IMF WORKING PAPER

WP/95/14

INTERNATIONAL MONETARY FUND

Research Department

Long-Run Exchange Rate Modeling: A Survey of the Recent Evidence

Prepared by Ronald MacDonald 1/

Authorized for distribution by Peter B. Clark

January 1995

Abstract

In this paper we survey the recent literature on long run, or equilibrium, exchange rate modeling. In particular, we review the voluminous literature which tests for a unit root in real exchange rates and the closely related work on testing for a unit root in the residual from a regression of the nominal exchange rate on relative prices. We argue that the balance of evidence is supportive of the existence of some form of long-run exchange rate relationship. The form of this relationship, however, does not accord exactly with a traditional representation of the long-run exchange rate. We offer some potential explanations for this lack of conformity.

JEL Classification Number:

F31

1/ This paper was begun when I was a visiting scholar in the Research Department. I am grateful to Peter Clark for his very helpful comments on an earlier draft of the paper.

Contents

		Page
Summary		iii
I.	Introduction	1
II.	Absolute and Relative Purchasing Power Parity	2
III.	The Balance of Payments and Purchasing Power Parity	5
	1. Absolute PPP and the current account	7
	2. The capital account of the balance of payments and efficient markets PPP	8
	3. The balance of payments and the specification of the long-run exchange rate equation	10
IV.	Modeling the Long-Run Nominal Exchange Rate	11
	1. Absolute purchasing power parity and cointegration	11
	2. The nominal exchange rate and relative excess money supplies	19
V.	Modeling Long-Run Real Exchange Rates: Efficient Markets and the Random Walk Real Exchange Rate Model	21
	1. The span of the data--low frequency and cross sectional aspects	24
	2. The power of unit root tests	26
	3. The real exchange rate and cointegration	32
	4. The real interest rate/exchange rate link--some further evidence	33
VI.	Concluding Comments	34
Text Tables		
	1. Engle-Granger Two-Step Cointegration Tests	14
	2. Johansen Multivariate Cointegration Tests	17
	3. Tests for a Unit Root in the Real Exchange Rate Series	23
	4. Variance Ratio Statistics - Consumer Prices	29
	5. Variance Ratio Statistics - Wholesale Prices	30
Charts		
	1a. Canadian Dollar - U.S. Dollar	4a
	1b. French Franc - U.S. Dollar	4a
	1c. German Mark - U.S. Dollar	4b
	1d. Italian Lira - U.S. Dollar	4b
	1e. Japanese Yen - U.S. Dollar	4c
	1f. Netherlands Guilder - U.S. Dollar	4c
	1g. Swedish Krona - U.S. Dollar	4d

Contents Page

1h. Swiss Franc - U.S. Dollar 4d
1i. Pound Sterling - U.S. Dollar 4e

References 36

Summary

Recently there has been a revival of interest in the determinants of long-run exchange rates. This interest has been generated in large part by developments in the time-series literature, particularly those relating to unit root and cointegration testing. The form of the long-run exchange rate that has received most attention is based on the doctrine of purchasing power parity (PPP). This paper presents an overview of the large number of contemporary tests of PPP.

Recent tests of PPP have been conducted in one of two ways. One approach involves examining whether nominal exchange rates are cointegrated with relative prices, while the other (which is complementary) seeks to determine if real exchange rates contain a unit root. This paper demonstrates that each of these approaches may be derived from a particular account of the balance of payments: generally speaking, the former kind of test stems from current account transactions, while the latter emanates from the capital account. Focusing on one or the other account of the balance of payments, however, may result in a misspecified relationship, especially when a researcher is using data from the recent floating experience. It is suggested that when data for this period are being used to test PPP, it would be better to consider the total balance of payments.

The paper identifies a general trend in recent empirical work on long-run exchange rate modeling, which is that PPP does seem to have some long-run validity. In particular, many currencies are found to have a unique cointegrating relationship between an exchange rate and relative prices, and real exchange rates display mean reverting behavior (two pieces of evidence that are complementary and supportive of a traditional form of PPP). Using a new data base and a variety of estimation techniques, this paper confirms these findings. However, the form of the long-run exchange rate relationship unearthed by recent work does not conform exactly to what many would understand as "traditional PPP." Specifically, there appear to be extremely long-lived deviations from PPP, and the restrictions of symmetry and homogeneity of degree one often associated with PPP are usually rejected. The paper offers some explanations for the apparent discrepancy between the empirical and traditional versions of PPP.

I. Introduction

In this paper we survey the recent literature on long-run, or equilibrium, exchange rate modeling. 1/ Although, of course, interest in the determinants of long-run exchange rates is not new, the topic has been revived by recent developments in the time series literature, particularly those relating to cointegration and unit root testing. The tenor of the conclusions contained in this paper may be summarized in the following way. Ten years or so ago, the consensus view in the economics profession was that, from an empirical perspective, the long-run exchange rate was not well defined. 2/ Today, the evidence summarized in this paper would lead one to the opposite conclusion. There is now overwhelming evidence to indicate that economists can say something positive about long-run exchange rates. This is clearly a relief. Such long-run relationships relate either directly or indirectly (in the case of the monetary model) to some form of absolute purchasing power parity (PPP): the hypothesis that a long-run exchange rate is determined by domestic prices relative to foreign prices. The existence of empirically verifiable long-run exchange rate relationships provides a firm foundation on which to build what may be referred to as medium-run exchange rate models; that is, exchange rate models which capture exchange rate movements over the economic cycle. Recently such modeling has also gained a new lease of life, and we briefly note some of the relevant papers in our concluding section.

The paper has four main components. In Section 2, we present an overview of the concepts of absolute and relative PPP and some general issues concerning their validity. The second component, contained in Section 3, relates the concepts of absolute and efficient markets PPP (EMPPP) to a standard balance of payments equilibrium condition. Although this condition has become an unfashionable framework of late, we, nevertheless, believe it is insightful in the current context. Thus, it may be used to demonstrate the fact that absolute PPP emanates from goods arbitrage on the current account, and the strict conditions necessary for this to hold, while EMPPP is generated from arbitrage on the capital account. The use of the balance of payments condition is also helpful in distinguishing between the concepts of statistical and 'true' economic equilibria (a distinction which is important when trying to interpret recent empirical work). The third part of the paper, contained in Section 4, details recent empirical work on absolute PPP using cointegration methods; we demonstrate that there is now considerable support for some form of long-run PPP. The fourth main component of the paper concerns the empirical evidence on the efficient markets view of PPP and, in particular, the time

---

1/ There are a number of surveys of the earlier literature on long-run modeling--see, for example, Frenkel (1981), Mussa (1979), and MacDonald (1988). Froot and Rogoff (1994) also provide a survey of some of the earlier literature and, additionally, some of the more recent material.

2/ See, for instance, the discussion in Frenkel (1981) and Krugman (1978).

series properties of the real exchange rate; this evidence is discussed in Section 5. We demonstrate in Section 5 that the real exchange rate is a mean-reverting series and this may be interpreted as further evidence in favor of absolute PPP, but unfavorable to efficient markets PPP. Also considered in Section 5 are a number of papers which seek to model the real exchange rate. The paper closes with some concluding comments, which incorporates a discussion of how the findings of this paper relate to the rapidly growing literature on dynamic exchange rate modeling. In particular, this involves a discussion of how an exchange rate moves from a position of short-run dynamic disequilibrium to the kinds of long-run equilibrium discussed in this paper.

II. Absolute and Relative Purchasing Power Parity

The condition of absolute PPP is usually derived in a two country setting in which the home and foreign country each produce a range of homogeneous tradeable goods (by which we mean a U.S. produced refrigerator is identical to an Italian produced refrigerator), the 'law of one price' holding for each of the goods:

$$p_t^i = S_t p_t^{i*}, \tag{1}$$

where p_t^i denotes the domestic price of good i, S_t denotes the home currency price of a unit of foreign currency and an asterisk denotes a foreign magnitude. Condition (1) is maintained by arbitrage. Thus, if for some reason the left hand side in (1) is greater than the right hand side, it would be profitable to ship the good from the foreign country to the domestic country thereby forcing the domestic currency value of the foreign good up (by a rise in S_t and/or p_t^{i*}) and the domestic price of the good down, until equality between the two prices is restored. By summing the prices of all of the traded goods in each country, and giving each price the same weight in the sum, we obtain the condition of absolute PPP.

$$S_t = P_t/P_t^*, \tag{2}$$

where $P_t=\sum_{i=1}^{n}\alpha^i p_t^i$, $P_t^*=\sum_{i=1}^{n}\alpha^i p_t^{i*}$ and α denotes a weight. An increase in the domestic price level, generated, say, by a monetary expansion should result in an equiproportionate depreciation of the exchange rate. The restrictiveness of the absolute PPP hypothesis is clear: even if it were possible to contruct prices in the manner suggested by (2) (we return to this point below) the existence of transportation costs and other impediments to trade, such as tariffs and quotas, will prevent (1) or (2) from holding exactly. However, if such factors are assumed constant over

time, then either (1) or (2) would be expected to hold up to a constant factor Π. 1/2/

$$S_t = \Pi P_t/P^*_t, \tag{2'}$$

or in logs

$$s_t = \pi + p_t - p^*_t, \tag{3}$$

where lower case letters now indicate that the level of the variable has been transformed using the natural logarithm operator. On expressing the terms in (3) in changes we may obtain a weaker version of PPP, which is usually labeled relative PPP

$$\Delta s_t = \Delta p_t - \Delta p^*_t, \tag{4}$$

which states that the percentage exchange rate depreciation is equal to the difference between domestic and foreign inflation. Note that (4) may be rearranged to produce an expression for the change in the real exchange rate: conditional on relative PPP holding, the real exchange rate change should equal zero.

Often a proponent of PPP is understood as someone who believes that expressions like (2) and (4) hold continuously and at all times. However, it is worth remarking at this stage that it is clear from the writings of Cassel, and other prominent proponents of PPP, that the concept is often taken to be one to which an exchange rate gravitates. Thus there are seen to be an array of factors, such as central bank intervention and long and short-term capital flows (see Officer (1976) for a review of these factors) which keep the actual exchange rate away from its PPP determined rate. Eventually, however, the exchange rate should move in line with its PPP rate. To use a time series expression, discussed in some detail later, the real exchange rate under this view is mean-reverting. We label this view of PPP the 'Cassellian' view. The distinction will be useful below when we come to discuss the efficient markets approach to PPP.

The last point also raises the issue of causation. In both of the concepts of PPP discussed above, causation supposedly runs from prices to the exchange rate. However, in circumstances where we have short-run real exchange rate changes it is possible for causation to run in the opposite direction. For example, consider the situation where from a position of absolute PPP holding there is a one-shot capital outflow from the domestic country, thereby depreciating the nominal rate and, with sticky prices in the short run, the real rate. In the Cassellian view this change in the

---

1/ This, of course, still relies on the efficient functioning of goods markets.

2/ The factor Π could also incorporate constant differences in weights across countries.

real rate should be corrected, but how? It is clearly possible that this adjustment takes place, at least in part if not wholly, by prices reacting to the initial exchange rate change--indeed this is essentially the story portrayed in the seminal article of Dornbusch (1976). This reverse causation, and joint endogeneity of exchange rates and prices, is especially likely to be a feature of actual data from the recent floating period.

In attempting to test either absolute or relative PPP, a researcher is immediately confronted with the issue of the appropriate price series to use. If one could construct price series consisting of the prices of homogeneous internationally-traded goods, testing PPP would be relatively clean and straightforward. However, in practice such price measures are not available and researchers usually use consumer or wholesale price series. 1/ Since both of these measures incorporate prices of nontraded goods, it is unlikely that their use in an empirical test would produce the symmetry and homogeneity implied by (1) and (4), although these conditions are perhaps more likely to hold for tests constructed using wholesale prices, a series which contains a relatively large traded goods element. There are a number of other well-known problems which occur in trying to test PPP, and we discuss these in more detail in the next section.

It is worth noting that even if there are substantial nontraded elements in the price series used in an empirical test, relative PPP may still hold if the overall prices are homogeneous of degree one in monetary impulses. Thus, the so-called homogeneity postulate suggest that an increase in the money supply should leave equilibrium relative prices unchanged and should increase all prices by the same amount (this is discussed in more detail in Section 4.2 below).

Throughout the paper we illustrate some of the key tests that have been employed in the recent literature using a standardized data base. In common with other researchers we use a data set consisting of bilateral U.S. exchange rates and relative consumer and wholesale prices for nine currencies, over the period March 1973 to December 1992. 2/ Given the points made above we must sound a cautionary note regarding the usefulness of such data in testing PPP. However, we believe that such tests are useful for illustrative purposes. Further, implementing a selection of tests on a unified data set should help to clarify whether or not the range of results reported in the literature, using different sample periods and different estimation methods, are sample-specific or a function of the estimator used.

In Charts 1a-1i, we present plots of our group of exchange rates and relative domestic-U.S. consumer prices (the wholesale price plots are similar and are therefore not reported). For most of the currencies we note

---

1/ Some empirical work has been conducted on the law of one price relationship and this is discussed below.

2/ The data is that used in MacDonald and Moore (1994), the majority of which has been extracted from the IMF's International Financial Statistics.

