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Banks' Precautionary Capital and Credit Crunches

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Banks' Precautionary Capital and Persistent Credit Crunches

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Abstract

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Periods of banking distress are often followed by sizable and long-lasting contractions in bank credit. They may be explained by a declined demand by financially impaired borrowers (the conventional financial accelerator) or by lower supply by capital-constrained banks, a “*credit crunch*”. This paper develops a bank model to study credit crunches and their real effects. In this model, banks maintain a precautionary level of capital that serves as a smoothing mechanism to avert disruptions in the supply of credit when hit by small shocks. However, for larger shocks, highly persistent credit crunches may arise even when the impulse is a one time, non-serially correlated event. From a policy perspective, the model justifies the use of public funds to recapitalize banks following a significant deterioration in their capital position.

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I. INTRODUCTION

The financial turmoil which originated with the collapse of the subprime mortgage market² in the US and spread through the globe has claimed more than US\$ 600 bn. in losses to financial firms and a significant contraction in bank lending standards. More recently, the US Congress passed a bailout package to alleviate capital-constrained financial institutions in a market intervention not seen since Franklin Roosevelt's New Deal, aimed at avoiding the potentially catastrophic consequences of a severe and protracted credit crunch. This paper contributes with a model to study credit crunches, which can be used as a laboratory to explore policy options. It belongs to the growing literature on the linkages between real and financial factors.

The link between financial factors and the real economy relies on the breakdown of the Modigliani and Miller (1958) theorem.³ By now, there is significant empirical evidence suggesting the failure of the Modigliani and Miller (1958) theorem—see for instance Bernanke, Gertler, and Gilchrist (1996)—and substantial theoretical work modeling the dynamic interaction between borrowers' access to credit and the value of collateral, to show that credit market imperfections amplify and propagate shocks to the economy. Important contributions along these lines include Bernanke and Gertler (1989, 1990), Kiyotaki and Moore (1997), Gertler (1992), Greenwald and Stiglitz (1993), Fuerst (1995), Carlstrom and Fuerst (1997, 2001), and Bernanke, Gertler, and Gilchrist (1999);⁴ Chari, Kehoe, and McGrattan (2006) argue that such frictions are not important once time-variation in other factors such as efficiency and labor wedges are taken into account. However, they potentially understate the importance of credit market frictions by ignoring their interactions with other efficiency wedges, as suggested by Christiano and Davis (2006). Levin, Natalucci, and Zakrajsek (2004) estimate the magnitude and cyclical behavior of financial frictions using publicly traded debt in a sample of U.S. firms and find that they are statistically significant and economically important.

There are also a number of empirical studies providing evidence in rejection of the Modigliani and Miller (1958) theorem in the context of banks. Bernanke and Lown (1991), Peek and Rosengren (1994, 1995), Hancock and Wilcox (1994), Kashyap and Stein (1995, 2000) are examples of studies showing that banks' lending decisions depend on their balance sheet structure. Moreover, Bernanke (1983), Dell'Ariccia, Detragiache, and Rajan (2005), Gibson (1995, 1997), Klein, Peek, and Rosengren (2002), Peek, Rosengren, and Tootell (2003), Peek and Rosengren (1997, 1999, 2000) and others have provided evidence

² See Gorton (2008) for a description of how it all started.

³ In a world of perfect capital markets, the Modigliani and Miller (1958) theorem implies that the capital structure of firms is uninformative for real economic decisions. However, this conclusion may be invalid when information about the default risk of borrowers cannot be costlessly acquired by lenders (Blinder and Stiglitz (1983)).

⁴ For an open economy extension of this literature see Aghion, Bacchetta, and Banerjee (2004), Caballero and Krishnamurthy (2001), Paasche (2001), Burnside, Eichenbaum, and Rebelo (2000), Mendoza (2004) and Gertler, Gilchrist, and Natalucci (2001).

suggesting the detrimental economic effects of financial distress in the banking industry. More recently, Adrian and Shin (2008a and 2008b) argue that marked-to-market leverage of financial firms is strongly procyclical and constitutes an important amplification mechanism during the ongoing crisis.

The key contribution of this paper is the development of a tractable framework to analyze banks' intertemporal decisions in a world of imperfect capital markets, in line with existing empirical work. Two of its key advantages over existing bank models are its infinite-horizon feature and non-linear solution. Models that include a bank, such as Bernanke and Gertler (1987), Holmstrom and Tirole (1997), Chen (2001), Meh and Moran (2004), and Christiano, Motto, and Rostagno (2004) have been developed in general equilibrium framework. However, in the cases where they have a dynamic nature, the solution arises from linearized versions of the first order conditions, eliminating thus important aspects of banks' behavior such as their precautionary motive. Other models such as Stein (1998), Diamond and Dybvig (1983), and Diamond and Rajan (2001, 2003a, b) offer important insights about equilibrium outcomes; however, their static or short-horizon nature—as it is also the case for Holmstrom and Tirole (1997)—do not permit exploring the dynamic adjustment towards equilibrium. Models closely related to the one presented in this paper are Van Den Heuvel (2002), which examines the role of regulatory constraints on bank behavior, and Geiregat (2001), which models the bank in a social planner setup.

The model developed in this paper has two key set of players: borrowers and a monopolistic bank. The borrower-bank relationship is modeled as a risky debt contract subject to information problems. In order to focus the attention entirely on the bank's optimal decisions, borrowers' creditworthiness is constant over time—that is, the Bernanke, Gertler, and Gilchrist (1999)'s financial accelerator is shut down. The bank faces constraints in raising funds, as a result of credit market imperfections, implying that its funding costs increase when solvency declines. This setup generates an optimal financial structure and permits examining how the bank adjusts to deviations from it. For instance, when a negative shock deteriorates bank capital, it is optimal for the bank to reduce dividends in order to restore solvency to its optimal level. If such strategy is not enough to restore equilibrium, the bank reduces lending, triggering a credit crunch. This credit crunch can be quite persistent because the bank restores solvency only gradually through retained earnings. This result is crucial because the persistent credit crunch arises even when the original impulse is a transitory, one-period i.e.—interest rate or productivity—shock. Another interesting feature of the model is that the bank exhibits a precautionary motive: when credit risk increases, the optimal level of capital—and hence solvency—is higher.

The results obtained from numerical simulations of the model suggest that the financial health of the banking system may be a significant contributor to the propagation of economic shocks, especially negative ones. Banks' precautionary motive insulates lending from shocks up to some size, but for larger shocks the economic consequences of the ensuing credit crunch may be significant. In this context, the results suggest that government bailouts of distressed banks—in the form of recapitalization policies—may be a reasonable response during episodes of systemic financial pressures.

This paper is organized as follows: the next section provides a review of the literature on the real effects of financial distress; section 3 presents the model; section 4 discusses some quantitative experiments; section 5 discusses bank recapitalization in the context of the model; and section 6 concludes.

II. BANKS AND THE REAL ECONOMY

Periods of significant financial distress are often followed by sizable and long-lasting contractions in bank credit, as shown in Figure 1. Bernanke and Gertler (1995) and others have argued that credit market imperfections are an important factor behind the magnitude and persistency of these contractions. Under imperfect capital markets, the cost of external finance for borrowers includes a wedge that varies countercyclically with their net worth, enhancing macroeconomic shocks. When the initial impulse corresponds to a monetary policy shock, the mechanism is referred to as the credit channel of monetary policy—see Bernanke, Gertler, and Gilchrist (1996) for empirical evidence in support of this theory and Gertler (1988) for a nice survey of the literature.

However, credit contractions may also be driven by changes in the financial condition of lenders—the focus of this paper and more related to the ongoing global financial turmoil. Nonetheless, the separation of these two channels is subject to a fundamental identification problem, that of disentangling demand from supply. For that reason, it is useful to subdivide the second channel into two questions: how does lending respond to the financial structure of banks? And a broader one, whether changes in the financial condition of banks have exogenous real effects. The first question has been addressed widely with bank-level data, and the consensus is that lending responds significantly to the financial structure of the lender. For instance: Bernanke and Lown (1991), Peek and Rosengren (1994, 1995), and Hancock and Wilcox (1994) are some examples studying how bank capital shortfalls affect lending, and the results show statistically significant effects. Kashyap and Stein (1995) find evidence suggesting a different impact of monetary policy on the security and loans portfolio of small and large banks. Similarly, Kashyap and Stein (2000) find that the impact of monetary policy on lending is stronger for banks with less liquid balance sheets. Cetorelli and Goldberg (2008) find a substantially more active lending channel than that documented in Kashyap and Stein (2000), but conclude that within the United States it is declining in strength as banking becomes more globalized.

