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How Does Learning Affect Inflation
After a Shift in the Exchange Rate Regime?

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Abstract

This paper analyzes the consequences of a shift from a floating to a pegged exchange rate regime on the actual and expected inflation rate, in an environment of asymmetric information. Policymaking is endogenous and the public learns rationally. There are two main findings. First, there is a "honeymoon effect" after the regime change, where inflation is lower than in the long run. Second, the asymmetric information outcome converges to that of symmetric information in the long run.

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Summary

Stabilization programs often include a fixing of the exchange rate. One of the theoretical justifications for this can be found in the literature on policy games. In that context, it has been shown that time-consistent inflation would be lower with a fixed exchange rate system rather than a free float system where purchasing power parity always holds. However, when a stabilization program is implemented, there is often great uncertainty about the policymakers' intentions and some authors have identified these information asymmetries between the government and the public as a possible cause of the slow decline in inflation.

This paper challenges this view. It analyzes the consequences on the actual and expected inflation rate of a shift from a floating rate regime--where purchasing power parity always holds--to a pegged exchange rate regime, in an environment of asymmetric information. The framework of the analysis is a policy game in which policymaking is endogenous and the public learns rationally.

The paper presents two main findings. First, that there is a "honeymoon effect" after the regime change, where inflation is lower than in the long run. Second, that the asymmetric information outcome converges to that of symmetric information in the long run. Hence, the paper concludes that information asymmetries are not responsible for lengthy disinflations. Rather, the private sector's uncertainty about government preferences results in lower time-consistent inflation than that prevailing once private agents have learnt the policymakers' preferences.

I. Introduction

The fixing of the exchange rate is often part of stabilization programs. One of the theoretical justifications for this can be found in the literature on policy games. In that context, Giavazzi and Pagano (1988) show that time-consistent inflation would be lower with a fixed exchange rate system rather than a free float system where purchasing power parity (PPP) always holds. The rationale is that, assuming that policymakers care about their country's competitiveness, inflation becomes more costly when the nominal exchange rate is fixed, as it reduces competitiveness, while in a PPP regime the real exchange rate is constant and cannot be affected by the authorities' decision to create inflation.

However, when a disinflation program is implemented, there is often great uncertainty over policymakers' intentions and some authors have identified these information asymmetries between the government and the public as a possible cause of the slow decline in inflation. This paper challenges this view.

The aim of this paper is to assess whether the public's lack of information about the government's preferences can help explain lengthy disinflations. This is explored in the context of a policy game that analyzes the public's inflationary expectations and the authorities' decision to create inflation in the presence of information asymmetries. This paper focuses on the strategic interactions between the public, which learns rationally, and the government, which sets policy endogenously.

There are two main results. First, there is a "honeymoon effect" after the regime change, when inflation is lower than in the long run. Hence, there is a short term benefit in terms of lower inflation stemming from the public's lack of information. Second, the asymmetric information outcome converges to that of symmetric information in the long run.

1. Review of the literature

Examples of the view that the public's slow adjustment of expectations and learning process could explain lengthy disinflations are found in the debate about the effectiveness of the ERM as an anti-inflationary device.

Driffill and Miller (1991) and Giovannini (1990) deal with learning following an exchange rate regime shift similar to that assumed here. In Driffill and Miller's paper, the public learns about the probability of the occurrence of a realignment. The private sector starts from a high prior, and then gathers information from the observation of realignments. The Bayesian learning process is then embodied in a model of overlapping contracts a' la Calvo. Wage contracts, in turn, depend on the likelihood of a realignment and directly affect actual inflation. Due to the high probability initially attached to the occurrence of a realignment and to the gradual revision of expectations, actual inflation declines slowly, competitiveness worsens and output falls below potential.

Giovannini adopts a Barro-Gordon type model, where inflation is replaced by the exchange rate and the latter is the policymakers' choice variable. It is assumed that a change in the exchange rate arrangement, from floating, in which the government manipulates the exchange rate, to fixed, takes place. Expectations about the exchange rate level are formed looking at the past and result in being initially higher than the actual value and then adjust gradually, provided the parity is not changed. As a result of this gradual convergence, an output loss results.

Both models, Driffill and Miller's and Giovannini's, have a weakness: they neglect the interaction of the public's expectations with the government's strategy; i.e. policy is not endogenous.

Elsewhere, it has been pointed out that ERM membership performs a signalling role of the new anti-inflationary stance and that a credibility bonus would ensue for inflation-prone countries. Similarly, some have highlighted the possibility that governments implement policies which convey information about their preferences to private agents, i.e. they signal their priorities among the different objectives.

Vickers (1986) presents a closed economy, two-period model of monetary policy. There are two possible types of policymakers, wet and dry, and a unique separating equilibrium results. Winckler (1991), in a framework similar to Vickers' applied to an open economy, finds that a dry government may want to signal the policy shift by setting the exchange rate parity at an appreciated level. 1/ However, Winckler's analysis does not deal with learning and the modelling of the private agents' behavior is somewhat undeveloped.

