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## The Impact of Public Capital, Human Capital, and Knowledge on Aggregate Output

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## **IMF Working Paper**

IMF Institute

### **The Impact of Public Capital, Human Capital, and Knowledge on Aggregate Output**

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#### **Abstract**

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This paper investigates the impact of public capital on private sector output by testing and estimating an aggregate production function for the U.S. economy over the postwar period augmented to include the stock of public capital as an additional factor input. We use patent applications to proxy for knowledge/technology stocks and adjust labor hours for changes in human capital or skill. Using Johansen's (1988 and 1991) multivariate cointegration analysis, we find a positive and significant long run effect of public capital, private capital, skill-adjusted labor, and technology/ knowledge on private sector output. We find that public capital accounts for about half of the post-1973 productivity slowdown, but only plays a minor role in the partial recovery of labor productivity growth since the mid 1980s. The largest contribution to that (partial) recovery comes from the knowledge stock and human capital.

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## I. Introduction and Contribution

Public capital has been referred to as the “wheels of economic growth” by the World Bank (1994). This claim seems to be an issue for empirical investigation. Indeed there has been a fair amount of debate and empirical work to date on the potential influence of public capital on private sector economic performance. That research was triggered by David Aschauer’s (1989) observation that the U.S. post-1973 productivity slowdown was “*matched or slightly preceded by a precipitous decline in additions to the net stock of public non-military structures and equipment.*” Motivated by this observation, Aschauer estimated a standard Cobb-Douglas production function that included the stock of public capital as an additional factor input. His estimation results over the period 1949-1985 suggested a strong positive relationship between public capital and private sector output. A 1% increase in the public capital stock was estimated to increase private sector output by 0.39%. Consequently, Aschauer argued that much of the slowdown in U.S. productivity growth in the 1970s and 1980s could potentially be explained by the declining rates of public investment spending. The policy implications seemed to be clear: public investment should increase to give a boost to the economy.

Many authors have subsequently criticized Aschauer’s results mainly on econometric grounds. The major issues that have been raised concern the non-stationarity of the data and potential spurious correlation, as well as the potential endogeneity of public capital. The critics argued that these two econometric problems could potentially explain the large public capital elasticity (0.39) found by Aschauer (1989) [For excellent surveys of the public capital literature, see Munnell (1992), Gramlich (1994), and Romp and de Haan (2007)]. Despite the fact that many have identified these important econometric problems, not much has been done to properly correct for them.

In this paper, we estimate an aggregate production function for the U.S. economy over the postwar period augmented to include the public capital stock as an additional factor input. Our aim is to estimate the long run impact of public capital on aggregate output while fully accounting for the econometric issues regarding spurious correlation and endogeneity. To that end, we estimate the aggregate production function in a cointegrating framework, and hence avoid the spurious regression problem typically encountered with non-stationary variables while preserving (and actually estimating) the long run level relationship in the data. The paper uses Johansen’s (1988, 1991) multivariate cointegration procedure that tests for and estimates cointegrating relationships in a Vector Error-Correction Model (VECM). By using Johansen’s procedure, the paper adequately addresses issues related to dynamics, endogeneity, and the potential presence of multiple cointegrating vectors. In doing so, the paper improves on the existing studies in the literature that use a single-equation approach for cointegration.

We also improve on the previous public capital literature in the treatment of technology, and the labor input. As technology is not directly measurable, it has been typically approximated by a deterministic time trend in production function specifications, and the labor input has been typically measured by raw hours worked. Instead, we use patent applications to proxy for knowledge/technology stocks, and adjust labor hours for changes in human capital or skill.

To our knowledge, we believe we are the first to estimate the effect on output of public capital, human capital/skill-adjusted labor, and the measured knowledge stock *in a unified multivariate cointegrating framework*. We also use a longer span of data than prior research (1948 to 2004), and we do not impose any a priori restrictions on the returns to scale in the production function. We also control for recessions where energy shocks are considered a significant contributing factor through the use of dummy variables in the cointegrated VAR system.

We find evidence of an aggregate Cobb-Douglas production function with constant returns to scale with respect to private capital and skill-adjusted labor. We also find that the estimated long run elasticity of output with respect to public capital is the same as that for private capital. There is a positive and significant long run effect of public capital, skill-adjusted labor, and technology/ knowledge on private sector output, with estimated long run elasticities of 0.39, 0.61, and 0.13 respectively. Our estimated long run elasticity for public capital is the same as that of Aschauer (1989). When these numbers are used to conduct a simple growth accounting exercise for the postwar U.S. economy, we find that public capital accounts for about half of the post-1973 productivity slowdown, but only plays a minor role in the partial recovery of labor productivity growth since the mid 1980s. The largest contribution to that (partial) recovery comes from the knowledge stock and human capital.

The rest of the paper is organized as follows: Section II provides a critical review of the relevant literature. Section III describes our model and Section IV discusses the data set and measurement issues. Section V conducts the initial data analysis and reduction of the VAR system. Section VI contains the cointegration analysis of the data. This includes testing for cointegration, estimation of the long run aggregate production function and the speed of adjustment parameters. This is followed by tests of weak exogeneity and hypotheses tests on the aggregate production function. Section VII uses the estimated parameters of the production function to conduct a growth accounting exercise for the U.S. economy over the postwar period. Finally, section VIII offers some concluding remarks.

## II. Literature Review

Aschauer's (1989) estimates of the productivity impact of public capital were supported by his subsequent work [Aschauer (1990)] and by that of Munnell (1990), whose research yielded an estimate of the elasticity of output with respect to the public capital stock of about

0.34. Ford and Poret (1991), Holtz-Eakin (1992), and Fernald (1993) reached similar conclusions supporting a strong positive impact of the public capital stock on private sector output.

The above elasticity estimates and papers in turn generated criticism from authors such as Aaron (1990), Eisner (1991), Hulten and Schwab (1991, 1993), Tatom (1991, 1993), among others. These authors pointed out that the econometric methodology of estimating production functions followed by Aschauer and his supporters was flawed. Two major econometric problems were identified. The first of these problems emerged from the observation that all the variables included in the production function, inputs and output, were non-stationary in levels. Like many macroeconomic variables, output, capital (private and public), as well as labor tend to drift upward over time away from their initial values with no tendency to revert to a mean value. In the terminology of the time series literature, the series exhibit non-stationary behavior and are said to be integrated or to contain unit roots with a drift.

The finding of unit roots made Aschauer's results, derived using data in levels, suspicious due to potential *spurious correlation* [see, for example, Tatom (1993)]. Hence, the critics argued that spurious correlation between data on output and public capital in levels could potentially explain the large public capital elasticity found by Aschauer and his supporters.

The second major econometric problem is *endogeneity*. That is, part of the positive correlation observed between output and the public capital stock could actually reflect feedback or "causality" from output into public capital investment rather than the other way around. In other words, public capital investment is potentially an endogenous variable that goes hand in hand with economic activity. This makes single-equation estimation of the production function potentially subject to simultaneous equation bias. [Musgrave (1990), Eisner (1991), Hulten and Schwab (1993)].

Despite the fact that many have identified these important econometric problems, not much has been done to properly account for them. To correct for spurious correlation, and hence determine the true relationship between public capital and output, researchers have typically specified the production function in terms of first differences of the data. Estimating the production function in first differences yielded results that suggested a very small effect of public capital, usually not even statistically significant [for example, Hulten and Schwab (1991), Tatom (1991), and Evans and Karras (1994)].

While first differencing the data removes any spurious correlation, it creates a bigger problem. Economic theory suggests that the production function is a long run phenomenon between the levels of inputs and output. First differencing destroys any long run relationship that may exist amongst the levels of the variables of interest, which is exactly the relationship one is typically interested in identifying and estimating [Munnell (1992)].

One fruitful way to eliminate spurious correlations while maintaining the true long run relationship between inputs and output is to estimate the production function in a cointegration framework. Intuitively, two or more non-stationary series are said to be cointegrated if they “move” closely together in the long run, even though they may drift apart in the short run. In other words, and if we apply this to the production function, despite the fact that inputs and output are individually non-stationary, trending away from their initial values, they maybe linked together in a long run relationship. When this is true, inputs and output are said to be cointegrated, and the production function can be identified as the long run cointegrating relationship. Despite its appeal, relatively few studies have actually used cointegration in estimating production functions particularly in the public capital literature.

Among those who did use cointegration to estimate production functions are: Tatom (1991), Bajo-Rubio and Sosvilla-Rivero (1993), Sturm and de Haan (1995), Hamilton (1996), Batina (1999), Pereira and Flores de Frutos (1999), and Everaert and Heylen (2001). However, these authors typically adopted a single-equation residual based test of cointegration. That is, an aggregate production function was estimated as a single equation and the residuals of that equation were tested for stationarity to assess whether or not inputs and output were cointegrated. Tatom (1991), Sturm and de Haan (1995), and Pereira and Flores de Frutos (1999) did not find evidence for cointegration for the U.S. economy, while Hamilton (1996) and Batina (1999) did. Hamilton’s (1996) estimate of the elasticity of output with respect to public capital was 0.17 while Batina (1999) reported estimates in the range of 0.14 to 0.375. Bajo-Rubio and Sosvilla-Rivero (1993), and Everaert and Heylen (2001) did find evidence for cointegration for Spain and Belgium with an estimated public capital elasticity of 0.19 and 0.29 respectively. It appears that the single equation cointegration approach yielded estimates of the public capital elasticity smaller than that of Aschauer (1989).

While the single equation cointegration approach eliminates spurious correlations while maintaining and estimating the long run level relationship in the data, it has its own drawbacks. First, the single equation approach does not fully address the second econometric problem highlighted in the literature, that is, the endogeneity of the public capital stock. If there is feedback from output into the public capital stock, inference based on a single equation approach may be invalid [Ericsson and Irons (1994)]. This is true even if corrections for bias due to potential endogeneity problems are made to single equation estimators <sup>2</sup>[see Inder (1993)].

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<sup>2</sup> Fully Modified OLS estimators are typically used in single equation (static) cointegrating models. These estimators apply non-parametric adjustments to the OLS estimates of the cointegrating vector and the associated t-statistics to account for any bias in the OLS residuals due to endogeneity and autocorrelation [ See, for example, Phillips and Hansen (1990)]. Inder (1993) found that the modified OLS estimates of the cointegrating vectors made little or no improvement in precision relative to the standard OLS estimates. In many of the Monte Carlo experiments he conducted, bias remained a problem. Furthermore, Inder (1993) claims that ( p.66) “ Modified OLS gives t-statistics whose sizes are generally no better than the OLS results.... The poor performance of such t-statistics suggests that in this case a very large sample is required for the asymptotics to take effect”.



Second, the single equation cointegration approach does not allow for multiple cointegrating vectors. If multiple vectors are present and only one is assumed, then this will lead to inefficiency when estimating the single equation model in the sense that all that can be obtained is an estimate of a linear combination of the vectors. In this case, the single equation approach can not validly identify and estimate the long run relationships among variables. But even if there is only one cointegrating vector, a single equation approach is potentially inefficient. There is loss of information unless each endogenous variable is explicitly modeled as part of a system of equations. The only exception is when all right-hand-side variables—the inputs of the production function—are weakly exogenous. Testing for weak exogeneity, however, requires a multivariate framework or a system type approach to cointegration.<sup>3</sup>

The Johansen's maximum likelihood procedure is a much more powerful approach for cointegration than the single-equation approach for two major reasons. First, it is based on a system or VAR approach that treats all variables in the model as potentially endogenous and explicitly estimates and allows for testing any potential feedback from output into the public capital stock (among other potential feedbacks). Second, it allows for multiple cointegrating relationships. If more than one cointegrating relationship is present, inference based on a single-equation approach for cointegration is invalid [Ericsson and Irons (1994)]. Whether or not there is more than one cointegrating relationship is testable with the Johansen's procedure. In addition, the Johansen procedure can address issues related to the dynamic interaction of variables and forms of endogeneity and "causality."