Addressing the implications of the above results for the real economy is a challenge, because the health of the banking system remains intertwined with the overall performance of the economy. Despite the difficulties, there are authors who have provided evidence consistent with the hypothesis that deterioration in banks' financial health has important real effects. Bernanke (1983) asserts that the length and depth of the Great Depression cannot be explained solely on the base of monetary factors. He concludes that weakness of financial institutions had real effects. In exploring the amplification mechanisms that played a role in the ongoing financial turmoil that began in 2007, Adrian and Shin (2008a and 2008b) present evidence that marked-to-market leverage of financial firms is strongly procyclical and argue that such behavior has aggregate consequences.

Dell’Ariccia, Detragiache, and Rajan (2005) explore whether the decline in credit and growth that follows a banking crisis can be partially explained by the weakening of the banking sector. They find evidence suggesting that sectors more dependent on external finance perform relatively worse after banking crises. Using two different sample periods, Gibson (1995, 1997) find that Japanese firms whose main bank had financial problems had investment levels substantially lower than firms whose bank was not in trouble or those who were listed on the stock market. The overall impact, however, was found to be small because of the relative size of bank-dependent borrowers. Klein, Peek, and Rosengren (2002), using firm-level and bank-level data, find that financial difficulties at banks were economically and statistically important in reducing the number of FDI projects by Japanese firms into the United States. Peek, Rosengren, and Tootell (2003) identify loan supply shocks using CAMEL ratings of U.S. banks and find that bank health has economically significant effects on the U.S. economy. Moreover, they also find that their loan supply measure is particularly important for understanding movements in inventories. Driscoll (2004), on the other hand, uses a panel of state-level U.S. data to test whether changes in bank loan supply affect output and find that loans have small, often negative and statistically insignificant effects on output.

Peek and Rosengren (1997, 1999) investigate whether the sharp stock market decline in Japan was transmitted to the United States via U.S. branches. They find statistically and economically significant results suggesting that binding risk-based capital requirements associated with the fall of stock prices resulted in a reduction in lending by Japanese banks in the United States. On a similar venue, Peek and Rosengren (2000) also find that loan supply shocks emanating from Japan and transmitted to U.S. markets—where Japanese banks had significant market penetration—impacted construction activity in U.S. commercial real estate markets. Caballero, Hoshi, and Kashyap (2008) provide an explanation to the decade-long economic stagnation of Japan based on continued lending by banks to insolvent firms “*zombies*”, which in turn crowded out good firms.

With the caveats regarding the identification problem mentioned earlier, the existing empirical evidence seems to suggest that banks financial health matters for economic activity. In this paper, we proceed to develop a model that captures the main conclusions obtained from empirical work.

III. THE MODEL

The credit market includes a monopolistic bank and a continuum of ex-ante identical borrowers. The borrower-bank relationship is modeled as a risky debt contract with costly monitoring. The existence of a bank in this environment is justified by its cost efficiency in ex-ante evaluation and ex-post monitoring of borrowers’ investment projects, relative to individual investors. While a monopolistic bank is an unrealistic assumption, only a much more complicated industry structure would capture reality more closely. However, one may justify it by thinking of a regional monopoly or as the bank having substantial informational advantages over some segment of the pool of borrowers.

A. The Loan Contract

The loan agreement takes the form of a standard debt contract with maximum equity participation,⁵ as described in Gale and Hellwig (1985), and similar to the one in Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999). The demand side of credit comes from a continuum of entrepreneurs whose individual size is negligible relative to that of the bank. Entrepreneurs live for only two periods⁶ and receive at birth a common endowment of resources, which for simplicity is normalized to 1. The endowment, together with loans from the bank, is used to purchase capital, k , at a unit price. It is assumed that entrepreneurs have access to a common production technology which uses only capital as an input. For simplicity it is assumed that production takes two periods, with capital depreciating fully at the end of it. It is also assumed that entrepreneurs derive utility only from consumption in the second period of their lives. An entrepreneur's production y , at time $t+2$, is given by (1)

$$y_{t+2} = \alpha_{t+2} \Phi_{t+2} k_{t+2} \quad (1)$$

where $k_{t+2} = l_t + 1$ with α and Φ denoting i.i.d., lognormal⁷ stochastic idiosyncratic and aggregate productivity shocks respectively, with $E[\Phi] = 1$ and $E[\alpha] > 0$, for all t , and supports $\alpha \in (0, \infty)$ and $\Phi \in [\underline{\Phi}, \overline{\Phi}]$. It is assumed that there exists a minimum scale for entrepreneurs' projects, requiring a level of investment that is strictly larger than the endowment.

There are no ex-ante informational asymmetries—productivity is unknown when investment is made—however, following Townsend (1979), Gale and Hellwig (1985), and Williamson (1987), once idiosyncratic productivity, α , is realized, it is assumed to remain the private information of entrepreneurs. The bank may observe the productivity realization of an entrepreneur only after paying monitoring or bankruptcy costs $1 \geq u > 0$, expressed as a fraction of a borrower's project value. An entrepreneur's return is equal to the outcome of the investment minus the amount—principal plus interest—owed to the bank: $R_t l_t$, as indicated in (2)

$$\pi(R_t, l_t, \alpha_{t+2}, \Phi_{t+2}) = (l_t + 1) \alpha_{t+2} \Phi_{t+2} - R_t l_t \quad (2)$$

⁵ Standard means that the amount of repayment is fixed in the absence of bankruptcy. Maximum equity participation refers to the use of all resources available to the entrepreneur in the investment project.

⁶ This assumption rules out the need to keep track of the entire credit history of borrowers. Alternatively we could assume that there is enough anonymity among borrowers that only their endowment at the moment of applying for a loan matters for credit decisions.

⁷ The assumption of lognormally distributed productivity guarantees a non-rationing outcome. See Bernanke, Gertler, and Gilchrist (1999).

where R_t and l_t denote the interest rate and loan amount agreed on the debt contract. Since α is assumed to be continuously distributed over a non-negative support, there exists a cutoff value $\underline{\alpha} \in [0, \infty)$ such that the return to an entrepreneur is equal to zero.

$$(1_t + 1)\alpha_{t+2}\Phi_{t+2} - R_t l_t = 0 \quad (3)$$

$$\underline{\alpha}(\Phi_{t+2}, R_t, l_t) = \frac{R_t l_t}{(1_t + 1)\Phi_{t+2}} \quad (4)$$

Productivity realizations for an entrepreneur above $\underline{\alpha}$ imply that he consumes the surplus after honoring the contract, otherwise—with limited liability—default occurs. In such an event, the bank seizes the project and pays monitoring costs to observe the outcome.

Conditional on the realization of α and limited liability, the return to an entrepreneur can be summarized by

$$\pi(\alpha_{t+2}, R_t, l_t, \Phi_{t+2}) = \begin{cases} \alpha_{t+2} \Phi_{t+2} (1 + l_t) - R_t l_t & \text{if } \alpha_{t+2} \geq \underline{\alpha}(R_t, l_t, \Phi_{t+2}) \\ 0 & \text{if } \alpha_{t+2} < \underline{\alpha}(R_t, l_t, \Phi_{t+2}) \end{cases} \quad (5)$$

At the moment the contract is signed, the expected—with respect to α —return to a borrower is denoted by $E_t[\pi(R_t, l_t, \alpha_{t+2}, \Phi_{t+2})]$. The bank makes “*take-it-or-leave-it*” offers to entrepreneurs that include an amount l_t and an interest rate R_t . Since borrowers are ex-ante identical, they all are offered the same contract. If a borrower accepts the offer, he starts a two-period investment project, otherwise he can deposit the endowment at the bank and expect a return $E_t[(1 + \rho_{t+1})(1 + \rho_{t+2})] = (1 + \bar{\rho})^2$, which corresponds to the exogenously assumed i.i.d deposit interest rate, compounded for two periods, and unknown at the moment of making borrowing decisions. The problem for an entrepreneur is given by (6).

$$\underset{\{accept, reject\}}{Max} \left\{ E_t[\pi(\alpha_{t+2}, R_t, l_t, \Phi_{t+2}), (1 + \bar{\rho})^2] \right\} \quad (6)$$

Participation of any entrepreneur is subject to a rationality constraint requiring that the rate of return from the project is at least as good as his opportunity cost, that is

$$E_t[\pi(\alpha_{t+2}, R_t, l_t, \Phi_{t+2})] \geq (1 + \bar{\rho})^2 \quad (7)$$

With the “*take-it-or-leave-it*” assumption, and the existence of an interior solution, constraint (7) holds with equality at the levels of lending and interest rate that solve the bank’s problem—presented in the next section. If not, the bank could always charge a slightly higher interest rate and would still have a borrower accepting the offer.⁸ Thus, equation (7) implicitly defines the interest rate schedule $R(l, \Phi)$ charged by the bank, which is written contingent on aggregate productivity to preserve the optimality of the loan contract—see

⁸ This is a standard result in principal-agent problems of this type. See Mas-Colell and Green (1995).