Britton (1992) discusses the importance of connecting the private sector's revision of beliefs with the policymakers' strategy in order to understand the consequences of a policy shift, like that occurring when a country joins the ERM.

No analytical model has been developed which analyzes the disinflation process following the adoption of a pegged exchange rate regime embodying both rational learning and endogenous policy. This paper is an attempt to fill this gap.

2. Outline of the paper

The aim of this paper is to examine how the change in the exchange rate regime and the asymmetric information structure affect the public's inflationary expectations and the authorities' decision to create inflation. In this paper, it is assumed that in the initial exchange rate regime PPP always holds. The public cannot gather information about the policymakers' preferences about competitiveness, as the crawling of the nominal exchange

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1/ Besides this separating equilibrium, there exists also a pooling equilibrium.

rate offsets domestic inflation, keeping the real exchange rate constant. Thus, the policymakers' decision to create inflation cannot affect competitiveness. However, once the pegged exchange rate regime is in place, the government's choice to inflate entails worsened competitiveness and thus reflects its intentions about the real exchange rate. The public can now learn about the authorities' willingness to endure misalignments in the competitive position of their country. In addition, since the government is aware of the private sector's learning, it will take into account the information content of its actions in deciding its policy. 1/

The basic model used here is an open economy version of the Barro-Gordon model (Barro and Gordon 1983) used by Giavazzi and Pagano (1988), in which stochastic realignments are introduced. This appears to be the natural framework in which to imbed the asymmetric information feature and analyze the interactions between the authorities and the public. The model consists of a dynamic non-cooperative game of asymmetric information between two players, the government and the private sector.

The paper is organized as follows. The basic model with stochastic realignments is laid out in section II, and the solution for the symmetric information case is derived. Section III is divided into three subsections. In the first, the asymmetric information structure is introduced and the solution procedure is illustrated. In the second, the public's learning problem is solved; in the third, the government's optimal policy is worked out. Section IV concludes.

II. The Basic Model

The basic model is an amended version of Giavazzi and Pagano's model with full capital mobility (Giavazzi and Pagano 1988). The novelty is the introduction of stochastic realignments and the government's imperfect control of inflation via the fiscal lever. The result of this model is that the time-consistent inflation rate in the pegged exchange rate regime is lower than that prevailing under floating with PPP.

The model has two goods: one is traded and its price, p^*, is that prevailing on international markets and is an exogenous variable. The price of the tradable good in domestic currency also depends on the exchange rate at time t, s_t, defined as the price of foreign currency in terms of the domestic currency. The other good is not traded and its price, p_t, is set as a mark-up over wages, w_t; for simplicity p_t equals w_t. Wages are equal in the two sectors of the economy, and wage setting is based on: the price level expected by private agents for time t, $p^e{}_t$; on the level of demand, y_t; and on a supply shock, z_t, modelled as a Weiner process, which cannot be immediately offset by the government. Aggregate demand depends on real

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1/ A similar approach has already been employed in a closed economy model of monetary policy by Cripps (1991) and Cripps and Papi (1991).

government spending, $g_t - p_t$. Competitiveness, or the real exchange rate, q_t, is an increasing function of p^* and s_t and a decreasing function of p_t. Variations in competitiveness affect the profitability of the tradable sector and lead to changes in the composition of output. As will be seen later, competitiveness enters the government's objective function. All this can be formalized in logarithms as follows:

$$p_t = w_t \tag{1}$$

$$w_t = p_t^e + \beta y_t + z_t \tag{2}$$

$$y_t = g_t - p_t \tag{3}$$

$$q_t = p^* + s_t - p_t. \tag{4}$$

The model elaborated so far can be described in two equations. The first reduced-form equation is:

$$y_t = \frac{1}{\beta}\left(p_t - p_t^e\right) - \frac{1}{\beta}z_t = \frac{1}{\beta}\left(\pi_t - \pi_t^e\right) - \frac{1}{\beta}z_t , \tag{5}$$

which shows that output is a function of price, or inflationary, surprises, and it increases with these. Furthermore, output depends negatively on the supply shock. The second reduced-form equation is:

$$p_t = \frac{1}{(1+\beta)}\left[\beta g_t + p_t^e + z_t\right] . \tag{6}$$

Equation (6) shows that the price level is affected by the public's price expectations and the government's choice of public spending. The expectation of the price level, $p^e{}_t$, or equivalently, the expectation of inflation, $\pi^e{}_t$, represents the public's strategy, which aims at minimizing the mean square error of the inflation forecast. [1]/ The public moves first, then, the government fixes its strategy, which is nominal public spending, g_t. In this section, symmetric information is assumed and the only uncertainty is represented by the supply shock.

