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On the Dynamics of Economic Growth

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Abstract

This paper examines the dynamics of economic growth. First, it demonstrates that the standard neoclassical growth model with constant elasticity of intertemporal substitution is not consistent with the patterns of development we observe in the real world, once we consider the initial conditions. Second, it examines an alternative growth model, which is consistent with endogenously determined initial conditions and also generates dynamics that are in accord with the historical patterns of growth rates, capital flows, savings rates and labor supply. The alternative model is a generalized version of the neoclassical growth model, with increasing rates of intertemporal substitution due to a Stone-Geary type of utility.

JEL Classification Numbers:
O11, O41

1/ This paper is a revised version of the first chapter of my Ph.D. thesis, presented to Harvard University in July 1994. I would like to thank all the participants in the Macro-Growth Seminar at Harvard, and especially Professor Robert Barro, for helpful comments and discussions.

Contents

Summary

In examining the dynamics of economic growth, this paper first takes a closer look at the standard neoclassical growth model with constant elasticity of intertemporal substitution. It demonstrates that this model can generate either increasing growth rates and international divergence in income levels, or diminishing growth rates and international convergence. The actual outcome will depend on the initial conditions that existed before the process of economic growth began. Furthermore, the paper shows that both theoretical arguments and empirical facts point to initial conditions that are exactly the opposite of those needed to generate the standard predictions of the model.

Second, the paper examines an alternative growth model. The alternative model is a generalized version of the neoclassical growth model, with increasing rates of intertemporal substitution due to a Stone-Geary type of utility. Both micro and macro studies in recent literature find strong evidence for this type of preferences. The dynamics of economic growth generated by this growth model are consistent with endogenously determined initial conditions. Moreover, they are in accord with the historical patterns of growth rates, capital flows, savings rates, and labor supply. This growth model generates these dynamics without the need to assume heterogenous preferences, externalities, or increasing returns.

This paper has important policy implications regarding the effectiveness of different strategies of development, explaining, for example, why a strategy of development based on capital flows to developing countries cannot have long-term effects. The model presented in the paper also suggests an alternative strategy of development: assist poor countries in getting richer by helping them to increase the level of technology and of knowledge, including human capital.

I. Introduction

The simplest version of the neoclassical growth model assumes constant technological progress and identical preferences across people in different countries. Under these assumptions, all economies share a common steady state and the only difference among them is their position along the path leading to this steady state. This simple version is widely believed to generate dynamics of growth associated with monotonically decreasing rates of growth and therefore monotonic international convergence in the standard of living. These dynamics are in accord with empirical evidence from developed countries, but not with evidence from less developed countries. However, as this paper reveals, the absolute convergence result in the standard model is true only in the particular case of constant elasticity of intertemporal substitution. Even then, it depends on a particular arbitrary choice of initial conditions. The paper proves that once we let these initial conditions be endogenously determined, the predictions of the standard neoclassical growth are reversed to absolute divergence, contradicting most of the stylized facts of economic growth.

Another version of this model allows for different rates of technological progress or for different values for the preference parameters. In this case, each economy will converge to its own steady state. Instead of generating absolute convergence, this version generates conditional convergence. The dynamics generated by this version of the model are in accord with cross-country data, but not with long time-series data. Moreover, the assumption of permanent differences in the rate of technological progress or of different preferences across countries is not theoretically appealing.

This paper develops a different version of the neoclassical growth model, characterized by increasing elasticity of intertemporal substitution. This version is consistent with endogenously determined initial conditions and also generates dynamics that are in accord with a large array of historical evidence on growth rates, capital flows, savings rates and labor supply. It is also in accord with studies concerned with individual preferences. As opposed to the conditional convergence approach, it assumes that all people have identical preferences and that differences in the rate of technological progress across countries cannot be permanent.

We examine the theoretical predictions of the model and also simulate its dynamics. The model predicts patterns of growth with rates of growth first increasing and then decreasing. This humped shape of growth rates can explain why various studies reached contradictory conclusions about convergence. The model can generate absolute convergence between middle-income countries and rich countries, as well as divergence between middle-income countries and poor countries. It can also explain why the growth rates in the industrial countries increased over the last 300 years but decreased over the last 50 years. Furthermore, in sharp contrast to other growth models, it explains why capital does not flow from rich countries and from middle-income countries to poor countries, why the savings rate increases dramatically in the first stages of economic development but is stable or even decreases in the latter stages, and why there is a positive correlation between savings rates and leisure in the process of economic development.

Section 2 presents the standard neoclassical growth model with exogenous and constant technological progress, population growth and elasticity of intertemporal substitution (*CEIS*). It concentrates on the dynamics of the output/capital ratio and the consumption/capital ratio, enabling the standard model to be used as a reference point for the next sections.

Section 3 discusses the main implications of the *CEIS* model. In particular, it shows why this growth model can not perform a good job in describing the patterns of growth we observe in the real world, either from an empirical or from a theoretical perspective.

Section 4 introduces a modified neoclassical model, having increasing elasticity of intertemporal substitution (*IEIS*), due to Stone-Geary preferences. This model addresses some of the weaknesses of the standard model, while providing a good fit for the stylized facts of economic development.

Section 5 reviews the historical evidence on the patterns of growth, as reflected in the recent growth literature and in long-run data on growth rates. It discusses the endogenous growth models with externalities and the conditional convergence tests of the neoclassical growth model and how they relate to the results of this paper.

Section 6 presents concluding remarks and discusses policy implications.

II. The Standard Neoclassical Growth Model with Constant Elasticity of Intertemporal Substitution (*CEIS*)

The standard model assumes a Cobb-Douglas production function and Hicks-neutral technological progress. 1/ The production function is assumed to be:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{1}$$

where

$$A_t = A_0 e^{xt} \tag{2}$$

The assumption made here is that the technological progress represents accumulation of knowledge. This increase in knowledge is due to a higher level of education, results of research and development, learning by doing, etc. 2/ This paper assumes that the rate at which knowledge increases over

1/ The assumption about the nature of the technological progress is irrelevant in the case of Cobb-Douglas technology.
2/ In the growth literature, it is usually defined as the "Solow Residual" or the "Total Factor Productivity Growth".

time, x, is constant and exogenous. Human capital is assumed to be an implicit part of "knowledge", and therefore it does not enter the production function explicitly. Therefore, we can concentrate in our analysis on the accumulation of physical capital only. 1/

The assumption of constant technological progress (or advance in knowledge) enables a clear distinction to be made between the patterns of growth that are a result of the transitional dynamics generated by the neoclassical model and the patterns of growth associated directly with differences in the advance of knowledge. 2/ Each individual supplies one unit of labor. Omitting the time subscripts, the production function in per capita terms is:

$$y = A\, k^{\alpha} \tag{3}$$

The model presented in this section assumes a constant elasticity of intertemporal substitution (*CEIS*). The utility function and the budget constraint take the usual form:

$$MAX \int_0^{\infty} e^{-\rho t} \frac{c^{1-\theta}}{1-\theta}\, dt \tag{4}$$

$$s.t.\quad \dot{k} = y-c-(\delta+n)k \;;\; y = Ak^{\alpha} \tag{5}$$

The solution to the model is in the Appendix. The main characteristic of the solution is the existence of two differential equations, for the consumption/capital and the output/capital ratios:

$$\frac{(\dot{\frac{c}{k}})}{(\frac{c}{k})} = \frac{c}{k}-\frac{1}{\theta}[(\theta-\alpha)\frac{y}{k}-(\theta-1)(\delta+n)+\rho] \tag{7}$$

and

$$\frac{(\dot{\frac{y}{k}})}{(\frac{y}{k})} = (1-\alpha)[\frac{c}{k}+\delta+n-\frac{y}{k}+g] \tag{8}$$

where

$$g = \frac{x}{1-\alpha} \tag{9}$$

1/ An alternative way is to assume that human and physical capital are perfect complements in the production function. In this case the two types of capital will be accumulated in fixed proportions and again in our analysis we can concentrate on the accumulation of physical capital.

2/ In the case of less developed countries, this progress in knowledge can be interpreted as the rate of assimilation of the knowledge that flows from more developed countries.

