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**Long-Run Determinants of the Real Exchange Rate:  
A Stock-Flow Perspective**

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**Abstract**

This paper examines the long-run determinants of the real exchange rate from a stock-flow perspective. The empirical analysis estimates a long-run relationship between the real exchange rate, net foreign assets and other factors affecting trade flows. Using postwar data for the United States and Japan, cointegration analysis supports the finding that the structural factors underlying each country's net trade and net foreign asset positions determine the long-run path for the real value of the dollar and the yen. The empirical analysis also provides estimates for the underlying stochastic trend in each real exchange rate series.

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### Summary

This study investigates the sources of long-run movements in the real exchange rate. For the United States and Japan, significant trends in their real exchange rates remain as prominent stylized facts of the postwar era, although the long-term drift in each case has been in opposite directions. In the case of the United States, there has been a steady overall decline in the real value of the dollar, whereas Japan has experienced extraordinary real appreciation in the yen since World War II.

To account for long-run relative price movements, this paper implements a version of the macroeconomic balance approach, emphasizing the stock-flow determination of the real exchange rate compatible with internal and external balance. Viewing purchasing power parity (PPP) as a fixed steady-state condition rather than as a long-run equilibrium condition, the analytical framework allows the long-run real exchange rate to be affected by real disturbances--representing fundamental shifts in the relative prices compatible with international equilibrium.

Using postwar data for the United States and Japan, cointegration analysis is used to examine the long-run co-movements between the real exchange rate and a set of fundamental determinants. Cointegration tests suggest a deterministic long-run relationship between the structural components in the current and capital accounts--underlying a country's net trade and net foreign asset positions--and the real exchange rate for these countries.

Specifically, cointegration estimates provide strong evidence that productivity differentials explain a significant portion of the trend variation in the real value of both the dollar and the yen. For the United States, there is also supporting evidence that the stock of net foreign assets has had an impact on the long-run real exchange rate. In both cases, there is little empirical support for the terms of trade determining the long-run path. The empirical analysis also provides estimates for the underlying stochastic trend in the real exchange rate for the United States and Japan, conditional on the empirically relevant fundamentals.



## I. Introduction

As the most widely accepted indicator of international competitiveness, the real exchange rate occupies a prominent place in the discourse on macroeconomic policy and the interests of economic research. Meanwhile, the doctrine of purchasing power parity (PPP) has maintained a central role in the discussion and analysis of real exchange rates. As a theory of exchange rate determination, PPP posits an underlying tendency for movements in the nominal exchange rate to offset movements in the ratio of national price levels, assuring constancy of the real exchange rate. <sup>1/</sup> Based on this static measure of long-run relative prices, deviations in the real exchange rate from its PPP benchmark can then be viewed as gains or losses in external competitiveness.

However, real exchange rate movements do not completely coincide with perceived changes in competitiveness, reflecting a basic flaw in the PPP-indicators approach. Instead, the likely effects of exchange rate changes on the trade balance are often difficult to predict without further information regarding the source of the shock. Moreover, it may be quite misleading to view the real exchange rate as an isolated measure of external competitiveness without further reference to developments within the overall macroeconomic environment. <sup>2/</sup>

Like other relative prices, real exchange rates are affected by real disturbances. In turn, real exchange rate movements stemming from real shocks may often represent fundamental shifts in the relative prices compatible with international equilibrium. <sup>3/</sup> Hence, a more general view of real exchange rate determination is needed than the one offered by PPP. Specifically, a distinction needs to be made between relative-price movements that represent lasting changes in the level of competitiveness and short-term fluctuations that reflect transitory departures from a given PPP

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<sup>1/</sup> See Dornbusch (1988) for a survey on PPP. See Breuer (1994) for a recent survey on the empirical evidence. In keeping with this literature, the real exchange is defined here as the currency-adjusted ratio of national price levels.

<sup>2/</sup> In a normative context, this basic identification problem also arises. As a policy target, PPP exchange rate rules provide an anchor to minimize potential misalignments. However, efforts to stabilize an inappropriate target for the real exchange rate have sometimes lead to increased macroeconomic instability. See Aghevli, Khan and Montiel (1991), Montiel and Ostry (1991), and Calvo, Reinhart, and Vegh (1994).

<sup>3/</sup> As for the trade balance, whether equilibrium changes in the real exchange rate accompany lasting changes in net trade flows depend upon the nature of the shock. Disturbances that alter the underlying savings-investment balance may effect permanent changes in trade flows and relative prices, whereas other types of shocks may lead to long-run changes in the real exchange rate alone. In either instance, the implications for competitiveness stemming from equilibrium developments are inherently different from the case where exchange rate fluctuations simply reflect transitory deviations from PPP.

level. Consequently, whether the long-run real exchange rate actually remains constant over a given time horizon (and PPP obtains) depends upon the behavior of its underlying economic determinants.

Viewing PPP as a fixed steady-state condition rather than as a long-run equilibrium condition, this paper investigates the sources of trend variation in the real exchange rate. Focusing on the United States and Japan, the empirical analysis examines the long-run relationship between the real exchange rate and the fundamental determinants which underlie the trend decline in the real value of the dollar and the trend appreciation in the real yen over the postwar period.

In developing the relationship between the real exchange rate and its fundamental determinants, this paper implements a version of the macroeconomic balance approach <sup>1/</sup> to exchange rate determination. In this framework, the sustainable real exchange rate is broadly defined as that value or path consistent with internal and external macroeconomic balance. Internal balance corresponds to output being at its potential level in conjunction with a non-accelerating rate of inflation. External balance requires a balance of payments position in which any current account imbalance is financed by a sustainable rate of capital flows.

Since capital flows are simply international transfers of financial claims, sustainability of the capital account in turn rests upon the desired net holdings of assets and liabilities between nations. Thus, stock variables play an important role in the determination of real exchange rates in addition to the conditions supporting flow equilibrium and macroeconomic balance. Moreover, changes in the long-run level of net foreign assets, reflecting changes in its underlying fundamentals, may have long-run consequences for relative prices (and net trade flows) as part of the adjustment mechanism toward equilibrium.

To further illustrate this point, the well-known transfer problem provides a useful example. Consider a country that experiences a steady-state decline in its stock of net foreign assets. The expenditure-reducing impact of this redistribution of wealth on domestic spending predominantly affects demand for domestically produced goods, and conversely abroad. In other words, changes in net external assets affect the relative demand for domestic and foreign goods when national spending patterns differ. Hence, this transfer of wealth must be accompanied by a real depreciation at home and expenditure-switching toward home goods to allow adjustment at full employment (internal balance) and an improved trade position to offset lower interest income from abroad (external balance).

Integrating stock variables and stock-equilibrium relationships directly into the analysis thus has the further advantage that flow

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<sup>1/</sup> See IMF (1984), Edwards (1989), Williamson (1990), and Bayoumi, et al. (1994) for discussion regarding the macroeconomic or underlying balance approach.

equilibrium must follow as a necessary condition. Hence, a sustainable real exchange rate within a stock-flow framework can in principle account for both internal and external macroeconomic (flow) balance.

Flow considerations reflecting real factors that affect the trade balance may also have long-run consequences for the real exchange rate. Among these factors, "the productivity approach" based on the seminal work of Balassa (1964) and Samuelson (1964) has perhaps received the most attention. 1/ The Balassa-Samuelson hypothesis maintains that differential rates of productivity growth across sectors may generate secular trend movements in the relative price of traded versus nontraded goods and, hence, the long-run real exchange rate. 2/

Other structural determinants including the stance of trade policy, variations in the terms of trade, and the composition of fiscal spending may also have long-run effects on the real exchange rate. In a developing country context, Edwards (1989) presents an empirical analysis of exchange rate determination emphasizing these and other factors affecting equilibrium. 3/

Using cointegration techniques, this paper examines empirically the long-run determinants of the real exchange rate for the United States and Japan over the postwar era. In particular, structural components in both the current and capital accounts--underlying each country's net trade and net foreign asset positions--are shown to influence the path of the long-run real exchange rate for each country. The empirical analysis also provides estimates for the sustainable real value of the dollar and the yen over the postwar period, conditional on the stock of net foreign assets and real factors affecting trade flows.

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1/ See for example Hsieh (1982), Marston (1987), De Gregorio, Giovannini and Wolf (1994), and Asea and Mendoza (1994) and the references cited therein for empirical and theoretical background on the Balassa-Samuelson effect.

2/ With labor mobility and wage equalization across sectors, an increase in productivity in the traded goods sector raises the real wage in both sectors, leading to an increase in the relative cost and price of nontraded goods. In a more general sense, both supply and demand factors should impact on the relative price of nontradables whereas the Balassa-Samuelson effect focuses on the former. Asea and Mendoza (1993) show that as a long-run proposition, however, sectoral productivity differentials determining the relative price of nontradables (along the long-run growth path) may be appropriate. See also De Gregorio, Giovannini and Wolf (1994) for further long-run empirical evidence.