Notice that the supply shock, z_t, affects the price level, so that, given the public's strategy, the authorities' choice of g_t does not

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1/ The public's objective function is specified in the next section, equation (24).

determine a unique value for p_t. Equation (6) can be rewritten in terms of rates of change as follows:

$$dp_t = \frac{1}{(1+\beta)}\left(\beta dg_t + dp_t^e + dz_t\right) . \tag{7}$$

In the rest of the paper, for simplicity, the choice of planned inflation, $\pi^p{}_t$, is the authorities' strategy, instead of g_t. The distinction between planned inflation, which is determined by the government, and actual inflation is necessary because of the government's imperfect command of inflation, via fiscal policy. Actual and planned inflation differ by a white noise disturbance, i.e. the increment of a Weiner process, so that the evolution of the price level is described by the following equation:

$$dp_t = \pi_t^p \, dt + dz_t . \tag{8}$$

The government is assumed to have some standard preferences. The authorities like to create inflation surprises, as these raise output. They also dislike inflation *per se* and its damaging effects on the profits of the export sector, which depend on the level of competitiveness. The government maximizes the following objective function:

$$V_0 = E_{gov}\left[\int_0^\infty e^{-\rho t}\left[hq_t + c(\pi_t-\pi_t^e) - \frac{1}{2}(\pi_t^p)^2\right]dt\right], \tag{9}$$

where ρ is the authorities' discount rate, and E_{gov} is the expectation operator with respect to the government's information set (which, in this basic model, coincides with the public's information set).

The real exchange rate, which is the state variable in this problem, can be expressed in the following way:

$$q_t = q_0 - \int_{t_1}^t \pi_s ds , \tag{10}$$

where q_0 is the level of competitiveness at time 0, i.e., when cumulated inflation is 0, which equals p^* plus s_0, and is assumed to be the level of the real exchange rate prevailing in the floating regime, that is the PPP

level. For simplicity, q_0 is assumed to be 0. As time goes by, competitiveness is eroded by inflation.

Realignments are the other crucial variable, besides domestic inflation, in determining the real exchange rate. In equation (10), t_1 denotes the date of the last realignment, so that the integral represents cumulated inflation from that date. This is so because it is assumed that at each realignment the initial level of competitiveness, q_0, is restored. To capture the fact that realignments occur infrequently, a Poisson process has been chosen for the variable X, denoting the number of realignments. The probability of having a parity change is constant over time and, in an interval of time which tends to 0, equals

$$\text{prob}(X=1) = \lambda \Delta t + \circ (\Delta t) \tag{11}$$

where λ is the parameter of the Poisson process, which is an institutional feature of the monetary system, and hence, outside the authorities' discretion.

Each period between realignments is treated independently. This is intended to model a situation in which, after each realignment, everything goes back to the beginning. 1/ In this case, the problem becomes separable between parity changes. Therefore, only one of these intervals of time is considered, as the problem is the same in each. The government's objective function is modified accordingly, that is, by neglecting temporarily the distinction between π_t and π^p_t and by assuming that π_t is a deterministic process, it is:

$$V_0 = E_{gov} \left[\int_0^{\tau_N} e^{-\rho t} \left[hq_t + c(\pi_t - \pi_t^e) - \frac{1}{2}(\pi_t)^2 \right] dt \right], \tag{12}$$

where the date of the last realignment has been normalized to 0 and τ_N denotes the date of the next realignment. The latter is a stopping time defined by:

$$\tau_N = \inf [t > 0; X_t = 1], \tag{13}$$

in words, it is the first time the random variable X takes the value 1.

1/ It is possible to show that, in the deterministic case, the solution obtained when each period between realignments is treated independently holds also when links between realignments are taken into account. However, this is no longer true when asymmetric information is introduced.

Before proceeding with the optimization, it is necessary to calculate the expectation of the whole objective function, which is a function of the random time τ_N. The result is presented in the lemma below.

Lemma

Given that the density of the random time τ_N is:

$$\lambda\, e^{-\lambda \tau_N} , \qquad (14)$$

and letting

$$Q_t = hq_t + c(\pi_t - \pi_t^e) - \frac{1}{2}\pi_t^2 , \qquad (15)$$

the expectation of the objective function with respect to the next realignment date is:

$$\int_0^\infty e^{-(\rho+\lambda)t}\, Q_t\, dt . \qquad (16)$$

Proof

The expectation of the objective function with respect to the random time can be computed as follows:

$$E_{gov}\left[\int_0^{\tau_N} e^{-\rho t}\, Q_t\, dt\right] = \int_0^\infty \lambda e^{-\lambda \tau_N}\left[\int_0^{\tau_N} e^{-\rho t}\, Q_t\, dt\right] d\tau_N . \qquad (17)$$

Equation (16) is obtained by reversing the order of integration in the right hand side of equation (17).

