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Endogenous Time Preference and Endogenous Growth

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Abstract

The present paper develops a one-sector aggregate endogenous growth model with intertemporal preference dependence. The resultant model possesses the fundamental property of growth convergence, in the sense that countries with identical parameters regarding technology, preference, and government policy will converge to a steady state with the same (positive) growth rate. A notable tax policy implication of the model is that, even in the absence of externalities, the growth effects of an income tax are shown to be a priori ambiguous and dependent on the relative magnitudes of the tax rate and the tax elasticity of the savings rate.

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H20, O41

1/ The author thanks Bin Xu for helpful discussions. The views expressed herein are the author's and do not necessarily reflect those of the IMF or its staff.

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Summary

The paper develops a one-sector aggregate endogenous growth model with intertemporal preference dependence. In addition to its conceptual appeal, the endogenous time preference construct provides a rigorous basis for generating meaningful transitional dynamics even within the confines of a simple one-sector model. The model has two important attractive attributes: (i) in contrast to those of the neoclassical and most other endogenous growth models with constant time preferences, its transitional dynamics do not instantaneously vanish in the face of a perfect international capital market, because the implied intertemporal elasticity of substitution with a time-varying time preference changes along the path of transition to the steady state; and (ii) it avoids the necessity of employing more complex, multi-sector models in analyzing many growth issues, which invariably diminish analytical transparency and tractability.

The specific model examined possesses the fundamental property of growth convergence, that is, countries with identical parameters regarding technology, preference, and government policy will converge to a steady state with the same (positive) growth rate. At the same time, the model is not necessarily inconsistent with the large differences in cross-country growth rates which may be observed over prolonged periods of time, since such differences could well be attributable to the lengthiness of the transition, among other things. However, the plausibility of this explanation will not be known until quantitative simulations of the model based on specific parameter values have been carried out.

A notable tax policy implication of the model concerns the growth effects of an income tax. In virtually all endogenous growth models in the literature, an income tax, by reducing the net-of-tax return to capital, exerts a negative impact on growth, unless the tax revenue is used to finance some productive public good, such as infrastructure. In a model with endogenous time preference, income tax will still lower the net-of-tax return to capital at an unchanged rate of savings. However, the rate of savings will be raised, as the tax depresses consumption, which in turn has a positive impact of growth. Hence, even in the absence of externalities, the model shows that the growth effects of an income tax are ambiguous and dependent on the relative magnitudes of the tax rate and the tax elasticity of the savings rate. *At a minimum, this illustrates that the relationship* between income taxation and growth may not be as straightforward as the existing literature seems to suggest.

I. Introduction

There are broadly three strands of models in the recent literature on endogenous growth. 1/ One strand generates sustained, positive steady-state growth through the accumulation of rivalrous and excludable factors of production (for example, embodied human capital) that display constant returns in the aggregate, even though the underlying technology may be time-invariant (King and Rebelo (1990) and Rebelo (1991)). These models are compatible with a Pareto-optimal decentralized equilibrium because private and social returns do not diverge. A second strand emphasizes productivity shifts, brought about by the accumulation of nonrival and nonexcludable inputs (for example, general knowledge or disembodied human capital), in sustaining positive growth (Uzawa (1965), Romer (1986), and Lucas (1988)). Since externalities or spillover effects are present, a decentralized equilibrium in these models is sub-optimal, thus providing a normative basis for government intervention (Barro (1990) and Jones, Manuelli, and Rossi (1993)). The third strand also focuses on technological change as the engine of growth, but the advancement in technology is modeled as an increase in the number of new product designs (Romer (1987, 1990)). Because the benefits of new designs are at least partially excludable, a decentralized equilibrium in these models is characterized by some degree of monopolistic power.

While differing in their conceptual frameworks, all three strands of models share a common focus on the nature of the production process as the key to analyzing economic growth. In particular, the assumption of nondiminishing returns to the factor(s) which is (are) being accumulated is the critical element that enables them to break out of the Solow-Swan neoclassical straightjacket of zero per capita growth (or positive per capital growth with exogenous technological progress) in the steady state. With their focus exclusively placed on the production side, the preference functions in these models are, almost without exception, taken to be of the time-separable, constant-elasticity form with a constant subjective rate of discount. These preference functions allow a constant optimal rate of consumption growth as long as the interest rate is constant and exceeds the subjective discount rate, and are, therefore, compatible with positive steady-state growth.

The basic appeal of the preference functions of the type noted above lies in their apparent conformity with some of Kaldor's celebrated stylized facts of economic development (Romer (1989)). However, their use in endogenous growth models does imply--if these models are to be believed--that observed (large) differences in cross-country growth rates are entirely due to (large) differences in rates of returns to capital and not to

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1/ For a survey of this literature, see Barro and Sala-i-Martin (1992b) and Xu (1994).

differences in income levels. 1/ With international mobility of capital, rates of returns to capital are (instantaneously) equalized across countries, and all countries should, therefore, (instantaneously) grow at the same rate (Rebelo (1992)). This implication, which applies with equal force to common endogenous growth models both with and without transitional dynamics, is clearly not supported by available growth data. 2/

Although the existence of growth convergence has met with strong skepticism (Romer (1989)), there is now a growing body of empirical evidence which tends to affirm the basic convergence property of the neoclassical growth model, provided that the accumulation of human capital is properly taken into account (Baumol (1986), Barro (1991), Barro and Sala-i-Martin (1991, 1992a), and Mankiw, Romer, and Weil (1992)). For endogenous growth models to be consistent with these findings, not only must they possess transitional dynamics, these transitional dynamics must also survive the presence of international capital mobility so that they do not degenerate into the case of instantaneous convergence. 3/

The present paper develops a one-sector aggregate endogenous growth model with transitional dynamics. These dynamics arise from a modification to the standard specification of the preference function. Specifically, the model incorporates an endogenous time preference formulation as studied by Uzawa (1968), Epstein and Hynes (1983), Epstein (1987), and Obstfeld (1990). Despite the intuitive appeal of intertemporal preference dependence (surely most will accept as a matter of fact that one's valuation of future relative to current consumption cannot be the same at different levels of income), and several empirical studies of consumption behavior that have questioned the plausibility of intertemporal preference independence (see the citations and discussions in Obstfeld (1990)), conventional specifications of the consumer's intertemporal utility maximization problem continue to invoke the

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1/ Of course, growth rates can be different because of differences in the preference parameters. But resorting to this type of explanation is economically uninteresting (aside from being obvious).

