IMF WORKING PAPER

WP/94/64

INTERNATIONAL MONETARY FUND

Research Department

Are the Unemployed Unemployable?

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June 1994

Abstract

This paper develops a matching model of the labor market under wage rigidity when hiring decisions are irreversible. There are two types of workers, the skilled and the unskilled. The model is used to analyze whether technological advances may have increased unemployment. It is shown that it is likely to be so if they are associated with an increase in the productivity and/or the supply of skilled workers relative to unskilled workers. These effects are stronger when hiring decisions are more irreversible.

JEL Classification Nos.
J64, J65, J68

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Summary

The problem of persistent and high unemployment is particularly acute in Europe and does not seem about to disappear. A common view is that the unemployed are "unemployable": they lack the skills that are demanded in the labor market because of technological advances.

This paper provides some theoretical foundations for this view and suggests that it is more valid if the labor market is rigid in the sense that hiring decisions are irreversible. A matching model in the spirit of Diamond (1982), Blanchard and Diamond (1990), and Pissarides (1990) is developed where real but not relative wages are rigid, and where it is costly or impossible to fire workers. This irreversibility, along with the assumption of decreasing returns and real wage rigidity, generates an arbitrage condition in terms of the relative unemployment rate of the unskilled: the unskilled must be relatively more abundant than the skilled in order for firms to want to hire them. The main results are the following: first, when the relative productivity of the skilled increases, the unskilled unemployment rate increases, the skilled unemployment rate decreases, and aggregate unemployment unambiguously increases if firing costs are high enough. Second, when the proportion of skilled workers in the labor force rises, the unemployment rate for both the skilled and the unskilled increases. As a result, aggregate unemployment will only decrease if the initial proportion of skilled workers is quite high. Third, these effects are weaker when firing costs go down.

The model implies that technological advances may generate unemployment through two channels: by increasing the productivity of the skilled relative to the unskilled, and by increasing the supply of skilled workers in the economy. What renders the unskilled "unemployable" is that the firm makes less profit from them and is stuck with them. Hiring an unskilled worker entails a capital loss which is greater when their relative productivity is lower and when the skilled are found more easily.

Another implication of this model is that the return to becoming skilled increases when there are more skilled workers.

Training programs for the unskilled have been advocated as a means to reducing unemployment. However, the model suggests that the effect may be perverse unless the productivity of the whole of the unskilled labor force is increased. This suggests that an improvement in the quality of the school system has a better chance of reducing unemployment than limited training programs.

I. Introduction

The problem of persistent and high unemployment is particularly acute in Europe and does not seem about to disappear. A common view about this problem is that the unemployed are "unemployable": they lack the skills which are demanded on the labor market. This view is often associated with the idea that technological advances have increased skill requirements to such a point that relatively uneducated people do not qualify and therefore end up being unemployed.

This paper develops the point that labor market institutions may play an important role in that process. The kind of labor market institution I focus on is firing restrictions. It is shown that such restrictions tend to depress the demand for unskilled workers relative to skilled workers because they create an "option value" of maintaining a vacancy idle until it is filled by a skilled worker rather than filling it now with an unskilled worker.

We show that this option value makes the arbitrage condition between hiring a skilled vs. an unskilled worker much less favorable to the latter. Furthermore, it is shown that the unskilled's unemployment rate rises when the proportion of skilled workers in the economy increases. When firing costs are large, this effect is much stronger. This is because the likelihood of filling a vacancy with a skilled worker increases, which has a strong positive effect on the option value. When this likelihood passes a certain threshold, the demand for unskilled labor falls to zero because the value of an empty vacancy in the skilled labor market is higher than the value of a job filled by an unskilled. In the absence of firing costs this cannot happen.

The existence of this "supply" effect is suggestive that the increase in the number of educated workers may have contributed to the unskilled's unemployment problem, particularly so in countries where firing costs are large. It is also shown that aggregate unemployment is likely to increase when there are more educated workers in the economy. The model therefore raises doubt about the desirability of training policies as a tool to reduce unemployment, and suggests that there may be increasing net returns to education if hiring decisions are irreversible.

The analysis relies on a matching model in the spirit of Diamond (1982), Blanchard and Diamond (1990) and Pissarides (1990) where real wages fail to adjust to clear the labor market, and where it is costly or impossible to fire workers.

The paper is organized as follows: the basic model, where firms cannot fire workers, is set up in section II. Section III studies the impact of relative productivity levels and of the skill composition of the labor force. Section IV considers what happens if firms can fire workers at a cost, thus being able to replace unskilled workers with skilled workers.