3/ See also Ostry (1988). For industrial countries, several studies have used a simulations approach to calculate real exchange rate trajectories compatible with macroeconomic balance. See for example Williamson (1990) and Bayoumi, et al. (1994) and the references therein.

The paper is organized as follows. Section II presents a simple model emphasizing the stock-flow determination of the real exchange rate. Based on this general conceptual framework, Section III outlines an econometric approach based on cointegration to estimate long-run relationships between the real exchange rate and its fundamental determinants. Section IV presents empirical results for the dollar and yen real exchange rates over the postwar period based on Johansen's (1988) cointegration procedure. Section V offers some concluding remarks.

## II. Illustrative Model

Consider a world economy consisting of two countries--designated as home and foreign--engaging in the trade of two distinct goods and one financial asset. <sup>1/ 2/</sup> The home country produces and consumes a domestic good, and purchases the foreign good through trade with the rest of the world. With the price of the latter good serving as the numeraire, variables are expressed in real terms (measured in units of the foreign good) unless specified otherwise, and output is taken to be fixed at its full employment level.

Assets pay a fixed real rate of return,  $r$ , and the net stock of real assets held by home country is denoted by  $f$ . By assumption, the large foreign country absorbs any excess spending or saving in the home country through a flow of securities without affecting its demand for the home good. From goods market equilibrium, the trade balance for the home country, which also equals the difference between the value of domestic output and domestic spending, depends on the relative price of home and foreign goods plus an exogenous shift parameter. Specifically, net exports  $nx$  for the home country can be written as:

$$nx = -\gamma q + x; \quad \gamma > 0, \tag{1}$$

where  $q$  is the (log) real exchange rate defined as the real price of the domestic good so that an increase denotes a real appreciation at home, and  $x$

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<sup>1/</sup> The model is a continuous-time version of Mussa (1984), which also provides extensions concerning nontradable goods, price stickiness and disequilibrium dynamics. Although the theoretical analysis here only focuses on factors determining the relative price of imports versus exports, the subsequent empirical analysis generalizes the measure of international relative prices to also take into account factors which affect the relative price of traded versus nontraded goods.

<sup>2/</sup> The analysis assumes imperfect substitutability in goods but not assets. In the case of imperfect substitutability in assets, portfolio effects provide the channel linking asset stocks and the exchange rate. Changes in asset positions reflecting changes the relative supplies of domestic and foreign debt, require changes in either the relative yield (risk premium) or the relative valuation (real exchange rate) to restore portfolio balance.



represents the shift parameter incorporating exogenous factors that affect the relative demand and supply of domestic and foreign goods, and thus the trading position of the home country. 1/ Note that equation (1) embodies the traditional elasticities approach to the balance of payments, allowing the contemporaneous relative price of exports versus imports (abstracting from J-curve effects) to impact on the trade balance, where the parameter  $\gamma$  captures the familiar Marshall-Lerner condition. 2/

Abstracting from detailed aspects of the service account, the current account is defined simply by the net trade in goods plus the interest income received (or paid) on a country's net foreign asset (or debt) position:

$ca = nx + r f$ . The current account balance also equals the rate of accumulation of net foreign assets held domestically: 3/

$$\dot{f} = -\gamma q + x + r f, \quad (2)$$

where dot variables throughout denote time derivatives, i.e.,  $\dot{y} = dy/dt$ . Hence,  $\dot{f}$  in equation (2) represents the instantaneous change in the stock of net foreign assets held by the home country resulting from a given current account position.

Determining the equilibrium real exchange rate over the medium term requires addressing the issue of sustainability. In regards to external balance, a viable balance of payments position reflects a corresponding current account position that is financed by a desired or sustainable rate of capital flows. 4/ In turn, a sustainable capital account position rests on the underlying determinants of net foreign asset equilibrium. That desired rate of net foreign asset accumulation (or decumulation), which mirrors a desired amount of excess saving (or spending), is characterized by the following behavioral equation:

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1/ See Ostry (1988), Edwards (1989), and Khan and Ostry (1991) for further discussion regarding the effects of tariff changes and terms-of-trade disturbances on the equilibrium real exchange rate.

2/ Equivalently, equation (1) can be interpreted from the absorption approach. As in the transfer example, countries favor their own goods in consumption. Hence, an increase in domestic real absorption--relative to (fixed) output--must be accompanied by an increase in the relative price of domestic goods to ensure goods market equilibrium.

3/ Net foreign assets here refer to both private and official holdings. Note that the real analysis here applies to both fixed and floating nominal exchange rate regimes.

4/ In general, the underlying balance approach defines the equilibrium exchange rate as that value or trajectory consistent with both internal and external macroeconomic balance. Here, transitory or short-run fluctuations in income are neglected but could be added--see Mussa (1984). In that case, the equilibrium real exchange rate would explicitly require both full-employment output and balance of payments equilibrium.

$$\dot{f}^d = \delta(r - \rho) + \phi(f^d - f); \delta, \phi > 0, \quad (3)$$

where the desired rate of accumulation  $\dot{f}^d$  is a function of the difference between the domestic real interest rate  $r$  and the domestic long-run rate  $\rho$ , and the difference between the target level  $f^d$  and the actual level of net foreign assets. The target variable measures the stock of net foreign assets that domestic residents would prefer to hold if the short-run rate of interest equaled  $\rho$ . 1/ As a baseline case, the long-run real interest rate is assumed fixed equal to the world rate of interest ( $\rho = r$ ). 2/

In addition, the prevailing domestic rate of interest  $r$ , which influences desired consumption and savings decisions implicit in equation (3), reflects the *ex ante* rate of asset returns measured in units of the domestic good:

$$r = r - \alpha E_t[\dot{q}], \quad (4)$$

where  $\alpha$  is the expenditure share of domestic goods in home consumption, and  $E_t[\cdot]$  is the rational expectations operator conditional on the information set at time  $t$ . From equation (4), an anticipated increase in the price of the domestic good (real appreciation) indicates a lower expected real return (in terms of the domestic good) from holding assets which pay a fixed nominal rate of return (in terms of the foreign good), and consequently affects the desired rate of asset accumulation.

A sustainable balance of payments position, associated with flow equilibrium over the medium term, is identified by the relation  $\dot{f} = \dot{f}^d$ . In conjunction with internal balance, this condition ensures that the corresponding real exchange rate represents a sustainable equilibrium value or path consistent with underlying macroeconomic balance. Using equations (2), (3), and (4), this relation can be written as:

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1/ Including this target level pins down the steady-state level of net foreign assets, avoiding the indeterminacy feature associated with standard infinite horizon models with representative agents, where any distribution of wealth is self-replicating (i.e., multiple stationary states). See Giavazzi and Wyplosz (1984). Constructing a utility-maximizing model that generates a unique steady-state requires some form of heterogeneity across agents such as overlapping generations or introducing uncertainty (precautionary savings).

2/ Setting  $\rho = r^*$  assures a stable level for the real exchange rate in steady state. Otherwise, there would exist a steady-state rate of real appreciation or depreciation equal to the long-run real interest rate gap in equation (4).

$$-\gamma q + x + r f = -\alpha \delta E_t[\dot{q}] + \phi(f^d - f), \quad (5)$$

The balance of payments equilibrium condition in (5) requires that the net flow of goods and services be equal to the rate of desired excess spending over income (desired current account). Equivalently, from a flow of funds perspective, this condition specifies that the actual rate of net foreign asset accumulation be consistent with the desired net flow of financial claims (desired capital account). Hence, based on equation (5), the current account position over the medium term is financed by a sustainable rate of international capital flows.

Equations (2) and (5) together form a system of simultaneous linear equations consisting of two endogenous state variables  $f$ ,  $q$ , and two exogenous or forcing variables  $f^d$ ,  $x$ . Conditional on initial and terminal conditions for  $f$  and  $q$  respectively to ensure an economically sensible, nonexplosive solution, <sup>1/</sup> the fundamental solution for  $q$ , derived in the appendix, is given by:

$$q(t) = \bar{q}(t) + \sigma[f(t) - \bar{f}(t)]; \sigma > 0, \quad (6)$$

$$\bar{q}(t) = \frac{r}{\gamma} \bar{f}(t) + \frac{1}{\gamma} \bar{x}(t), \quad (7)$$

$$\bar{f}(t) = \lambda \int_t^\infty e^{-\lambda(s-t)} E_t[f^d(s)] ds, \quad (8)$$

$$\bar{x}(t) = \lambda \int_t^\infty e^{-\lambda(s-t)} E_t[x(s)] ds. \quad (9)$$

In expressions (6)-(9), bars over variables indicate long-run (stock) equilibrium values, while other variables reflect current (flow) equilibrium values. The forward-looking nature of the solution depicted above incorporates the fact that anticipations of future economic conditions are

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<sup>1/</sup> See Appendix for details. For a general discussion regarding continuous-time systems see Buiter (1989), and Blanchard and Khan (1981) for discrete-time analysis.

important for current variables, and, thus, the exchange rate is affected by market expectations. 1/

From equation (8), equilibrium holdings of net foreign assets  $\bar{f}(t)$  depend on the expected forward evolution in the target level of net foreign assets  $\{f^d(t)\}_t^\infty$ . Similarly, as seen from equation (9), the exogenous permanent component in net exports  $\bar{x}(t)$  relies on the present discounted value of the expected path of future trade disturbances  $\{x(t)\}_t^\infty$ . Finally, the central result is seen by equation (7), defining the long-run equilibrium real exchange rate  $\bar{q}(t)$  as a function of these underlying components in both the current account and the net foreign asset position.