The two equations that define the steady state and the dynamics of the model are:

$$(\frac{\dot{c}}{k}) = 0 \qquad (\frac{\dot{y}}{k}) = 0 \tag{10}$$

These two equations are time invariant and they can be represented in a phase diagram (Figure 1).

The slope of the y/k equation is exactly 1. The slope of the c/k equation is $1 - \underline{\alpha}/\underline{\theta}$, i.e., positive but less than 1, given the usual assumption that $\underline{\theta}>\underline{\alpha}$.

In calculating the phase diagram, the following parameters were assumed:

$$\theta=1.5 \quad \alpha=0.4 \quad \delta=0.04 \quad n=0.0 \quad \rho=0.02 \quad x=0.01 \tag{11}$$

The corresponding steady state values are

$$\frac{y}{k}=0.213 \quad \frac{c}{k}=0.156 \quad s=0.267 \quad \frac{\dot{y}}{y}=0.0167 \quad r=0.045 \tag{12}$$

The dynamics of the model define a unique stable arm. We can think about it in the following way: any economy, at any moment in time, and in particular at time 0, has two endowments. The first endowment is the level of capital per person (k), and the second is a certain level of technology (A). 1/ The production function $y = Ak^{\underline{\alpha}}$ determines the output. Then the y/k ratio (and the horizontal position in the phase diagram) is determined. The only "choice" this economy makes is its level of consumption. This determines in turn the c/k ratio (and the corresponding vertical position in the phase diagram). The uniqueness of the stable arm means that there is only one choice of consumption that does not violate the assumptions of the model and the implicit transversality conditions. From the moment this "choice" is made and the economy is on the stable arm, the dynamics of the model take it towards the steady state.

We use a computer program to simulate the dynamics of the system. Economy A starts with a y/k ratio above the steady state, while economy B starts below. We assume $y_0=200$, $k_0=200$ for economy A, and $y_0=200$, $k_0=1000$ for economy B. The four panels in Figure 2 describe the simulated stable arm, the log of output per person as a function of time, the growth rate as a function of the rate of return and the rate of return as a function of time. Figure 3 shows the y/k ratio, the c/k ratio, the growth rate and the savings rate as a function of the log of income.

1/ In every other respect, except the population size, all economies are assumed to be identical.