The problem has thus been transformed from a stochastic one, due to the presence of a stopping time as the horizon of the optimization, into a deterministic one, where the parameter of the Poisson process, λ, appears as additional discounting in the government's objective function. 1/

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1/   If the simplifying assumptions of having $\pi_t = \pi_t^p$ and π_t deterministic were removed, the problem would be stochastic and have the same solution for the government's policy. In the rest of this paragraph these assumptions are maintained because the solution for the optimal policy is the same, whilst the distinction between actual and planned inflation and the stochastic nature of π_t will be reintroduced in the next paragraph.

The government's optimization can now be tackled. Formally, the problem is the following:

$$\underset{\pi_t}{\text{Max}} \quad \int_0^\infty e^{-(\rho+\lambda)t} \left[hq_t + c(\pi_t-\pi_t^e) - \frac{1}{2}\pi_t^2 \right] dt \qquad (18)$$

$$\text{s.t.} \quad \dot{q} = -\pi_t , \qquad (19)$$

where equation (19) is obtained from equation (10). The time-consistent solution for the inflation rate, π_t^*, can be easily obtained as:

$$\pi_t^* = c - \frac{h}{(\lambda+\rho)} . \qquad (20)$$

The optimal inflation rate is time-invariant and shows that a country has always lower inflation when adopting a pegged exchange rate regime than under a free floating regime with PPP. In fact, under the latter regime the real exchange rate cannot be affected by policymakers' actions, so that the time-consistent inflation rate coincides with the closed economy solution, which is $\pi_t^* = c$. In the solution obtained here, under the pegged exchange rate regime, the reduction in the inflationary bias is larger, the smaller are the discount rate and the probability of having a realignment. [1]/ If realignments are more likely, and hence the expected date of the next realignment is closer, the period during which inflation costs are incurred is shorter. Hence, the tighter the institutional constraint, the greater the benefit of lower inflation resulting from "tying one's hands." Also, the larger h is, i.e. the more the government cares about competitiveness, the lower inflation is.

III. The Model with Asymmetric Information

1. The information structure and the solution procedure

In this section, the information asymmetry between the government and the private sector is introduced.

In a country which has recently changed the exchange rate regime, from floating to a peg, it is likely that the public does not know how much the

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1/   Notice that in this model it is not necessarily the case that there is a positive inflationary bias; in fact it is possible to have 0 as a time consistent inflation rate, or even a negative rate.

government cares about competitiveness. 1/ Thus, the private sector's uncertainty concerns the parameter h in the authorities' objective function, because the public never had the opportunity to gather information about this preference parameter in the flexible exchange rate regime with PPP. It is assumed that the distribution from which h has been drawn is common knowledge and it is:

$$h \sim N(\bar{h} , \sigma_h^2) , \tag{21}$$

where the mean of the distribution is the public's prior and σ_h^2 is the variance of the distribution, which is an indicator of the public's degree of uncertainty.

In this section, the assumption of imperfect controllability of inflation by the government is made; thus the evolution of the price level, or actual inflation, is described by equation (8). Furthermore, planned inflation, which is the authorities' strategy, is the government's private information, whereas actual inflation is observed by all agents in the economy. It is clear that in this set up the information sets of the two players differ: the public's information set is a subset of the government's, and compared to the latter, it lacks h and π^P_t.

From the observation of actual inflation, the private sector gathers information about h, the unknown element of the policymakers' preferences, and hence optimally revises its expectations of inflation, using a Kalman-Bucy filter.

The analysis is aimed at examining how the public's inflationary expectations and the government's choice of optimal planned inflation are modified by the asymmetric information structure and the stochastic environment, given that both players act strategically.

The public's filtering problem and the government's optimization can be solved in a sequence of stages because the separation principle applies; it applies given the linearity of the stochastic differential equations, which represent the constraints, and the fact that the objective function is quadratic in the control and linear in the state variables (see Davis 1977). The solution procedure begins by postulating the government's optimal policy. The following form is postulated:

1/ As already pointed out in the introduction, in the floating regime assumed here, PPP holds at all times, so that the government cannot affect the real exchange rate. In the pegged exchange rate system instead, the government's decision to create inflation reduces competitiveness.

$$\pi_t^p = cA_t - \frac{h}{(\lambda+\rho)} B_t \ , \tag{22}$$

where A_t and B_t are undetermined coefficients which will be derived once the government's optimal strategy has been solved for. A_t and B_t are assumed to be common knowledge.

The stochastic differential equation which describes the public's updating of beliefs, which is the evolution of h_t, becomes part of the policymakers' optimization. In fact h_t becomes the second state variable, in addition to q_t , in this asymmetric information context. The resulting equilibrium is a rational expectations equilibrium and the equilibrium of this game is a perfect Bayesian equilibrium.

