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Asymmetric Effects of Economic Activity on Inflation:
Evidence and Policy Implications

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Abstract

This paper examines the evidence on asymmetries in the effects of activity on inflation. Data for the G-7 countries are found to strongly support the view that the inflation-activity relationship is nonlinear, with high levels of activity raising inflation by more than low levels decrease it. In the face of such asymmetries, the average level of output in an economy subject to demand shocks will be below the level of output at which there is no tendency for inflation to rise or fall, contrary to the implications of linear models. One implication of these results is that policymakers can raise the average level of output over time by responding promptly to demand shocks, thus reducing the variance of output around trend.

JEL Classification Numbers:

C51; E31; E52

1/ David Rose is a Research Adviser at the Bank of Canada. Irene Chan, currently a graduate student at Queen's University, worked with us in the first phase of this research and co-authored a paper reporting preliminary estimation results. This paper has benefitted from helpful comments by Bijan Aghevli, Tamim Bayoumi, Bankim Chadha, Peter Clark and Steven Symansky.

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Summary

This paper examines the evidence on asymmetries in the relationship between economic activity and inflation. As previous theoretical and empirical work provides little guidance as to the form that such asymmetries might take, alternative models are estimated using pooled Group of Seven data for the period 1967-91. The paper finds strong evidence in favor of a nonlinear relationship, in which the effect of excess demand in raising inflation is much stronger than that of excess supply in reducing it. In addition, the preferred specification implies an upper limit on the level of output in the short run: as output approaches this level, inflationary pressures rise without bound. The paper also shows that the absence of strong evidence in favor of nonlinearities in previous studies may have been due to a misspecification of the level of potential output. In particular, in a stochastic economy with an asymmetric inflation-activity trade-off, trend output lies below the level of output at which there is no tendency for inflation to either rise or fall; ignoring the difference between these two concepts is shown to reduce the power of tests of the nonlinear hypothesis.

The existence of inflation-activity asymmetries has important implications for demand-management policies. Simulations of a small macroeconomic model indicate that, in a linear world, it may be desirable for policymakers to postpone responses to positive shocks to aggregate demand. In a nonlinear world, in contrast, it is preferable to respond quickly to incipient inflationary pressures, as deep recessions are needed to offset periods of mild excess demand. Minimizing the initial rise in demand thus reduces the cumulative loss in output. This is an example of a more general proposition about the role of demand-management policies. Specifically, in a nonlinear world, the average level of output lies below its potential level by an amount that depends on the variance of output and the degree of convexity of the inflation-activity trade-off. By adopting rules that minimize the variance of output, policymakers can raise the average level of output of the economy over time.

I. Introduction

In 1953 I realized that the straight line leads to the downfall of mankind. The straight line has become an absolute tyranny. The straight line is something cowardly drawn with a rule, without thought or feeling; it is the line which does not exist in nature. And that line is the rotten foundation of our doomed civilization. Even if there are places where it is recognized that this line is rapidly leading to perdition, its course continues to be plotted. (Hunderwasser, as quoted in Kennedy (1992), page 102).

The link between economic activity and prices is fundamental to the study of business cycles, as it determines how changes in aggregate demand affect real versus nominal variables. At one extreme, classical economists believed that prices would fully accommodate shifts in nominal demand, leaving real variables unchanged. The Keynesian revolution reversed this split--the assumed unresponsiveness of prices and wages to economic conditions meant that demand shocks primarily affected real activity. An intermediate view, embodying an apparent tradeoff between activity and inflation, was advanced in the late 1950s in the form of the original "Phillips curve" (Phillips (1958)): the observation that stronger activity was associated with higher inflation seemed to imply that policymakers could choose between high employment and high inflation, or low employment and low inflation. In the late 1960s and 1970s, however, this simple tradeoff was discarded on theoretical and empirical grounds in favor of the "expectations-augmented" Phillips curve, $\frac{1}{\pi}$ where inflation varied relative to its expected level in response to changes in activity. As expected inflation is endogenous, attempts to raise activity through demand stimulus would lead to rising inflation expectations and ever-accelerating inflation.

The broad acceptance of the expectations-augmented Phillips curve--and the associated "natural rate" hypothesis--led to the important conclusion that a long-run tradeoff between activity and inflation did not exist. Subsequent research on output-inflation linkages has focussed on how expectations are formed and the reasons for the price "stickiness" that causes real variables to respond to nominal shocks. Almost all of this work, however, has been predicated on the assumption that the tradeoff between activity and inflation is linear; that is, that the response of inflation to a positive gap between actual and potential output is identical to the response to a negative gap of the same size.

The presumption of linearity reflects several considerations, including: its simplicity; the tractability it affords in deriving analytical solutions to models; and its statistical robustness to mismeasurement of the level of potential output. At the same time, the linear model ignores much of the historical context underlying the original split between classical and Keynesian economics: under conditions of full employment, inflation appeared to respond strongly to demand conditions,

1/ The most prominent critic of the initial relationship was Friedman (1968).

while in deep recessions, it was relatively insensitive to changes in activity. Indeed, the original article by Phillips emphasized such an asymmetry, where excess demand had a much stronger effect in raising inflation than excess supply had in lowering it. Over the past two decades, deep and protracted recessions have been required in industrialized countries to reverse inflationary forces generated during periods of economic overheating.

Reflecting this experience, policymakers have become more aware of the need to avoid excess demand pressures. Yet the conventional theory--based on a linear tradeoff between activity and inflation--provides no basis for this aversion: in such a world, inflation can be as easily "wrung out" of the economy as it was initially generated. The effect on output of periods of excess demand and supply tends to "average out" over time when the longer-run rate of inflation remains unchanged. Thus, while it might be desirable to avoid sharp swings in aggregate demand for other reasons, such an approach cannot be motivated by the model linking activity to prices--a point made forcefully by DeLong and Summers (1988). A further, related, difficulty is that the policy advice implied by traditional linear models is independent of the state of the business cycle. Specifically, because the effects on inflation of aggregate demand policies do not depend on whether output is initially above or below potential, there is no inherent reason for taking the state of the cycle into account when pursuing such policies. Finally, the assumption of linearity imposes no "upper bound" on the short-run effect on output of stimulative policies: theoretically, a sufficiently large demand shock could raise output by, say, 10 or 20 percent relative to potential. Experience, however, suggests that inflation starts to increase sharply with much smaller positive output gaps.

These considerations point to an uneasy relationship between, on the one hand, the implications of a linear output-inflation tradeoff, and, on the other, the stylized facts of business cycles and the associated policy advice. This paper explores an alternative specification based on an *asymmetric* relationship between output and inflation. Specifically, we assume that the effect on inflation of deviations in output from potential rises, at the margin, the higher is the level of output. At the limit, there is a "wall" at which real activity cannot rise further, regardless of the size of the demand shock: any nominal stimulus beyond this point is translated directly into inflation.

The implications of such a model for macroeconomic policy differ sharply from those of the linear model. Because excess demand raises inflation by more than excess supply lowers it, policies that allow output to temporarily rise above potential necessitate a stronger monetary contraction in the future to contain inflationary pressures. Indeed, a delayed monetary policy response to a temporary period of excess demand will generally lead to a cumulative loss in output, in contrast to the implications of the linear model. Another implication of the asymmetric output-inflation tradeoff is that, the greater is the *variance* of output, the lower will be the *average level* of output. Thus, the nonlinear model

provides a fundamental motivation internal to the model for pursuing stable aggregate demand policies that is absent from linear models.

The paper is organized as follows. The next section discusses the empirical evidence on output-inflation tradeoffs. A model is estimated using pooled data for the G-7 countries; the results support strongly a nonlinear relationship with the properties described above over a linear alternative. In the third section, a small stylized model is used to explore the implications of the nonlinear tradeoff. The results show that policies that respond quickly to demand shocks minimize the ultimate loss in output. This illustrates the more general conclusion that reducing the variance of output raises its mean level when the output-inflation tradeoff is asymmetric. The final section summarizes the results and suggests possible directions for further research.

II. Estimation

While the stylized facts of business cycles suggest the existence of an asymmetric output-inflation tradeoff, 1/ the theoretical literature provides little guidance as to the form such a nonlinearity might take. 2/ At an empirical level, the literature on nonlinear price equations is also relatively undeveloped. In the absence of strong theoretical or empirical priors concerning the precise form of the nonlinearity, two approaches might be taken to specifying a function suitable for estimation. The first is to use a flexible approximation to any general nonlinear function. The second is to "prespecify" a functional form that satisfies conceptual priors about the nature of the nonlinearity, while allowing enough flexibility to let the data tie down its precise shape. Here we explore both approaches. To summarize the results of this section, the linear model can be conclusively rejected in favor of *either* a flexible nonlinear approximation or a specific function that satisfies conceptual priors. Both imply strong convexity in the inflation-activity relationship.

1. Alternative characterizations of the output-inflation tradeoff

In terms of flexible approximations to general nonlinear functions, the most common and easy to implement is a power series expansion. Representing the function relating the effect on inflation of the output gap as $f(\text{gap})$, such an expansion implies:

$$\text{effect on inflation} = f(\text{gap}) \approx \alpha \text{gap} + \beta \text{gap}^2 + \delta \text{gap}^3 + \dots \quad (1)$$

1/ See, for instance, DeLong and Summers (1988) for evidence on asymmetries in business cycles characteristic of such nonlinearities.

2/ Tsiddon (1993) and Ball and Mankiw (1994) provide theoretical models capable of generating asymmetries in the inflation-output process.

where *gap* is the deviation between actual output and its potential level. 1/ In estimation, the testing down from the general expansion in equation (1) to a specific approximation can be based on statistical criteria. It is interesting though, to consider *a priori* the properties of some specific cases of this class of approximations. One is the function consisting only of the linear and quadratic terms:

$$f(\text{gap}) = \alpha \text{gap} + \beta \text{gap}^2 . \quad (2)$$

As illustrated in the upper left-hand panel of Chart 1, this approximation implies a convex function, with the effect on inflation tapering off as the *gap* becomes negative. 2/ Another, even more parsimonious, alternative would be to simply raise the *gap* to an odd integer--the power must be odd to preserve the appropriate sign of the inflation-output relationship. 3/ An example is the cubic function: 4/

$$f(\text{gap}) = \alpha \text{gap}^3 . \quad (3)$$

This specification would cause the effect on inflation to increase rapidly as the *gap* rose in size, while the slope of the function would approach zero when the *gap* was small. For negative *gaps*, the function would have the same shape as for positive *gaps*--that is, downward pressure on inflation would become large as the negative *gap* increased in size (Chart 1, upper right-hand panel). This limiting case, then, has the disadvantage that, while nonlinear, it is symmetric: positive and negative *gaps* have the same effect on inflation. Thus, it cannot explain apparent asymmetries in the inflation-activity relationship.

Alternatively, specific functional forms can be chosen *ex ante* that embody conceptual priors about the shape of the inflation-activity tradeoff. The latter approach was adopted by Chadha, Masson, and Meredith (1992)--henceforth CMM--in estimating alternative Phillips curves for the G-7 countries. The functional form employed by CMM to represent the nonlinearity is the (modified) hyperbola:

$$f(\text{gap}) = \beta (\omega^2 / (\omega - \text{gap}) - \omega) , \quad (4)$$

1/ In what follows, we use the relationship $\text{gap} = \log(y/y^*)$, so that the deviation of actual from noninflationary output is expressed in proportional terms. Positive output *gaps* tend to raise inflation relative to its expected level, while negative *gaps* tend to reduce it.

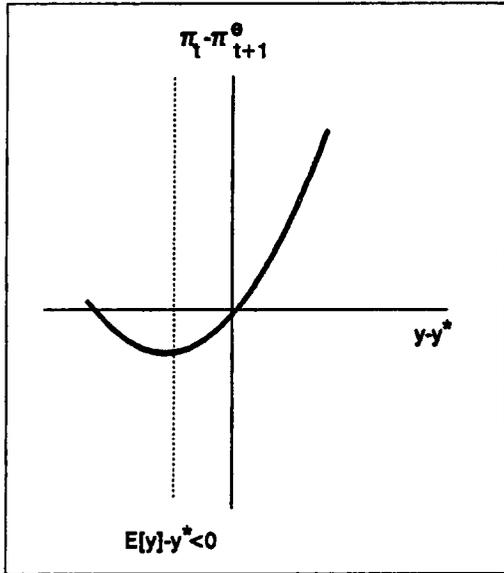
2/ Less plausibly, this function also implies a region where downward pressure on inflation declines as excess supply increases beyond a certain level.

3/ The effect on inflation should have the same sign as the output *gap* itself, which would not be the case if the *gap* were raised to an even power.

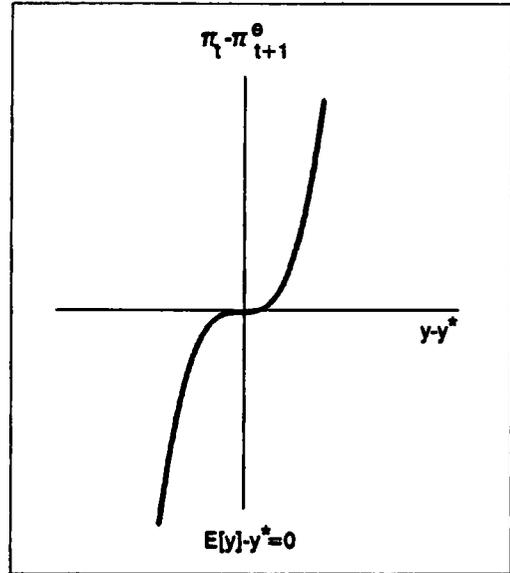
4/ See Masson and Meredith (1990) for an example of the estimation of this functional form for the G-7 countries.