Chart 1a.
Canadian Dollar - U.S. Dollar

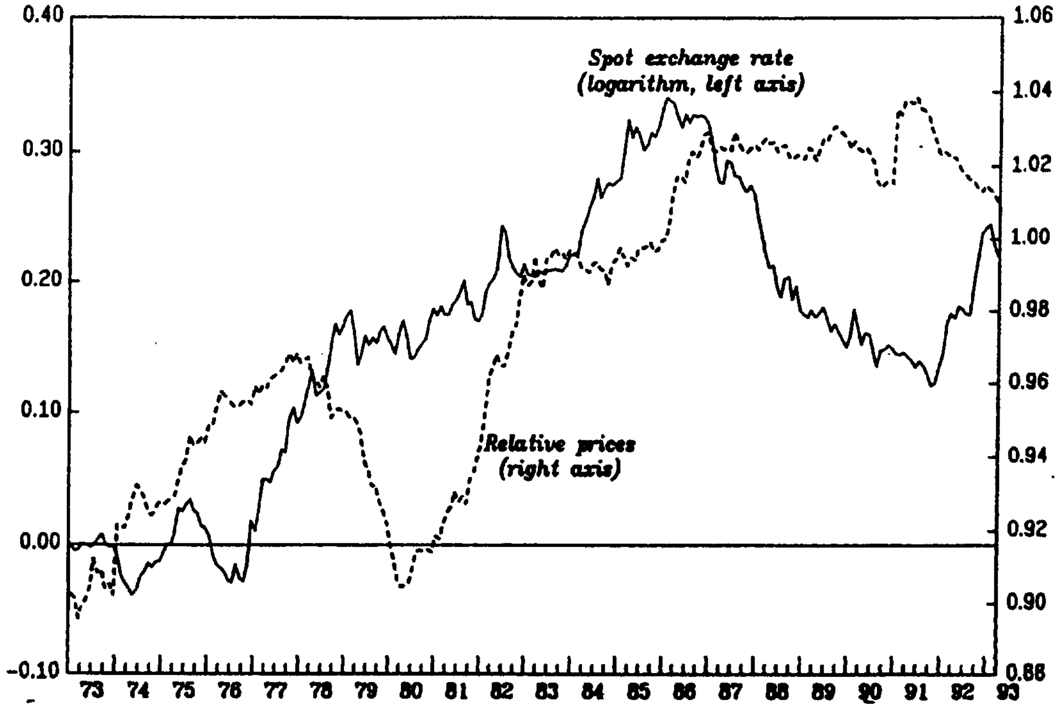


Chart 1b.
French Franc - U.S. Dollar

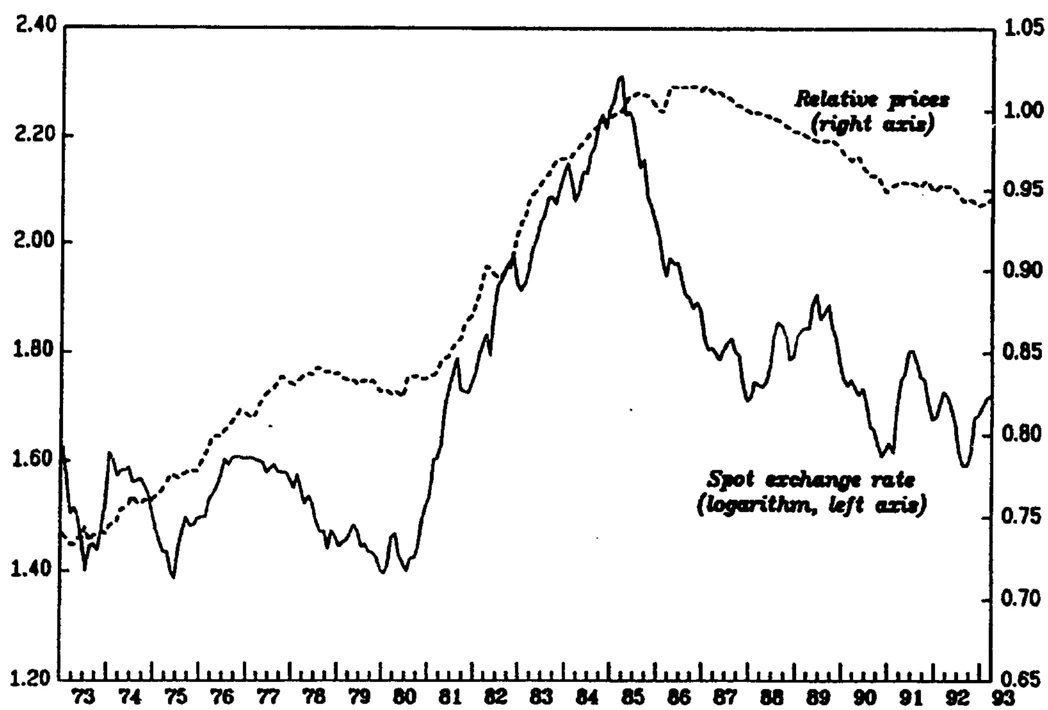


Chart 1c.
German Mark - U.S. Dollar

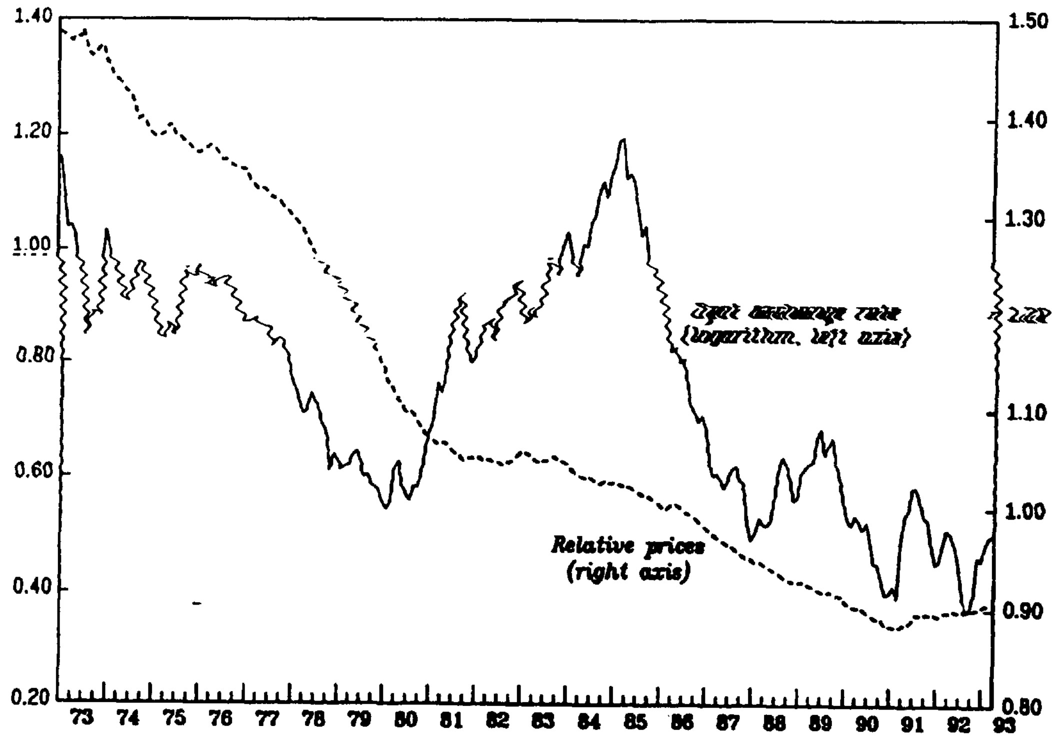


Chart 1d.
Italian Lira - U.S. Dollar

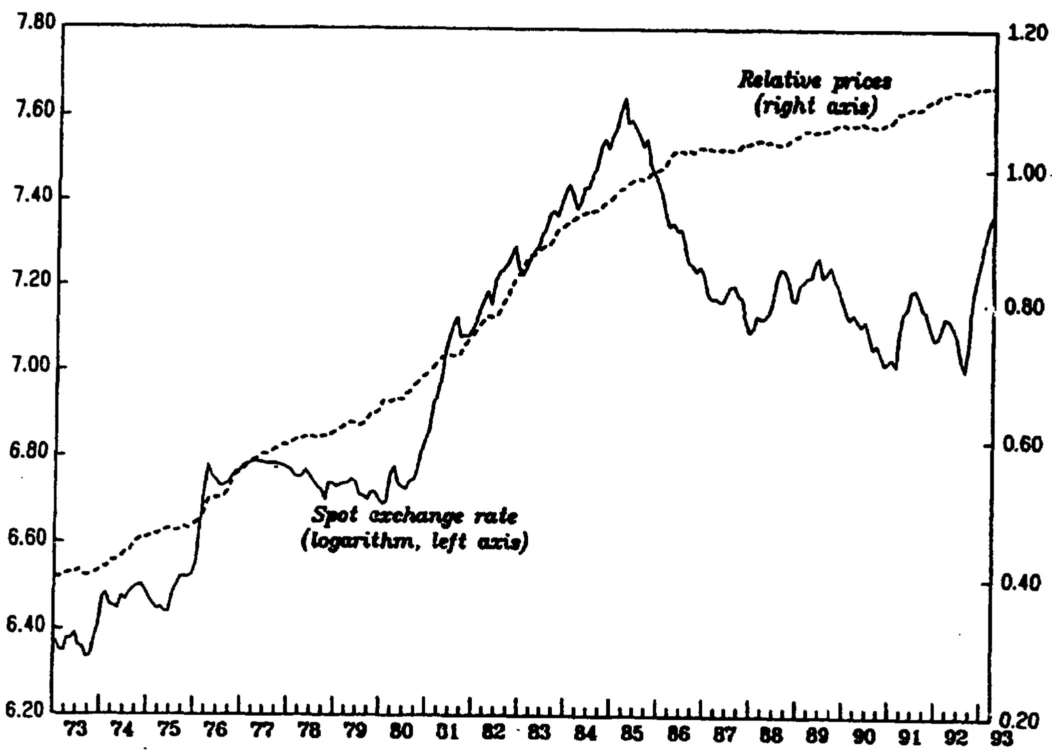


Chart 1e.
Japanese Yen - U.S. Dollar

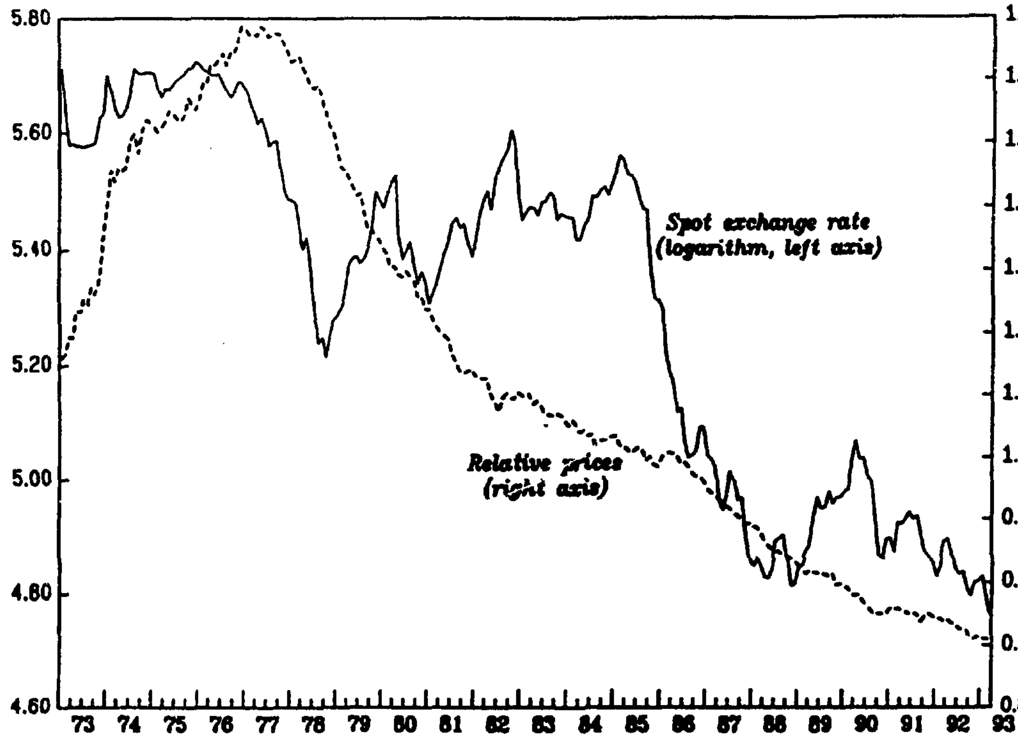


Chart 1f.
Netherlands Guilder - U.S. Dollar

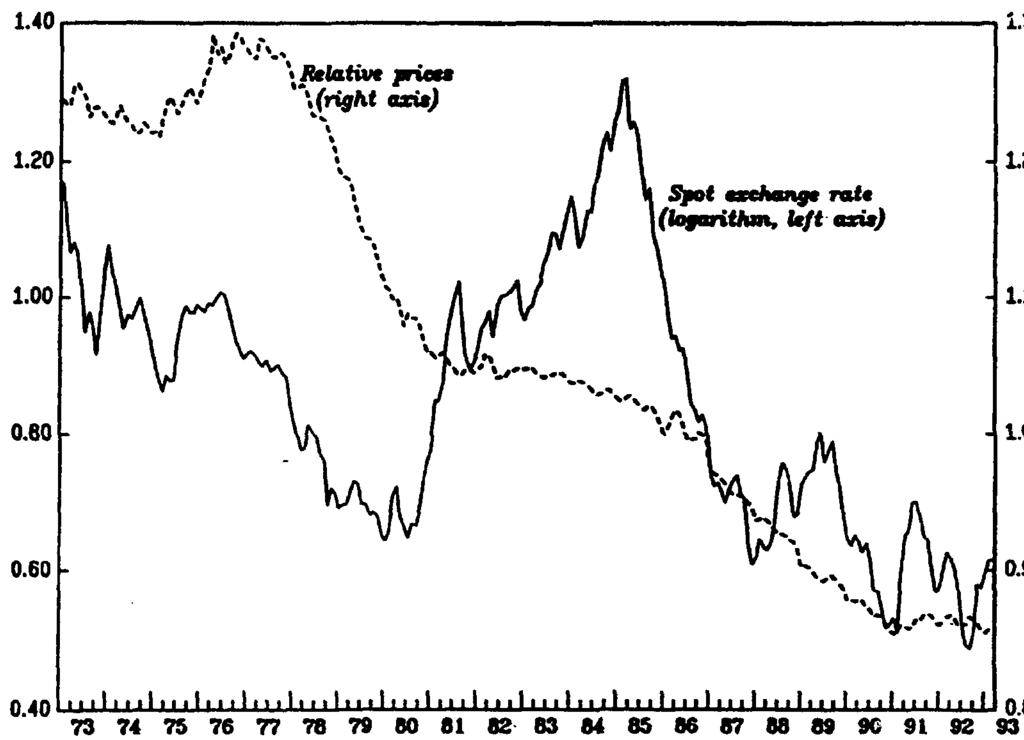


Chart 1g.
Swedish Krona - U.S. Dollar

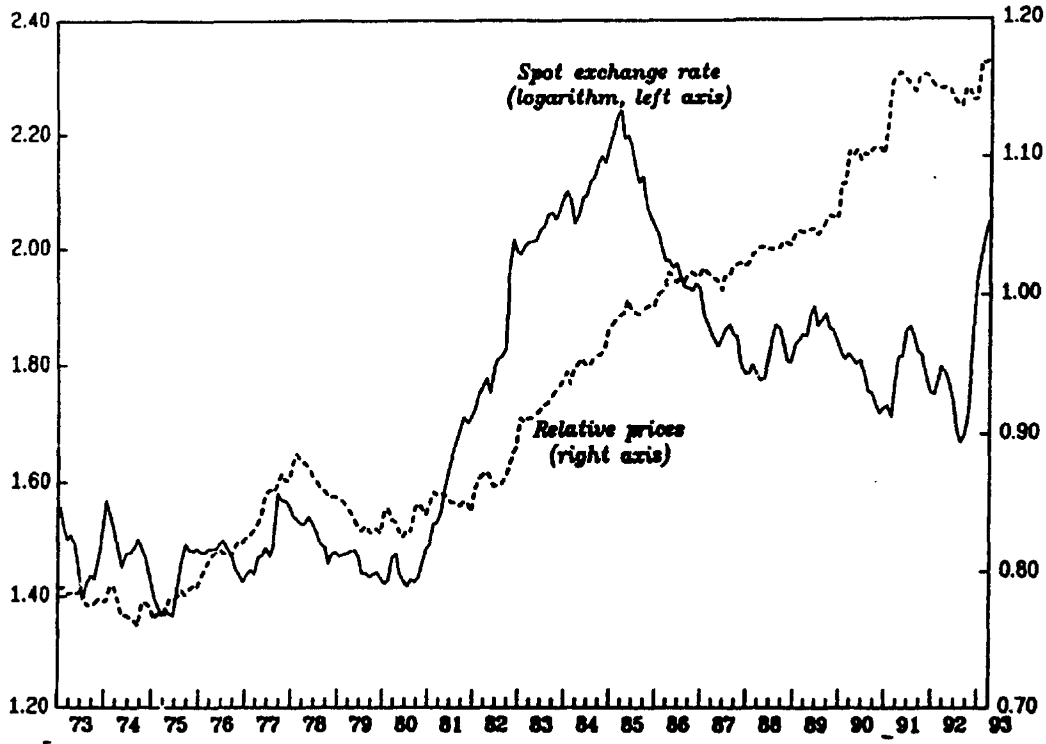


Chart 1h.
Swiss Franc - U.S. Dollar

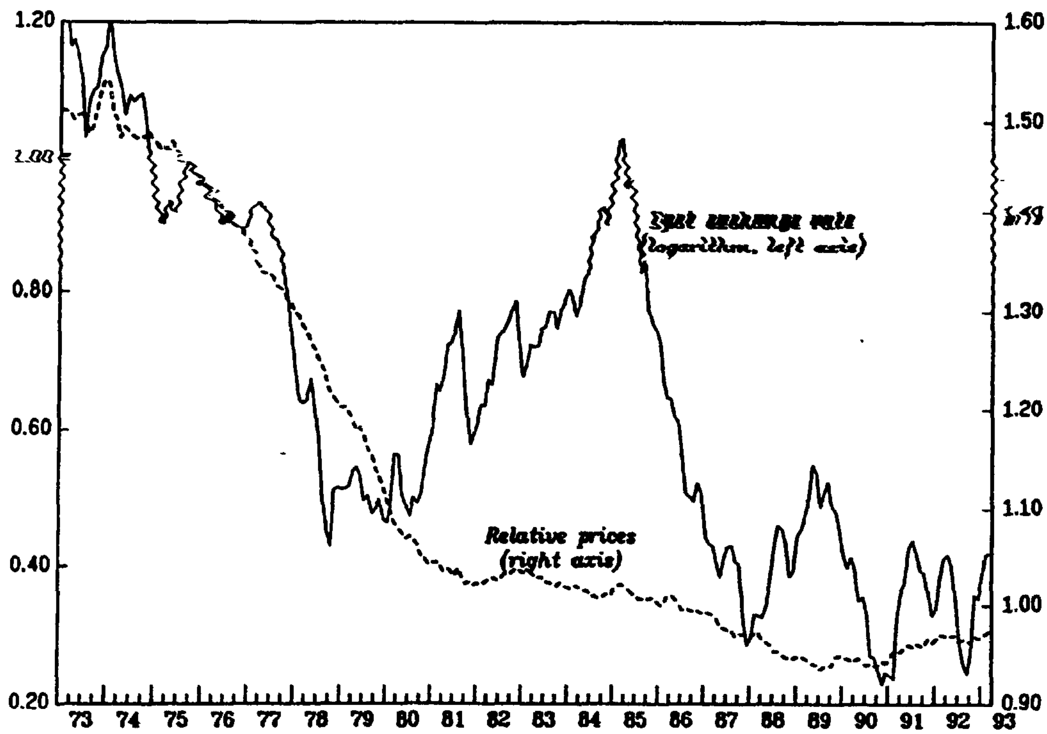
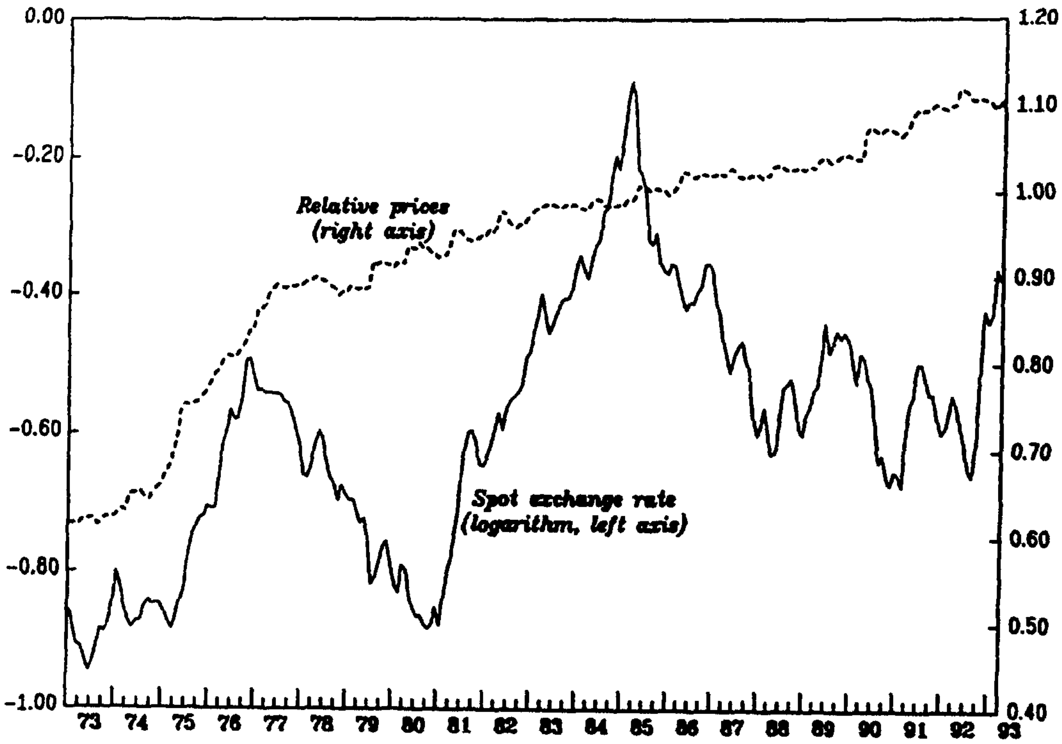


Chart 1i.
Pound Sterling - U.S. Dollar



that there are lengthy periods in which an exchange rate tracks relative prices reasonably well and, in particular, there are often relatively long periods in which the trend behavior of the two series are very similar. However, there are also currencies (for example, the U.K. pound sterling) and periods (particularly post-1986) for which the correspondence does not appear close.

A further common feature of these plots is that the variability of the exchange rates appears greater than that of relative prices. This 'stylized fact' has led some commentators to argue that there is more to exchange rates than relative prices. This is certainly true in the short run--on a month to month, quarter to quarter basis--but is it equally true in the 'long run'? That is the topic of this paper. In particular, we consider whether the large array of sophisticated econometric tests that have been conducted on the exchange rate/relative price relationship reveal more than our simple visual inspection of the data.

III. The Balance of Payments and Purchasing Power Parity

A useful focal point for our discussion of long-run exchange rate modeling is the familiar balance of payments equilibrium condition under floating exchange rates.

$$ca_t + cap_t - \Delta f_t = 0, \tag{5}$$

where ca_t denotes the current account balance, cap_t denotes the capital account balance and Δf_t denotes the change in reserves. Under freely floating exchange rates, the conventional balance of payments view of the determination of the exchange rate suggests that the exchange rate moves to equilibrate the sum of the current and capital accounts of the balance of payments, thereby ensuring that the change in reserves equals zero (see MacDonald 1988). A model in whi h balance of payments equilibrium is linked to conditions in asset markets, and shows how the exchange rate moves to ensure both stock and flow market equilibrium (where the latter is consistent with desired magnitudes), has been developed by Mussa (1984) and Frenkel and Mussa (1988) 1/. There are two main reasons for using (5) as the focal point of our discussion. First, tests of absolute purchasing power parity can be shown to be related in a straightforward way to the current account, while tests focussing on the real exchange rate take as their starting point the capital account. 2/ Second, the use of (5) also illustrates the ways in which simple PPP models, and other (related) real

---

1/ The empirical implications of this model have been investigated by Faruqee (1994) and MacDonald (1994a).

2/ Not all researchers motivate their PPP modeling in quite this way. However, we believe that no great injustice is done by using this framework; the expositional advantages outweigh any disadvantage of presenting each researchers account using his or her, perhaps slightly different, framework.

exchange relationships, may be deficient, and offers an appropriate way of testing PPP. The following equations summarize the assumptions regarding the current and capital accounts 1/

$$ca_t = nx_t + i^* A_t, \tag{6}$$

$$nx_t = \alpha(s_t + p_t^* - p_t) + \beta z_t; \quad \alpha > 0 \quad \beta = ?, \tag{7}$$

$$cap_t = \mu(i_t - i_t^* - \Delta s_{t+k}^e); \quad \mu > 0, \tag{8}$$

where, of terms not previously defined, nx_t denotes net exports, A_t is the stock of net foreign assets, z_t captures exogenous influences on net exports, i_t is the nominal domestic interest rate, Δ is the first difference operator, an ^{e} denotes a subjective expectation, an asterisk denotes a foreign magnitude and lower case letters indicate that the level of the variable has been transformed using the natural logarithm operator (apart from the interest rate terms which are expressed as proportions).

Equation (7) indicates that net exports are dependent on the real exchange rate, or competitiveness, and exogenous factors. For purposes of exposition, we have assumed that a country's competitiveness is a function of an overall price index, such as the CPI, which includes both traded and nontraded goods. The parameter α is the elasticity of net exports with respect to competitiveness (see discussion below). The z_t variable is an exogenous 'catch-all' term which captures, for example, expenditure effects from government and private consumers and productivity differences in the manufacture of traded goods between the home and foreign country. Equation (8) is a standard capital account relationship describing the flow of capital as a function of the expected excess yield on domestic relative to foreign assets. The parameter μ captures the mobility of international capital. If $\mu \to \infty$ then capital is perfectly mobile and (8) collapses to uncovered interest rate parity. If, however, $\mu < \infty$ we have imperfect capital mobility and the term in parentheses may be thought of as a risk premium. On substituting (6), (7) and (8) in (5), and solving for the exchange rate we obtain:

$$s_t = p_t - p_t^* - (\beta/\alpha).z_t - (i^*/\alpha).A_t - (\mu/\alpha).(i_t - i_t^* - \Delta s_{t+k}^e). \tag{9}$$

which may be thought of as a reduced form balance of payments equation for the exchange rate. Equation (9) is useful for motivating the two versions of PPP which have been widely tested in the recent exchange rate literature, namely traditional absolute PPP and the EMPPP.

1/ Since the exchange rates considered in this paper are all bilateral rates, the implicit assumption in our discussion here is that the balance of payments accounts are defined on a bilateral basis, rather than on the conventional multilateral basis.