Section V analyzes the model's implications for the returns to education. Section VI contains concluding comments.

II. The Basic Model

The model is a continuous-time matching model in the fashion of Pissarides (1990, 1992) with two types of workers: high-skill workers (type 1) and low-skill workers (type 2). Each type of worker has a separate labor market and on any market i (i=1,2) the number of matches per unit of time is determined by a matching function $m(U_i,V_i)$ where U_i is the number of type i unemployed workers and V_i is the number of vacancies in this market. That is, we assume that vacancies must be directed at a specific type of skill ex-ante and that firms cannot post a "general" vacancy and decide which type of worker to hire in the course of the application process. This is a realistic feature provided the skills are differentiated enough: skill requirements are often associated with jobs, and jobs advertisement always specify what type of worker is needed, including degrees, work experience, etc.

The total number of jobs is held fixed and equal to K. K may alternatively be thought of as the capital stock or an index of aggregate activity. Each job may be held by either type of worker. Low-skill workers are less productive in any given job than high-skill workers.

The assumption that K is fixed is an extreme form of decreasing returns. Decreasing returns are important for our results because they generate a linkage between the two labor markets. In the case of a fixed number of jobs, it implies that each job held by a low-skill worker decreases the number of jobs available for high-skill workers by exactly one unit. With weaker forms of decreasing returns, it will decrease the productivity of high-skill workers.

Because K is fixed, for each vacant job firms have to decide whether they try to hire a low-skill worker or a high-skill worker. This will generate an arbitrage condition between the value of a vacancy in market 1 and the value of a vacancy in market 2.

I also assume that firms cannot post more than one vacancy per job. That is to say, the matching process is really defined in terms of jobs rather than vacancies. In Pissarides, what prevents the number of vacancies from becoming infinite is the fact that vacancies are costly so that their value eventually becomes negative as the labor market becomes tighter. Here vacancies are costless but they cannot exceed K. As a result the total number of vacancies is simply equal to the number of vacant jobs. 1/

Last, I assume a simple form of wage bargaining: the output of any worker is equally split between the firm and the worker. I assume that a skilled worker produces 2 units of good, and an unskilled 2 ρ. As a result firms make a profit per unit of time equal to 1 for each job held by a type 1 worker, and $\rho < 1$ for each job held by a type 2 worker. (This is the option taken in Pissarides (1992)).

Alternatively one can assume Nash Bargaining, which allows for wages reacting to labor market conditions. Intuitively this weakens the results (since part of higher unemployment is absorbed by lower wages), but do not make them disappear. This is indeed the case: the model's solution under continuous-time Nash-Bargaining with permanent renegotiation is derived in the Appendix. It turns out that the results are not qualitatively affected.

Bargaining implies that firms make less profit out of an unskilled worker than out of a skilled worker. Therefore, they would like to replace low-skill workers with high-skill workers. In this section, I assume that firing costs are high enough to prevent that; the opposite case is analysed in section IV.

Although I will write the equations of the model in a dynamic form, I will limit myself to a comparison of steady-states, the dynamic system having too high a dimension.

1. The arbitrage condition for vacancies

Since ρ is less than one, firms will prefer that a job be held by a high-skill worker rather than a low-skill worker. As a result, if their chances of hiring a type one worker were the same as those of hiring a type two worker, they will prefer to post vacancies in market 1; but this, of course, tightens market 1 and slackens market 2. As a result type 2 vacancies are filled more quickly than type 1 vacancies. This raises the value of type 2 vacancies and decreases the value of type 1 vacancies. The process will continue until the relative tightness of the two markets is such that firms are indifferent between either type of vacancies. Hence, because of wage rigidity, the arbitrage between the two types of workers is realised through arrival rates instead of wages.

1/ This implies a positive relationship between vacancies and unemployment; therefore, the shifts in u and v considered in this paper correspond to shifts of the Beveridge curve, not shifts along the Beveridge curve. Shifts along the beveridge curve are generated by changes in the index of aggregate labor demand, K. For evidence and discussions about whether the Beveridge curve has shifted, see Bean (1992), Layard and Nickell (1991), Blanchard and Diamond (1990b).

More formally, let λ_{1t} be the probability per unit of time of getting a type 1-applicant for a type 1-vacancy, and λ_{2t} the arrival rate of type 2-applicants. The value to the firm of a type 1 vacancy is therefore:

$$VAC_{1t} = (1 - rdt)\,[\lambda_{1t}dt \,.\, J_{1t+dt} + (1-\lambda_{1t}dt)\,VAC_{1t+dt}] \tag{1}$$

Where dt is a small time interval, r is the discount rate, and J_{1t} is the PDV to the firm of a job held by a type 1-worker.