Batina (1998), Kamps (2005), and Pina and Aubyn (2005) use the Johansen procedure to investigate the cointegration properties of data on output, labor, private capital and public capital. Batina (1998) and Kamps (2005) use data for the U.S. and OECD countries respectively. Both find evidence of multiple cointegrating vectors, but no attempt for the identification of those vectors is made. Also, there is no mention of speed of adjustment coefficients. Simply the unidentified vectors are taken as is and then a cointegrated VAR is used to conduct impulse responses. Furthermore, there is no adjustment made for the skill level of the labor force. There is no treatment of technology in Batina's (1998) model, while Kamps (2005) captures technology only by a deterministic time trend allowed to enter the cointegrating space. Pina and Aubyn (2005) look at data for Portugal. They adjust the labor input by the average years of schooling. However, no measure for technology is used, not even a trend term restricted in the cointegrating space. Also, the production function is assumed to have constant returns to scale with respect to physical capital, public capital, and labor. This restriction is imposed rather than tested for. The Johansen approach yields one cointegrating vector, which they interpret as a production function. However, skill-adjusted

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<sup>3</sup> Urbain (1992) suggests that weak exogeneity tests based on single equation models—a Wu-Hausman-type orthogonality test for example—do not give clear results. He suggests testing for weak exogeneity in a system type framework.

labor and public capital are not significant, raising concerns of such interpretation. Also, there is no discussion of speed of adjustment coefficients.

Hamilton (1996) looks at data for the U.S.. He incorporates a measure for human capital based on educational costs, and proxies for the stock of knowledge using R&D stocks. Human capital is not significant. As will be discussed below, our human capital measure incorporates experience in addition to education, and hence is a conceptually better measure than that of Hamilton. Also, Hamilton estimates a single equation cointegrating model using the Fully Modified OLS estimator. This estimator is less powerful than Johansen's estimator [see text on the comparison of the single equation approach to cointegration to the system type approach of Johansen; see also footnote 2].

### III. The Model

Our approach and methodology address the criticism of Aschauer's approach as well as the shortcomings of the incomplete cointegration analysis used in subsequent work. In addition, we augment the production function to include measures for labor skills and technology. We aim to test (the existence of) and estimate a Cobb-Douglas specification for the aggregate production function:

$$Y_t = A_t^{\beta_1} KP_t^{\beta_2} KG_t^{\beta_3} L_t^{\beta_4} e^{\varepsilon_t} \quad (1)$$

where  $Y$  denotes output,  $A$  is the stock of knowledge/technology,  $KP$  is private capital,  $KG$  is public capital,  $L$  is human capital or skill-adjusted labor,  $\varepsilon_t$  is a white noise disturbance term, and the  $\beta$ s are elasticities to be freely estimated.

As mentioned earlier, we depart from the previous public capital literature in the treatment of the labor input. Straight labor hours (or employment) have been typically used in that literature, with no adjustment made to the skill or human capital embodied in those hours. This appears to be inconsistent with human capital-driven endogenous growth models [see, for example, the pioneering work of Lucas (1988)]. As the set of skills embodied in people, human capital has been hypothesized as an input into production that, alongside with physical capital and raw labor, increases a nation's output of goods and services. This paper improves on the previous public capital literature by adjusting hours worked for changes in human capital/skill. We use an interesting measure developed by the Bureau of Labor Statistics (1993) that broadens the concept of skill to include not only education, but also work experience. This is appealing since the level of work experience essentially captures skills accumulated outside the formal educational system, such as on-the-job training or learning by doing.

We also depart from the previous public capital literature in the treatment of technological progress. That literature typically approximated technological progress by including a time

trend in the production function, which imposes a deterministic growth rate to the pace of technological change. Instead, this paper follows the approach of Abdi and Joutz (2006) and exploits the historical time series of patent applications at the U.S. Patent Office to construct technology/knowledge stocks. This allows for modeling technology endogenously as part of a system of equations within the Johansen's cointegrating framework, and also allows for the estimation of the long run impact of the technology stock on aggregate output. Our treatment of technology is closer in spirit to the endogenous growth models of Romer (1990), Grossman and Helpman (1991a, 1991b), Aghion and Howit (1992), and Jones (1995) than the previous literature. These models emphasize "ideas" or knowledge as determinants of long run growth and model knowledge endogenously. Our approach proxies the stock of ideas/knowledge by the stock of patented ideas and also empirically models that stock as a potentially endogenous variable.

We control for recessions where energy shocks are considered a significant contributing factor through the use of dummy variables in the cointegrated VAR system. The early public capital literature, especially Tatom (1991), includes energy prices in the production function specification. This approach has been heavily criticized by Gramlich (1994) because it mixes production functions with cost functions. Since prices belong to a cost rather than a production function, the interpretation of Tatom's (1991) single equation cointegration analysis is not straightforward. Our approach avoids the inclusion of energy prices in the long run production function while still accounting for their effect.

#### IV. Data

The information or data set is annual and covers the postwar period for the U.S. economy, 1948 to 2004.

##### A. Output, Private Capital, and Public Capital

The measures for output, private capital, and public capital are fairly standard in the literature. Output  $Y_t$  depicts real production in the private business sector as compiled by the Bureau of Labor Statistics (BLS). It is measured in billions of chained 2000 dollars. Private capital in a given year  $t$ ,  $KP_t$ , is measured as the real net non-residential stock of private fixed assets (structures, equipment and software) at the end of the previous year, multiplied by the Federal Reserve Board's capacity utilization rate.<sup>4</sup> The capacity utilization adjustment is intended to capture the service flows from the level of the stock available [see, for example, Tatom (1991), and Hamilton (1996)]. Public capital in a given year  $t$ ,  $KG_t$ , is measured as the real net non-residential non-military stock of (federal, state, & local) structures, equipment and software at the end of the previous year. This series essentially captures the stock of public infrastructure in the U.S. economy. It includes highways, streets and roads, mass

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<sup>4</sup> The capacity utilization rate is for the manufacturing sector since this data for the total private industry is only available back to 1967.

transit and airport facilities, educational buildings, electric, gas and water supply facilities and distribution systems, and wastewater treatment facilities, among others. Both the private and public stocks are measured in billions of chained 2000 dollars and are obtained from the Bureau of Economic Analysis (BEA).

### B. Skill-Adjusted Labor

We measure labor  $L_t$  following the methodology of the Bureau of Labor Statistics (1993). Traditionally, aggregate labor input has been measured by the sum of hours worked by all workers (or the number of people employed). Virtually all studies that estimate production functions for the U.S. economy use this measure for labor. This approach implicitly assumes that workers are perfect substitutes or homogeneous, in the sense that an hour of work is weighted equally for different workers in the aggregate labor input measure. However, workers have different characteristics and are in that sense heterogeneous. The fact that firms pay workers different wages suggests that firms do treat workers as having different characteristics and different contributions to output.

The Bureau of Labor Statistics (1993) [henceforth BLS] starts with the premise that workers are heterogeneous and are of different skill levels—skill being defined as the education and experience achieved by workers. Then, it aggregates these heterogeneous workers with varying skill levels into an overall index of labor input. Specifically, the BLS compiles data on hours of work cross-classified by education and work experience for each sex. The hours of work of these different types of workers are then aggregated into an overall measure of the labor input  $L_t$ :

$$L_t = g(h_{1t}, h_{2t}, \dots, h_{nt}) \quad (2)$$

where  $h_{it}$  is hours of workers of type  $i$  at time  $t$  ( $i=1,2,\dots,n$ ). For example, men with a college degree and 10 years of experience constitute one type of workers, and say  $h_1$  denotes their hours. Women with a high school degree and five years of experience constitute another type, and say  $h_2$  denotes their hours, etc. There are 72 levels of work experience (0 to 71 years), and 7 levels of education for each sex<sup>5</sup>. There is, therefore,  $2*7*72=1008$  types of

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<sup>5</sup> The levels of education or categories of years of schooling are the following: 0-4 years of completed schooling, 5-8 years, 9-11 years, 12 years, 13-15 years, 16 years, and 17 years and over. As for work experience, The BLS collects information on work histories of individuals by exploiting data from Social Security records and Internal Revenue Service records. This information is matched with data on the individuals' demographic characteristics from the Current Population Survey. These characteristics include years of schooling, and potential experience (age minus years of schooling minus six), among others. This wealth of data makes it possible for the BLS to estimate a level of work experience for each worker based on that worker's demographic characteristics. For more details, see BLS (1993), pages 56-65.

workers (i.e.  $n$  in equation 2 is 1008). The BLS then calculates the growth rate of the labor input  $L_t$  using a Tornqvist index formula<sup>6</sup>, as follows:

$$\ln\left(\frac{L_t}{L_{t-1}}\right) = \sum_i s_{it}^* \ln\left(\frac{h_{it}}{h_{i,t-1}}\right), \text{ where} \quad (3)$$

$$s_{it}^* = \frac{s_{it} + s_{i,t-1}}{2}, \quad i = 1, 2, \dots, n, \text{ and} \quad (4)$$

$$s_{it} = \frac{w_{it} h_{it}}{\sum_i w_{it} h_{it}}, \quad i = 1, 2, \dots, n. \quad (5)$$

In equation (5),  $w_{it}$  denotes the wage rate per hour of workers of type  $i$  at time  $t$ . Therefore,  $s_{it}$  is the share of the total wage bill that is paid to workers of type  $i$  at time  $t$ , and  $s_{it}^*$  in equation (4) is the average share between period  $t-1$  and  $t$ . The growth rate of the labor input is then, according to equation (3), a weighted average of the growth rates of hours of the different types of workers, where the weights are the (average) shares of each type in total wage compensation.

With data on hours of the different types of workers compiled for each year, the implementation of equation (3) requires the computation of wages (hourly earnings) to calculate compensation shares, which are in turn used as weights on the growth rates of hours of each type of worker. The BLS estimates wages (hourly earnings) by adopting a standard Mincer (1974) type wage model that relates hourly earnings to years of education and experience. The fitted values of the wage model are used as estimates of the wage rates for the various types of workers. Hence, and intuitively speaking, the labor input  $L_t$  weighs the hours of a worker with a given level of education and experience by the rewards/productivity due to that level of education and experience.

The labor input can be expressed as the product of the total number of hours of all workers and an index that captures the skill composition of those hours. That is,

$$L_t = H_t LC_t \quad (6)$$

where  $L_t$  is again the labor input at time  $t$ ,  $H_t$  is the total number of hours of all workers at time  $t$ , i.e.  $H_t = \sum_i h_{it}$ , and  $LC_t$  is what the BLS calls the “labor composition index.” It essentially captures the skill (education and experience) composition of hours worked, and

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<sup>6</sup> In the index number literature, the Tornqvist index is sometimes called the Divisia index. For more details, see Diewert (1976).

can be interpreted as measuring the average skill/human capital level embodied in an hour of work. Given this interpretation, then as implied by equation (6), the labor input  $L_t$  can be thought of as measuring total skill or aggregate human capital in the economy. As mentioned above, virtually all studies estimating production functions for the U.S. economy use hours  $H_t$  as a measure of labor. This paper improves on them by using  $L_t$  that adjusts those hours for changes in human capital/skill.

The labor composition index  $LC_t$  generally increases due to two things. First, it increases when the distribution of hours changes toward workers that are more skilled. Intuitively, such a change in the distribution of hours leads to an increase in the mean of the distribution, that is, an increase in the average quantity of skill embodied in the workforce. Second, increases in the relative wages in favor of the skilled can in principle increase  $LC_t$  through their impact on compensation shares. Hence, the labor composition index reflects not only changes in the average *quantity* of skill of the workforce, as measured by the *quantity* (years) of education and work experience, but also changes in the *relative rewards* received for those skills. In other words, the labor composition index reflects changes in the average skill of the workforce, where both the *quantity of and return to* a worker's education and experience are regarded as measures of the underlying skills.

