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Monetary Aggregation:  
A Reconciliation of Theory and Central Bank Practice

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Abstract

Monetary assets have different characteristics which make them more or less useful in facilitating transactions. Academic economists have consistently argued that these differences should be incorporated in monetary aggregates by assigning assets different weights. However, central banks continue to use conventional aggregates with equal weights for all assets. For a transactions model of money, which the academic view implicitly embodies, weighted aggregates, although imperfect, are certainly superior. However, once this structural model is abandoned in favor of alternatives where monetary assets play a different role, central banks' continued use of simple sum measures of money may be justified.

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	<u>Page</u>
Summary	iii
I. Introduction	1
II. The Rationale for Divisia Indices	2
III. Microeconomic Foundations of Aggregation Theory	5
1. Restrictions through time	5
2. Restrictions over assets	6
3. Restrictions over individuals	7
IV. The Advantages of a Divisia Monetary Index	8
V. Alternative Models of Money	10
VI. Conclusions	12
Appendix	14
References	16

### Summary

While academic economists have exerted considerable influence over the evolution of many areas of public policy in recent years, this has been far from the case in the field of monetary aggregation. Over the last two decades, academics have consistently advocated the use of weighted aggregates, notably the Divisia Index, that purport to measure the services provided by monetary assets to the depositors who hold them. However, over the same period, central banks have remained wedded to conventional simple-sum aggregates in presenting policy and implementing monetary targets. This paper offers an explanation of why academic economists have failed to influence central bankers' choice of monetary aggregates.

Weighted monetary aggregates, such as the Divisia Index, attempt to measure the transactions services provided by monetary assets. Implicitly, advocates of such aggregates are assuming that the transactions model of money is the correct specification at a macroeconomic level. However, this need not be the case; there are alternative models of the role of money and the banking system in the macroeconomy that attribute a central role to the aggregate size of the banking system's balance sheet. This paper suggests that a conventional simple-sum aggregate may provide the appropriate measure of money in such a model.

Identification problems make testing between the alternative structural models essentially impossible in macroeconomic time series. Central bankers--who are primarily interested in the "information content" of monetary data--are content to rely on estimates of reduced-form relationships that reveal the statistical indicator properties of various monetary measures. The paper concludes that until weighted measures perform unambiguously better on such criteria, central bankers may be justified in continuing to use the conventional simple-sum aggregates.



## I. Introduction

The academic literature on monetary aggregation (comprehensively reviewed by Barnett (1980) and Barnett *et al* (1992)) has consistently advocated the use of aggregation procedures that assign different weights to the various component assets of money (see for example, Belongia and Chalfont (1989), Belongia and Chrystal (1991) and Ford *et al* (1992)), where the weights are intended to reflect the differing characteristics of the assets. Typically, the Divisia (1925) weighting scheme has been proposed, although variants have been suggested (Rotemberg *et al* (1991)). Such an approach is founded on a desire to base the aggregation procedure on formal microeconomic grounds. Academics have been dismissive of the official simple sum monetary aggregates that have few, if any, theoretical foundations.

Nevertheless, to the extent that targets or monitoring ranges for monetary growth are still publicly announced by central banks, they continue to be expressed using simple sum aggregates. 1/ The extensive academic literature on monetary aggregation appears to have had little influence on central banks' choice of monetary indicators.

This paper attempts to explain the observed divergence between academic proposals and central bank practice. 2/ It critically evaluates the academics' claims for weighted monetary aggregates, especially those based on the Divisia approach. Whilst such claims are generally found to be internally consistent, they may simply be irrelevant from a practical policy making perspective. The paper suggests that recent developments in the macroeconomic literature offer a plausible justification of central bankers' continued reliance on simple sum measures of money.

The paper is organized as follows. Section II outlines the rationale for adopting a Divisia weighting procedure to measure money and defines the Divisia index. Sections III and IV assess the microeconomic arguments in favor of Divisia aggregates in the context of a traditional transactions model of money. Section V discusses alternative models of money where weighted aggregates may not provide an appropriate measure. Finally, some conclusions about central bank practice are drawn in the light of the discussion in the main body of the paper.

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1/ For example, M0 and M4 in the United Kingdom, M3 in Germany, ALP in Spain, and M2 in the United States.

2/ The distinction between academic and central bank views in this paper is a gross oversimplification used for purposes of exposition. The views of neither group are monolithic. Many academic studies continue to use simple sum measures, and many central banks have published Divisia monetary aggregates and monitored their behavior over certain periods. We simply use the distinction between academic economists and central bankers as a convenient way of presenting two sides of the controversy concerning monetary aggregation.

