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## A Model of Sovereign Debt in Democracies

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**IMF Working Paper**

African Department

**A Model of Sovereign Debt in Democracies**

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**Abstract**

**This Working Paper should not be reported as representing the views of the IMF.**

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This paper develops and empirically tests a political economy model of sovereign debt. The main incentive for repaying sovereign debt is to maintain access to international capital markets. However, in a democracy, one generation may choose default regardless of its consequences for future generations. An old generation with little concern for its country's access to capital markets can force a default on debt if it has the majority of voters. On the other hand, if the younger generation is more numerous, it can force repayment of previously defaulted debt. Other voter heterogeneities, such as in income, can generate similar results.

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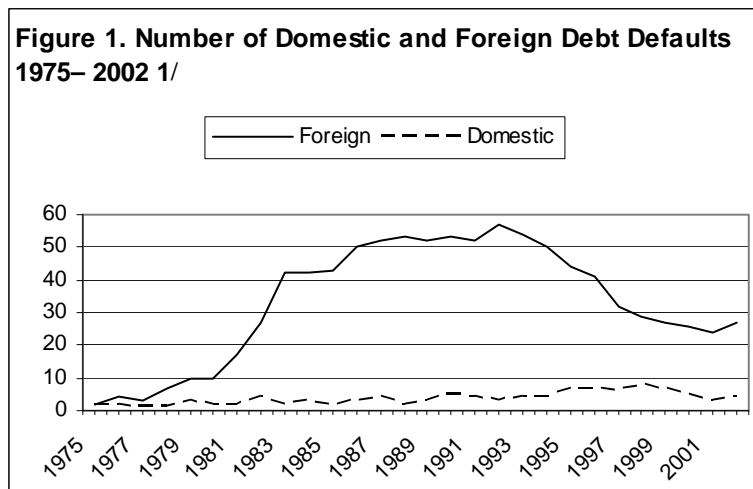
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## I. INTRODUCTION

**Sovereign governments borrow from international capital markets for smoothing purposes.** They borrow in bad crop years, and are expected to repay back their debt in good crop years. In this way, continued access to international capital markets serves as insurance for countries against future bad shocks.

**Many episodes of default—broadly defined as not fully respecting the repayment schedule for sovereign debt—have been recorded.** Figure 1 shows the number of defaults on sovereign external<sup>2</sup> and domestic debt for 1975–2002, a period when default on external debt was considerably more frequent than on domestic debt.



<sup>1</sup> Source: Beers and Chambers (2003)

**Our paper studies how countries decide whether to respect the schedule for repaying sovereign debt or to default.** It is very important for lenders to understand how this decision is made or they might face losses from potentially avoidable defaults. Like most previous contributions we assume that debtors default because they *do not want* to repay, not because they *cannot*. We ask what mechanism enforces repayment of debt, and why it is not always successful.

**There is no direct mechanism for enforcing repayment of sovereign external debt.** This makes it remarkably different from domestic or other types of debt. Lenders usually do not hold any collateral to be confiscated upon default, and long gone are the times when lenders

<sup>2</sup> In this paper sovereign debt is used interchangeably for sovereign external debt, unless otherwise mentioned.

could justify military invasion of defaulting sovereigns. Yet more often than not the debt is repaid. The logical next step, then, is to look for an indirect mechanism that enforces repayment of debt.

**The literature has focused on “reputational” tools for enforcing repayment of sovereign debt:** Lenders can threaten to cut or limit a defaulting country’s access to international capital markets. They can also threaten to impose trade sanctions. Borrowers pay back debt to have a good reputation with lenders and avoid punishments. Threats or actual punishments, however, are not always effective; many countries refuse to clear external arrears even after having been cut off from international capital markets for a long time.

**We argue that political economy factors can also serve as enforcement mechanisms in the case of sovereign debt.** The literature has largely neglected the influence of democratic forces in shaping a country’s decisions about debt. The democratic forces represent the preferences of individuals and are exerted through democratic processes like voting or lobbying. Our basic insight here is that different individuals have different attitudes to their country’s reputation in capital markets. Some care more than others about access to them as insurance against future bad shocks to the economy. For example, a young voter might value such insurance more than a non-altruistic older, simply because she expects to live longer. If the young are politically stronger than the old, we can expect to see repayment. Conversely, if politically the old dominate, default will be more likely.