The relationship between sustainable adjustment over the medium term and long-run equilibrium is captured by equation (6). The sustainable (saddle) path for the real exchange rate  $q(t)$ --associated with internal and external macroeconomic balance--differs from its long-run value  $\bar{q}(t)$  until full stock equilibrium is attained.

In transition, the real exchange rate may move away from its long-run equilibrium value to assure a convergent path for net foreign assets toward its steady-state value. For example, a permanent increase in the target level of net foreign assets which requires an eventual and lasting real appreciation, initially depreciates the real exchange rate in order to improve the trade balance and increase the current stock of net foreign assets toward its higher desired long-run level. 2/

Steady state equilibrium, characterizing a stable level of net foreign assets or liabilities, implies:  $\dot{f} = \dot{f}^d = 0$ . With exogenous forcing variables being constant at their steady-state values ( $f^d(t) = \bar{f}$ ,  $x(t) = \bar{x}$ ), equilibrium in stationary state is summarized by the following set of conditions:

1/ The forward-looking nature of the solution results from the fact that desired excess spending is affected by the expected rate of depreciation. An "asset price" interpretation for this condition treats the expectations term as the anticipated capital gains from holding the foreign asset. Alternatively, from an intertemporal viewpoint, the expected future real exchange rate (i.e., intertemporal relative prices) influences an agent's optimal consumption allocations that maximize lifetime utility. See Mussa (1984).

2/ The two variables  $q$ ,  $\bar{f}$  initially move in opposite directions in this case as evident from equation (6) and from the equivalent solution for  $q$  shown in the appendix.

$q(t) = \frac{r}{\gamma} \bar{f} + \frac{\bar{x}}{\gamma}$ ,  $f(t) = \bar{f}$ ,  $nx(t) = -r \bar{f}$ ,  $ca(t) = 0$ . 1/ Thus, only when the economy reaches steady state and fundamentals have settled down to their stationary values does PPP obtain in terms of constancy of the real exchange rate. Meanwhile, in the presence of long-run movements in the fundamentals, a clear distinction exists between equilibrium exchange rate movements, along changing saddle paths and steady-states, and a constant PPP value, associated with a particular stationary state.

In steady-state, the trade balance is determined solely by the equilibrium level of net foreign assets. This result can be interpreted as a "stock" version of the absorption approach. The desired net foreign asset position anchors the sustainable series of net savings flows ( $Y-A$  or  $S-I$  balances) and trade balances. In steady state, net exports attain a sufficient "primary" surplus (deficit) to offset interest obligations (receipts) on the stable level of external debt (assets). Consequently, those disturbances that impact on the current account over the short term without affecting net foreign assets in the long run, translate into changes in the real exchange rate, without affecting net exports, in steady-state equilibrium. 2/

Notice that determinants of the equilibrium real exchange rate summarized in equation (7) include factors that affect both the net trading position of the home country in world markets, as well as the underlying propensity of the home country to be a net lender or borrower of capital. In other words, the interaction between the permanent, structural components in both the current and capital account jointly determine the sustainable real exchange rate.

On the trade side, determinants that primarily operate through the current account include variables such as productivity growth differentials that impact on the relative price of nontraded goods, commodity-price shocks that vary the terms of trade, or commercial policy changes that alter the

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1/ The analysis can also be revised to account for steady-state growth rather than a stationary level of income, in which case a non-zero current account can exist in steady state. In stationary equilibrium (constant real exchange rate but income growth), the trade balance and current account as a share of GNP depend on the stable ratio of net foreign assets to income along the balanced growth path.

2/ Following the discussion in Ostry (1988), this result suggests that the direct effect of a permanent  $x$  shock on the trade balance is fully offset by the indirect effect via the change in the real exchange rate.

trade regime. 1/ On the finance side, fundamentals that essentially determine the economy's long-run net foreign asset position may include variables such as demographic factors, which reflect the age-structure of the population, or the stock of government debt, which affects net national borrowing in the absence of Ricardian equivalence. 2/

### III. Econometric Methodology

Of course, determining the relevant set of economic variables that underlie the sustainable real exchange rate remains an empirical issue, and devising an econometric framework based on the preceding theoretical discussion becomes the focus here. The central considerations involve the identification and estimation of the long-run relationship between the real exchange rate and its fundamental determinants.

In that regard, cointegration analysis provides a natural conceptual framework for examining long-term co-movements between a set of time-series variables. As a matter of definition, a set of  $N$  difference-stationary variables are said to be cointegrated if there exists at least one linear combination--i.e., cointegrating vector--of these variables that is stationary, defining their long-run relationship(s). 3/

Intuitively, cointegrated variables may drift apart temporarily, but must converge systematically over time. Hence, any model that imposes a deterministic long-run relationship between a set of integrated economic variables, while allowing those variables to deviate over the short term, will exhibit cointegration.

In the context of exchange rate determination, the long-run relationship between the real exchange rate and its fundamental determinants is summarized by equation (7). Meanwhile in the short-run, the presence of speculative factors--reflecting asset market disturbances--and cyclical

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1/ The model characterizes two basic types of fundamental shocks: those that affect the short-run trade balance (flow shocks) and those that affect the long-run net foreign asset position (stock disturbances), where only the latter type affect both  $q$  and  $nx$  in steady-state. In practice, however, variables rarely fit neatly into either category and the empirics to follow does not require these types of shocks to be orthogonal. For the purposes of exposition, a fundamental variable is referred to as a determinant operating through the trade balance or net foreign assets depending on the primary channel through which that factor impacts on the real exchange rate.

2/ See Masson, Kremers and Horne (1993) for empirical evidence on the effects of debt and demographics on net foreign assets.

3/ Moreover, the number of independent cointegrating vectors  $r$  must be such that  $0 < r < N$ . If there were exactly  $N$  such linearly independent combinations, then the set of variables must all be stationary ( $I(0)$ ). If no combinations exist ( $r = 0$ ), the series are independent difference-stationary ( $I(1)$ ) variables.

factors--given the sluggish adjustment of prices and wages--may cause the real exchange rate to deviate temporarily from its sustainable path, defined by the movement of its (nonstationary) fundamentals.

Cointegration analysis also provides empirical estimates for the long-run path of the real exchange rate, conditional on the time-series evolution of its fundamentals. Using the estimated cointegrating vector to identify the underlying stochastic trend, observed exchange rate movements can be decomposed into its transitory and permanent components (cycle and trend).

Annual data for the United States and Japan were obtained for the postwar period. 1/ For the real exchange rate, a CPI-based index of the real effective exchange rate (*REER*) was used. 2/ Explanatory variables included stock data on net foreign assets as a share of GNP (*NFA*) 3/ and a terms of trade index (*TOT*)--constructed as the ratio of export unit value to import unit value. 4/ As for productivity, two measures were implemented. First, following Kakkar and Ogaki (1993), a comparative index of the relative price of traded versus nontraded goods (*TNT*)--composed of the ratio of the domestic CPI to WPI relative to the corresponding (trade-weighted) index for the remaining G7 countries (except Canada)--was constructed. 5/ Second, a comparative index of labor productivity levels (*PROD*), constructed from rates of growth in real output per manhour in manufacturing at home versus the (trade-weighted) values for the rest of the G7, was also used. 6/

In other contexts, the variable *TNT* representing the relative price of nontraded goods may actually serve as a measure of the real exchange rate. Of course, the two variables *TNT* and *REER* should in principle be closely

1/ Plots of all the data series subsequently listed are presented for each country in the Appendix in Figures A1 and A2.

2/ All variables measured as indices are expressed in log-levels using 1985 as the base year (data source for *REER*: International Financial Statistics).

3/ Obtained from Masson, Kremers, and Horne (1993).

4/ Source: International Financial Statistics.

5/ Source: International Financial Statistics. Canada was excluded due to data availability. Note that the wholesale price index predominantly measures traded goods prices, while the consumer price index has a significant component of services--which are generally not traded. Hence, the ratio of the two indices compared to its foreign counterpart serves as a proxy for the relative price structure in the U.S. and Japan compared to its major trading partners. Specifically, if  $CPI = (P_t)^\alpha (P_n)^{1-\alpha}$  and  $WPI = (P_t)^\beta (P_n)^{1-\beta}$ , where  $\beta > \alpha$ , then the ratio will be an increasing function of the relative price of nontraded goods at home:  $(P_n/P_t)^{\beta-\alpha}$ .