## II. The Rationale for Divisia Indices

Advocates of weighted monetary aggregates typically have a transactions model of money in mind. They argue that money is held largely to facilitate transactions. Consequently, money holdings are related to the level of real activity and are likely to have good indicator properties. In this context, the "transactions services" offered by money holdings are the object of interest, not the stock of money holdings measured as a simple sum. Monetary assets vary widely in their ability to facilitate transactions. Clearly, not all monetary assets are perfect substitutes for transactions purposes--notes and coin provide quite different transactions services from interest bearing bank accounts. In consequence, a measure of transactions services should accord different monetary assets different weights.

Given this framework, the central issue is how to choose the weighting procedure. Two approaches present themselves. The microeconomic theory of aggregation suggests that a parametric "aggregator function" should be estimated. Alternatively, index numbers of various forms have been suggested by statisticians. For brevity, and in common with most of the existing literature, we focus on the Divisia weighting scheme as the preferred alternative to simple sum measures of money. The Divisia index has achieved prominence because it offers a link between these two approaches to choosing the weighting scheme, as shown by Diewert (1976). Much of the discussion, however, is equally applicable to alternative weighting procedures.

Aggregation theory describes the conditions under which a group of goods can be aggregated together and treated as if they were a single good. If these conditions do not hold the demand for the aggregate will depend not only the aggregate's own price, but also on the relative prices of its component assets. In such circumstances, a well-defined demand function for the aggregate will not exist. To construct a monetary aggregate, the relationship between component assets--specifically the extent to which assets are substitutes--has to be ascertained. This implies estimation of the parameters of an aggregator function. Since the resulting aggregate will then depend on the choice of estimator and the chosen functional form--both of which are to some extent arbitrary--this procedure is unlikely to prove popular for official measures of money. 1/

Whereas an economic quantity aggregate depends only on the component quantities and unknown parameters, a statistical index depends only on component quantities and their prices--both of which are observable (at least in principle). Consequently, a statistical quantity index avoids the need to estimate parameters of the aggregator function.

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1/ Note that simple sum aggregation entails exactly this choice, albeit by default since the parameters are not estimated but arbitrarily set equal to unity.

Diewert (1976) defined a class of "superlative" index numbers which approximate arbitrary "exact" aggregator functions whenever the data are consistent with maximizing behavior. Hulten (1973) showed that in continuous time the Divisia index would satisfy this criterion for any weakly separable aggregator function. In discrete time these exact index numbers do not exist. However, Diewert (1976) shows that the Tornquist-Theil Divisia index (i.e., the discrete time version of the Divisia formula) will provide a second-order approximation to any linearly homogeneous aggregator function.

A Divisia quantity index is a chain linked statistical index where the growth rate of the aggregate is a weighted sum of the growth rates of the component assets. The weights are expenditure shares in total implicit spending on the aggregate. Divisia measures of money are therefore given by the formula:

$$\frac{\dot{D}}{D} = \sum_{n=1}^N S_n \frac{\dot{M}_n}{M_n}$$

$$S_n = \frac{P_n M_n}{\sum_{n=1}^N P_n M_n}$$

where assets are indexed by  $n=1, \dots, N$ .

- $M_n$  = stock of asset  $n$ .
- $\dot{M}_n$  = growth rate of asset  $n$ .
- $P_n$  = price (user-cost) of asset  $n$ .
- $D$  = Divisia index level.
- $\dot{D}$  = growth rate of Divisia index.

We now assess the arguments for and against Divisia measures of money.

### III. Microeconomic Foundations of Aggregation Theory

A central claim of the advocates of the Divisia measure is that the construction of the index is grounded in an explicit microeconomic optimization problem. The relevant issues are then as follows. What are the restrictions on a representative consumer's maximization problem required to derive the optimality of the Divisia approach to monetary aggregation? Are these restrictions both plausible in the light of developments observed in the monetary and financial system and consistent with the existing theoretical and empirical literature on money?

We start by taking a generic consumer intertemporal optimization problem, one that imposes very few restrictions on the data. The representative consumer maximizes utility with respect to an intertemporal budget constraint, facing exogenously given prices: 1/

$$\max U(\bar{C}, \bar{M}) \quad (1)$$

subject to

$$\int_0^{\infty} e^{-rt} P_{ct} C_t dt + \int_0^{\infty} e^{-rt} P_{mt} M_t dt = Z_0 + \int_0^{\infty} e^{-rt} P_{ct} Y_t dt \quad (2)$$

where

- $P_c$  = price of commodities;
- $Y$  = income;
- $P_m$  = price of monetary assets (the "user costs");
- $\bar{C}$  = vector of consumption commodities;
- $\bar{M}$  = vector of monetary asset holdings;
- $r$  = interest rate; and
- $z_0$  = initial asset position.