**We develop a political economy model of sovereign debt that belongs to the class of reputational models,** which hold that the only possible benefit of debt repayment is to keep a good reputation in capital markets. The advantage of our model over others is that we include heterogeneities that influence the decisions of individuals—voters—on default. The model accounts for episodes of default and repayment of international debt better than pure reputational models.

The rest of the paper is organized as follows. Section II briefly reviews the relevant literature and outlines our main contributions. We detail our empirical methodology and theoretical model in Section III. Section IV reports the results, and Section V draws conclusions.

## II. RELEVANT LITERATURE<sup>3</sup> AND OUR CONTRIBUTION

**Eaton and Gersovitz (1981) provided the seminal contribution on sovereign debt.** They offer the first full-fledged model with reputation as the main debtor motivation for repaying sovereign debt.

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<sup>3</sup> For a comprehensive literature review, see Eaton and Fernandez (1995). Obstfeld and Rogoff (1996), offer a textbook treatment of the topic.

**Bulow and Rogoff (1989) made another important contribution.** They critique the reputational explanation of Eaton and Gersovitz, arguing that countries default on their outstanding debt to a certain lender if they have access to alternative lenders or asset markets. For countries that have alternative sources of financing, bad reputation in international capital markets will not be enough of a concern to make them repay debt. The following example illustrates how the reputational explanation can collapse despite its intuitive appeal.

Suppose a country has \$1 million of loans due this year. Credit-constrained and unable to borrow more, it can choose from two options:

- (a) Repay \$1 million this year
- (b) Default (do not repay \$1 million) this year and save \$1 million in a safe foreign bank with a positive interest rate.

If it chooses (a), the country will maintain a good reputation in international capital markets and can get a loan of \$1 million whenever it needs one. If it chooses (b), it will not get any loans in the future. Which will it choose? The country will choose (b) because both choices offer the same amount of funds in future, but choice (b) also yields interest income and choice (a) does not. This result hinges on the assumption that even if the country has a good reputation, it will not be entitled to more than \$1 million of loans in future. While this assumption does not hold universally, it is realistic if lenders set loan limits (e.g., the IMF sets a maximum access to funds for each member that depends on the member's quota share).

**Amador (2003) offered a counter example to Bulow and Rogoff (1989).** He shows that political uncertainty can affect repayment of debt. The argument is based on a famous result from the political economy literature that the uncertainty of politicians about reelection reduces their savings. Therefore, even if the country has access to an alternative saving option, politicians choose not to save, so they still need to build a good reputation with lenders to secure future loans.

There are other models, but each is generally similar to one of those just discussed.

**The empirical literature has not closely tracked the theoretical literature.** Most studies have used ad hoc sets of variables for the analysis. Among all of them, we find Van Rijckeghem and Weder (2005) most interesting. It studies the effect of a large number of macroeconomic and political variables on default and finds many variables that contribute to default, among them debt/GDP, GDP growth, currency crises, openness, and democracy.

**The consensus in the literature seems to be that the incentive to repay sovereign debt is very sensitive to international conditions.** Eaton and Fernandez (1995) in their review of the literature conclude that the incentive for honoring external liabilities is subtle and sensitive to the environment provided by the international financial system.

**Here we study the problem of sovereign debt in isolation**, though sovereign debt relationships can be interwoven with other relationships. For example, if a country defaults, it might face trade sanctions or other economic punishments. Rose (2002) offers evidence that countries face more restrictions on international trade if they have defaulted on sovereign debt. While these channels are potentially important, they are beyond the scope of this paper, which aims to make two major contributions to the literature:

- **We develop a comprehensive model of sovereign debt that explains both episodes of default and episodes of repayment.** Such a model is unique. Models that explain default (e.g., Bulow and Rogoff, 1989) find it difficult to explain repayment, and models that explain repayment (e.g., Eaton and Gersovitz, 1981) find it difficult to explain default. Our research will fill this gap.
- **We show that domestic forces within borrowing economies can significantly affect decisions about default or repayment.** Individuals help shape these decisions through the democratic process of voting for repayment or default.

**The paper consists of two parts: theoretical and empirical. In the theoretical part we create a political economy model by employing a standard overlapping generations (OLG) framework.** This model makes the following novel predictions:

- Citizens shape a country's decisions about debt repayment by voting.
- Macroeconomic conditions affect individual voting decisions.
- Individual heterogeneities, such as age and income, also affect voting decisions.
- Individuals consider voting for repayment of sovereign debt only to the extent they care about their country's reputation in international capital markets. Continued access to international capital markets insures against unfavorable future shocks.