### C. The Knowledge Stock

We use counts of patent applications to proxy for technology/knowledge stocks. One must acknowledge upfront that patents are not a perfect measure of knowledge or technological innovation. For example, not all “knowledge” or “ideas” are patentable. Innovations such as the mass production system, new personnel and accounting practices, improvements in management practices, and financial innovations and new credit instruments such as credit cards are usually not embodied in a good and hence hard to patent. These innovations, however, are extremely hard to measure. There is simply no systematic data about innovations that are not patented [see Comin and Mulani (2005) for a discussion on this issue].

Although imperfect, patent applications do serve as a valuable resource for measuring innovative activity and have been extensively used in the patent literature as measures of technological change [see, for example, Hausman, Hall, and Griliches (1984), Griliches, Pakes, and Hall (1987), and Kortum (1997)]. Also, Griliches (1989 and 1990) argues that the *aggregate count of patents* can serve as a measure of shifts in technology. Joutz and Gardner (1996) and Abdih and Joutz (2006) argue that patent application trends are a good approximation for technological output over the long run. Firms have invested resources in developing a new technology, which they feel has economic value and they are willing to submit an application to capture rents from their initial investments. In addition, several studies have argued that (the inverse of) the relative price of capital is a good indicator of the quantity of economically useful knowledge [for example Krusell (1998), and Cummins and

Violante (2002)].<sup>7</sup> Samaniego (2005) actually compares (the inverse of) the relative price of capital with patent applications for the US economy over the post-war period and finds that the two series are highly positively correlated, which is supportive of the use of patent applications as a measure of knowledge. As such, this paper follows the patent literature and uses patent applications to construct knowledge stocks.<sup>8</sup>

The U.S. Patent Office provides information on the number of patent applications filed from 1840 to present. These include patents for invention, designs, and plants. We sum patents for invention, designs, and plants to get the total number of patent applications. Then we cumulate the total number of patent applications into a patent stock measure using the perpetual inventory method with an assumed depreciation rate of 15%.<sup>9</sup> The stock measure is constructed as an end of period stock. We lag it one period to reflect the stock in place. That is, we measure the knowledge stock available at time  $t$ ,  $A_t$ , by the amount in existence at the end of the previous time period  $t-1$ .

Using a particular rate of depreciation (usually 15 % for the U.S.) in constructing stocks is typical in the patent literature [e.g., Griliches (1989), Joutz and Gardner (1996), and Abdih and Joutz (2006)]. While this approach is ad-hoc and not necessarily justified by theory, researchers have typically checked the robustness of their results to changes in the depreciation rate. We experimented with constructing stocks using 0%, 5%, and 10% depreciation rates, and the results of the model(s) estimated below are robust to using these alternative depreciation rates.

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<sup>7</sup> However, while the relative price of capital has merit in so far as it captures embodied knowledge in the capital stock, it does not capture the sources of new knowledge which are not in physical capital.

<sup>8</sup> Note that we measure knowledge/technology using patent applications rather than patent grants. The lag between application and grants could be quite long and it varies over time partly due to changes in the availability of resources to the U.S. Patent Office. This notion is best articulated by Griliches (1990): “A change in the resources of the patent office or in its efficiency will introduce changes in the lag structure of grants behind applications, and may produce a rather misleading picture of the underlying trends. In particular, the decline in the number of patents granted in the 1970s is almost entirely an artifact, induced by fluctuations in the U.S. Patent Office, culminating in the sharp dip in 1979 due to the absence of budget for printing the approved patents.” This paper views patent applications as a much better measure of knowledge/ technology than patent grants. Also, It is widely believed that patent application data is a better measure of new knowledge produced in an economy than R&D expenditures [see, e.g., Joutz and Gardner (1996), and Abdih and Joutz (2006)]. The reason is that R&D expenditures are more properly thought of as inputs to technological change while patents are an output. Hence patent applications more closely approximate the output of the knowledge production function in standard R&D-based growth models [e.g., Romer (1990), and Jones (2002)] than R&D expenditures.

<sup>9</sup> We are aware that patents are heterogeneous in how much they contribute to knowledge. An ideal measure would probably weigh patents by their “importance” or “quality”. The work of Jaffe, Trajtenberg, and Henderson (1993) and Jaffe, Trajtenberg, and Fogarty (2000) have attempted to do that at the micro level, usually for a sample of patents in a cross-sectional framework, using patent citations as a measure of quality or importance. However, to our knowledge, there has been no attempt to address or measure how the distribution of patent quality has evolved at the aggregate level over time for the entire postwar period in the U.S. The data is simply not available. This probably explains why virtually all aggregate growth studies that use patents as an indicator of technology/knowledge have relied on patent counts following the tradition of Griliches (1990). Our paper follows this tradition.

## V. Initial Data Analysis and Reduction of the System

The objective is to estimate the long run impact of public capital on aggregate output within the theory of reduction [Hendry (1986)]. We begin with an information set that may capture the data generating process for aggregate production and economic growth. The data series are a priori assumed to be jointly distributed in a dynamic system environment. Causal links among the variables are not imposed but rather tested conditional on the information set. These links may relate to long-run relations and endogeneity. They can be direct and indirect. The notion of an aggregate production function and its components are estimated in this framework. We employ Johansen's (1988, 1991) maximum likelihood procedure that tests for and estimates cointegrating relationships in a Vector Error-Correction Model.

### A. Initial Plots and Integration Tests

Figure 1 shows the five series after transformation to natural logarithms. All the series exhibit upward trends and appear to more than double over the sample. Output grows by an average annual rate of about 3.5%; there are a few small dips for the recessions in 1974, 1980-82, 1991, and 2001. Private capital grows by about 3.1% annually on average and appears more volatile around the recessions where it is accentuated by the capacity utilization factor. The labor input grows by 1.4%, and falls during recessions. The growth appears to pick up in the 1980s and the 1990s supported in part by a strong growth in the labor composition index: relative wages increased significantly in favor of the highly educated and experienced; the average years of experience increased as the baby boomers matured; and the average years of schooling continued to rise during that period.

Public capital grows at an average annual rate of 3.1% over the entire sample. The growth rate is relatively high in the 1950s through the late 1960s, at about 4.4%, thereafter it seems to slow down significantly to 2.4%. The sharp rise of the public capital stock in the 1950s and 1960s was partly due to the building of the interstate highway system and the building of educational structures to accommodate the rapid increase in the school and college age population caused by the post World War II baby boom. Since 1970, the growth in the stock of highways and streets, and educational buildings has slowed down significantly, which contributed to the decline in the overall growth of public capital.<sup>10</sup>

The patent stock series grows at an average annual rate of 2.4% over the entire sample. However, prior to the mid 1980s the average annual growth was fairly steady at 1.2%. Thereafter, it increased sharply to 4.8%, reflecting a surge in discovery and innovation as documented in the literature: First, Greenwood and Yorukoglu (1997) document that the 1980s and 1990s witnessed "an explosion of formation of new firms and innovation in the high-tech industries, particularly in the information technology, biotechnology, and software

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<sup>10</sup> In 2004, the stock of highways and streets, and educational buildings accounted for nearly 50 percent of the total non-residential, non-military stock of public capital. Prior to that, the combined share of these two components was at least 50 percent.



industries.” Hence, the sharp increase in patenting may indicate a “technological revolution” as emphasized by these authors. Second, it is quite possible that the use of information technology itself in the discovery of new ideas might have substantially boosted research productivity. Arora and Cambardella (1994) argue that this was an important source of accelerating technological change. A third possibility, emphasized by Kortum and Lerner (1998), is that the sharp increase in patenting since the mid 1980s indicates an increase in innovation driven by improvements in the management of R&D. In particular, there has been a reallocation of resources from basic research toward more applied activities and hence a resulting surge in patentable discoveries. As Kortum and Lerner (1998, p. 287) point out “Firms are restructuring, redirecting and resizing their research organizations as part of a corporate-wide emphasis on the timely and profitable commercialization of inventions combined with the rapid and continuing improvement of technologies in use.”<sup>11</sup>

We now take a closer look at the univariate time series properties of the data, where we test for unit roots or the order of integration of the various series under consideration [All series in levels have been transformed to natural logarithms]. We perform the standard Augmented Dickey-Fuller (ADF) tests in both the levels and first differences of the variables of interest. Tables 2.A and 2.B provide the results for the variables in levels. Table 2.A uses a constant and trend in the ADF test while Table 2.B uses only a constant in the ADF regression. The tests are repeated for the first differences of the series in Tables 3.A and 3.B. There are five columns in the tables. The first column lists the variables [In all the tables in this paper, we use  $Lx$  to denote the natural logarithm of variable  $x$ ]. Columns two and three provide the ADF t-statistic and the implied coefficient on the lagged level term (equal to unity under the null hypothesis of a unit root). The forth column reports the number of lags on the dependent variable chosen using the Akaike Information Criterion (AIC), whose value is reported in the last column.

We cannot reject the null hypothesis of a unit root for all variables in levels. We, however, reject the null of a unit root for the first differences of output, labor, and private capital, suggesting that these variables are integrated of order one,  $I(1)$ , in levels.

The knowledge stock and public capital appear possibly  $I(1)$  in first differences – or equivalently  $I(2)$  in levels— since the null of a unit root cannot be rejected for the first differences of these variables. However, the plots of these two series in first differences (of the natural logarithms) reveal what appears to be a permanent shift in the mean starting in the mid 1980s for the patent stock and 1970 for the public capital stock. A single break in a stationary  $I(0)$  process can lead to non-rejection in the ADF test suggesting the series is  $I(1)$ . We examine this feature of the data generating process (DGP) for both series and test for it in

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<sup>11</sup> Samaniego (2005) compares (the inverse of) the relative price of capital— an alternative indicator of the quantity of economically useful knowledge — with patent applications for the US economy over the post-war period and observes that the growth of both series accelerated starting in the 1980s. This is consistent with the argument above that the surge in patenting, that started in the 1980s, is not “spurious” but rather reflects an actual increase in the rate of innovation in the U.S. economy.

two ways. First, a recursive regression analysis of the ADF equation reveals that the coefficient on the lagged level term is non constant with a break right where one might expect it: 1985 for the knowledge/patent stock and 1970 for the public capital stock. This approach does not place restrictions a priori on where the break occurs. Second, we use the Perron (1989) structural break procedure to test for whether there was a mean shift in the first difference processes that caused the I(1) findings. For either the patent stock or the public capital stock, we could not reject the (null) hypothesis of stationarity in the first difference process after correcting for the (structural) mean shift. We conclude that all the series are best characterized as I(1) in levels, or equivalently as stationary in first differences.

### B. VAR Model Specification and Estimation

Our analysis of the aggregate production function begins with the assumption that there is a multivariate distribution process among the variables, they are non-stationary, and a system approach is the appropriate specification. These features are testable. The production function is estimated beginning with an information set assumed to capture the local data generating process. The information set and assumptions have implications for the appropriate statistical methodology. While focusing on changes in output or production eliminates the problem of spurious regressions, it also results in a potential loss of information on the long-run interaction of variables [see for example, Davidson, Hendry, Srba, and Yao (1978)]. We examine the hypothesis of whether there exist economically meaningful linear combinations of the I(1) series: real private business sector output, the stock of knowledge (captured by patents), the stock of real private capital, the stock of real public capital, and skill-adjusted labor that are stationary or I(0). The Johansen (1988 and 1991) maximum likelihood procedure is used for the analysis. The procedure begins with specifying a VAR system,

$$Y_t = \sum_{i=1}^p \Pi_i Y_{t-i} + \Psi D_t + e_t ; \quad (7)$$

$$Y_t = \begin{bmatrix} \text{Ln Real Private Business Sector Output}_t \\ \text{Ln Patent Stock}_t \\ \text{Ln Real Private Capital Stock}_t \\ \text{Ln Real Public Capital Stock}_t \\ \text{Ln Skill - Adjusted Labor}_t \end{bmatrix}, e_t \sim IN(0, \Omega), \text{ and } D_t = \begin{bmatrix} \text{Constant} \\ \text{Stepdum86}_t \\ \text{Impulse74808291}_t \\ \text{Impulse97}_t \end{bmatrix}$$

where  $Y_t$  is  $(5 \times 1)$  and the  $\Pi_i$ s are  $(5 \times 5)$  matrices of coefficients on lags of  $Y_t$ .  $D_t$  is a vector of deterministic variables that can contain a constant, dummy-type variables, or other regressors considered to be fixed and non-stochastic. Finally,  $e_t$  is a  $(5 \times 1)$  vector of independent and identically distributed errors assumed to be normal with zero mean and

covariance matrix  $\mathbf{\Omega}$ —that is,  $e_t \sim \text{i.i.d. } N(\mathbf{0}, \mathbf{\Omega})$ . As such, the VAR comprises a system of 5 equations, in which the right-hand side of each equation includes a common set of lagged variables and deterministic regressors.