Figure 1

The Phase Diagram

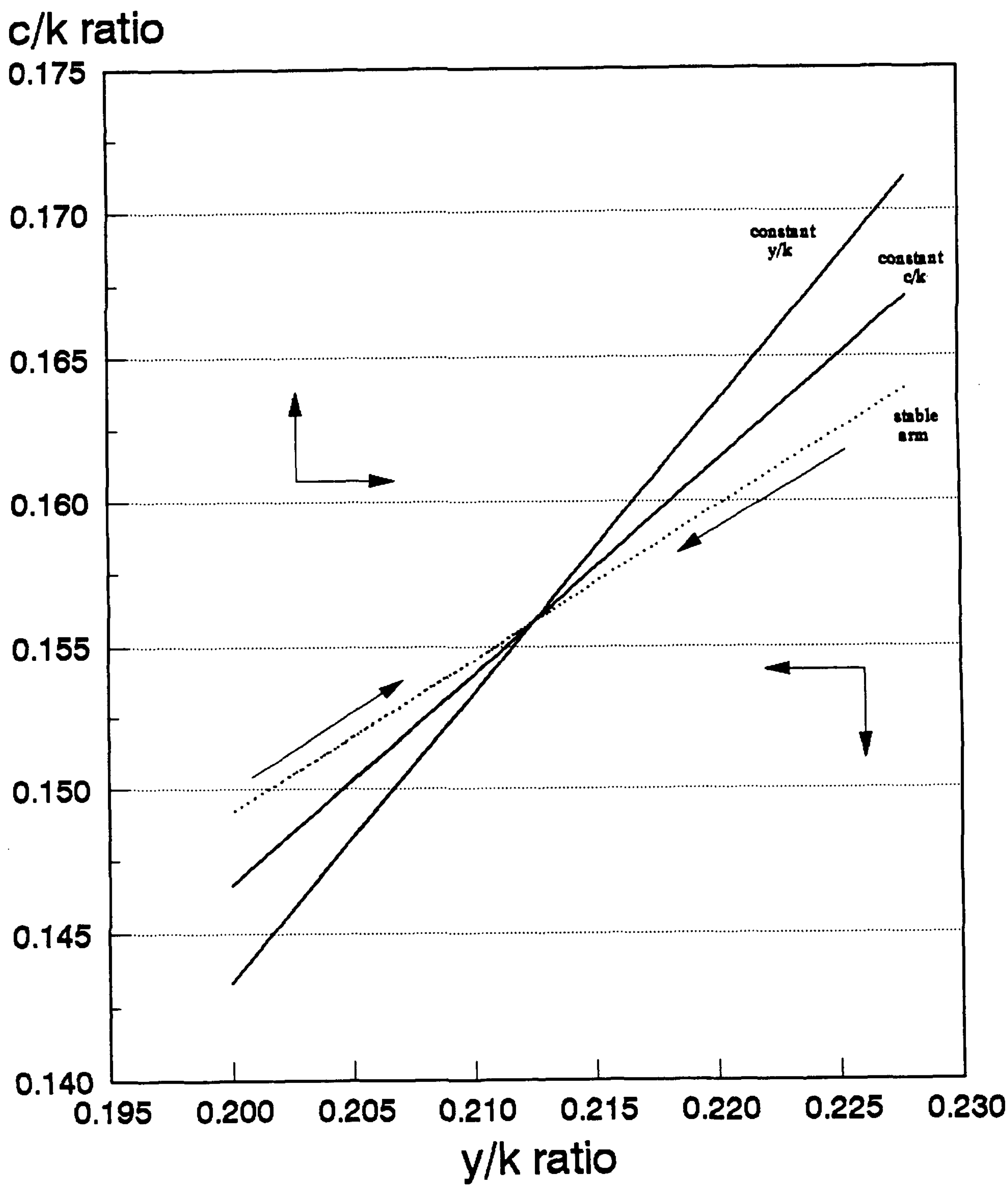


Figure 2 (a)
CEIS

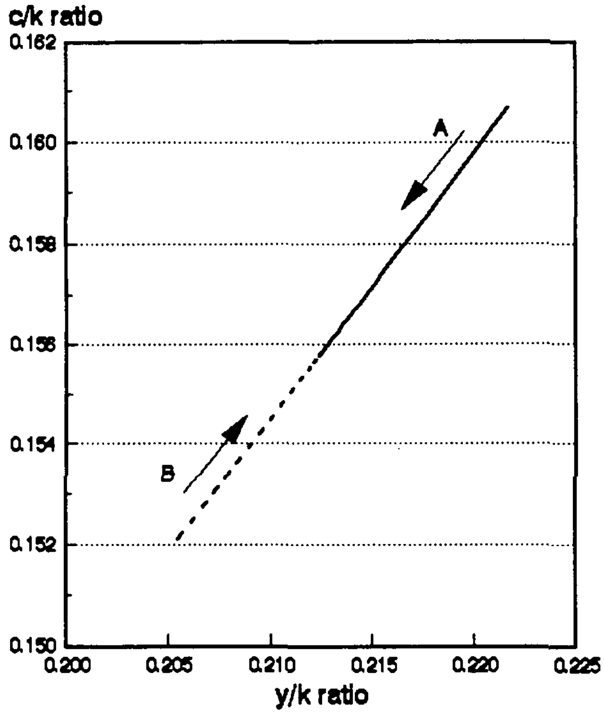


Figure 2 (b)
CEIS

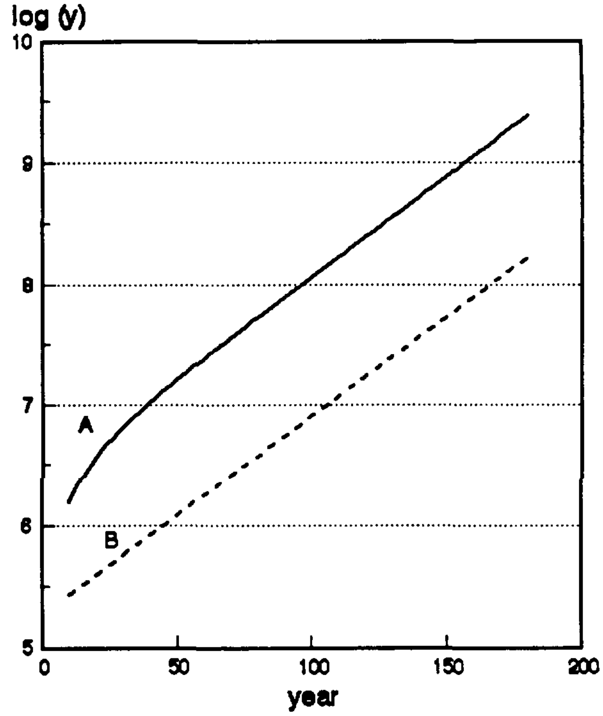


Figure 2 (c)
CEIS

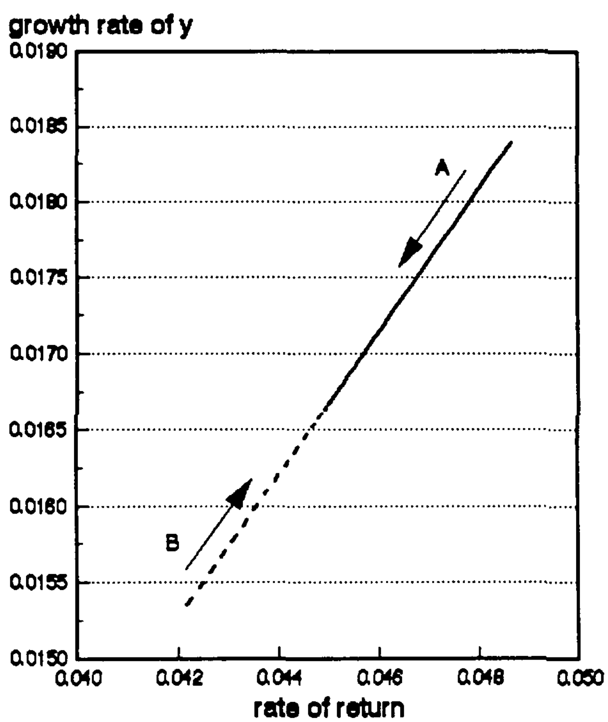


Figure 2 (d)
CEIS

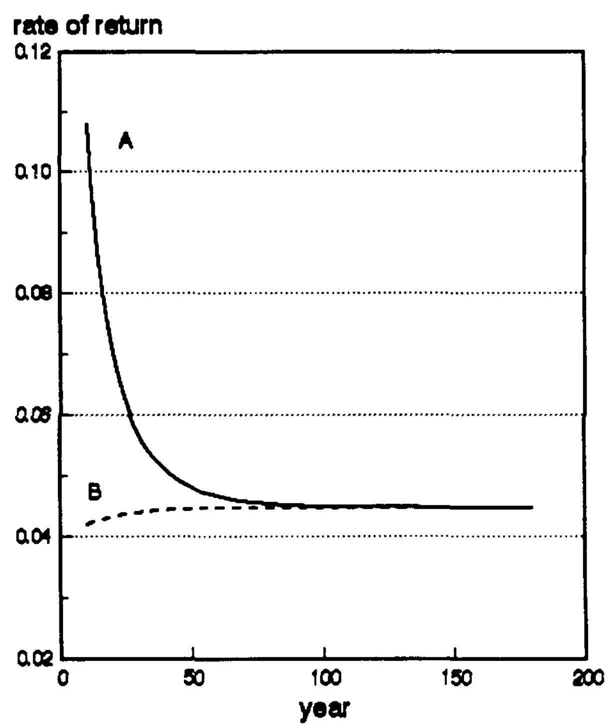


Figure 3 (a)
CEIS

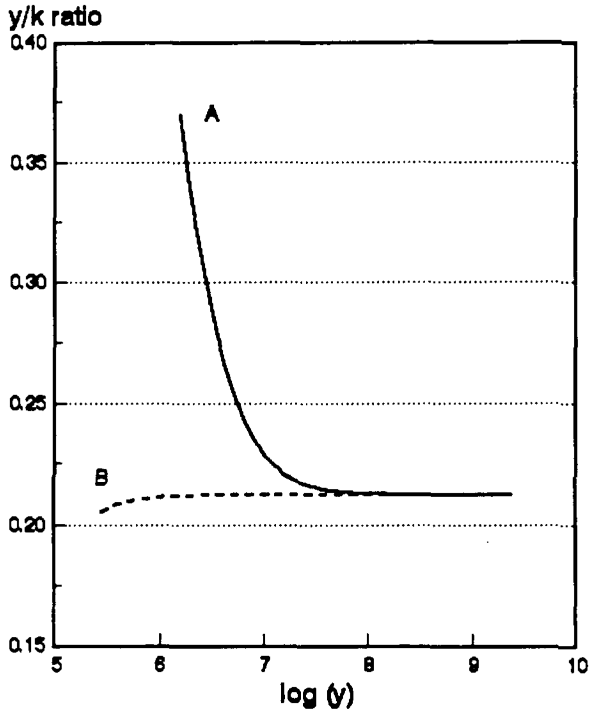


Figure 3 (b)
CEIS

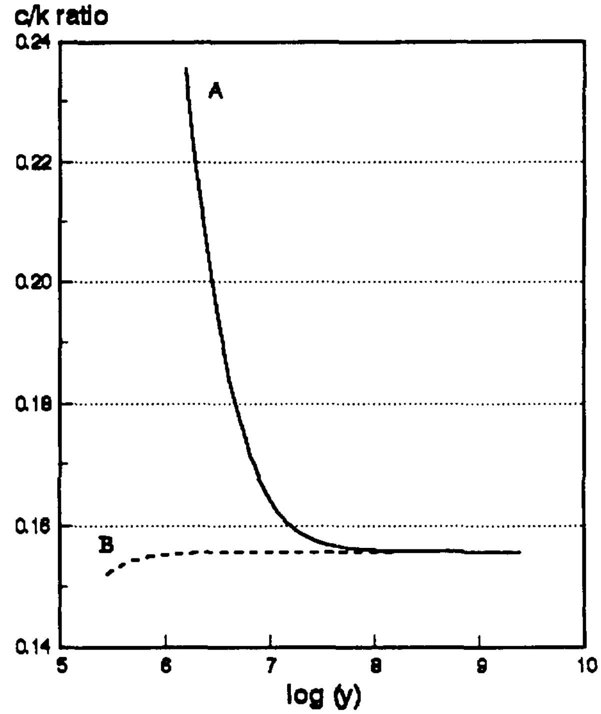


Figure 3 (c)
CEIS

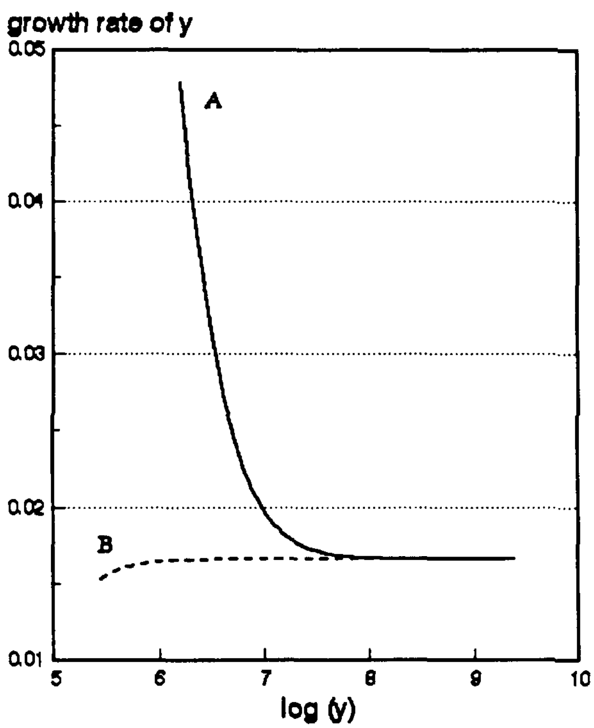
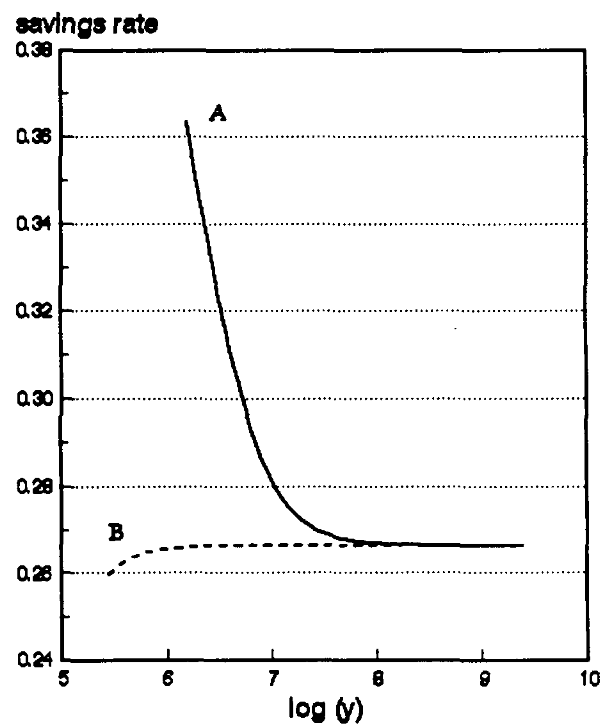