In order to make this generic optimization problem tractable and operational at the macroeconomic level, we need to impose restrictions along three dimensions, namely through time, across assets, and across individuals. We consider each of these dimensions in turn. 2/

1/ The prices of the monetary assets are "user-costs," normally measured as the interest foregone through holding a monetary asset rather than a financial asset that offers a higher return but provides no monetary services. Justification for this measure of user costs is provided by Barnett (1978), although Donovan (1978) provides a more accessible explanation. More sophisticated theories of money (such as that offered by Kiyotaki and Wright (1989, 1991) discussed below) suggest alternative prices for monetary assets. However, it is unclear how these could be incorporated into the Divisia scheme.

2/ Since Divisia advocates have claimed the Divisia measure remains stable in the face of financial innovation--a central concern of the theoretical and practical literature of the 1980s--restrictions relevant to this issue, which impinge to some extent on all three dimensions, are discussed in Section IV.

### 1. Restrictions through time

From the definition of the Divisia index described above, it is apparent that the index is based on aggregating the rates of change of component assets. For this procedure to result in a meaningful index, we require the growth of the components to be related to the growth of monetary services in a simple way. Specifically, it is necessary to impose the restriction that the intertemporal utility function in the generic optimization problem is time-additive separable. This requires the representative consumer's problem to have the restricted form:

$$\max \int_0^{\infty} U_t(\tilde{C}_t, \tilde{M}_t) dt \quad (3)$$

subject to the original budget constraint (2).

The significance of this restriction is its potential conflict with two propositions of "traditional" monetary literature. Recent empirical work on the demand for money has adopted the error correction approach, as refined by cointegration analysis. This assumes that money balances adjust slowly to some desired long-run level because of (non-trivial) adjustment costs. Adjustment costs in the consumer optimization framework could be modelled explicitly by including a rate of change variable in the objective function. However, this would violate the time additivity restriction. The additive separability restriction also rules out "learning by doing" effects which have been found to be important in some studies and offer an alternative explanation of slow adjustment to the long run. 1/

In practice, these objections may not be too damning. One could view the consumer as solving a long-run problem that abstracts from such adjustment costs. The optimization problem described above could be interpreted as determining the desired long-run level of money holdings, while the adjustment process towards this desired long run is estimated in a purely statistical, atheoretical way. On this interpretation, policymakers would have to be wary of over-interpreting short-run changes in the growth of Divisia money.

Typically, to simplify the algebraic derivations, the additional restriction that intertemporal utility is the discounted value of a time

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1/ Consumers are likely to obtain more monetary services for a given holding of a monetary asset as, by using the asset over time, they become increasingly adept at economizing on the holdings of monetary assets required to facilitate transactions. Consequently, the monetary services obtained from current asset holdings are likely to depend on holdings in previous periods, violating the time separability restriction.

invariant static utility function has been imposed. The consumer's problem is then:

$$\max \int_0^{\infty} e^{-\rho t} U(\tilde{C}_t, \tilde{M}_t) dt \quad (4)$$

subject to the original budget constraint (2).

In fact, this is slightly more restrictive than necessary. Barnett (1990) has shown that the Divisia index for money remains a conceptually correct measure of monetary services in the face of Hicks-neutral technological change. Changes to the form of the utility function that affect all of its arguments in the same manner do not disrupt the properties of the final aggregate. Intuitively, the Divisia index measures monetary services in terms of foregone consumption. Any change to the utility function that leaves the trade off between consumption and the provision of monetary services unchanged will not alter the properties of the resulting aggregate.

Equally, changes to the utility function that do alter this trade off, such as those caused by non-neutral technological change, will result in the Divisia aggregate mis-measuring monetary services. Many of the recent forms of financial innovation represent non-neutral technological changes. We return to these issues in Section IV.

## 2. Restrictions over assets

A group of goods can be considered as an aggregate--as if it was an elementary good--if the decision regarding the level of the aggregate to be held is independent of its composition. This requires weak separability of the group of assets in the utility function. In the absence of such separability, changes in the relative prices of its components that left the aggregate's overall price index unchanged would imply different levels of demand for the aggregate as a whole. In this case, a stable microeconomic demand function for the aggregate could not exist. Thus weak separability is a necessary condition for any collection of assets to be considered as an admissible monetary aggregate.