The deterministic regressors in the VAR include a constant and three dummy variables. The first dummy variable is *Stepdum86*, which takes the value of one after 1985 and zero otherwise. The inclusion of this variable is intended to capture the dramatic increase in measured knowledge growth—given here as patent activity—since the mid 1980s, as discussed in detail above. While we have adjusted for economic fluctuations and their impact on firms’ choices for employing private capital and labor in multiplicative transformations, there appears to be important information in a few recessions that is important for understanding the time series dynamics. The second dummy variable is closely aligned with recessions in 1974, 1980, 1982, and 1991 where energy shocks are considered a significant contributing factor. The individual impulse dummies for those years had the correct sign and could be combined into one dummy variable with no loss of explanatory power in the VAR—*Impulse74808291*, which takes the value of one in 1974, 1980, 1982, and 1991 and zero otherwise. The recessions in 1958, 1962, and 2001 do not have a similar effect. The third dummy variable is *Impulse97*, which is zero except for unity in 1997. This impulse captures institutional changes in the U.S. patent policy: the movement toward the typical international patent system policy of granting 20-year awards instead of 17-year awards, and the fact that 12-year patent renewal fees were collected for the first time in the United States [Kortum 1997], increasing the cost of patent applications. Statistically, the inclusion of *Stepdum86*, *Impulse74808291*, and *Impulse97* in the VAR results in a substantial improvement in the fit of the model and much better residual diagnostics, and ensures a statistically stable/constant VAR.

### C. Lag length selection of the VAR

The Johansen procedure assumes that the parameters of the VAR are constant over time and VAR residuals are white noise. In addition, the lag-length of the VAR is not known a priori, so some testing of lag order must be done. Therefore, prior to conducting the cointegration tests, the appropriate lag-length of the VAR must be determined and a constant/stable model found. VAR residuals should also appear close to the assumption of white noise.

A large number of lags is likely to produce an over-parameterized model. Too few lags, on the other hand, may induce autocorrelation in the residuals and hence violate the assumption of white noise. Our selection methodology starts with a VAR with an initial maximum of  $p$  lags, which is assumed here to be four given considerations for data frequency and sample size. Then we estimate VARs that include three lags, two lags, and one lag on each variable. We use information criteria and finite sample F-tests to guide the lag-length selection process.

For each VAR, Table 4.A reports values for the Schwartz Criterion (SC), the Hannan-Quinn Criterion (HQ), and the Akaike Information Criterion (AIC). These are used as alternative criterion in testing. The three statistics do not always agree on the same number of lags. That is the case here. The Akaike Information Criterion is minimized at four lags; the Hannan-Quinn value is smallest at three lags, and the Schwartz Criterion is minimized at two lags. Since we are working with limited number of annual observations a finite sample test may shed more light.

Table 4.B reports the results of lag length analysis using finite sample F-tests. There are three columns with a heading for the number of unrestricted lags. Below each is the realized F-statistic with the p-value reported in brackets for reducing the number of lags to the number in that row. We cannot reject the reduction from a VAR with four lags to a VAR with three lags; the F statistic is 1.38 with an associated p-value of 0.14. However, we strongly reject reductions to VARs with less than three lags. Hence, we proceed the analysis with a VAR that includes three lags on each variable.<sup>12</sup>

#### **D. Residual Diagnostics from the VAR Model**

Table 5 contains residual diagnostic tests for the VAR with three lags. These consist of tests performed on each equation of the VAR separately, and vector tests performed on the entire system. The individual equation tests cover a Breusch-Godfrey Lagrange Multiplier (LM) test for serial autocorrelation up to the second lag; the Jarque-Bera test for normality; an autoregressive conditional heteroscedasticity (ARCH) test; and White's test for heteroscedasticity. The null hypotheses for these tests are, respectively, no residual autocorrelation exists, the residuals are normally distributed, no residual ARCH process exists, and no residual heteroscedasticity exists. The realized values of the various test statistics and the associated tail probabilities are given in columns 4 and 5, respectively. None of the diagnostic tests reject the null hypothesis at the 5 percent significance level. Hence, the VAR residuals appear serially uncorrelated, normal, and homoscedastic.

The bottom partition of Table 5 presents the vector tests. The first test is for autocorrelation across all equations up to the second lag. The null hypothesis of no own and cross-equation autocorrelation cannot be rejected at least at the 5 percent significance level. The vector normality test statistic is 9.30 with 10 degrees of freedom and a p-value of 0.50, suggesting that the third and fourth moments reasonably capture a normal distribution. Finally, the vector test for heteroscedasticity does not reject the null hypothesis of homoscedasticity.<sup>13</sup>

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<sup>12</sup> When information criteria give different values for the lag-length of the VAR, it is common practice to prefer the Hannan-Quinn (HQ) criterion [see Johansen et al. (2000)]. It is interesting to note that the HQ criterion was minimized with a VAR that includes three lags on each variable. This is exactly the result we get based on the finite sample F-tests.

<sup>13</sup> For more details on the test statistics, see Doornik and Hendry (2001).

We conclude that the VAR residuals are consistent with the assumption of white noise disturbances.

### **E. Recursive Analysis for Model Constancy and Stability**

While the VAR residual properties appear to meet the white noise standard, the data generating process may have undergone changes, raising concerns about the stability of the coefficient estimates. An important aspect of diagnostic checking is then to test for model constancy/stability. For that purpose, we employ recursive estimation techniques and conduct recursively estimated Chow tests.

The basic idea behind recursive estimation is to fit the VAR to an initial sample of  $m-1$  observations, and then fit the VAR to samples of  $m, m+1, \dots$ , up to  $T$  observations, where  $T$  is the total sample size. Figure 2 shows the results from recursively estimating the VAR. Specifically, the figure shows two types of recursively estimated Chow tests for the VAR system, the 1-step Chow test (denoted *1up CHOWs*) and the N-down Chow test (denoted *Ndn CHOWs*). The 1-step Chow statistic tests whether the model based on the sample  $Y_1, \dots, Y_{m-1}$  is good at forecasting  $Y_m$  for every feasible  $m$ . The N-down or Break-Point Chow statistic tests whether the model based on the sample  $Y_1, \dots, Y_{m-1}$  is good at fitting all the remaining variables  $Y_m, \dots, Y_T$ . In other words, and for a given year, the N-down Chow statistic tests the null hypothesis that the coefficients estimated up to that year are the same as those estimated for the entire sample. The Chow statistics are normalized by the one percent critical value, so that the horizontal line at 1 gives the critical value. The results from the plots strongly suggest that the estimated coefficients of the VARs are constant over time.<sup>14</sup>

In summary, the above analysis indicates that the VAR is empirically well behaved and hence is a suitable starting point for the cointegration analysis. The cointegration analysis proceeds in several steps: testing for the existence of cointegration, interpreting and identifying the relationship(s), and inference tests on the coefficients from theory and weak exogeneity. Testing permits reduction of the unrestricted general model to a final restricted model without loss of information.

## **VI. Cointegration Analysis**

### **A. Testing for Cointegration**

Following Johansen and Juselius (1990), the VAR model in equation 7 provides the basis for cointegration analysis. Adding and subtracting various lags of  $Y_t$  yields an expression for the VAR in first differences:

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<sup>14</sup> See Doornik and Hendry (2001) for more details on the Chow tests.

$$\Delta Y_t = \pi Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \Psi D_t + e_t$$

where

( 8 )

$$\Gamma_i = -(\pi_{i+1} + \dots + \pi_p), \quad i = 1, \dots, p-1 \quad \text{and} \quad \pi \equiv \left( \sum_{i=1}^p \pi_i \right) - I.$$

If  $\pi$  is a zero matrix, then modeling in first differences is appropriate. There is no cointegration or long-run relation in this case. The matrix  $\pi$  may be of full rank or less than full rank, but of rank greater than zero. When  $\text{rank}(\pi) = 5$ , then the original series are not  $I(1)$ , but in fact  $I(0)$ ; modeling in differences is unnecessary. But, if  $0 < \text{rank}(\pi) \equiv r < 5$ , then the matrix  $\pi$  can be expressed as the outer product of two full column rank  $(5 \times r)$  matrices  $\alpha$  and  $\beta'$  where  $\pi = \alpha\beta'$ . This implies that there are  $(5-r)$  unit roots in  $\pi Y$ . The VAR model can then be expressed in error correction form. That is,

$$\Delta Y_t = \alpha\beta' Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \Psi D_t + e_t \quad (9)$$

The matrix  $\beta'$  contains the cointegrating vector(s) and the matrix  $\alpha$  has the weighting elements for the  $r$ th cointegrating relation in each equation of the VAR. The matrix rows of  $\beta' Y_{t-1}$  are normalized on the variable(s) of interest in the cointegrating relation(s) and interpreted as the deviation(s) from the long-run equilibrium condition(s). In this context, the columns of  $\alpha$  represent the speed of adjustment coefficients from the long-run or equilibrium deviation in each equation. If the coefficient is zero in a particular equation, that variable is considered to be weakly exogenous and the VAR can be conditioned on that variable.

The results of the Johansen cointegration test are presented in Table 6. The table is partitioned into two components. The first provides the tests for existence of cointegration and the second presents reduced rank standardized coefficients when there is a single cointegrating relation that is normalized on real output.

The top half of Table 6 reports the null hypothesis for the rank of the  $\pi$  matrix, the sorted eigenvalues of the  $\pi$  matrix, the Log likelihood value, the Trace and Max-Eigenvalue asymptotic test statistics (together with their associated  $p$ -values), and the Trace and Max-Eigenvalue statistics that make finite sample corrections by adjusting for degrees of freedom [see Reimers (1992)]. The Trace statistics test the null hypothesis that there are at most  $r$  cointegrating vectors against the alternative that there are more than  $r$  vectors, whereas the Max-Eigenvalue statistics test the null that there are  $r$  cointegrating vectors against the alternative that  $r+1$  exist.

The null hypothesis of no cointegration ( $r = 0$ ) is strongly rejected at the 1 percent significance level across all test statistics. The asymptotic Trace statistic rejects the null hypothesis of at most one cointegrating vector against the alternative that more than one vector exist. The asymptotic Max-Eigenvalue statistic rejects the null hypothesis of one cointegrating vector against the alternative that two vectors exist. However, Reimers (1992) suggests that in small samples these asymptotic tests tend to over-reject the null when it is true. Thus he suggests making finite sample corrections by adjusting these tests for degrees of freedom. Using Reimers adjusted Trace (Max-Eigenvalue) statistic, we cannot reject the null hypothesis of at most one (one) cointegrating vector against the alternative that more than one (two) exist. The adjusted Trace and Max-Eigenvalue statistics are 41.73 and 24.18 respectively with associated p-values of 0.39 and 0.17. We, therefore, conclude that there is a single cointegrating relation among the variables.