2. The public's learning

The public uses a Bayesian algorithm, i.e. the Kalman-Bucy filter, to obtain and revise the estimate of the parameter h and then sets its strategy as:

$$\pi_t^e = cA_t - \frac{\hat{h}_t}{(\lambda+\rho)} B_t \ . \tag{23}$$

The chosen strategy is measurable with respect to the information field generated by all the variables of the model which can be observed by private agents. It minimizes the public's objective function, which is:

$$J = E\left[\int_0^\infty (\pi_t - \pi_t^e)^2 \ dt\right] . \tag{24}$$

In other words, the private sector aims at minimizing the mean square error of the forecast of inflation, which, given the information structure assumed here, is equivalent to minimizing the mean square error of the forecast of the parameter h. Formally, the filtering can be expressed by two equations. The first, equation (25), describes the system, which is the parameter h. The second equation (26) describes the evolution of the observations, namely actual inflation or the variations of the price level. The equations are as follows:

$$dh = 0 \ ; \quad E_0 (h) = \bar{h} \ , \quad E_0 \left[h - E_0 (h)\right]^2 = \sigma_h^2 \ . \tag{25}$$

$$dp_t = \pi_t^p dt + dz_t = \left[cA_t - \frac{h}{(\lambda+\rho)} B_t \right] dt + dz_t . \tag{26}$$

Since the above equations are linear, the estimate can be computed with the Kalman-Bucy procedure. The estimate of h, $\hat{h}_t$, is the best estimate based on the available observations. In fact, the filter at time t is adapted to the σ-algebra generated by the continuous observations of actual inflation from time 0 to time t and minimizes the mean square error of the forecast of h. The filter, $\hat{h}_t$, satisfies the following stochastic differential equation:

$$d\hat{h}_t = - \frac{B_t S_t}{(\lambda+\rho)} \left[dp_t - \left(cA_t - \frac{B_t}{(\lambda+\rho)} \hat{h}_t \right) dt \right] . \tag{27}$$

where $S_t = E[(h - \hat{h}_t)^2]$ solves the deterministic Riccati equation shown below:

$$\dot{S}_t = - \frac{B_t^2 S_t^2}{(\lambda+\rho)^2} ; \quad S_0 = E[h-E_0(h)]^2 = \sigma_h^2 . \tag{28}$$

The solution to equation (28) is:

$$S_t = \left[\frac{1}{(\lambda+\rho)^2} \int_0^t B_s^2 ds + \frac{1}{\sigma_h^2} \right]^{-1} = K_t^{-1} . \tag{29}$$

Notice that the speed at which the public learns depends on two things, besides the parameters of the model. (i) The term in brackets in equation (27), which is the discrepancy between actual and expected inflation, and hence, between the true and estimated value of h. This element becomes smaller and smaller as time goes by, because the public is learning. The learning process is thus faster at the beginning when each observation conveys a lot of information, and slows down over time. (ii) B_t, that is the undetermined coefficient to be chosen by the government. The smaller B_t, the slower the adjustment of expectations, because the greater becomes the relative size of the noise in the observations. 1/ Likewise, the larger B_t, the stronger the signal the government sends, and hence, the easier for the public to extract information.

1/ The B_ts also affect S_t in an inverse way; the larger the B_ts are, the smaller S_t is; however, if one considers only one B_t at a time, keeping the others fixed, the effect on S_t is infinitesimal, whereas there is obviously a finite effect on B_t and therefore the latter effect dominates.

Finally, solving the stochastic differential equation of the filter, the following expression for the estimate of h is obtained:

$$\hat{h}_t = \frac{\frac{1}{(\lambda+\rho)}\left[c\int_0^t B_s A_s ds - \int_0^t B_s dp_s\right] + \frac{\bar{h}}{\sigma_h^2}}{\frac{1}{(\lambda+\rho)^2}\int_0^t B_s^2 ds + \frac{1}{\sigma_h^2}} . \qquad (30)$$

It is possible to see that at time 0, the public's estimate of h is just the prior; as time tends to infinity, i.e. in the steady state, $\hat{h}_t$ tends to the true value.

Notice that the public sector's strategy, π^e_t, is not fully worked out yet, since it includes both A_t and B_t which will only be determined in the government's optimization. This is done in the next section.

3.   The government's optimization

The government's optimization problem in the asymmetric information environment can now be tackled, as all the elements needed have been worked out. These are: (i) the government's objective function; (ii) the evolution of the first state variable, competitiveness; and (iii) the evolution of the second state variable, the public's estimate of the parameter h. The second state variable appears only in the asymmetric information case, because the authorities now have to take into account the effects of their decisions on the public's learning process. The strategic interactions between the two players can thus be examined. Formally, the government's optimization is:

$$\max_{\pi^P_t} E_{gov}\left[\int_0^\infty e^{-(\lambda+\rho)t}\left[hq_t + c(\pi_t-\pi^e_t) - \frac{1}{2}(\pi^P_t)^2\right]dt\right], \qquad (31)$$

$$s.t. \quad dq_t = -\pi^P_t dt - dz_t , \qquad (32)$$

$$d\hat{h}_t = -\frac{B_t}{(\lambda+\rho)K_t}\left[\left[\pi^P_t - \left[cA_t - \frac{B_t \hat{h}_t}{(\lambda+\rho)}\right]\right]dt + dz_t\right], \qquad (33)$$

where π^e_t is given by equation (23) and K_t is defined in equation (29).