2/ In models without transitional dynamics, such as Rebelo's (1991) one-sector *Ak* model, there is no growth convergence irrespective of whether economies are closed or open. In models with transitional dynamics, such as the two-sector model studied by Mulligan and Sala-i-Martin (1992), there is growth convergence in a closed-economy setting, but the convergence would be instantaneous if capital is internationally mobile.

3/ It is well known that (nondegenerate) transitional dynamics are present in the standard neoclassical growth model in a close-economy setting, although the recent study by King and Rebelo (1993) has cast doubt on their significance in explaining observed differences in cross-country growth rates. In an open-economy setting, however, even the neoclassical growth model cannot survive the test of complete capital mobility. But it can be rescued by the assumption that capital is only partially mobile (Barro, Mankiw, and Sala-i-Martin (1992)): mobility applies to physical capital (through lending and borrowing) but not to human capital.

constant time preference formulation for its mathematical simplicity. However, the justification for adhering to it, even on grounds of technical convenience, has been increasingly weakened by the growing number of studies which have demonstrated that the endogenous time preference formulation can provide a tractable theoretical framework for analyzing a variety of problems that cannot be meaningfully tackled if the time preference is constant (see, for example, its use by Obstfeld (1981, 1982) to achieve convergence to stationary equilibria in small, open economies with perfect capital markets; and by Shi and Epstein (1993) to study habit formation).

The endogenous growth model with endogenous time preference examined in the present paper suggests that countries with identical parameters regarding technology, preference, and government policy (for example, taxation) will converge to a steady state with the same (positive) growth rate. Moreover, this convergence will not be instantaneous even in the presence of a perfect international capital market, since the implicit intertemporal elasticity of substitution with time-varying pure subjective rate of time preference is not constant along the path of transition. 1/ Hence, it is possible for a model of this type to generate meaningful transitional dynamics necessary for convergence.

While the absence of externalities in the present model precludes any normative (efficiency) justification for government intervention, positive analyses of the growth effects of certain public policies, such as those related to taxation, can still be carried out. Because the consumer's time preference is endogenously determined, it could be affected by taxation, and could be affected differently by different taxes (such as a tax on income versus a tax on consumption). The changes in time preference induced by taxation in turn will affect savings, capital accumulation, and ultimately growth. It is shown below that the growth effects of an income tax in this model can be quite different from those conventional endogenous growth models with a constant time preference.

The general theoretical framework is laid out in the next section. Section III analyzes a specific version of the model. Tax policy implications are discussed in Section IV. Section V concludes the paper. Conditions for the existence of a stable steady state are provided in the Appendix.

II. The General Theoretical Framework

Consider an economy whose productive technology is given by

$$y(t) = f[k(t)],\ f' > 0,\ f'' \geq 0, \tag{1}$$

1/ Rebelo (1992) also obtained a nonconstant intertemporal elasticity of substitution by way of a Stone-Geary utility function.

where $y(t)$ is per capita output, $k(t)$ is per capita stock of capital, and t is a time index. Without loss of generality, it is assumed that the population size is constant and that capital does not depreciate. Capital, therefore, accumulates according to

$$\dot{k}(t) = y(t) - c(t), \quad k(0) \text{ given}, \tag{2}$$

where $c(t)$ is per capita consumption and a dot over a variable denotes its time derivative. In contrast to the standard neoclassical growth model, positive per capita output growth in the steady state is possible in this model because the marginal product of capital, as stipulated in equation (1), is nondiminishing.

An infinitely lived representative consumer seeks to maximize, subject to equations (1) and (2), his overall utility

$$U = \int_0^\infty v[c(t)]\cdot e^{-\rho(t)\cdot t}dt, \; v' > 0, \; v'' < 0, \; \rho(t) \geq 0 \text{ for all } t, \tag{3}$$

where v is his momentary utility function and $\rho(t)$ is a time-varying subjective rate of discount. If the consumer's pure subjective rate of time preference at any given instant s is given by $\phi(s)$, then $\rho(t)$ can be interpreted as the average pure subjective rate of time preference between the time interval 0 and t:

$$\rho(t) = (1/t)\cdot\int_0^t \phi(s)ds, \; \phi(s) \geq 0 \text{ for all } s. \tag{4}$$

The endogeneity of $\phi(t)$ is captured by the specification

$$\phi(t) = \phi[\gamma(t)], \tag{5}$$

where the pure subjective rate of time preference is postulated to depend on the rate of consumption, $\gamma(t) \equiv c(t)/y(t)$. If $\phi(t) = \phi$ is an exogenously given constant, then by equation (4) $\rho = \phi$ would also be a constant, and equation (3) would revert to the usual set-up. 1/ An important difference between the specification of $\phi(t)$ employed here and that found in the endogenous time preference literature is that the latter typically specifies $\phi(t)$ to be a function of the level of consumption (for example, Uzawa (1968) and Obstfeld (1990)). Such a specification, while workable in the neoclassical model in which per capita growth is zero in the steady state, is clearly not compatible with models that permit positive steady-state growth. Specifying $\phi(t)$ as a function of the rate, rather than the level, of

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1/ Sign restrictions on $\phi(t)$ are discussed in the context of a specific model in the next section.

consumption solves the technical problem of requiring $\phi(t)$ to be constant in the steady state.