This equation can be rewritten:

$$0 = -(r+\lambda_{1t})\,VAC_{1t} + \lambda_{1t}J_{1t} + dVAC_{1t}/dt \tag{2}$$

Similarly, one has, for market 2:

$$0 = -(r+\lambda_{2t})\,VAC_{2t} + \lambda_{2t}J_{2t} + dVAC_{2t}/dt \tag{3}$$

Provided type 2 workers are not all unemployed in equilibrium, firms must be indifferent between either type of vacancy. 1/ This implies:

$$VAC_{1t} = VAC_{2t} = VAC_t \tag{4}$$

for all t. This may be rewritten:

$$\lambda_1[J_1 - VAC] = \lambda_2[J_2 - VAC] \tag{5}$$

This states that the arrival rate of unskilled workers must exceed that of skilled workers by the ratio of the capital gain from hiring a skilled worker over the capital gain from hiring an unskilled worker.

I then assume that there is an exogenous quit rate equal to s. The value to the firm of a job held by a skilled worker is thus defined by:

$$0 = 1 - (r+s)\,J_1 + dJ_1/dt + sVAC \tag{6}$$

1/ The corner case is examined below.

The similar formula for type 2-workers is:

$$0 = \rho - (r+s) J_2 + dJ_2 / dt + sVAC \tag{7}$$

In steady state, eliminating J_1 between (6) and (2) yields:

$$VAC = \lambda_1 / [r(r+\lambda_1+s)] \tag{8}$$

and symmetrically for unskilled workers:

$$VAC = \lambda_2\rho / [r(r+\lambda_2+s)] \tag{9}$$

Confronting (8) and (9) yields the arbitrage condition in terms of the arrival rates:

$$\lambda_1 / (r+\lambda_1+s) = \lambda_2\rho / (r+\lambda_2+s) \tag{10}$$

Equation (10) is the first fundamental equation of the model. It may be rewritten:

$$\lambda_2 = \lambda_1 / (\rho - \lambda_1(1-\rho) / (r+s)) \tag{10'}$$

It defines a curve AA in the (λ_1, λ_2) locus (Fig.1). AA is convex, upward-sloping. It goes through the origin. Its slope is greater than 1 and it is always above the 45° line. This reflects the fact that since the firms make more money on skilled workers, they must lose less time in finding unskilled workers if these are demanded. Also, AA has a vertical asymptote at:

$$\lambda_1 = \lambda_1^* = \rho(r+s) / (1-\rho) \tag{11}$$

This means that if λ_1 is greater than λ_1^*, a type 1 vacancy has a greater value than a type 2 job. As a result no level of λ_2 can match VAC_1 and VAC_2, so that the arbitrage condition (4) no longer holds and has to be replaced by an inequality: $VAC_1 \geq VAC_2$. In this zone all type 2 workers are unemployed.

2. The flow equilibrium locus

I now close the model and derive the second relation between λ_1 and λ_2. Let U_i be the number of unemployed workers of group i, and V_i the number of vacancies for this group. I assume that the number of matches per unit of time in market i is described by a constant-returns, concave matching function $m(V_i, U_i)$. 1/ In market i the application rate λ_i is the number of matches divided by the stock of vacancies, that is:

$$\lambda_i = m(V_i, U_i) / V_i = m(1, U_i / V_i) \tag{12}$$

where I have used the constant returns assumption. Note that the matching function is assumed to be the same in both markets: there is no a priori reason to believe that there is a systematic correlation between skill levels and the efficiency of the matching process.

Equation (12) may be inverted as:

$$U_i = V_i h(\lambda_i) \tag{13}$$

where $h = m(1,.)^{-1}$ is a convex, increasing function. For each type of workers, the change in employment is just hirings minus quits, that is:

$$dL_i / dt = -sL_i + \lambda_i V_i = -sL_i + m(V_i, U_i) \tag{14}$$

The total number of vacancies must equal the number of available jobs:

$$V_1 + V_2 = K - L_1 - L_2 \tag{15}$$

Where L_i is the number of type i workers who are employed. I normalize the total labor force to 1 and assume there is a proportion x of skilled workers. Hence:

$$U_1 = x - L_1 \tag{16}$$

1/ On matching functions, see Pissarides (1990).