6/ Source: Bureau of Labor Statistics. Note that in levels, output per manhour is not directly comparable across countries (index level is arbitrarily chosen), however, trend comparison can be made.

related, depending on the source of the shock. Specifically, shocks that irreversibly alter the relative price of tradables versus nontradables should be manifested in the stochastic trend in each series, reflecting the influence of the fundamentals common to both. <sup>1/</sup> It is precisely for this reason that including *TNT* as a proxy for trends in sectoral productivity may help explain long-run trends in the real exchange rate. <sup>2/</sup>

Under the assumption that (average) labor productivity in manufacturing reflects overall productivity in traded goods, the variable *PROD* provides a more direct measure of existing productivity differentials in tradables at home and abroad. Unfortunately, the equivalent measure for nontradables, which is inherently more difficult to define and measure, is not available. Since productivity in traded versus nontraded goods is the critical comparison, it should be noted that the measure *PROD* may be appropriate only under the further assumption that trend movements in relative productivity in services are insignificant among the major industrial countries.

#### IV. Empirical Results

Cointegration estimation is conducted using the multivariate maximum likelihood estimation (MLE) technique proposed by Johansen (1988). The Johansen procedure provides test statistics for the number of cointegrating

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<sup>1/</sup> To make explicit the relationship between *REER* and *TNT*, note that the latter by definition can be expressed:  $\ln(\text{CPI}/\text{ECPI}^*) - \ln(\text{WPI}/\text{EWPI}^*)$ , where  $*$  indicates foreign variables and  $E$  is the nominal exchange rate. The first term in this expression is in fact *REER* (ignoring coverage issues). If long-run PPP were to hold in tradables (i.e.,  $\ln(\text{WPI}/\text{EWPI}^*) \sim I(0)$ ), while the Balassa-Samuelson effect was the main source of secular trends in the real exchange rate, then *REER* and *TNT* alone would cointegrate with a coefficient of unity. However, if permanent shocks to tradables cause  $\ln(\text{WPI}/\text{EWPI}^*)$  to be nonstationary as well, then *REER* and *TNT* will cointegrate only when some other measure(s) is included, capturing permanent movements in the relative price of traded goods. The model in the previous section highlights potential sources for these latter long-run movements.

<sup>2/</sup> See De Gregorio, Giovannini, and Wolf (1994), and Micosi and Milesi-Ferreti (1994) for recent evidence on the relation between the relative price of nontraded goods and sector productivity differentials.



relationships that may exist, as well as empirical estimates for each of the cointegrating vectors. 1/

Prior to estimating the cointegration parameters, Augmented Dickey-Fuller (ADF) test statistics were calculated to indicate the order of integration in each of the univariate time series. The results of unit root tests--based on a unit-root null versus a trend-stationary alternative--are reported in Table 1. In every case, the ADF tests are consistent with each series being characterized as  $I(1)$  variables. Specifically, the ADF test fails to reject the presence of a unit root for each series in levels, but not in first differences. 2/

# 1. United States

The test statistics for cointegration for the United States based on the Johansen procedure are reported in Table 2a, using *TNT*. Tests for the number of cointegrating relationships in the data consist of the maximal eigenvalue and trace test statistics, where  $\lambda$ MAX tests for at most  $r$  cointegrating vectors against a point alternative of exactly  $r+1$  cointegrating relationships, while TRACE tests for at most  $r$  cointegrating vectors against an alternative of at least  $r+1$  vectors.

The null hypothesis of no cointegration ( $r=0$ ) among the four time series in each table is soundly rejected by both the TRACE and  $\lambda$ MAX

---

1/ The general empirical framework involves estimating a vector error correction model (VECM) of the form:

$$\Delta Y_t = \Pi Y_{t-1} + \sum_{j=1}^k \Gamma_j \Delta Y_{t-j} + \epsilon_t; \Pi = \alpha \beta',$$

where  $Y_t$  is a  $n \times 1$  vector of  $I(1)$  variables. Under cointegration, the long-run impact matrix  $\Pi$  has non-zero rank  $r < n$ , equal to the number of linear combinations of the series in  $Y_t$  which are stationary. In that case, the singular matrix can be decomposed into two distinct  $n \times r$  matrices  $\alpha$  and  $\beta$ , where the latter is a matrix of cointegrating vectors such that  $\beta' Y_t \sim I(0)$  and the former is a matrix of error-correction coefficients. See Campbell and Perron (1991) for a general discussion of the Johansen method.

2/ For the dollar real exchange rate, a case could be made for rejecting the null in favor of a trend-stationary alternative ( $p$ -value near 0.10) given the low power of unit root tests. This result only highlights the near observational equivalence between trend-stationary and difference-stationary processes in finite samples. See Perron and Campbell (1991). However, the steady-state implications of a deterministic trend are quite unappealing.

Table 1. Tests of Order of Integration

Variable	<u>ADF(k) Test Statistic</u>	
	United States, 1950-90	Japan, 1951-90
<i>REER</i>	-3.31 (k=1)	-2.64
$\Delta REER$	-3.85 <sup>a</sup>	-4.11 <sup>b</sup> (k=3)
<i>NFA</i>	-1.44 (k=1)	-2.46 (k=1)
$\Delta NFA$	-3.65 <sup>a</sup>	-4.71 <sup>b</sup>
<i>TOT</i>	-1.53	-2.68 (k=1)
$\Delta TOT$	-6.61 <sup>b</sup>	-4.98 <sup>b</sup>
<i>TNT</i>	-1.25	-2.77 (k=1)
$\Delta TNT$	-6.02 <sup>b</sup>	-5.03 <sup>b</sup> (k=1)
<i>PROD</i>	-0.69	-0.92
$\Delta PROD$	-5.63 <sup>b</sup>	-5.84 <sup>b</sup>

<sup>a</sup> indicates significance at 5 percent level;

<sup>b</sup> indicates significance at 1 percent level; and

\* Mackinnon (1991) critical values.

1/ The null hypothesis is a unit root versus a trend-stationary alternative. The ADF(k) test statistic for a variable  $x_t$  is given by the t-statistic on the estimated coefficient  $\pi_2$  in the following auxiliary regression (including constant and trend):

$$\Delta x_t = \pi_0 + \pi_1 trend + \pi_2 x_{t-1} + \sum_{j=1}^k \gamma_j \Delta x_{t-j},$$

where k is determined by the highest order lag for which the corresponding  $\gamma_j$  is significant. Note: If the underlying data generating process is an AR(p), then  $k=p-1$ . See Perron and Campbell (1991). In Table 1,  $k=0$  (DF test) unless specified otherwise.

Table 2a. Johansen Maximum Likelihood Tests: 1/ United States, 1950-90

(Eigenvalues in descending order: 0.553, 0.391, 0.373, 0.190, 0.000)

No. of Cointegrating Vectors: Null Hypothesis	$\lambda$ MAX	Trace
$r=0$	29.82 <sup>a</sup>	73.30 <sup>b</sup>
$r \leq 1$	18.35	43.47 <sup>b</sup>
$r \leq 2$	17.30 <sup>a</sup>	25.12 <sup>b</sup>
$r \leq 3$	7.82	7.82

<sup>a</sup> indicates significance at 5% level; <sup>b</sup> indicates significance at 1 percent level; and \* Critical values based on Johansen and Juselius (1990).

1/ Estimation involved a VAR with four lags and a restricted constant in the cointegrating vector. The Jacque-Bera test for normality and the Box-Pierce test against serial correlation (not reported) suggest that the selection of lag length is suitable. As a check for robustness, alternate lag length specifications were tested and do not affect the results.

Table 2b. Estimated Cointegration Vector: United States, 1950-90

(Corresponding maximal eigenvector)

	<i>REER</i>	<i>NFA</i>	<i>TOT</i>	<i>TNT</i>	<i>Constant</i>
Unrestricted	-39.38	41.56	6.03	35.20	-10.86
Normalized	-1.00	1.06	0.15	0.89	-0.28

Restricted Estimates:

$$REER_t = 1.54NFA_t + 0.91TNT_t - 0.30;$$

(Exclusion on *TOT*,  $\chi^2(1)=1.95$ )

$$REER_t = 1.47NFA_t + TNT_t - 0.30;$$

(Exclusion on *TOT* & Homogeneity on *TNT*,  $\chi^2(2)=3.33$ )

statistics. 1/ Indeed, based on the test statistics, multiple cointegrating relationships may possibly exist. 2/ The cointegrating vector corresponding to the maximal eigenvalue (i.e., the dominant long-run relationship) is reported in Table 2b. The long-run coefficients have the correct signs, and the point estimates on the normalized coefficients are of plausible magnitude. 3/

Tests of exclusion restrictions reported in Tables 2c confirm that each variable enters the reported cointegrating vector and thus shares a deterministic long-run relationship with the real exchange rate, with the exception of the terms of trade measure *TOT*. Moreover, the test results of joint exclusion restrictions shown in Table 2d support the finding that neither *NFA* nor *TNT* alone can explain permanent movements in *REER*. 4/ Both productivity differentials and net wealth are found to be relevant in the long-run determination of the U.S. real exchange rate.