Define a subjective time variable $\Delta(t)$ at any objective time instant t as the cumulated pure subjective rate of time preference, that is,

$$\Delta(t) = \rho(t)\cdot t. \tag{6}$$

It immediately follows, from equation (4), that

$$\dot{\Delta}(t) = \phi(t). \tag{7}$$

The consumer's maximization problem now involves two equations of motion, one related to capital accumulation (equation (2)) and the other to the accumulation of his pure subjective rate of time preference (equation (7)). Since the maximization problem is autonomous, it can be simplified by employing the Uzawa (1968) procedure that transforms its time dimension from t (objective time) to Δ (subjective time). Using equations (1) - (2) and (5) - (7), equation (3) can be rewritten as

$$U = \int_0^\infty \{v[\gamma(\Delta)\cdot y(\Delta)]/\phi(\Delta)\}\cdot e^{-\Delta}d\Delta, \tag{8}$$

while equation (2) can be similarly restated as

$$\frac{dk(\Delta)}{d\Delta} = \frac{y(\Delta)\cdot[1 - \gamma(\Delta)]}{\phi(\Delta)},\ k(0) \text{ given.} \tag{9}$$

The transformed problem is then one of maximizing equation (8), subject to equation (9), and the control variable is now the rate, rather than the level, of consumption. Its first-order conditions are derivable by use of the maximum principle. 1/ The current-value Hamiltonian is given by

$$H = [v(\gamma,k) + \lambda\cdot y\cdot(1 - \gamma)]/\phi, \tag{10}$$

where λ is the costate variable measuring the shadow price of capital. The necessary conditions for maximizing H are

$$\phi\cdot(v_\gamma - \lambda\cdot y) = [v + \lambda\cdot y\cdot(1 - \gamma)]\cdot\phi', \tag{11}$$

$$d\lambda/d\Delta = \lambda\cdot[1 - (1 - \gamma)\cdot f'/\phi] - v_k/\phi, \tag{12}$$

and the transversality condition

$$\lim_{\Delta\to\infty} e^{-\Delta}\cdot\lambda\cdot k = 0, \tag{13}$$

1/ For notational simplicity, the time index on all variables will be omitted as long as no ambiguity arises.

where υ_γ and υ_k are, respectively, $\partial\upsilon/\partial\gamma$ and $\partial\upsilon/\partial k$. Equation (11) can be used to solve for λ explicitly. After some algebraic rearrangement, it can be expressed as

$$\lambda(\gamma,k) = \frac{\phi\cdot\upsilon_\gamma - \phi'\cdot\upsilon}{[\phi + (1 - \gamma)\cdot\phi']\cdot y} \,. \tag{14}$$

Taking the time derivative of equation (14) yields

$$\dot{\lambda} = (\partial\lambda/\partial\gamma)\cdot\dot{\gamma} + (\partial\lambda/\partial k)\cdot\dot{k}, \tag{15}$$

which can be rearranged to get

$$\dot{\gamma} = [\dot{\lambda} - (\partial\lambda/\partial k)\cdot\dot{k}]/[(\partial\lambda/\partial\gamma)], \tag{16}$$

where $\dot{k}$ follows directly from equation (2):

$$\dot{k} = y\cdot(1 - \gamma). \tag{17}$$

The evolution of the costate variable, $\dot{\lambda}$, can be solved by first substituting equation (1) into equation (12), and then using equation (7) to reverse the earlier time transformation, to obtain

$$\dot{\lambda} = \lambda\cdot[\phi - (1 - \gamma)\cdot f'] - \upsilon_k. \tag{18}$$

Equations (16) and (17) are the two key equations of the model governing the behavior of, respectively, the rate of consumption and capital accumulation over time. As stated, however, they are in too general a form to be useful in understanding the mechanics of the growth process. To gain additional analytical insight into the above theoretical framework, the production function f, the momentary utility function υ, and the pure subjective time preference function ϕ are all given simple, specific functional forms.

III. A Specific Model

Rebelo's (1991) *Ak* model provides the simplest specification of the production function f which will permit positive steady-state growth:

$$y = A\cdot k,\ A > 0. \tag{19}$$

The assumption that the productive technology displays constant returns to scale in capital can be justified on grounds that k is a measure of broad concept of capital, encompassing human capital (among other things) in addition to physical capital (Barro (1990)). The momentary utility function is assumed to take the conventional constant elasticity form:

$$\upsilon = (\gamma\cdot y)^{1-\sigma}/(1-\sigma),\ 1 \neq \sigma > 0. \tag{20}$$

For the pure subjective time preference function ϕ, it is convenient for it to have the specification

$$\phi = \alpha \cdot \gamma^{1-\theta}, \ \alpha > 0, \tag{21}$$

so that ϕ has the constant elasticity $(1 - \theta)$ with respect to the rate of consumption. A crucial aspect of equation (21) relates to the stipulation of the sign of θ and its magnitude relative to σ. Technically, the endogenous time preference literature has long recognized that it is necessary for ϕ to be a positive function of the level of consumption (increasing impatience) to ensure convergence in infinite-horizon models (Lucas and Stokey (1984)). While the concept of increasing impatience to consume may not easily conform to one's intuition, at least at low consumption levels, about how a change in the current level of consumption would alter one's valuation of future relative to current consumption, Epstein (1987) and Shi and Epstein (1993) noted that it becomes more plausible at high consumption levels. In any event, its use in the literature has been clearly compelled more by technical reasons than by economic intuition. However, with ϕ now specified as a function of the rate of consumption in a framework in which the returns to capital are nondiminishing, it is no longer immediately obvious that increasing impatience remains a mathematical necessity. Nevertheless, in the context of the specific model analyzed here, the condition

$$\sigma > 1 - \theta > 1 \tag{22}$$

is sufficient to ensure strict concavity of the Hamiltonian at the optimal solution with respect to the control variable γ within the meaningful range $1 > \gamma > 0$ (see below). For this reason, condition (22) will be imposed from now on, which implies $\theta < 0$, or $\phi' > 0$, so that the behavior of ϕ conforms to the concept of increasing impatience.