Chart 1. Alternative Nonlinear Functional Forms

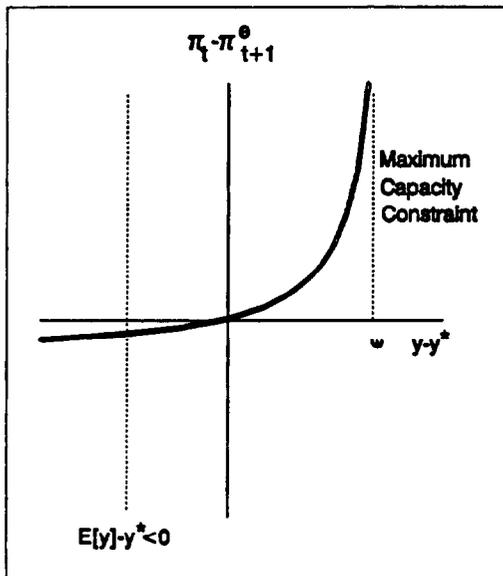
1a: Linear and Quadratic Function



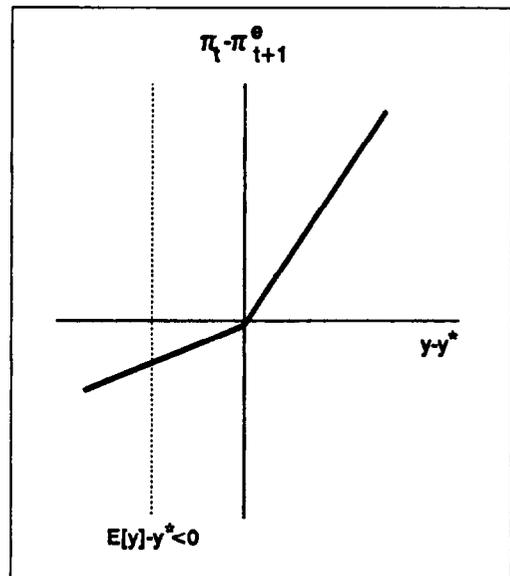
1b: Cubic Function



1c: CMM Function



1d: Kinked Function



where β and ω are parameters to be estimated. This function is graphed in the lower left-hand panel of Chart 1. Its relevant properties can be derived by looking at the first derivative of $f()$ --that is, the slope of the inflation-activity tradeoff:

$$f'(gap) = \beta \omega^2 / (\omega - gap)^2 . \quad (5)$$

The limiting values of this derivative (and of the function itself) for some specific values of ω and gap are:

$$\lim_{\omega \rightarrow \infty} f'() = \beta , \quad (5a)$$

$$\lim_{gap \rightarrow \omega} f'() = \infty , \quad f() = \infty , \quad (5b)$$

$$\lim_{gap \rightarrow -\infty} f'() = 0 , \quad f() = -\beta\omega , \quad (5c)$$

$$f'(0) = \beta , \quad f(0) = 0 . \quad (5d)$$

Equation (5a) shows that, as the parameter ω becomes large, the CMM function approaches a linear relationship (as ω decreases, in contrast, it can be shown that the function approaches a reverse "L" shaped relationship). Equation (5b) indicates that the effect on inflation rises without bound as the output gap approaches ω : in other words, ω represents a "wall" beyond which output cannot increase in the short run. As the gap becomes negative, the slope of the function decreases; equation (5c) shows that, at the limit, there is a lower bound to the effect on inflation of $-\beta\omega$ as the gap becomes highly negative. When the gap is zero, the effect on inflation is also zero, and the slope of the tradeoff is β (equation (5d)).

Another example of a specific functional form is Laxton, Rose, and Tetlow (1993)--henceforth LRT--using Canadian data. LRT used a "kinked" function to represent the nonlinearity, with a discontinuity in its slope as the output gap changes sign: 1/

$$\begin{aligned} f(gap) &= \alpha_1 gap & \text{if } gap \leq 0 & \\ &= \alpha_2 gap & \text{if } gap > 0 , & \end{aligned} \quad (6)$$

where α_1 and α_2 are parameters to be estimated, and the value of α_2 exceeds that of α_1 (Chart 1, lower right-hand panel).

The limited theoretical literature on nonlinearities provides little guide as to the preferred specification. On conceptual and empirical

1/ The actual function estimated by LRT also had a quadratic term in the region of positive excess demand that generated increasing marginal pressure on inflation.

grounds, however, some discrimination is possible. The function consisting only of the cubic term, for instance, has the conceptual drawback that, while nonlinear, it is symmetric: negative gaps reduce inflation by as much as positive gaps raise it. Empirically, the implication that downward pressure on inflation increases exponentially as economic slack rises appears to conflict with the experience of large industrial countries that have experienced deep recessions in recent years. 1/ The "kinked" function used by LRT is both transparent and consistent with the proposition that excess supply has a smaller effect on inflation than does excess demand. However, it has the weakness that no upper bound is imposed on output. Operationally, the discontinuity implied by the kink makes some aspects of estimation problematic. It is also difficult to rationalize the abrupt change in the value of α when the gap changes sign, given that the aggregate gap reflects the average of conditions in many different markets. The quadratic function also fails to impose an upper bound on output, while the upward-sloping region in the area of significant excess supply is implausible on conceptual grounds.

From these points of view, the CMM function seems the most suitable of the nonlinear alternatives: it is asymmetric, has a continuously differentiable slope, and implies an upper bound on output. Nevertheless, in the absence of clear predictions from theory, considerable weight must be given to empirical evidence in selecting the appropriate functional form. As shown below, the CMM function performs well in fitting the data for the major industrial countries. We therefore focus on it in the following discussion and in the simulation exercises in the next section.

2. Specification of the price equation

Our price equation is a reduced-form representation of wage and price dynamics, in which the response of wages to labor market conditions is subsumed in the response of inflation to deviations in output from potential. To focus on the broadest possible measure of output prices, but to exclude the supply shocks associated with movements in oil prices in the 1970s and 1980s, we use the rate of change in the non-oil GDP deflator (π) as the dependent variable. Inflation is explained by: a weighted average of past and expected future growth in output prices; growth in the contemporaneous absorption deflator (π^a) relative to output prices; and terms in contemporaneous and lagged output gaps. We allow the slope parameter β in equation (4) to differ between the two output gap terms, whereas the "wall" parameter ω is constrained to be the same. The general form of the equation is then:

$$\pi_t = (1-\delta) \pi_{t-1} + \delta \pi_{t+1}^e + \lambda (\pi_t^a - \pi_t) + \beta_1 f(\text{gap}_t) + \beta_2 f(\text{gap}_{t-1}). \quad (7)$$

1/ From a policy perspective, this function implies that the average level of output is independent of its variance, negating a role for policies designed to smooth demand fluctuations.

To make equation (7) suitable for estimation, the contemporaneous value of π on the right-hand side can be eliminated by bringing it to the left-hand side and dividing through by $(1+\lambda)$, yielding:

$$\begin{aligned}\pi_t &= (1/(1+\lambda)) ((1-\delta)\pi_{t-1} + \delta\pi^e_{t+1} + \lambda\pi^a_t + \beta_1 f(\text{gap}_t) + \beta_2 f(\text{gap}_{t-1})) \\ &= (1-\lambda') ((1-\delta)\pi_{t-1} + \delta\pi^e_{t+1}) + \lambda'\pi^a_t + \beta_1' f(\text{gap}_t) + \beta_2' f(\text{gap}_{t-1}) \quad (7')\end{aligned}$$

where:

$$\begin{aligned}\lambda' &= \lambda / (1+\lambda) \\ \beta_1' &= \beta_1 / (1+\lambda) \\ \beta_2' &= \beta_2 / (1+\lambda).\end{aligned}$$

Here, we summarize the rationale for this specification; a more detailed discussion is provided in Chadha, Masson, and Meredith (1992).

Traditional, "backward-looking," Phillips curves relied on past inflation to reflect inertia in the wage and price contracting process as well as to proxy for expectations of future inflation. In contrast, more recent theoretical models of overlapping contracts with forward-looking agents (such as Calvo (1983)) represent inflation as a function of its expected future realization based on all available information about the state of the economy. The inclusion in our specification of a weighted average of past and expected future inflation reflects elements of both approaches, with the importance of each determined empirically. ^{1/} Expected future inflation is constructed by regressing actual inflation on a set of past information known at the time expectations are formed; the fitted values from this regression are then used as proxies for inflation expectations. The estimated weights on past and expected future inflation determine the relative importance of the "forward" and "backward" looking components of the inflation process. The constraint that the parameters sum to unity ensures that no long-run tradeoff exists between the level of inflation and excess demand pressures.

The term in relative absorption inflation captures the effect on wage demands of changes in consumption prices. Theoretically, it could enter the equation in either levels or growth rates, depending on the assumptions of the model. In the Calvo (1983) model, for instance, wages are fixed in levels for the life of the contract, implying that the relevant variable is the level of the absorption price. In other models, where contract wages grow over time, it is the growth rate that is relevant. Following the CMM finding that the growth rate specification works better empirically for the G-7 countries, we retain that specification here.

^{1/} A more formal justification for the presence of lagged inflation is given by Taylor (1980) in a model of overlapping wage contracts expressed in growth rates.

The output gap term is a (nonlinear) function of the deviation of actual output from its "potential" level. 1/ A complication arises in implementing this approach given our assumption of a nonlinear output-inflation tradeoff. Specifically, as shown formally in Appendix I, the average observed level of output will generally be below the potential level of output. Constructing proxies for potential by detrending movements in actual output will result in downward-biased estimates of potential. To correct for this bias, we introduce a parameter, α , in the output gap term that indicates the average deviation of trend output from the conceptual level of potential in the absence of stochastic demand shocks:

$$\begin{aligned} \text{gap} &= \log (y / y^*) \\ &= \log (y / (1+\alpha)\tilde{y}) , \end{aligned} \tag{8}$$

where y is real GDP, y^* is potential output, \tilde{y} is trend GDP, and α is the percent deviation between y^* and \tilde{y} .

Both the contemporaneous and lagged values of the output gap are included in equation (7'). Such dynamics in the response of inflation to demand conditions can be motivated by traditional backward-looking models of inflation. 2/ They are also consistent with forward-looking behavior with adjustment costs. A particular case arises when costs are associated with changing the level of output, as reflected in an upward-sloping marginal cost curve. This would imply that the change in the markup of output prices over variable costs depends on the *change* in the level of capacity utilization.

3. Estimation results for the G-7 countries

Data for the estimation of the inflation equation were obtained from the MULTIMOD database for the G-7 industrial countries. 3/ Specifically, annual data were available from 1965 to 1993 for: the non-oil GDP deflator; the deflator for total domestic demand (i.e., absorption); and real GDP. In addition, it was necessary to construct a measure of trend output for each country. Two methods were used for this purpose. The first involved smoothing (the logarithm of) actual output using the filter developed by Hodrick and Prescott (1980) in the context of the analysis of real business cycles. The second--more direct--method involved taking a two-sided moving average of the logarithm of actual output; a five-year "window" was chosen

1/ We use the term "potential" output to refer to the level of output at which there is no tendency for inflation to either rise or fall. It should be emphasized, though, that this definition of potential will *not* correspond to the average attainable level of output in a stochastic economy.

2/ See, for instance, the derivation of CMM's equation (4).

3/ For a description of MULTIMOD see Masson, Symansky, and Meredith (1990). The database is constructed primarily from conventional national accounts data for the G-7 countries; non-oil GDP deflators are derived using OECD data on oil production for the G-7 countries.

for this purpose. 1/ Both methods yielded plausible measures of output gaps based on the stylized facts of G-7 business cycles. While the estimation results were also similar, those using the moving-average filter yielded somewhat more precise parameter estimates; as it is also the more direct of the two measures to construct (and easily replicated by other researchers), we use it in the estimation results described below.

In addition, it was necessary to estimate auxiliary equations using instrumental variables to deal with the endogeneity of absorption inflation, and to construct proxies for expected future inflation. Following the strategy pursued by CMM, the instruments used were: the lagged output gap; lagged growth in the non-oil GDP deflator; the second lag of growth in the absorption deflator; lagged money growth; and the lagged ratio of real government expenditures to GDP. 2/ Auxiliary equations were estimated using these variables as regressors--the fitted values from these regressions were then used in the estimation of equation (7').

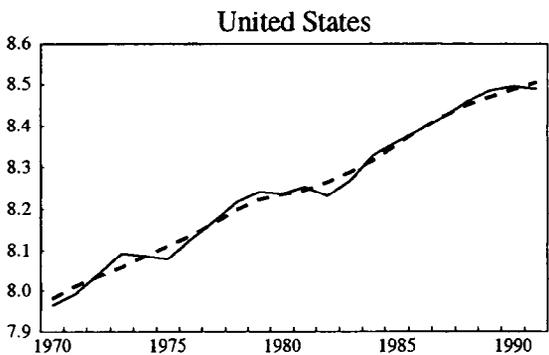
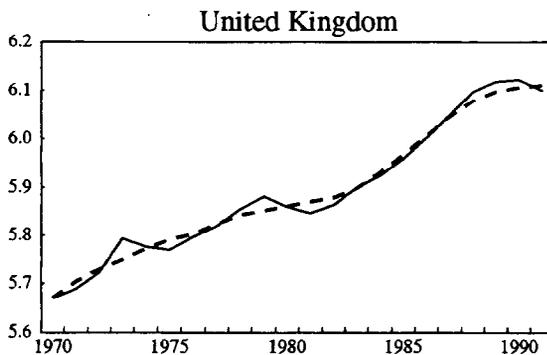
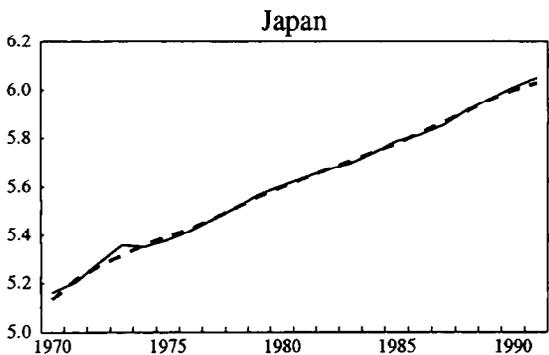
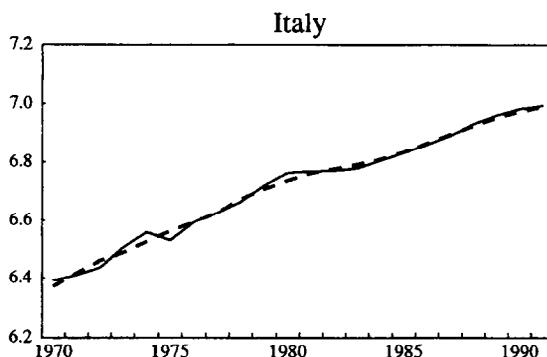
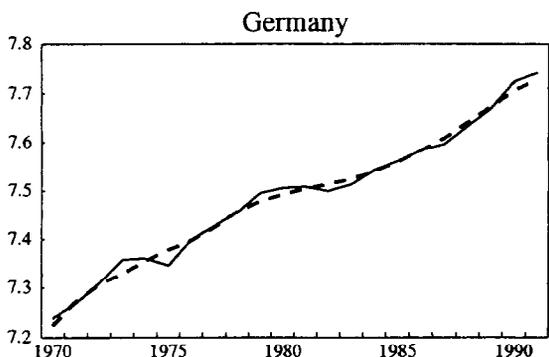
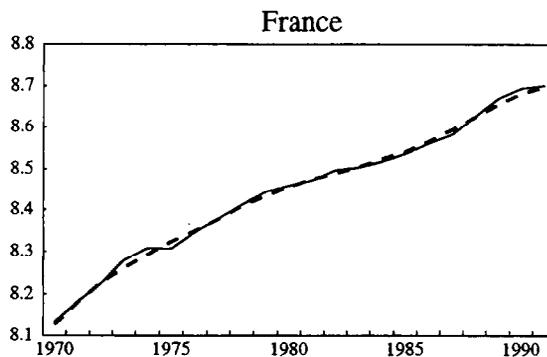
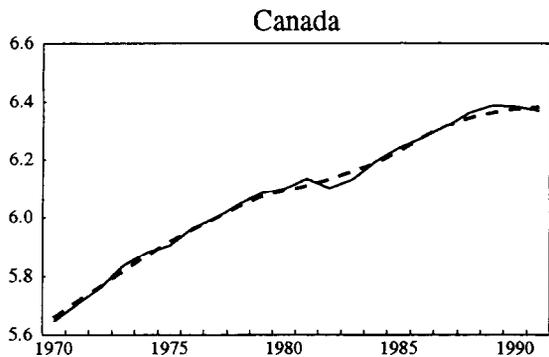
After allowing for leads and lags in the constructed data, 25 observations remained for each G-7 country, extending from 1967-1991. The data on actual and trend output for the G-7 countries are shown in Chart 2(a), while Chart 2(b) compares the associated output gaps with the inflation rates observed in these countries. For most countries there appears to be a systematic relationship between output gaps and changes in the inflation rate. However, given the limited number of observations for each country, it is not surprising that empirical research using individual country data has failed to find consistent evidence in favor of nonlinearities. This problem has been compounded by imprecise measures of the gap. 3/ However, even if the output gaps were measured precisely, there are too few disinflationary/inflationary episodes in the individual country experiences to develop reliable estimates of the degree of convexity. However, taken together, the experiences of all countries in the G-7 may be sufficient to detect significant convexity in the inflation-output process. Consequently, to enhance the efficiency of the estimation process, data for the G-7 countries were pooled and equation (7')