$$U_2 = 1 - x - L_2 \tag{17}$$

Using (13),(14),(16) and (17) it is possible to solve for U_i, V_i and L_i in steady state. If u_i and v_i are the unemployment and vacancy rates for type i (relative to labor supply), one gets:

$$u_i = h(\lambda_i) / [\lambda_i / s + h(\lambda_i)] \tag{18}$$

$$v_i = 1 / [\lambda_i / s + h(\lambda_i)] \tag{19}$$

Clearly, u_i is increasing and v_i decreasing in λ_i : more unemployed people mean larger arrival rate given vacancies, and larger arrival rates mean less vacancies given the employment level. From this one can compute the aggregate and individual unemployment and vacancy rates:

$$u = x u_1 + (1 - x)\, u_2 \tag{20}$$

and:

$$v = x v_1 + (1 - x)\, v_2 \tag{21}$$

It is then possible to express (15) in terms of λ_1 and λ_2 :

$$x \frac{1 + \lambda_1 / s}{\lambda_1 / s + h(\lambda_1)} + (1 - x) \frac{1 + \lambda_2 / s}{\lambda_2 / s + h(\lambda_2)} = K \tag{22}$$

This defines a locus BB which we will call the *flow equilibrium locus* (See Figure 1). Equation (22) may be rewritten as follows:

$$x \, . \, w(\lambda_1) + (1 - x) \, . \, w(\lambda_2) = K$$

where $w(\lambda_i) = (1 + \lambda_i / s) / (\lambda_i / s + h(\lambda_i)) = 1 - u_i + v_i = w_i$ is the steady state *job rate* for type i workers, i.e., the total number of jobs available for this type, whether vacant or not, divided by the supply of workers of this type. (22) obviously states that the total number of jobs must be equal to K.

BB is downward sloping in the (λ_1, λ_2) plane and admits two asymptotes at $\lambda_1 = \overline{\lambda}_1$ and $\lambda_2 = \overline{\lambda}_2$ respectively. The $\overline{\lambda}_i$'s are defined by:

$$x\frac{1+\bar{\lambda}_1/s}{\lambda_1/s+h(\lambda_1)}=K \quad and \quad (1-x)\frac{1+\bar{\lambda}_2/s}{\lambda_2/s+h(\lambda_2)}=K$$

In the case depicted figure 1, $\bar{\lambda}_1<\lambda_1^*$ and there is a unique equilibrium determined by the intersection of AA and BB. Since AA is above the 45° line, the equilibrium satisfies $\lambda_1<\lambda_2$. Hence the unemployment rate for the unskilled is higher than for the skilled. The converse is true for vacancy rates.

Whenever $\lambda_1^*<\bar{\lambda}_1$, AA and BB do not cross. The unemployment rate for type 2 is thus equal to 1, so that $\lambda_2=+\infty$ and $\lambda_1=\bar{\lambda}_1$. The unskilled are then literally unemployable.

In the next section, I study the comparative statics with respect to ρ and x.

III. Impact of the Productivity Differential and the Skill Composition of the Population

The two main comparative statics exercises we are interested in are with respect to ρ and x. These two variables capture the transmission of technical progress to unemployment via the heterogeneity of skills and the composition of the workforce. In conformity with the debate exposed in the introduction, I will work under the hypothesis that technical advances have made skilled labour more valuable relative to unskilled labor, and has consequently increased the relative supply of skilled labor. I am therefore interested in the impact of a decline in ρ and a rise in x.

1. Impact of a change in ρ

Concerning the impact of a change in ρ, it should first be noted that ρ captures the relative productivity differential of unskilled labor. Multiplying the profits from both types of labor by the same constant would leave (10) unaffected. 1/ Therefore, an increase in the productivity of skilled relative to unskilled labor implies a decline in ρ, even though the absolute productivity of type 2 workers may rise.

The impact of a decline in ρ is clear from equation (10) and figure 2: the AA locus shifts upwards. Firms require a greater probability of finding an unskilled worker in order to compensate for the lower profits they make relative to type 1 workers. Consequently λ_1 declines and λ_2 rises. As a result the unskilled unemployment rate rises and the skilled unemployment rate goes down (equation (18)). What happens to the aggregate unemployment rate? Interestingly, it is possible to show that it goes up unambiguously:

1/ This would also be true if there was a vacancy cost proportional to productivity.

PROPOSITION 1 - $\partial u/\partial \rho < 0$

PROOF : See Appendix

Proposition 1 tells us that in some sense there should be a correlation between workforce heterogeneity and unemployment: the more the unskilled are remote from the skilled in terms of productivity, the higher the unemployment rate.

A close inspection of the proof reveals that proposition 1 is essentially a "mismatch" result: an decrease in ρ increases unemployment in the slack labor market, while it reduces it in the tight labor market. Because of congestion effects in the search process, the Beveridge curve is convex. As a result a given increase (resp. fall) in labor demand reduces (resp. raises) unemployment by a lower (resp. larger) amount when unemployment is initially high (resp. low). Therefore, the increase in u_2 generated by the fall in ρ will typically weigh more in aggregate unemployment than the concomitant fall in u_1.