It is readily verifiable from equations (19) - (21) that

$$f' = A > 0, \tag{23}$$

$$\upsilon_\gamma = \gamma^{-\sigma} \cdot y^{1-\sigma} > 0, \tag{24}$$

$$\upsilon_k = \gamma^{1-\sigma} \cdot A \cdot y^{-\sigma} > 0, \text{ and} \tag{25}$$

$$\phi' = \alpha \cdot (1 - \theta) \cdot \gamma^{-\theta} > 0. \tag{26}$$

Substituting equations (19) - (21) and (23) - (26) into (14) yields

$$\lambda = \frac{(\theta - \sigma) \cdot \gamma^{1-\sigma} \cdot y^{-\sigma}}{(1 - \sigma) \cdot [1 - \theta \cdot (1 - \gamma)]} > 0, \tag{27}$$

so that the shadow price of capital is positive. To check that the solution to the first-order condition of maximizing the Hamiltonian H with respect to

γ (equation (11)) indeed provides a maximum, partially differentiate equation (10) with respect to γ twice and evaluate it at the solution implied by equation (11) to obtain

$$\partial^2 H/\partial\gamma^2 = -(\lambda\cdot y\cdot[\phi' + \phi''\cdot(1 - \gamma)] + \alpha\cdot y^{1-\sigma}\cdot\gamma^{-\sigma-\theta}\cdot q)/\phi^2, \tag{28}$$

where $q \equiv -[\sigma\cdot(1 - \sigma) - \theta\cdot(1 - \theta)]/(1 - \sigma) > 0$ in view of condition (22).

Furthermore, since

$$\phi'' = -\alpha\cdot(1 - \theta)\cdot\theta\cdot\gamma^{-\theta-1} > 0, \tag{29}$$

it immediately follows from equation (28) that $\partial^2 H/\partial\gamma^2 < 0$, which indicates that H is strictly concave at the solution, given λ and k, as required.

From equation (27), it is straightforward to derive the following expressions:

$$\partial\lambda/\partial\gamma = \lambda\cdot[(1 - \sigma)\cdot(1 - \theta) - \sigma\cdot\theta\cdot\gamma]/\{\gamma\cdot[1 - \theta\cdot(1 - \gamma)]\} \text{ and} \tag{30}$$

$$\partial\lambda/\partial k = -\lambda\cdot\sigma/k. \tag{31}$$

Substituting equations (30) and (31) into equation (16) and using equations (17), (18), and (27), the change in the rate of consumption over time can be stated, after some algebraic manipulation, as

$$\dot{\gamma} = z\cdot(\phi - \Omega), \tag{32}$$

where $z \equiv \gamma\cdot[1 - \theta\cdot(1 - \gamma)]/[(1 - \sigma)\cdot(1 - \theta) - \sigma\cdot\theta\cdot\gamma] < 0$ by condition (22), and

$$\Omega \equiv A\cdot(1 - \sigma)\cdot[1 - \sigma\cdot(1 - \gamma)]/(\theta - \sigma). \tag{33}$$

The significance of equation (32) becomes clear when note is taken of the fact that the growth rate of capital, $g \equiv \dot{k}/k$, is expressible from equation (17) and with the use of equation (19) as

$$g = A\cdot(1 - \gamma). \tag{34}$$

Suppose capital grows at the constant rate g^* in the steady state. Then by equation (34) the rate of consumption in the steady state, γ^*, must be a constant. This in turn implies that $\dot{\gamma} = 0$ in the steady state. Hence, from equation (32), γ^* is that value for γ which satisfies

$$\phi = \Omega. \tag{35}$$

Once solved from equation (35), γ^* can be substituted into equation (34) to obtain the steady-state growth rate g^*.

Equation (35) is the fundamental equation for the steady-state solution in the present model. Due to its degree of nonlinearity, a closed-form solution cannot be obtained for γ^*. Nevertheless, the economic intuition underlying it can be easily illustrated graphically. Given that $\phi' > 0$ (equation (26)) and $\phi'' > 0$ (equation (29)), ϕ is upward sloping and strictly convex with respect to the origin, with $\phi = 0$ as $\gamma = 0$ and $\phi = \alpha$ as $\gamma = 1$. Moreover, $\phi'(\gamma = 0) = 0$ and $\phi'(\gamma = 1) = \alpha\cdot(1 - \theta)$. The complete behavior of ϕ over the range $1 \geq \gamma \geq 0$ is shown in Figure 1. For the behavior of Ω over the same range, note that from equation (33),

$$\Omega' = A\cdot(1 - \sigma)\cdot\sigma/(\theta - \sigma) > 0, \tag{36}$$

and that $\Omega = A\cdot(1 - \sigma)^2/(\theta - \sigma) < 0$ as $\gamma = 0$ and $\Omega = A\cdot(1 - \sigma)/(\theta - \sigma) > 0$ as $\gamma = 1$. This is illustrated in Figure 2. A simple economic interpretation can be given to Ω. In any given instant t, the physical marginal product of capital is A. However, with increasing impatience to consume, the "subjective" value of A rises as the rate of consumption increases. Hence, equilibrium is achieved only when this subjective value of the marginal product of capital, given by Ω, is equated with the pure subjective rate of time preference, ϕ.

Since both ϕ and Ω slope upward with respect to γ, the properties of a stable steady-state solution, if it exists, must be ascertained from the model's transitional dynamics, given by equation (32). Local stability requires that the derivative of $\dot{\gamma}$ with respect to γ, when evaluated at the steady state, be negative, that is, $d\dot{\gamma}/d\gamma < 0$ at $\phi = \Omega$. Because $z < 0$, a locally stable steady state must be characterized by

$$\phi' > \Omega', \tag{37}$$

which says that the slope of ϕ must be greater than that of Ω in the neighborhood of that steady state, such as the one illustrated in Figure 3 by the point S where ϕ intersects Ω from below. To the left of γ^*, $\Omega > \phi$, and, therefore, by equation (32), $\dot{\gamma} > 0$ and the rate of consumption will be increasing. The converse is true for any rate of consumption to the right of γ^*. With such a configuration of ϕ and Ω at the point S, the economy will converge to the steady state given by S in the neighborhood of γ^*. In contrast, the steady state given by the point T is unstable. 1/ Note also that the convergence to a (new) steady state following an arbitrary change in the marginal returns to capital, representable graphically by a shift in the Ω curve, will not be instantaneous. This implies that the transitional dynamics in the present model can survive the test of international capital mobility.

Once γ^* is determined at the appropriate intersection of ϕ and Ω, the steady-state growth rate g^* can be found graphically from the line g

1/ Necessary restrictions on the parameters to ensure the existence of a stable steady state are provided in the Appendix.

representing equation (34). Since by stipulation $0 > (1 - \sigma)$, the consumer's utility is always bounded in any steady state with positive growth.