1/ In other words, $\bar{y}_t = 0.2 (y_{t-2} + y_{t-1} + y_t + y_{t+1} + y_{t+2})$.

2/ All obtained from the MULTIMOD database.

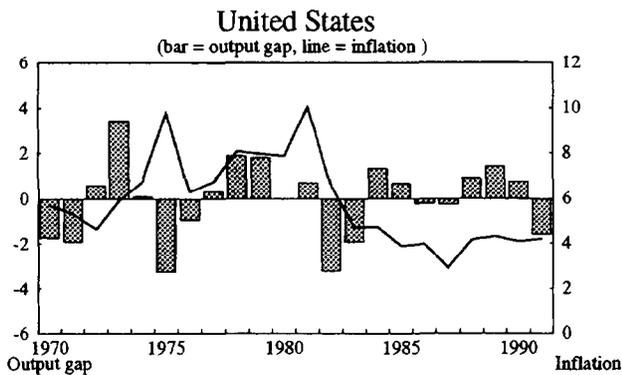
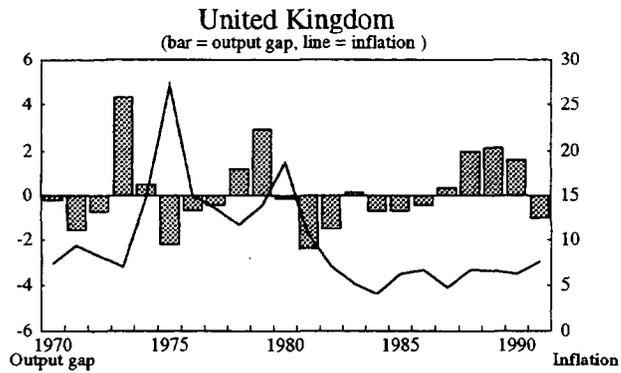
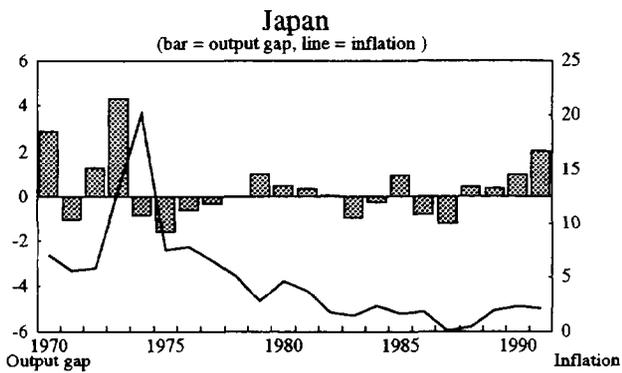
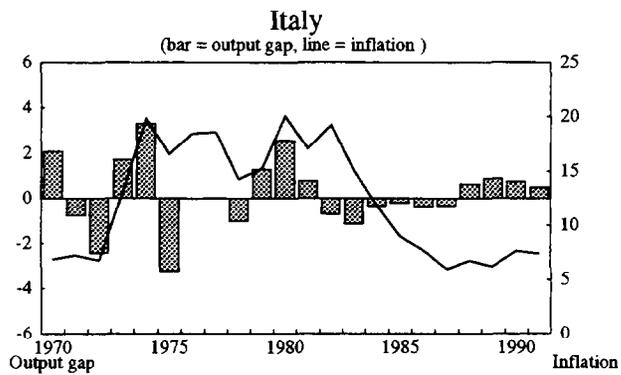
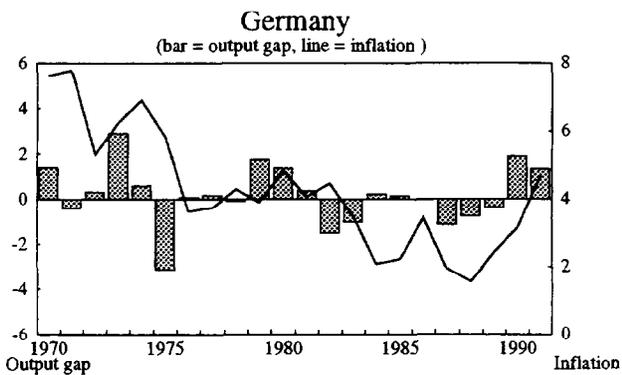
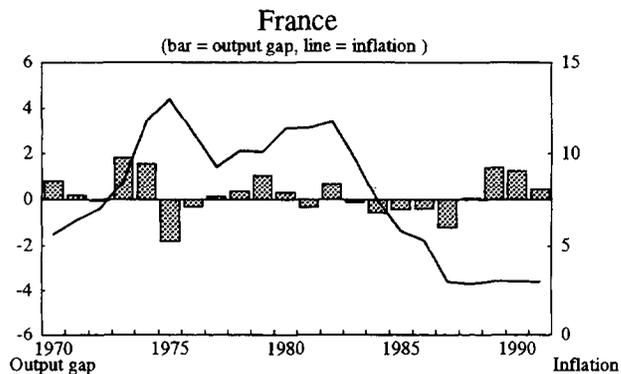
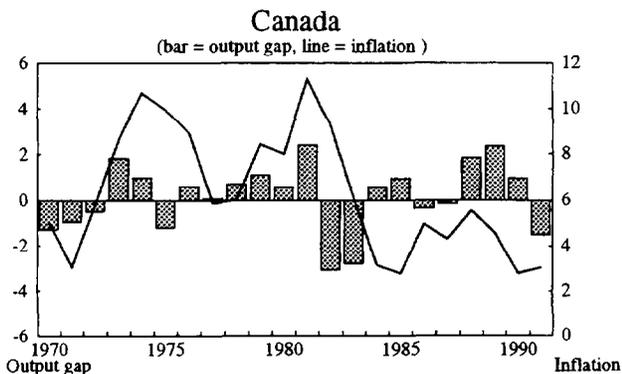
3/ Laxton, Rose and Tetlow (1993a) demonstrate using Monte Carlo techniques that statistical tests have been biased against finding convexity because researchers have typically employed mean-square-error criteria to measure the output gap. The intuition behind this bias is as follows. If excess demand is more inflationary than excess supply is deflationary, the non-inflationary level of output must be greater than its mean level (see Appendix I). If a mean-squared-error criteria is used to measure the latter, estimates of excess supply will be too small, on average, while estimates of excess demand will be too large. In the artificial economies studied by Laxton, Rose and Tetlow, this measurement bias substantially reduced the power of statistical tests of nonlinearities. The approach used in this paper to dealing with this issue is discussed below.

Chart 2a: Real GDP and Trend GDP in the G-7 Countries
(In natural logarithms)



— Real GDP
--- Trend GDP

Chart 2b: Inflation and Output Gaps in the G-7 Countries (In percent)



was estimated as a nonlinear system with common parameters across countries. 1/

Table 1 compares results for the CMM model with those for the linear alternative. 2/ In the case of the CMM function, the nonlinearity in the functional form implied by ω --the "wall" parameter--prohibited its joint estimation with the other parameters using conventional nonlinear methods. Instead, a grid search was used where ω was varied from 0.04 to 0.10 in increments of 0.001. Line (1) shows the results for the unconstrained model with nonlinear CMM terms in the contemporaneous and lagged output gaps. The parameters on both terms are economically large and statistically significant. The value of the "wall" parameter, ω , that maximizes the log likelihood function is 0.049, implying that the effect on inflation becomes unbounded as the output gap, measured relative to potential output, approaches 5 percent of GDP. However, the parameter α has a value of 0.006, indicating an average difference of 0.6 percent between the hypothetical level of output at which the effect on inflation is zero and the constructed "trend" level of output. 3/ Thus, measured relative to trend output, inflation becomes infinite as the output gap approaches 5 1/2 percent of GDP.

The resulting nonlinearity in the relationship between output gaps and the effect on inflation, based on the estimated parameters, is illustrated in the upper two panels of Chart 3. Looking, for example, at the contemporaneous effect, a positive output gap of 3 percent would tend to raise the inflation rate by about 1 1/2 percentage points, while a negative gap of the same size would reduce inflation by less than 1/2 percentage point. The effects of the lagged gap are larger given that β_2' exceeds β_1' , but the overall shape of the function is the same.

The weight on the forward-looking component of the inflationary process is 0.414, while that on the backward-looking component is 0.586 (i.e., $1-0.414$); both are (highly) significantly different from zero, with values similar to those found in other studies embodying forward- and backward-looking elements in the inflationary process. The parameter on contemporaneous absorption inflation is 0.193 with a t-statistic of slightly below 2, suggesting a limited impact of shocks to the relative price of absorption on output prices.

Line (2) shows the results obtained when the linear version of the output-inflation tradeoff is substituted for the CMM function (equivalent to imposing a large value for ω in the estimation procedure). For testing the

1/ Tests of the pooling restrictions are presented in Chan and others (1994).

2/ As discussed above, the linear function is simply a nested version of the CMM function with the parameter ω set to infinity.

3/ As discussed in the next section, this provides a measure of the amount by which the average level of output was lowered over the sample period by the volatility of shocks to aggregate demand.

Table 1. Estimation Results for CMM Versus Linear Functional Forms

(t-statistics in parentheses)

Estimated equation: $\pi_t = (1-\lambda')(\delta\hat{\pi}_{t+1} + (1-\delta)\pi_{t-1}) + \lambda'\hat{\pi}_t^a + \beta_1'f(\text{gap}_t) + \beta_2'f(\text{gap}_{t-1})$