Bernanke, Gertler, and Gilchrist (1999).⁹ It is assumed that when indifferent, an entrepreneur always prefers to invest.

Using the interest rate function previously derived, revenues for the bank are then given by (8).

$$g(\alpha_{t+2}, l_t, \Phi_{t+2}) = \begin{cases} R(l_t, \Phi_{t+2})l_t & \text{if } \alpha_{t+2} \geq \underline{\alpha}(R(l_t, \Phi_{t+2}), l_t, \Phi_{t+2}) \\ \alpha_{t+2} \Phi_{t+2} (1 + l_t)(1 - u) & \text{if } \alpha_{t+2} < \underline{\alpha}(R(l_t, \Phi_{t+2}), l_t, \Phi_{t+2}) \end{cases} \quad (8)$$

Following Gale and Hellwig (1985), the contract is incentive-compatible because in the non-default state, the payment to the lender is fixed and strictly larger than the value of the project in the default state.¹⁰ The assumption of a continuum of borrowers and the law of large numbers imply that the bank can perfectly diversify the idiosyncratic component of risk. Denoting the mean of a variable across borrowers with $M[\cdot]$, ex-post revenues for the bank are given by (9).

$$G(l_t, \Phi_{t+2}) = M[g(\alpha_{t+2}, l_t, \Phi_{t+2})] \quad (9)$$

Notice that with a fixed endowment and i.i.d shocks, expected revenues are time-invariant and we are shutting down Bernanke, Gertler, and Gilchrist (1999)'s financial accelerator effect.

B. The Bank's Optimization Problem

It is assumed that risk-neutral stockholders¹¹ simultaneously choose amounts of lending (l_t), dividends (d_t), and deposits (c_t), in order to maximize the market value of the bank, given by

$$\text{Max}_{\{l_s, d_s, c_s\}} E_t \sum_{s=t}^{\infty} d_s \quad (10)$$

where β denotes the discount factor. For convenience, it is assumed that decisions are made in the middle of each period and uncertainty is revealed between periods. Table 1 shows the sequence of events for the bank. As seen on the table, the bank has two state variables: the

⁹ Furthermore, $\underline{\Phi}$ and $E[\alpha]$ are chosen so that the interest rate is a real number. It requires assuming $E[\alpha_t] > (1 + \bar{\rho})^2$.

¹⁰ Similar to Bernanke, Gertler, and Gilchrist (1999), we assume $(1 - u)E[\alpha_t] < (1 + \bar{\rho})^2$, to rule out the possibility of having the bank making unbounded profits on a contract where an entrepreneur has a probability of default equal to 1, since $\lim_{\alpha \rightarrow \infty} g(\alpha_{t+2}, l_t, \Phi_{t+2}) = (1 - u)E[\alpha_t]$.

¹¹ We abstract from modeling any conflicts of interest between managers and stockholders by assuming that the bank is managed by its owners, or simply that stockholders' and managers' interests are aligned.

book value of bank capital, n , and outstanding loans, o . Recall that loans are extended with a two-period maturity. Their respective transition equations are given by

$$n_{t+1} = G(o_t, \Phi_{t+1}) + l_t - (1 + \rho_{t+1})c_t - f_t c_t \quad (11)$$

$$o_{t+1} = l_t \quad (12)$$

where (11) denotes the difference in book value of assets and liabilities. $G(o_t, \Phi_{t+1})$ is simply equation (9), which corresponds to revenues from loans made in period $t-1$ as suggested by equation (12). $(1 + \rho_{t+1})c_t$ denote obligations to depositors, where ρ is the interest rate on deposits, at which funds are supplied infinitely elastically by depositors. Finally, it is assumed that credit market frictions affect the bank in two ways: the bank pays costs f_t that are linked to the bank's leverage¹² and equity finance is ruled out by assuming that it is infinitely costly to the bank to issue equity.

$$d_t \geq 0 \quad (13)$$

These assumptions invalidate Modigliani and Miller (1958)'s theorem. The assumption of costs f_t may be interpreted as agency problems between regulators and the bank—in the presence of deposit insurance—or between depositors and the bank in the absence of it.

These agency problems are modeled by assuming they are a function of the bank's leverage

$f\left(\frac{c}{l+o}\right)$ increasing and at least twice continuously differentiable in deposits. Without loss

of generality, possible interpretations of $f(\cdot)$ include: deposit insurance premium, costs associated with intensified regulatory scrutiny as the bank's financial position weakens, or simply a deadweight loss associated with information problems just as the case of the borrower-bank relationship. Under this interpretation—elaborated further in the appendix—“ f ” arises endogenously, embedded in the contractual deposit rate as expected bankruptcy costs. Because the objective of this paper is not to argue in favor of some specific form of friction, it suffices to use the reduced-form modeling device introduced through f . We impose a final restriction that requires deposits to be at least as good as the funding needs of the bank, given by the difference between the book value of assets and bank capital, net of dividends. This restriction is summarized by (14)

$$c_t \geq o_t + l_t + (n_t - d_t) \quad (14)$$

With the assumption of f_t and no return on cash balances, equation (14) holds with equality, otherwise the bank can always increase profits by reducing the amount of deposits.

Equations (15) to (19) summarize the bank's problem, written in Bellman's equation form.

¹² Berger and Bonaccorsi (2002) find evidence consistent with the hypothesis that increases in leverage of U.S. banking firms raises agency costs.

$$V_t(n_t, o_t) = \text{Max}_{\{d_t, l_t\}} \{d_t + E_t \beta V_{t+1}(n_{t+1}, o_{t+1})\} \quad (15)$$

s.t.

$$c_t = o_t + l_t + (n_t - d_t) \quad (16)$$

$$d_t \geq 0 \quad (17)$$

$$n_{t+1} = G(o_t, \Phi_{t+1}) + l_t - (1 + \rho_{t+1})c_t - f_t c_t \quad (18)$$

$$o_{t+1} = l_t \quad (19)$$

A final assumption, necessary to guarantee a solution to the above problem, is that stockholders are impatient. This assumption requires $1/\beta > 1+\rho$, and it guarantees that the bank does not fund itself entirely with internal funds. Intuitively, one could argue that the opportunity cost of stockholders is higher than what they would earn on a bank deposit, or that they want to reduce potential agency problems with managers by not accumulating too much internal funds. An alternative approach to generate a similar outcome is to introduce taxes, in which case the bank will always find optimal to hold debt to benefit from the deduction of interest expenses.

It is worth emphasizing that the focus of this model is entirely on solvency. Therefore, liquidity considerations as in Diamond and Dybvig (1983) or Kashyap, Rajan, and Stein (1999) are ignored for the sake of simplicity. However, it is feasible to enrich the structure of the model to analyze both liquidity and solvency together.

C. Solution

The model is solved numerically using the logic of endogenous gridpoints developed in Carroll (2006).¹³ The appendix provides the assumed values for the parameters of the model and a detailed description of the solution algorithm. For intuition purposes, it is useful to write the problem in terms of middle-of-period and end-of-period state variables. Using equation (16) to substitute out deposits and outstanding loans, and denoting end-of-period variables with Gothic letters, the problem can be written as

$$V_t(n_t, l_{t-1}) = \text{Max}_{\{q_t, l_t\}} \{q_t + \mathfrak{V}_t(q_t, l_t, l_{t-1})\} \quad (20)$$

$$n_t - q_t \geq 0 \quad (21)$$

¹³ The endogenous gridpoints method involves using end-of-period values of the state variables, the marginal value functions, and first order conditions to construct middle-of-period levels of the state variables.

Where bank capital as of the end of the period is defined as $q_t = n_t - d_t$ and the end-of-period value function, $V_t(q_t, l_t, l_{t-1})$, given by

$$V_t(q_t, l_t, l_{t-1}) = E_t \beta V_{t+1}(n_{t+1}, l_t) \quad (22)$$

s.t.

$$n_{t+1} = G(l_{t-1}, \Phi_{t+1}) + l_t + (1 + \rho_{t+1} + f_t) [(l_t + l_{t-1} - q_t) / (l_t + l_{t-1})] (l_t + l_{t-1} - q_t) \quad (23)$$

with first order conditions

$$\partial V_t(q_t, l_t, l_{t-1}) / \partial q = 1 \quad (24)$$

$$\partial V_t(q_t, l_t, l_{t-1}) / \partial l = 0 \quad (25)$$

Where

$$\partial V_t(q_t, l_t, l_{t-1}) / \partial q = E_t \beta [(1 + \rho_{t+1} + f_t) - (l_t + l_{t-1} - q_t) \partial f / \partial q] \partial V_{t+1}(n_{t+1}, l_t) / \partial n \quad (26)$$

$$\partial V_t(q_t, l_t, l_{t-1}) / \partial l = E_t \beta \{ [-\rho_{t+1} - f_t - (l_t + l_{t-1} - q_t) \partial f / \partial l] \partial V_{t+1}(n_{t+1}, l_t) / \partial n + \partial V_{t+1}(n_{t+1}, l_t) / \partial l \} \quad (27)$$

Equation (24) tells us that the amount of dividends distributed is such that their marginal value equals that of bank capital. The right-hand side of the equation shows that the marginal value of bank capital is affected by changes in the financial structure of the bank through $\partial f / \partial q$. The latter enters into play because changes in bank capital affect leverage and ultimately funding costs. In absence of credit market frictions, that is $f = 0$, and permitting issuing equity, the value of an additional dollar of bank capital is simply given by the exogenous interest rate ρ . Under such scenario, the financial structure of the bank would play no role and in the event of an adverse shock that deteriorates capital, the bank instantaneously substitutes raising debt for issuing equity.