2. Impact of a change in x

The x variable is of interest because it allows us to analyze how the skill composition of the population affects unemployment; in particular whether the increase in the number of people with higher education has had a positive or adverse effect on unemployment. A change in x affects unemployment through a shift in the flow equilibrium locus; more precisely:

PROPOSITION 2 - When x increases, the BB locus rotates clockwise around its intersection with the 45-degree line. As a result both λ_1 and λ_2 increase (Figure 3).

PROOF - At $\lambda_1 = \lambda_2$ the LHS of (22) is independent of x. Therefore the new BB locus has the same intersection with the 45 degree line. Consider a point of BB above the 45 degree line, i.e., such that $\lambda_2 > \lambda_1$. Since $(1+\lambda/s)/(\lambda/s+h(\lambda))$ decreases with λ, it is lower for λ_2 than for λ_1. As a result an increase in x tends to increase the LHS of (22) at this point. To compensate and maintain it equal to K, λ_2 has to increase for the same value of λ_1. Therefore BB is above its previous location when x increases. The converse holds in the zone where $\lambda_2 < \lambda_1$.

Since unemployment rates are increasing functions of the λ's, proposition 2 tells us that the unemployment rate will increase for *both* types of workers. That it increases for skilled workers is not surprising, since they are relatively more abundant. But because of the arbitrage condition, it will also increase for unskilled workers: because employers have more chances to hire a skilled worker, they will want to hire unskilled workers only if their market slackens enough, so that their unemployment rate indeed rises.

What does this imply for the aggregate unemployment rate? Since the unemployment rate for the skilled is lower than for the unskilled, the two negative effects on skill-specific unemployment rates are balanced by a positive composition effect: there are more people in the low unemployment portion of the population. Therefore the aggregate unemployment rate may either rise or decline when x increases. It is possible to show that both things must happen for $x \in [0,1]$. The argument is simple: since, as argued above, only relative productivity matters, the model with x=0 is essentially the same as with x=1 (or ρ=1). Therefore, aggregate unemployment is the same for x=0 and x=1. By continuity, it must go both up and down as x moves from 0 to 1. Numerical simulations with a Cobb-Douglas matching function indicate that it first rises and then declines; and that it declines over a smaller range than it rises (an example is given in figure 4). 1/

This again suggests that unemployment is higher when there is more heterogeneity in the workforce, i.e., for intermediate rather than extreme values of x. Given that one would rather believe that x is relatively low, part of the rise in unemployment may be ascribed to an increase in the proportion of workers with higher education.

Before concluding this section, one should also note that the "unemployability" of the unskilled becomes a more severe problem as x increases, since λ_2 / λ_1 increases as one moves along the AA locus. In fact, an increase in x may lead the economy to the "corner solution" zone where u_2=1.

IV. The Flexible Regime and the Role of Irreversibility

Until now, I have assumed that it was not profitable for firms to post a type-1 vacancy for jobs held by type 2-workers and replace them when an applicant comes. In this section, I assume it is possible to do so provided the firms entails a "firing cost" F when it gets rid of the unskilled worker. If F is not too high, then it will indeed be optimal for firms to post vacancies for jobs held by type 2-workers and replace them with type 1 workers. This will be referred to as the "flexible regime".

This exercise allows to analyze how the option value of leaving a vacancy idle until a skilled worker is found affects the arbitrage condition. As will be clear, much of the steepness and convexity of the arbitrage condition is due to the option value. In the absence of firing costs, not only the unskilled unemployment rate would be lower but the

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1/ An element of intuition for this hump-shaped response is as follows: when x is small, a given increase in x has large impacts on the skilled's arrival rate λ_1 because it is large in relative terms. Because AA's slope is greater than 1, this must be matched by a large effect on λ_2. Since most of the workers are unskilled, this generates a large effect. The argument is reversed for x large.

adverse effect of an increase in the proportion of skilled workers would be much smaller.

How is the above analysis affected if one assumes that firms have the option of replacing unskilled workers with skilled workers? The value of an unskilled job must now embody this possibility. As a result, equation (7) is replaced with:

$$0=\rho-(r+s)\,J_{2t}+sVAC_t+dJ_{2t}/dt+Max[0,\lambda_{1t}(J_{1t}-F-J_{2t})] \tag{23}$$

where the second term in the brackets is the value of posting a type 1 vacancy for this job and replacing the type 2-worker when an applicant is found.

