cannot be a zero vector. For instance, if  $h_{2k} = 0$ , the last equation of (A7) yields  $(w_\omega - \mu_k)h_{4k} = 0$ , from which it follows that  $h_{4k} = 0$ .

APPENDIX II

$$trM = \Lambda_\lambda + \kappa_Z + F_F + w_\omega \stackrel{>}{<} 0, \quad (A8)$$

while its determinant is given by

$$\begin{aligned} detM = w_Z \left\{ q'_E (\Lambda_\lambda \kappa_a - \kappa_\lambda \Lambda_F) - \kappa_\omega (\Lambda_\lambda F_F - F_\lambda \Lambda_F) \right\} \\ + w_\omega \left\{ \Lambda_\lambda (\kappa'_Z F_F - F_Z \kappa_a) - \Lambda_Z (\kappa_\lambda F_F - F_\lambda \kappa_a) + \Lambda_F (\kappa_\lambda F_Z - F_\lambda \kappa'_Z) \right\}. \end{aligned}$$

A necessary condition for the number of positive roots to be equal to three is that  $detM$  must be negative. Direct inspection of the above expression indicates that this requires in turn the coefficient  $\xi$  to be sufficiently high. 1/ To rule out the case in which  $detM < 0$  implies the existence of three negative roots, write the characteristic polynomial of the coefficient matrix as

$$\Pi(\mu) = k_4 \mu^4 + k_3 \mu^3 + k_2 \mu^2 + k_1 \mu + k_0, \quad (A9)$$

where

$$k_0 = detM,$$

$$k_1 = - (\Lambda_\lambda + F_F) (w_\omega \kappa'_Z - w_Z \kappa_\omega) - (\kappa'_Z + w_\omega) (\Lambda_\lambda F_F - \Lambda_F F_\lambda),$$

$$k_2 = (\Lambda_\lambda + F_F) (\kappa'_Z + w_\omega) + (\Lambda_\lambda F_F - \Lambda_F F_\lambda) + (w_\omega \kappa'_Z - w_Z \kappa_\omega),$$

$$k_3 = - trM,$$

$$k_4 = 1.$$

By definition,  $k_0$  is equal to the product of all the roots (the determinant of the matrix of coefficients),  $k_1$  is equal to the sum of all products of three of the roots,  $k_2$  is equal to the sum of all products of two of the roots, and  $k_3$  is equal to minus the sum of all

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1/ If  $\xi \rightarrow 0$ , so are  $\kappa_\lambda, \kappa_a$ . Thus  $detM \rightarrow (w_\omega \kappa'_Z - w_Z \kappa_\omega) (\Lambda_\lambda A_a - \Lambda_a A_\lambda)$ . It is readily verified that  $(\Lambda_\lambda A_a - \Lambda_a A_\lambda) < 0$ . Since  $w_\omega = -\kappa_\omega$  and  $w_Z + \kappa'_Z < 0$ , the first term in brackets is also negative, implying that  $detM$  is always positive if  $\xi \rightarrow 0$ .

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products of one of the roots (or minus the trace). By Descartes' rule of signs, to obtain three positive roots, given that  $k_0 < 0$  and  $k_4 > 0$ , requires either one of the following conditions to be satisfied:  $k_1 > 0$ ,  $k_2 > 0$ , and  $k_3 < 0$ ;  $k_1 > 0$ ,  $k_2 < 0$ , and  $k_3 > 0$ ; or  $k_1 > 0$ ,  $k_2 < 0$ , and  $k_3 < 0$ .

Using the initial condition on  $F_0$  in (A5) yields

$$C_1 = (h_{31})^{-1}(F_0 - \tilde{F}).$$

The solutions can be written as

$$\begin{aligned} a_t &= \tilde{a} + (F_0 - \tilde{F})(h_{31})^{-1} \exp(\mu_1 t), & \lambda_t &= \tilde{\lambda} + h_{11}(F_t - \tilde{F}), \\ z_t &= \tilde{z} + h_{21}(F_t - \tilde{F}), & \omega_t &= \tilde{\omega} + h_{41}(F_t - \tilde{F}), \end{aligned}$$

from which the impact effects of a disturbance at time  $t = 0$  to variable  $x$  on, say, the real exchange rate is given by

$$\frac{dz_0^+}{dx} = \frac{d\tilde{z}}{dx} - h_{21}(h_{31})^{-1} \left( \frac{d\tilde{F}}{dx} \right).$$