In the case of lending, equation (25) dictates that optimal lending is such that the marginal value of outstanding loans equals the marginal cost of raising funds. This is better appreciated by rewriting equations (25) and (27) as

$$E_t [[\rho_{t+1} + f_t + (l_t + l_{t-1} - q_t) \partial f / \partial l] \partial V_{t+1}(n_{t+1}, l_t) / \partial n] = E_t [\partial V_{t+1}(n_{t+1}, l_t) / \partial l] \quad (27)$$

In equation (28), the left-hand side corresponds to the marginal cost of lending, which is determined by the increase in funding costs—given by $[\rho_{t+1} + f_t + (l_t + l_{t-1} - q_t) \partial f / \partial l]$ —because of the additional deposits needed to fund new lending and the impact that new lending has on the future's profitability of the bank, $\partial V_{t+1}(n_{t+1}, l_t) / \partial n$ and $\partial V_{t+1}(n_{t+1}, l_t) / \partial l$. The latter is affected because loans extended in the current period yield revenues two periods later. Furthermore, loans extended today affect the future financial structure of the bank, which in turn affect its future funding costs. Notice again that ruling out credit market imperfections breaks any existing link between the bank's lending decisions and its financial structure, in line with the propositions of the Modigliani and Miller (1958) theorem. The assumption of having the

bank facing credit market imperfections invalidates the irrelevance of the financial structure of the bank for lending decisions as implied by Modigliani and Miller (1958). By introducing this assumption, the lender no longer plays a passive role as in Bernanke, Gertler, and Gilchrist (1999).

Figure (2) shows the optimal decision rules $d(n_t, l_{t-1})$ and $l(n_t, l_{t-1})$ for the problem, with the dots indicating the target values or ergodic means for each variable. In both cases one can notice a kink in the policy functions, which is precisely the point where the constraint on dividends is binding. Furthermore, they are non-decreasing in bank capital because as the latter increases there are more resources for dividend distribution, but also solvency improves, which in turn reduces the marginal cost of lending. They are also non-increasing in outstanding loans because as they increase, solvency decreases and funding costs rise.

The non-linearity of the policy functions suggests also asymmetry in lending dynamics: for a bank starting at the target, negative shocks may trigger sharp contractions in lending, while positive ones do not generate the opposite effect. Dell’Ariccia and Garibaldi (2005) estimate gross credit flows for the U.S. banking system between 1979 and 1999 and find that for any given rate of change of net credit, gross flows are larger in a recession than in a boom and that credit contractions are more volatile than credit expansions. This asymmetry in responses suggested by the model, could also be related to the asymmetric effects of monetary policy on output that some economists have documented, such as Cover (1992).

D. Risk and the Target Level of Solvency

An interesting implication of the model is the existence of a target level of bank capital, which results from the interaction between the impatience assumption and credit market imperfections. To see this point more clearly, Figure 3 plots the marginal value of bank capital—shown in equation (26)—normalized by the level of assets to express it in terms of solvency. For low values of bank capital—and therefore solvency—its marginal value is high, therefore it pays off to retain earnings. On the other extreme, for sufficiently high levels of bank capital, its marginal value is low, lower than the time preference rate. Therefore, it is optimal for the impatient stockholders to distribute dividends.¹⁴ The target is then the point at which these two forces are exactly balanced.

Figure 3 permits also to illustrate the relevance of the imposed assumptions on f and impatience. Suppose first that impatience is maintained, but f is linear. In that case, the marginal value of bank capital would be uniformly higher (lower) than the time preference rate, $(1/\beta - 1)$, and the bank would hold no deposits (no positive capital). Suppose now that f has the desired differentiability properties—at least twice continuously differentiable—but impatience is decreased. It is easy to see that, as impatience decreases, the horizontal line at

¹⁴ If bank capital is large enough so that the constraint on dividends is not binding, the Envelope theorem implies that $\partial V_{t+1}(n_{t+1}, l_t)/\partial n = 1$. If in addition to that, we know that as $q \rightarrow \infty$ we have that $f \rightarrow 0$, then the marginal value of bank capital is simply given by the deposit interest rate. However, keep in mind that q is an endogenous variable, and therefore its limit was taken only for expositional clarity purposes.

$(1/\beta - 1)$ approaches $\bar{\rho}$, and consequently, the target level of solvency increases. This result arises because with a lower degree of impatience, stockholders have less incentive to distribute dividends. In the extreme case of no impatience, dividends and internal funds are perfect substitutes and a non-uniqueness problem arises. The dots plotted on the policy functions (see Figure 2) denote the corresponding targets for each variable.

One important feature of the model that can be appreciated in Figure 3 is the existence of a precautionary motive: the bank raises its target level of solvency as a response to increases in risk. Figure 3 shows how by increasing the variance of productivity shocks, the marginal value of bank capital shifts upwards, inducing the bank to accumulate earnings. Valencia (2006) tests this hypothesis in a sample of US banks and finds that if risk were to decrease for all banks to the lowest level observed in the sample, one would observe a median reduction in bank capital of 8 percent. This behavior is closely related to that of a consumer who self-insures against income uncertainty—see Carroll (2004).

This analysis leads to the conclusion that the role of bank capital in this model is that of a cushion against unexpected shocks that would otherwise hinder the bank's lending operations, consistent with Diamond and Rajan (1999) and Berger, Herring, and Szeg (1995).

IV. QUANTITATIVE EXPERIMENTS

This section presents some quantitative experiments using the optimal decision rules shown in Figure (2).

The first experiment consists of a negative productivity shock for one period. Figure 4 plots the corresponding results. The figure shows two cases distinguishing one another only in the size of the shock. In both cases, the initial conditions correspond to the target levels of each variable. The transitory shock to productivity causes a decline in revenues which deteriorates bank capital, as illustrated in Figure 6a. As it was mentioned during the analysis of the first order conditions, the deterioration in bank capital raises funding costs and the bank finds optimal to reduce dividends aiming at restoring solvency back to its optimal level. For the scenario where the shock is only one standard deviation, solvency returns to the target quite rapidly, with most of the adjustment carried through dividends.

For the case of the stronger shock, the larger deterioration in bank capital triggers much higher borrowing costs than those arising in the previous scenario, and consequently, reducing dividends is not enough to restore solvency. The bank finds optimal to cut lending and gradually restore bank capital through the accumulation of retained earnings. Notice however, that it takes several periods for lending to return to the target, despite the fact that the ex-ante financial condition of borrowers has not changed—recall that the financial accelerator effects that arise in Bernanke, Gertler, and Gilchrist (1999) are shut down in this model. As a result, the bank works as a propagation mechanism causing a one-period i.i.d impulse to have a long-lasting effect on the economy.

The response of the bank is similar in the case of an interest rate shock. Figure 5 plots the responses to a one-period change in the deposit rate ρ . An increase in the interest rate raises

funding costs and deteriorates bank capital because of the mismatch in maturity between loans and deposits. As before, a credit crunch arises until the bank restores solvency.¹⁵

Recall that the two key assumptions that generate the targeting behavior—as illustrated in Figure 3—were impatience and a decreasing marginal value of bank capital, with the latter induced by imperfect credit markets. Table 3 shows a few more scenarios to analyze the sensitive of these results to changes in the intensity of these two assumptions. For a 2-standard-deviation negative productivity shock, the table shows that the contraction of lending--on impact--more than doubles when the degree of impatience increases from 1 percent to 3 percent. Clearly, as stockholders become more impatient, they are less willing to accumulate a large capital buffer at the bank. Therefore, when the shock hits, the bank's buffer is not large enough to weather the bad times just as well as the baseline case. When we examine an upward shift in financial frictions “*f*” the difference in results is more notorious at low levels of impatience. With a low degree of impatience, stockholders shield lending quite effectively: the contraction in lending is halved relative to the baseline case.