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1/ Replacing *TNT*, cointegration estimates using *PROD* (not reported) also support a finding of cointegration. However, the latter measure of productivity consistently overstates the comparative decline in the relative price of nontraded goods in the U.S. until the late 1960s (normality of the residuals is overwhelmingly rejected due to subsample bias). Hence, subsequent analysis is conducted with *TNT* as the proxy for productivity in the case of the U.S.

2/ The presence of multiple cointegrating vectors suggest the presence of multiple long-run economic relationships between the set of variables or some subsets thereof. For example, if the fundamentals influence one another in a long-run sense, these variables may cointegrate separately from the real exchange rate. Hypothesis testing for exclusion restrictions is conducted to further examine this issue.

3/ Interpreting the cointegrating vector as the empirical analog to equation (7), the point estimate on the long-run coefficient of *NFA* suggests a real interest rate of around 5 percent, based on an empirical estimates of  $\gamma$  obtained from regressing the trade balance (as a share of GNP) on lagged cyclical fluctuations in the real exchange rate. Meanwhile, the estimated coefficient on *TNT* is close to unity as expected.

4/ This second finding may have the following economic interpretation. Broadly speaking, permanent movements in *TNT* involve factors which affect the relative price of nontraded goods without necessarily affecting the relative price across traded goods. Meanwhile, equilibrium changes in *NFA* require movements in the relative price of imports versus exports without necessarily affecting the price of nontraded goods relative to traded goods (under the proviso that wealth effects are not biased in that regard).

Table 2c. Tests of Exclusion Restrictions 1/  
United States, 1950-90

$$\text{Model: } \beta_1 REER + \beta_2 NFA + \beta_3 TOT + \beta_4 TNT + \mu \sim I(0)$$

Null Hypothesis	LR(1) Statistic 2/ Assuming 1 Cointegrating Vector	LR(2) Statistic Assuming 2 Cointegrating Vectors
$\beta_1=0$	9.96 <sup>a</sup>	10.18 <sup>a</sup>
$\beta_2=0$	7.81 <sup>a</sup>	8.83 <sup>a</sup>
$\beta_3=0$	1.95	2.13
$\beta_4=0$	8.78 <sup>a</sup>	8.88 <sup>a</sup>

<sup>a</sup> indicates significance at 1 percent level.

1/ Also, exclusion restrictions for the constant  $\mu$  are rejected at the 1 percent level of significance assuming either a single or multiple cointegrating vectors.

2/ The likelihood ratio test statistic  $LR(k)$  is distributed as  $\chi^2(rk)$ , where  $k$  is the number of restrictions and  $r$  is the number of cointegrating vectors.

Table 2d. Tests of Joint Exclusion Restrictions,  
United States, 1950-90

$$\text{Model: } \beta_1 REER + \beta_2 NFA + \beta_3 TOT + \beta_4 TNT + \mu \sim I(0)$$

Null Hypothesis	LR(2) Statistic Assuming 1 Cointegrating Vector
$\beta_2=\beta_3=0$	10.08 <sup>a</sup>
$\beta_3=\beta_4=0$	9.75 <sup>a</sup>

<sup>a</sup> indicates significance at 1 percent level.

Based on the estimated vectors of cointegration reported in Table 2b, estimates for the trend component of dollar real exchange rate can be computed. <sup>1/</sup> The underlying stochastic trend depicting the long-run path for the dollar real exchange rate is calculated based on the second set of restricted estimates reported in Table 2b.

Figure 1 displays the observed dollar real exchange rate and its estimated trend component over the sample period. Based on the fitted trend, the variance ratio of permanent (trend) innovations to actual innovations in the real exchange rate is around 30 percent for the entire sample, and 20 percent for the subsample under floating exchange rates (1973-90). In other words, about one-fifth of the variation of observed real exchange rate changes can be attributed to permanent shocks and the variation of changes to the long-run real exchange rate.

The most salient feature of the time-series behavior of the U.S. real exchange rate over the postwar period is the overall steady decline in both its actual and sustainable values as evident in Figure 1. Understanding this steady long-run real depreciation in the dollar becomes more clear upon examining the path of its underlying fundamentals.

A well-known stylized fact of the postwar era is that industrial countries have experienced more rapid productivity growth and a tendency toward convergence in per capita income vis-à-vis the United States. <sup>2/</sup> In turn, economic convergence among this group in the aftermath of World War II has had important consequences for the real value of the dollar.

With productivity gains accruing mainly in the traded goods sector, the relative price of traded versus nontraded goods declined more slowly in the United States (CPI/WPI ratio rose less quickly) than in the rest of the world. Consequently, the measure *TNT* exhibits a steady trend decline over the sample, only leveling off since the mid-1970s in conjunction with the productivity slowdown. <sup>3/</sup> This downward secular trend, resulting from differential rates of biased productivity growth at home and abroad, appears to have been largely responsible for the declining real value of dollar since the Second World War.

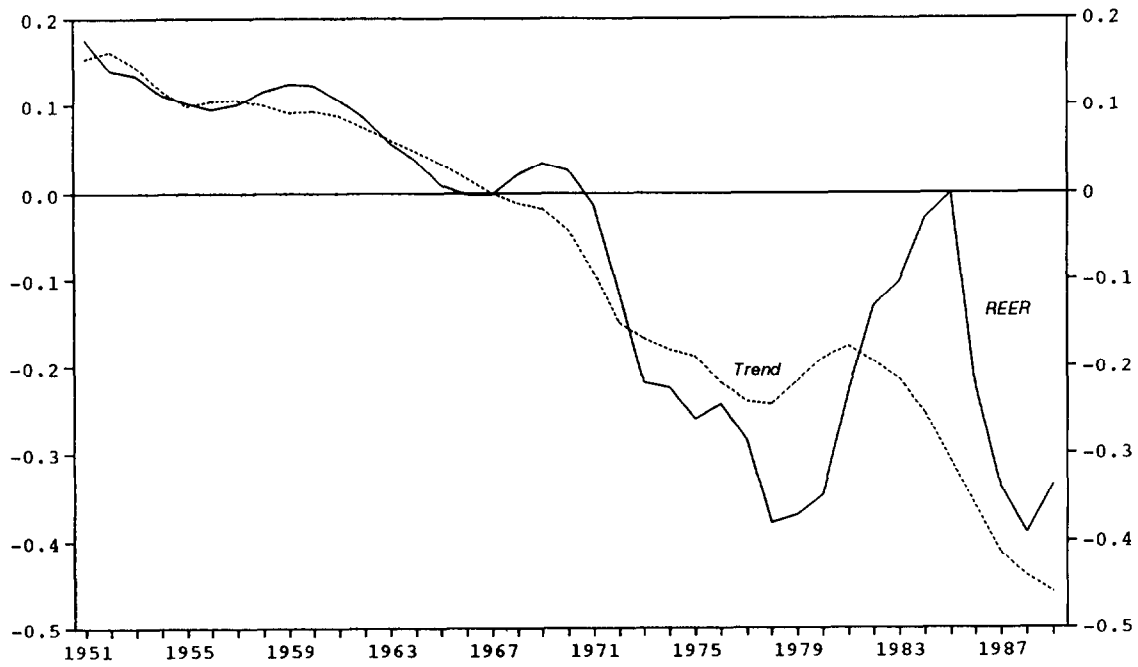
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<sup>1/</sup> Since the long-run coefficients are based on the co-movements between the series over the entire sample period, substituting observed values of the fundamentals directly into the estimated cointegrating relationship neglects the effects of stationary short-run noise. See Edwards (1989). To compensate, filtered estimates of the fundamentals are subsequently used. Specifically, the permanent component is smoothed using a three-year moving average, including one lead and one lag (the estimated coefficient of error correction for the dollar real exchange rate is  $\alpha_1 = -0.8$ ).

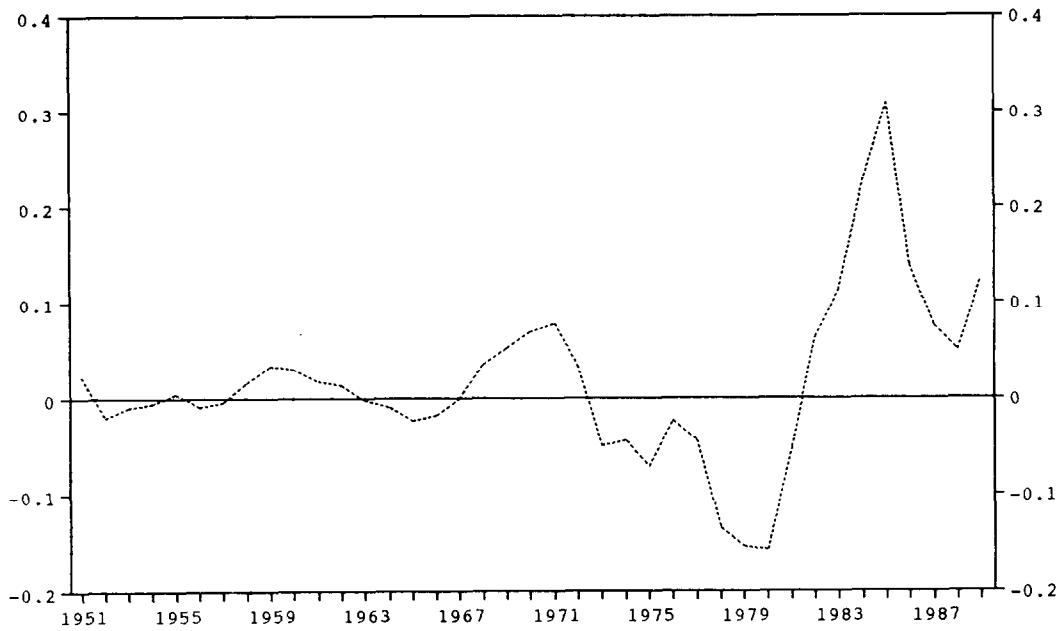
<sup>2/</sup> The empirical literature on convergence documents the significant narrowing in the initial dispersion of income per capita and productivity measures across industrial countries over the postwar period until 1973. See Dorwick and Nguyen (1989) and the references therein for a review.