The solution procedure involves the use of the Hamilton-Jacobi-Bellman (HJB) equation. It provides the optimal government policy, that is the optimal path for planned inflation, given the private sector's strategy. Since the government's decision satisfies the HJB, it is time-consistent. The strategy is measurable with respect to the policymakers' information field, generated by the history of all the variables of the model, excluding the current value of the supply shock. The resulting equilibrium is a perfect Bayesian equilibrium.

Proposition

The equilibrium strategy of the government, $(\pi^P_t)^*$, obtained from the game outlined above is:

$$(\pi^P_t)^* = c \left[1 - \frac{e^{(\lambda+\rho)t}}{(\lambda+\rho)^2} \left[\bar{C} - \int_0^t \frac{e^{-(\lambda+\rho)s}}{K_s} ds \right] \right] - \frac{h}{(\lambda+\rho)} \tag{34}$$

$$\text{where} \quad \bar{C} = \lim_{t \to \infty} \int_0^t \frac{e^{-(\lambda+\rho)s}}{K_s} ds \ . \tag{35}$$

Proof

The solution is derived by postulating the following form for the value function:

$$V_t (q_t , \hat{h}_t) = e^{-(\lambda+\rho)t} W_t (q_t, \hat{h}_t) \ , \tag{36}$$

where W_t is a linear function of the state variables, q_t and $\hat{h}_t$, that is:

$$W_t = \mu_{0t} + \mu_{1t} q_t + \mu_{2t} \hat{h}_t \ . \tag{37}$$

The HJB equation is then a partial differential equation in V and results in being:

$$(\lambda+\rho)W_t - \dot{\mu}_{0t} - \dot{\mu}_{1t} q_t - \dot{\mu}_{2t} \hat{h}_t =$$

$$\max_{\pi^P_t} \ c\pi^P_t - c \left[cA_t - \frac{\hat{h}_t B_t}{(\lambda+\rho)} \right] - \frac{1}{2}(\pi^P_t)^2 + hq_t - \mu_{1t}\pi^P_t \tag{38}$$

$$- \mu_{2t} \frac{B_t}{(\lambda+\rho)K_t} \left[\pi^P_t - \left[cA_t - \frac{\hat{h}_t B_t}{(\lambda+\rho)} \right] \right] ;$$

from which the first order condition yields:

$$(\pi_t^p)^* = c - \mu_{1t} - \frac{\mu_{2t}B_t}{(\lambda+\rho)K_t} \ . \tag{39}$$

The next step consists of the determination of the undetermined coefficients of the value function, μ_{1t} and μ_{2t} , by equating the terms in q_t and in h_t respectively, on the left hand side and the right hand side of the HJB equation. This gives two deterministic differential equations, one in μ_{1t} and the other in μ_{2t}. The solutions to these equations, which are fully worked out in the Appendix, are, respectively:

$$\mu_{1t} = \frac{h}{(\lambda+\rho)} \ , \tag{40}$$

$$\mu_{2t} = \frac{e^{(\lambda+\rho)t}K_t c}{(\lambda+\rho)} \left[\bar{C} - \int_0^t \frac{B_s e^{-(\lambda+\rho)s}}{K_s} \ ds \right] \tag{41}$$

where

$$\bar{C} = \lim_{t \to \infty} \int_0^t \frac{B_s e^{-(\lambda+\rho)s}}{K_s} \ ds \ . \tag{42}$$

The coefficients μ_{1t} and μ_{2t} are then substituted in the optimal control, equation (39). The government's strategy results in being of the form initially postulated in equation (22), so that it is the rational expectations equilibrium solution, which will be complete after the determination of the coefficients A_t and B_t, which appear in the conjectural function. The solutions for A_t and B_t are shown below.

$$B_t = 1 \ ; \tag{43}$$

$$A_t = 1 - \frac{e^{(\lambda+\rho)t}B_t}{(\lambda+\rho)^2} \left[\bar{C} - \int_0^t \frac{e^{-(\lambda+\rho)s}B_s}{K_s} \ ds \right] = $$

$$1 - \frac{e^{(\lambda+\rho)t}}{(\lambda+\rho)^2} \left[\bar{C} - \int_0^t \frac{e^{-(\lambda+\rho)s}}{K_s} \ ds \right] < 1 \ . \tag{44}$$

The two equations above, substituted in equation (22) to give equation (39), complete the proof.