To summarize, shocks that deteriorate the bank capital position of the bank trigger higher borrowing costs. Due to the inability of issuing equity, the bank returns only gradually to its optimal solvency level. This sluggish adjustment has been documented in empirical work. Using quarterly US data, Hancock, Laing, and Wilcox (1995) study the dynamic responses of securities and loans to bank capital shocks. They find that most loan categories take two to three years to complete their adjustment.

V. BANK RECAPITALIZATION

The quantitative experiments presented in the previous section show that a transitory shock is capable of generating a persistent credit crunch. Its effects on the real economy could be important, especially when firms are highly dependent on bank-intermediated credit. For the economy to escape the credit crunch, the solvency of the banking system needs to be restored. If shareholders are unable to inject fresh capital, as it has been assumed throughout this paper, public funds may be used to accelerate the recovery—as it has been done in banking crises episodes such as those shown in Table 2.

Bank recapitalization with public funds is a complex issue, since it has to be carefully designed in order to minimize moral hazard problems. Elaborating on the many aspects it involves is beyond the scope of this paper. However, the interested reader may refer to Enoch, Garcia, and Sundararajan (2002) for a description of available strategies and to Laeven and Valencia (2008) for a compilation of country experiences. Our interest here is a simple quantitative application of the framework we have just developed. We treat recapitalization as a one time, unexpected event from the perspective of the bank. Otherwise, if the bank assigns a positive probability to being recapitalized, its behavior

¹⁵ In this model, credit cycles are induced by exogenous shocks; Gorton and He (2004) provide empirical evidence consistent with the hypothesis that competition among banking firms could enhance these cycles. An interest extension of this model could be to analyze both mechanisms together.

would change. Alternatively, one could think of such strategy as being accompanied of enough restrictions than even if anticipated, banks would try to avoid it at all cost.¹⁶

Imagine that the economy experienced a large negative shock leading to a substantial deterioration in bank solvency—a banking crisis like the ongoing global turmoil. Figure 6 shows the adjustment with and without capital injections.¹⁷ In absence of capital injections, lending returns to normal levels in 9 periods—which with the assumed parameters could be interpreted as 9 years--whereas when the bank is recapitalized the recovery is much faster—depicted by the dashed line. The size of recapitalization is obtained by using the first order conditions of the model—equation (25)—to solve for the level of capital that is needed to have lending equal to its long-run target, conditional on the amount of outstanding loans at the time the shock hits. For recapitalization levels below the amount determined above, the credit crunch is not fully eliminated. Figure 7 plots the relationship between the severity of the credit crunch and the size of recapitalization, expressed as a percentage of the target level of capital. The credit crunch severity is computed as the cumulative difference between the long-run and actual levels of lending.¹⁸

The exercise yields some interesting implications. There is a minimum level of bank capital, q^l , below which the bank collapses without recapitalization. In this case, raising deposits becomes expensive enough that makes lending unprofitable, causing the bank to collapse.¹⁹ Similarly, there is a level of bank capital, q^h , above which the bank restores solvency by cutting dividends without inducing a persistent credit crunch, and recapitalization is not needed. Finally, for levels of bank capital q_t such that $q^l < q_t < q^h$ the bank may recover without recapitalization, but only gradually. These solvency regions are depicted in Table 4 and the respective thresholds can be solved numerically for a given calibration.²⁰

VI. CONCLUSIONS

The ongoing banking crisis in mature markets has brought to the fore once again questions about appropriate policies to resolve the crisis and avoid a deep and prolonged recession. This paper contributes with a policy tool that can be used to analyze the interactions between banks' solvency and the real economy, and hence simulate the effects of different policies at hand. In this framework, the bank is modeled as a firm in a world of imperfect capital markets, with a key feature being the existence of an optimal financial structure. This optimal financial structure depends on the perceived risk of lending activities, creating a precautionary motive: when risk increases, banks' hold additional capital as a buffer against

¹⁶ For instance, the new shareholders—possibly the state—would require board participation, veto power on certain decisions, dividend restrictions, etc.

¹⁷ The shock corresponds to 4 standard deviations.

¹⁸ This cumulative difference is exactly zero when the size of recapitalization reaches 63 percent of target capital.

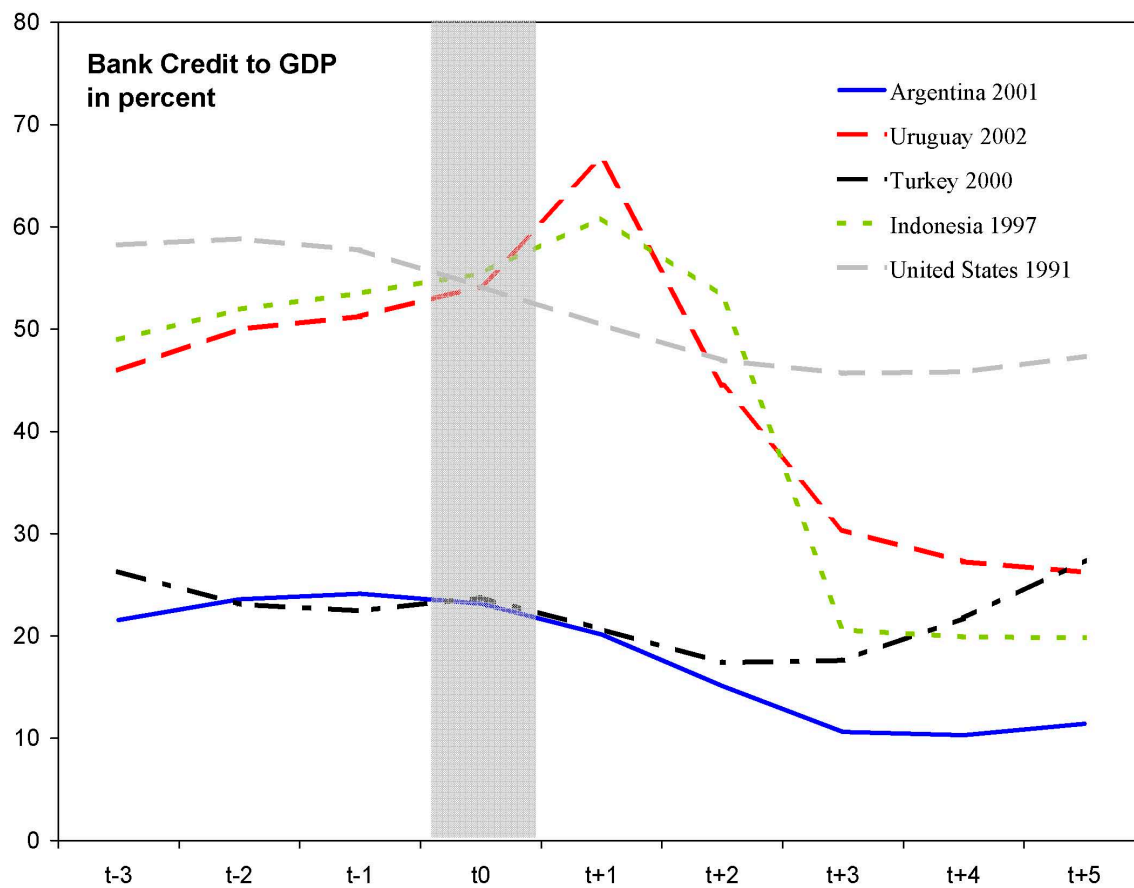
¹⁹ This is an absorbing state because the bank cannot generate profits without lending, and thus cannot restore solvency.

²⁰ The thresholds can be found by solving: $\partial \mathcal{V}_t(q^l, 0, l_{t-1}) / \partial l = 0$, and $\partial \mathcal{V}_t(q^h, l^*, l_{t-1}) / \partial l = 0$ using the converged marginal value function, and with l^* denoting the target level of lending.

shocks that would deteriorate its capital position and hence affect the supply of loans. Therefore, when faced with a small shock, the bank effectively shields lending. However, for larger shocks, a persistent credit crunch may arise even when the shock is a one time, non-serially correlated event. Consequently, i.i.d. shocks generate a serially correlated response in lending, and therefore persistent effects on the economy. Moreover, the propagation mechanism is much stronger for negative shocks than positive ones. The results derived from this model are a familiar outcome in models with market frictions; the novelty, however, lies in that this propagation mechanism stems from the bank instead of the borrower as it had been shown in previous work.

From a policy perspective, as illustrated in the previous section, the results presented in this paper suggest that banking bailouts in periods of significant financial distress may be justified to avoid an economically costly and persistent credit crunch.