(23) implies that the firm will do so if and only if:

$$J_{1t}-F-J_{2t}\geq 0 \tag{24}$$

If (24) is satisfied, the economy is in the flexible regime. In steady state, (2) and (6) determine J_1 and VAC as a function of λ_1, with no change with respect to section II :

$$VAC=\lambda_1/[r\,(r+\lambda_1+s)] \tag{8}$$

implying :

$$J_1=(r+\lambda_1)/(r\,(r+\lambda_1+s)) \tag{25}$$

(3) and (23) now determine VAC and J_2 as a function of λ_2, λ_1 and J_1:

$$VAC=[\lambda_2\rho+\lambda_1\lambda_2(J_1-F)]/[(r+\lambda_2)(r+\lambda_1)+rs] \tag{26}$$

$$J_2=[\rho+sVAC+\lambda_1(J_1-F)]/(r+s+\lambda_1) \tag{27}$$

Eliminating (VAC) and (J_1) between (8),(25) and (26) yields the new arbitrage condition :

$$\lambda_2=\lambda_1/(\rho-F\lambda_1) \tag{28}$$

Solving for J_2, condition (24) may be rewritten :

$$F < (1-\rho)/(r+s) \tag{29}$$

(29) states that the economy will be in the flexible regime if the firing cost is less than the capital gain from replacing an unskilled worker with a skilled worker, which is equal to the PDV of the profit differential, $(1-\rho)/(r+s)$. Therefore, the flexible regime is more likely whenever (i) Firing costs are lower, (ii) ρ is lower (iii) interest rates are lower, and (iv) quits are lower.

Note that if one replaces F with $(1-\rho)/(r+s)$ in (28), one gets exactly (10'): the discounted loss from not being able to replace a type 2 worker with a type 1 worker plays, in the rigid regime, the same role as the firing cost in the flexible regime. Since (29) has to hold, in the flexible regime the AA curve (defined by (28)) is flatter than and below its rigid counterpart (defined by (10')); and its asymptote is for a larger value of λ_1; in other words, firms are more willing to hire type 2 workers because of the option value of replacing them - hence, they require a lower arrival rate to post type 2-vacancies.

If F is equal to 0, equation (28) collapses to $\lambda_2 = \lambda_1/\rho$. The arbitrage condition would then be linear and would not have an asymptote. The unskilled's unemployment problem is therefore made much more severe by firing costs. 1/ Figure 5 depicts the arbitrage condition for several values of the firing cost, including 0. The option value's contribution to that condition is measured by the vertical distance between the condition and the $\lambda_2 = \lambda_1/\rho$ line. It is steeply increasing in λ_1 and F.

V. Implications for the Returns to Education

Whenever AA is steep enough, an increase in the proportion of skilled workers will have a large impact on the unskilled's unemployment rate. An interesting implication is that this may increase the net returns to becoming skilled. The model therefore suggests that there may be increasing returns to education, more so when there is more irreversibility.

Assume for example that quits are into retirement, which has a zero present discounted value, and that this outflow is matched by an inflow of new entrants into the labor force who become unemployed before getting a job. Let $g(\lambda_i)$ be an unemployed's flow probability of getting a job in

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1/ Note that the flow equilibrium locus is also different in the flexible regime. In the CEPR working paper version of this paper (Saint-Paul (1992)), it is shown that it is qualitatively similar to the one derived in the previous section.

market i. Formally one has $g(\lambda_i) = m(1/h(\lambda_i),1)$, where h is defined in (13). Clearly one has $g' < 0$. Given that the skilled earn 1 and the unskilled earn ρ, the present discounted value of entering the labor force in market 1 is :

$$W_1 = \frac{g(\lambda_1)}{(r+s)(r+g(\lambda_1))} \tag{30}$$

Similarly, the PDV of entering the labor force in market 2 is :

$$W_2 = \frac{g(\lambda_2)\rho}{(r+s)(r+g(\lambda_2))} \tag{31}$$

When x increases, λ_2 and λ_1 increase as the economy moves along AA. This will increase the returns to education if $dW_1 - dW_2 > 0$. Differentiating (30)-(31) shows that this will be the case if:

$$-\rho g'(\lambda_2)(r+g(\lambda_1))^2 d\lambda_2/d\lambda_1 + g'(\lambda_1)(r+g(\lambda_2))^2 > 0 \tag{32}$$

Given that g' is negative, (32) is more likely to hold if $d\lambda_2/d\lambda_1$ is large, i.e if AA is steep. Consider for example the case where $m(u,v) = (uv)^{0.5}$. Then one has $g(\lambda) = 1/\lambda$. In the absence of firing costs, one has $d\lambda_2/d\lambda_1 = 1/\rho$. Straightforward computations show that the above inequality is then equivalent to $(1+r\lambda_1)^2 > (1+r\lambda_2)^2$, which is never true since $\lambda_2 > \lambda_1$. Consider now the case with a firing cost equal to F. Then differentiating (28), plugging it into (32), and substituting for λ_2 yields:

$$\rho^2(1+r\lambda_1)^2 > (\rho+(r-F)\lambda_1)^2 \tag{33}$$

which, given that $\lambda_1 < \rho/F$, is equivalent to $F > r(1-\rho)$.