The second or bottom part of Table 6 presents the unrestricted standardized coefficients from the cointegrated VAR model. It reports the standardized eigenvector or cointegrating  $\beta$  vector normalized on real output, together with the associated standard errors. Our hypothesis is that the long-run or equilibrium relation may explain an aggregate production function.<sup>15</sup> Table 6 also reports the estimated speed of adjustment coefficients,  $\alpha$ , and their associated standard errors.

### **B. Cointegration, Weak Exogeneity, and Testing Restrictions on the Production Function**

We interpret the cointegrating relationship as a long run production function and the  $\beta$  coefficients as long run elasticities of private output with respect to the stock of knowledge, private capital, public capital, and skill-adjusted labor. All appear to be significant and the four inputs have signs consistent with theory. The knowledge elasticity is about 17%. The private and public capital stocks have elasticities of about 0.4 each, and the labor elasticity is 0.49.

Below, we interpret and compare these estimates to the literature after we test and impose acceptable restrictions on the model. The fourth column of Table 6 shows the estimated speed of adjustment coefficients,  $\alpha$ . If the cointegrating relation we have specified is appropriate (and stationary), then the speed of adjustment coefficient for output must be negative. In this case, the estimate is -0.71 and significant.

We can test if the remaining speed of adjustment coefficients are significant, that is whether the equations for the inputs of the production function are influenced by the cointegrating

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<sup>15</sup> We find that the cointegrating specification with a constant included provides the most stable results. Intuitively, we believe that while the series are I(1) with drift, there is a stochastic and deterministic component to the trend growth of the variables.

relation. Individually, the estimates are less than their respective standard errors for the stock variables (knowledge, private and public capital). Intuitively, this makes sense since the stock measures by definition are based on the beginning of period values. The speed of adjustment for labor is barely larger than its standard error. It may be weakly exogenous [Formal testing and discussion of weak exogeneity will be presented below].

Table 7 presents results from hypotheses tests about the cointegrating relation. There are four parts: hypotheses tests on the cointegrating  $\beta$  vector or production function relation, hypotheses tests on the speed of adjustment coefficients,  $\alpha$ , joint tests on both vectors, and the final reduced rank specification. These tests are distributed as Chi-squared under the null hypothesis.

First, we perform Johansen's (1995) multivariate stationarity tests for each variable in the second column. Testing for stationarity for a given variable amounts to testing the restriction that the cointegrating vector contains all zeros except for a unity corresponding to the designated variable. Empirically, we strongly reject the null hypothesis of stationarity for all variables. The Johansen test has higher power than the univariate ADF tests because it is multivariate and so involves a larger information set. Moreover, the null hypothesis is the stationarity of a given variable rather than non-stationarity. Stationarity may be a more appealing null hypothesis. That said, these rejections of stationarity are consistent with our individual equation ADF test results.

Then we conduct hypotheses tests for the individual  $\beta$  coefficients followed by tests relating to the production function. Table 6 suggested that each input in the production function has the correct sign. We now formally test the null hypothesis for whether there is zero explanatory power for each variable individually. All variables are strongly significant with p-values of less than 1 percent.<sup>16</sup> Two interesting hypotheses are further tested. The unrestricted estimates for private capital and public capital are numerically close. We test if they are equal; the test statistic is 0.22 with a p-value of 0.64. Theory is replete with references to Cobb-Douglas specifications with constant returns to scale with respect to the private inputs. Initially, we test if the elasticities for the private capital stock and skill-adjusted labor sum to unity. The Chi-square statistic is 1.90 with a p-value of 0.17 suggesting the hypothesis may be true. Then we combine the two tests for equal capital elasticities and constant returns to the private inputs. The test statistic is 2.32 with a p-value of 0.31. Thus, there is evidence in favor of the elasticities for private capital and public capital being the same and the aggregate production function exhibiting constant returns to scale with respect to private capital and skill-adjusted labor.

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<sup>16</sup> Knowledge, labor, private and public capital are also jointly strongly significant. The test statistic is 37 with a p-value of 0.000.



The second part of Table 7 reports tests on the  $\alpha$  or speed of adjustment coefficients without any restrictions on the  $\beta$  terms. We begin by testing whether the individual coefficients are statistically significant. If they are not, this suggests that that variable can be treated as weakly exogenous for further modeling of the cointegrating relation. Output is the only variable for which weak exogeneity is rejected: the test statistic is 5.58 with a p-value of 0.018. This is at least necessary for interpreting the equilibrium correction mechanism as one in terms of output.<sup>17</sup> The finding of weak exogeneity for the two physical capital stocks (and knowledge) is consistent with investment theory. Changes in the capital stocks (net investment expenditures) are the result of long term planning and time-to-build processes [See, for example, the seminal work of Kydland and Prescott (1982)] . A deviation from long-run output in one year is unlikely to impact investment expenditures in that year.<sup>18</sup>

Also, skill-adjusted labor appears to be weakly exogenous when tested individually, but not jointly with the other inputs. To show this, we conduct two additional tests. First, we test the null hypothesis that all four inputs are jointly weakly exogenous. This hypothesis is strongly rejected:  $\chi^2(4) = 14.00$  [0.0073] (the degrees of freedom are in parentheses and the p-value is in square brackets). Second, we test that the three stock series (knowledge, private and public capital) are weakly exogenous. The hypothesis is not rejected:  $\chi^2(3) = 1.97$  [0.58].

The third part of Table 7 presents joint tests for the  $\beta$  and  $\alpha$  vectors. The first joint hypothesis is that the production function exhibits constant returns to scale with respect to private capital and skill-adjusted labor, and the three stock variables are weakly exogenous. Public capital elasticity is unrestricted. This hypothesis cannot be rejected:  $\chi^2(4) = 4.10$  [0.39]. Then, we augment the previous hypothesis by restricting public capital and private capital to have the same elasticity. The chi-square statistic with five degrees of freedom has a p-value of 0.44 and hence the hypothesis cannot be rejected. Figure 3 presents a recursive plot of the likelihood ratio test statistics with these five joint restrictions. The restrictions on the cointegrating space appear to be stable with no values exceeding the critical value(s). Augmenting the above five restrictions with weak exogeneity of skill-adjusted labor is strongly rejected by the data:  $\chi^2(6) = 16.90$  [0.0096]. This confirms the above discussion that skill-adjusted labor may not be weakly exogenous. Therefore, our final model imposes only the five statistically accepted restrictions: constant returns to scale with respect to private capital and skill-adjusted labor; equal elasticities for the public and private capital stocks; and weak exogeneity of private capital, public capital, and the knowledge stock.

### C. The Final Long run Aggregate Production Function

The bottom of Table 7 presents the final reduced rank estimates for the aggregate production function system. We can express the relation in natural logarithms as follows:

<sup>17</sup> When we started testing the coefficients jointly, the output rejection continued to hold.

<sup>18</sup> Recall that all stock variables are constructed as beginning of period stocks.

$$\text{Output}_t = 0.13 \text{ Knowledge Stock}_t + 0.39 \text{ Private Capital}_t + 0.39 \text{ Public Capital}_t \\ + 0.61 \text{ Skill-Adjusted Labor}_t - 2.44$$

with Speeds of Adjustment

$$\alpha_{\text{output}} = -0.65 \text{ and } \alpha_{\text{labor}} = -0.28$$

The data appears to suggest there is evidence of an aggregate Cobb-Douglas production function with constant returns to scale to the private inputs. Hence, there is increasing returns to all inputs—inclusive of public capital and the knowledge stock— which can be motivated by (1) the possibility of considerable economies of scale resting behind the provision of public capital [Achauer (1989)] and (2) the notion that ideas/knowledge are non-rival [Romer (1990), and Jones (1995)]. The estimated elasticities of skill-adjusted labor and private capital roughly match their shares in national income, about 60% and 40% respectively.<sup>19</sup> It appears that private capital and public capital complement each other with equal estimated coefficients. In addition, the estimated knowledge/technology stock elasticity is 0.13, which matches the estimates found by Adams and Coe (1990) and Abdih and Joutz (2006) at the aggregate level for the U.S. economy (0.135 and 0.12 respectively). Our estimate also lies approximately at the upper end of the range of estimated elasticities typically found in studies based on firm- and industry-level data— between 0.01 and 0.1, as summarized by Griliches (1988). This is consistent with the idea that an aggregate production function may be better able to capture spillovers that increase the social and hence aggregate return to R&D. Finally, our estimate of the public capital elasticity is consistent with estimates for the U.S. economy found by Munnell (1990), Ford and Poret (1991), Crowder and Himarios (1997), and Batina (1999) [0.34, 0.30, 0.36 and 0.38 respectively], and is identical to that found by Aschauer (1989).

Our results indicate a strong long run effect of public capital on output. This strong and significant effect provides evidence counter to those studies that estimated production functions in differences and typically obtained insignificant coefficients for public capital [for example, Hulten and Schwab (1991), Tatom (1991), and Evans and Karras (1994)]. Once multivariate cointegration techniques are utilized to incorporate the long run information in the data, as this paper does, the public capital stock appears to exert a strong long run effect.

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<sup>19</sup> The estimated elasticity of skill adjusted labor, 0.61, is consistent with the coefficient on human capital typically used in growth accounting exercises. For example, Bosworth and Collins (2003) use a coefficient of 0.65. It is also consistent with recent estimates of the elasticity of output with respect to average educational attainment for OECD countries obtained by De la Fuente and Domenech (2002). The range of plausible values reported by the authors include 0.6, which they claim is consistent with available evidence on the returns to schooling from microeconomic wage equations.

Our results also provide some evidence against those studies that found no evidence of cointegration using a single equation residual-based test for the US economy [for example, Tatom (1991), Sturm and de Haan (1995), and Pereira and Flores de Frutos (1999)]. Our study differs from those studies in two major aspects. First, we use the multivariate Johansen test for cointegration, which is conceptually more powerful than the single equation residual-based test used by them. Second, we also show that the stock of patents/knowledge and labor skills belong to the cointegration space. In contrast, measured knowledge and labor skills are ignored in those studies. This may partially explain their results of lack of cointegration.<sup>20</sup>

The speeds of adjustment for output and labor are -0.65 and -0.28 respectively. They are negative (and significant), which is consistent with theory and error correction. If output is below its long run equilibrium path in the previous period, then a higher growth rate of labor and hence output is needed in the current period to move the system back to equilibrium. In other words, in the face of lagged disequilibrium in the cointegrating relation, labor and output jointly move the system back to equilibrium. This finding supports our system approach to estimating the production function.

If a single equation approach had been adopted instead, we would have invalidly conditioned on labor; the result would have been biased and inconsistent estimates of the parameters of the production function. The magnitudes of the speed of adjustment coefficients suggest that shocks to the economy are less persistent for output than labor. Labor market fluctuations are not as volatile relative to output fluctuations as firms adjust labor more slowly for reasons like hoarding. Moreover, the ratio of the speeds of adjustment is similar to the Okun's Law relation.

Figure 4 shows the short run changes or deviations in output from its long run path given by the aggregate production specification. The movements are close to output fluctuations of the last 55 years. We see slowdowns or recessions in 1954 following the Korean War, 1958, 1963, 1969, 1974-75, 1980, and 1991. In addition, the slow recoveries in the early 1980s and 1990s are evident.

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<sup>20</sup> Hamilton (1996) found evidence of cointegration using a single equation residual-based test for the U.S. economy. He also incorporated in the production function a measure for human capital—constructed as the stock of accumulated educational costs for primary, secondary, and college education—and a measure for the stock of knowledge using R&D stocks. Moreover, Hamilton estimated a single equation cointegrating model using the Fully Modified OLS estimator. Human capital was included as a separate input and not multiplicative with raw hours, which is not very intuitive. It was not significant in the estimated regression. Our human capital measure is conceptually better, since it exploits the experience dimension of skill in addition to education, and it enters multiplicatively with raw hours. Also, the R&D stock elasticity in Hamilton (1996) was 0.04, much less than estimates reported in the literature for the US at the aggregate level. For example, Adams and Coe (1990) and Abdi and Joutz (2006) report estimates of 0.135 and 0.12 respectively. The public capital elasticity was 0.17, less than Aschauer's (1989) estimate and ours. Finally, Hamilton uses the Fully Modified OLS estimator, which Inder (1993) found to have made little or no improvement in precision relative to the standard OLS estimator. In many of the Monte Carlo experiments he conducted, bias remained a problem [see also footnote 1].