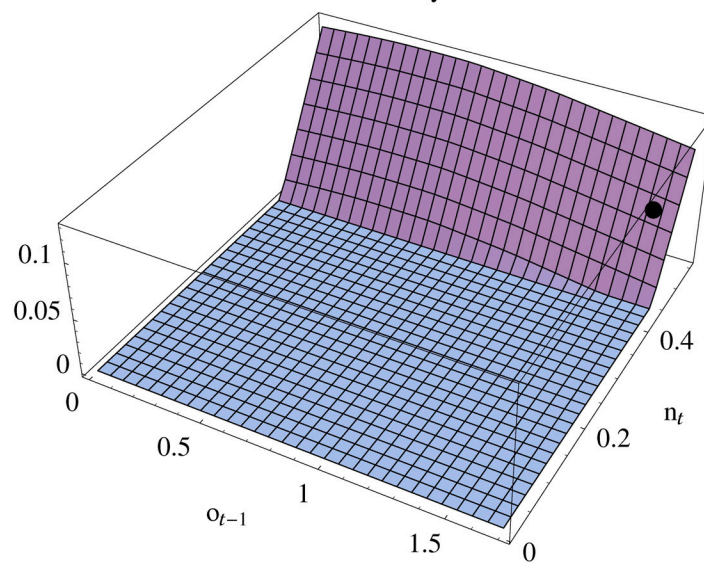
Figure 1. Bank Credit as Percentage of GDP, Selected Countries.



Source: IMF International Financial Statistics.

Figure 2. Optimal Policy Functions.

A. Dividends Policy Function



B. Lending Policy Function

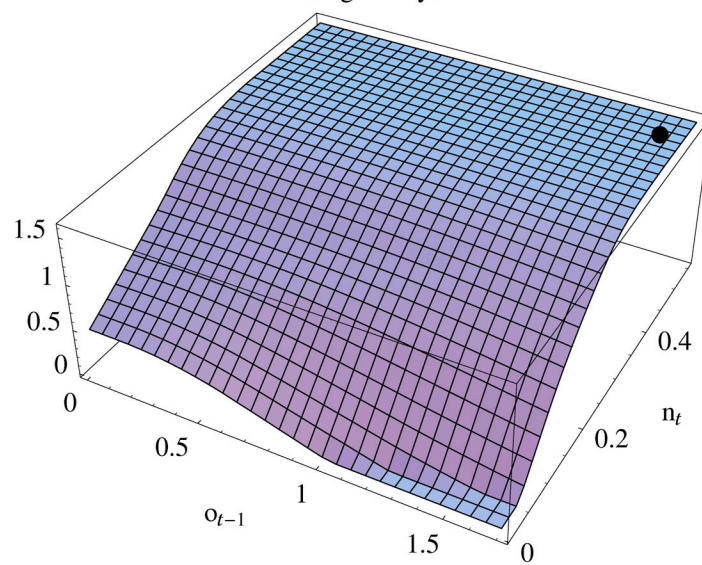


Figure 3. Target Level of Solvency.

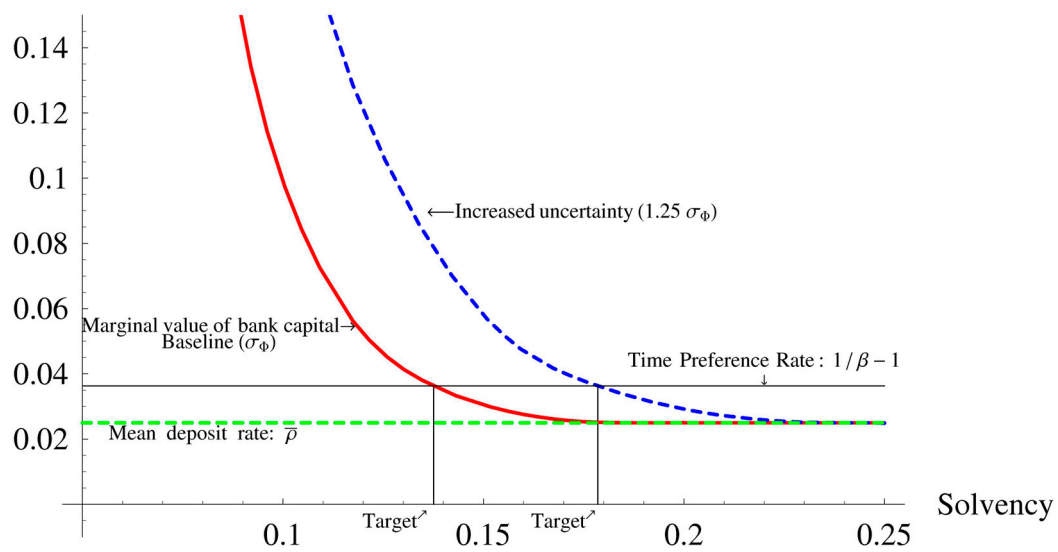


Figure 4. Responses to a Negative Transitory Productivity Shock.

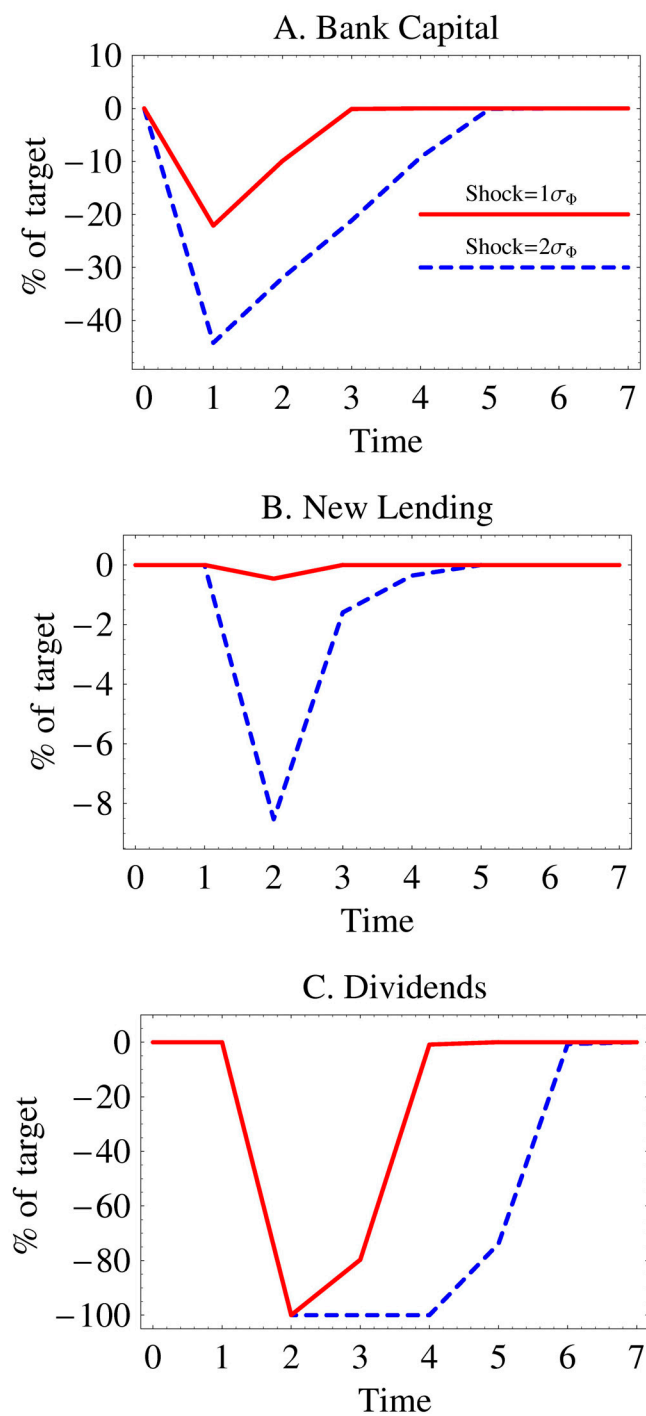


Figure 5. Responses to an Interest Rate Increase.