**The empirical part tests the theoretical model's predictions on actual country data.** We investigate how macroeconomic variables, structural variables, and political economy variables affect the probability of default.



### III. THE MODEL

Our data set consists of macroeconomic variables (GDP growth rate and external debt); structural variables (population, age distribution, and income distribution); and political economy variables (democratic structure). It also covers countries that have had at least one episode of external debt default in their history through 2003. Sources and brief explanations for variables are shown in Table 1.

**Table 1. Sources and Explanations of Variables**

	Source	Explanation
<b>Default</b>	S&P	Binary: 1 if the country defaulted on its external debt; 0 otherwise.
<b>Debt/GDP</b>	WEO	Total External Debt as a Percentage of GDP.
<b>Unemployment</b>	WEO	Rate of unemployment
<b>GDP/Capita</b>	WEO	Real GDP per Capita in year-2000 USD.
<b>Democracy</b>	WB, Database of Political Institutions	Binary: 1 if the president/prime minister has finite term in office; 0 otherwise.
<b>Population 15-59 (%)</b>	UN Population Database	Population of the "young", as a percentage of total population of adults.

**Our estimation method is a pooled panel probit regression as follows:**

$$Default_{it} = \alpha_{it} \cdot Macro_{it} + \beta_{it} \cdot Political_{it} + error_{it}$$

where

$Default_{it}$  is a binary variable set equal to one if country  $i$  defaulted on its external debt at period  $t$ , and zero otherwise.

$Macro_{it}$  is the vector of macroeconomic and structural variables, which contains GDP growth per capita, debt/GDP and unemployment.

$Political_{it}$  is the vector of political variables for country  $i$  at period  $t$ .<sup>4</sup> The regression results have been reported only for variable *Democracy*, which is binary and equal to one if the president or prime minister has a finite term in office and zero otherwise.

$\alpha_{it}$  and  $\beta_{it}$  are the coefficient vectors, and  $error_{it}$  is the error term.

Since data on most structural and political variables are scarce for the countries in our sample, this methodology is the only feasible way to estimate our model.

**The empirical model has been derived from the theoretical model developed next.**

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<sup>4</sup> Apart from capturing degree of democracy, this variable makes it possible to capture transitions from autocracy to democracy and vice versa. This is interesting for studying the effects of different democratic regimes on a country's behavior toward debt.

## Theoretical Model

We consider a discrete time overlapping-generations model. The economy consists of  $N$  young and  $N$  old individuals at each period.<sup>5</sup> Each individual lives for exactly two periods. There is no altruism between or within generations. A young person at time  $t \in \{1, 2, \dots\}$ , will be old at time  $t + 1$ , and is known as a member of generation  $t$ .

There is a single (perishable) consumption good in the economy. Individuals get utility from consumption of this good. An individual's Bernoulli utility function of consuming  $c$  units of the good at each period is  $u(c)$  with the following standard concavity properties:  $u' > 0$  and  $u'' < 0$ . Young individuals discount future utility by  $\beta > 0$ .

At the beginning of each period  $t$ , each individual (young or old) receives  $w$  units of the consumption good. This endowment is stochastic:  $w \in \{w^h, w^l\}$ . The simplifying assumption that everybody receives the same amount of endowment ensures that there is always aggregate uncertainty in the economy, even for a large  $N$ .<sup>6</sup> We assume  $0 < w^l < w^h$ . The assumption of non-zero endowments is consistent with the common perception that defaulting countries generally choose not to repay debt even when they can, at least partially, afford it. At each period,  $w^h$  and  $w^l$  happen with respective probabilities of  $\pi^h$  and  $\pi^l$  (such that  $\pi^h + \pi^l = 1$  and  $0 < \pi^h, \pi^l < 1$ ). Information is symmetric among all the decision makers of each period.

### A. Insurance Contracts

Insurance contracts are rules that are described as follows.

The country, as a whole, can buy insurance from a risk-neutral foreign insurer. However, no proper subset of the population can sign any insurance contract.

The only insurance contract available at time  $t$  is a one-period insurance contract

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<sup>5</sup> For the most part, we keep the assumption that the population of each generation is fixed at  $N$ . At the end of this section, we discuss the case of different sizes for different generations. The model can also be extended to incorporate more than two overlapping generations. However, this complication would not change our qualitative results.

<sup>6</sup> At the end of this section we introduce income heterogeneity to the model and show that our main results do not hinge on this assumption.