1. Absolute PPP and the current account

What then is the process that allows us to move from expression (9) to the absolute PPP condition (2), re-expressed here in logs

$$s_t = p_t - p_t^*. \tag{2'}$$

First, in some time frame, which may be referred to as long run, it would be expected that net capital flows are zero (perhaps because net savings are at their desired level) and therefore the last term in (9) goes to zero; balance of payments equilibrium reduces to current account balance (this is the kind of stock-flow equilibrium captured in the portfolio balance model--see MacDonald (1988)). Focusing on the current account items in (9), we note that strict absolute PPP requires z_t and A_t to be zero. One way of obtaining this would be to assume that α, the elasticity of net exports with respect to relative prices, is infinitely large. This assumption is often made in textbook expositions of PPP; however, it is has no empirical support (see Goldstein and Khan (1985)).

Non-zero values of A_t and z_t will produce a real exchange rate configuration which is not equal to zero. Hence even with full long-run price flexibility changes in net excess demands for domestic goods can alter the relative price of traded to nontraded goods and hence the real exchange rate. Examples of this would be the well-known Balassa-Samuelson productivity bias or changes in government and/or private consumption in favor of, say, domestic goods (see Hallwood and MacDonald (1994)). This effect is likely to be most important when comparing countries at different stages of development, but less important for countries at a similar level of development. 1/ However, even if z_t is zero notice that any net foreign asset accumulation that has occurred in the move from equilibrium to equilibrium will require a value of the nominal exchange rate which does not simply reflect relative prices. This is one of the insights of the portfolio balance approach to the exchange rate and what Isard (1977) refers to as the 'knockout punch' to absolute PPP. It is worth noting at this stage that even in a long-run context a researcher will face a standard omitted variable bias in estimating a long-run PPP relationship, to the extent that (i^*/α), A_t and z_t are not included in the estimating equation. This makes clear the restrictiveness of the assumptions necessary for the purest form of PPP, even as a long-run concept.

1/ Although even here it may be important. For example, comparisons of countries within the European Community are unlikely to generate any strong biases, whereas comparisons of EC countries, and the United States, with Japan may well produce very marked productivity biases for the post-war period (see discussion on page below).

A final point to note is that even if α were high, as long as μ, the capital mobility parameter, is also high, capital flows would be an important reason for the violation of absolute PPP--this is essentially the Cassellian view outlined earlier. The Cassellian view may be expressed more formally as positing although there may be disturbances that push the nominal exchange rate away from the relative price configuration, these will not be permanent and will eventually be offset in the long term. To use a time series expression, the real exchange rate under the Cassellian view is mean-reverting:

$$q_t = \rho q_{t-1} + \epsilon_t, \qquad 0 < \rho < 1 \tag{10}$$

where q_t denotes the real exchange rate, $(s+p^* -p)_t$, ϵ_t is a random error term 1/ and ρ should lie in an interval between zero and unity. An alternative approach to defining PPP, which is actually diametrically opposite to the Cassellian view, asserts that ρ in (10) is unity: shocks to the real exchange rate are permanent. This view of PPP, which we label the efficient markets view of PPP, is due to Adler and Lehman (1983), Roll (1979) and MacDonald (1985) and relies for its derivation purely on the capital account of the balance of payments.

2. The capital account of the balance of payments and efficient markets PPP

The efficient markets view of PPP asserts that in a world of high or perfect capital mobility it is not goods arbitrage that matters for the relationship between an exchange rate and relative prices but interest rate arbitrage. The concept, which gives a fundamentally different prediction for the behavior of the real exchange rate than absolute PPP, may be illustrated in the following way. In (9) assume that capital is perfectly mobile and therefore $\mu \rightarrow \infty$. As in the Mundell-Fleming and Dornbusch (1976) models this immediately focuses attention on the capital account of the balance of payments, and, in a sense, it becomes the tail wagging the current account dog. If one views the capital account as reflecting the desired actions of agents, as in the models of Mussa (1984) and Frenkel and Mussa (1988) such tail-wagging may not be all that unappealing. 2/ On assuming perfect capital mobility we have:

$$i_t - i_t^* = \Delta s_{t+k}^e, \tag{11}$$

which is the condition of uncovered interest rate parity (UIP). 3/

1/ To allow for heteroscedastic disturbances ϵ_t need not be iid.

2/ Although it surely is unappealing in the context of models in which the stock-flow repercussions of capital account imbalances are ignored, such as in the original textbook Mundell-Fleming model.

3/ See Engel (1994) for a useful survey of the UIP condition.

The relative nominal interest rate term in (11) may be decomposed into a real and expected inflation component using the Fisher decomposition

$$i_t - i_t^* = (r_t^e - r_t^{e*}) + (\Delta p_{t+k}^e - \Delta p_{t+k}^{e*}), \tag{12}$$

where r_t denotes the real interest rate and $\Delta p^e{}_{t+k}$ denotes the expected inflation rate. On using this in the UIP condition we obtain:

$$(s-p+p^*)_t = -(r^e-r^{e*})_{t,t+k}+(s^e-p^e+p^{e*})_{t+k}, \tag{13}$$

or

$$q_t = -(r^e - r^{e*})_{t,t+k} + q_{t+k}^e. \tag{13'}$$

Equation (13') states that the current real exchange rate is determined by the (negative of) the expected real interest differential and the expected real rate in period $t+k$ (this is sometimes interpreted as the equilibrium rate; that is, $q_{t+k}^e = \bar{q}$ where $\bar{q}$ denotes the equilibrium real rate).