Previous criticisms of the single equation aggregate production function models have stressed that these models ignore feedback effects running from output to capital. In particular, several studies have argued that the large estimated elasticity of output with respect to public capital found by Aschauer (1989) may capture reverse causation or a feedback effect running from output to public capital [for example, Musgrave (1990), Eisner (1991), Hulten and Schwab (1993), among others].

Our system modeling approach, in contrast, incorporates the dynamic relations between output and the inputs in the estimation process. We test whether such a feedback effect exists by performing block exogeneity or Granger causality tests in the cointegrated VAR model. The estimated coefficients on the once and twice lagged changes in output in the equation explaining the current change in public capital are 0.07 [0.016] and 0.006 [0.88] respectively; the numbers in square brackets are p-values for testing the null of zero coefficients on those variables individually. The test statistic for the null hypothesis that lagged changes in output do not jointly explain current changes in the public capital stock is a chi-square with two degrees of freedom. The test statistic is 5.89 with a p-value of 0.053. Hence, there is some evidence that public investment responds positively to private output changes, as suggested by the critics.<sup>21</sup> However, this effect can exist without necessarily tainting the coefficient on public capital in our estimated production function.

It is interesting that although we account for the econometric problems highlighted in the literature, including the potential endogeneity of public capital, we still obtain an estimated public capital elasticity the same as that of Aschauer (1989). Aschauer estimated a static production function that involved non-stationary variables. Under cointegration, the OLS estimate of the estimated elasticity of public capital is superconsistent, and hence asymptotically, the potential endogeneity of public capital may not matter. This may partially explain why we obtain the same estimated elasticity of public capital as Aschauer (1989). However, a caveat to this argument is the finite sample bias typically found in OLS estimates of static regressions of the cointegrating relation.<sup>22</sup>

## VII. Growth Accounting for the Postwar U.S. Economy

The estimated parameters of the aggregate production function may be used to investigate the post-1973 slowdown in productivity and its partial recovery since the mid 1980s. We show these calculations in Table 8. Exploiting constant returns to scale with respect to private

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<sup>21</sup> Coupled with the finding of weak exogeneity of public capital, this means that public capital may not be strongly exogenous. We also found a feedback effect running from output to private capital. The test statistic for the null hypothesis that lagged changes in output do not explain current changes in private capital is a chi-square with two degrees of freedom. The statistic is 22.23 with p-value of 0.00. The joint test for whether there is no impact of past changes in output on the current change in private and public capital is a chi-square with four degrees of freedom, has value of 24.67, and is significant at one percent.

<sup>22</sup> We thank Peter Pedroni for these comments.

capital and skill-adjusted labor, and recalling that skill-adjusted labor is the product of hours worked and the labor composition index, the long run production function can be written (in natural logarithms) as:

$$\text{Output per hour}_t = 0.13 \text{ Knowledge Stock}_t + 0.39 \text{ Public Capital}_t + 0.39 \text{ Private Capital per hour}_t + 0.61 \text{ Labor Composition}_t - 2.44$$

The average annual growth rate of output per hour (labor productivity) fell from 3.25 percent in the period 1949-1973 to 1.62 percent in the period 1973-1985. The average annual growth rates of the knowledge stock, public capital, private capital per hour, and labor composition also fell between these two subsamples by 0.23, 1.98, 1.74, and 0.05 percentage points respectively. Given the estimated parameters of the aggregate production function, this implies that the contributions of the knowledge stock, public capital, private capital per hour, and labor composition to the 1.63 percentage point decline in labor productivity growth are 2, 48, 42, and 2 percent respectively. These numbers are broadly consistent with prior research. Griliches (1988), Dean and Kunze (1988), and Baily and Chakrabarti (1988) find that R&D did not play a major role in the productivity slowdown. The BLS (1993) find similar results regarding labor composition. In fact, our estimate of the percent contribution of labor composition to the productivity slowdown is about the same as theirs. Our relatively large estimate of the percent contribution private capital per hour to the slowdown is consistent with Baily (1981) who argued that the flow of services from private capital has deteriorated significantly during that period and that this deterioration may account for a significant portion of the productivity slowdown. The public capital contribution of 48 percent to the slowdown is in the range of the estimates reported in the literature. At the upper end of the reported estimates is the study of Munnell (1990)—about 78 percent. Hamilton (1996) and Lynde and Richmond (1993) report more conservative estimates of about 27 percent and 40 percent respectively.

The average annual growth rate of output per hour increased from 1.62 percent in the period 1973-1985 to 2.27 percent in the period 1985-2004. The average annual growth rates of the knowledge stock, public capital, private capital per hour, and labor composition also increased between these two subsamples by 3.61, 0.13, 0.01, and 0.30 percentage points respectively. While public capital accounted for about half of the productivity slowdown, it only accounted for about 8 percent of the 0.65 percentage point subsequent increase in labor productivity growth. The largest contribution came from the knowledge/patent stock, 70 percent, followed by labor composition, 28 percent. Private capital per hour accounted for a mere 1 percent of the increase in labor productivity growth. These contributions were partially offset by a 7 percent negative contribution of a residual term.

The above numbers suggest that the partial rebound of labor productivity growth is mostly due to a strong growth in knowledge and human capital/labor composition witnessed over the period 1985-2004. The strong knowledge/patent growth captures the surge in discovery and

innovation that we documented in section IV. As for labor composition, the strong growth is due to the fact that: relative wages increased significantly in favor of the highly educated and experienced; the average years of experience increased as the baby boomers matured; and the average years of schooling continued to rise during that period.

### **VIII. Concluding Remarks**

In this paper, we estimate an aggregate production function for the U.S. economy over the period 1948-2004 incorporating human capital in the labor input, the stocks of private capital and public capital, and the stock of knowledge (patents). Our results indicate that there is a strong long run positive effect of public capital, human capital, and the stock of knowledge on aggregate output.

We find that the estimated long run elasticity of output with respect to the public capital stock is positive, large and significant. The point estimate is 0.39, consistent with the earlier estimates found by Aschauer (1989) and Munnell (1990). The result of a strong positive impact of public capital on aggregate output provides some evidence against those who attributed this impact to spurious correlations [for example, Aaron (1990), and Tatom (1991,1993)] and/or endogeneity of the public capital stock [Musgrave (1990), Eisner (1991), Hulten and Schwab (1993), among others]. The paper explicitly addresses these two potential problems by utilizing the Johansen's system cointegrating approach, and still finds a large positive and significant impact. In particular, and as far as the endogeneity issue is concerned, the paper finds some evidence that public investment responds positively to private output changes. However, the paper also finds that this effect can exist without necessarily tainting the coefficient of public capital in an estimated production function.

The paper also sheds some light on the importance of the stock of knowledge and human capital for aggregate output in the long run. Both variables are significant when incorporated in the aggregate production function. The estimated long run elasticity of output with respect to the stock of knowledge (proxied for by the patent stock) is 0.13, which is consistent with both the macroeconomic and microeconomic studies on the return to R&D. The estimated long run elasticity of output with respect to human capital/skill-adjusted labor is 0.61, which is consistent with human capital-driven endogenous growth models [see, for example, Lucas (1988)], and estimates of the share of labor in national income.

Our research also demonstrates that the production function should be looked at as a multivariate cointegrating system, and hence the results here may invalidate results based on the estimation of production functions in first differences, or even using single-equation-based cointegration tests. The former approach typically found insignificant coefficients on public capital [for example, Hulten and Schwab (1991), Tatom (1991), and Evans and Karras (1994)]. The latter approach either found no evidence for cointegration with U.S. data [for example Tatom (1991), Sturm and de Haan (1995), and Pereira and Flores de Frutos (1999)],

or detected cointegration but with a typically smaller public capital elasticity than Aschauer's (1989) [see Hamilton (1996) and footnote 20].

Our results also indicate that the short run changes or deviations in output from its long run path given by the aggregate production specification are close to output fluctuations of the last 55 years. Moreover, the error correction components or speed of adjustment coefficients for output and labor reveal an Okun's Law relation.

When the parameters of the aggregate production function are used to conduct a simple growth accounting exercise for the postwar U.S. economy, we find several interesting results: public capital accounts for about half of the post-1973 productivity slowdown, but only plays a minor role in the partial recovery of labor productivity growth since the mid 1980s. The largest contribution to that (partial) recovery comes from the knowledge stock and human capital.

An implication one might draw from this paper is that skills, technology, and capital—whether physical or public—are important components in determining and explaining economic growth. Advocates of only one type are mistaken. We see the changing importance and role of each as the U.S. economy evolves.

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**Table 1. The Information Set: Data Series from 1948 to 2004**

Variable	Description
$Y_t$	Real output in the private business sector measured in billions of (chained/constant) 2000 dollars. Source: Bureau of Labor Statistics (BLS).
$KP_t$	<p>Real net stock of nonresidential private fixed assets at the end of the previous year, multiplied by the Federal Reserve Board's capacity utilization rate.</p> <p>The real net stock of nonresidential private fixed assets is measured in billions of chained 2000 dollars; it consists of structures, equipment and software, and is obtained from the Bureau of Economic Analysis (BEA):  <a href="http://www.bea.gov/bea/dn/FA2004/DownSS2.asp">http://www.bea.gov/bea/dn/FA2004/DownSS2.asp</a></p> <p>Capacity utilization rates ( output as a fraction of capacity) are for the manufacturing sector since this data for the total private industry is only available back to 1967. This data is obtained from various issues of <i>The Economic Report of the President</i> , available on-line at:  <a href="http://www.gpoaccess.gov/eop/download.html">http://www.gpoaccess.gov/eop/download.html</a></p>
$KG_t$	<p>Real net stock of nonresidential, non-defense government (federal, state, local) structures, equipment and software at the end of the previous year. This data is measured in billions of chained 2000 dollars and is obtained from the Bureau of Economic Analysis (BEA):  <a href="http://www.bea.gov/bea/dn/FA2004/DownSS2.asp">http://www.bea.gov/bea/dn/FA2004/DownSS2.asp</a></p>
$L_t$	<p>Skill-adjusted labor input / human capital is the product of the number of total hours worked in the private business economy (<math>H_t</math>) and the labor composition index (<math>LC_t</math>). That is, <math>L_t = H_t LC_t</math>. The labor composition index captures the average skill level embodied in an hour of work, where both the quantity of and the return to education and experience are regarded as measures of the underlying skills. Data for hours, labor composition, skill-adjusted labor input, and private sector output are obtained from Larry Rosenblum at the Office of Productivity and Technology, the Bureau of Labor Statistics, U.S. Department of Labor.</p>
$A_t$	<p>Stock of Knowledge is the stock of total patent applications at the end of the previous year. This series is constructed by cumulating the numbers of total patent applications filed at the U.S. Patent Office into a stock measure, using the perpetual inventory method with a 15% depreciation rate. The patent applications data is obtained from the U.S. Patent Office:  <a href="http://www.uspto.gov/web/offices/ac/ido/oeip/taf/h_counts.htm">http://www.uspto.gov/web/offices/ac/ido/oeip/taf/h_counts.htm</a></p>

**Table 2.A. Augmented Dickey-Fuller Tests for Variables in Levels, Sample 1953-2004, Constant and Trend Included**

Critical Values 5% = -3.50, 1% = -4.14				
Variable	t-ADF	$\pi+1$	Lags	AIC
LY	-3.268	0.646	1	-7.277
LA	1.875	1.029	2	-9.615
LKP	-2.869	0.691	0	-6.197
LKG	-3.440	0.986	3	-12.180
LL	-2.530	0.835	0	-7.605

**Table 2.B. Augmented Dickey-Fuller Tests for Variables in Levels, Sample 1953 – 2004, Constant Included**

Critical Values 5% = -2.92, 1% = -3.56				
Variable	t-ADF	$\pi+1$	Lags	AIC
LY	-0.199	0.999	0	-7.153
LA	2.261	1.015	1	-9.642
LKP	-0.770	0.990	0	-6.088
LKG	-2.702	0.997	2	-12.090
LL	0.675	1.009	2	-7.513

Notes:

(1) For a given variable  $x$ , the Augmented Dickey-Fuller equation with a constant and trend included has the following form:

$$\Delta x_t = \pi x_{t-1} + \sum_{i=1}^p \theta_i \Delta x_{t-i} + a + \delta t + \varepsilon_t$$

and the specification with only a constant included simply excludes the trend term.