$(\tau_{t+1}(w^h), \tau_{t+1}(w^l))$ . We call this contract the “time  $t + I$ ” insurance contract. This is an agreement to be signed between the country and the foreign insurer at time  $t$ . It calls for a net transfer of  $\tau_{t+1}(w)$  for each realization of  $w \in \{w^h, w^l\}$  from the foreign insurer to the country at time  $t + I$ , where  $\tau_{t+1}(w^h) \leq 0 \leq \tau_{t+1}(w^l)$ . An unpaid (defaulted) insurance payment continues to be outstanding until it is paid off. An insurance contract is offered at each period only if the country does not have any outstanding debt from previous insurance contracts. As soon as the country pays off all its previous debt (if any) at time  $t$ , the “time  $t + I$ ” insurance contract becomes available. For simplicity of analysis, we assume zero interest rates and no late fees for repayment of debt.

For simplicity of notation all payments are in units of the consumption good per person. Since the population in each period is fixed at  $2N$ , the total payments would be  $2N\tau(w^h)$  or  $2N\tau(w^l)$ .

Any positive inflow to the country will be distributed equally among all living individuals as a poll subsidy. Likewise, any positive outflow from the country is financed by taxing all living individuals equally through poll taxes. We assume the tax/subsidy system has no transaction costs.

In order to concentrate on sovereign debt we assume there are no domestic insurance markets. Throughout most of the paper domestic insurance markets are ruled out because the young are identical. When we introduce income heterogeneity, the structure of the model will permit domestic insurance markets, but we assume it away.

We have assumed this insurance rule, rather than perfect competition in the insurance market, to sidestep the complications arising from strategic interaction of different foreign insurers.

## B. Definition of Default

At each period  $t$ , after realization of the endowment,  $w \in \{w^h, w^l\}$ , the country chooses one of two alternatives:

- Default: not make the transfer payment of  $\tau_t(w_t)$  on the outstanding debt due to previous insurance contracts. This in turn makes the country unable to sign an insurance contract for period  $t + I$ .

Notice that for realization of  $w_t = w^l$ , default is unlikely because in this case the country is entitled to receive funds from the foreign insurer. The case is trivial and not interesting for purposes of this chapter. However, our definition of default is general

enough to capture it, as well as the more interesting realization of  $w_t = w^h$ , following which default may take place.

- Repay: make the transfer payment of,  $\tau_t(w_t)$  the outstanding debt from previous defaults, or both. This entitles the country to sign an insurance contract for  $t + 1$ .

### C. Sequence of Events

The sequence of events in each period is as follows:

1. All the previous period's young become old ( $N$  individuals) and a new generation of young with the same population ( $N$ ) becomes eligible to vote.
2. The endowment is realized and observed by everybody.
3. Individuals vote for default or repayment. The outcome is determined by a simple majority rule and executed via poll taxes and subsidies (more details will be given later, when we explain the country).
4. Individuals consume whatever is left over for them.
5. If the country has chosen repayment, it signs an insurance contract for the next period. Otherwise, no insurance contract is offered.

### D. The Foreign Insurer

For simplicity of analysis, we assume that the foreign insurer has a limited strategic role in the model. It offers time-independent but (first order) history-dependent insurance contracts: For period  $t \in \{0, 1, 2, \dots\}$ , it offers insurance contract  $(\tau^h, \tau^l)$  if the country does not have any debt outstanding due to prior defaults. Otherwise, the contract is offered only after the country has repaid all of its debt. The insurance contract is a purely reputational contract because it does not require any premium payment and is not enforceable by a third party.

### E. The Country

#### *The Voting Process*

At each period  $t$ , after the endowment,  $w_t$ , is realized, the country has to decide whether to default or repay debt.<sup>7</sup> The binary choice is modeled by a simple majority rule within the country. This rule requires that whenever more than 50 percent of the voters vote for default, the country defaults. Therefore, for the baseline model, a default occurs only if both the young and the old vote for it. Otherwise, the country does not default.

### *The Turn-Out Rate*

Not all individuals turn out to vote. In each period, the turn-out rate of the young is bigger or equal to the turn-out rate of the old with probability of  $\gamma$ .