Therefore, *there exists a threshold value of the firing cost above which there will be increasing returns to education.*

VI. Conclusion

The main implication of the model is that if labor market rigidities are important, advances in productivity may have adverse effects on unemployment. First, if technological change is such that it increases the productivity of the skilled more than that of the unskilled, the unemployment of the latter, and often aggregate unemployment, are likely to rise. Second, and even more strikingly, an increase in the proportion of skilled workers among the labor force will increase the unemployment rate for both types of workers and, over a large range, will also increase aggregate unemployment. This model is therefore an element of explanation for the rise in European unemployment.

The model could be extended in several directions. First, the total number of jobs could be endogenized, for example through the dynamics of capital accumulation under installation costs. Second, the decision of getting educated - hence x - could also be endogenized. This paper shows that in a world with real rigidities in the labor market, the net return to education may well increase with the supply of skilled workers. Endogenizing x could therefore generate multiple equilibria.

The model also sheds light on the effect of training programs on unemployment. Since the unemployment rate is higher in low skill categories, training programs have been much advocated as a means to reduce unemployment. 1/ The model suggests that the effect of these programs depend on how they are implemented, and may indeed be quite perverse. If a training program works in such a way that it turns a fraction of the unskilled population into skilled workers, i.e., if it increases x, it will make the economy move up the AA curve: the unemployment rate will increase both for the skilled and the unskilled, and the aggregate unemployment rate may well increase; it is true that those who benefited from the program will on average have lower unemployment rates; but the rest of the labor force will lose. 2/ If on the contrary, the productivity of the *whole* of the unskilled labor force is increased (i.e., ρ increases), then their unemployment rate will go down, and there are good chances that aggregate unemployment will also go down. This suggests that an improvement in the quality of the schooling system has better chances to reduce unemployment than the limited training programs that are often implemented.

1/ Their popularity among politicians may also stem from the fact that they are a secure way to reduce the unemployment *statistics* before an election.

2/ Doubts about the efficiency of active labor market policies as tools to reduce unemployment are expressed in Calmfors (1993).

Proof of proposition 1

We have to prove that $du/d\rho > 0$. The aggregate unemployment rate is:

$$u = x . h(\lambda_1) / [\lambda_1/s + h(\lambda_1)] + (1-x) . h(\lambda_2) / [\lambda_2/s + h(\lambda_2)] \tag{A1}$$

How does u vary when ρ changes, i.e., when one moves along the BB curve? Differentiating (A1) one gets:

$$du = x . \mu(\lambda_1) d\lambda_1 + (1-x) . \mu(\lambda_2) d\lambda_2 \tag{A2}$$

where μ is defined by:

$$\mu(\lambda) = \frac{(\lambda h'(\lambda) - h(\lambda)) / s}{(\lambda/s + h(\lambda))^2}$$

Differentiating the equation for BB yields a relationship between $d\lambda_1$ and $d\lambda_2$:

$$0 = x . \varphi(\lambda_1) d\lambda_1 + (1-x) . \varphi(\lambda_2) d\lambda_2 \tag{A3}$$

where φ is defined by:

$$\varphi(\lambda) = \frac{(h(\lambda) - \lambda h'(\lambda)) / s - (h'(\lambda) + 1/s)}{(\lambda/s + h(\lambda))^2}$$

Plugging (A3) into (A2) one sees that

$$du = x . \mu(\lambda_1) . [1 - \mu(\lambda_2) . \varphi(\lambda_1) / (\mu(\lambda_1) \varphi(\lambda_2))] . d\lambda_1 \tag{A4}$$