Figure 3 (d)
CEIS



The growth and the savings rates can be written as

$$\frac{\dot{y}}{y} = x+\alpha\frac{\dot{k}}{k} = x+\alpha\left[\frac{y}{k}-\frac{c}{k}-(\delta+n)\right] \tag{13}$$

and

$$s = 1-\frac{c}{y} = 1-\frac{(\frac{c}{k})}{(\frac{y}{k})} \tag{14}$$

Thus, once y/k and c/k are determined at each moment in time by the position of the economy on the stable arm, everything else, including the growth rate and the savings rate, is determined endogenously.

The real rate of return implied by the production function is a linear function of the y/k ratio:

$$r = MPK-\delta = A\alpha k^{\alpha-1}-\delta = \alpha\frac{y}{k}-\delta \tag{15}$$

A given economy may start with a high level of y/k (meaning that it is relatively more endowed in technology than in capital), or with a low level of y/k. In each case, the economy eventually converges along the stable arm, which has a positive slope, to the steady state level of y/k. Thus, the y/k ratio and the rate of return always increase or decrease monotonically during the transition to the steady state. 1/

The growth rate of consumption is:

$$\frac{\dot{c}}{c} = \alpha\frac{y}{k}-\delta-\rho-n \tag{16}$$

Therefore, the monotonic trend in the output/capital ratio also implies a monotonic trend in the growth rate of consumption. The growth rate of consumption always decreases or always increases but never changes direction. For most parameter values, the growth rate of output closely resembles the growth rate of consumption.

Another important implication is the prediction of capital flows. It was previously mentioned that the real rate of return implied by the

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1/ We strongly believe that the methodology we use to analyze the system (using output/capital and consumption/capital ratios) is superior to the traditional methodology (that uses a normalization of k defined as "capital per effective worker"). The reasons are: (1) many variables in the model are linear in y/k, both during transition and in the steady state; (2) the y/k and c/k ratios have a strong intuitive meaning, as opposed to the "capital per effective worker" variable. However, all the results in our analysis can easily be replicated using the traditional methodology.

production function is a positive linear function of the output/capital ratio. If international movements of capital are allowed, we expect capital to flow from economies with low output/capital ratios and low growth rates to economies with high output/capital ratios and high growth rates. This speeds the rate of convergence towards the steady state level of y/k, because for any given technology A, having a higher capital stock k reduces the y/k ratio.

III. Two Versions of the Initial Position in the *CEIS* Model

Most of the modern economic thinking about the dynamics of economic growth is based on the *CEIS* model. But this model can only determine what the dynamics of growth will be if the economy starts with a high or a low output/capital ratio. It can not determine the initial output/capital ratio. For this reason, it is crucial to determine the initial position of an economy in order to conclude what the predictions of the standard neoclassical growth model are. Different assumptions about this initial position generate two distinct versions of the *CEIS* model.

1. The standard version

The standard assumption about the initial position is that poor economies are relatively scarcer in capital than in output and have higher output/capital ratios than rich economies. In this case, the process of economic development can be described as a continuous accumulation of capital (relative to output) and a decrease in the output/capital ratio until it asymptotically reaches the steady state.

However, the assumption that poor countries start the process of economic growth with high output/capital ratio is, at best, questionable. Except in certain circumstances, it is difficult to justify this assumption in a historical perspective. 1/ The only evidence supporting this view is data from the last several decades which tends to show that the output/capital ratio declines with the level of development, both in cross-section and in time-series data. This kind of evidence is not robust due to the great difficulty of measuring capital stocks. 2/ In addition, a systematic bias is introduced by using historical investment and geometric depreciation rates -- the usual way to estimate capital stocks. This method ignores land and natural resources, which represent a large fraction of the capital stock in poorer countries. Assuming land and natural resources are a substitute for physical capital, it is easy to see why the output/capital ratio in poor countries is much lower than the usual method of measuring

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1/ Special circumstances may include the aftermath of a major war that devastates the capital stock but not the technological know-how.

2/ For example, analyzing the original data on output and capital stocks in the United States used by Solow (1956), one can easily check that the output/capital ratio actually increased over the sample period (1909-1949).

capital would imply. Moreover, if one accepts the historical evidence about an agricultural revolution leading to a food surplus, rural-to-urban migration and industrialization, then it is difficult to accept a decreasing output/capital ratio as a stylized fact. 1/ However, the main problem with the standard version is not its lack of historical foundations but rather the lack of any theoretical justification for its main assumption.

2. The alternative version

An alternative way to think about the initial position is to start from a theoretical argument. For every economy, let us divide the history into two periods: the old period and the modern period. In the old period, all economies were poor and did not enjoy any growth. In the modern period, they started to grow and to increase their incomes. Accordingly, there are two steady states: one corresponding to the old period and one to the modern period. We assume that the only difference between the characteristics of the "old" steady state and of the "modern" steady state is the rate of technological progress. Hence, we can use the same growth model to calculate the position of the old steady state. The result is that the old steady state is characterized by a **lower** output/capital ratio than the modern steady state. 2/

Using this historical view of the world, the process of development can be viewed as a transition from an old steady state (which characterized the economy before the modern process of economic growth began) to a modern steady state (which corresponds to a positive rate of advance in knowledge). First, economies are in a steady state that corresponds to $x=0$. The y/k ratio, the income level and the consumption level are constant. The growth rate is *0*. Then, the rate of technological progress jumps to a positive level. This jump is exogenous and can be explained by various historical and sociological events, such as the development of democracy, property rights, patent laws, literacy, typography, roads and basic infrastructure, inter-regional and international trade. These and other factors can positively effect the dispersion of knowledge and ensure that the progress in knowledge can occur in cumulative steps as opposed to cycles. This discrete jump in x marks the division between the old period and the modern

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1/ If most capital is in the form of land and this land produces food, then an increase in the production of food means that the output/capital ratio must also increase, at least in the beginning of the development process.

2/ Under our assumptions regarding the parameter values, the output/capital ratio that corresponds to a rate of technological progress of 1% is 0.2125, while the one that corresponds to a zero rate of technological progress is only 0.1000. In general, the output/capital ratio in the steady state is always a positive function of the rate of technological progress. See the Appendix for proof.

period and can occur in different economies at different points in time. 1/ At this point, the economy jumps to a new stable arm and develops according to the dynamics of the system until it reaches asymptotically the steady state value of y/k that corresponds to the new rate of technological progress. During the transition to the new steady state there are various dynamics of the growth rate, savings rate and the other variables. In the final stage, the economy is in the new steady state. The growth rate, the savings rate, the y/k ratio, the c/k ratio and the rate of return are all constant.

This alternative version of the *CEIS* model predicts that during the transition, the growth rate, the output/capital ratio and the rate of return increase monotonically. But this prediction is not supported by observed stylized facts of economic growth. In fact, we saw previously that exactly the opposite is usually believed to be true.