A_t and B_t can now also be substituted in the equations of the learning process. For convenience, the equilibrium strategies of the two players are rewritten below:

(i) the government's strategy:

$$(\pi_t^p)^* = c \left[1 - \frac{e^{(\lambda+\rho)t}}{(\lambda+\rho)^2} \left[\bar{C} - \int_0^t \frac{e^{-(\lambda+\rho)s}}{K_s} ds \right] \right] - \frac{h}{(\lambda+\rho)} ; \qquad (45)$$

(ii) the private sector's strategy:

$$(\pi_t^e)^* = c \left[1 - \frac{e^{(\lambda+\rho)t}}{(\lambda+\rho)^2} \left[\bar{C} - \int_0^t \frac{e^{-(\lambda+\rho)s}}{K_s} ds \right] \right] - \frac{\hat{h}_t}{(\lambda+\rho)} , \qquad (46)$$

where

$$\hat{h}_t = \frac{\frac{ht}{(\lambda+\rho)^2} + \int_0^t dz_s + \frac{\bar{h}}{\sigma_h^2}}{K_t} , \qquad (47)$$

$$K_t = \frac{t}{(\lambda+\rho)^2} + \frac{1}{\sigma_h^2} , \qquad (48)$$

and

$$\bar{C} = \lim_{t\to\infty} \int_0^t \frac{e^{-(\lambda+\rho)s}}{K_s} ds . \qquad (49)$$

Some observations on the government's optimal policy are now warranted. First of all, the coefficient, A_t, is less than one; this means that planned inflation is lower in the presence of asymmetric information than in the case of symmetric information. The informational advantage allows the government to sustain lower time-consistent inflation rates. The path of the time-consistent inflation rate for a country which switches from a flexible exchange rate regime, where PPP always holds, to a pegged exchange rate system, in the presence of asymmetric information, shows an initial decline which exceeds the long run reduction due to the change in the exchange rate regime. It is possible to label this an "overshooting" of the

inflation rate owing to the informational asymmetry, or a "honeymoon effect."

Immediately after the regime shift, i.e. at time 0, the value of A_0 is:

$$A_0 = 1 - \frac{\bar{C}}{(\lambda+\rho)^2} , \qquad (50)$$

and hence inflation equals:

$$\pi_0^p = c \left[1 - \frac{\bar{C}}{(\lambda+\rho)^2} \right] - \frac{h}{(\lambda+\rho)} . \qquad (51)$$

The above formula shows that the initial value of A_t is further from 1 the greater is σ^2_h , because the greater is σ^2_h , the larger $\bar{C}$. This means that the more the public is uncertain about h, the lower the time-consistent inflation rate, so that the overshooting effect is larger. The value of A_0 also depends on ρ and λ the more the government discounts the future payoffs and the more likely a realignment, the closer A_0 is to 1, i.e., the smaller the additional bonus in terms of reduced inflation. Given the initial value of π^p_t, inflation then increases towards the long run level prevailing under the new exchange rate regime, which is lower than that resulting under the free float system assumed here.

The second observation concerns B_t which is unity; this shows that the second term in the planned inflation formula, equation (45), is unchanged with respect to the symmetric information scenario (see equation (20)). This second element of planned inflation appears with the introduction of the pegged exchange rate regime and can be labelled the "discipline effect" of pegged exchange rate system. The finding that B_t equals one signifies that the discipline effect is not affected by the asymmetry in information.

Furthermore, there is another important consequence of the fact that B_t is unity, namely, the fact that policymakers do not manipulate the public's learning process, neither in the form of signalling their preferences (and hence speeding up the process of learning which would have required a decreasing sequence for the B_ts), nor in the form of slowing down the revision of inflationary expectations (which would have required an increasing sequence for the B_ts). It is worth pointing out that the government is subject to two types of pressures. On one hand, it would like to conceal its preferences in order to slow down the learning. On the other hand, it is eager to behave according to its preferences. In this case, the two forces are perfectly balanced. In other words, the government chooses not to manoeuvre the speed at which private agents glean information. The latter only depends on the discrepancy between actual inflation and its

estimated value, and is therefore greater in the early stages of the game and decrease as the degree of the public's uncertainty is reduced.

The public's expectations of inflation immediately after the regime switch equal:

$$\pi_0^e = c \left[1 - \frac{\overline{c}}{(\lambda+\rho)^2} \right] - \frac{\overline{h}}{(\lambda+\rho)} \quad . \tag{52}$$

It easy to see that the discrepancy between actual and expected inflation, at time 0, depends on the gap between the true value of h and the private agents' prior. The unconditional mean of the distribution from which h has been drawn is not explained in this model. Therefore, it is only possible to say that if the prior is higher than the true value of h, for example because it is based on the past history of high inflation, then the public will over-estimate inflation. Similarly, if the prior is lower than the actual h, because, for example, the private sector considers the regime shift as the beginning of a new game, inflation will result in being under-estimated. In expected terms, the public will continue, also after the initial moment after the regime shift, to overpredict the inflation rate, in the first case, and to underpredict in the second case. Although prediction errors disappear when the private sector has learnt the true h, during the learning phase, prediction errors have implications for output.