IV. Implications for Tax Policy

The model laid out above has implications for tax policy that are quite different from those of the standard models of endogenous growth. Consider the long-run effects of an income tax at the ad valorem rate τ^y and those of a consumption tax at the ad valorem rate τ^c. For simplicity, the effects of government expenditure are ignored, so that the tax revenue enters into neither the production nor the utility function.

With the presence of taxes, the appropriate concept of the rate of consumption that enters into the consumer's pure subjective time preference function as its argument is one that is defined on the basis of net-of-income tax but gross-of-consumption tax, that is,

$$\hat{\gamma} = c\cdot(1 + \tau^c)/[y\cdot(1 - \tau^y)], \tag{38}$$

so that the corresponding time preference function becomes

$$\hat{\phi} = \alpha\cdot\hat{\gamma}^{1-\theta}. \tag{39}$$

Capital now grows at the rate

$$\hat{g} = A\cdot(1 - \tau^y)\cdot(1-\hat{\gamma}), \tag{40}$$

instead of that given by equation (34). Clearly, the fundamental equation for the steady-state solution (equation (35)) must also be modified accordingly:

$$\hat{\phi} = \hat{\Omega}, \tag{41}$$

where

$$\hat{\Omega} = A\cdot(1 - \tau^y)\cdot(1 - \sigma)\cdot[1 - \sigma\cdot(1 - \hat{\gamma})]/(\theta - \sigma). \tag{42}$$

All sign restrictions on the parameters are as previously stipulated.

A tax on income ($\tau^c = 0$)

The impact of an income tax at the rate τ^y on the rate of consumption can be derived from taking the total differential of equation (41) to obtain

$$d\hat{\gamma}/d\tau^y = -\hat{\Omega}/[(1 - \tau^y)\cdot(\hat{\phi}' - \hat{\Omega}')] < 0, \tag{43}$$

since by the stability requirement $\hat{\phi}' > \hat{\Omega}'$. Hence, a tax on income decreases the steady-state rate of consumption. Whether this translates