The main reason for the popularity of the standard *CEIS* model is its possibility of delivering some of the stylized facts of economic growth, but this is true only if we assume that the various economies approach the steady state from the right (decreasing y/k ratios). This fact was assumed to be true in the standard version, without much theoretical or historical justification. However, in the alternative version of the model, based on endogenously determined initial conditions, the economies approach the steady state from the left. In this case, the same *CEIS* model has implications that are necessarily the opposite of the ones that are considered the main strength of the model!

IV. The Model with Increasing Elasticity of Intertemporal Substitution (*IEIS*)

Even if we accept the hypothesis about the continuous decrease in the output/capital ratio in the last decades and the evidence presented to support it, we would like to find a more profound theoretical reason for it, than simply to view it as an historical accident. Ideally, we would like to get this type of approach towards the steady state as an endogenous result of a growth model that incorporates consistent assumptions about the initial conditions. This section develops an alternative model that:

(1) Delivers endogenously the result of decreasing output/capital ratios in economies close to their steady state, even if the economies started their growth having a lower y/k ratio than in the final steady state.

1/ This is the explanation for the present differences in income per person across countries: they are all identical, but some started to grow earlier than others.

(2) Delivers convergence in the steady state neighborhood when the economies are relatively rich. Further in the past, when the economies are poorer, it delivers divergence.

(3) Delivers a possibility of different dynamics of the growth rate, in which the sequence of the growth rate pattern is typically *low-high-low* as a country develops and is not necessarily decreasing (*high-low*), or increasing (*low-high*) as in the *CEIS* model.

(4) Delivers an explanation why capital does not flow from rich countries to poor countries. In addition, allowing for capital flows does not necessarily speed convergence.

(5) Delivers dynamics of savings rates and labor supply in accordance with the stylized facts.

The only change we will introduce in the basic model is to drop the assumption of constant elasticity of intertemporal substitution. Instead, we assume that the utility function takes the more general Stone-Geary form:

$$U(c) = \frac{(c-c_0)^{1-\theta}}{1-\theta} \tag{17}$$

where c_0 can be interpreted as a fixed quantity of consumption needed for survival. This utility function generates an increasing elasticity of intertemporal substitution (*IEIS*). This elasticity is equal to $1/\theta$ only asymptotically when c_0/c approaches 0:

$$\sigma = \frac{1}{\theta}\left(1-\frac{c_0}{c}\right) \tag{18}$$

The Stone-Geary preferences represent the simplest case of increasing elasticity of intertemporal substitution while having a strong intuitive appeal. Poor people allocate a significant fraction of their income to basic survival necessities. At low income levels, the propensity to save is very low, because biological needs place a tight limit on the possibility of decreasing consumption today for higher consumption tomorrow, or vice-versa. As a result, the elasticity of intertemporal substitution is extremely low when the level of consumption is close to the survival level.

The direct empirical evidence for this type of preferences is quite favorable. Giovannini (1985) has estimated for a large set of less developed countries an elasticity of intertemporal substitution which is very close to zero. Ogaki, Ostry and Reinhart (1994) attempt to quantify the response of household saving to changes in real interest rates. In the process, they estimate the intertemporal elasticity of substitution for 14 countries during the period 1960-1992, and find evidence that supports increasing rates of intertemporal substitution. Other studies (e.g., Ogaki and Atkeson (1993) and Atkeson and Ogaki (1993)) found similar results.

Indirect empirical evidence for this type of preferences can be found in countless studies that document a strong relationship between savings rates and income and between expenditures on food and other necessities and income, especially at the lower spectrum of income. For example, the study by The World Bank (1994) that examines sub-Saharan African countries concludes that the median gross domestic savings rate over the period 1981-1986 was 5.3 percent for low-income countries and 25.2 percent for middle income countries (Table A.24). The figures for the period 1987-1991 are, respectively, 5.6 percent and 19.0 percent. The same study examines the food expenditures of poor, rural smallholders in 11 selected countries and regions in sub-saharan Africa. The food expenditures in this sample range from 59 percent to 82 percent of total expenditures, and the median is 67 percent (Table A.27).

This particular form of utility function has been used in some previous studies of the dynamics of growth, such as the one by King and Rebelo (1989). The result of that study, presented in Figure 6 in their paper, *shows that Stone-Geary preferences can generate a humped shape of growth* rates. The real interest rate, however, starts from an unrealistically high level of about 40 percent and monotonically decreases, thus generating dynamics that are very similar to the standard version of the *CEIS* model. Moreover, King and Rebelo claim that the Stone-Geary assumption "generates a longer period with high real interest rates in initial stages of development". 1/

In another paper, Rebelo (1992) presents new arguments in favor of the Stone-Geary preferences. He finds that "it is consistent with evidence that poor countries save less than rich countries and implies that a liberalization of capital flows would have negligible short-term effects on the rate of growth".

In the manuscript for their book on economic growth, Barro and Sala-i-Martin (1993) present a different argument in favor of increasing elasticity of intertemporal substitution. They analyze the leisure-consumption trade-off and show that a constant elasticity of intertemporal substitution necessarily results in a pattern of growth in which the amount of work effort and the savings rate rise or fall together, contrary to the evidence that during the process of economic development the savings rate increased and the work effort decreased. They conjecture that Stone-Geary preferences can generate dynamics in accordance to the empirical evidence on savings and work effort.

In general, the case of constant elasticity of intertemporal substitution can be viewed as a particular case of the Stone-Geary preferences for the case where $c_0=0$. It does not have any intuitive reasons and is not supported by empirical evidence. Its only advantage is that it produces relatively neat results in complex economic models.

1/ This result of course makes the Stone-Geary preferences an unattractive alternative to the standard model. Simulating the *IEIS* model, we will prove that this is not a general result of using these preferences.

The model with Stone-Geary preferences is shown in the Appendix. The two differential equations that characterize the solution are:

$$\frac{(\dot{\frac{y}{k}})}{(\frac{y}{k})} = (1-\alpha)\,[\frac{c}{k}+\delta+n-\frac{y}{k}+g] \qquad (19)$$

$$\frac{(\dot{\frac{c}{k}})}{(\frac{c}{k})} = \frac{c}{k}-\frac{1}{\theta}\,[(\theta-\alpha z_1)\,\frac{y}{k}-(\theta-z_1)\,(\delta+n)+z_1\rho] \qquad (20)$$

where

$$z_1 = 1-\frac{c_0}{c} \qquad (21)$$

Now, the differential equation for the consumption/capital ratio depends not only on the variables c/k and y/k, but also on the absolute level of consumption. Consequently, the system can no longer be represented in a phase diagram with constant dynamics. However, the steady state corresponding to positive technological progress is identical to the steady state in the standard model. The reason for this is simple: after a long period of positive growth, consumption is high and the variable z_1 approaches asymptotically the value *1*, thus creating a convergence between the two models. Moreover, the steady state value of y/k is identical in the two models also in the case in which the rate of technological progress is *0*. 1/ Therefore, the economy must have a smaller output/capital ratio in the old steady state (before the modern process of economic growth began) than in the final steady state.

The dynamics of the *IEIS* model can be simulated using the Euler equations developed in the Appendix. The main difference, however, between the dynamics in this model and the dynamics in the *CEIS* case, is that now the initial starting y/k ratio, as determined by the stock of capital and by the available technology, determines the whole path towards the steady state and not just the point where the system joins a unique stable arm. In particular, for reasonable starting points, such as the one corresponding to the old steady state, the growth rate increases initially but decreases in the new steady state's neighborhood. A typical path resulting from this type of simulation (assuming the initial y/k ratio corresponds to the old steady state) is shown in Figures 4 and 5. We assume the same parameter values that we did when simulating the *CEIS* model, in Figures 2 and 3.

The dynamics of growth in the *IEIS* model are completely different than those implied by the standard *CEIS* model. In fact, these dynamics overturn almost all the well-known implications of the usual neoclassical growth model:

1/ See the Appendix for proof.

(1) The growth rate of output accelerates at the beginning, then it decreases as the economy approaches the steady state. The initial growth rate may even be negative despite technological progress. There may be a portion in which the growth rate is negatively correlated with the output/capital ratio (or the rate of return).