In terms of the consumer optimization problem discussed above, weak separability requires that the vector of monetary assets is the argument of an aggregator function ( $\phi$ ) which is itself an argument of the utility function as a whole. The consumer's utility function can then be written in the form:

$$U(\tilde{C}_t, \tilde{M}_t) = U(\tilde{C}_t, \phi(\tilde{M}_t)) \quad (5)$$

This formulation implies that the marginal rate of substitution between different monetary assets is independent of the consumption of non-monetary commodities. Thus changes in the relative prices of the component assets will not alter the desired holding of the monetary aggregate, as required by the economic theory of aggregation.

### 3. Restrictions over individuals

If the above restrictions are satisfied for the representative consumer, the maximization problem can be solved to obtain a Marshallian microeconomic demand for money equation for a specific monetary aggregate. Moreover, duality theory implies that any aggregate constructed from the component monetary assets has a price dual associated with it. The price dual is constructed by weighting the prices of the monetary assets. There is a one-to-one relationship between the weighting scheme used for the quantity aggregate and that used for its price dual.

As with any Marshallian demand function, the demand for the monetary aggregate ( $M_D$ ) will have the own price (here the price dual ( $p_M$ )), the prices of other goods, and income (typically some measure of "permanent" income ( $\bar{y}$ )), as arguments.

$$M_D = M(\bar{P}_C, P_M, \bar{y}) \quad (6)$$

This representative agent approach is only applicable under very restrictive assumptions. The simplest--but also the least plausible--assumption is that all consumers have identical preferences, identical initial endowments, and identical income streams. As a general proposition, aggregate behavior does not have to obey microeconomic restrictions except under extremely implausible assumptions about the cross-sectional characteristics of individual consumers. This immediately raises the issue of whether microeconomic foundations are a fruitful starting point for policy-related empirical work on monetary aggregates.

Moreover, many of the less restrictive aggregation theorems require the absence of externalities or spillover effects from one individual's holdings of monetary assets to the utility derived by another individual from his asset holdings. This rules out effects that are typically central to the literature on the fundamental origin of money. For example, Kiyotaki and Wright (1989, 1991) emphasize the existence of externalities in the demand for money. In this literature, money has value because it is a *commonly* accepted medium of exchange. That is, there is something intrinsically social about the origin and use of money that implies that externalities are

pervasive. 1/ This is at odds with the requirements of cross-sectional aggregation over individual consumers.

Advocates of Divisia indices have argued that the implausibility of the assumptions implied by these restrictions is equally damning to all forms of monetary aggregates. Criticizing Divisia indices on these grounds is unfair because the same criticisms apply to all weighting schemes, including the traditional simple sum aggregates. Such arguments overlook the fact that Divisia's claims to be superior rest on such microeconomic foundations, whereas those of simple sum aggregates may not. For example, once the "transactions" model of money is abandoned in favor of alternative structural models which may be less well grounded in microeconomic theory, the argument is far less convincing. At present however, we are evaluating Divisia indices on microeconomic criteria. In this context, the argument has considerable force. To pose the issue in a slightly different way, we must consider what advantages Divisia offers over alternative weighting procedures.

#### IV. The Advantages of a Divisia Monetary Index

We have already noted that a Divisia index offers a non-parametric method for constructing a monetary aggregate. Since there is no need to assume a functional form for the aggregator function ( $\phi$ ), the possibility of specification error in measurement is ruled out. 2/ Other weighting procedures implicitly assume a specific functional form for the aggregator function. Errors in the measurement of transactions services will arise if the wrong functional form is chosen. By avoiding the need to make choices of this type, Divisia measures surmount the problem. When the aggregator function is linearly homogeneous, the Divisia index will exactly reflect the growth of transactions (or, more generally, monetary) services in the economy. This is illustrated by the simple derivation in the Appendix.

The essence of the case made by advocates of the Divisia is that the Divisia approach offers a substantial gain--namely that the resulting aggregate is a non-parametric index and thus free of specification error in measurement. This gain, however, comes at the expense of making a specific assumption about the aggregator function. If one is prepared to make all the prior microeconomic assumptions described above--and remember that all these restrictions apply in principle to any aggregate, including the

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1/ A simple example illustrates this point. The value of a bank checking account to a depositor will be very low if no one else has an account into which they can pay the checks he writes for them. As others also open checking accounts such that checks are a more widely acceptable means of payment, the value of the account to the original depositor will rise.