Expression (13') may be rearranged to give a measure of the evolution of the expected real exchange rate:

$$[\Delta s^e - \Delta p^e + \Delta p^{e*}]_{t+k} = (r^e - r^{e*})_{t,t+k}, \tag{14}$$

or

$$\Delta q_{t+k}^e = (r^e - r^{e*})_{t,t+k}. \tag{14'}$$

On further assuming that the subjective expectations in (14) are equal to their rational counterparts, we may obtain: 1/

$$\Delta s_{t+k} - \Delta p_{t+k} + \Delta p_{t+k}^* = [E_t\, r_t - E_t\, r_t^*] + \omega_{t+k}, \tag{15}$$

where the left hand side is simply the change in the real exchange rate and this is driven by a real interest differential and a random error term. This may be referred to as the base-line real exchange rate model, due to Roll (1979) and Adler and Lehmann (1983) and MacDonald (1985a and b). It assumes that real interest rates are exactly equalized across countries and therefore:

$$q_t - q_{t-1} = \omega_t. \tag{16}$$

1/ Where we have made use of the following expressions:

$$\Delta s_{t+k} = E_t \Delta s_{t+k} + \epsilon_{t+k}, \Delta p_{t+k} = E_t \Delta p_{t+k} + \varphi_{t+k}, \Delta p_{t+k}^* = E_t \Delta p_{t+k}^* + \xi_{t+k} \text{ and } \omega_{t+k} = \epsilon_{t+k} - \varphi_{t+k} + \xi_{t+k}.$$

This model implies, therefore, that the real exchange rate follows a random walk. 1/ It is worth noting the similarity between the EMPP view of the evolution of the real exchange rate and that implied by traditional relative PPP as given by (4). Superficially the two concepts appear to be very similar. Thus relative PPP as given by (4) implies that the change in the real exchange rate equals zero: here EMPPP gives the same kind of story, apart from a random error. Note, however, that the story underpinning (4) is radically different to that generating (11). In (4) it is goods arbitrage that gives relative PPP, whereas (16) exists through arbitrage on the capital account of the balance of payments. These differential theoretical underpinnings mean that the error term in (16) is far from an innocuous appendage. Thus, the existence of the error term indicates that a disturbance to the real exchange rate will have a permanent effect, whereas in (4) such disturbances are ruled out by definition. This difference may be seen more clearly by means of an example. In a world of short-run sticky prices, a monetary surprise, which pushes the nominal exchange rate away from the relative price relationship will from (16) result in a permanent change in the real exchange rate. In the traditional Cassellian view the monetary disturbance although not immediately forcing (2) or (4) would eventually lead to such relationships being restored; real exchange rates would be mean-reverting.

Also the implications of equation (13) for the current account are rather odd in that it suggests that the real exchange rate will only by chance be at a level consistent with a zero current balance and therefore in equilibrium the current account presumably does not matter. However, it appears more reasonable to suppose that a country cannot go on accumulating or decumulating foreign assets without bound.

From a time series perspective notice that the derivation of (13) presupposes that real interest rates and the real exchange rates are integrated of different orders, or that the real exchange rate is I(1) and real interest rates are cointegrated (we shall return to this point again below).

3. The balance of payments and the specification of the long-run exchange rate equation

In Section 3.1 we noted potential ways in which the balance of payments equilibrium condition may influence the specification of a long-run exchange rate model:- productivity differences between countries or net asset accumulation can twist the exchange rate/relative price relationship. However, when comparing countries at a similar level of development, which nearly all of the existing research does, (and which we do in this paper)

1/ An alternative, less restrictive version of the EMPPP, would have interest rates being equalized up to a constant differential, in which case a constant would be added to the right hand side of (13).

this is unlikely to be important. 1/ The foreign asset accumulation channel is unclear in the current context, although, again, for countries at a similar level of development it may not be that important. However, there is another way in which (9) may be useful in defining a long-run equilibrium exchange rate and this relates to the distinction between what may be referred to as a statistical and 'true' long-run (the latter concept has been referred to as a long long-run by Breuer (1994)).

In many of the papers discussed in this survey, when a researcher refers to an estimated long-run exchange rate relationship, he or she is referring to the existence of a statistical equilibrium; that is, one which is consistent with the particular estimator used (for example, the Johansen maximum likelihood procedure). This may or may not conform to a 'true' equilibrium position, which is one defined by economic theory. Indeed, the kind of long-run equilibrium which many researchers have estimated for the recent float does not actually conform to what most would understand by equilibrium (i.e., a 'true' equilibrium). We suggest that the balance of payments equilibrium condition may be a useful way of understanding the empirical findings reported in this paper and reconciling the distinction between measures of true and statistical equilibrium. This is a topic to which we return to in the concluding section.

IV. Modeling the Long-Run Nominal Exchange Rate

1. Absolute purchasing power parity and cointegration

Much recent work on modelling equilibrium exchange rates has focused on testing equation (2'). Since the variables contained in (2') are likely to be nonstationary, such tests have focussed on exploiting the cointegration methods proposed by Engle and Granger (1987). Since this and its associated methods are now well documented we will not give a detailed account here (see, for example, Banerjee et al (1993) for a comprehensive survey). A brief account is, however, beneficial not least from the point of view of introducing terminology, and we focus our discussion on equation (17), which is the regression equation analogue to (2').

$$s_t = \beta + \alpha_0 p_t + \alpha_1 p_t^* + \phi_t. \qquad (17)$$

If the variables entering (17) are all first-order nonstationary, 2/ then they are integrated of order 1, I(1). If there is no 'long-run' relationship between the exchange rate and relative prices, the residual

---

1/ Japan, however, is likely to be the exception--see Faruqee (1994).
2/ This is not uncontroversial since there is some evidence to suggest that prices are I(2) processes (exchange rates being I(1)); see, for example, MacDonald (1993).

series in (17) would also be nonstationary, I(1). 1/ If there is a long-run relationship between an exchange rate and relative prices, which we note from our previous discussion is what proponents of PPP have in mind, then the appropriate way to capture it is to use cointegration methods. In the current context cointegration could be said to exist when, despite the variables entering (17) being individually nonstationary, there exists some linear combination which transforms the residuals to an I(0) series. There are now a plethora of different ways of testing for cointegration, each of which has been developed from the initial testing methods of Engle and Granger (1983). Cointegration-based tests of PPP have followed, in a chronological sense, the development of cointegration tests and, as we shall see, the more recent cointegration tests facilitate a more powerful test of PPP than the base-line Engle-Granger method.

In the context of the cointegration literature, the existence of long-run PPP amounts to the satisfaction of three conditions. First, and most importantly, the errors, φ_t, from an estimated version (17) should be stationary; that is, they should be I(0). If they are not then there will be a tendency for the exchange rate and relative prices to drift apart without bound, even in the long run. Second, the α_0 and α_1 coefficients should enter (17) with an equal and opposite sign (the condition of symmetry) and, third, they should be equal to plus and minus unity (the condition of proportionality). 2/

The Engle-Granger cointegration method simply entails estimating (17) by OLS and subjecting the residuals to a variety of diagnostic tests of which the most popular has proven to be the Augmented Dickey-Fuller test. This amounts to estimating an equation of the form:

$$\Delta\varphi_t = \nu_1\varphi_{t-1} + \nu_i\sum_{i=2}^{p} \Delta\varphi_{t-i+1} + \epsilon_t \tag{18}$$

If the null hypothesis of no cointegration is valid--the residuals are I(1)--then ν_1 should be insignificantly different from zero and this may be tested using a t-test, denoted τ. Under the altarnative hypothesis of stationarity ν_1 is expected to be significantly negative. As the distribution of t is nonstandard, Engle and Granger have tabulated the appropriate critical values (other sets of critical values are given by

---

1/ However, in such circumstances there may still be a short-run relationship and an appropriate way to capture this would be to specify the regression in first differences (which would amount to a test of relative PPP--see Frenkel (1981), Krugman (1978) and MacDonald (1988b) for such estimates).

2/ The distinction between symmetry and proportionality is, we feel, rather artificial, but is one which is often made in the literature.

Engle and Yoo (1987) and MacKinnon (1991)). One advantage of the Engle-Granger approach, as highlighted by Stock (1987), is that if cointegration exists, then even if all the variables entering (17) are I(1), the coefficient estimates approach their asymptotic values at a rate equal to T rather than the conventional $T^{1/2}$, where T denotes the number of observations; that is, the estimates are super-consistent. This information may be helpful in allowing a researcher to gauge how far away from symmetry and proportionality her estimates are, although one problem with the Engle-Granger approach is that it does not allow one to draw any inferences on the basis of these values (thus the fact that the variables entering (17) are all nonstationary means that standard statistical inference is not valid).

For the recent experience with flexible exchange rates, the Engle-Granger two-step cointegration method has been applied to aggregate price data by Enders (1988), Mark (1990), Patel (1990) and Taylor (1988). Because in their initial paper Engle and Granger only computed critical values for τ for a regression equation with two variables Enders, Mark and Taylor constrain the coëfficient on the relative price terms to be equal and opposite (that is they impose symmetry). A paper by Engle and Yoo (1987) tabulates critical values for τ from a regression of up to 5 variables and these are used by Patel to estimate (17) in unconstrained fashion. Enders (1988) estimates (18) with relative wholesale price terms constructed for Canada, Germany, and Japan against the United States for the period January 1973 to November 1986 and is unable to reject the null of no cointegration in any instance. Taylor comes to the same conclusion using a similar data base. Mark (1990) investigates a number of OECD bilateral rates based on, respectively, the U.S. dollar, U.K. pound, and Japanese yen as the home currency for the period June 1973 to February 1988 (consumer prices) and finds only one instance (out of 13) when the null of no cointegration is rejected. 1/ Patel uses a quarterly data base spanning the period 1974-86 for Canada, Germany, Japan, the Netherlands, and the United States (a variety of bilateral exchange rate combinations are considered for these countries) and reports that the null is rejected in only four instances out of a total of fifteen. In sum, we interpret this group of papers as suggesting that there is no long-run tendency for exchange rates and relative prices to settle down on an equilibrium track. 2/ One disadvantage of these studies, as we indicated above, is that the use of the two-step methodology precludes an actual test of the proportionality and symmetry of the α's with respect to the exchange rate, although the estimated values are often far from 1 and -1.

In Table 1 we present some estimates of (17) and (18) using the data set discussed in the introduction. With the WPI as the price measure we

---

1/ This one significant value is less than the number that would be expected to occur by chance for this group of exchange rates.

2/ Choudry, McNown, and Wallance (1991), using the Engle-Granger method, find some evidence of cointegration for the Canadian dollar in the 1950s.

Table 1. Engle-Granger Two-Step Cointegration Tests

	CPI α_0	CPI $\alpha 1$	CPI ADF	WPI α_0	WPI α_1	WPI ADF
Canada	-0.0001	0.223	-2.400	0.783	-0.672	2.100
France	3.157	-3.662	-2.260	1.709	-1.192	-2.350
Germany	5.552	-3.266	-1.620	3.144	-2.250	-1.420
Italy	1.916	-2.892	-2.050	0.668	-0.365	-1.470
Japan	1.088	-1.390	-2.670	2.161	-1.621	-2.920
Netherlands	1.681	-1.275	-2.150	2.111	-1.593	-1.310
Sweden	-0.318	0.921	-1.950	0.929	-0.809	-1.180
Switzerland	0.382	-0.758	-2.360	2.254	-1.355	-1.650
United Kingdom	0.614	-0.579	-2.350	0.517	-0.478	-2.060

Note: The countries in column one denote the home currency component of the nominal exchange rate used in the Engle-Granger two-step regression (in all cases the foreign currency is the U.S. dollar). The entries in the columns labeled a and a denote estimated coefficients and ADF denotes the augmented Dickey-Fuller statistic calculated from the residuals of the cointegration regression. The critical value for the latter is -2.98 (source MacKinnon (1991)). The labels CPI and WPI indicate the use of a consumer or wholesale price measure in the cointegrating regression.

note that all of the α_0 and α_1 coefficients are correctly signed (positive and negative, respectively) while with the CPI seven out of the nine currencies produce correctly signed values of these coefficients. However, most of the estimated coefficients are far from their hypothesized values of plus and minus one. Of most concern, though, is the fact that none of the estimated augmented Dickey-Fuller statistics are significant at the 5 percent level (indeed none are significant at even the 10 percent level). These results confirm, albeit with a longer sample period, the research discussed in the last paragraph.

The single equation estimator of Engle and Granger, however, poses the researcher with very real practical problems. Thus, Banerjee et al (1986) have noted that the small sample properties based on an equation like (17) are poor. Additionally, if the regressors in (17) are endogenous (which, as we suggested earlier, are highly likely to be in our application) and (or) the errors exhibit serial correlation (which again is very likely in the current application) then the asymptotic distribution of $T(\hat{A} - A)$, where T denotes the number of observations, $A' = [\alpha_0 \ \alpha_1]$ and $\hat{A}$ is an estimate of A, which will depend upon nuisance parameters. The Full Information Maximum Likelihood (FIML) method of Johansen (1988, 1991) and Ahn and Reinsel (1988), which imposes unit roots on the variables, will produce asymptotically optimal estimates because they incorporate a parametric correction for serial correlation (which comes from the assumed underlying VAR structure--see below) and, since it is a systems method, it can handle the endogeneity of the regressors (to the extent the implied price equations are plausible).

Since the method of Johansen is now well-known we do not discuss it here, rather we simply note two tests statistics which we and other researchers have used to test for the number of cointegrating vectors. In our application the likelihood ratio, or Trace, test statistic for the hypothesis that there are at most r distinct cointegrating vectors is:

$$LR1 = T \sum_{i=r+1}^{N} \ln(1 - \hat{\lambda}_i) \tag{19}$$

where $\hat{\lambda}_{r+1}, \ldots, \hat{\lambda}_N$ are the N-r smallest squared canonical correlations between X_{t-k} and ΔX_t series (where $X_t = [s_t, p_t, p_t^*]'$ and where all of the variables entering X are assumed I(1)), corrected for the effect of the lagged differences of the X process (for details of how to extract the λ_i's see Johansen 1988, and Johansen and Juselius, 1990). Additionally, the likelihood ratio statistic for testing at most r cointegrating vectors against the alternative of $r+1$ cointegrating vectors--the maximum eigenvalue statistic--is given by (20):

$$LR2 = T \ln(1 - \lambda_{r+1}) \tag{20}$$

Johansen (1988) shows that (19) and (20) have a nonstandard distribution under the null hypothesis. He does, however, provide approximate critical values for the statistic, generated by Monte Carlo methods.

A number of researchers have argued that the failure to find a cointegrating relationship between relative prices and an exchange rate may be due to the econometric method used, rather than the absence of a long-run relationship. For example, Cheung and Lai (1993), Kugler and Lenz (1993), MacDonald (1993) and MacDonald and Marsh (1994) all advocate using the Johansen cointegration method to test for the number of cointegrating vectors amongst relative prices and exchange rates for bilateral U.S. dollar exchange rates (MacDonald (1993), MacDonald and Marsh (1994) and Cheung and Lai (1993)) and German mark bilateral dollar rates (Kugler and Lenz (1993), MacDonald (1993) and MacDonald and Marsh (1994)). A considerable amount of evidence in these papers supports the contention that there is indeed a long-run PPP relationship for a variety of currencies in the sense that most bilateral currency/price combinations exhibit cointegration. However, often the restrictions of symmetry and proportionality are rejected in these studies (especially when U.S. dollar bilateral rates are used).