(2) The savings rate may have distinctive dynamics, as opposed to the monotonic trend in the standard model. 1/

(3) The rate of return (and the y/k ratio) is low at the beginning. Then it increases over a long range, achieves a maximum, and finally decreases as the economy approaches the steady state. The final value is higher than the initial value.

The last point can also explain the patterns of capital flows. If *capital flows are a function of the difference between the rates of return* in two economies, then the model predicts capital flows from poor countries to both rich and middle income countries and from rich countries to middle income countries. This prediction is very different from the one derived in the standard neoclassical model and is much in accordance with the facts we observe in the real world. In addition, the *IEIS* model can explain the positive correlation between the historical patterns of leisure and savings rates, a fact that the *CEIS* model could not explain. 2/

The intuition for the dynamics of the output/capital ratio is very simple. There are two forces at work. One dominates when the economy is relatively poor, and the other dominates when the economy is richer. The production function implies diminishing returns to capital. When the economy is rich in capital, adding an extra unit of capital does not increase output much. Therefore the y/k ratio declines. But when the economy is poor, the amount of consumption is very close to the level that is necessary for survival and the additional utility derived from the marginal unit of consumption is extremely high. In this case, any increase in output is allocated for immediate consumption and not for investment. Therefore, at low income levels, the increase in output resulting from the technological progress cannot be matched by a similar increase in capital stocks and the y/k ratio increases.

1/ In general, as noted by Rebelo (1992), the saving rate will be extremely low for low-income economies that are in the first stages of development.

2/ Introducing leisure in the utility function and work in the production function and taking first order conditions with respect to consumption and leisure, we obtain the following expression:

$$leisure = \beta\, z_1\, (1-s) \,/\, [\beta\, z_1\, (1-s) + (1-\alpha)]$$

where s is the saving rate, β is a positive parameter and $z_1 = 1 - c_0 / c$. In the *CEIS* model $z_1 = 1$ and this implies a negative correlation between leisure and savings. In the *IEIS* model, on the other hand, z_1 is between 0 and 1 and increasing. In this case, a positive correlation between leisure and savings is possible.