2/ Note that this specification error in measurement is related to the arbitrary choice of functional form for the aggregator function. It is different from model expectation errors that are not necessarily ruled out by non-parametric methods.

conventional simple sum aggregates--then the trade off between assuming linear homogeneity (a relatively weak restriction in this context) and obtaining a non-parametric aggregate (a strong and potentially powerful result) appears favorable to Divisia.

Divisia aggregates may also have an advantage over simple sum measures in dealing with the process of financial innovation, which has posed considerable problems in the empirical estimation of money demand equations. Proponents of Divisia have argued that the ability to account for financial innovation, particularly the introduction of interest-bearing accounts, is one of the main advantages of assigning different weights to the individual components of monetary aggregates. To assess this argument it is helpful to distinguish two forms of financial innovation--product innovation and technological innovation.

Product innovation arises when banks use existing technology to introduce new types of accounts--the innovation essentially offers existing financial asset characteristics in a different combination. Such product innovation, which has been extensive in a number of industrial countries, is to a large extent the result of increased competition and liberalization in the financial services industry. The Divisia index should in principle be able to account for this form of innovation since the trade-off between the consumption of commodities and the consumption of transactions services will not be affected. Asset holders will reallocate their money holdings without altering the aggregate consumption of transactions services.

The effects of non-neutral technological innovations may change the parameters of the aggregator function, in which case the trade-off between the consumption of commodities and the consumption of transactions services will change. An example of this would be new technology, such as the introduction of ATMs, which increase the transactions services provided by existing asset holdings without necessarily increasing the user cost. To the extent that such innovations are not reflected in equilibrium interest rates, the Divisia index will mis-measure the growth of transactions services. 1/

Another form of financial innovation is the increasing use of money substitutes to effect transactions. Under the restrictions discussed above, Divisia will measure the transactions services supplied by money correctly. However, money may provide a varying proportion of the total transactions services consumed in the economy. Transactions services are also supplied by non-money phenomena, such as credit cards and charge cards. In relating

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1/ The endogenous growth literature has shown that technological innovation must be considered either costless or, if there is a return to "research and development," can only be explained by relaxing the assumption of perfect competition (Romer (1990)). In either case, the impact of innovation will not be reflected in equilibrium interest rates (market prices), implying that Divisia indices will mis-measure the transactions services provided by monetary assets.

it to broad measures of transactions, a Divisia index for money will not be able to internalize and thus account for such substitution effects. Unless we can explain the substitution between monetary and non-monetary transactions media, any relationship between the Divisia index and a broad activity measure is likely to be unstable over time.

Divisia money can account for some forms of financial innovation, namely "product innovation," but not for others ("technological innovation"). This is an improvement over simple sum aggregates which can account for neither (other than "by accident")--but it does not rule out the need to have *ad hoc* explanations of financial innovation in demand for Divisia money equations.

#### V. Alternative Models of Money

The microeconomic arguments in favor of Divisia indices are compelling. Divisia measures of money can account for certain forms of financial innovation where simple sum aggregates cannot. Moreover, Divisia measures are not susceptible to specification or estimation errors since they are index numbers and thus depend only on observable prices and quantities. However, it should be noted that Divisia indices measure the flow of services (mainly transactions services) arising from money holdings. A Divisia measure of money is only interesting at a macroeconomic level in so far as this service flow is important to macroeconomic developments.

Much of the academic literature has a specific structural model of money in mind. It is concerned with a "transactions" model of money. The Divisia index of transactions services is the appropriate measure of money for this structural macroeconomic model, given the microeconomic assumptions described above. Although many of these assumptions are implausible, it would be wrong to conclude that a Divisia index is inferior to the conventional monetary aggregates as a measure of transactions money. First, the theoretical difficulties may not be severe; and second, some of the difficulties apply at least as severely to simple sum aggregates. Even a Divisia index that captures transactions services imperfectly is likely to provide a better measure of transactions money than alternative aggregates.