### *The Old Voters*

At time  $t$ , if the endowment realization of  $w_t = w^h$  is observed, an old voter votes for default to avoid payment to the foreign insurer. If the endowment realization of  $w_t = w^l$  is observed, there are two possibilities to consider:

- The country has outstanding debt due to prior defaults and does not have any period  $t$  insurance. Therefore, the old vote for default on the outstanding debt.
- The country does not have any outstanding debt and has a period  $t$  insurance contract. In this case the old vote against default because repayment entitles them to a positive transfer payment from the foreign insurer.

### *The Young Voters*

A young voter votes for default only if the lifetime benefits of default to her outweigh its costs. The young discount their old time consumption at rate  $\beta$ . We study the problem of the young following low and high endowment shocks separately.

#### *Case $w_t = w^l$*

Suppose at time  $t$ , the endowment realization of  $w^l$  is observed and the country has no outstanding debt and does have a period  $t$  insurance contract. Obviously, in this case the young do not vote for default because the insurance contract entitles them a positive transfer payment from the foreign insurer. However, at time  $t$  if the country has outstanding debt of  $-\tau^h$  from previous defaults, after observing the endowment realization of  $w_t = w^l$ , there are two possibilities:

- If  $\pi^l$  is large enough, a young voter votes for default. For example, if  $\pi^l > \pi^h$ , then  $\tau^l < -\tau^h$ . In this case, the young are not willing to pay  $(-\tau^h)$  at time  $t$  in order to buy insurance  $(\tau^h, \tau^l)$  for period  $t + 1$ .

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<sup>7</sup> The choice of default or repay as a binary variable has been made for simplicity of analysis. One might model default as a continuous variable to allow for partial default, but we will avoid such technical difficulties by simply modifying our binary setup when we study partial default.

- If  $\pi^l$  is not large enough,  $\tau^l$  will be enough bigger than  $-\tau^h$  and therefore a young individual votes for repayment.

The following assumption will be made throughout the paper to simplify these cases:

**Assumption.** If the country has outstanding debt of  $-\tau^h$  from previous defaults and  $w_t = w^l$ , the young vote for default at time  $t$ . While this assumption is reasonable intuitively—it says in bad crop years arrears are not cleared—it can be formalized by assuming certain attitudes of voters toward risk or certain shock distributions, such as  $\pi^l$  sufficiently big.

*Case  $w_t = w^h$*

The following lemma identifies conditions in which the young vote for repayment, following a high endowment realization ( $w^h$ ).

**Lemma 1.** Suppose  $w_t = w^h$ . The young vote for repayment only if  $Z \geq 1$ , where  $Z$  is defined as follows:  $Z(w^h, w^l, \tau^h, \tau^l) = \beta[\pi^l \frac{u(w^l + \tau^l) - u(w^l)}{u(w^h) - u(w^h + \tau^h)} - \gamma\pi^h]$ .

**Proof.** All the proofs can be found in the Appendix.

$Z$  is interpreted as a measure of the desire of the young for insurance. A higher  $Z$  implies a higher desire for insurance. If the young are indifferent about insurance, then  $Z = 1$ . For  $Z \geq 1$ , the young have enough desire for insurance to vote for repayment. Notice that for this condition it does not matter whether the country has outstanding debt from a prior default or a current high crop year because in either case the country has the same obligation of  $-\tau^h$ .

Notice that  $\frac{\partial Z}{\partial \gamma} < 0$ . This might seem counterintuitive because it suggests that a higher turn-out of the young causes them to desire insurance less. The reason for this result is as follows:  $\gamma$  is the turn out rate for every period. Expecting a higher turn-out in the next period, a young individual this period finds it more likely that her next period vote for default will fall into the minority. This decreases her desire to buy insurance this period.

## F. Contract Concepts

We define and study three types of contracts: default-free, default repayment, and default renegotiation.

### *Default-Free Contract*

Envision an ideal world in which countries can always benefit from risk-sharing in international markets. In this world, an insurance contract is offered, accepted, and honored every period, so that both old and young maximize utility and the insurer breaks even.

More formally:

Bundle  $(\tau^h, \tau^l)$  is a default-free contract if it is offered by the insurer and accepted by the country at every period  $t \in \{0, 1, 2, \dots\}$  such that:

- The foreign insurer breaks even in expected value at each period:  $\pi^h \tau^h + \pi^l \tau^l = 0$ . Since we have assumed  $w^l < w^h$  and  $0 < \pi^l, \pi^h < 1$ , it should be the case that  $\tau^h < 0$  and  $\tau^l > 0$ . This simply says that if the country experiences a low endowment shock, it will be entitled to receive a payment from the foreign insurer. On the other hand, after a high endowment shock the country must make a payment to the insurer.
- Old and young voters maximize their expected lifetime utility.