<sup>3/</sup> Correspondingly, the evidence for catch-up and convergence has been much weaker since the productivity slowdown after 1973. See again Dorwick and Nguyen (1989). Perron (1990) estimates the slowdown in income growth using a nonlinear (breaking) trend for the G7.

**Figure 1 - US Real Exchange Rate**  
**Actual and Trend Values, 1951-1989**



**Figure 2 - US Real Exchange Rate**  
**Cyclical Component, 1951-1989**







Meanwhile, net foreign assets *NFA* remained relatively stable over the entire sample until the 1980s. Since that time, however, the U.S. net foreign asset position has declined significantly, representing the transformation of the United States from the world's largest creditor to the world's largest debtor country. <sup>1/</sup> The consequence of which has been a further decline in the sustainable value of the dollar real exchange rate toward the end of the period.

Based on estimates of the permanent component, cyclical fluctuations in the dollar real exchange rate--obtained as the difference between the actual and trend values--are shown in Figure 2. The vertical axis is once again measured in percentage terms. This (stationary) residual component can be interpreted as transitory deviations from the long-run path, resulting from short-term cyclical and speculative factors. <sup>2/</sup>

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<sup>1/</sup> Masson, Kremers, and Horne (1993) largely attribute the sustained decline in net foreign assets to the overall stance of U.S. fiscal policy and the increase in stock of public debt.

<sup>2/</sup> Permanent real exchange rate movements--which reflect sustainable adjustment towards steady state, conditional on macroeconomic balance--are positively related to changes in net foreign assets, reflected by the positive long-run coefficient on *NFA*. Meanwhile, over the cycle with macroeconomic imbalance, the real exchange rate (an increase denotes appreciation) and *NFA* can be shown to be negatively related:

$$\Delta NFA = -0.10\Delta\tilde{q} - 0.09\Delta\tilde{q}_{-1} - 0.08\Delta\tilde{q}_{-2} - 0.23\Delta\tilde{q}_{-4};$$

(2.28)    (1.99)    (1.92)    (4.51)

$$\text{adj } R^2=0.45, \text{ s.e.e.}=0.013, \text{ DW}=1.57, \text{ LM}[\chi^2(1)]=-1.98, \text{ LM}[\chi^2(4)]=-4.56,$$

where  $\tilde{q}$  is the cyclical component shown in Figure 2 and absolute t-statistics are given in parentheses. Using actual *REER* instead in the above regression would change the sign on some coefficients, significantly lower the  $R^2$ , and lead to serial correlation in the residuals due to misspecification (i.e., trend component in *REER* is dependent on *NFA*).

From Figure 2, notice that the real dollar appears to have been above trend on more than one occasion during the Bretton Woods period. 1/ This result may not seem surprising considering the dollar's unique role as the reserve currency under the gold-exchange standard. Under the Bretton Woods system, U.S. payments deficits were essentially financed through an accumulation of dollar reserves abroad, as central banks maintained fixed parities vis-à-vis the dollar. Although in principle, dollar reserves could have been converted into gold to offset the overall increase in world reserves, authorities generally accepted the increase in currency reserves as part of the mechanism providing international liquidity in a growing global economy. 2/

As evident in Figure 2, the largest divergence in the dollar real exchange rate relative to trend during the Bretton Woods period appears to have occurred towards the end of the regime. Substantiating this result, the last few years of the fixed exchange rate system witnessed a tremendous increase in outstanding dollar liabilities (up 250 percent between 1969-72), as the system ultimately collapsed. Note that although the observed value of the real exchange rate had actually showed a general decline up until that time, the critical fact remains that the underlying trend value of the dollar fell even further. 3/

In comparing the behavior of transitory fluctuations across fixed and floating exchange rate regimes, a sharp difference is apparent in Figure 2. In particular, cyclical variation in the dollar real exchange rate has, not

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1/ A caveat is warranted regarding the interpretation of the cyclical component as a measure of disequilibrium. To the extent that the stochastic trend captures changing steady-states, deviations between the actual and trend values do not necessarily represent (flow) disequilibria, as shown in the model (stock vs. flow shocks). If, on the other hand, trend movements reflect sustainable adjustment (along the saddle path to a given steady-state), deviations do reflect transitory (flow) disequilibria. The estimation of the trend cannot further distinguish between these sources of long-run variation, without a priori information regarding equilibrium in the fundamentals. Although given the length of time that is likely needed to reach steady-state, the view in the previous footnote seems reasonable (predominance of flow disturbances). Alternatively, "cycles" can certainly be interpreted here in a longer-run sense regarding stock disequilibrium.

2/ Data from 1950 to 1972 confirm that U.S. payments deficits were largely accommodated by an increase in dollar reserve holdings abroad. The increase in dollar liabilities was around \$60 billion, while the decline in the gold stock was around \$12 billion for that period.

3/ This result provides empirical support for the view summarized by Krugman (1990):

Arguably it was the secular decline in the equilibrium real dollar that really broke up Bretton Woods: the overvaluation of the dollar in 1971 owed little to faster US inflation since 1960, and much to a decline in the real dollar compatible with international equilibrium. (p. 168)

surprisingly, been much higher since 1973. 1/ With regards to the more recent behavior of the dollar, notice that the period of massive nominal (and real) appreciation from 1980 to 1985 reflects a large divergence in the real dollar from its estimated long-run path. 2/ Of course, *ex post*, this episode in fact proved to be unsustainable in a long-run sense. 3/

## 2. Japan

The test statistics for cointegration in the case of Japan are reported in Table 3a, using *PROD*. The null hypothesis of no cointegration among the four time series is again rejected by the TRACE statistic at the one percent level and by  $\lambda$ MAX statistic at the ten percent level of significance (critical value=24.9), with the possibility of two cointegrating vectors. The vector corresponding to the maximal eigenvalue for Japan is reported in Table 3b. Note that only the long-run coefficient on *NFA* has the correct sign, but the point estimate is implausibly large. However, the point estimates and coefficient signs appear quite sensitive to the choice of lag length of the VAR, and must be interpreted carefully.

Results of exclusion restrictions shown in Table 3c indicate that, in the presence of exactly one stationary linear combination, no single variable need enter the cointegrating vector, including *REER* itself. However, each variable must enter at least one of the vectors in the presence of two such long-run relationships. This finding suggests that different subsets of the real exchange rate and the explanatory variables are probably cointegrated. Based on tests of joint exclusion reported in Table 3d, only productivity and the real exchange rate cointegrate alone, with the restricted estimates reported in Table 3b. 4/

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1/ By construction, the cyclical component is stationary, and thus has a well-defined (time-invariant) second moment. Comparing variances before and after 1973 indicates that the Transitory fluctuations (obtained from the fitted trend) in the dollar real exchange rate have been 16 times more variable under floating exchange rates.

2/ Reiterating an earlier caveat: the estimated cyclical component for the period from 1980-85 need not reflect completely a divergence from a value compatible with flow equilibrium. To the extent that the desired U.S. net foreign asset position also declined around that time (see Figure A1), real exchange rate overshooting would in part be a necessary element of the adjustment process; a stronger currency is needed initially to induce a sufficient current account deficit to ensure a convergent path in net external assets towards its new stock equilibrium value.

3/ Stein (1994) provides estimates of the medium-term dollar real exchange rate, which tracks its actual path more closely. Not surprisingly, the empirics there place much greater emphasis on flow rather than stock variable in the analysis.

4/ The estimated cointegrating vector excluding *PROD* retains a large coefficient on *NFA* and an opposite sign on the coefficient for *TOT*:  $REER_t = 3.03NFA_t - 0.56TOT_t$ . The large coefficient on net foreign assets reflects the fact that its upward trend has been clearly out-paced by the rate of real appreciation in the yen.

Table 3a. Johansen Maximum Likelihood Tests 1/  
Japan, 1951-90

(Eigenvalues in descending order: 0.522, 0.490, 0.130, 0.096)

No. of Cointegrating Vectors: Null Hypothesis	$\lambda_{MAX}$	Trace
$r=0$	26.62	59.50 <sup>b</sup>
$r \leq 1$	24.26 <sup>a</sup>	32.88 <sup>a</sup>
$r \leq 2$	8.62	8.62
$r \leq 3$	3.61	3.61

<sup>a</sup> indicates significance at 5 percent level; <sup>b</sup> indicates significance at 1 percent level; and \* Critical values based on Johansen and Juselius (1990).