In Table 2 we present estimates of the PPP relationship for our data set using the Johansen method. The Table should be read in the following way. The columns under the heading 'Trace' contain our estimates of (19), while the entries in the columns under 'λMax' contain our estimates of (20). The estimates of the normalized cointegration vector are contained in the two columns under β, the entries in the α column are the estimated α coefficients in the exchange rate equation, and the entries in the LR3 and LR4 columns are likelihood ratio test statistics for testing, respectively, proportionality and symmetry. An asterisk denotes that a statistic is significant at the 5 percent significance level. Note that on the basis of the Trace and λMax statistics that there is evidence of at least one cointegrating vector for each currency apart from Sweden. Note, further, that although many of the estimated coefficients (in the β columns) are correctly signed, there are a number which are wrongly signed and also many coefficient values are far from their numerical values of unity, in absolute terms. It is not surprising, therefore that the proportionality and homogeneity restrictions are convincingly rejected. This evidence is consistent with the body of research discussed in the last paragraph: there is, in contrast to Engle-Granger tests of PPP, strong evidence of a long-run relationship between exchange rates and relative prices, but this relationship does not conform exactly to that defined in equation (2'). In the concluding section we offer an interpretation of this conflict.

A third approach, which is quite illuminating in the present context since it facilitates a test of the stability of the PPP relationship, is the application of MacDonald and Moore (1994) [hereafter, MM]. They take

Table 2. Johansen Multivariate Cointegration Tests

	Trace			λMax			β		α	LR3	LR4
Canada											
CPI	5.43	13.65	35.54	2.53	8.22	21.88	-10.88	-0.12	-0.02	11.9*	12.2*
WPI	2.53	9.76	39.72	2.53	7.23	29.96	-1.12	0.01	0	15.0*	25.0*
France											
CPI	4.04	15.02	35.45	4.04	10.98	20.43	-2.95	0.03	-0.76	0.1	0.3
WPI		-			-		-	-	-	-	-
Germany											
CPI	0.04	6.59	19.62	0.04	6.56	13.03	-0.37	0.01	-0.02	-	-
WPI	0.00	9.26	24.16	0.00	9.25	14.90	-84.6	0.46	-0.00	-	-
Italy											
CPI	6.17	14.91	35.79	6.17	8.74	20.88	-5.8	0.09	-0.01	10.6	11.1*
WPI	0.82	6.25	25.06	0.82	5.43	18.81	7.11	-0.10	-0.01	-	-
Japan											
CPI	4.64	18.29	45.98	4.64	13.65	27.69	-22.9	0.11	0.01	0.02	6.18
WPI	1.76	4.33	21.59	1.76	2.57	17.67	-2.27	-0.02	-0.23	-	-
Netherlands											
CPI	4.21	17.15	36.92	4.21	12.73	19.77	6.60	-0.00	0.00	0.01	5.51
WPI	1.13	11.49	36.3	1.13	10.36	24.90	20.46	-0.10	-0.01	9.09*	15.97*
Sweden											
CPI	0.07	17.53	23.37	1.07	4.76	17.53	2.84	-0.04	0.07	-	-
WPI	2.34	12.52	26.52	2.34	10.19	13.99	8.49	-0.13	-0.01	-	-
Switzerland											
CPI	0.16	7.60	27.03	0.16	7.43	19.43	2.28	-0.00	-0.05	6.23*	11.27*
WPI	0.92	11.38	41.43	0.92	10.46	30.05	4.38	0.00	-0.01	9.67*	19.41*
United Kingdom											
CPI	10.08	24.05	43.79	10.08	13.98	19.74	-0.59	0.01	-0.06	0.34	6.96*
WPI	3.35	16.28	40.91	3.35	12.93	24.63	0.27	0.01	-0.03	0.01	3.17

Notes: The first column denotes the country used in the Johansen test, while the second column denotes the relevant price series. Entries in the columns directly below TRACE and λMax are the estimates of the Trace (21) and λMax (22) statistics discussed in the text. The estimates of the normalized (on the exchange rate) cointegration statistics are contained in the two columns headed by β, and the entries in the α column are the estimated α coefficients from the exchange rate equation. LR3 and LR4 are, respectively, likelihood ratio tests for symmetry and proportionality. An * denotes significance at the 5 percent level.

cointegration as the null hypothesis and use the fully modified estimator of Phillips and Hansen (1990) as extended by Hansen (1992). The Phillips-Hansen component of the estimator produces estimates of the coefficients and standard errors in (17) which are fully modified (using a nonparametric correction) in the sense that they are robust to serial correlation of the disturbances and to potential endogeneity of the regressors. The Hansen (1992) contribution to the estimator concerns the derivation of an algorithm which allows one to interpret rejections of the null hypothesis of cointegration in terms of coefficient instability. In the present application the idea may be seen in the following way.

As we have noted, if s_t, p_t and p_t^* are cointegrated the error term, φ_t, in (17) should be I(0). If, however, s_t, p_t and p_t^* are not cointegrated then φ_t is I(1) and we may think of it as containing a random walk component, D_t, and a stationary term, v_t (i.e., $\varphi_t = D_t + v_t$). Under these conditions we may rewrite (17) as:

$$s_t = \beta_t + \alpha_0 p_t + \alpha_1 p_t^* + v_t \tag{17'}$$

where $\beta_t = \beta + D_t$. Hence the alternative hypothesis considered by MM is equivalent to the intercept term in (17) following a random walk.

MacDonald and Moore's tests are implemented for three groups of bilateral currencies, based on the German mark, Japanese yen and U.S. dollar (consumer price indices are used in the comparisons), using data for the recent float. The other countries involved in these bilateral relationships are Canada, France, Italy, the Netherlands, Sweden, Switzerland, and the United Kingdom. Overall, their results may be summarized by saying they indicate a remarkable degree of stability for all three bilateral groupings, in the sense that β does not follow a random walk. Interestingly, it is only the Japanese Yen--U.S. dollar relationship that exhibits any evidence of instability (a finding MM attribute to persistent long-term capital flows from Japan to the United States over the period and the continuing productivity bias in favor of Japanese goods). 1/ Additionally, for some of the bilateral pairings (particularly against the mark and yen) one cannot reject the symmetry/homogeneity restrictions.

The empirical evidence relating to (17) may be summarized in two ways. First, there is now mounting evidence to suggest that the residual in an estimated version of (17) is a mean-reverting process; that is, it is stationary, although the deviations from PPP seem to be relatively long-lived (see MacDonald, 1993). Second, the relationship between exchange rates and relative prices rarely obeys the symmetry and degree one homogeneity restrictions which is suggestive of real factors (such as net asset accumulation, perhaps) requiring real exchange rate adjustments for

---

1/ This finding is consonant with Faruqee's (1994) findings for the real effective value of the Japanese yen over a period encompassing the floating period and part of the Bretton Woods period.

the particular sample periods considered by the above researchers.

2. The nominal exchange rate and relative excess money supplies

A long-run model of the exchange rate which is closely related to PPP is the flex-price monetary model (useful surveys of this approach are to be found in Frenkel and Mussa (1988), MacDonald (1988a,1990) and Frankel and Rose (1994)). This model essentially appends a theory of the determination of the price level to absolute PPP. In particular, assume that the domestic and foreign demand for money functions may be written using standard Cagan log-linear specifications

$$m^D - p = \alpha_0 y - \alpha_1 i, \tag{21}$$

$$m^{D*} - p^* = \alpha_0 y^* - \alpha_1 i^*, \tag{21'}$$

where, of terms not previously defined, m is the logarithm of the demand for money, y is the logarithm of income, i is the nominal interest rate and α_0 and α_1 are, respectively, the income elasticity and the interest semi-elasticity of the demand for money (we have assumed these to be identical across countries). The money market equilibrium conditions for the home and foreign country are given by (22):

$$m^D = m, \tag{22}$$

$$m^{D*} = m^*, \tag{22'}$$

where m denotes the supply of money. On using (22) in (21) and solving for the relative price level we obtain the long-run relative price relationship

$$p - p^* = m - m^* - \alpha_0(y - y^*) + \alpha_1(i - i^*), \tag{23}$$

which posits that the relative price of home to foreign goods is determined by the excess of money supply over money demand. On substituting this expression in (16) we obtain:

$$s = m - m^* - \alpha_0(y - y^*) + \alpha_1(i - i^*), \tag{24}$$

which is the (continuous) solution for the flex-price monetary model (Frenkel (1976) and Hodrick (1978)) and the long-run solution for the sticky price model (see Dornbusch (1976) Frankel (1979) and Buiter and Miller (1981)).

A number of researchers have tested (24), or variants thereof, using the Engle-Granger two-step procedure. For example, Boothe and Glassman (1987) test for cointegration of the U.S. dollar/Deutschmark exchange rate and only the relative money supply and are unable to reject the null hypothesis of no cointegration. However, it is not entirely clear that Boothe and Glassman exploit a potentially valid cointegrating set since the appropriate long-run monetary model emphasizes relative excess money

supplies; that is, relative money supplies adjusted for, at least, relative incomes and perhaps also interest rates. 1/ Meese (1987) tests the monetary model for U.S. bilateral dollar rates of the Deutschmark, the pound sterling and the yen, but he is unable to unearth a valid cointegrating set. 2/ Kearney and MacDonald (1990) test for cointegration between the Australian dollar/U.S. dollar exchange rate and relative money, income and interest rates and are unable to reject the null of no cointegration. Finally, Baillie and Selover (1987) test whether a version of the sticky price variant of the monetary model is able to produce a valid cointegrating set, for the bilateral U.S. dollar rates of the Canadian dollar, French franc, German mark, Japanese yen and U.K. pound. In common with the other Engle-Granger based studies, these authors are also unable to reject the null of no cointegration. The combined impression one obtains from the above-noted results would therefore seem to suggest that the monetary model does not even have empirical support as a long-run relationship.

Paralleling the recent cointegration literature on PPP, MacDonald and Taylor (1991) have criticized the use of the two-step procedure to test the monetary model and have, instead, advocated the Johansen (1988) maximum likelihood method. Interestingly, in using this approach to test the monetary model for three currencies (dollar-mark, dollar-sterling and dollar-yen) MacDonald and Taylor demonstrate that there is very strong support for the monetary model as a long-run relationship (indeed for the dollar-mark exchange rate they show that all of the restrictions implied by the monetary model are accepted by the data). 3/ Moosa (1994) also uses the Johansen method to test the validity of the monetary model for the pound sterling, mark and yen (against the U.S. dollar) for the period January 1975 to December 1986. Moosa's estimated version of the monetary model differs from that of MacDonald and Taylor in that he distinguishes between traded and nontraded goods in his specification of (23). This extension to the model also produces strong evidence of cointegration (although he finds that the monetary restrictions are rejected).

1/ In the forward-looking, or rational expectations, version of (24), the exchange rate is the present discounted value of expected future relative money supplies and income levels and the appropriate cointegrating relationship does not include interest rates. The 'static' version of the monetary model, given by (24), suggests that interest rates should be included in the cointegrating set.

2/ Meese tests the forward monetary model and has in his cointegrating set, the exchange rate and relative money supplies and income levels.

3/ In particular, they find evidence of degree one homogeneity between the exchange rate and relative money supplies and coefficients on relative income and interest rate terms which are correctly signed and have plausible magnitudes.

V. Modeling Long-Run Real Exchange Rates: Efficient Markets and the Random Walk Real Exchange Rate Model

An alternative to testing for cointegration between a nominal exchange rate and relative prices is to actually impose the symmetry and homogeneity restrictions and test if a real exchange rate contains a unit root. The null hypothesis therefore is given by (16) which we repeat here with a drift term

$$\Delta q_t = a + \omega_t, \tag{16'}$$

where Δ is the first difference operator, a is a drift term, which captures, perhaps, the failure of real interest rates to be equalized across countries, and ω_t is a stationary process. An alternative hypothesis to (16') is that the real exchange rate exhibits temporary deviations around a trend; that is, it is trend stationary

$$q_t = \gamma_o + \gamma_1 t + \epsilon_t, \tag{25}$$

where $\gamma_1 t$ denotes the time trend. The null hypothesis may be thought of as the efficient markets null while the alternative hypothesis may be interpreted as traditional absolute PPP.

The standard test of the above null hypothesis against the trend stationary alternative may be understood using the following ARMA specification for the real exchange rate (as may some of the other tests considered in this section)

$$\phi(B)q_t = a + \theta(B)\epsilon_t, \tag{26}$$

where B denotes the lag operator and a $= \gamma_o + \gamma_1 t$. The following sets of tests are all dependent in some form or other on (26).

The easiest way to motivate a test for a unit root in q_t is to assume that the real exchange rate has a purely autoregressive representation which will be the case if the moving average polynomial, $\theta(B)$, in (26) is invertible. Given this assumption we may reparameterize (26) as

$$\Delta q_t = \gamma_o + \gamma_1 t + (\beta_o - 1)q_{t-1} + \sum_{j=1}^{n-1} \beta_{j+1}\Delta q_{t-j} + \nu_t, \tag{26'}$$

where

$$\beta_i = \sum_{j+1}^{n} \phi_j;\ i=1,\ \ ,n,$$

since $\phi(B)-1$ will contain a unit root if $\sum_1^n \phi=1$ the presence of a unit root is formally equivalent to a test of whether β_0 =1 or (β_0 -1) = 0. This hypothesis may be tested using a standard t-test, although as Dickey and Fuller (1979) and many others note, this will have a nonstandard distribution and therefore one has to use the percentiles tabulated by Fuller (1976). One may test for two unit roots in the real exchange rate by estimating (26') with all of the real exchange rate terms first differenced again. The inclusion of the correct specification of deterministic variables in (26) is crucial to the power of the test (see Banerjee et al (1993)).

Much as in the cointegration literature on testing for absolute PPP, papers which test for a unit root in real exchange rates may be placed into two groups; the first, chronologically earlier, group yield evidence which is favorable to the hypothesis, while the second, more recent, group are unfavorable. The first group, essentially uses standard (classical) unit root test statistics, as given by (26') to test the null hypothesis that a variety of bilateral and effective real exchange rates contain a unit root (see, inter alia, Roll (1979), Frenkel (1981), Darby (1981), Adler and Lehmann (1983), MacDonald (1985a and b), Meese and Rogoff (1988), Enders (1988), Mark (1990) and Edison and Pauls (1993); 1/ the vast majority of such tests, which all use monthly data, are unable to reject the null.

In Table 3 we illustrate the findings of this group of papers by presenting our tests for one and two unit roots in our real exchange rate data. We constructed two real exchange rates for each country: one based on the CPI, the other based on the WPI. Each real exchange rate is expressed in natural logarithms. The results, regardless of the deterministic specification used, strongly indicate the presence of a single stochastic unit root. These results are therefore consistent with the results of other researchers noted in the previous paragraph.

How then should these results be interpreted? Do they really indicate that EMPPP is indeed a valid description of the evolution of the real exchange rate? In fact, much as in the cointegration literature, the above-noted unit root tests may be regarded as rather primitive and analogous to the original Engle-Granger two-step tests in their power at

---

1/ The papers of Roll (1979) and Adler and Lehmann (1983) both use an autoregression of the differenced real exchange rate to test the hypothesis that the sum of the coefficients on the autoregressive terms are jointly equal to zero.