where: $\text{gap} = \log(y/y^*)$

$y^* = (1 + \alpha) \bar{y}$

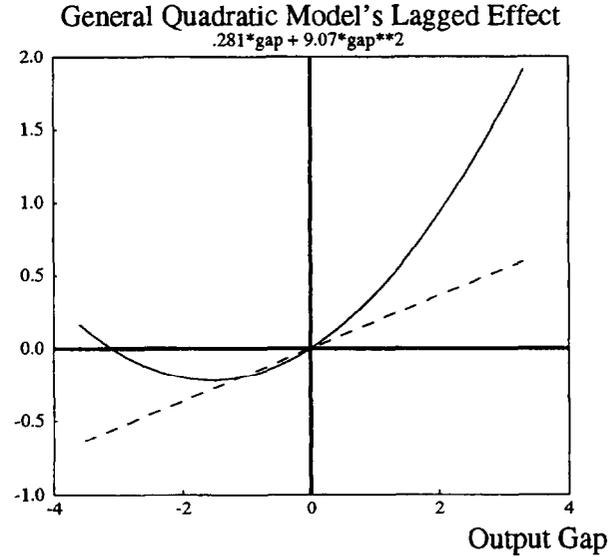
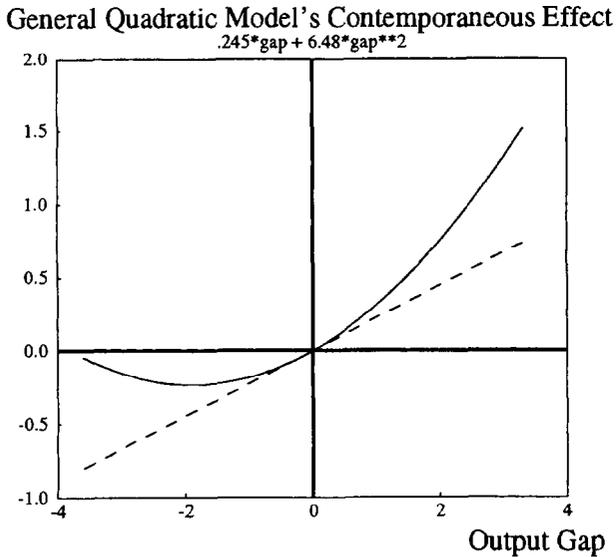
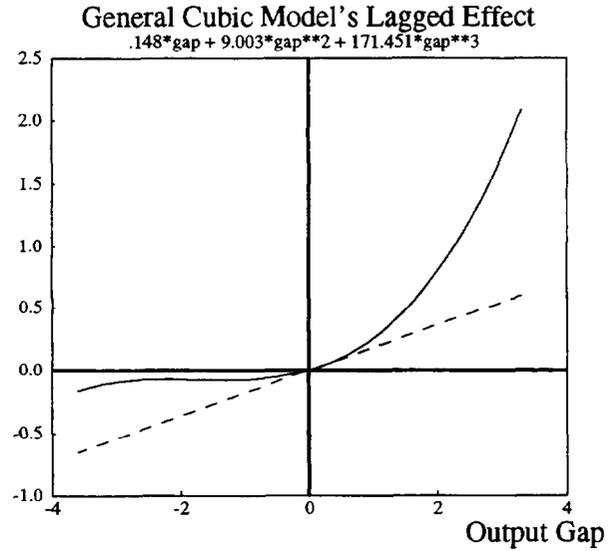
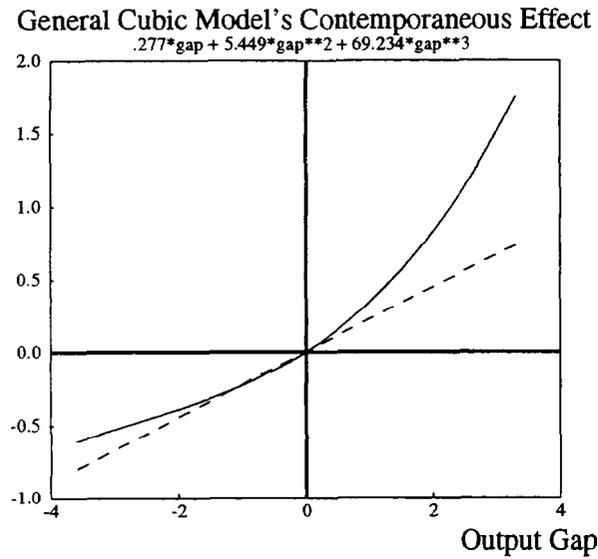
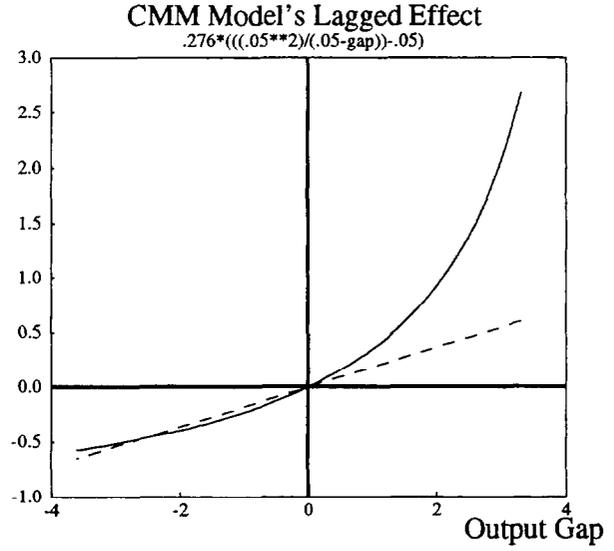
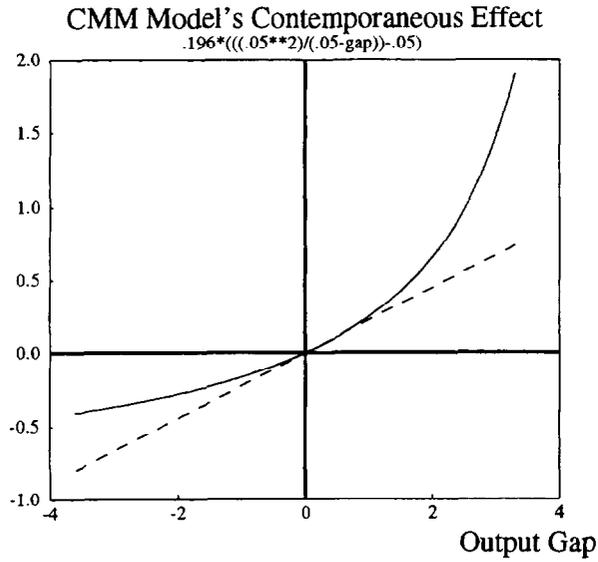
$f(\text{gap}) = (\omega^2 / (\omega - \text{gap})) - \omega$

- π = change in the log of the non-oil GDP deflator
- π^a = change in the log of the absorption deflator
- \underline{y} = real GDP
- \bar{y} = trend real GDP
- y^* = level of GDP at which the effect on inflation is zero
- α = gap between trend GDP (\bar{y}) and level of GDP at which the effect on inflation is zero (y^*)
- ω = parameter identifying maximum short-run level of GDP
- δ = weight on forward-looking component of inflation expectations
- λ' = weight on absorption deflator
- $\hat{}$ = denotes fitted value from auxiliary regression (see text)

Data: Annual, 1967-91, pooled G-7 countries.

| | δ | λ' | α | β_1' | β_2' | ω | LLF |
|---|-----------------|-----------------|-----------------|-----------------|-----------------|------------------|--------|
| (1) Nonlinear CMM function | 0.414 (7.33) | 0.193 (1.96) | 0.006 (3.04) | 0.196 (3.55) | 0.276 (3.96) | 0.049 (7.96) | 495.28 |
| (2) Linear function ($\omega=\infty$) | 0.417 (6.18) | 0.258 (2.56) | 0.002 (0.67) | 0.224 (3.00) | 0.180 (2.28) | large | 489.43 |
| (3) Nonlinear CMM function ($\alpha=0$) | 0.444 (7.76) | 0.194 (1.94) | -- | 0.091 (2.24) | 0.160 (2.60) | 0.053 (11.38) | 492.59 |
| (4) Linear function ($\omega=\infty, \alpha=0$) | 0.423 (6.39) | 0.248 (2.48) | -- | 0.219 (2.91) | 0.173 (2.19) | large | 489.21 |

Chart 3: Estimated Nonlinear Functional Forms Versus Linear Model Contemporaneous and Lagged Effects on Inflation



hypothesis of nonlinearity, the most important aspect of these results is the drop in the value of the log of the likelihood function (LLF) when linearity is imposed. In the event, twice the difference in the LLF gives a test statistic of 11.8, which is distributed $\chi^2(1)$ under the null hypothesis of a linear tradeoff. As the critical value of the $\chi^2(1)$ distribution at the 99 percent confidence level is 6.6, the linear hypothesis is overwhelmingly rejected by the data. Otherwise, the parameters are similar to those estimated using the CMM function, although the parameters on the lagged output gap (β_2') and the difference between the potential and trend levels of output (α) are both somewhat smaller and less significant.

The above results, indicating a strong rejection of the linear model in favor of the nonlinear alternative, raise the question of why earlier estimation exercises have not found conclusive evidence of nonlinearities. For example, CMM (1992) were able to reject the linear model at the 95 percent confidence level, but not at the 97.5 percent level. An indication of how these earlier results can be reconciled with the current evidence is provided in Lines (3) and (4) of Table 1, which show estimation results for the nonlinear and linear models when α is constrained to zero. This parameter, which measures the gap between the potential and trend levels of output, allows for a horizontal "shift" in the output-inflation tradeoff such that inflation tends to fall when output is at its trend level. In the earlier work of CMM, α was implicitly constrained to zero. As indicated by the drop in the LLF in Line (3) from that in Line (1), this constraint significantly worsens the fit of the nonlinear specification. The linear specification, in contrast, fits roughly as well with or without the constraint on α , as shown by a comparison of Line (4) with Line (2). 1/ Comparing Lines (3) and (4), then, the evidence in favor of nonlinearities is much less strong when α is constrained to zero than when it is a free parameter. From this, we conclude that the empirical case for nonlinearity becomes more robust when the specification of the functional form is made fully consistent with theoretical priors.

Further evidence on this point is provided in Table 2, which shows the value of the log likelihood function for a grid of values for α and ω . Consistent with the above results, the value of the likelihood function is maximized at 495.26 when α is 0.006 and ω is 0.05. However, the slope of the likelihood function does not remain constant as we move away from this point: when α is at its optimal value, the LLF declines faster as the relationship becomes more linear (i.e., as ω becomes larger) than when α is zero. For example, the LLF falls to 491.44 when α equals 0.006 and ω moves from 0.05 to 0.10, producing a χ^2 statistic of 7.64. When α is zero, in contrast, the decline in the LLF as ω moves from 0.05 to 0.10 is only 1.62 (492.49-490.87), and the test statistic falls to 3.24. This confirms the earlier result, that the power of tests of linear versus nonlinear

1/ These results are consistent with the hypothesis that the linear specification is more robust to the mismeasurement of potential.

Table 2. Values of the Log Likelihood Function for Various Values of ALPHA and OMEGA

Estimated equation: $\pi_t = (1-\lambda')(\delta\hat{\pi}_{t+1} + (1-\delta)\pi_{t-1}) + \lambda'\hat{\pi}_t^a + \beta_1'f(\text{gap}_t) + \beta_2'f(\text{gap}_{t-1})$

where: $\text{gap} = \log(y/y^*)$

$y^* = (1 + \alpha) \bar{y}$

$f(\text{gap}) = (\omega^2 / (\omega - \text{gap})) - \omega$

π = change in the log of the non-oil GDP deflator

π^a = change in the log of the absorption deflator

y = real GDP

\bar{y} = trend real GDP

y^* = level of GDP at which the effect on inflation is zero

α = gap between trend GDP (\bar{y}) and level of GDP at which the effect on inflation is zero (y^*)

ω = parameter identifying maximum short-run level of GDP

δ = weight on forward-looking component of inflation expectations

λ' = weight on absorption deflator

$\hat{}$ = denotes fitted value from auxiliary regression (see text)

Data: Annual, 1967-91, pooled G-7 countries.

| α | Values of ω | | | | | | |
|----------|--------------------|------------|------------|------------|------------|------------|------------|
| | <u>.04</u> | <u>.05</u> | <u>.06</u> | <u>.07</u> | <u>.08</u> | <u>.09</u> | <u>.10</u> |
| 0.000 | 487.11 | 492.49 | 492.24 | 491.71 | 491.34 | 491.07 | 490.87 |
| 0.001 | 488.04 | 492.95 | 492.82 | 492.33 | 491.95 | 491.65 | 491.42 |
| 0.002 | 489.17 | 493.46 | 493.42 | 492.94 | 492.51 | 492.16 | 491.87 |
| 0.003 | 490.71 | 492.01 | 493.99 | 493.45 | 492.93 | 492.50 | 492.15 |
| 0.004 | 488.95 | 494.55 | 494.43 | 493.74 | 493.10 | 492.58 | 492.17 |
| 0.005 | 490.86 | 495.01 | 494.61 | 493.72 | 492.96 | 492.37 | 491.91 |
| 0.006 | 491.94 | 495.26 | 494.44 | 493.35 | 492.51 | 491.90 | 491.44 |
| 0.007 | 492.87 | 495.18 | 493.91 | 492.71 | 491.86 | 491.26 | 490.83 |
| 0.008 | 493.66 | 494.72 | 493.10 | 491.89 | 491.10 | 490.56 | 490.17 |
| 0.009 | 494.20 | 493.92 | 492.15 | 491.02 | 490.32 | 489.84 | 489.51 |

models are weakened by misspecification of the level of potential. ^{1/} Since traditional detrending techniques tend to produce estimates of excess demand that are, on average, too large when the world is truly nonlinear, their naive use can result in false rejections of nonlinear models.

Turning to the estimation of approximations to general functional forms, Table 3 provides results for the power series expansion discussed above. The "general" cubic model shown in Column (1) incorporates contemporaneous and lagged values of three terms in the output gap: linear, quadratic, and cubic (higher-order terms were statistically insignificant). Compared with the pure linear model, this specification contains four additional free parameters. Taking twice the difference in the value of the log likelihood function for this model versus the linear alternative (Column (4)) yields a test statistic of 11.5 distributed $\chi^2(4)$: as the critical value at the 99 percent confidence level is 13.3, the linear model cannot be conclusively rejected in favor of this general alternative. However, the critical value at the 97.5 percent confidence level is 11.1, so the evidence against the linear model remains strong by classical standards. The estimated relationship between output gaps and inflation using this general nonlinear approximation is shown in the middle two panels in Chart 3--the shape of the function is similar to the estimated CMM relationship, supporting the view that the latter provides a useful characterization of the convexity.

Setting the parameters on the cubic terms to zero yields the general quadratic model shown in Column (2) (these estimates are graphed in the lower panels of Chart 3). Comparing the LLF for this model with the general model indicates that the exclusion of the cubic terms implies only a small loss of fit. Not surprisingly, then, the evidence in favor of the quadratic model versus the linear model is stronger than that for the general cubic model: twice the difference in the LLF yields a test statistic of 10.0 distributed $\chi^2(2)$, exceeding the critical level of 9.2 at the 99 percent confidence level.

One implication of these results using flexible approximations is that the rejection of the linear model using the CMM function is not due to idiosyncracies of that specification: the linear model is also rejected using more general nonlinear alternatives. At the same time, comparing the fit of the quadratic model with that of the CMM function indicates that the latter fits the data better with a more parsimonious specification. In addition, the CMM function does not present the conceptual problem discussed earlier: in some regions, the quadratic function implies that the downward effect on inflation declines, rather than increases, as excess supply rises (see bottom panels of Chart 3). For these reasons, we retain the CMM function as the preferred nonlinear specification.

^{1/} Such a result is also consistent with the Monte Carlo evidence reported in Laxton, Rose and Tetlow (1993b).