Figure 4 (a)
IEIS

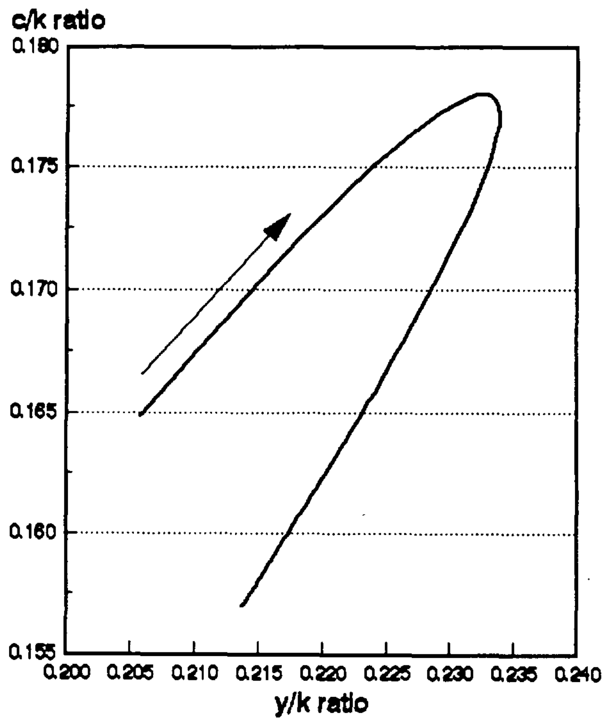


Figure 4 (b)
IEIS

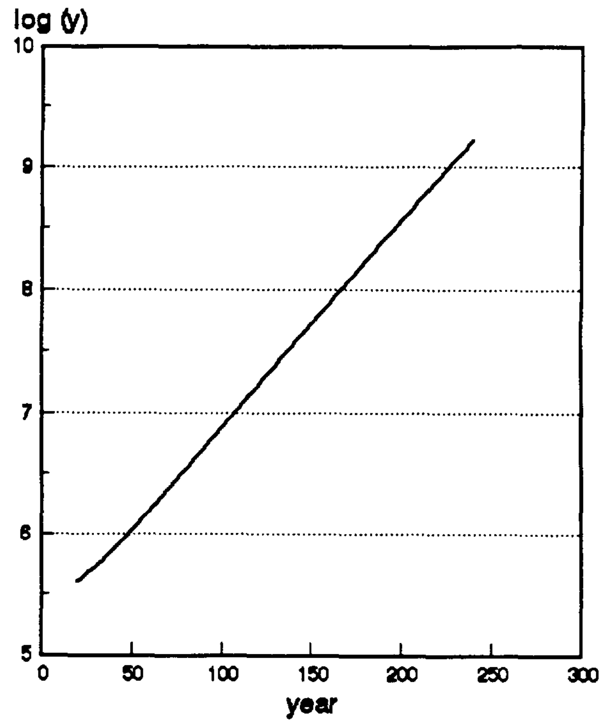


Figure 4 (c)
IEIS

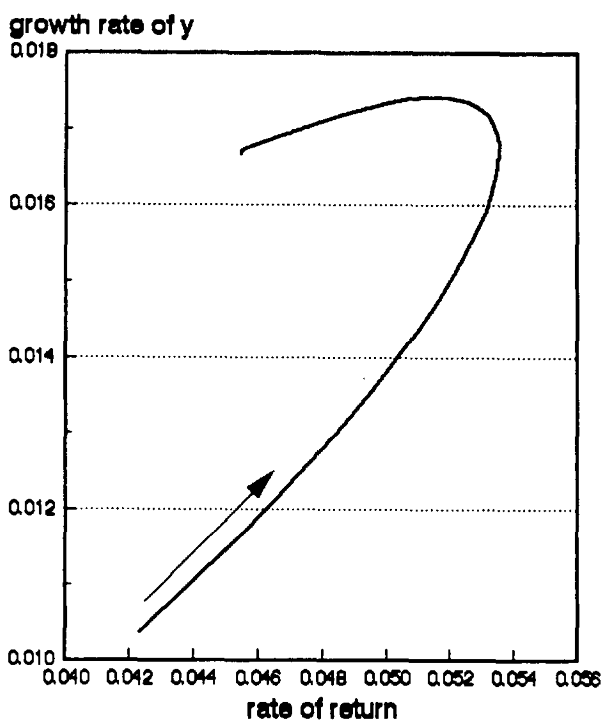


Figure 4 (d)
IEIS

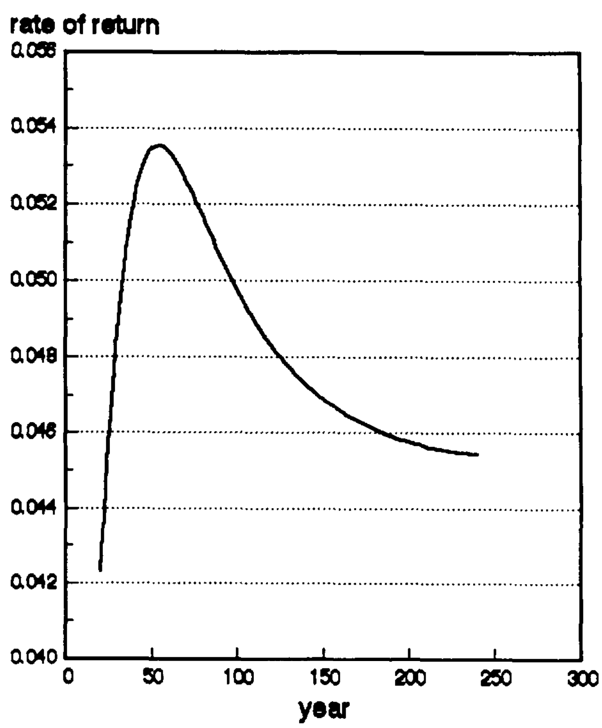


Figure 5 (a)
IEIS

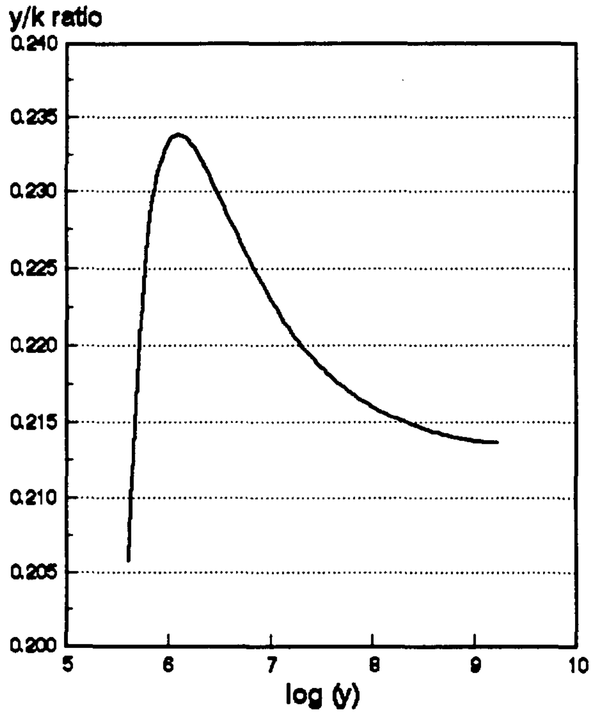


Figure 5 (b)
IEIS

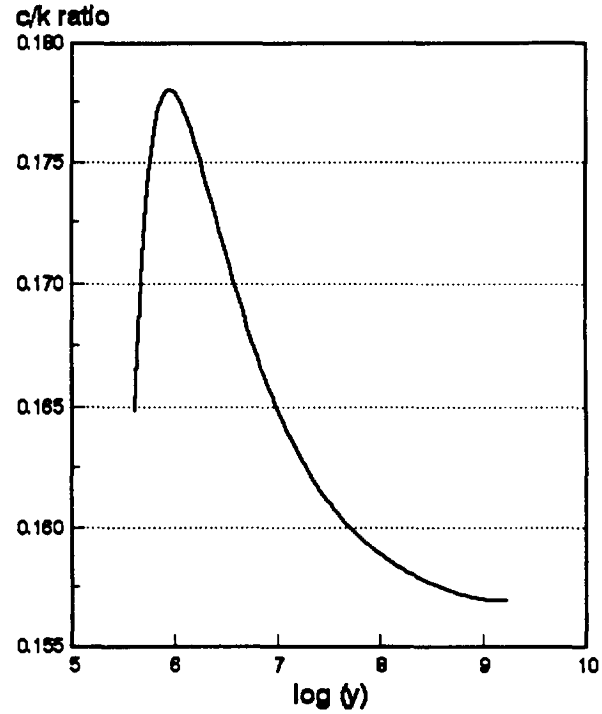


Figure 5 (c)
IEIS

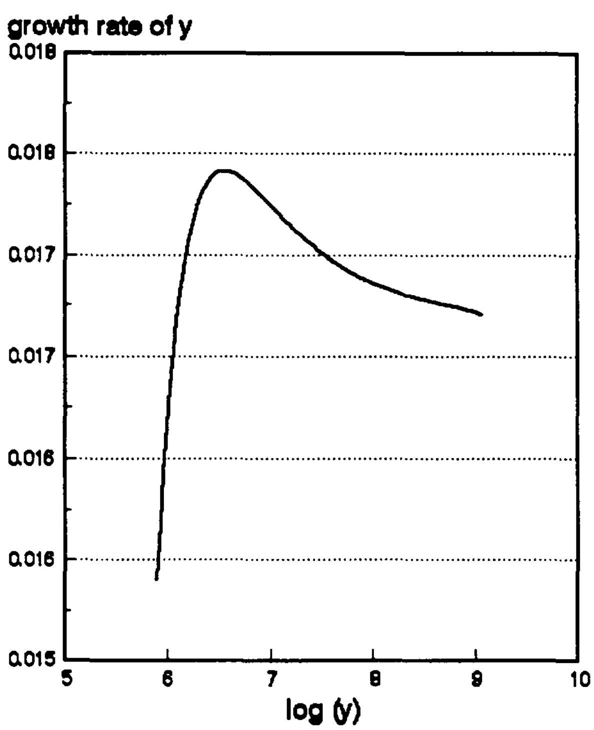
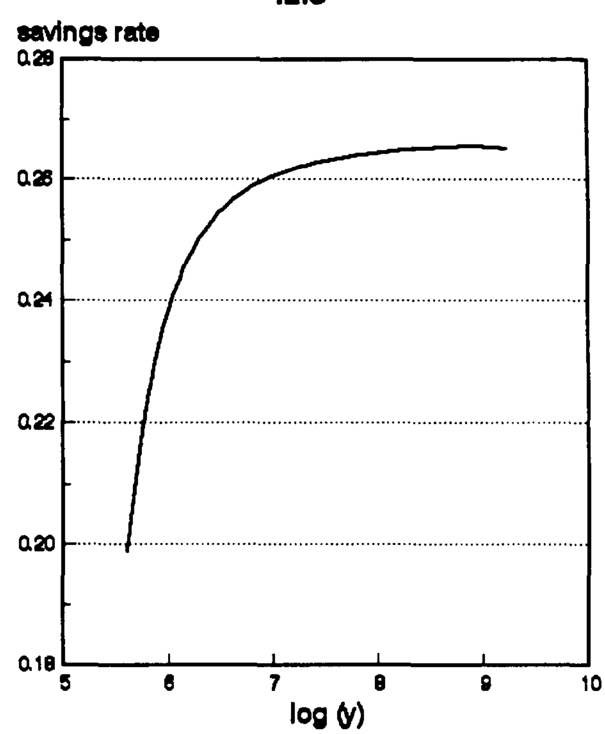


Figure 5 (d)
IEIS



V. Historical Patterns of Growth and Recent Growth Literature

After several decades of research in the field of economic growth, economists still disagree about one of the most important stylized facts of this field, the dynamics of growth rates. Some studies of economic growth have focused on cross-countries or cross-region empirical evidence. These studies usually find absolute convergence between relatively rich countries and regions but not between poorer countries and regions. 1/ The same evidence led to the development of the "conditional convergence" hypothesis. The idea behind the conditional convergence hypothesis is that the different countries or regions grow according to the neoclassical growth model but do not share the same preferences and rate of technological progress. Instead, each one of these economies follows its own path to its own steady state, one that is defined by its own set of technology and preference parameters. Barro and Sala-i-Martin ((1991) and (1992)), among others, tested for conditional convergence and indeed found robust evidence in both cross-country and cross-region data. This finding was interpreted as strong evidence for this heterogenous version of the neoclassical growth model. Other studies looked at longer time series data and found that rates of growth actually accelerated in the last 300 years. For example, Romer (1986) using data from Maddison ((1979) and (1982)) presents compelling evidence that the growth rates did indeed increase over this period. Convergence findings appear only if one looks at cross-country databases and includes in the sample countries or regions that are viewed as industrialized at the end of the sample period, thus creating a selection bias. This evidence contradicts the standard neoclassical growth model and supports endogenous growth models with constant or increasing rates of growth as advanced by Romer (1986), Lucas (1988) and others. These models usually incorporate some form of increasing returns or externalities in production or find another way to avoid the problem of diminishing returns.

The development of both the heterogenous version of the neoclassical growth model (which implies conditional convergence but not absolute convergence) and of the endogenous growth models with increasing returns and externalities (which imply constant or increasing rates of growth) are the direct result of the inability of the simple *CEIS* model to generate complex dynamics of growth to fit the empirical evidence. These two classes of theory are fundamentally different but neither can generate complex dynamics of growth in accordance with the full array of empirical evidence, with growth rates first increasing and later decreasing in the process of development and with systematic convergence between the rich economies but

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1/ A good example of this kind of study is Barro and Sala-i-Martin (1992). Figures 1, 2 and 3 in their paper show convergence between rich countries and regions, while Figure 4 shows no evidence of convergence between poorer countries. Figure 5 is especially interesting, showing no convergence between the poorest countries but strong convergence among the richest countries. The emphasis in their paper, however, is on the conditional convergence findings and therefore they do not discuss this feature of the absolute convergence findings.

not between the poor ones. The empirical evidence that supports this pattern of growth can be easily observed in both cross-section and time-series data. For example, Easterly (1994) refers to studies by Baumol, Blackman and Wolff (1989) and by Dollar (1992) and concludes "Empirical results show that growth of growing countries first accelerates and then falls as income rises". 1/

The results from the analysis of the *IEIS* model suggest dynamics of growth that are in more complete accordance with the stylized facts of economic growth. There is no need to sacrifice either the assumption of homogenous preferences, or the assumption of constant returns without externalities. The *IEIS* model, which is based on the Stone-Geary generalization of preferences, generates these complex dynamics while preserving the basic neoclassical approach to economic growth.