Although a Divisia index could measure transactions services correctly, the measure may be of little relevance at the macroeconomic level if the underlying structural model of transactions money is mis-specified. Recent developments in macroeconomic theory [Bernanke and Blinder (1988), Kashyap and Stein (1993)] have emphasized the special role of the commercial banking system in the transmission mechanism of monetary policy. Individuals and small companies are unable to issue primary securities because of asymmetric information problems. Consequently they are reliant on banks for external finance. Unavailability of bank credit will curtail the real activities of these groups. An expansion of bank credit will relax the constraints on these groups, allowing them to expand real activity. Thus the ability of the monetary sector to expand domestic credit is central to the explanation

of domestic output. The size of the banking system's consolidated balance sheet may have good indicator properties. 1/

Macroeconomic models of this form typically have few explicit microeconomic foundations in the sense used earlier in this paper. Whilst the microeconomic theory of credit rationing is relatively well developed [Stiglitz and Weiss (1981)], it has yet to be fully integrated into a coherent macroeconomic model. Therefore, although microeconomic justifications of the importance of bank credit (surveyed in Dale and Haldane (1993)) are reasonably convincing, they do not provide the formal basis for analysis comparable to that undertaken with Divisia indices. This suggests that the microeconomic foundations approach is, as yet, less relevant for alternative structural models. Consequently the claim that the Divisia measure is preferable because of its firm grounding in microeconomic theory is less compelling when the possibility of alternative structural models of money is entertained.

Consider a closed economy 2/ where the government refrains from borrowing from the commercial banking system. 3/ In such models, where the size of the consolidated balance sheet of the commercial banking system is of importance, simple sum aggregates may well be the appropriate measure of money. Weighted aggregates based on Divisia indices do not capture the concept of "loanable funds" available to expand bank credit and thus are not the relevant measure. 4/

The question of how to measure money correctly is logically distinct from the issue of whether a stable relationship exists between a monetary aggregate and other macroeconomic variables [Pill and Pradhan (1994)]. What constitutes a sensible measure of money is largely dependent on what structural model of the role of money one has in mind. Divisia indices--which purport to measure the transactions services associated with money holdings--are only appropriate to a transactions structural model. Alternative models necessitate the use of alternative measures, including at

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1/ "Good" in this context suggests both that the indicator tracks variables of interest closely and that, because the relationship is "structural" (in the sense it describes genuine economic behavior), it should be stable over time.

2/ Closed in the sense that there are effective capital controls preventing the flow of deposits at domestic banks to or from the rest of the world.

3/ This effectively implies that fiscal deficits are financed entirely by debt sales to the non-bank private sector. An example would be the "full fund rule" operated by the U.K. authorities during 1985-92.

4/ In countries that have well developed capital markets, the appropriate measure of loanable funds will include credit extended through securities markets as well as bank credit. In this case, even simple sum measures of bank liabilities may be poor indicators.

times traditional simple sum measures, such as those employed by central banks.

## VI. Conclusions

From a policymaker's perspective, a measure of money is only useful in so far as it imparts information about the contemporaneous or future behavior of objective variables, such as prices, output, and national income. The authorities use monetary data--which are typically reported promptly and accurately--to assess the behavior of other variables, which are generally only available after a longer time lag and are subject to considerable revision.

Policymakers are typically agnostic about the structural model of money. If they were convinced that the transactions services model of money is a good representation of the observed data, then presumably they would be less averse to adopting weighted aggregates such as Divisia. However, in general, policymakers are not convinced that any one structural model is infallible. They are especially wary of structural models associated with the monetary aggregates following the problems with monetary targeting encountered in the early 1980s in the United Kingdom, the United States, and elsewhere.

At a macroeconomic level, it is impossible to separate the issue of measurement from that of model specification. Implicitly, any attempt to estimate a demand for money equation using macroeconomic data is testing a joint hypothesis, namely that the aggregate correctly measures the monetary concept of interest and that the structural model is appropriate. In the context of Divisia, even a perfect measure of transactions money may not yield a useful relationship with objective variables because the underlying model is misspecified. Equally, simple sum aggregates may result in a more useful--from an information content perspective--relationship with prices or income, not because they offer a better measure of transactions money but because the transactions theory of money alone may be an inadequate description of the macroeconomic data and behavior.

Since any macroeconomic model involves "incredible" identifying assumptions [Sims (1980)], tests of model specification are inevitably of very low power. It is essentially impossible to determine whether the transactions theory of money--or any other theory for that matter--is right or wrong in aggregate time series. This leaves two possible approaches. One is to allow the data to "speak"; to concentrate attention on useful 1/ reduced form relationships, in the knowledge that they may break down in the presence of future structural or policy shocks. The other is to accept a certain model specification on the basis of *a priori* reasoning and explore the restrictions imposed by that model on the data.

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1/ What constitutes "useful" is determined by an exogenous criterion, typically based on indicator or information content properties.