Figure 1. Pure Subjective Rate of Time Preference

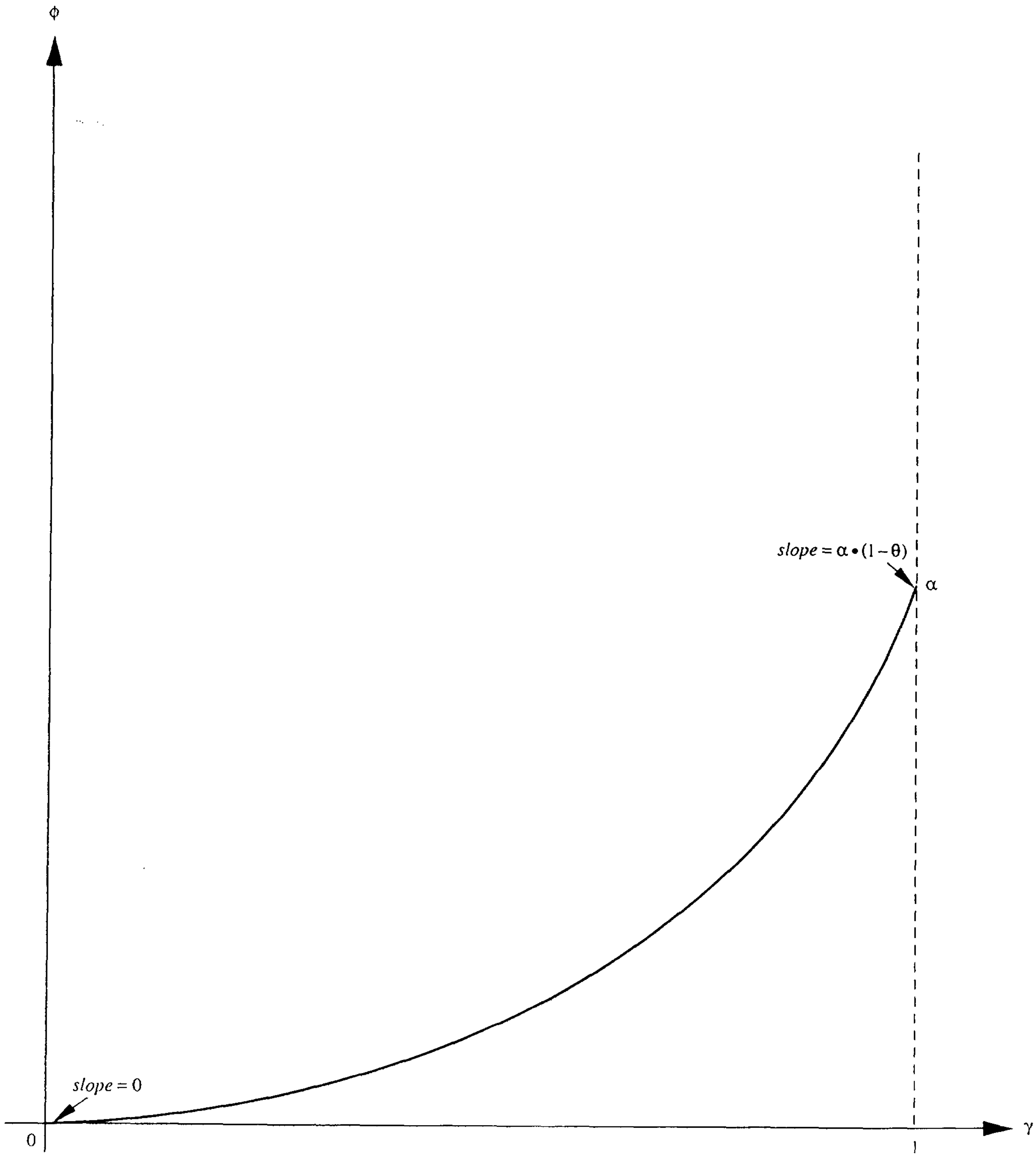


Figure 2. Subjective Value of marginal Product of Capital

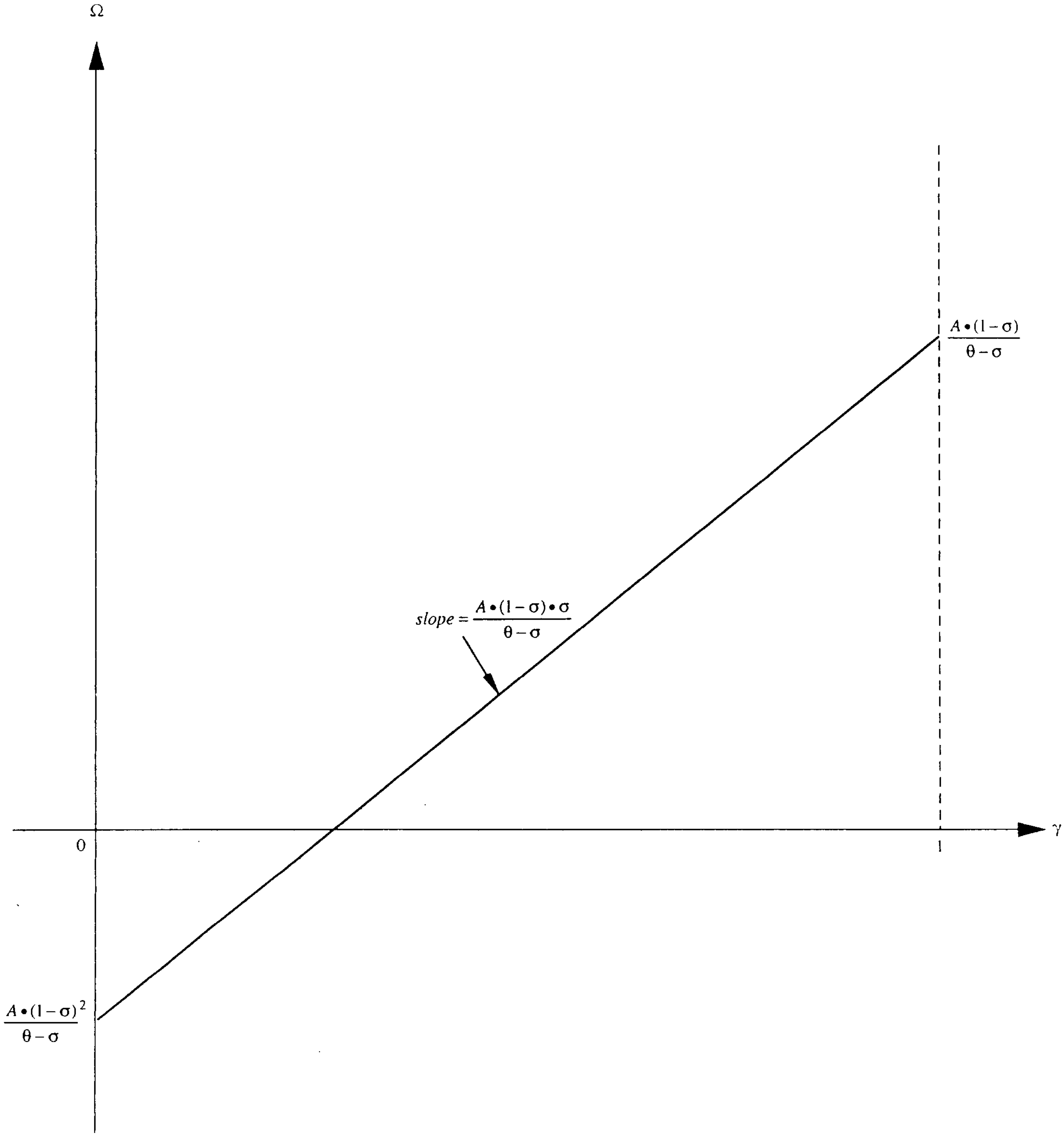
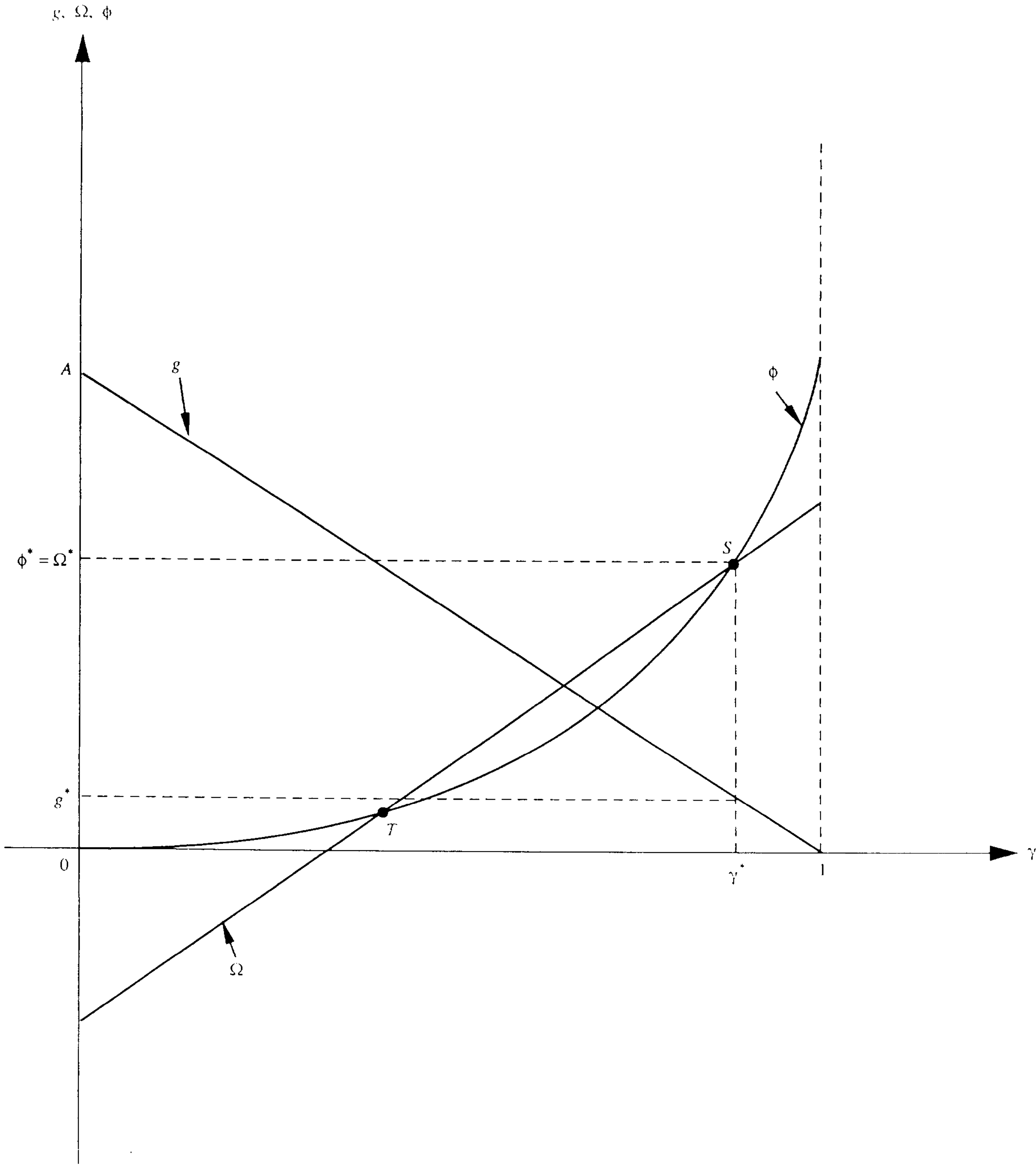