Negative inflationary surprises entail an output loss, whereas positive inflationary surprises boost income. Notice, however, that the path of planned inflation is independent of the prior. In this regard, it is worthwhile pointing out that, if the pegged exchange regime is adopted as an anti-inflationary strategy, while in a symmetric information scenario the reduction in the time-consistent inflation rate occurs with no output loss, in an incomplete information setup it is possible that, although the government's policy involves lower inflation, expected inflation remains high. If this occurs, the sacrifice ratio is greater than in the symmetric information framework.

The solution to the game when the public's uncertainty tends to zero, converges to the solution of the symmetric information case presented in Section II. This can be shown by computing the limiting value for the control equation (39) as time tends to infinity. In fact, since μ_{2t} is finite and K_t tends to infinity, as time tends to infinity, as shown in the Appendix, the third term on the right hand side tends to 0 and equation (39) reduces to equation (20).

IV. Concluding remarks

This paper analyses the consequences of a shift in the exchange rate regime, from a free float with PPP to a pegged exchange rate regime with realignments, on the actual and expected inflation rate. The main feature

of the model is the presence of uncertainty and asymmetric information concerning the government's preferences about competitiveness. In this framework, the public's learning and the government's optimal policy are analyzed.

The main findings of this paper are: (i) immediately after the regime shift, there is a "honeymoon effect" where inflation drops to a level that is lower than that which prevails once private agents have learnt the true value of the preference parameter. In other words, there is an "overshooting" of planned inflation, due to the authorities' information advantage which allows them to sustain lower time-consistent inflation rates. 1/ Hence, in the presence of asymmetric information, the advantage of tying one's hands is temporarily increased.

(ii) The reduction of actual inflation might be accompanied by output losses for certain values of the public's prior concerning the unknown preference parameter. (iii) The policymakers' optimal strategy does not involve either signalling or concealing of the government's preferences. The speed of learning is not affected by the authorities' policy. (iv) As time passes and private agents learn, inflation converges to the symmetric information equilibrium which is characterized by lower inflation compared to that of the initial free float regime. Hence, policymakers cannot conceal their preferences indefinitely.

These results, which take fully into account the strategic interactions between the government and the private sector, challenge the view that informational asymmetries and the public's learning could be responsible for the slow decline in inflation, after a shift to a pegged exchange rate regime. The public's inflationary expectations, however, have an important role to play also in the model developed here, since they determine, together with actual inflation, the level of output. In fact, while the government's strategy does not depend on the private sector's prior for the unknown preference parameter, the path of expected inflation does depend on it.

1/ The reduction in planned inflation entails also a reduction in actual inflation in expected terms.

The solution to the differential equation for μ_{1t}
The differential equation for μ_{1t} is:

$$(\lambda+\rho)\mu_{1t} - \dot{\mu}_{1t} = h \ . \tag{53}$$

The solution to this linear deterministic differential equation is straightforward:

$$\mu_{1t} = \frac{h}{(\lambda+\rho)} + \left[\mu_{10} - \frac{h}{(\lambda+\rho)}\right] e^{(\lambda+\rho)t} \tag{54}$$

and the only value for μ_{10} which ensures that μ_{1t} is always finite makes μ_{1t} time-invariant, i.e.:

$$\mu_{1t} = \frac{h}{(\lambda+\rho)} \ . \tag{55}$$

The solution to the differential equation for μ_{2t}
The differential equation for μ_{2t} is as follows:

$$\dot{\mu}_{2t} = \left[(\lambda+\rho) + \frac{B_t^2}{(\lambda+\rho)^2 K_t}\right] \mu_{2t} - \frac{cB_t}{(\lambda+\rho)} \ . \tag{56}$$

Recalling that

$$K_t = \frac{1}{(\lambda+\rho)^2} \int_0^t B_s^2 ds + \frac{1}{\sigma_h^2} \tag{57}$$

it is possible to see that:

$$\frac{B_t^2}{(\lambda+\rho)^2 K_t} = \frac{d \log K_t}{dt} \ . \tag{58}$$

It follows that the solution to equation (56) is:

$$\mu_{2t} = \frac{e^{(\lambda+\rho)t} K_t c}{(\lambda+\rho)} \left[\bar{C} - \int_0^t B_s \frac{e^{-(\lambda+\rho)s}}{K_s} ds \right] \tag{59}$$

where $\bar{C}$ is an arbitrary constant. The choice of $\bar{C}$ is made to ensure that μ_{2t} is finite at all times. There is a unique constant which satisfies this requirement and this is the following:

$$\bar{C} = \lim_{t \to \infty} \int_0^t B_s \frac{e^{-(\lambda+\rho)s}}{K_s} ds \; . \tag{60}$$

It is possible to show, using De L'Hopital rule, that the limiting value for μ_{2t} as time tends to infinity is:

$$\lim_{t \to \infty} \mu_{2t} = \frac{cB_t}{(\lambda+\rho)^2} \; . \tag{61}$$

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