$\varepsilon_t$  is assumed to be white noise. For a given variable, the table reports the number of lags on the dependent variable,  $p$ , chosen using the Akaike information Criterion (AIC) (whose value is also reported), and the augmented Dickey-Fuller statistic, t-ADF, which is the t-ratio on  $\pi$ . The statistic tests the null hypothesis of a unit root in  $x$ , i.e.  $\pi = 0$ , against the alternative of stationarity. The table also reports the coefficient on the lagged level of  $x$  in the above specification where the dependent variable is  $x$ , i.e.,  $\pi+1$ .

(2) The symbols \* and \*\* denote rejection of the null hypothesis at the 5% and 1% critical values respectively.



**Table 3.A. Augmented Dickey-Fuller Tests for Variables in First Differences, Sample 1953-2004, Constant and Trend Included**

Critical Values 5% = -3.50, 1% = -4.14					
Variable	t-ADF		$\pi+1$	Lags	AIC
DLY	-6.808	**	0.028	0	-7.115
DLA	-2.549		0.778	0	-9.607
DLKP	-5.796	**	-0.614	2	-6.066
DLKG	-2.338		0.904	1	-12.030
DLL	-6.031	**	-0.121	1	-7.529

**Table 3.B. Augmented Dickey-Fuller Tests for Variables in First Differences , Sample 1953 – 2004, Constant Included**

Critical Values 5% = -2.92, 1% = -3.56					
Variable	t-ADF		$\pi+1$	Lags	AIC
DLY	-6.878	**	0.028	0	-7.153
DLA	-1.780		0.901	0	-9.581
DLKP	-6.415	**	-0.339	1	-6.093
DLKG	-1.266		0.963	1	-11.990
DLL	-5.915	**	-0.080	1	-7.542

Notes:

(1) For a given variable x, the Augmented Dickey-Fuller equation with a constant and trend included has the following form:

$$\Delta x_t = \pi x_{t-1} + \sum_{i=1}^p \theta_i \Delta x_{t-i} + a + \delta t + \varepsilon_t$$

and the specification with only a constant included simply excludes the trend term.

$\varepsilon_t$  is assumed to be white noise. For a given variable, the table reports the number of lags on the dependent variable, p, chosen using the Akaike information Criterion (AIC) (whose value is also reported), and the augmented Dickey-Fuller statistic, t-ADF, which is the t-ratio on  $\pi$ . The statistic tests the null hypothesis of a unit root in x, i.e.  $\pi = 0$ , against the alternative of stationarity. The table also reports the coefficient on the lagged level of x in the above specification where the dependent variable is x, i.e.,  $\pi+1$ .

(2) The symbols \* and \*\* denote rejection of the null hypothesis at the 5% and 1% critical values respectively.

**Table 4.A Lag Length Analysis: Selected Statistics**

<b>Lags</b>	<b>Log-Lik</b>	<b>SC</b>	<b>HQ</b>	<b>AIC</b>
4	1012.5509	-29.220	-31.966	-33.681
3	981.92743	-29.937	-32.111	-33.469
2	946.53153	-30.474	-32.076	-33.077
1	854.30035	-28.867	-29.896	-30.54

Notes :

The VARs include the variables (LY, LA, LKP, LKG, LL), a constant (restricted), and three dummy variables: Stepdum86, Dum97, and Dum74808291. The sample is 1952 through 2004.

**Table 4.B. Lag Length Analysis: F-Tests for Model Reduction**

<b>Restricted Models</b>	<b>Unrestricted Models</b>		
	<b>4 Lags</b>	<b>3 Lags</b>	<b>2 Lags</b>
<b>3 Lags</b>	1.3774 [0.1372]		
<b>2 Lags</b>	1.7062 [0.0099]**	1.9549 [0.0095]**	
<b>1 Lag</b>	4.0973 [0.0000]**	5.2565 [0.0000]**	8.1651 [0.0000]**

Notes :

The VARs include the variables (LY, LA, LKP, LKG, LL), a constant (restricted), and three dummy variables: Stepdum86, Dum97, and Dum74808291. The sample is 1952 through 2004.

**Table 5. Individual Equation and Vector Misspecification Tests for the VAR Model of the Production Function**

Equation	Test	Statistic	Value	P-Value
LY	AR 1-2 test:	F(2,33)	0.4302	0.654
LA	AR 1-2 test:	F(2,33)	2.3728	0.1089
LKP	AR 1-2 test:	F(2,33)	0.8982	0.417
LKG	AR 1-2 test:	F(2,33)	1.7477	0.1899
LL	AR 1-2 test:	F(2,33)	2.9450	0.0666
LY	Normality test	Chi <sup>2</sup> (2)	0.1467	0.9293
LA	Normality test	Chi <sup>2</sup> (2)	5.7030	0.0578
LKP	Normality test	Chi <sup>2</sup> (2)	1.5407	0.4628
LKG	Normality test	Chi <sup>2</sup> (2)	4.6064	0.0999
LL	Normality test	Chi <sup>2</sup> (2)	0.9867	0.6106
LY	ARCH 1-1 test:	F(1,33)	0.9574	0.335
LA	ARCH 1-1 test:	F(1,33)	0.6796	0.4156
LKP	ARCH 1-1 test:	F(1,33)	0.0683	0.7954
LKG	ARCH 1-1 test:	F(1,33)	0.1107	0.7414
LL	ARCH 1-1 test:	F(1,33)	0.0024	0.9609
LY	hetero test:	F(30,4)	0.1676	0.9988
LA	hetero test:	F(30,4)	0.2749	0.9844
LKP	hetero test:	F(30,4)	0.2789	0.9833
LKG	hetero test:	F(30,4)	0.2390	0.9918
LL	hetero test:	F(30,4)	0.2154	0.9951
Vector Tests		Statistic	Value	P-Value
Vector AR 1-2 test:		F(50,99)	1.4679	0.0532
Vector Normality test:		Chi <sup>2</sup> (10)	9.2996	0.5039
Vector hetero test:		Chi <sup>2</sup> (450)	467.87	0.2709

Notes :

The VARs includes three lags on each of the variables (LY, LA, LKP, LKG, LL), a constant (restricted), and three dummy variables: Stepdum86, Dum97, and Dum74808291. The sample is 1951-2004.

**Table 6. Cointegration Analysis with Johansen's Test**

H0: rank	Eigenvalue	Loglik for rank	Trace test	[ Prob]	Max test	[ Prob]	Trace test (T-nm)	[ Prob]	Max test (T-nm)	[ Prob]
0		939.352	122.47	[0.000]**	64.68	[0.000]**	88.45	[0.004]**	46.72	[0.001]**
1	0.698	971.694	57.78	[0.021]*	33.47	[0.008]**	41.73	[0.391]	24.18	[0.171]
2	0.462	988.431	24.31	[0.448]	13.96	[0.478]	17.56	[0.857]	10.08	[0.823]
3	0.228	995.412	10.35	[0.612]	8.63	[0.486]	7.47	[0.860]	6.23	[0.760]
4	0.148	999.727	1.72	[0.825]	1.72	[0.824]	1.24	[0.903]	1.24	[0.903]
5	0.031	1000.585								

**Reduced Rank Standardized Coefficients**

Variable	Beta Vector	Std Err	Alpha Vector	Std Err
LY	1	0	-0.706	0.285
LA	-0.168	0.032	-0.033	0.085
LKP	-0.438	0.071	-0.063	0.493
LKG	-0.370	0.046	0.026	0.030
LL	-0.487	0.091	-0.336	0.251
Constant	2.698	0.258		

Notes :

The VARs includes three lags on each of the variables (LY, LA, LKP, LKG, LL), a constant (restricted), and three dummy variables: Stepdum86, Dum97, and Dum74808291. The sample is 1951-2004.

**Table 7. Hypotheses Tests on the Cointegrating Relation**

Hypotheses Tests for the Beta Vector				
Variable	Variable Stationarity Chi^2(4)		Zero Beta Coefficient Chi^2(1)	
LY	37.006	[0.0000]**		
LA	39.851	[0.0000]**	13.120	[0.0003]**
LKP	36.054	[0.0000]**	10.283	[0.0013]**
LKG	34.091	[0.0000]**	24.057	[0.0000]**
LL	39.035	[0.0000]**	19.653	[0.0000]**
Constant			18.352	[0.0000]**
Linear Beta Restrictions				
LKP = LKG	Chi^2(1)	0.219	[0.6402]	
LKP + LL = 1	Chi^2(1)	1.9045	[0.1676]	
LKP = LKG and LKP + LL = 1	Chi^2(2)	2.319	[0.3137]	
Hypotheses Tests for the Alpha Vector: Weak Exogeneity				
Variable	Zero Alpha Coefficient [Chi^2(1)]			
LY	5.577	[0.0182]*		
LA	0.16203	[0.6873]		
LKP	0.01627	[0.8985]		
LKG	0.933	[0.3340]		
LL	1.917	[0.1662]		
Joint Zero Alpha Coefficients [Chi^2(.)]				
LA = LKP = LKG = LL = 0	Chi^2(4)	14.002	[0.0073]**	
LA = LKP = LKG = 0	Chi^2(3)	1.971	[0.5784]	
Joint Hypothesis Tests for the Alpha and Beta Vectors: Weak Exogeneity and Linear Restrictions for the Aggregate Production Function				
(a) Beta Restrictions: LKP + LL = 1				
Alpha Restrictions: LA = LKP = LKG = 0	Chi^2(4)	4.096	[0.3932]	
(b) Beta Restrictions: LKP + LL = 1; LKP = LKG				
Alpha Restrictions: LA = LKP = LKG = 0	Chi^2(5)	4.781	[0.4432]	
(c) Beta Restrictions: LKP + LL = 1; LKP = LKG				
Alpha Restrictions: LA = LKP = LKG =LL= 0	Chi^2(6)	16.904	[0.0096]**	
Final Reduced Rank Standardized Coefficients (b)				
Variable	Beta Vector	Std Err	Alpha Vector	Std Err
LY	1	0.000	-0.652	0.072
LA	-0.12693	0.016	0.000	0.000
LKP	-0.39253	0.000	0.000	0.000
LKG	-0.39253	0.000	0.000	0.000
LL	-0.60747	0.005	-0.280	0.084
Constant	2.43840	0.197		

Notes :

(1) The VARs includes three lags on each of the variables (LY, LA, LKP, LKG, LL), a constant (restricted), and three dummy variables: Stepdum86, Dum97, and Dum74808291. The sample is 1951-2004.