1/ Estimation involved a VAR with four lags and an unrestricted constant to allow for possible deterministic trends. The Jacque-Bera test for normality and the Box-Pierce test against serial correlation (not reported) suggest that the selection of lag length is suitable. Similar results obtain using *TNT* instead of *PROD*.

Table 3b. Estimated Cointegration Vector  
Japan, 1951-90

(Corresponding maximal eigenvector)

	<i>REER</i>	<i>NFA</i>	<i>TOT</i>	<i>PROD</i>
Unrestricted	12.95	-104.99	16.92	13.31
Normalized	-1.00	-8.11	1.31	1.03

Restricted Estimates:  
 $REER_t = 0.66PROD_t$ ;  
 (Exclusion on *TOT* & *NFA*,  $\chi^2(2)=1.95$ )

Table 3c. Tests of Exclusion Restrictions  
Japan, 1951-90

$$\text{Model: } \beta_1 REER + \beta_2 NFA + \beta_3 TOT + \beta_4 PROD \sim I(0)$$

Null Hypothesis	LR(1) Statistic	LR(2) Statistic
	Assuming 1 Cointegrating Vector	Assuming 2 Cointegrating Vector
$\beta_1=0$	1.89	21.07 <sup>a</sup>
$\beta_2=0$	1.61	20.27 <sup>a</sup>
$\beta_3=0$	1.76	19.39 <sup>a</sup>
$\beta_4=0$	0.81	19.78 <sup>a</sup>

<sup>a</sup> indicates significance at 1 percent level.

Table 3d. Tests of Joint Exclusion Restrictions  
Japan, 1951-90

$$\text{Model: } \beta_1 REER + \beta_2 NFA + \beta_3 TOT + \beta_4 PROD \sim I(0)$$

Null Hypothesis	LR(2) Statistic
	Assuming 1 Cointegrating Vector
$\beta_2-\beta_3=0$	1.95
$\beta_3-\beta_4=0$	10.29 <sup>a</sup>
$\beta_2-\beta_4=0$	15.57 <sup>a</sup>

<sup>a</sup> indicates significance at 1 percent level.

Cointegration estimates using *TNT* instead of *PROD* in the case of Japan (not reported) yield very similar results. With either measure of productivity, the results of various exclusion tests appear somewhat sensitive (unlike the case of the United States) to the selection of the lag length of the VAR. In particular, exclusions restrictions on *NFA* and *TOT* may or may not be rejected under different specifications.

However, the empirical results on the role of *PROD* (and *TNT*) are robust, consistently rejecting its exclusion from any long-run relationship with the real exchange rate for Japan, as well as yielding consistent parameter estimates in the restricted vector of cointegration. In combination with the results for the United States, the empirical findings thus lend strong support for the "productivity approach" as described in Hsieh (1982), Marston (1987) and others, recast here in a cointegration framework. <sup>1/</sup>

As a part of the convergence club, Japan like Western Europe has experienced a period of sustained productivity catch-up versus the United States--the likely result of faster capital-deepening, technological spillovers or some combination thereof. Moreover, productivity gains in Japan have been more heavily concentrated in tradables than in other industrial countries, as evidenced by the manufacturing data. The comparatively faster rate of productivity growth for Japan in traded goods underlies the yen's real appreciation vis-à-vis the remainder of the G7 in addition to the United States over the postwar period.

Trend-cycle decompositions for the real value of the yen are shown in Figures 3 and 4. The (filtered) trend components in Figure 3 are based on the long-run relationship between *REER* and *PROD* in Table 3b and the corresponding estimates using *TNT*. <sup>2/</sup> Both measures yield similar results, although the long-run estimates based on *PROD* generate a larger transitory component, probably as a result of omitting comparative productivity in nontradables. Based on the fitted trends, the variance ratios of innovations in the stochastic trend to innovations in the observed real exchange rate are 16 and 27 percent, using *PROD* and *TNT* respectively.

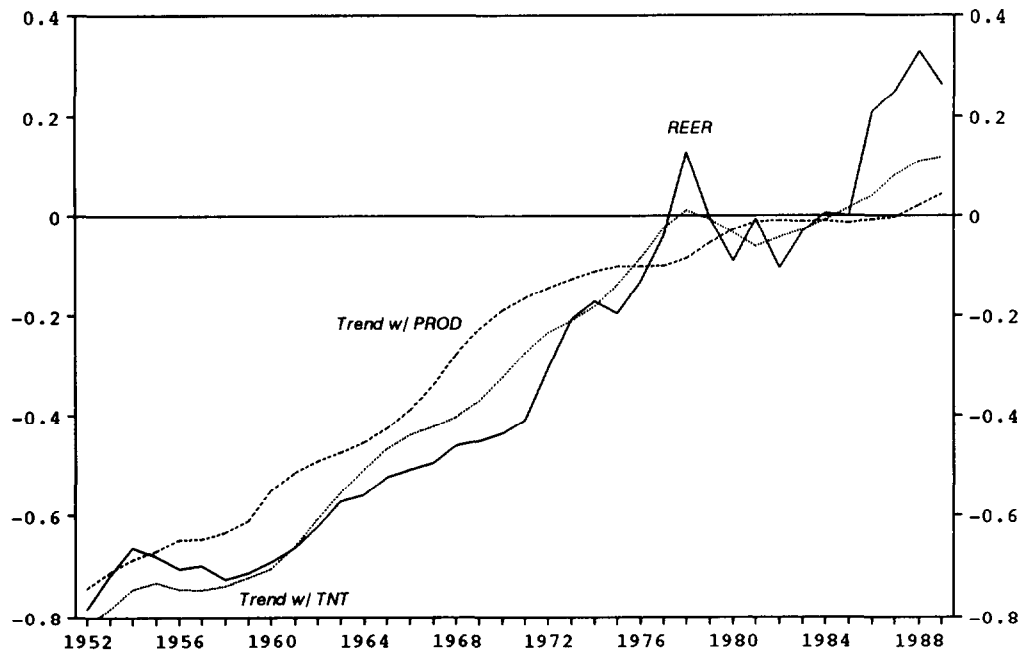
Examining both estimates of the cyclical component for Japan in Figure 4, indicate interestingly that the largest disparity in the actual rate relative to trend during the Bretton Woods period also occurred towards the end of the regime. Also of note, the quantitative estimates of misalignment for both the dollar and the yen real exchange rates during the breakup of Bretton Woods are broadly in line with the simulation results reported in Bayoumi, et al. (1994).

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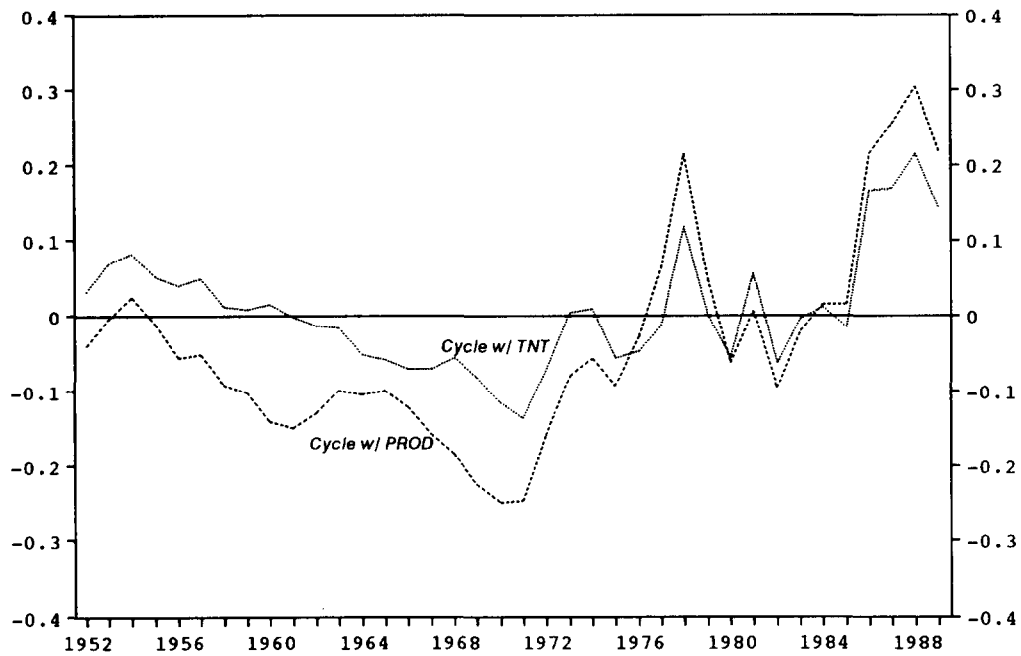
<sup>1/</sup> See also Kakkar and Ogaki (1993).