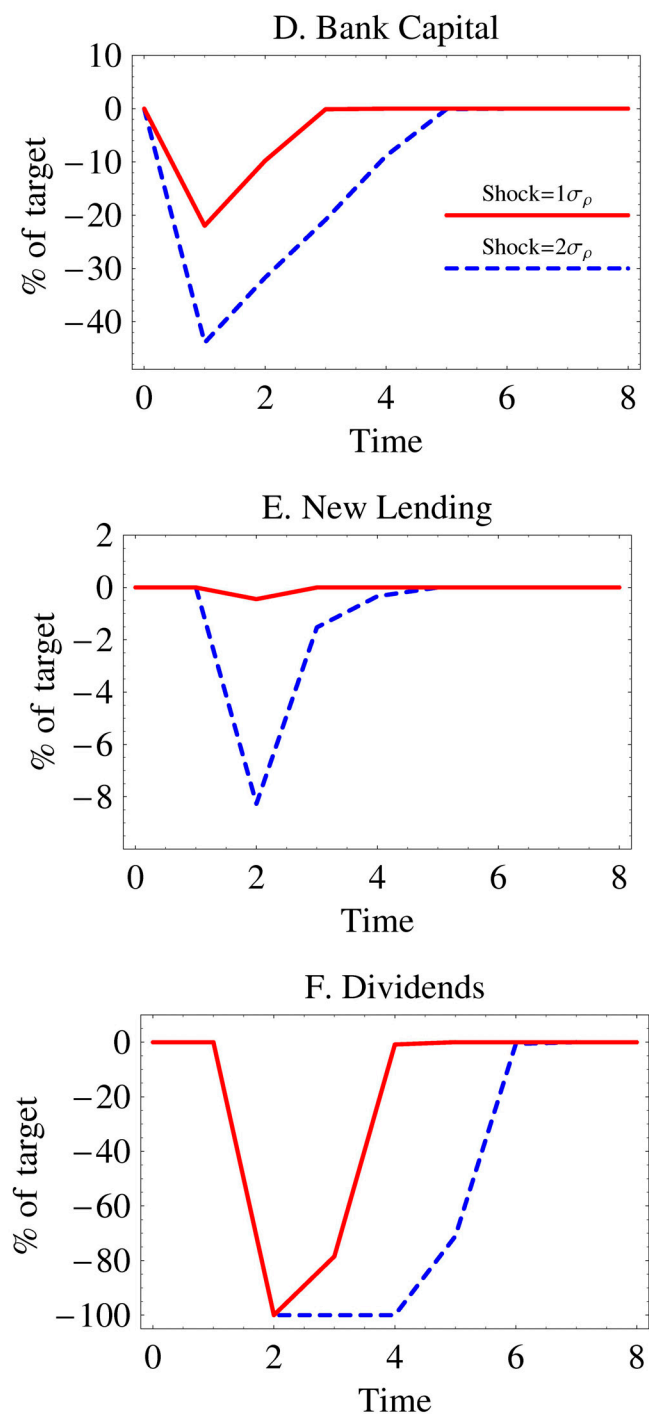


Figure 6. Responses to a Large Negative Shock, With and Without Recapitalization

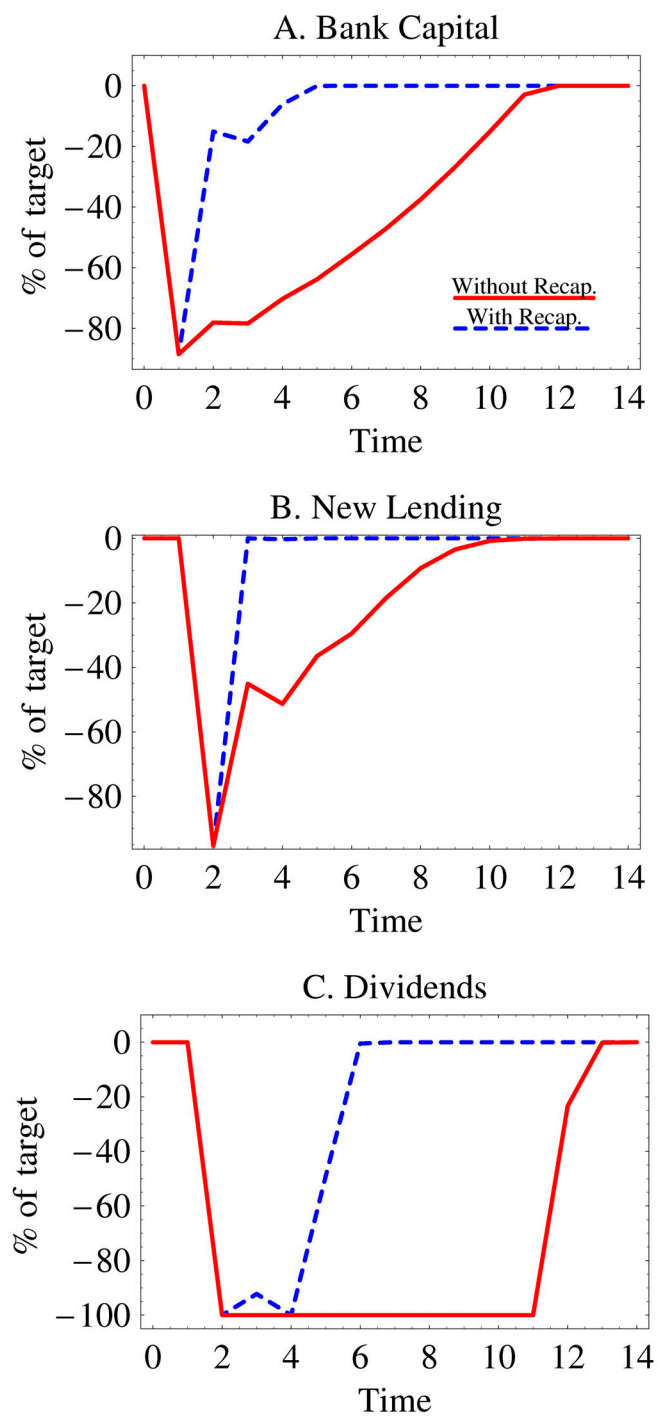


Figure 7. Credit Crunch Severity and Bank Recapitalization.

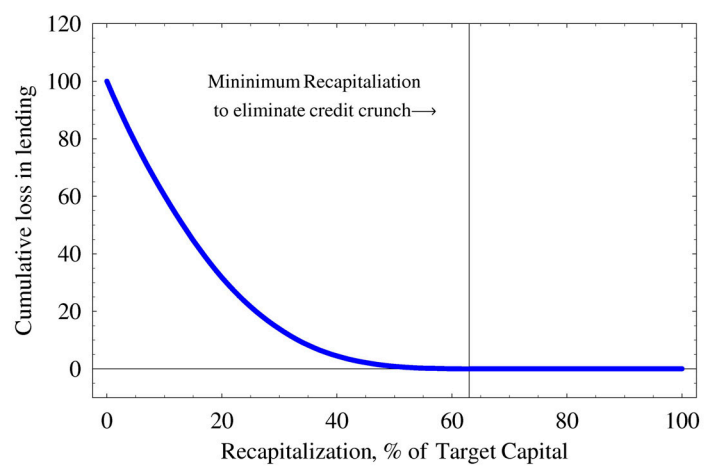


Table 1. Bank's Sequence of Events

t - 1	t	t + 1
<p>State Variables after decisions were made: o_{t-1}, l_{t-1}, c_{t-1}</p>		
	<p>Uncertainty is realized: Φ_t, ρ_t</p>	
	<p>State Variables: o_t, n_t Control Variables: c_t, d_t, l_t</p>	
		<p>State variables after decisions were made: o_t, l_t, c_t</p>
		<p>Uncertainty is realized: Φ_{t+1}, ρ_{t+1}</p>

Table 2. Public Recapitalization Costs for Selected Crises Episodes

Country	Year ^a	Public Gross Recapitalization Costs In percent of GDP
Indonesia	1997	37.3
Turkey	2000	24.5
Korea	1997	19.3
Malaysia	1997	16.4
Argentina	2001	9.6
Thailand	1997	8.5
Uruguay	2002	6.2

^a First year of crisis episode.

Source: Laeven and Valencia (2008)

Table 3. Sensitivity Analysis to a 2- σ Productivity Shock

Degree of impatience: $1/\beta - 1 - E[\rho]$				
		Baseline		
		1%	2%	3%
Baseline "f"	Target solvency	13.50%	12.70%	11.40%
	Contraction in new lending on impact	8.70%	16.80%	32.90%
	Length of credit crunch in periods	4	4	4
Upward shift in "f" ^a	Target solvency	14.70%	12.80%	11.50%
	Contraction in new lending on impact	4.80%	16.40%	33.10%
	Length of credit crunch in periods	4	4	4

^a Computed as a 25% increase in "f" for all levels of solvency

Table 4. Bank's Solvency Regions.