Table 3. Estimation Results with Quadratic and Cubic Functional Forms

(t-statistics in parentheses)

Estimated
equation:

$$\pi_t = \alpha + (1-\lambda) (\delta \hat{\pi}_{t+1} + (1-\delta)\pi_{t-1}) + \lambda \hat{\pi}^a + \tau_1 \text{gap}_t + \tau_2 \text{gap}_t^2 + \tau_3 \text{gap}_t^3 + \beta_1 \text{gap}_{t-1} + \beta_2 \text{gap}_{t-1}^2 + \beta_3 \text{gap}_{t-1}^3$$

where: π = change in the log of the non-oil GDP deflator
 π^a = change in the log of the absorption deflator
 y = real GDP
 \bar{y} = trend real GDP
 gap = $\log(y/\bar{y})$
 δ = weight on forward-looking component of inflation expectations
 λ = weight on absorption deflator
 α = constant term
 $\hat{\pi}$ = fitted value from auxiliary regression (see text)

Data: Annual, 1967-91, pooled G-7 countries.

| | (1) General Cubic Model | (2) General Quadratic Model | (3) Constrained Cubic Model | (4) Linear Model |
|-----------|-------------------------------|-----------------------------------|-----------------------------------|---------------------|
| α | -0.004 (2.97) | -0.004 (3.33) | -0.001 (0.56) | -0.001 (0.67) |
| τ | 0.187 (1.87) | 0.188 (1.93) | 0.300 (2.88) | 0.257 (2.56) |
| δ | 0.500 (7.24) | 0.436 (7.63) | 0.384 (5.46) | 0.417 (6.19) |
| τ_1 | 0.277 (2.09) | 0.245 (3.51) | ... | 0.223 (2.97) |
| τ_2 | 5.449 (1.73) | 6.480 (2.19) | ... | ... |
| τ_3 | 69.234 (0.53) | ... | 268.011 (3.09) | ... |
| β_1 | 0.148 (1.32) | 0.281 (3.82) | ... | 0.181 (2.29) |
| β_2 | 9.003 (3.05) | 9.070 (3.17) | ... | ... |
| β_3 | 171.451 (1.34) | ... | 276.458 (3.07) | ... |
| LLF | 495.02 | 494.29 | 489.46 | 489.27 |

Finally, Column (3) of Table 3 shows the results for the constrained cubic model with no linear or quadratic terms. The fit of this model is similar to that of the linear model, with the same number of free parameters. The marginal improvement in the fit of this cubic model--which is symmetric in terms of the effects of positive and negative output gaps--compared with the large improvement shown by other nonlinear models suggests that an important role is played by asymmetries, and not just nonlinearities, in the output-inflation relationship. This result, then, supports the view that the effect of positive output gaps on inflation exceeds that of negative gaps. 1/

III. Illustrative Simulations of a Small Model

This section explores some policy implications of asymmetries in price adjustment of the type estimated above. We do so by simulating the effects of demand shocks in a small macroeconomic model, contrasting the results using a linear price adjustment equation with those using our preferred nonlinear specification. Of particular interest is how the results change for the two models as we delay the response of monetary policy to the demand shock.

The model consists of an equation describing aggregate demand (and the way monetary policy influences aggregate demand); a policy reaction function; and alternative inflation equations. The aggregate demand equation is specified and calibrated to reflect the stylized facts of the U.S. economy. The policy reaction function is similar to those used in the simulations described in Bryant, Hooper, and Mann (1993). The price adjustment equations are simplified versions of our estimated equations from the first two lines of Table 1.

Our model economy is assumed to experience stochastic shocks to demand. Of central interest is the effect of these shocks on output and inflation, and, in particular, the interaction between these effects and the reactions of policymakers. The simulations show that, when the output-inflation tradeoff is nonlinear, shocks that create excess demand lead to permanent output losses as the monetary authority responds to prevent an acceleration of inflation. The nonlinear economy also has the property that it is important for monetary policy to tighten quickly in the face of inflationary shocks: when the monetary response is delayed, output losses are larger, since a more severe tightening is required to combat higher inflation expectations. These results contrast with the situation when the output-inflation tradeoff is linear. In this case, delaying the reaction of interest rates to an increase in aggregate demand is actually desirable, because it can result in a *positive* cumulative effect on output. Given such a dramatic difference, it is clear that the issue of the form of the link

1/ Further tests supporting this conclusion are provided in Chan and others (1994).

between excess demand and inflation is of great practical importance to policymakers. These results suggest that the asymmetric formulation provides a logic for stabilization policy more consistent with the approach of policymakers than does the linear alternative.

1. A small simulation model

As discussed above, the core of our model consists of an aggregate demand equation, a Phillips curve, and a policy reaction function. In order to examine the implications of nonlinear price adjustment for forward-looking asset prices, we also include an auxiliary equation that links the five-year bond yield to expected future one-year yields.

The aggregate demand equation determines the deviation of output from its supply-determined value (the latter is assumed to be exogenous). Our specification reflects two assumptions that are critical to the ability of policymakers to control the economy. First, there are assumed to be significant lags between changes in interest rates and their full effects on aggregate demand. Second, there is persistence in movements in the output gap, implying that shocks to aggregate demand propagate to future periods. These assumptions are important because they make the model economy more difficult to control than if the dynamics linking demand shocks, interest rates, and output were purely contemporaneous. 1/

Specifically, the output equation is:

$$gap_t = 0.304 gap_{t-1} - 0.098 rr_t - 0.315 rr_{t-1} + \epsilon_t, \quad (9)$$

where: gap = output gap,
 rr = short-term real interest rate,
(measured as a deviation from equilibrium)
 ϵ = a stochastic disturbance to aggregate demand.

Rather than attempting to estimate equation (9) directly from observed annual data, the parameter values were obtained by simulating a quarterly equation of the relationship between the output gap and the short-term real interest rate using U.S. data. (Details are provided in Appendix II). 2/ The quarterly responses were then converted to an annual frequency by

1/ Indeed, when the lagged effects of the policy instrument exceed the contemporaneous effect, attempts to fully offset demand shocks will generally lead to "instrument instability," characterized by explosive oscillations in interest rates.

2/ The estimation of equation (9) using annual data is complicated by an identification problem: real interest rates tend to rise in the face of positive shocks to aggregate demand, generating a positive correlation between the contemporaneous real interest rate and the disturbance term. The use of higher frequency data gets around this problem to a large extent by more efficiently "time-ordering" the relationship between output and interest rates.

time-averaging the simulation results obtained from a number of shocks to the real interest rate. These results were consistent with those obtained from simulations of the Federal Reserve Board's quarterly MPS model of the U.S. economy. They show that, while there is some contemporaneous effect of real interest rates on output, most of the response occurs with a one-year lag. In addition, there is inertia in the output gap, as reflected in the coefficient of 0.304 on the lagged gap. Taken together, these coefficients imply that output would eventually fall about 0.6 percent below potential in the face of a persistent 100 basis points rise in short-term real interest rates above their equilibrium value.

The model is completed with equations that define the ex ante real rate, inflation expectations, and a policy rule. The definition of the ex ante real interest rate is:

$$rr_t = rs_t - \pi^e_{t+1} , \quad (10)$$

where: rs = short-term nominal interest rate,
 π^e_{t+1} = expected inflation in period t+1.

For inflation expectations, we rely on the estimates of the backward- and forward-looking components model implied by the estimated Phillips curve. This representation is used widely in policy simulation models. 1/ For the CMM specification, this implied the following weights on the model-consistent solution for future inflation and lagged inflation: 2/

$$\pi^e_{t+1} = 0.41 \pi_{t+1} + 0.59 \pi_{t-1} . \quad (11)$$

In keeping with the traditional approach to implementing policy feedback rules in simulation models, our policy reaction function is represented by an interest rate rule: interest rates rise when inflation is above target and fall when it is below. To incorporate developments in real activity (which signal changes in the future inflation rate) in the reaction function, the gap between actual and potential output is also included. The resulting policy reaction function is slight generalization of the one employed extensively in model simulation work reported by Bryant, Hooper and Mann (1993). This basic rule has been shown to have desirable properties compared to other simple alternatives such as money control and fixed exchange rates in a wide class of macroeconomic models. The parameters were chosen to reestablish the initial level of inflation within three years following a shock to aggregate demand, without inducing ongoing cycles in the economy. In our model, experiments pointed to weights of 3.0 on the deviation of inflation from the target and 1.0 on the output gap:

1/ For example, McKibbin and Sachs (1991), Masson, Symansky and Meredith (1990), Laxton and Tetlow (1991), and Laxton, Rose and Tetlow (1993c).

2/ Model-consistent means that π^e_{t+1} is derived from the actual model solution for future inflation rates.

$$rs_t - rsbar = 3.0 (\pi_t - \pi^*_t) + 1.0 gap_t , \quad (12)$$

where: π^* = inflation target,
 $rsbar$ = baseline interest rate.

In experiments where we want to delay the interest rate response of the authorities, the reaction function is temporarily "turned off" in the first year of the shock by exogenizing the level of the short-term interest rate. Of course, pegging the short-term interest rate is only possible in the short run; attempting to fix interest rates indefinitely in accelerationist models results in indeterminacy because no mechanism exists to bound future inflation expectations. In this sense, the monetary authority must eventually take actions to anchor longer-run inflation expectations.

Long-term rates are linked to short-term rates according to the expectations theory of the term structure. We could add a fixed term premium, but nothing is lost here by ignoring this complication. Thus, the long-term (five-year) rate is a geometric average of expected short-term rates:

$$1+r_l = \{(1+rs_t) (1+rs_{t+1}) (1+rs_{t+2}) (1+rs_{t+3}) (1+rs_{t+4})\}^{0.2} , \quad (13)$$

where r_l is the long-term interest rate. The simulation model then has equations (9)-(13) and the Phillips curve estimated in the previous section. For convenience, these equations are repeated in Table 4.

2. Simulation results

Some important policy implications of the asymmetries in the Phillips curve can be seen from conducting simple deterministic simulations on this small model. All variables are initially set at their deterministic equilibrium values--that is, the "control" solution is a deterministic steady state. We then consider temporary shocks to aggregate demand, varying the sign and size of the shock.

We start with the linear model. Chart 4 shows the shock-minus-control results for two shocks to aggregate demand. The solid lines indicate the results for a 2 percent positive shock when monetary policy responds immediately, while the dashed lines show the results for the same shock when the monetary authority delays increasing the short-term interest rate for one year. In each case, the shock is applied only in the first year of the simulation--thereafter, the responses reflect solely the dynamics of the model.

The short-term interest rate response is illustrated in the top left-hand panel. Consider, first, the results where the monetary authority reacts immediately to the shock. Short-term rates rise by slightly over 3 1/2 percentage points in the first year, blunting, but not eliminating, the initial effects of the demand shock on output. The initial change in output is reduced to about 80 percent of the shock itself (middle left-hand panel). However, this leaves significant inflationary pressures: as shown

Table 4. A Small Simulation Model

Nonlinear Inflation Model:

Phillips Curve: $\pi_t = \pi_{t+1}^e + .196 f(gap_t) + .276 f(gap_{t-1})$

Inflation Expectations: $\pi_{t+1}^e = .414 \pi_{t+1} + (1-.414) \pi_{t-1}$

Convex Function: $f(gap) = \omega^2 / (\omega - gap) - \omega$, $\omega = .049$

Real Interest Rate: $rr_t = rs_t - \pi_{t+1}^e$

Aggregate Demand Equation: $gap_t = .304 gap_{t-1} - .098 r_t - .315 rr_{t-1} + e^{gap}$

Long-Term Interest Rates: $rl_t = \prod_{i=0}^5 (1 + rs_{t+i})$

Policy Reaction Function: $rs_t = \lambda \bar{rs}_t + (1-\lambda) 3 (\pi_t - \pi_t^*) + \lambda gap_t$

- | | |
|------------------------------------|---|
| π = inflation | λ = variable to make rs exogenous |
| gap = output gap ($y^d - y^s$) | rr = real interest rate |
| rs = short-term interest rate | π_{t+1}^e = inflation expectations |
| rl = long-term interest rate | ω = wall capacity constraint |
| π^* = inflation target | |

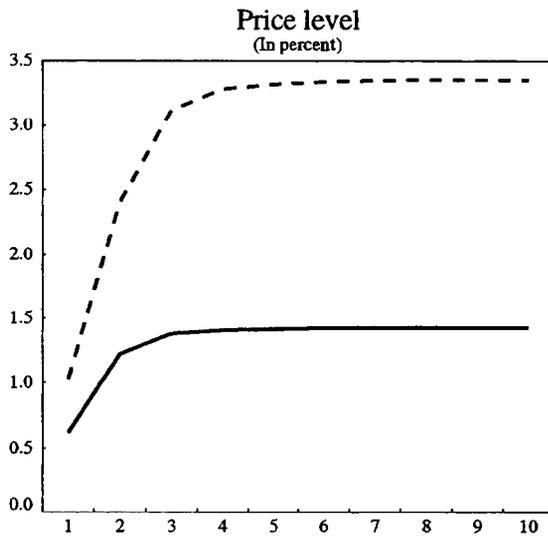
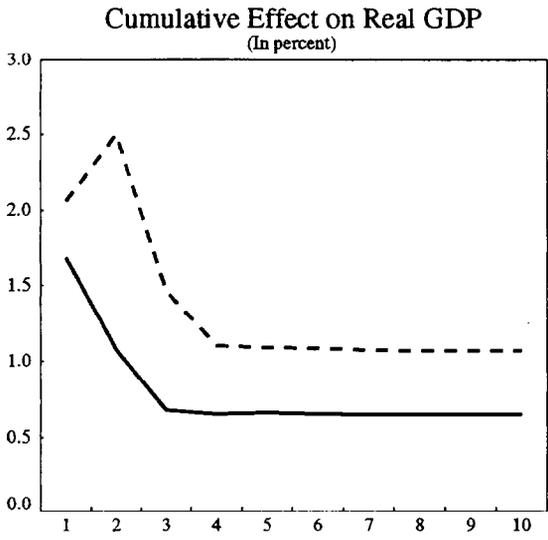
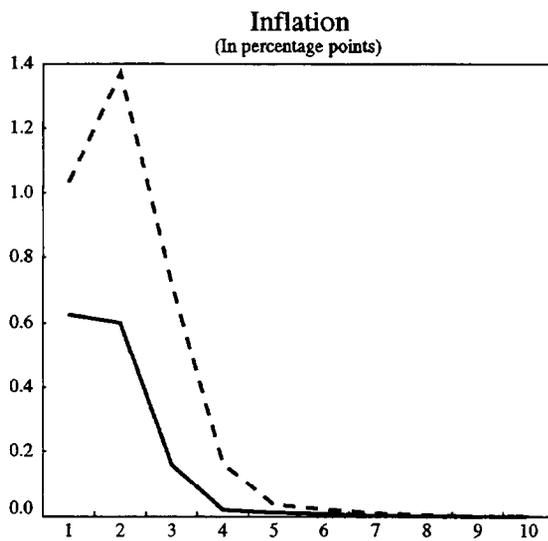
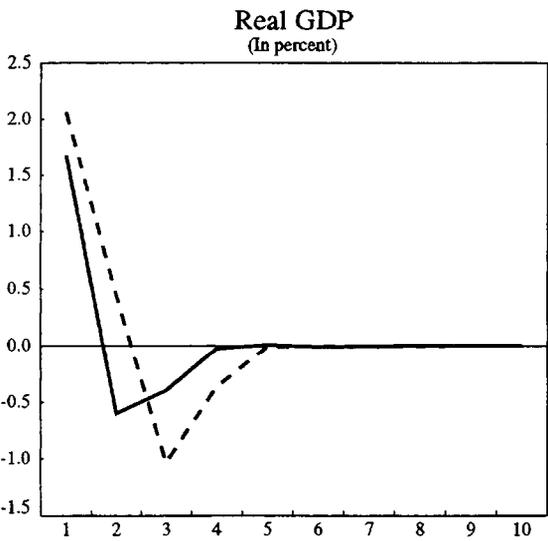
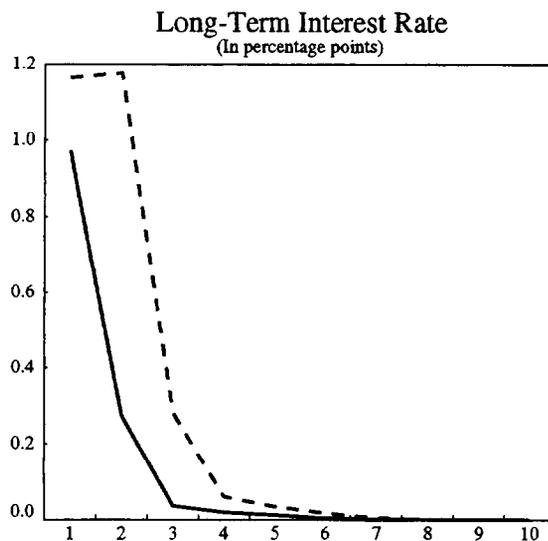
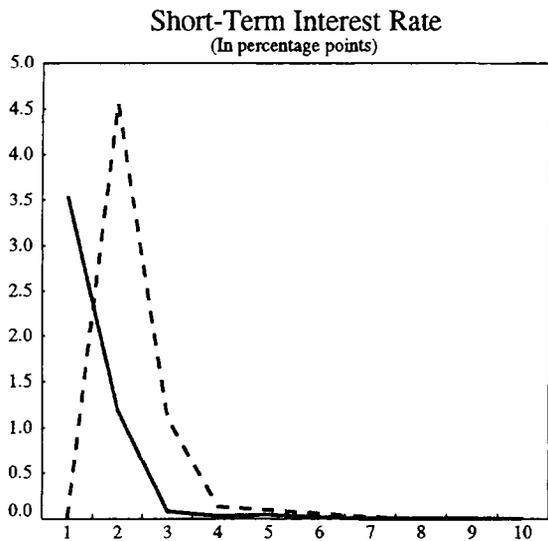
Estimated Model with a Linear Phillips Curve:

Phillips Curve: $\pi_t = \pi_{t+1}^e + .223 gap_t + .181 gap_{t-1}$

Inflation Expectations: $\pi_{t+1}^e = .417 \pi_{t+1} + (1-.417) \pi_{t-1}$

Chart 4: Linear Model Responses to Positive Demand Shocks (Deviation: shock minus control)

— 2% positive shock --- 2% positive shock with delayed policy response



in the middle right-hand panel, inflation jumps by about 0.6 percentage points in the first year. Since the shock lasts only one period, and the lagged effects of the initial rise in short-term rates are felt in the second year, the monetary authority can lower rates sharply in year two. Output falls below control, consistent with inflation returning to its target level. By the fourth year, inflation, output, and interest rates have all converged close to their control levels.

When the interest rate response is delayed by one year, the initial rise in output is slightly larger than the shock itself. This reflects the additional stimulus to aggregate demand of the larger drop in short-term real interest rates, which is caused by a larger rise in inflation (now more than 1 percentage point above control in the first year). In the second year, the reaction function causes interest rates to increase by almost 5 percentage points in response to higher inflation. Output remains above control, however, as the contemporaneous effect of higher interest rates is limited; as a result, inflation rises even further above its original level. In the third year, lagged interest rate effects "kick in," causing output to decline 1 percent below control, and inflation drops sharply. Nevertheless, it is only by the fifth year of the simulation that output, inflation, and interest rates converge back to their original values.

These results suggest--not surprisingly--that the amplitude and length of the cycles generated by demand shocks are larger when the response of monetary policy is delayed. Of particular interest in the current context, however, are the *cumulative* effects of the shocks on real output, as shown in the lower left-hand panel of Chart 4. In the linear model, a delayed policy response in the face of a positive shock to aggregate demand results in a larger cumulative rise in output than if there is no delay. 1/ Specifically, there is a cumulative increase in GDP of 1 1/4 percent when the policy response is delayed, compared with 3/4 percent when the response is immediate. 2/ On the basis of the linear model, then, there are potential benefits to delaying the reaction to demand shocks. 3/ Thus, the linear model does not support the view that policies should react quickly to preempt economic overheating--indeed, it points to the opposite conclusion.

1/ Since the monetary target is expressed in terms of inflation as opposed to the price level, there is also cumulative drift in prices. As shown in the lower right-hand panel of Chart 4, this drift amounts to 1 1/4 percent when the monetary authorities react by raising interest rates immediately versus 3 1/4 percent when they delay the increase in short-term interest rates.

2/ Appendix III discusses the relationship between cumulative output gaps and changes in the inflation rate in accelerationist models.

3/ The effects of negative shocks in the linear model are the mirror image of the responses in Chart 4. Thus, when the shock to aggregate demand is negative, the cumulative loss in output is greater when the monetary response is delayed.

Chart 5 reports the results for the same experiments done with the same model, except that the asymmetric inflation equation estimated in Line (1) of Table 1 is substituted for the linear equation. When policies respond immediately, output rises by 1 1/2 percent above control in the first year, similar to the response from the linear model. The inflation rate, however, jumps by 1 percentage point, almost double the effect in the linear model, reflecting the stronger effect of the positive output gap on inflation. The larger rise in inflation also causes the interest rate to increase by more on impact than when the model is linear. In the second and third years of the simulation, output must fall further below control than in the linear case to bring inflation back to its target level, reflecting both the larger impact effect of the shock on inflation and the smaller effect of negative output gaps on inflation. In the event, the secondary decline in output exceeds the initial increase, leaving a (small) negative cumulative output response.

When the monetary response is delayed, the impact effect on output of slightly over 2 percent is similar to that obtained with the linear model, while inflation rises by almost twice the linear response. In the second year of the simulation, then, interest rates also rise by almost twice as much. Nevertheless, output remains above control in the second year, given the small contemporaneous effect of interest rates on demand. Inflation rises further in the second year, peaking at almost 3 percentage points above control. The consequences for output in the third and fourth years of the simulation are dramatic, with large negative output gaps being required to reverse inflationary forces. The end result is a larger cumulative output loss than when the monetary authority responds immediately, amounting to almost 1 percent of GDP.

To summarize, in both the linear and nonlinear models, delaying the policy response to a positive demand shock increases the variance of the effects on output and inflation. When the model is linear, however, the cumulative increase in output is larger when the response is delayed than when it is immediate. In the nonlinear model, the reverse is true: the cumulative loss in output is exacerbated by a delayed policy response.

Chart 6 reports the same experiments with the nonlinear model, but with a 2 percent *negative* (as opposed to positive) shock to aggregate demand. While the size of the initial effects on output in the two simulations are similar to those when the shock is positive (with the sign reversed), the drop in inflation is much smaller than the increase associated with a positive demand shock, reflecting the nonlinear price response. The increases in real GDP above control beyond the first year of the simulation are smaller than in the case of a positive shock, resulting in larger cumulative losses in output. It still holds, however, that the output loss is greater when the policy response is delayed than when it is immediate.

These experiments illustrate one of the most important implications of asymmetries in the Phillips curve: monetary policy should respond quickly to inflationary pressures. Allowing inflationary conditions to persist triggers the unfavorable consequences of asymmetry, exacerbating the

Chart 5: CMM Model Responses to Positive Demand Shocks
(Deviation: shock minus control)

— 2% positive shock - - - 2% positive shock with delayed policy response

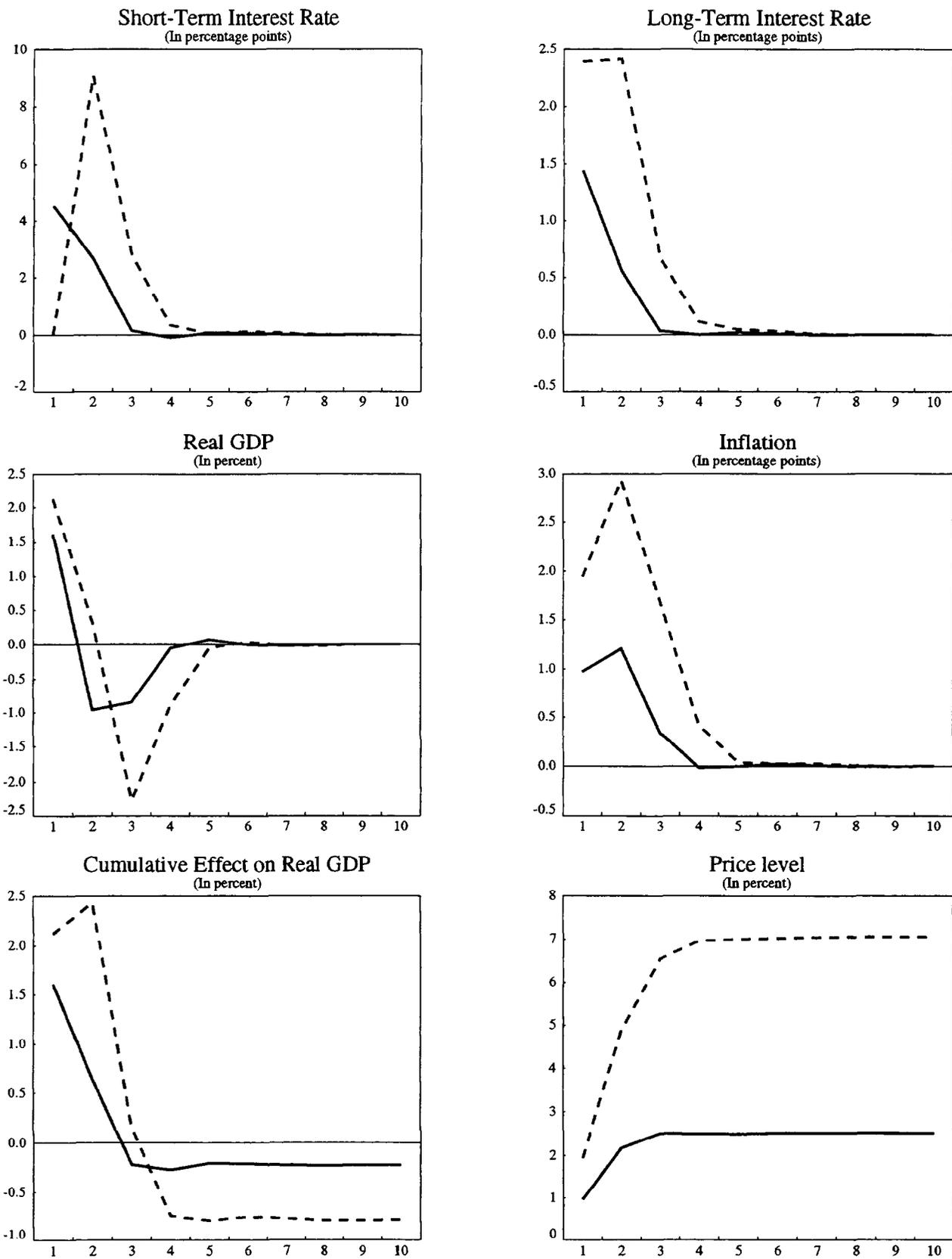
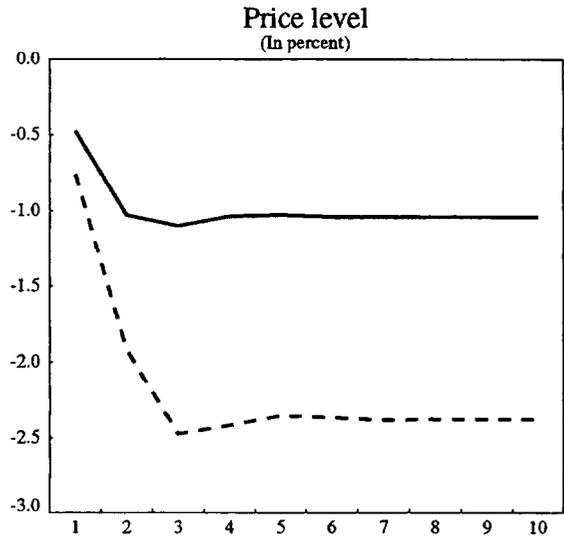
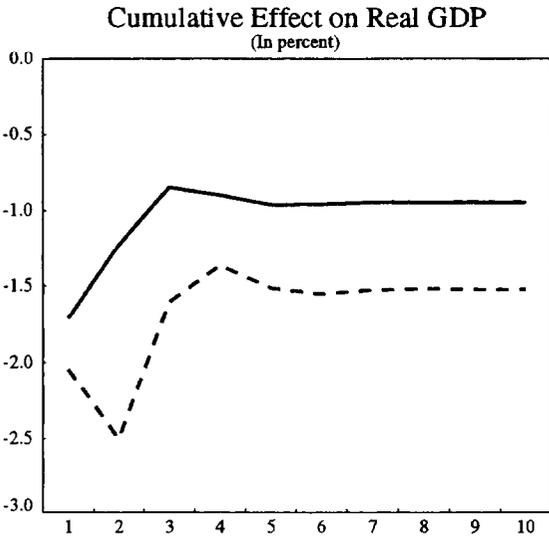
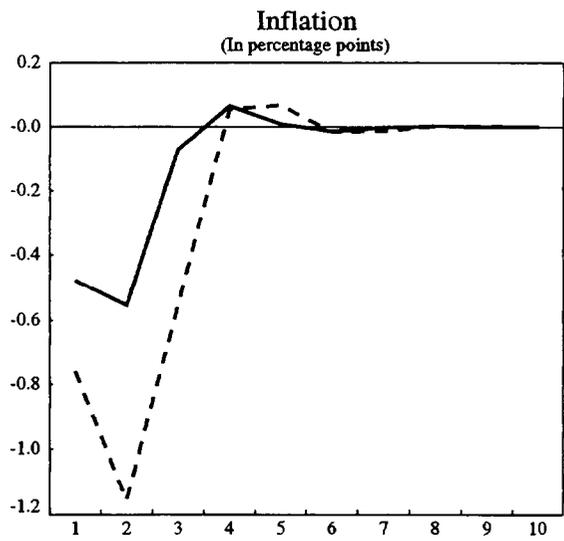
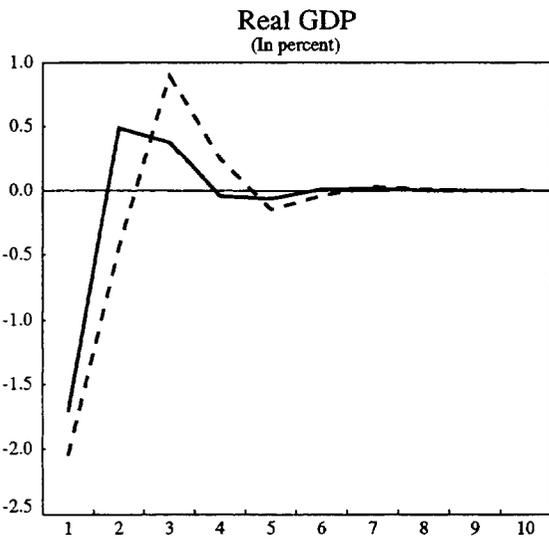
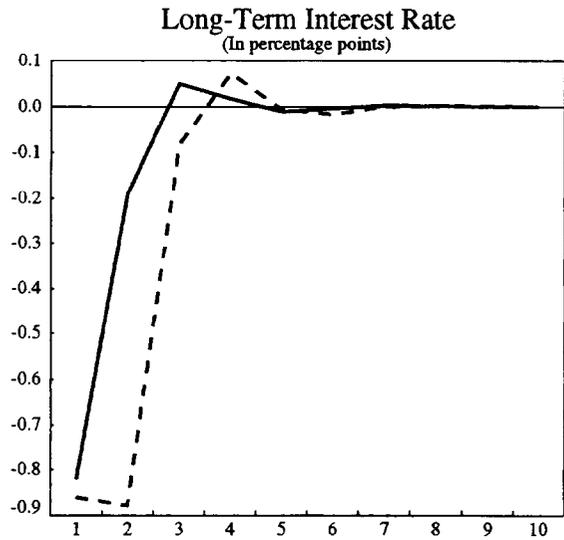
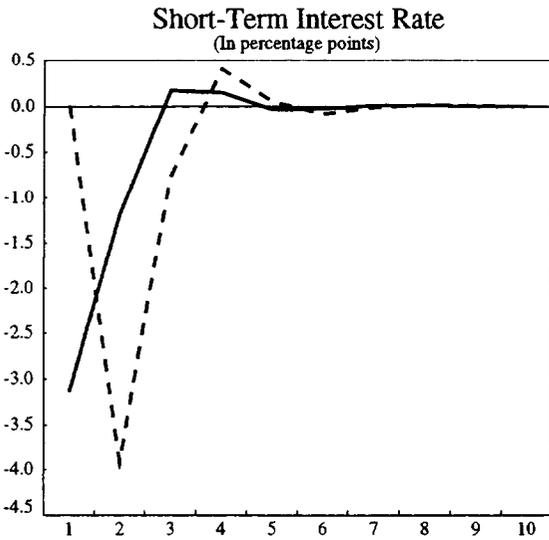