Table 3. Tests for a Unit Root in the Real Exchange Rate Series

		L		ΔL	
		t	τ	t	τ
Canada	CPI	-2.18	-2.19	-3.54	-3.76
	WPI	-2.14	-2.11	-3.93	-4.08
France	CPI	-1.92	-1.93	-3.21	-3.23
Germany	CPI	-2.00	-1.89	-3.15	-3.23
	WPI	-1.76	-1.80	-3.29	-3.43
Italy	CPI	-1.72	-2.05	-3.17	-3.29
	WPI	-1.23	-1.69	-3.22	-3.46
Japan	CPI	-1.92	-2.82	-3.51	-3.51
	WPI	-2.87	-2.01	-3.82	-3.88
Netherlands	CPI	-1.98	-1.98	-3.17	-3.18
	WPI	-1.66	-1.58	-3.21	-3.33
Sweden	CPI	-1.59	-1.61	-3.03	-3.19
	WPI	-1.28	-1.23	-3.35	-3.54
Switzerland	CPI	-2.25	-2.31	-3.61	-3.59
	WPI	-2.07	-2.07	-3.84	-3.87
United Kingdom	CPI	-2.29	-2.32	-3.33	-3.35
	WPI	-1.36	-2.13	-3.28	-3.33

Note: The exchange rates are real bilateral U.S. dollar rates of the countries listed in column 1, and the price levels used to construct a real rate - CPI or WPI - are defined in column 2. The numbers in the columns headed t and τ are, respectively, the estimated t-ratios from (29') when a constant (t) and constant plus time trend (τ) are included as the deterministic variables. The two columns under L denote a test for a unit root in the level of the real exchange rate while the two columns under ΔL denote a test for a unit root in the first difference of the real rate. The 5 percent critical values for t and τ are, respectively, -2.79 and -3.09 (source Dickey and Fuller (1981)).

detecting the 'true' underlying relationship. We now consider a number of ways in which the above unit root tests are not the last word on the stochastic properties of the real exchange rate.

1. <u>The span of the data--low frequency and cross sectional aspects</u>

It has long been recognized that relatively high frequency data, such as monthly data, has a low signal-to-noise ratio compared to annual data. Thus, in considering two data sets, each containing the same series and each having the same number of observations, the only difference being the observational frequency, a researcher should presumably always choose the lower frequency data set since it will have a higher informational content. This kind of argument has been formalized in the context of unit root testing by Shiller and Perron (1985).

One of the first tests of the importance of the span of the data on the mean-reverting properties of real exchange rates was conducted by Frankel (1986, 1988). Frankel demonstrated that when he moved from a monthly post Bretton Woods data base to a long-run annual data base, for the U.K. pound--U.S. dollar (period 1869-87), the estimated ρ coefficient from (10) changed from being insignificantly different from unity to significantly below unity. Interestingly, though, the real exchange rate series, even over this long time span, contained considerable persistence in the sense that only 16 percent of a deviation is extinguished per year. Grilli and Kaminsky (1991) also utilize annual data for the U.S. dollar-U.K. pound real exchange rate, over the period 1885-86, and test for a unit root using Phillips-Perron adjusted Dickey-Fuller statistics. 1/ They find that the null of a random walk is rejected for the full sample period but not for a variety of subsamples.

Kim (1990) employs annual data for a wider range of currencies than the studies of Frankel and Grilli and Kaminsky. In particular, he examines the U.S. bilateral real rates of the Canadian dollar, French franc, Italian lira, Japanese yen and U.K. pound for the period 1900-87 (using CPI data) and 1914-87 (using WPI data). He demonstrates that the null hypothesis of a random walk may be rejected in all cases apart for the CPI-based real Canadian dollar, yen and pound. An interesting finding in the light of the panel unit root tests discussed below is that the null of a unit root can only be rejected for the yen-dollar rate when a time trend is included in the regression. 2/ The results of Frankel, Grilli and Kaminsky and Kim would seem to support the view that having as long a time span as possible

---

1/ They also use variance ratio tests which are supportive of the Phillips-Perron tests--see our discussion below.

2/ Kim also conducts cointegration tests using this larger span of data and is able to reject the null of non-cointegration (using the Engle-Granger two-step method) in six out of ten currency/price combinations. Furthermore, he obtains estimated coefficients on the price ratios which are numerically very close to unit for all of the cointegrating sets.

is important in discriminating between unit root and near unit root behavior. 1/

In the context of testing for PPP, an alternative way of addressing the span of the data has been advocated by MacDonald (1988b). This paper, although not strictly speaking comparable to unit root testing methods of this section, exploits the span of the data in a novel way and one which is useful for motivating some panel unit root tests considered below. MacDonald suggests using annual average data on relative prices and exchange rates for the recent floating period; as we have argued, such data should be more appropriate for picking up the low frequency determinants of the exchange rate. However, one problem with this is that in contrast to the studies using, say, one hundred years of annual data, it severely constrains the available degrees of freedom. 2/ MacDonald therefore advocates pooling across currencies to obtain the requisite degrees of freedom. The results are interesting in that the symmetry/homogeneity restrictions cannot be rejected. A representative result is reported here as equation (27)

$$\Delta s_t = \underset{(8.37)}{-1.199} - \underset{(3.58)}{1.040}\Delta(p - p^*)_t, \tag{27}$$

where the exchange rates are defined as the foreign currency price of a unit of home currency (in particular, these are U.S. dollar--domestic currency rates) and therefore the coefficient on relative prices is expected to be minus one rather than plus one. In fact, the hypothesis that the slope coefficient is minus one cannot be rejected (the appropriate t-ratio is 0.13). 3/ Flood and Taylor (1994) have also used a time averaged/pooled approach in analyzing PPP for the recent float, and their results basically corroborate those of MacDonald (1988b), although they find that the time averaging has to be implemented over a longer period to satisfy the homogeneity/symmetry restrictions.

The idea of increasing the data span of an annual data set by using pooled cross-section time series data has been applied to the construction of unit root tests by Quah (1990) and Levin and Lin (1994). The latter authors, for example, demonstrate that implementing a unit root test on a pooled cross-section data set, rather than performing separate unit root tests for each individual series, can provide 'dramatic improvements in statistical power'. The Levin-Lin test is designed to evaluate the null

---

1/ Some other papers which also use long runs of data to test for unit roots are considered in the next section.

2/ As the PPP relationship is parsimonious regarding degrees of freedom, it may be argued that even for the recent float there are sufficient degrees of freedom to estimate it using annual data.

3/ The pooled timewise autoregressive/cross section heteroscedastic-consistent estimator of Kmenta (1986) was used to estimate (27).

hypothesis that each individual series is I(1), versus the alternative hypothesis that all the series considered as a panel are stationary. The Levin-Lin test may be viewed as especially attractive since it facilitates the incorporation of a wide variety of individual-specific, or heterogeneous, effects under the null. Their testing method produces a single t-ratio for the panel and this statistic is shown to have a standard normal distribution. MacDonald (1994b) applies the Levin and Lin method to two panel data sets consisting of real exchange rates, defined using both WPI and CPI price measures, for the recent float. 1/ The results turn out to be similar to those using long runs of annual data: the null hypothesis that each real exchange rate contains a unit root is rejected in favor of the alternative hypothesis that real exchange rates are stationary. Our discussion of panel unit root tests has raised the issue of the power of unit root tests. We now focus our attention specifically on this issue.

2. The power of unit root tests

In the last section our explanation for the finding of a unit root in real exchange rates lay in the type of data set used by researchers. An alternative, although not mutually exclusive explanation, is to be found in the power of the kind of tests employed by the researchers noted above. As is now increasingly well known, one disadvantage of unit root tests based on (29') or variants thereof is that they have relatively low power to test alternatives of near stationary behavior (see Cochrane (1988) and Campbell and Perron (1992)). One way of thinking about this is to say that it may take a long time for real rates to exhibit mean-reverting behavior and such behavior will certainly not be picked up by the lag lengths conventionally used in an estimated version of (26). 2/ A better way of picking up long autocorrelations may be to use the variance ratio test, recently introduced into the economics literature by Cochrane. This test uses the insight that if a series does indeed follow a random walk (the null hypothesis) then the variance of the kth difference of the series should equal k times the first difference. That is, if equation (16) truly is the time series representation of the real exchange rate, then

$$Var\,(q_t - q_{t-k}) = k.\ Var(q_t - q_{t-1}), \tag{28}$$

or

1/ More specifically, the data sets are for OECD currencies and consist of 17 real rates based on the WPI and 23 real rates based on the CPI, over the period 1973 to 1992 (annual data).

2/ Fama and French (1987), for example, suggested that stock prices contain slowly decaying stationary components which induce negative serial correlation into long-holding-period returns.

$$V_k = \frac{1}{k}\,\frac{Var(q_t - q_{t-1})}{Var(q_t - q_{t-k})} = 1, \tag{28'}$$

where V_k denotes the variance ratio, based on lag k. 1/ Lo and MacKinlay have demonstrated that the variance ratio is asymptotically equal to 1 plus a weighted average of the first k-1 autocorrelation coefficients of $q_t - q_{t-1}$. If the average of these autocorrelations is zero, V_k will be unity. If, however, there is a preponderance of negative autocorrelations this will produce a value of V_k less than one and we have mean reversion. Conversely, if positive autocorrelations predominate this will give a value of V_k above one and we have super-persistence--a tendency for the series to cumulatively move above the mean. 2/ The key insight of the variance ratio test is that it may be necessary to include a large number of autocorrelations to pick up such mean reverting behavior. Standard unit root tests or tests of persistence based on short-term ARMA models (such as those proposed by Campbell and Mankiw (1987)) may fail to capture this mean reverting behavior.

Huizinga (1988) calculates the variance ratio test (28) for ten dollar bilateral exchange rates for part of the recent floating experience. He does not use the significance tests devised by Lo and MacKinlay, but rather uses the standard $T^{1/2}$ formula to construct standard errors. Huizinga reports evidence of mean reversion in an economic, or qualitative, sense. By this he means that for all currencies the variance ratio (with a ten year lag) is numerically below unity (the average across the ten currencies is 0.65 after ten years); however, none of the ratios are statistically different from unity at the 5 percent level. Interestingly, for lags up to around five years the variance ratio for all currencies is above unity, which indicates positive autocorrelation and what we have referred to as super-persistence: the plotted V_k's exhibit a hump-shaped profile. One problem, however, with the kind of variance ratio tests implemented by Huizinga is that under the null hypothesis it is assumed the errors in the autoregressive representation are iid. This may not in fact be a good working assumption given the evidence that exchange rates (both real and nominal) display heteroscedasticity.

Glen (1992) has calculated variance ratios, and the corresponding significance tests of Lo and MacKinlay, for a broad selection of U.S. dollar bilateral exchange rates using monthly data. For lags up to and including

---

1/ In an empirical implementation of the variance ratio test two sources of small sample bias must be corrected for. Cochrane (1988) provides a formula for an unbiased estimator of (28).

2/ Lo and MacKinlay (1989) have derived two test statistics which facilitate testing the significance of V_k (and, in particular, whether V_k is significantly different from unit or not). The first test statistic is calculated on the assumption that the error term in the real exchange rate process is iid; their second statistic is robust to non-iid errors.

32 months Glen finds values of V_k which are significantly above unity and he is therefore able to reject the random walk null in favor of the alternative of positive serial correlation, or super-persistence. Note, however, that this finding is not supportive of traditional long-run PPP since such super-persistence indicates that a shock to the real exchange rate, rather than pushing it back to its initial value, results in further movements in the same direction. Using an annual data set, however, for the period 1900-87, evidence of mean reversion is found after 4 years, and by 16 years the ratio is 0.433.

Glen's failure to find negative autocorrelation in the monthly data set, which is similar to that used in our multivariate cointegration tests, is rather worrisome since it suggests an important inconsistency in the two sets of results. This inconsistency may reflect the fact that Glen's lag horizon in the monthly data base is simply not long enough to pick up significant mean reversion. Huizinga, as we have noted, required 10 year lags--120 lags with monthly data--to produce mean reversion. In Tables 4 and 5 we therefore present our own estimates of V_k and the associated significance levels for our currency sets, using lags of 12 through to 120. In terms of the WPI (Table 5), we note that 6 out of the 8 currencies 1/ display the humped shaped pattern noted by Huizinga; Glen's results, therefore, seem to stem from too short a truncation of the lag length. Of the 6 currencies which exhibit mean reversion after 10 years, three of the variance ratios are significantly below unity at the 5 percent level (for the United Kingdom, Japan, and Switzerland). 2/ There is slightly less evidence of mean reversion for the real exchange rates based on CPI's (Table 4); 5 out of the 9 rates display mean reverting behavior after 10 years and 2 of these are significantly below unity. It is interesting to note the very different patterns in the Japanese yen rate using the CPI and WPI measures. Thus with the WPI, as we have seen, the variance ratio is significantly below unity by lag 120, whereas with the CPI it is still above unity (although not significantly so). This finding confirms Kim's (1990) result for the Japanese yen, noted above.

Abuaf and Jorion (1990) advocate testing for a unit root in real exchange rates by estimating (10) directly (rather than imposing a unit root on the testing method as in (26). They propose increasing the efficiency of the estimates by stacking the autoregressive equations for each country into a system (a ZSURE system) and estimating them jointly. Following this method, Abuaf and Jorion show that ρ lies in the range of 0.98 to 0.99 when monthly data are used. 3/ Although these point estimates are extremely close to one, they are not exactly one, indicating that there is some

---

1/ The French WPI data stops in 1985 and we have omitted it from our sample.

2/ The German mark is significant at the 10 percent significance level.

3/ With these numbers, it would take between 3 and 5 years for a 50 percent over-appreciation of a currency to be cut in half.

Table 4. Variance Ratio Statistics - Consumer Prices

		Lag Length										
		2	12	24	36	48	60	72	84	96	108	120
Canada	V	0.96	1.14	1.39*	1.17*	1.82*	1.78*	1.66*	1.52*	1.64*	2.02*	2.19*
France	V	0.98	1.33*	1.73*	2.03*	2.09*	2.14*	2.10*	1.97*	1.63*	1.31*	0.88
Germany	V	0.98	1.29*	1.66*	1.86*	1.91*	2.08*	2.15*	2.02*	1.58*	1.25	0.95
Italy	V	1.03	1.49*	1.79*	1.89*	2.00*	2.21	2.28*	2.09*	1.55*	1.37*	1.22
Japan	V	1.08	1.64*	1.86*	2.03*	1.97*	1.79*	1.83*	1.76*	1.35*	1.77	1.24
Netherlands	V	0.89	1.29*	1.62*	1.83*	1.84*	1.98*	2.01*	1.89*	1.50*	1.18	0.89
Sweden	V	1.04	1.19	1.61*	2.14*	2.44*	2.70*	2.85*	2.91*	2.61*	2.01*	1.23
Switzerland	V	1.03	1.26*	1.45*	1.46*	1.28*	1.37*	1.43*	1.27*	0.86*	0.65*	0.58*
United Kingdom	V	1.09	1.16	1.29*	1.54*	1.66*	1.49*	1.22	0.94	0.61*	0.45*	0.26*

Notes: The country names in column one denote the home country currency relative to the U.S. dollar. The entries in the rows labelled V are the estimates of (28') (with an appropriate small sample correction), and an * denotes significance at the 5 percent level, or better, on the basis of the Lo and MacKinlay (1988) Z2 statistic. The latter statistic is a test for deviations of V from unit, and is robust to non-lid errors. The numbers at the top of each column denote the lag length used to construct the variance ratio.