Because of the convexity of h, $\mu(.)>0$. Since $d\lambda_1 /d\rho > 0$, $du/d\rho$ has the sign of the term in brackets. It will be negative if:

$$\varphi(\lambda_1) / \varphi(\lambda_2) > \mu(\lambda_1) / \mu(\lambda_2)$$

This is equivalent to:

$$\frac{\lambda_1 h'(\lambda_1) - h(\lambda_1) + sh'(\lambda_1) + 1}{\lambda_2 h'(\lambda_2) - h(\lambda_2) + sh'(\lambda_2) + 1} > \frac{\lambda_1 h'(\lambda_1) - h(\lambda_1)}{\lambda_2 h'(\lambda_2) - h(\lambda_2)}$$

or, rearranging:

$$\frac{\lambda_2 h'(\lambda_2) - h(\lambda_2)}{sh'(\lambda_2) + 1} > \frac{\lambda_1 h'(\lambda_1) - h(\lambda_1)}{sh'(\lambda_1) + 1} \tag{A5}$$

Differentiating this functional form with respect to λ, one sees that it is increasing. Therefore, (A5) is always true since $\lambda_2 > \lambda_1$. This establishes the claim that $du/d\rho < 0$. Q.E.D

Derivation of the results under Nash Bargaining

I now show that the results are qualitatively unaffected when wages are determined by Nash bargaining rather than the split-of-the-pie rule. Specifically I show that the AA locus has the same shape in the (λ_1, λ_2) plane. Clearly BB is unaffected by wage formation.

Let w_i be the bargained wage for type i worker. Then (2) and (3) are unaffected, but (6) and (7) become:

$$0 = 2 - w_1 - (r+s)\, J_1 + dJ_1/dt + sVAC \tag{6'}$$

$$0 = 2\rho - w_2 - (r+s)\, J_2 + dJ_2/dt + sVAC \tag{7'}$$

Let PV_i (resp. PU_i) the value of being employed (resp. unemployed) for type i. Suppose the unemployed of type i earn ωw_i where ω is the replacement ratio, set by law. Then the Bellman equations for PU and PV are:

$$0 = \omega w_i - (r+\mu_i) PU_i + \mu_i PV_i + dPU_i/dt \tag{A6}$$

$$0 = w_i - (r+s)\, PV_i + sPU_i + dPV_i/dt \tag{A7}$$

In (A6) μ_i is the flow probability of finding a job for an unemployed of type i. One has:

$$\mu_i = m(v_i, u_i) / u_i = m(1/h(\lambda_i), 1) = \lambda_i / h(\lambda_i) = g(\lambda_i)$$

Clearly, $g' < 0$.

The employer's threat point is the value of a vacancy VAC, while the employed's threat point is the value of being unemployed PU_i. Consequently, wage formation is determined by maximization at each instant of time of the following Nash product:

$$\varphi\, Log(J_i - VAC) + (1-\varphi)\, Log(PV_i - PU_i) \tag{A8}$$

Where maximization takes place with respect to w_i. The first-order conditions are given by:

$$\varphi / (J_i - VAC) = (1-\varphi) \,/\, (PV_i - PU_i) \tag{A9}$$

Now, subtracting (2) or (3) from (6') or (7') and (A6) from (A7) and plugging the results on both sides of (A9) allows to solve for w_1 and w_2 in steady state. This yields:

$$w_1 = \frac{2\,(1-\varphi)\,(r+s+\mu_1)}{\varphi\,(1-\omega)\,(r+s+\lambda_1) + (1-\varphi)\,(r+s+\mu_1)}$$

$$w_2 = \frac{2\rho\,(1-\varphi)\,(r+s+\mu_2)}{\varphi\,(1-\omega)\,(r+s+\lambda_2) + (1-\varphi)\,(r+s+\mu_2)}$$

It is then possible to substitute these expressions into (6') and (7') and to compute the equilibrium values of J_i. Next, plugging them into (2),(3), and (4) yields the new equation for AA:

$$\frac{\lambda_1}{\varphi(1-\omega)\,(r+s+\lambda_1) + (1-\varphi)\,(r+s+\mu_1)} = \frac{\rho\lambda_2}{\varphi(1-\omega)\,(r+s+\lambda_2) + (1-\varphi)\,(r+s+\mu_2)} \tag{A10}$$

Clearly (A10) is identical to (10) if $\varphi=1$. It can be rewritten:

$$\psi(\lambda_1) = \rho\psi(\lambda_2)$$

Where $\psi(\lambda)=\lambda/(\varphi(1-\omega)(r+s+\lambda)+(1-\varphi)(r+s+g(\lambda)))$. ψ is strictly increasing and bounded from above. This in turns implies that (AA) has the same qualitative properties as in the text, namely that (i) it is above the 45½ line, (ii) it is increasing, and (iii) it has a vertical asymptote at λ_1^* such that $\psi(\lambda_1^*)=\rho.\lim_{\lambda\to\infty}\psi(\lambda)$.

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