<sup>2/</sup> The estimated cointegrating vector obtained by replacing *PROD* with *TNT* in the system is given by:  $REER_t = 1.19TNT_t$ . The long-run coefficient on *TNT* is near unity as expected and smaller than the coefficient on *PROD* (due to faster trend growth in the latter series). Note that the above vector happened to be rejected by exclusion tests (for lag length=4). However, that result is not robust to alternate lag-length specifications.

**Figure 3 - Japan Real Exchange Rate**  
Actual and Trend Values, 1952-1989



**Figure 4 - Japan Real Exchange Rate**  
Cyclical Component, 1952-1989







## V. Concluding Remarks

Viewing PPP ultimately as a steady-state condition rather than as a long-run equilibrium condition, this paper has sought to explain long-run movements in the real exchange rate from a stock-flow perspective. Focusing on the United States and Japan, the empirical methods have applied recent cointegration techniques to examine the long-run determinants of the real exchange rate, in order to understand trend movements in the real value of the dollar and the yen over the postwar period.

For the United States, cointegration tests suggest that net foreign assets and productivity differentials share a long-run relationship with real exchange rate. This finding supports the proposition that the structural components in both the current and capital accounts--underlying a country's net trade and net foreign asset positions--jointly determine the long-run sustainable real exchange rate.

For Japan, the results are a bit less clear, except for the fact that productivity certainly matters in the long run. Cointegration tests for Japan suggest that various measures of productivity differentials share a long-run relationship with the real exchange rate. As a unique member of the convergence club, Japan has enjoyed tremendous productivity growth (particularly in manufacturing), leaving room for little else to explain the extraordinary postwar real appreciation of the yen.

On the other side of convergence, the relative gains that industrial countries have made compared to the United States in terms of productivity and output explain much of the downward secular trend in the dollar real exchange rate since World War II. Thus, the empirical findings firmly support the view that sectoral productivity differentials explain a large portion of the trend variation in the real exchange rate for the United States and Japan.

For both countries, there is little empirical support for the terms of trade having had a significant long-run impact on the real exchange rate once the effects of productivity and net wealth have been accounted for. As an extension, one could possibly use other terms-of-trade measures, such as the world price of oil, in the estimation. A further extension would involve incorporating more direct measures of the fundamentals, particularly those underlying external debt, to identify more closely the sources of long-run movements in the real exchange rate.

### Solution Method

The system of equations defined by (2) and (5) can be written in matrix form as follows:

$$\begin{bmatrix} \dot{f}(t) \\ E_t \dot{q}(t) \end{bmatrix} = A \begin{bmatrix} f(t) \\ q(t) \end{bmatrix} + B \begin{bmatrix} x(t) \\ f^d(t) \end{bmatrix} \quad (10)$$

where  $A$  and  $B$  are conformably partitioned matrices of coefficients for the state and forcing variables, respectively. The solution method proceeds as follows. The matrix  $A$  can be diagonalized:

$$A = V^{-1} \Lambda V; \quad \Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \quad (11)$$

where  $\Lambda$  is a diagonal matrix of eigenvalues of  $A$ . It is straightforward to show that:

$$\lambda = .5 \left[ r + \gamma/\alpha\delta \pm \sqrt{(r + \gamma/\alpha\delta)^2 + 4\gamma\phi/\alpha\delta} \right] \quad (12)$$

where  $\lambda_1 < 0$  and  $\lambda_2 > 0$  (appropriate discount factor), so that the number of stable and unstable roots equals with the number of predetermined and non-predetermined state variables, and the unstable root  $\lambda_2$  corresponds to the state variable. <sup>1/</sup> The matrix  $V$  is composed of linearly independent left eigenvectors of  $A$ , and, along with the matrix of coefficients on forcing variables  $B$ , takes the general form:

$$V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \quad (13)$$

The minimal state (fundamental) solution to this linear rational expectations system in equation (A1) for the non-predetermined variable  $q$  has the following general and specific forms:

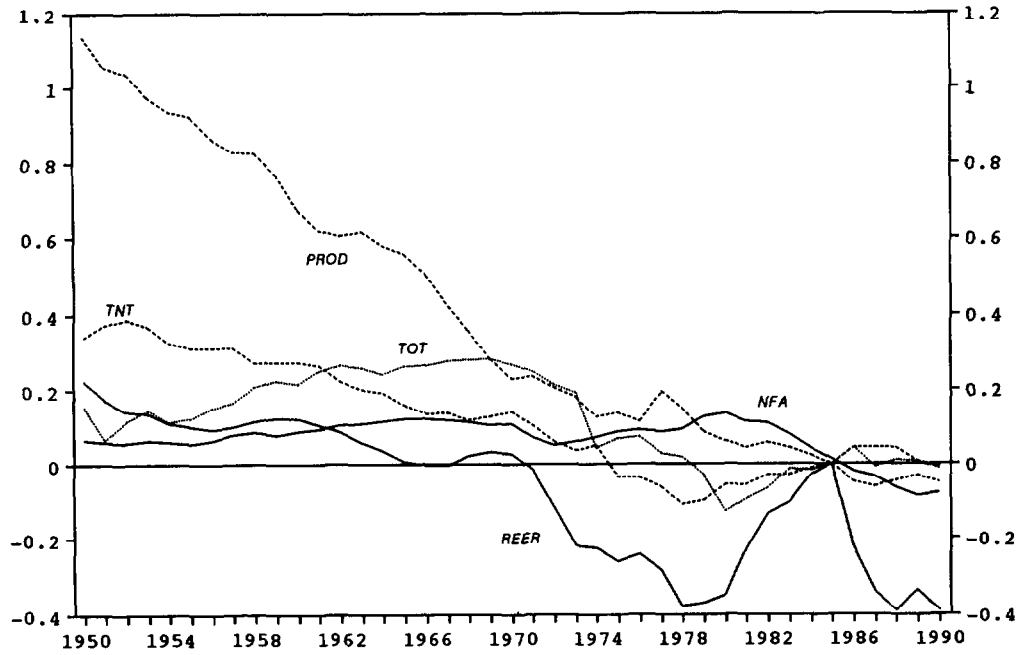
where  $Z$  is the vector of forcing variables in (A1) and where  $C = V_{21}B_1 + V_{22}B_2$ ,

and  $V_{21} = \frac{\phi + r}{\alpha\delta(r - \lambda_2)}$ ,  $V_{22} = 1$ ,  $B_1 = [1 \ 0]$ ,  $B_2 = [-\frac{1}{\alpha\delta} \ \frac{\phi}{\alpha\delta}]$ . See Buiter (1989).

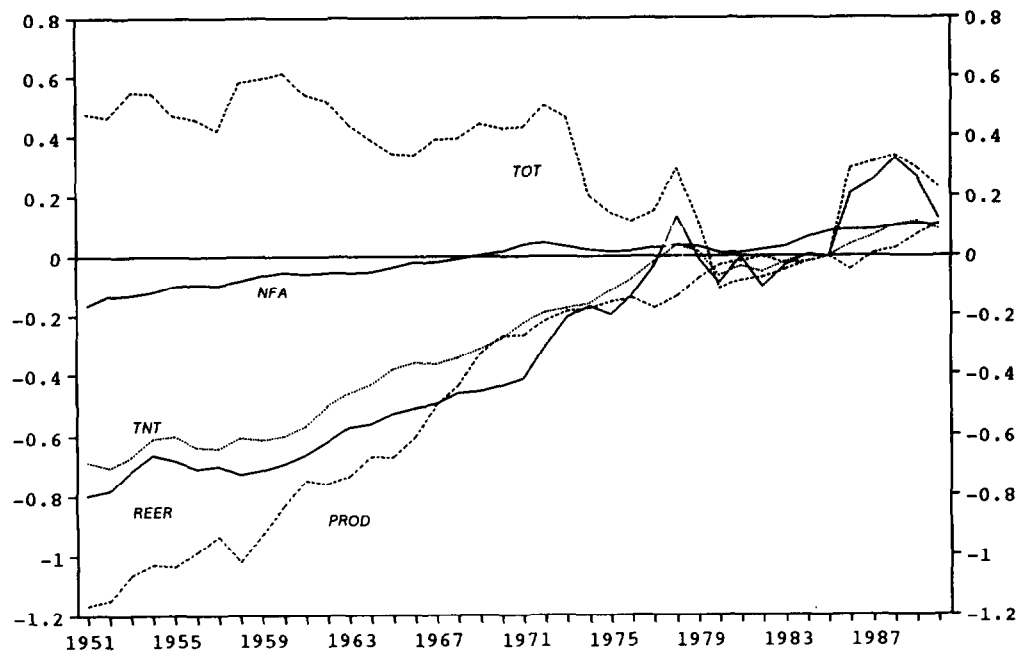
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<sup>1/</sup> Hence, the steady state is indeed unique, avoiding indeterminacy problems (zero roots in the transition matrix) in the long-run determination of the real exchange rate like those discussed in Giavazzi and Wyplosz (1984).

*Figure A1 - Time Series Data  
United States, 1950-1990*



*Figure A2 - Time Series Data  
Japan, 1951-1990*





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