**Table 8. Growth Accounting for the Postwar U.S. Economy**

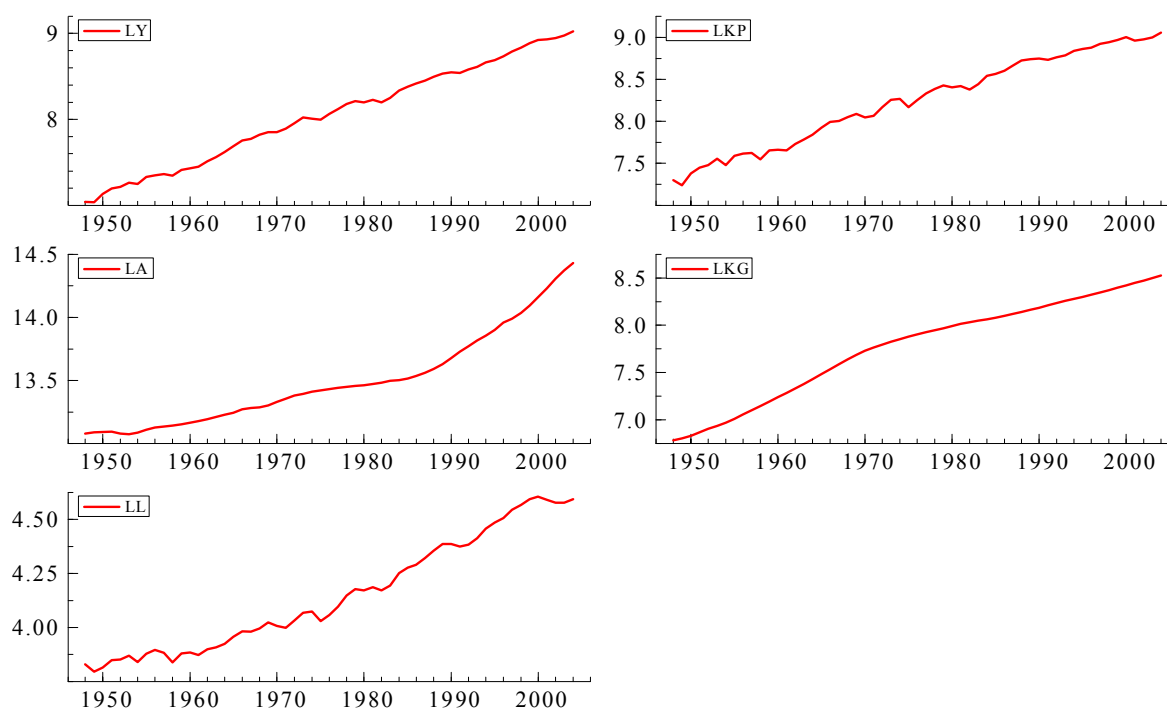
<b>Period</b>	<b>Average Annual Growth Rate of Output per hour (percent per year)</b>	<b>Contribution of (in percentage points):</b>				
		<b>Knowledge</b>	<b>Public Capital</b>	<b>Private Capital per hour</b>	<b>Labor Composition</b>	<b>Residual</b>
1949-2004	2.54	0.31	1.22	0.83	0.22	-0.040
1949-1973	3.25	0.16	1.63	1.23	0.16	0.063
1973-1985	1.62	0.13	0.86	0.54	0.14	-0.048
1985-2004	2.27	0.59	0.91	0.55	0.32	-0.092
Subsample change:						
(1973-1985 minus 1949-1973)	-1.63	-0.03	-0.78	-0.68	-0.03	-0.11
percentages		-2	-48	-42	-2	-7
(1985-2004 minus 1973-1985)	0.65	0.46	0.05	0.003	0.18	-0.04
percentages		70	8	1	28	-7

Notes:

(1) For any given period , the contribution of a given input to the growth of output per hour is calculated as the average annual growth rate of the input weighted by its estimated elasticity in the aggregate production function.

(2) Contributions may not sum to the growth of output per hour due to rounding.

**Figure 1. The Variables in Natural Logarithms**



**Notes:**

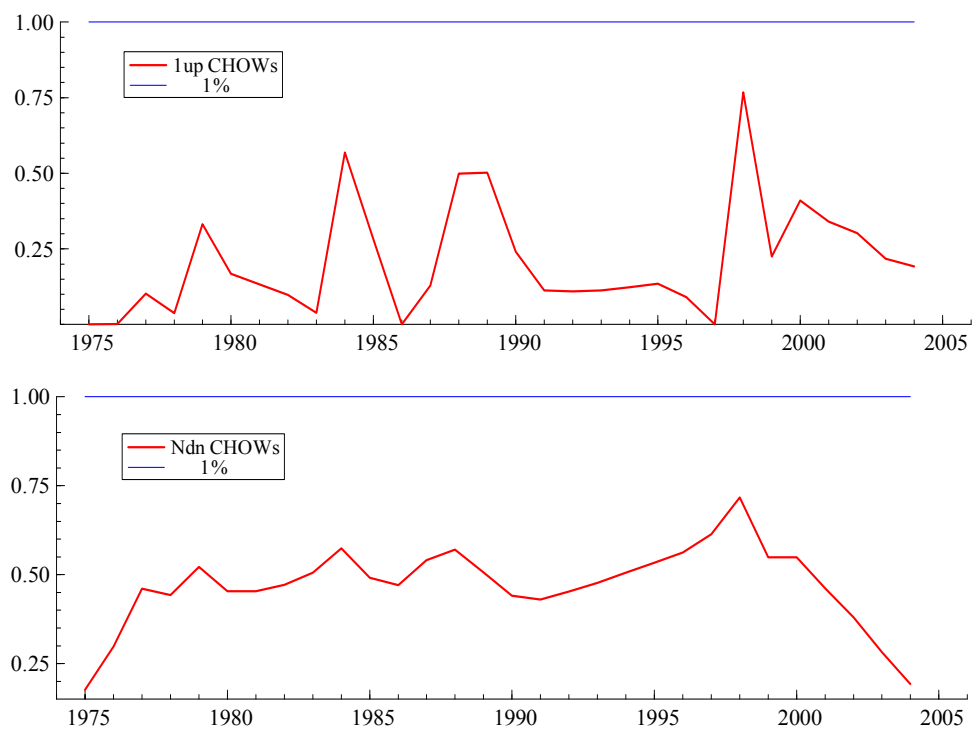
LY: Log of output in the private business sector.

LKP: Log of the private capital stock, adjusted for capacity utilization.

LKG: Log of the public capital stock.

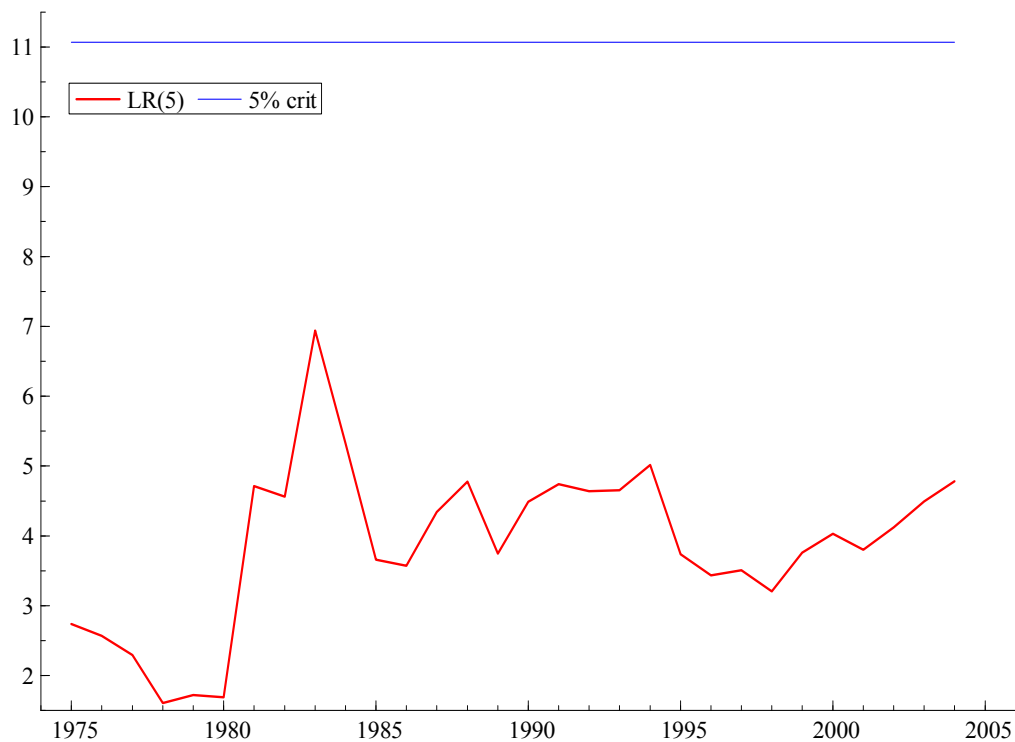
LA: Log of the stock of knowledge.

LL: Log of the Labor input.

**Figure 2. Recursive System Diagnostics for VAR(3) Model**



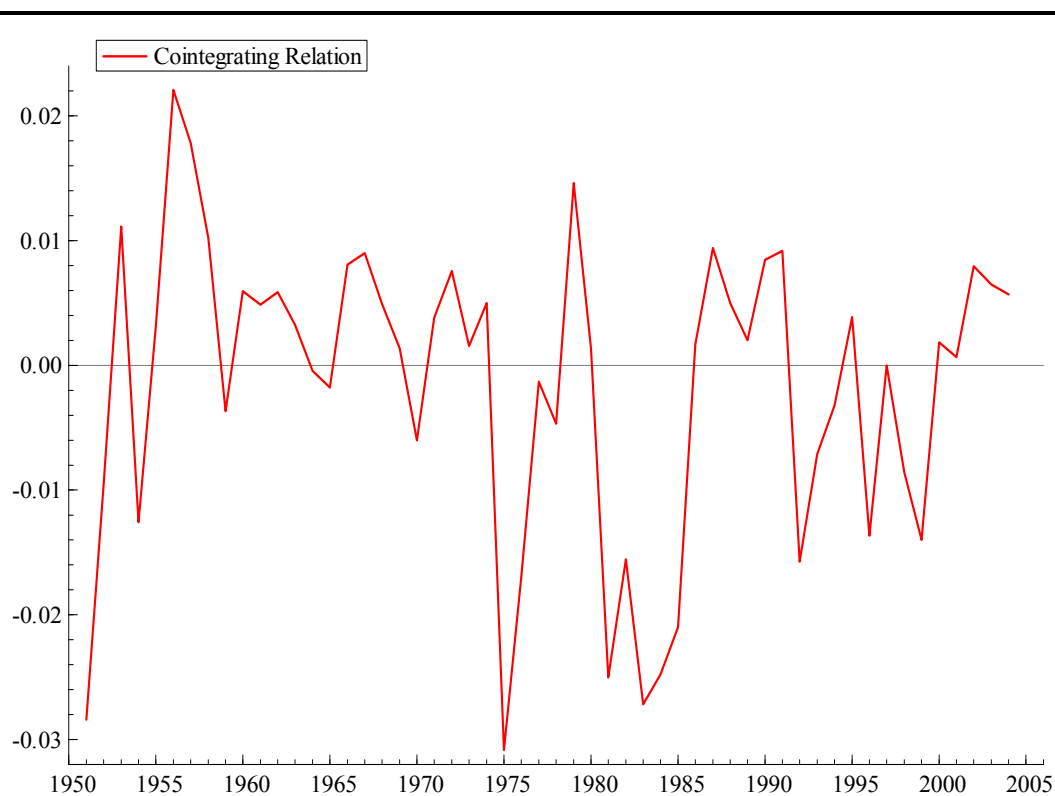
**Figure 3. Recursive Likelihood Ratio Test Statistic for Final Restrictions on the Cointegrating Space**



Notes:

The restrictions on the final model are: constant returns to scale for the private inputs ( labor and private capital), equal elasticities for private and public capital, and weak exogeneity for the knowledge, private capital and public capital stocks.

**Figure 4. Output Deviations from the Long Run Aggregate Production Function with Final Restrictions on the Cointegrating Space Imposed**



Notes:

The restrictions on the final model are: constant returns to scale for the private inputs ( labor and private capital), equal elasticities for private and public capital, and weak exogeneity for the knowledge, private capital and public capital stocks.