Table 5.   Variance Ratio Statistics - Wholesale Prices

	Lag Length										
	2	12	24	36	48	60	72	84	96	108	120
Canada	V 0.92	0.74*	0.83	0.93	0.98	1.00	0.97	0.94	0.98	1.06	1.11
Germany	V 1.00	1.08	1.34*	1.40*	1.39*	1.53*	1.56*	1.39*	1.00	0.83	0.75
Italy	V 0.98	1.22	1.52*	1.58*	1.66*	1.89*	1.95*	1.81*	1.41*	1.22*	1.07
Japan	V 0.98	1.13	1.19	1.26*	1.27*	1.13	1.07	0.95	0.66*	0.06*	0.55*
Netherlands	V 0.98	1.12	1.43*	1.59*	1.66*	1.83*	1.87*	1.69*	1.27*	1.01	0.87
Sweden	V 0.95	0.93	1.17	1.49*	1.66*	1.79*	1.82*	1.71*	1.44*	1.17	0.84
Switzerland	V 0.99	1.08	1.20	1.21	1.13	1.26*	1.24	1.01	0.64*	0.54*	0.51*
United Kingdom	V 1.08	1.18	1.33*	1.52*	1.62*	1.53*	1.43*	1.18	0.63*	0.53*	0.48*

Notes: See Table 5.

evidence of mean reversion. 1/ This may therefore suggest that it is the time averaging methods of MacDonald and Flood and Taylor which is the important factor in producing a satisfactory PPP result. Instead of moving to such pooled average data, however, Abuaf and Jorion move to using a long time span of annual data (1901-72) and report an average slope coefficient of around 0.78. This allows statistical rejection of the null of randomness and defines a half-life of 3.3 years, similar to that implied by the monthly data base.

Diebold, Husted and Rush (1991) propose a testing method which combines a long time span of data and the method of fractional differencing. The existence of a fractional difference (that is, a value of the difference operator, d, which lies between 0 and unity, rather than being exactly one), in the time series properties of a variable implies that the variable is stationary. Using data on 16 real exchange rates for the gold standard period (for most rates one hundred years of data was available) and a *maximum likelihood estimator, they find considerable evidence that d is* significantly below unity; indeed, in some instances the value of d is insignificantly different from zero. Such evidence is in accord with the view that real rates display mean-reverting behavior (indeed the results where d=0 are consistent with the level of the rate being stationary--a rather strong result). One telling piece of discussion in the paper is that standard unit root tests were unable to discriminate between a unit root and near unit root behavior even with such long spans of data. This tends to suggest that it is the type of test employed, rather than the observational frequency that is important.

Whitt's (1992) alternative to the standard unit root test is a Bayesian test, due to Sims (1988). The Sims-Bayesian approach demonstrates that the prior implicit in the classical unit root approach gives excessive weight to the unit root null. 2/ This alternative approach puts a prior on the autoregressive coefficient, r, which spreads the probability, α (where $0< \alpha <1$), of observing ρ, evenly between 0 and 1. The probability of observing a unit root (ρ=1) is $1-\alpha$, which as Whitt notes gives a limited advantage to the unit root hypothesis. Whitt employs two data bases to implement the Sims-Bayesian approach. In particular, the bilateral U.S. dollar rates of the French franc, German mark, Japanese yen, and Swiss franc (based on both CPI and WPI) real exchange rate data for two sample periods: one post Bretton Woods (monthly observational frequency), the other a period encompassing both Bretton Woods and post-Bretton Woods (annual observational frequency). Using a value for a of 0.8, Whitt is able to reject the unit root null for each of the real *exchange rates considered in each of the two* sample periods (although the rejection appears more straightforward for real

---

1/ See Moore (1993) for a critique of the method used by Abuaf and Jorion, (1990).

2/ Indeed, it gives substantial and excessive weight to values of ρ above unity; see Sims and Uhlig, 1988, p. 8).

rate measures using the WPI, perhaps indicating the problems, in terms of the traded/nontraded mix, in more general price series such as the CPI).

3. The real exchange rate and cointegration

As noted in Section 2.2, testing for a unit root in real exchange rates may be interpreted as a rather strict test of PPP. In particular, the condition that forces the real exchange rate to follow a random walk with drift is that ex ante real interest rates are equalized across countries. If one uses a sample period in which long term capital flows have equalized real rates then this is not a bad assumption to make. However, for the kinds of sample periods that researchers conventionally use this is probably an unreasonable assumption; long term capital flows do not, even on an average basis, reach a long-run equilibrium. One way of capturing the effect of capital flows on the real exchange rate would be to estimate a version of equation (13′), re-expressed here as a regression equation, 1/

$$q_t = \alpha + \beta(r - r^*)_t + \varphi_t. \tag{29}$$

This equation may be derived from (13′) by assuming the q^e_{t+k} term is constant 2/ and expectations are formed rationally. This basic relationship has been tested using the Engle-Granger two step method by Meese and Rogoff (1988), Coughlin and Koedijk (1990) and Edison and Pauls (1993) for the recent float. For example, for dollar-mark, dollar-pound and dollar-yen, over the period February 1974 through December 1985, Meese and Rogoff (1988) test (29) using the Engle-Granger two-step method and fail to find any evidence of cointegration. Edison and Pauls also estimate (29) for the dollar effective rate, over the period 1974, quarter 3 through to 1990, quarter 4. Using a variety of different proxies for expected inflation they cannot reject the null of no cointegration for the trade weighted value of the dollar and the bilateral U.S. dollar rates of the mark, yen, pound sterling and Canadian dollar. Furthermore, they also fail to reject the null of no cointegration when the potential non-constancy of $E_t\, q_T$ is allowed for (by assuming that $E_t\, q_T$ is a function of the cumulated current account and introducing this as an extra explanatory variable). The only paper in this genre to find some evidence of cointegration for (29) is that of Coughlin and Koedijk (1990), who report cointegration between the German mark-U.S. dollar rate and the real interest rate differential.

There are, however, at least two major problems with this kind of test, each of which may explain the failure to reject the null of noncointegration. First, we know from the nominal cointegration tests that

---

1/ One can think of this relationship as extending the cointegrating set. Thus to the extent that unit root tests on q indicate an absence of cointegration, adding in additional appropriate non-stationary variables to the cointegrating set may produce a cointegrating relationship.

2/ The plausibility of this assumption is doubtful and is one we discuss further below.

the real exchange rate is a stationary process and there is also evidence (noted below) to suggest that the real interest differential is stationary. What sense then does it make to model the real rate differential as an I(1) process (which is what is required for a cointegration test based on (29)). Second, all of the above-noted tests have involved the Engle-Granger two-step method; it is now widely accepted that this method is not a particularly powerful test of the null hypothesis of no cointegration when it is in fact false.

4. The real interest rate/exchange rate link--some further evidence

Before closing this section it is worth discussing some other research which examines the real exchange rate/real interest rate link using methods other than cointegration. Meese and Rogoff (1988) treat (29) as a standard regression equation and, assume that both of q and r-r^* are nonstationary (although, as we have noted this is not uncontroversial), regress the first difference of the real rate on the first difference of the real interest differential. 1/ Not only are Meese and Rogoff unable to find a value of β which is significantly above unity (for the three currencies noted above), they cannot reject the hypothesis that β is insignificantly different from zero. They also try 'sharpening up' the specification by modelling the equilibrium real rate (assumed equal to q^e_{t+k}) using home and foreign trade balances. However, these extended regressions do not result in any satisfactory improvement in the estimates.

A conclusion similar to that proposed by Meese and Rogoff is given by Campbell and Clarida (1987). Using an unobserved components model they demonstrate that the majority of movements in the real exchange rate (at least 79 percent) are driven by movements in the permanent component of the real rate (that is, $\bar{q}$-q^e_{t+k} the long-run component) and a very small component is due to real interest differentials (this is shown never to exceed 9 percent). Campbell and Clarida do, however, find that the implied value of β is greater than one in absolute terms, implying that the real exchange rate is more volatile than the real interest differential by a factor of about ten.

Baxter (1994) forcefully argues that the failure of studies like Meese and Rogoff and Campbell and Clarida to uncover any worthwhile relationships between real exchange rates and real interest differentials is due to the particular interpretation of the relationship. For example, by using a first difference transformation Meese and Rogoff presume that the relationship between the variables relates to the permanent elements (that

---

1/ The real interest rates are defined on an *ex post* basis; that is by subtracting the *ex post* realized inflation rate from the nominal interest rate. As shown in McCallum (1976), such an approach introduces a moving average error structure into the estimated equation and the OLS standard errors are corrected using a Generalized Method of Moments correction (which also corrects for heteroscedasticity).

is, assuming a unit root representation presuyposes that any change in each of the variables is a permanent change). As Baxter notes, however, the first difference operator, although removing the unit root from an economic time series, also removes most of the other low-frequency information. Moreover, she demonstrates that the key correlation (or prediction form the sticky price model) between q and r-r is between the temporary components of the real rate and the real differential. 1/ Using univariate and multivariate Beveridge and Nelson (1981) decompositions of the real exchange rate Baxter demonstrates statistically significant values for β, especially when the multivariate decomposition is employed.

VI. Concluding Comments

In this paper we have surveyed the recent empirical literature on the existence of a long-run exchange rate relationship. This literature has had something of a symbiotic relationship with recent developments in the time series literature and, in particular, the literature on cointegration. In summary, the literature presented in this paper, and our own empirical results, strongly suggest the existence of "some form" of long-run exchange rate relationship. 2/ The qualification reflects the fact that although real exchange rates appear to display mean-reverting behavior, and nominal exchange rates to be cointegrated with relative prices, the degree of mean eversion appears to be rather slow and the exchange rate/relative price relationship does not exhibit degree one homogeneity in the majority of cases. There would therefore seem to be 'something in the entrails' 3/ of the traditional PPP relationship that is unexplained. In fact, this statement is probably most relevant for the recent floating experience; with longer time spans of data the long-run exchange rate more closely conforms to traditional PPP. What then remains to be explained in the PPP relationship for the recent floating period?

We would argue that the explanation lies in the distinction between the concepts of 'true' and 'statistical' equilibrium, noted in Section 2.3. The true equilibrium is one which accords to what most economic models would posit as a long-run solution. Thus, the sticky-price monetary model of Dornbusch (1976) and the flex-price model of Frenkel (1976) and Hodrick (1978) both have absolute PPP as their long-run solution. 4/ The statistical equilibrium is simply the one which is captured by the particular econometric or statistical technique used to estimate the

---

1/ This framework makes clear that it does not make sense to test for cointegration between the level of a real exchange rate and the real interest differential.

2/ This evidence seems strongest for relationships based on the wholesale price measure, probably because the traded good component is much larger than for the CPI.

3/ MacDonald and Marsh (1994).

4/ In the flex-price monetary model the long-run holds continuously.

long-run exchange rate. Thus the Johansen method captures the mean-reverting properties of the nominal exchange rate with respect to relative prices for the recent floating experience, but it fails to capture the symmetry and homogeneity restrictions required by the true long-run equilibrium. The two concepts may, however, be reconciled by an explicit recognition of the distinction when undertaking an actual estimation. Thus in estimating PPP for the recent floating experience a suitable estimating equation should be derived from an equilibrium condition which conforms to the period studied, rather than from a 'true' equilibrium condition. Following MacDonald and Marsh (1994), we would argue that the relevant equilibrium for a period such as the recent floating period is the balance of payments equilibrium condition introduced in Section 3.

With freely floating exchange rates the exchange rate should move to ensure that the sum of the current and capital accounts, or the change in reserves, is equal to zero. However, most theoretical models of exchange rate determination would define a true long-run equilibrium as one in which the current account equals zero (and by implication net capital flows are zero). It is our contention that for a sample period such as the recent float, net capital flows will not go to zero and, therefore, they should be explicitly recognized in modeling the measure of the long-run exchange rate currently adopted in the literature. Of particular importance in this regard are long-term capital flows, which reflect productivity and thrift factors and also expected inflation, and which imply that a relationship which conditions exchange rates solely on relative prices will not tell the full story. MacDonald and Marsh (1994) have exploited this type of distinction for the recent float for the U.S. dollar bilateral rates of the German mark, Japanese yen and U.K. pound. They find that conditioning the exchange rate on relative prices and long-term bond yields produces cointegration and, crucially, a failure to reject the homogeneity and symmetry of the exchange rate with respect to relative prices. MacDonald and Marsh then use the estimated long-run relationships to produce short to medium-run dynamic exchange rate models which perform significantly better (in a statistical sense) than a simple random walk in terms of their out-of-sample forecasting accuracy. We believe that this kind of approach merits further attention especially when a researcher is limited to data from the recent floating experience. More generally, the recent success in modelling both short and long-run exchange rates is encouraging and should serve as a healthy counterpoint to the recent move away from fundamentals towards nonfundamental explanations of exchange rate behavior, such as chartism and market microstructure.

References

Abuaf, N., and P. Jorion, Purchasing power parity in the long run, The Journal of Finance, Vol. 45 (1990), pp. 157-74.

Adler, Michael and Bruce Lehmann, "Deviations from Purchasing Power Parity in the Long Run," The Journal of Finance, Vol. 38 (1983), pp. 1471-83.

Ahn, S.K., and G.C. Reinsel, Nested Reduced-Rank Autoregressive Models for Multiple Time Series, Journal of American Statistical Association, Vol. 83 (1988), pp. 849-56.

Baillie, Richard T., and David D. Selover, "Cointegration and Models of Exchange Rate Determination," International Journal of Forecasting, Vol. 3 (1987), pp. 43-52.

Banerjee, Aninda, Juan Dolado, David Hendry, and Gregor Smith, "Exploring Equilibrium Relationships in Econometrics Through State Models: Some Monte Carlo Evidence," Oxford Bulletin of Economics and Statistics, Vol 42 (1986), pp. 37-48.

Banerjee, Anindya, Juan Dolado, John Galbraith, and David Hendry Co-Integration, Error Correction, and the Econometric Analysis of Non-Satationary Data, Oxford: Oxford University Press (1993).

Baxter, Marianne, "Real Exchange Rates and Real Interest Rate Differentials: Have we Missed the Business-Cycle Relationship?" Journal of Monetary Economics, Vol. 33 (1994), pp. 5-38.

Bilson, John F.O., "Rational Expectations and the Exchange Rate," in J.A. Frankel, and H.G. Johnson (eds), The Economics of Exchange Rates, Reading (Massachusetts: Addison-Wesley, 1978), pp. 75-96.

Boothe, Paul and Debra Glassman "Off the Mark: Lessons for Exchange Rate Modeling," Oxford Economic Papers, Vol. 39 (1987), pp. 443-57.

Breuer, Janice, "Purchasing Power Parity: A Survey of and Challenge to Recent Literature," in Fundamental Equilibrium Exchange Rates (ed.) John Williamson (1994), forthcoming.

Buiter, Willem H., and Marcus H. Miller, "Monetary Policy and International Competitiveness: The Problem of Adjustment," Oxford Economic Papers, Vol. 33 (1981), pp. 143-75.