Policymakers have typically adopted the former approach whilst academic economists have taken the latter. Given the incentive structures facing the two groups, this is hardly surprising. Central banks will remain reluctant to use monetary aggregates constructed according to Divisia weighting schemes until they are convinced that the resulting measures offer unambiguously better statistical indicator properties than those constructed as simple sums. The evidence on whether this is the case remains, at best, mixed [Fisher, Hudson, and Pradhan (1993)]. In contrast, academics are reluctant to consider simple sum aggregates because, within the context of their chosen structural model, such aggregates are clearly so poorly constructed they are almost meaningless.

If one takes the transactions model of money as a starting point, Divisia indices for money appear to be far superior to the conventional simple sum aggregates. Academic criticism of central bank practice is vindicated. However, central bankers can coherently argue that the transactions model may be mis-specified or (to put it more directly) simply not very useful at a macroeconomic level. Once the transactions structural model is abandoned in favor of alternatives, there can no longer be any presumption that the Divisia measures of money are superior to conventional aggregates.

If one allows for the existence of alternative structural models of the role of money and the financial system in the macroeconomy, conventional measures may be appropriate. For example, when bank credit plays an important role, traditional simple sum measures--to the extent that aggregate bank liabilities are closely related to aggregate bank credit--are likely to capture the economically meaningful concept. Since direct tests of one structural model against another are impossible in macroeconomic data, alternative approaches have to be evaluated on the basis of statistical reduced form relationships. Until weighted aggregates perform better on such criteria, central banks are justified in continuing to rely on simple sum measures of money.

The Divisia Measure of Transaction Services

(i) To show that a Divisia monetary index measures transactions services, given a linearly homogenous aggregator function, consider a simple case of only two monetary assets, cash ( $C$ ) and one interest bearing asset ( $I$ ). Transactions services  $M$  will be a function of holdings of these two assets, where  $\phi(\cdot)$  is the aggregator function.

$$M = \phi(C, I) \tag{A1}$$

Total differentiation of (A1) and division by  $M$  yields:

$$\frac{dM}{M} = \phi_C \frac{dC}{M} + \phi_I \frac{dI}{M} \tag{A2}$$

where  $\phi_C$  and  $\phi_I$  are the partial derivatives of  $\phi$  with respect to  $C$  and  $I$ .

Multiplying the first term in (A2) by  $C/C$  and the second term by  $I/I$  gives:

$$\frac{\dot{M}}{M} = \phi_C \frac{C}{M} \frac{\dot{C}}{C} + \phi_I \frac{I}{M} \frac{\dot{I}}{I} \tag{A3}$$

By Euler's Law  $\phi_C C + \phi_I I = \phi(C, I) = M$ . Therefore,

$$\frac{\dot{M}}{M} = \frac{\phi_C C}{\phi_C C + \phi_I I} \left( \frac{\dot{C}}{C} \right) + \frac{\phi_I I}{\phi_C C + \phi_I I} \left( \frac{\dot{I}}{I} \right) \tag{A4}$$

In equilibrium consumers equate marginal utilities to the prices of assets, such that the marginal rate of substitution between any two assets will equal the ratio of their respective prices.  $\phi_C/\phi_I = p_C/p_I$ , where  $p_C$  and  $p_I$  are the user costs. Substituting these prices in place of the partial derivatives yields:

$$\frac{\dot{M}}{M} = S_C \left( \frac{\dot{C}}{C} \right) + S_I \left( \frac{\dot{I}}{I} \right) \tag{A5}$$

where:

$$S_C = \frac{P_C C}{P_C C + P_I I}, \quad S_I = \frac{P_I I}{P_C C + P_I I}$$

In continuous time  $\dot{M}/M \equiv \dot{D}/D$ , the growth of the Divisia index.

(ii) To show that the Divisia measure fails to account for 'technological' financial innovation, following Koenig and Fomby (1991), suppose the monetary services function is given by;

$$M = \phi(\alpha C, \beta I) \tag{A7}$$

where  $\alpha$  and  $\beta$  are time varying parameters reflecting the transactions technology (e.g., ATMs, direct debit facilities, etc.) which are not fully reflected in the own rates of return used to derive user costs.