Using the normalization rule  $h_{31} = 0$ , the coefficients  $h_{11}$ ,  $h_{21}$  and  $h_{41}$  are determined from (A6):

$$\begin{bmatrix} \kappa_\lambda & \kappa'_z - \mu_1 & k_\omega \\ F_\lambda & F_z & q'_E \\ 0 & w_z & w_\omega - \mu_1 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{41} \end{bmatrix} = \begin{bmatrix} -\kappa_a \\ \mu_1 - F_F \\ 0 \end{bmatrix},$$

which gives

$$h_{11} = \Delta^{-1} \left\{ -\kappa_a [F_z (w_\omega - \mu_1) - q'_E w_z] - (\mu_1 - F_F) [(\kappa'_z - \mu_1)(w_\omega - \mu_1) - w_z \kappa_\omega] \right\},$$

$$h_{21} = \Delta^{-1} (w_\omega - \mu_1) [\kappa_\lambda (\mu_1 - F_F) + \kappa_a F_\lambda],$$

$$h_{41} = -\Delta^{-1} w_z [\kappa_\lambda (\mu_1 - F_F) + \kappa_a F_\lambda] = -w_z h_{21} / (w_\omega - \mu_1),$$

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$$\Delta = -w_z[(\kappa'_z - \mu_1)q'_E - F_z \kappa_\omega] + (w_\omega - \mu_1)[\kappa_\lambda F_z - F_\lambda(\kappa'_z - \mu_1)].$$

It is readily verified that these coefficients are in general indeterminate.

Consider now the case of backward-looking contracts. The trace of the coefficient matrix remains ambiguous. To ensure that the number of positive roots is either one or three,  $\det M$  must be negative. In this case, it can be verified that if  $\xi$  is small,  $\det M$  is always negative since  $w_\omega = -\kappa_\omega$  and  $w_z + \kappa_z < 0$ . To rule out the case in which there exists only one negative root, we can again use Descartes' rule. Given an equation similar to (A9), by Descartes' rule of signs, to obtain three negative roots, given that  $k_0 < 0$  and  $k_4 > 0$ , requires now that  $k_1$ ,  $k_2$  and  $k_3$  be all positive or all negative. 1/

Using the initial conditions to solve for  $C_1$ ,  $C_2$  and  $C_3$  in the system (A6) yields

$$C_1 = [(z_0 - \tilde{z})(h_{32}h_{43} - h_{33}h_{42}) - (F_0 - \tilde{F})(h_{22}h_{43} - h_{23}h_{42}) + (\omega_0 - \tilde{\omega})(h_{22}h_{33} - h_{23}h_{32})]/\Omega, \quad (A10a)$$

$$C_2 = [- (z_0 - \tilde{z})(h_{31}h_{43} - h_{33}h_{41}) + (F_0 - \tilde{F})(h_{21}h_{43} - h_{23}h_{41}) - (\omega_0 - \tilde{\omega})(h_{21}h_{33} - h_{23}h_{31})]/\Omega, \quad (A10b)$$

$$C_3 = [(z_0 - \tilde{z})(h_{31}h_{42} - h_{32}h_{41}) - (F_0 - \tilde{F})(h_{21}h_{42} - h_{22}h_{41}) + (\omega_0 - \tilde{\omega})(h_{21}h_{32} - h_{22}h_{31})]/\Omega, \quad (A10c)$$

where

$$\Omega = h_{21}(h_{32}h_{43} - h_{33}h_{42}) - h_{22}(h_{31}h_{43} - h_{33}h_{41}) + h_{23}(h_{31}h_{42} - h_{32}h_{41}).$$

The coefficients  $h_{jk}$  can be solved for as before, given an appropriate normalization rule. In general, however, these solutions are very complex.

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1/ The negative roots can be either all real, or two of them can be a pair of complex conjugates with negative real parts.

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