VI. Conclusions

This paper examined theoretical implications of three versions of the neoclassical growth model.

First, it presented the standard version of the neoclassical growth model with constant elasticity of intertemporal substitution (*CEIS*).

Second, it presented an alternative version, based on the hypothesis that economies today are in transition between an old steady state and a modern steady state. The old steady state is characterized by zero rate of technological progress, while the new steady state by a positive rate. The paper proved that in this alternative version the familiar predictions of the standard model are reversed.

Third, it presented a version of the neoclassical growth model with increasing elasticity of intertemporal substitution (*IEIS*), due to Stone-Geary preferences. The dynamics of growth rates, savings rates, capital flows and labor supply that we observe in the world are in accordance with the predictions of the *IEIS* version, but not with the predictions of either version of the *CEIS* model. Moreover, the *IEIS* version generates these dynamics assuming initial conditions that are consistent with the theory. It also presents a simple explanation for the fact that middle-income countries are "catching-up" with the rich countries, while the poor countries are not and are even falling further behind. It combines the attractive implications of the conditional convergence hypothesis and of the

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1/ Anecdotal evidence for this pattern of growth is most evident in South-East Asia, first in Japan, and then in South-Korea, Taiwan, Singapore and Hong Kong. The experience of each individual country in this group confirms the hump-shape pattern of growth rates. Furthermore, each one of these countries achieved the peak in growth rates at a comparable income level. Recently, poorer countries such as China and Malaysia are achieving similar growth rates.

endogenous growth models, without the need to sacrifice the basic assumptions of the neoclassical approach to economic growth.

The theory developed in this paper has important policy implications. It helps explain why a strategy of development based on capital flows to developing countries cannot have long-term effects. The reason is that with a low propensity to save, most of these resources are consumed and are not invested. Moreover, because the poor countries have a lower rate of return than the richer countries, this additional investment necessarily crowds out other forms of investment. The model also suggests an alternative strategy of development: assist poor countries in getting richer by helping them to increase the level of technology and of knowledge, including human capital.

The *CEIS/IEIS* Model

This appendix presents both the *CEIS* and the *IEIS* models. It initially presents the *IEIS* model, which is more general. Then it solves the *CEIS* model as a special case in which $c_0 = 0$ and $z_1 = 1$.

The general problem can be written using a Hamiltonian:

$$H = e^{-\rho t}\,\frac{(c-c_0)^{1-\theta}}{1-\theta}+\mu\,[y-c-(\delta+n)\,k] \tag{22}$$

The first order conditions are:

$$H_c = 0: \quad e^{-\rho t}(c-c_0)^{-\theta} = \mu \tag{23}$$

$$H_k = -\dot{\mu}: \quad \delta+n-\alpha\frac{y}{k} = \frac{\dot{\mu}}{\mu} \tag{24}$$

From the budget constraint we get:

$$\frac{\dot{k}}{k} = \frac{y}{k}-\frac{c}{k}-\delta-n \tag{25}$$

From the first *FOC*:

$$-\rho-\theta\frac{\dot{c}}{c-c_0} = \frac{\dot{\mu}}{\mu} \tag{26}$$

Define:

$$z_1 = 1-\frac{c_0}{c} \tag{27}$$

Substituting the second *FOC*:

$$\frac{\dot{c}}{c} = \frac{z_1}{\theta}\,[-n-\rho-\delta+\alpha\frac{y}{k}] \tag{28}$$

Note that:

$$\frac{(\frac{\dot{c}}{k})}{(\frac{c}{k})} = \frac{\dot{c}}{c}-\frac{\dot{k}}{k} \tag{29}$$

Substitute and get:

$$\frac{(\frac{\dot{c}}{k})}{(\frac{c}{k})} = \frac{c}{k}-\frac{1}{\theta}\,[(\theta-\alpha z_1)\,\frac{y}{k}-(\theta-z_1)\,(\delta+n)+z_1\rho] \tag{30}$$

On the other hand, from the production function we get:

$$\frac{y}{k} = Ae^{xt}k^{\alpha-1} \tag{31}$$

$$\frac{(\frac{\dot{y}}{k})}{(\frac{y}{k})} = x-(1-\alpha)\frac{\dot{k}}{k} \tag{32}$$

Substitute and get:

$$\frac{(\frac{\dot{y}}{k})}{(\frac{y}{k})} = (1-\alpha)\,[\frac{c}{k}+\delta+n-\frac{y}{k}+g] \tag{33}$$

where

$$g = \frac{x}{1-\alpha} \tag{34}$$

Now, we have two differential equations in three variables, c/k, y/k and z_1:

$$(\frac{\dot{c}}{k}) = 0: \quad \frac{c}{k} = \frac{z_1}{\theta}\rho-\frac{\theta-z_1}{\theta}(\delta+n)+\frac{\theta-\alpha z_1}{\theta}\frac{y}{k} \tag{35}$$

$$(\frac{\dot{y}}{k}) = 0: \quad \frac{c}{k} = -[\delta+n+g]+\frac{y}{k} \tag{36}$$

The solution for y/k and c/k in the steady state is:

$$\frac{y}{k} = \frac{1}{\alpha}[\delta+n+\rho+\frac{\theta}{z_1}g] \tag{37}$$

and

$$\frac{c}{k} = \frac{1}{\alpha}[(1-\alpha)(\delta+n)+\rho+(\frac{\theta}{z_1}-\alpha)g] \tag{38}$$

1. The *CEIS* case

In the *CEIS* case, $z_1=1$ and we have two equations in two variables that can be represented in a phase diagram with constant dynamics. The output/capital and the consumption/capital ratios are constant in the steady state. Their respective values are:

$$\frac{y}{k} = \frac{1}{\alpha}[n+\delta+\rho+\theta g] \tag{39}$$

$$\frac{c}{k} = \frac{1}{\alpha}[(1-\alpha)(\delta+n)+\rho+(\theta-\alpha)g] \tag{40}$$

Output, capital and consumption in the steady state grow at an identical rate:

$$\frac{\dot{y}}{y} = \frac{\dot{k}}{k} = \frac{\dot{c}}{c} = g \tag{41}$$

The savings rate is defined as:

$$s = 1-\frac{c}{y} = 1-\frac{(\frac{c}{k})}{(\frac{y}{k})} = \frac{(\frac{y}{k})-(\frac{c}{k})}{(\frac{y}{k})} \tag{42}$$

Substituting the values for the steady state:

$$s = \alpha\,\frac{\delta+n+g}{\delta+n+\rho+\theta g} \tag{43}$$

The rate of return is defined as:

$$r = \alpha\,\frac{y}{k} - \delta \tag{44}$$

Substituting the value of y/k in the steady state:

$$r = n + \rho + \theta\, g \tag{45}$$

The steady state values of c/k, y/k and s are determined by the parameters α, x, n, θ, δ and ρ. The growth rate, however, is determined only by α and x.

2. <u>The *IEIS* case</u>

We simplify the analysis of the steady state in the *IEIS* case by observing that in the long run either $x = 0$ or $z_1 = 1$. The reason is that for a positive rate of technological progress, consumption eventually increases enough to drive asymptotically c_0/c to 0 and z_1 to 1. Only if there is no technological progress can the consumption stay constant and the value of z_1 remain indefinitely constant below 1. Therefore, if $x > 0$, the *IEIS* case converges asymptotically to the *CEIS* case and the steady states are identical. Moreover, if $x = 0$, the steady state values are also identical. The reason is that the term z_1 appears only in an expression that is multiplied by g, and $g = 0$ whenever $x = 0$. Therefore, if $x = 0$, the value of z_1 is irrelevant. The conclusion is that for any rate of technological progress, the steady state values of y/k and c/k are identical in both models.

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