Following the same procedure outlined above for the microeconomic derivation of the Divisia index, it can be shown that:

$$\frac{\dot{M}}{M} = S_C \left( \frac{\dot{C}}{C} \right) + S_I \left( \frac{\dot{I}}{I} \right) + S_C \left( \frac{\dot{\alpha}}{\alpha} \right) + S_I \left( \frac{\dot{\beta}}{\beta} \right) = \frac{\dot{D}}{D} + S_C \left( \frac{\dot{\alpha}}{\alpha} \right) + S_I \left( \frac{\dot{\beta}}{\beta} \right) \tag{A8}$$

The last two terms in (A8) capture the extent of the departure of the Divisia measure from the "true" growth of transactions services in the economy. To measure this divergence, some functional form for the payments technology ( $\alpha$  and  $\beta$ ) has to be specified and estimated.

References

- Barnett, W.A., "The User Cost of Money," *Economics Letters*, Vol. 1 (1978), pp. 145-9.
- , "Economic Monetary Aggregation: An Application of Aggregation and Index Number Theory," *Journal of Econometrics*, Vol. 14, No. 1 (September 1989), pp. 11-48.
- , "Developments in Monetary Aggregation Theory," *Journal of Policy Modeling*, Vol.12, No.2 (1990), pp. 205-57.
- , D. Fisher and A. Serletis, "Consumer Theory and the Demand for Money," *Journal of Economic Literature*, Vol. XXX (December 1992), pp. 2086-119.
- Belongia, M.T. and J.A. Chalfont, "The Changing Empirical Definition of Money: Some Estimates from a Model of the Demand for Money Substitutes," *Journal of Political Economy* (April 1989), pp. 387-97.
- , and K.A. Chrystal, "An Admissible Monetary Aggregate for the UK," *Review of Economics and Statistics*, Vol. 73 (1991), pp. 497-503.
- Bernanke, B.S. and A.S. Blinder, "Credit, Money and Aggregate Demand," *American Economic Review*, Vol.78, No. 2 (1988), pp. 435-39.
- Dale, S. and A.G. Haldane, "The Role of Commercial Banks in the Monetary Transmission Mechanism," *Bank of England Quarterly Bulletin* (November 1993).
- Diewert, W.E., "Exact and Superlative Index Numbers," *Journal of Econometrics*, Vol. 4 (1976), pp. 115-45.
- Divisia, F. "L'indice Monetaire et la Theorie de la Monnaie," *Revue d'Economie Politique*, Vol. 39 (1925), pp. 980-1008.
- Donovan, D.J., "Modeling the Demand for Liquid Assets: An Application to Canada," *Staff Papers*, Vol. 25 (Washington, D.C.: International Monetary Fund, 1978), pp. 676-704.
- Fisher P.G., S.L. Hudson, and M. Pradhan, "Divisia Indices for Money: An Appraisal of Theory and Practice," *Bank of England Working Paper*, No. 9 (April 1993).
- Ford, J.L., W.S. Peng, and A.W. Mullineux, "Financial Innovation and Divisia Monetary Aggregates," *Oxford Bulletin of Economics and Statistics*, Vol. 54, No. 1 (February 1992).
- Hulten, C.R., "Divisia Index Numbers," *Econometrica*, Vol. 41 (1973), pp. 1017-25.

- Kashyap, A.K., and J.C. Stein, "Monetary Policy and Bank Lending," *NBER Working Paper*, No. 4317 (1993).
- Kiyotaki, I.N. and R. Wright, "On Money as a Medium of Exchange," *Journal of Political Economy* (August 1989), pp 215-35.
- , "A Contribution to the Pure Theory of Money," *Journal Economic Theory*, Vol. 53 (April 1991), pp. 215-35.
- Koenig, E.F. and T.B. Fomby, "A New Monetary Aggregate," *Federal Reserve Bank of Dallas Economic Review* (May 1990), pp. 1-15.
- Pill, H.R. and M. Pradhan, "Divisia Indices for Money: A Theoretical Appraisal," in *Proceedings of the Business and Economics Section of the 1993 American Statistical Association Annual Conference* (March 1994).
- Romer, P.M., "Endogenous Technical Change," *Journal of Political Economy*, Vol. 98, No. 5, Part 2 (October 1990), pp. S71-S102.
- Rotemberg, J.J., J.C. Driscoll, and J.M. Poterba, "Money, Output and Prices: Evidence from a New Monetary Aggregate," *NBER Working Paper*, No. 3824 (1991).
- Sims, C.A., "Macroeconomics and Reality," *Econometrica*, Vol. 48 (1980), pp. 1-48.
- Stiglitz, J.E. and A. Weiss, "Credit Rationing in Markets with Imperfect Information," *American Economic Review*, Vol. 71, No. 2 (May 1981), pp. 393-410.