Campbell, John, and Richard Clarida, "The Dollar and Real Interest Rates," Carnegie-Rochester Series on Public Policy, Vol. 27 (1987), pp. 103-40.

———, and Gregory Mankiw, "Are Output Fluctuation Transitory?," Quartely Journal of Economics, 102 (1987), pp. 857-80.

———, and Pierre Perron, "Pitfalls and Opportunities: What Macroeconomists Should Know About Unit Roots," in O.J. Blanchard and S. Fischer (eds.), NBER Economics Annual 1991, MIT Press (1991).

Cheung, Y-W., and Lai, K.S., "Long-Run Purchasing Power Parity During the Recent Float," Journal of International Economics, Vol. 34 (1993), pp. 181-93.

Choudry, T., R. McNown, and M. Wallace, "Purchasing Power Parity and the Canadian Float in the 1950s," Review of Economics and Statistics, LXXIII, Vol. 3 (1991), pp. 558-62.

Cochrane, John H., How big is the Random Walk in GNP?, Journal of Political Economy, Vol. 96 (1988), pp. 893-920.

Coughlin, C., and Kees Koedijk, "What Do We Know About the Long-Run Real Exchange Rate?, St. Louis Federal Reserve Bank Review, Vol. 72, pp. 36-48.

Darby, M., Does Purchasing Power Parity Work?, NBER Working Paper, No. 607 (1980).

Dickey, David, and W.A. Fuller "Distribution of the Estimators for Autoregressive Time Series with a Unit Root," Journal of the American Statistical Association, Vol. 74 (1979), pp. 427-31.

Diebold, F.X., S. Husted, and M. Rush, 'Real Exchange Rates Under the Gold Standard,' Journal of Political Economy, Vol. 99 (1991), pp. 1252-71.

Dornbusch, Rudiger, "Expectations and Exchange Rate Dynamics," Journal of Political Economy, Vol. 84 (1976), pp. 1161-76

Edison, Hali (1985), Purchasing Power Parity: A Quantitative Reassessment of the 1920s Experience, Journal of International Money and Finance, Vol 4 (1985), pp. 361-72.

———, Edison Hali, and Dianne Pauls, "A Re-Assessment of the Relationship Between Real Exchange Rates and Real Interest Rates: 1974-90," Journal of Monetary Economics (1993), pp. 165-88.

Enders, Walter, "ARIMA and Cointegration Tests of PPP Under Fixed and Flexible Exchange Rates," Review of Economics and Statistics (1988), pp. 505-08.

Engle, Robert, and Clive Granger, "Cointegration and Error Correction: Representation, Estimation and Testing," Econometrica, Vol. 55 (1987), pp. 251-76.

———, and Bong Yoo, "Forecasting and Testing in Co-Integrated Systems," Journal of Econometrics, Vol 35 (1987), pp. 143-59.

Fama, Eugene and Kenneth French, "Permanent and Temporary Components of Stock prices," Journal of Political Economy (1987).

Faruqee, Hamid, "Long-Run Determinants of the Real Exchange Rate: A Stock-Flow Equilibrium Approach," IMF Working Paper No. 94/90 (Washington: International Monetary Fund, August 1994).

Fisher, and J.Y. Park (1991), 'Testing Purchasing Power Parity under the Null Hypothesis of Cointegration,' The Economic Journal, Vol. 101, pp. 1476-484.

Flood, Robert and Mark Taylor, "Exchange Rate Economics: What's Wrong With the Conventional Macro Approach?," (mimeographed: International Monetary Fund, 1994).

Frankel, Jeffrey A., "On the Mark: A Theory of Floating Exchange Rates Based on Real Interest Differentials," American Economic Review (1979), pp. 610-22.

———, "International Capital Mobility and Crowding-Out in the U.S. Economy: Imperfect Integration of Financial Markets or of Goods Markets," in R. Hafer, ed. How Open is the U.S. Economy? (Lexington, Massachusetts: Lexington Books, 1986).

———, "International Capital Flows and Domestic Economic Policies," in M Feldstein, ed. The United States in the World Economy (Chicago: Chicago University Press, 1988).

———, Andrew Rose, "A Survey of Empirical Research on Nominal Exchange Rates" NBER Working Paper No. 4865 (September 1994).

Frenkel, Jacob, "A Monetary Approach to the Exchange Rate: Doctrinal Aspects and Empirical Evidence," Scandinavian Journal of Economics, Vol. 78, No. 2 (1976) pp. 200-24.

Frenkel, J.A., "Exchange Rates, Prices and Money, Lessons From the 1920s," American Economic Association, Papers and Proceedings, Vol. 70, (1980) pp. 235-42.

Frenkel, Jacob A., 'Flexible Exchange Rates, Prices and the Role of "News": Lessons from the 1970s,' Journal of Political Economy, Vol. 89, No. 4 (1981), pp. 665-705.

Frenkel, Jacob, and Michael Mussa, "Exchange Rates and the Balance of Payments," in R. Jones, and P. Kenen, editors, Handbook of International Economics (Amsterdam: North Holland, 1988).

Froot, Kenneth A., and Kenneth Rogoff, "Perspectives on PPP and Long-Run Real Exchange Rates," (mimeographed, 1994).

Fuller, W.A, Introduction to Statistical Time Series (New York: John Wiley, 1976).

Glen, J., 'Real Exchange Rates in the Short, Medium and Long-Run,' Journal of International Economics, Vol. 33 (1992), pp. 147-66.

Goldstein, Morris, and Mohsin Khan, "Income and Price Effects in Foreign Trade," in R.W. Jones, and P.B. Kenen, editors, Handbook of International Economics, Vol. 2 (Amsterdam: North-Holland, 1985).

Grilli, Vittorio, and Graciela Kaminsky, "Nominal Exchange Rate Regimes and the Real Exchange Rates: Evidence for the United States and Greta Britain, 1885-1986," Journal of Monetary Economics, Vol. 27 (1991), pp. 191-212.

Hallwood, Paul, and Ronald MacDonald, "International Money and Finance, Second Edition (Oxford: Blackwell, 1994).

Hansen, Bruce E., "Tests for Parameter Instability in Regressions with I(1) Processes," Journal of Business and Economic Statistics, Vol. 10 (1992) pp. 321-35.

Hodrick, Robert J., "An Empirical Analysis of the Monetary Approach to the Determination of the Exchange Rate," in J.A. Frenkel, and H.G. Johnson, editors, The Economics of Exchange Rates (Reading, Massachusetts: Addison-Wesley, 1978) pp. 97-128.

Hooper, P., and Morton, J., 'Fluctuations in the Dollar: A Model of Nominal and Real Exchange Rate Determination,' Journal of International Money and Finance, Vol. 1, No. 1 (1982), pp. 39-56.

Huizinga, J., "An Empirical Investigation of the Long-Run Behaviour of Real Exchange Rates," in K. Brunner, and A.H. Meltzer, editors, Carnegie Rochester Conference Series on Public Policy (Amsterdam: North Holland, 1987), pp. 149-214.

Isard, P., "How Far Can We Push the Law of One Price?," American Economic Review, Vol. 67 (1977), pp. 942-8.

Johansen, Soren, "Statistical Analysis of Cointegrating Vectors," Journal of Economic Dynamics and Control, Vol. 12 (1988), pp. 231-54.

———, "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models," Econometrica, Vol. 59, No. 6 (1991), 1551-80.

Johansen, S., and K. Juselius, 'Maximum Likelihood Estimation and Inference on Cointegration--With Applications to the Demand for Money,' Oxford Bulletin of Economics and Statistics, Vol. 52 (1992), 169-210.

Kearney, Colm, and Ronald MacDonald, "Rational Expectations, Bubbles and Monetary Models of the Exchange Rate: The Australian/U.S. dollar Rate During the Recent Float," Australian Economic Papers (June 1990), pp. 1-20.

Kim, Yoonbai, "Purchasing power Parity in the Long-Run: A Cointegration Approach," Journal of Money Credit and Banking Vol. 22, No. 4, (1990), pp. 491-503.

Kmenta, J., "Elements of Econometrics," 2nd Ed., London (1986), pp. 617-618.

Kugler, Peter, and C. Lenz, "Multivariate Cointegration Analysis and the Long-Run Validity of PPP," Review of Economics and Statistics, LXXV (1993), pp. 180-87.

Krugman, Paul, "Purchasing Power Parity and Exchange Rates: Another Look at the Evidence," Journal of International Economics, Vol. 8 (1978), pp. 397-407.

Levin, Andrew, and Chien-Fu Lin, "Unit Root Tests in Panel Data: Symptotic and Finite Sample Properties," (mimeographed, Federal Reserve Board of Governors, 1993).

Lo, Andrew, and Craig MacKinlay, "Stock Prices Do Not Follow Random Walks: Evidence Form a Simple Specification Test," The Review of Financial Studies, Vol. 1 (1988), pp. 41-66.

MacDonald, Ronald, "Do Deviations of the Real Effective Exchange Rate Follow a Random Walk?," Economic Notes, Vol 14 (1985a), pp. 63-9.

———, "Are Deviations from Purchasing Power Parity Efficient?: Some Further Answers," Weltwirtschaftliches Archiv, Vol. 121, No. 4 (1985b), pp. 638-45.

———, Floating Exchange Rates: Theories and Evidence (London: Unwin Hyman, 1988a).

———, "Purchasing Power Parity: Some 'Long-Run' Evidence From the Recent Float," De Economist, Vol. 136 (1988b), pp. 239-52.

———, 'Exchange Rate Economics: An Empirical Perspective,' in G. Bird, ed., The International Financial Regime (Surrey University Press, 1990).

———, 'Long-Run Purchasing Power Parity: Is it for Real?," Review of Economics and Statistics (November 1994), pp. 690-95.

———, 'Asset Market and Balance of Payments Characteristics: An Eclectic Approach to Modeling the Exchange Rate (mimeographed, 1994a).

———, 'Random Walks, Real Exchange Rates and Panel Unit Root Tests,' (mimeographed, 1994b).

———, and Ian W. Marsh, "On Long- and Short-Run Purchasing Power Parity," eds., J. Kaehler P. Kugler, Econometric Analysis of Financial Markets (Heidelberg: Physica-Verlag, 1994), pp. 23-46.

———, and ———, "On Casselian PPP, Cointegration and Exchange Rate Forecasting," (mimeographed, University of Strathclyde, 1994).

———, and Michael Moore, 'Purchasing Power Parity and Structural Change,' mimeo, University of Strathclyde, 1994.

———, and Mark P. Taylor, "The Monetary Model of the Exchange Rate: Long-Run Relationships and Coefficient Restrictions," Economics Letters, Vol. 37 (1991), pp. 179-85.

———, and ———, "Exchange Rate Economics: A Survey," International Monetary Fund Staff Papers, Vol. 39, No. 1 (1992), pp. 1-57.

———, and ———, "The Monetary Approach to the Exchange Rate: Rational Expectations, Long-Run Equilibrium and Forecasting," International Monetary Fund, Staff Papers, Vol. 40 (1993), pp. 89-107.

———, and ———, "The Monetary Model of the Exchange Rate: Long-Run Relationships, Short-Run Dynamics and How to Beat a Random Walk," Journal of International Money and Finance, Vol. 13 (1994), pp. 276-90.

MacKinnon, James, "Critical Values for Co-integration Tests," in R.F. Engle, and C.W.J. Granger, editors, Long-Run Economic Relationships, (Oxford: Oxford University Press, 1991).

Mark, Nelson, "Real and Nominal Exchange Rates in the Long-Run: An Empirical Investigation," Journal of International Economics (1990), pp. 115-36.

Meese, Richard, "Testing for Bubbles in Exchange Rates: A Case of Sparkling Bubbles," Journal of Political Economy, Vol. 94, No. 2 (1987), pp. 345-73.

Meese, R.A., and Rogoff, K., 'Empirical Exchange Rate Mmodels of the Seventies: Do They Fit Oout of Sample,' Journal of International Economics, Vol. 14 (1983), pp. 3-24.

———, and ———, 'The Out of Sample Failure of Empirical Exchange Rate Models: Sampling Error or Misspecification?,' in J.A.tFrenkel, ed., Exchange Rates and International Macroeconomics (Chicago: NBER, 1984).

Meese, Richard, and Kenneth Rogoff, "Was it Real? The Exchange Rate-Interest Differential Relation over the Modern Floating-Rate Period," The Journal of Finance, Vol. XLIII, No. 4 (1988).

Moore, M.J. 'Systems of Bilateral Real Exchange Rates,' Applied Economics, Vol. 25 (1993), pp. 1161-66.

Moosa, I.A., "The Monetary Model of Exchange Rates Revisted," Applied Financial Economics, Vol. 4 (1994), pp. 279-87.

Mussa, M. 'Empirical Regularities in the Behaviour of Exchange Rates and Theories of the Foreign Exchange Market,' in Policies for Employment Prices, and Exchange Rates, ed., by K. Brunner, and A.H. Meltzer, Carnegie-Rochester Conference Series on Public Policy, supplement to the Journal of Monetary Economics, Vol. 11 (1979),. pp. 9-57.

Mussa, Michael, "The Theory of Exchange Rate Determination," in J.F.O. Bilson, and R. Marston, eds., Exchange Rate Theory and Practice, NBER Conference Report (Chicago: Chicago University Press, 1984).

Officer, Laurence. H., "The Purchasing Power Parity Theory of Exchange Rates: A Review Article," IMF Staff Papers, Vol. 23 (1976), pp. 1-60.

Patel, J., "Purchasing Power Parity As a Long-Run Relation," Journal of Applied Econometrics, Vol. 5 (1990), pp. 367-79.

Perron, P., and T.J. Vogelsang, "Non-Stationarity and Level Shifts with an Application to Purchasing Power Parity," Journal of Business and Economic Statistics, Vol. 10, No. 3 (1992), pp. 301-20.

Phillips, Peter, C.B., and Bruce, E. Hansen, "Statistical Inference in Instrumental Variables Regression with I(1) Processes," Review of Economic Studies, Vol. 58 (1990), pp. 407-36.

Quah, Danny, 'Exploiting Cross Section Variation for Unit Root Inference in Dynamic Data,' Economics Letters, Vol. 44 (1994), pp. 9-19.

Roll, Richard, "Violations of Purchasing Power Parity and Their Implications for Efficient International Commodity Markets," in M. Sarnat, and G. Szego, editors, International Finance and Trade 1, (Cambridge, Massachusetts: Ballinger, 1979).

Sims, Christopher, "Bayesian Skepticism on Unit Root Econometrics," Journal of Economic Dynamics and Control (1988), pp. 381-86.

———, and Harold Uhlig, "Understanding Unit Rooters: A Helicopter Tour," Discussion paper 4, Institute for Empirical Macroeconomics, Federal Reserve bank of Minneapolis (1988).

Stock, James H., "Asymptotic Properties of Least-Squares Estimators of Co-integrating Vectors," Econometrica, Vol. 55 (1987), pp. 1035-56.

Taylor, Mark P., "An Empirical Examination of Long-run Purchasing Power Parity Using Cointegration Techniques," Applied Economics, Vol. 20 (1988), pp. 1369-82

Whitt, J.A., 'The Long-Run Behaviour of the Real Exchange Rate: A Reconsideration,' Journal of Money Credit and Banking, Vol. 24, No. 1 (1992), pp. 72-82.