Figure 3. A Stable Steady State



into an increase in the steady-state growth rate will depend on the magnitude of the tax response from γ relative to the reduction in the net-of-tax marginal returns to capital. Differentiating equation (40) with respect to τ^y yields

$$d\hat{g}/d\tau^y = A\cdot(1 - \hat{\gamma})\cdot[\epsilon\cdot(1 - \tau^y)/\tau^y - 1], \tag{44}$$

where $\epsilon = [d(1 - \hat{\gamma})/d\tau^y]\cdot[\tau^y/(1 - \hat{\gamma})]$. Since $(1 - \hat{\gamma})$ is simply the rate of savings, ϵ is the income tax elasticity of the savings rate and has a sign opposite to that given by equation (43), that is, $\epsilon > 0$. Equation (44) underscores the importance of the magnitudes of ϵ and τ^y in determining the long-run growth effects of an income tax. In the case where $\epsilon = \tau^y/(1 - \tau^y)$, the steady-state growth rate is not affected by an income tax.

The above result can also be derived graphically by noting that, as given by equation (42), the presence of τ^y raises the vertical intercept of the Ω curve but lowers its right endpoint at $\gamma = 1$ (Figure 4). 1/ The new steady-state consumption rate is therefore unambiguously lower. The change in the steady-state growth rate is, however, ambiguous, since the $\hat{g}$ line now lies entirely below the earlier g line. Figure 4 illustrates the special case where the steady-state growth rate remains unchanged after the introduction of an income tax.

While the quantitative estimates of the impact of income taxation on growth have differed widely among recent studies, 2/ the qualitative predictions from most endogenous growth models without externalities are the same: an increase in the income tax rate lowers growth by reducing the after-tax returns to capital. 3/ As pointed out by Stokey and Rebelo (1993), this qualitative prediction seems to be at variance with the U.S. experience in the early 1940s, when a sharp rise in income tax revenue as a share of national income did not seem to have produced any negative impact on growth in subsequent periods. Such an empirical finding is, however, consistent with the present model, which shows that the growth effects of an income tax is a priori ambiguous when time preference is endogenous.

A tax on consumption ($\tau^y = 0$)

The impact of a consumption tax on growth in the present model is more conventional. Since the consumption tax rate τ^c enters equation (41) only through γ and not as an independent variable, it can affect neither the steady-state rate of consumption (gross of tax) nor the steady-state growth rate:

1/ In Figure 4, because the rate of consumption is measured on a net-of-tax basis along the horizontal axis, the position of the ϕ curve is the same as that of the earlier ϕ curve in Figure 3.

2/ See Stokey and Rebelo (1993) for a review.

3/ See Xu (1994) for a review.

$$d\hat{g}/d\tau^{c} = d\hat{\gamma}/d\tau^{c} = 0. \tag{45}$$

Given that there is no labor-leisure choice (by assumption), the consumption tax can only have a level, but not a growth, effect in the present model.