Chart 6: CMM Model Responses to Negative 2% Demand Shocks
(Deviation: shock minus control)

— 2% negative shock --- 2% negative shock with delayed policy response



consequent loss of output and making it more costly to re-establish the inflation target.

IV. Conclusions and Directions for Future Research

The estimation results presented in this paper provide strong evidence in favor of an asymmetric relationship between inflation and activity: using data for the major industrial economies, excess demand is found to raise inflation by more than excess supply acts to reduce it. This evidence is consistent with the characteristics of business cycles in these economies over the past two decades--short-lived but rapid increases in inflation have tended to be followed by protracted downturns in economic activity.

Thus, the asymmetric model of price adjustment provides at least a partial synthesis of classical and Keynesian views: under some circumstances, demand shocks will primarily affect prices, while under others they will have a large impact on activity. We have also shown that previous attempts to estimate asymmetric output-inflation tradeoffs may have failed to find strong supporting evidence because of a misspecification of output gaps. In particular, in a nonlinear world subject to stochastic shocks, "trend" output will be below the level of output at which there is no tendency for inflation to rise or fall. Failing to account for the gap between these two concepts reduces the power of tests of the nonlinear hypothesis.

The existence of an asymmetric output-inflation relationship has clear implications for demand management policies. In a linear world, there is no (direct) link between demand-side policies and the average level of output: positive and negative shocks to demand will have symmetric effects on inflation, with the net effect on output averaging to zero regardless of the stance of policies. In a nonlinear world, in contrast, positive shocks to demand raise inflation by more than negative shocks lower it. As a result, policymakers can raise the average level of output by reducing the variance of output around trend. In particular, prompt actions to offset positive demand shocks can reduce the need to take much stronger actions down the road to offset inflationary pressures. When demand shocks are in the other direction, large amounts of output slack do little to lower inflation--again, the nonlinear model provides a stronger justification for offsetting such shocks through demand stimulus. In short, while the "downfall of mankind" predicted by Hunderwasser in the introductory quote may overstate the dangers of ignoring nonlinearities, our model indicates that a failure to respond promptly to signs of economic overheating can lead to costly policy errors.

In terms of directions for future research, several areas appear promising. One would be to examine more systematically the desirability of alternative policy rules in a nonlinear framework using stochastic simulations, along the lines pursued in Bryant and others (1993). In this context, an interesting question is whether policy rules should themselves be asymmetric when the rest of the model is asymmetric; in particular, is

there a case for policymakers to react either more promptly or more forcefully to signs of economic overheating than to downturns? 1/

Another area for research involves the information content of financial variables. For instance, movements in long-term interest rates are often used as a guide to the desirability of policy-driven changes in short-term rates. Linear models shed little light on this issue, as the predicted change in long-term interest rates is independent of the state of the economy. In nonlinear models, in contrast, the response of long-term rates depends on whether output is initially above or below potential, which, in turn, determines the degree of price pressures associated with the policy innovation. Preliminary simulations of a fully-specified macroeconomic model (MULTIMOD) have generated interesting results in this area.

Finally, it would be desirable to derive more sophisticated measures of potential output than those based on the simple filtering techniques used here. In particular, even stronger inferences about the existence and nature of nonlinearities might be obtained by deriving "model-consistent" measures of the gap; for instance, by extending the work of Kuttner (1991) to allow for nonlinear dynamics.

1/ Further evidence on this point is provided by the stochastic simulation results in Laxton, Ricketts and Rose (1994).

Stochastic Equilibrium in an Economy with Asymmetric Nominal Dynamics

It has been suggested that in a stochastic environment with symmetric shocks, the presence of a convexity in the Phillips curve implies that the average level of output lies below the level attainable in an environment with no shocks. 1/ We establish this point formally here.

Consider a simplified version of an asymmetric Phillips curve:

$$\pi - \pi^e = f(y-y^*) + \epsilon, \quad \text{where } f(0) = 0 \quad \text{and } f'(\cdot) \geq 0. \quad (\text{I.1})$$

The variable ϵ represents a random error term with zero mean; output, y , is also stochastic. We are interested in the case where $f(\cdot)$ is continuous and globally convex:

$$(f(y_1-y^*) + f(y_2-y^*)) / 2 \geq f((y_1+y_2)/2 - y^*) \quad \forall \quad y_1, y_2 \quad (\text{I.2})$$

with strict inequality holding for (at least) some values of y_1, y_2 .

Consider the properties of a stochastic equilibrium, defined as a situation in which there is no systematic difference between π and π^e . Taking the unconditional expectation of equation (I.1), where $E(\cdot)$ denotes the expectations operator, and noting that $E(\epsilon)=0$, this implies:

$$E (f(y-y^*)) = 0. \quad (\text{I.3})$$

Given the continuity of $f(\cdot)$ and convexity (I.2), it follows from Jensen's inequality that: 2/

$$f(E(y-y^*)) = f(E(y)-y^*) \leq E (f(y-y^*)) = 0, \quad (\text{I.4})$$

with strict inequality holding if $f(\cdot)$ is strictly convex (and the variance of y is strictly positive, as discussed below).

Given the restriction that the effect on inflation cannot decrease as excess demand rises (i.e., that $f'(\cdot) \geq 0$), it follows from (I.4) that:

$$E(y) - y^* \leq 0, \quad (\text{I.5})$$

with strict inequality holding as long as $f(\cdot)$ is strictly convex and y has non-zero variance. Thus, the mean level of output in a stochastic economy with a convex Phillips curve lies below the equilibrium of the economy without shocks to output.

1/ See, for instance, DeLong and Summers (1988) and Laxton, Rose and Tetlow (1993c).

2/ Proofs are widely available. See, for example, Mood, Graybill and Boes (1974), page 72.

The relationship between the variance of output and the extent to which $E(y)$ falls below y^* is also of interest. To establish the basic point, take the special case of a quadratic function (i.e., a second-order approximation to a general $f()$ around 0), which we can solve analytically:

$$\pi - \pi^e = \alpha (y - y^*) + \beta (y - y^*)^2 + \epsilon, \quad \text{for } \alpha, \beta > 0. \quad (\text{I.6})$$

This function is not a sensible Phillips curve in all regions, because it is only well-behaved for $y - y^* > -\alpha/2\beta$, but it will suffice to demonstrate the basic point.

Defining the stochastic equilibrium as above, equation (I.6) implies:

$$\begin{aligned} 0 &= \alpha (E(y) - y^*) + \beta E[y - E(y) + E(y) - y^*]^2 \\ &= \alpha (E(y) - y^*) + \beta [\text{Var}(y) + (E(y) - y^*)^2] \end{aligned} \quad (\text{I.7})$$

Note that if output has zero variance, the solution to (I.7) collapses to $E(y) = y^*$. However, if $\text{Var}(y)$ is positive, then the second term is strictly positive and the solution must be characterized by $E(y) < y^*$. However, we seek an explicit solution for $E(y)$ in terms of its variance. Since equation (I.7) is a quadratic in $E(y) - y^*$, we have the solution:

$$E(y) - y^* = -\alpha/2\beta + \{(\alpha/2\beta)^2 - \text{Var}(y)\}^{1/2} \quad (\text{I.8})$$

where the other root is ruled out by the restriction on the sensible domain of definition of the function (i.e., $y - y^* > -\alpha/2\beta$). The term in $\{\}$ is a negative function of $\text{Var}(y)$: as the variance of output rises, $E(y)$ falls relative to y^* . Thus, a policy that reduces output fluctuations also raises the average attainable level of output. In the limit, if output can be perfectly controlled, the average attainable level of output in the stochastic economy is the same as in an economy without shocks.

This example also reveals the other determinant of the difference between $E(y)$ and y^* : the degree of convexity in $f()$. A relative measure of convexity is given by f''/f' , which here is simply $2\beta/\alpha$ (evaluated at $y = y^*$). Differentiating (I.8) with respect to $2\beta/\alpha$ yields:

$$d(E(y) - y^*)/d(f''/f') = (\alpha/2\beta)^2 - (\alpha/2\beta)^3 / \{(\alpha/2\beta)^2 - \text{Var}(y)\}^{1/2} \quad (\text{I.9})$$

Since α and β are positive, this expression has a maximum value when $\text{Var}(y) = 0$, where it collapses to zero; otherwise, it is strictly negative. Thus, the higher the degree of convexity in the Phillips curve, the greater the mean of output falls below y^* .

Specification of the Aggregate Demand Equation at an Annual Frequency

The annual aggregate demand equation discussed in the text was obtained by converting a quarterly model of the monetary transmission mechanism to an annual frequency. This approach was taken because of the well-known econometric difficulties associated with directly estimating such equations with annual data. The difficulties arise because, to contain inflationary pressures, interest rates must rise during periods of excess demand and fall during periods of excess supply. This endogeneity tends to produce positive signs on the contemporaneous real interest rate in equations estimated using ordinary least squares. Since this problem will generally be less severe with quarterly data, a quarterly equation was used to develop an annual representation of the monetary transmission mechanism.

One source of quarterly estimation results is Roberts (1994), who reports reduced-form regressions linking the quarterly percent change in real GDP to lagged changes in the federal funds rate. 1/ The key result is that there are significant lags from changes in short-term interest rates to real GDP growth: a change in the federal funds rate only starts to affect GDP after two quarters. A similar picture emerges from our own empirical work with reduced-form equations. For example, Table II.1 reports quarterly estimation results for the United States when we regress the output gap on its own lags as well as lagged values of the short-term real interest rate. 2/ The results for the unrestricted regression with two lags on the gap and eight on the real interest rate proxy are shown in Column (1). The only significant effects show up with a lag of two quarters. Indeed, if we constrain all other interest rate coefficients to zero, we end up with a parsimonious representation of the transmission mechanism (Column (2)).

To compare the estimated quarterly models more formally, and also to compare them with structural models, we conducted some simple simulations. Table II.2 reports the effects on real GDP for a temporary monetary-induced 100 basis points increase in the federal funds rate derived from two versions of the MPS model, as well as the results for the same experiment conducted on Roberts' reduced-form equation. The experiment involved a

1/ This approach follows work by Bernanke and Blinder (1983) and Bernanke and Kuttner (1992).

2/ The same centered two-sided filter was used to measure potential output as was used to estimate the inflation equation; however, in this case, because quarterly instead of annual data are used, we switched from a 5-year to a 21-quarter centered moving average filter. The real interest rate was constructed by taking the three-month CD rate and subtracting the Michigan Survey's measure of one-year-ahead inflation expectations (constructed by averaging monthly observations). Similar parameter values are obtained using the federal funds rate to construct the real interest rate.