$q_t < q^l$	$q^l \leq q_t < q^h$	$q^h \leq q_t$
Bank cannot recover without recapitalization	Bank recovers gradually and a credit crunch arises	Recapitalization not needed
Capital injection:	Capital injection:	Capital injection:
Min: $q^l - q_t$	Min: ...	Min: ...
Max: $q^h - q_t$	Max: $q^h - q_t$	Max: ...
With $q^l=7.7\%$ of long-run target q^* and $q^h=72.5\%$ of long-run target q^*		

APPENDIX

E. Calibration

This calibration aims at providing a quantitative flavor of the model presented in this paper, without targeting a specific goal of matching some specific circumstances. The numerical solutions are obtained using $1/\beta = 0.965$ and $1+E[\rho] = 1.025$. The deposit interest rate is assumed to be discretely distributed with three outcomes: $\{1.015, 1.025, 1.035\}$, with probability distribution $\{0.25, 0.50, 0.25\}$. The expected value of α is equal to 1.07 and $\sigma_\alpha = \sigma_\Phi = 0.04$. Bankruptcy costs are calibrated as in Carlstrom and Fuerst (1997), with $u = 0.20$, which falls within the range of estimates obtained in Levin, Natalucci, and Zakrajsek (2004). Finally, it is assumed that $f = 0.8(c/(1+o))^{100}$. These parameter values generate, at the target, a spread between the contractual lending and deposit rates of 3%, a solvency ratio of the bank of 13.5%, and a leverage ratio of borrowers of 61%.

F. Solution Algorithm

The starting point is to assume that the bank will be liquidated at some future time, T . Therefore, as of time T , the optimal decisions involve setting $d_T = n_T$, and to make no new loans. These decisions imply that $V_T(n_T, l_{T-1}) = n_T$. As of $T-1$, the problem becomes

$$V_{T-1}(n_{T-1}, l_{T-2}) = \underset{\{d_{T-1}\}}{\text{Max}} \{d_{T-1} + E_{T-1} \beta n_T\} \quad (28)$$

s.t.

$$d_{T-1} \geq 0 \quad (29)$$

$$c_{T-1} = l_{T-2} + (n_{T-1} - d_{T-1}) \quad (30)$$

$$n_T = G(l_{T-2}, \Phi_T) - (1 + \rho_T) c_{T-1} - f_{T-1} c_{T-1} \quad (31)$$

where the bank does not lend because it will be liquidated in the following period. The first order condition for the above problem is given by²¹

$$1/\beta = E_{T-1} [f_{T-1} + 1 + \rho_T - c_{T-1} \partial f / \partial c] \quad (32)$$

$$1/\beta - (1 + \bar{\rho}) = E_{T-1} [(1-a)f_{T-1}] \quad (33)$$

which yields the solution for dividends

²¹ (33) follows from the assumption of $f = v(c_{T-1}/l_{T-2})^a$, with the values of v and a given in the calibration section.

$$d = \text{Max}[0, n_{T-1} - l_{T-2} \{1 - (1/\beta - (1 + \bar{\rho})/((1-a)v))\}^{(1/a)}] \quad (34)$$

From period $T-2$ and backwards the bank will lend positive amounts. With the value function obtained in the previous step, we can write the problem as of $T-2$, whose structure is identical to the one shown in the main text—equations (15) through (19)—therefore it can be generalized to period t . The first step before applying backwards induction is to re-write the problem in terms of middle-of-period and end-of-period variables, as shown in the text—equations (20) through (27)—with their respective first order conditions.

The algorithm is implemented by specifying values for n_t and l_{t-1} collected in N and L respectively. For each value $l_{t-1} \in L$, a root-finding procedure is used to determine the values of q_t and l_t that satisfy the first order conditions (24) and (25). Define these values as q_t^* and l_t^* , for a given value of l_{t-1} . For increased numerical accuracy on the region where the constraint on dividends is binding, N is augmented with q_t^* . q_t^* is the optimal level of end-of-period bank capital. There is no change in outstanding loans between the middle and end of any period, therefore $l_{t-1} = l_t$. Now, for every pair $\{n_t, l_{t-1}\}$ such that $n_t \in N$ and $l_{t-1} \in L$ the solutions are obtained in the following way:

1. If $n_t \geq q_t^*$, the constraint on dividends is not binding, hence the optimal solutions are $d_t = n_t - q_t^*$ and $l_t = l_t^*$.
2. If $n_t < q_t^*$, the constraint on dividends is binding, therefore $d_t = 0$. The solution for lending is obtained by applying a root-finding procedure on equation (25), using the fact that the binding constraint implies that $q_t = n_t$.

The previous steps generate triples $\{l_t, n_t, l_{t-1}\}$ and $\{d_t, n_t, l_{t-1}\}$. We approximate the continuous policy functions $d_t(n, l)$ and $l_t(n, l)$ by piecewise linear interpolation. $\partial V_t(q_t, l_t, l_{t-1})/\partial q$ and $\partial V_t(q_t, l_t, l_{t-1})/\partial l$ are constructed using linear interpolation as well. This step completes the recursion. If $\|d_t(n, l) - d_{t-1}(n, l)\| \leq 0.0001$ and $\|l_t(n, l) - l_{t-1}(n, l)\| \leq 0.0001$ the algorithm stops, otherwise we move one period backwards and keep applying the same procedure until the convergence criteria is satisfied.^{22, 23}

G. Agency Costs in the Depositor-Bank Relationship

This section provides an alternative way to model credit market frictions. Following the costly state verification framework, let us assume that the realization of Φ is the private information of the bank. As before, there are no ex-ante information asymmetries, and the bank cannot issue equity. For simplicity, let us assume that the bank is closed down in period $t+1$ if the book value of capital is less than or equal to zero, that is

²² The expectations are approximated using a Gaussian quadrature—See Judd (1998)—with 3 points. There was no material difference when more points were used.

²³ The marginal value functions $\partial V_{t+1}(n_{t+1}, l_t)/\partial n$ and $\partial V_{t+1}(n_{t+1}, l_t)/\partial l$ are updated after each iteration according to: $\partial V_{t+1}(n_{t+1}, l_t)/\partial n = \partial V_{t+2}(n - d_{t+2}(n, l), l_{t+2}(n, l))/\partial q$ and $\partial V_{t+1}(n_{t+1}, l_t)/\partial l = \partial V_{t+2}(n - d_{t+2}(n, l), l_{t+2}(n, l))/\partial l$.

$$G(l_{t-1}, \Phi_{t+1}) + l_t - (1 + \rho_{t+1})c_t \leq 0 \quad (35)$$

There exists a value $\underline{\Phi}$ such that the book value of equity is exactly zero; thus,

$$G(l_{t-1}, \underline{\Phi}_{t+1}) + l_t - (1 + \rho_{t+1})c_t = 0 \quad (36)$$

If $\Phi_{t+1} \geq \underline{\Phi}$, the bank pays depositors the agreed amount $\rho_t c_t$ and remains open. If not, the bank goes bankrupt, depositors seize the bank and keep the liquidation value of its assets, distributed proportionally to all depositors. The return to depositors in period $t + 1$ is summarized by

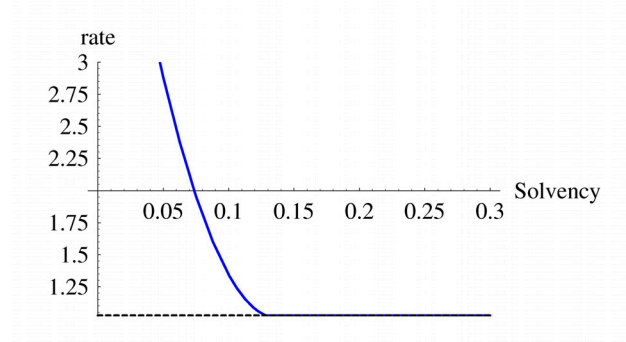
$$j(l_{t-1}, l_t, \Phi_{t+1}, c_t, \rho_t) = \begin{cases} \rho_t c_t & \text{if } \Phi_{t+1} \geq \underline{\Phi}(l_{t-1}, l_t, c_t, \rho_t) \\ (G(l_{t-1}, \Phi_{t+1}) + l_t)(1 - \lambda) & \text{if } \Phi_{t+1} < \underline{\Phi}(l_{t-1}, l_t, c_t, \rho_t) \end{cases} \quad (37)$$

where λ denotes monitoring costs paid by depositors in the event of bankruptcy of the bank. Under risk neutrality, depositors leave the money at the bank in period t if the return satisfies

$$E_t[j(l_{t-1}, l_t, \rho_t, c_t, \Phi_{t+1})] \geq \gamma_t c_t \quad (38)$$

where the expected return from a bank deposit should be at least as good as the opportunity cost of the depositor, denoted by γ_t . For simplicity, we can assume that there are infinitely many and small—relative to the bank—price-taker depositors. This assumption implies that equation (48) holds with equality, otherwise, the bank can always pay a slightly lower interest rate to depositors and would still have them leaving the money at the bank. Figure 8 plots the resulting interest rate at which (48) holds with equality, assuming $l_{t-1} = l_t = l^*$, which corresponds to the target value obtained in the baseline model, and for $\lambda = 0.40$, and $\gamma_t = 1.025$, while all other parameters are kept unchanged relative to the baseline version.

Figure 8. Deposit Interest Rate



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