V. Concluding Remarks

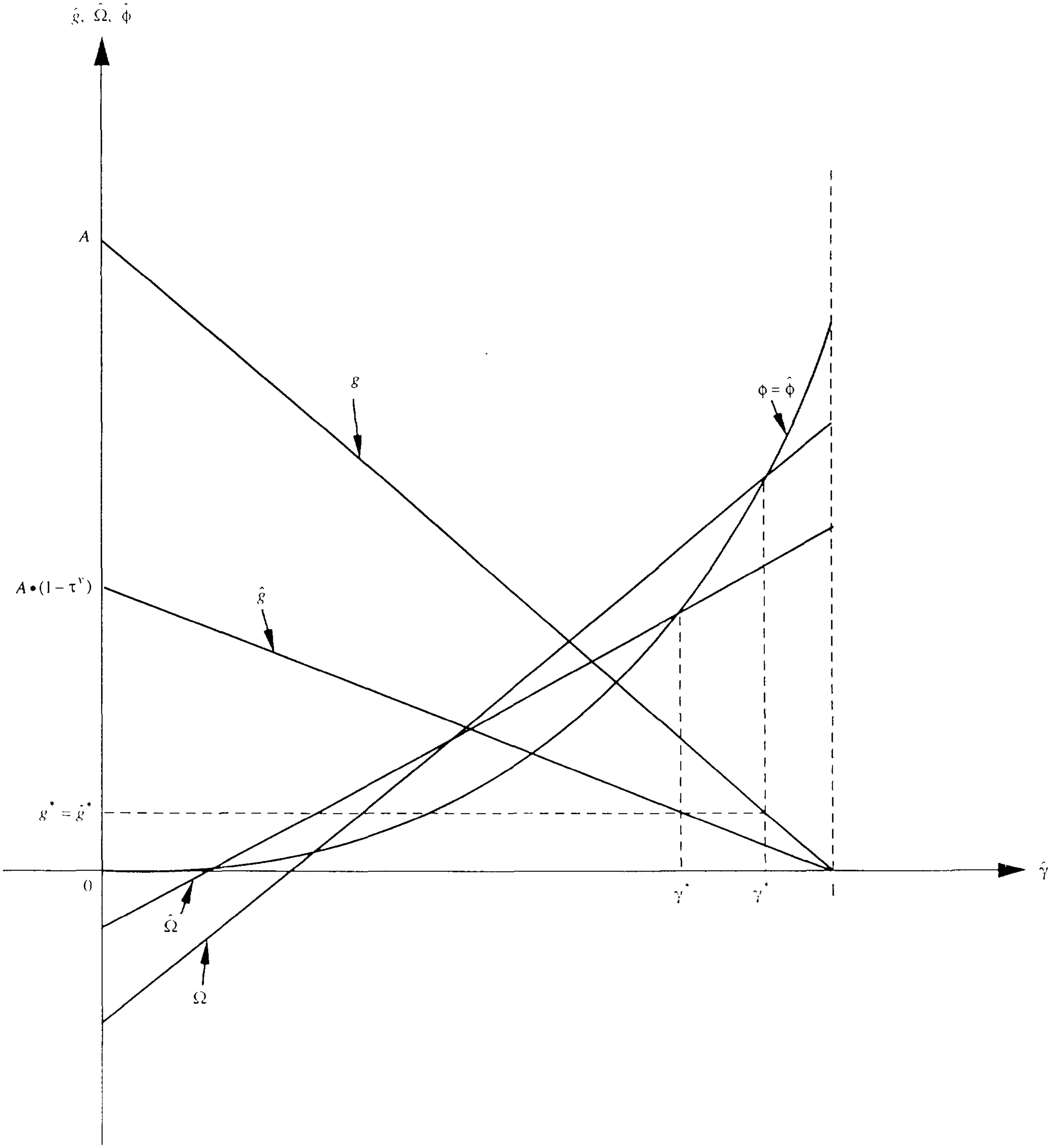
In addition to its conceptual appeal in allowing intertemporal preference dependence, the endogenous time preference construct provides a rigorous basis for generating meaningful transitional dynamics even within the confines of a simple one-sector endogenous growth model. As such, it has two important attractive attributes: (i) its transitional dynamics do not, in contrast to those of the neoclassical and most other endogenous growth models with constant time preferences, instantaneously vanish in the face of a perfect international capital market, since the implied intertemporal elasticity of substitution with a time-varying time preference changes along the path of transition; and (ii) it avoids the necessity of employing more complex, multi-sector models in analyzing many growth issues, which invariably diminish analytical transparency and tractability. 1/

As noted earlier in Section I, the specific model examined in the present paper possesses the fundamental property of growth convergence, in the sense that countries with identical parameters regarding technology, preference, and government policy will converge to a steady state with the same (positive) growth rate. At the same time, the model is not necessarily inconsistent with observed large differences in cross-country growth rates over prolonged periods of time, since such differences could well be attributable to the (possibly) lengthiness of the transition, among other things. However, the plausibility of this explanation must await quantitative simulations of the model based on specific parameter values.

A notable tax policy implication of the model concerns the growth effects of an income tax. In virtually all endogenous growth models in the literature, an income tax, by reducing the net-of-tax return to capital, exerts a negative impact on growth, unless the tax revenue is used to finance some productive public good, such as infrastructure. In a model with endogenous time preference, the income tax will still lower the net-of-tax return to capital at an unchanged rate of savings. However, the rate of savings will be raised by the tax as the latter depresses consumption, which in turn has a positive impact on growth. Hence, even in the absence of externalities, the present model shows that the growth effects of an income tax are a priori ambiguous and dependent on the relative magnitudes of the tax rate and the tax elasticity of the savings rate. At a minimum, this illustrates that the relationship between income taxation and growth may not be as straightforward as the existing literature seems to suggest.

1/ Indeed, this may explain why the one-sector neoclassical growth model is so much more popular than its two- (or more) sector counterparts.

Figure 4. Growth Effects of an Income Tax



Existence of a Stable Steady State

Given that ϕ is strictly convex and Ω is linear within the range $1 \geq \gamma \geq 0$, the existence of a stable steady state, such as the one illustrated by the point *S* in Figure 3, is ensured if and only if
(a) $\Omega(\bar{\gamma}) > \phi(\bar{\gamma})$, where $\bar{\gamma}$ is the value of γ at which $\Omega' = \phi'$; and
(b) $\Omega(\gamma = 1) < \phi(\gamma = 1)$. Moreover, when these conditions are satisfied, the stable steady state is also unique.

$\bar{\gamma}$ can be solved explicitly by equating equations (26) and (36) to get

$$\bar{\gamma} = \{A\cdot(1 - \sigma)\cdot\sigma/[\alpha\cdot(\theta - \sigma)\cdot(1 - \theta)]\}^{-1/\theta}. \tag{A1}$$

By substituting equation (A1) into Ω (equation (33)) and ϕ (equation (21)), it can be shown that the condition $\Omega(\bar{\gamma}) > \phi(\bar{\gamma})$ can be equivalently stated, after a fair amount of algebraic manipulation, as

$$\alpha/A > \left[\frac{1 - \sigma}{\theta - \sigma}\right]\cdot\left[\frac{\theta}{1 - \sigma}\right]^{-\theta}\cdot\left[\frac{\sigma}{1 - \theta}\right]^{1-\theta}. \tag{A2}$$

For the condition $\Omega(\gamma = 1) < \phi(\gamma = 1)$, it is easily seen that it is equivalent to

$$\alpha/A > (1 - \sigma)/(\theta - \sigma). \tag{A3}$$

The sign restrictions stipulated in condition (22) imply that the right-hand-side expression of inequality (A3), which is the same as the first bracketed term on the right-hand side of inequality (A2), is less than unity. The second and third bracketed terms on the right-hand side of inequality (A2) are, respectively, less than and greater than unity. Hence, satisfaction of inequality (A2) implies satisfaction of inequality (A3) if the product of the second and third terms on the right-hand side of inequality (A2) is greater than unity (and vice versa).

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