Table II.1: Estimation Results for Quarterly U.S. Output Gap Equations

(t-statistics in parentheses)

Estimated equation: $gap_t = \alpha + \tau_1 gap_{t-1} + \tau_2 gap_{t-2} + \beta_1 rr_{t-1} + \dots + \beta_8 rr_{t-8}$

where: α = constant term
 gap = $\log(y/\bar{y})$
 rr_t = real interest rate ($rs_t - \pi^e_{t+1}$)
 \bar{y} = 21-quarter centered moving average of U.S. quarterly real GDP
 y = U.S. quarterly real GDP
 rs_t = rate on three-month certificates of deposits in the secondary market
 π^e_{t+1} = one-year-ahead measure of inflation expectations taken from the Michigan Survey data

Data: Quarterly U.S. data, 1978Q1-1991Q4.

| | (1) | (2) |
|-------------|---------------|---------------|
| α | 0.486 (2.04) | 0.552 (3.11) |
| τ_1 | 1.01 (6.62) | 1.045 (8.65) |
| τ_2 | -0.254 (1.63) | -0.285 (2.43) |
| β_1 | 0.059 (0.70) | ... |
| β_2 | -0.211 (1.98) | -0.155 (3.71) |
| β_3 | 0.034 (0.30) | ... |
| β_4 | 0.026 (0.26) | ... |
| β_5 | -0.064 (0.64) | ... |
| β_6 | -0.067 (0.66) | ... |
| β_7 | 0.030 (0.30) | ... |
| β_8 | 0.058 (0.70) | ... |
| \bar{R}^2 | 0.805 | 0.820 |
| DW | 2.110 | 2.072 |

Table II.2: Lags in the Transmission Mechanism in MPS
Versus Simple Reduced-Form Models

(Percent response of real GDP)

The results for the MPS model and Robert's (1994) model are for a temporary four-quarter increase of 100 basis points in the federal funds rate. The results for our equation are for the same shock applied to the *real* short-term interest rate. For this reason, our results tend to be smaller because we are holding inflation expectations fixed. In other words, a monetary-induced shock to the nominal interest rate implies lower inflation expectations and a growing shock to the real interest rate.

We have included results for two versions of the MPS model because there is some uncertainty about the strength of real-financial linkages in the current version of the model. The results for the first version are obtained from a full-model simulation of the official model. The results for the second version are obtained by excluding the effects of the earnings-price ratio on investment, as there is some disagreement as to whether these effects are too large in the official model. Roberts' (1994) reduced-form equation produces estimates that fall between the two MPS estimates. In terms of timing, all four models suggest that there are significant lags in the transmission mechanism.

| Quarter | <u>Federal Reserve Board's MPS Model:</u> | | <u>Reduced-Form Models:</u> | |
|---------|---|-------------------|-----------------------------------|---------------------------------|
| | <u>Version #1</u> | <u>Version #2</u> | <u>Roberts (1994)¹</u> | <u>Our Equation²</u> |
| 0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 1 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2 | -0.1 | -0.1 | -0.3 | -0.2 |
| 3 | -0.2 | -0.1 | -0.4 | -0.3 |
| 4 | -0.3 | -0.2 | -0.5 | -0.4 |
| 5 | -0.4 | -0.2 | -0.7 | -0.4 |
| 6 | -0.5 | -0.3 | -0.5 | -0.3 |
| 7 | -0.6 | -0.3 | -0.5 | -0.3 |
| 8 | -0.7 | -0.3 | -0.5 | -0.2 |

^{1/} Results based on the reduced-form equation reported in Column (3) of Table 1 in Roberts (1994).

^{2/} Results based on the reduced-form equation reported in Column (2) of Table II.1.

100 basis point increase in the federal funds rate for four quarters. The results from the first version of the MPS model were derived from a full-model simulation on the current version of the model. The results for the second version were derived by eliminating the effects of equity prices on the cost of capital (the latter effects are absent in many other models of the U.S. economy, and may be too large in MPS.) As can be seen in Table II.2, the results of the adjusted MPS model are broadly consistent with Roberts' reduced-form evidence.

Table II.2 also includes results for an experiment using our equation when the shock is conducted on the short-term real interest rate. Naturally, the responses in the second year are smaller for a real interest rate shock, because a monetary-induced reduction in the nominal interest rate generates a rise in inflation expectations. In other words, the magnitude of the shock, in terms of changes in real interest rates, is slightly higher in the other models, and grows through time.

Finally, in order to derive the annual version of the output gap equation reported in the main text, we shocked the quarterly equation repeatedly with interest rate innovations, used the resulting time series to construct annual time series, and then used the latter to estimate a parsimonious annual model. ^{1/} While this process may seem circuitous, it avoids the econometric problem discussed above and makes our results easier to compare with the existing evidence.

^{1/} We assumed that the real interest rate shocks were drawn from a normal distribution with standard deviation of 2.34, based on the historical sample.

Cumulative Output Gaps and the Phillips Curve

One aspect of the model simulations that is emphasized in the main text is the cumulative effect on real output of demand shocks. Here we look analytically at how the specification of the Phillips curve influences this effect. Two aspects of the specification turn out to be important: the degree of convexity in the output-inflation tradeoff, and the extent to which expectations are forward-looking.

1. Backward-looking models

The traditional backward-looking Phillips curve with a linear output-inflation tradeoff can be represented as: 1/

$$\pi_t = \pi_{t-1} + \beta \text{gap}_t . \quad (\text{III.1})$$

One of the key properties of this equation is that, starting from an initial period 0, the cumulative output gap from period 1 to any future period N depends only on the change in inflation from period 0 to period N. This can be seen as follows:

$$\begin{aligned} \pi_1 &= \pi_0 + \beta \text{gap}_1 \\ \pi_2 &= \pi_1 + \beta \text{gap}_2 = \pi_0 + \beta (\text{gap}_1 + \text{gap}_2) \\ \pi_3 &= \pi_2 + \beta \text{gap}_3 = \pi_0 + \beta (\text{gap}_1 + \text{gap}_2 + \text{gap}_3) \\ \Rightarrow \pi_N &= \pi_0 + \beta \sum_{t=1}^N \text{gap}_t . \end{aligned} \quad (\text{III.2})$$

In the case where inflation returns to its initial level of π_0 by period N, equation (III.2) implies that the sum of the output gaps must be zero:

$$\pi_N = \pi_0 \quad \Rightarrow \quad \sum_{t=1}^N \text{gap}_t = 0 . \quad (\text{III.3})$$

This has been referred to as the "integral-gap" model because of property (III.3). Note, in particular, that this property is invariant to the specification of the policy rule, or, indeed, any other relationship in the model. As pointed out by, inter alia, Summers (1991), this result implies severe limits on the role of stabilization policies, because the mean level of output for a fixed inflation target does not depend on the path of output or inflation over time: in other words, there is no role

1/ We confine our analysis to "accelerationist" models where current inflation is homogeneous degree one in past and future inflation.

internal to the model for policies that attempt to minimize deviations in output around its trend level.

The results are somewhat different when the output-inflation relationship is nonlinear. Substituting $f(\text{gap}_t)$ for gap_t in equation (III.1) gives the following analog to equation (III.3):

$$\pi_N - \pi_0 \quad \Rightarrow \quad \sum_{t=1}^N f(\text{gap}_t) = 0 \quad (\text{III.3}')$$

$$\Rightarrow \quad \sum_{t=1}^N \text{gap}_t < 0, \quad (\text{III.4})$$

where equation (III.4) follows from equation (III.3') and the discussion in Appendix I: when $f()$ is convex, the sum of the gaps will be negative when the sum of $f(\text{gap}_t)$ is zero. More generally, policies that reduce the variance of fluctuations in output will raise the mean level of output.

2. Forward-looking models

The relationship between changes in inflation rates and cumulative output gaps is more complicated when we allow for inflation models with both forward- and backward-looking inflation components: 1/

$$\pi_t = \delta \pi_{t-1} + (1-\delta) \pi_{t+1} + \beta f(\text{gap}_t) . \quad (\text{III.5})$$

Solving this model for inflation in period N yields a solution that can be characterized as follows:

$$\pi_N = g(\delta, N) \pi_0 + (1-g(\delta, N)) \pi_{N+1} + \sum_{t=1}^N h(\delta, t, N) f(\text{gap}_t) , \quad (\text{III.6})$$

where the function $g(\delta, N)$ indicates that the relative weights on future and initial inflation depend on both δ and the length of the horizon N . The parameters on the $f(\text{gap}_t)$ terms are represented by $h(\delta, t, N)$; thus, the weight on each output gap term depends on the extent to which expectations are forward-looking (δ), the length of the horizon (N), and the position of the gap in time (t).

One important property of the function $h()$ is that it is increasing in t ; in other words, gaps further in the future have a higher weight. To illustrate this property, Chart III.1 graphs the relationship between

1/ The use of π_{t+1} as opposed to π_{t+1}^e in equation (III.5) is based on the assumption that all shocks to aggregate demand beyond period 0 are perfectly anticipated--i.e., these results hold along a "perfect foresight" path.

$h(\delta, t, N)$ and t for values of δ ranging from zero to one (N is fixed at 8 periods). In the purely backward-looking case ($\delta = 1$), the weights for all periods are identically equal to one, as discussed above. In the purely forward-looking case ($\delta = 0$), the weights on all gaps are zero except for period 8, when it equals one. For values of δ between these extremes, the weights on the gaps increase monotonically as t increases.

Assume that the horizon N is sufficiently long that inflation has stabilized, so that π_N equals π_{N+1} . Then equation (III.6) implies:

$$\pi_N - \pi_0 \quad \Rightarrow \quad \sum_{t=1}^N h(\delta, t, N) f(\text{gap}_t) = 0 . \quad (\text{III.7})$$

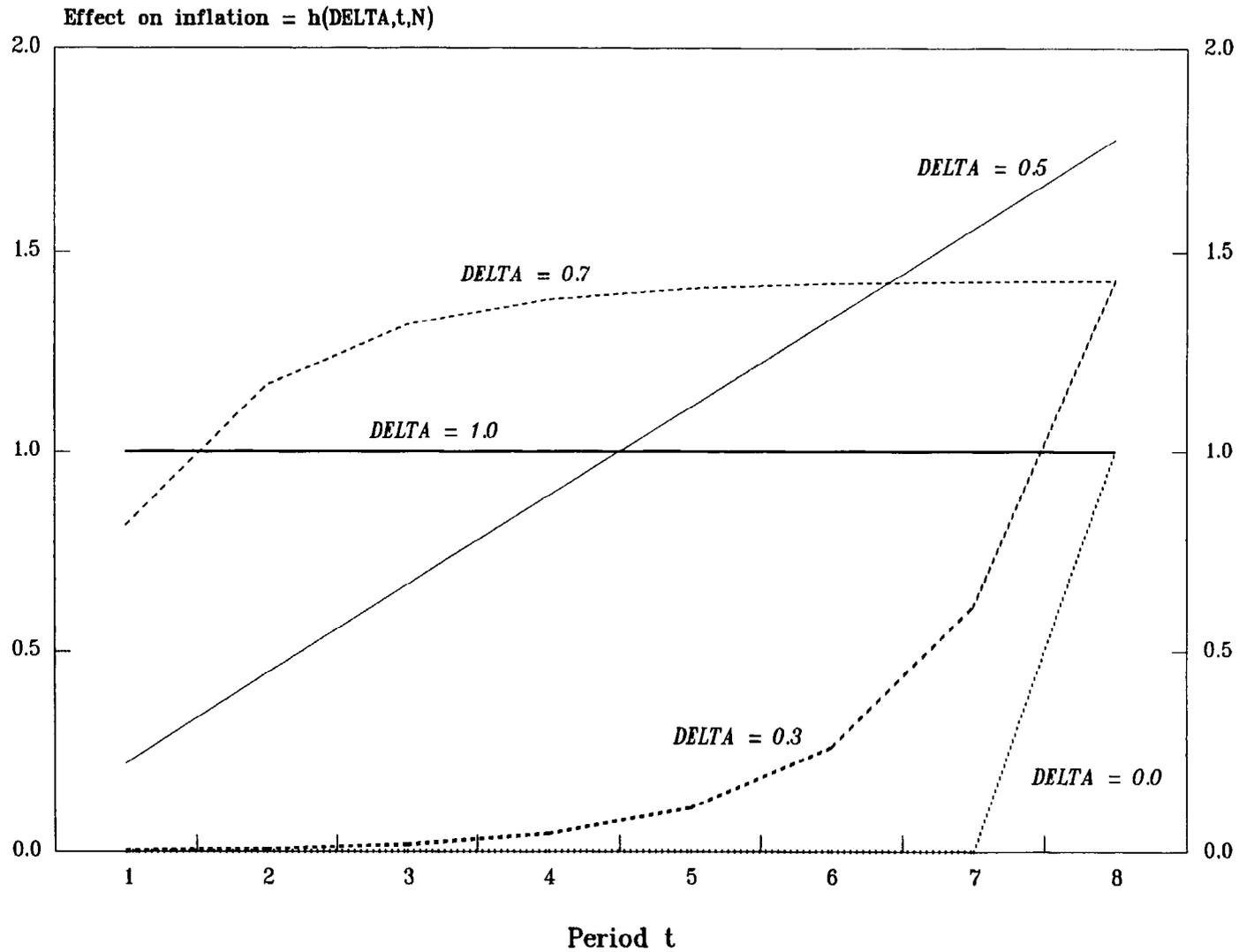
Because the weights on the output gaps in different time periods are not identical, a demand shock will result in a non-zero cumulative output gap even if the inflation equation is linear. The sign of the cumulative gap will depend on the sequence of the shocks to demand. In the case of a positive shock to demand in the current period, the cumulative sum of the gap terms will generally be positive, as future negative gap terms are weighted more heavily in equation (III.7). If $f(\text{gap})$ is linear, then the sum of the gaps themselves will be positive; if $f(\text{gap})$ is convex, the sum may be either positive or negative, depending on whether the convexity underlying equation (III.4) outweighs the difference in coefficients on the individual gap terms. Of course, if the demand shocks are negative, then both forces will work in the same direction, and the cumulative gap will also be negative.

A specific example illustrates the effect of a positive shock to aggregate demand in the linear version of equation (III.5). Suppose, in particular, that the monetary authority sets policy to "engineer" a rise in inflation of 1 percentage point in period 1, after which policy is set such that inflation returns to its control level. The path of inflation and the output gaps will be as follows (where the slope parameter on the gap is normalized to 1):

| Period | π_t | $\delta\pi_{t-1}$ | $(1-\delta)\pi_{t+1}$ | gap_t |
|--------|---------|-------------------|-----------------------|----------------|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 2 | 0 | δ | 0 | $-\delta$ |
| 3 | 0 | 0 | 0 | 0 |

It is evident that the cumulative output gap is $(1-\delta)$, consistent with the above model. When the Phillips curve is convex, the cumulative gap can be either positive or negative, depending on whether the convexity outweighs this effect.

Chart III.1
Coefficients on Output Gaps in (III.6) for Alternative Values of DELTA



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