**Proposition 1.**  $(\tau^h, \tau^l)$  is a default-free contract only if  $\gamma = 1$ ,

$$Z(w^h, w^l, \tau^h, \tau^l) = \beta[\pi^l \frac{u(w^l + \tau^l) - u(w^l)}{u(w^h) - u(w^h + \tau^h)} - \pi^h] \geq 1 \text{ and } \pi^h \tau^h + \pi^l \tau^l = 0.$$

In other words,  $(\tau^h, \tau^l)$  is a default-free contract only if the young are in the majority ( $\gamma = 1$ ) and have sufficient desire for insurance ( $Z(w^h, w^l, \tau^h, \tau^l) \geq 1$ ), and the foreign insurer breaks even ( $\pi^h \tau^h + \pi^l \tau^l = 0$ ).

**Corollary 1.** Given  $\beta, \pi^h, \pi^l, w^h, w^l$ , when the young are pivotal voters, the existence and uniqueness of a default-free contract depends only on the shape of the utility function

**Corollary 2.** There is a default-free contract for sufficiently risk-averse preferences.

This is an intuitive result. It suggests that a risk-neutral insurer is always able to insure a country that is sufficiently risk-averse.



### *Default-Free Contract for Quadratic Preferences*

Quadratic preferences are the only type of risk-averse preferences for which we have been able to characterize a default-free contract, as described in the example.

**Example 1.** For a country in which the young are pivotal and have quadratic preferences  $u(x) = ax - bx^2$ , a default-free contract exists for sufficiently high risk aversion (small  $\frac{a}{b}$ ) and small enough  $\pi^l$ .

### *Default-Repayment Contract*

A default-free contract sets a benchmark for an ideal world. However, in the real world countries sometimes default and lose access to foreign insurance markets, and at other times repay debt and regain access. A default-repayment contract is designed to capture this phenomenon.

Bundle  $(\tau^h, \tau^l)$  is a default-repayment contract if the insurance contract  $(\tau^h, \tau^l)$  is offered by the insurer and accepted by the country at some period  $t \in \{0, 1, 2, \dots\}$ , is defaulted upon at the subsequent period,  $t + 1$ , but is repaid at some period  $t + k$ , where  $k \in \{2, 3, 4, \dots\}$ , such that:

- The foreign insurer breaks even in expected value at each period that a contract is offered:  $\pi^h \tau^h + \pi^l \tau^l = 0$ .
- Both old and young voters maximize their expected utility.

### **Proposition 2.**

For  $0 < \gamma < 1$ ,  $(\tau^h, \tau^l)$  is a default-repayment contract if

$$Z(w^h, w^l, \tau^h, \tau^l) = \beta[\pi^l \frac{u(w^l + \tau^l) - u(w^l)}{u(w^h) - u(w^h + \tau^h)} - \gamma\pi^h] \geq 1 \text{ and } \pi^h \tau^h + \pi^l \tau^l = 0.$$

### **Remark 1.**

The case of  $\gamma = 0$  is trivial: The old are always in the majority and default on any payment. As a result, no contract is offered in the first place.

### *Comparison of Contract Conditions*

Sustaining a default-free contract requires that the young always be in the majority ( $\gamma = 1$ ). However, for a default-repayment contract, sometimes the young and sometimes the old are in the majority.

**Example 2.** Default-Repayment Contract for Quadratic Preferences

For quadratic preferences, there is a default-repayment contract for sufficiently small  $\pi^l$ .

*Default-Renegotiation Contract*

Often defaulted debt is renegotiated and repaid in installments. In this part, we allow for partial default by introducing a default-renegotiation contract, which is a more general version of a default-repayment contract. For simplicity of analysis we restrict attention only to the following form of renegotiation: The country is allowed to pay an installment of  $f < -\tau^h$  at one period and the remainder of its outstanding debt in another installment in a later period. A payment of the amount of  $f$  will entitle the country to an insurance contract of  $(\tau^h, \tau^l + \tau^h + f)$ .

Bundle  $(\tau^h, \tau^l)$  is a default-renegotiation contract if the insurance contract  $(\tau^h, \tau^l)$  is offered by the insurer and accepted by the country at some period  $t \in \{0, 1, 2, \dots\}$ , is defaulted upon at the subsequent period,  $t+1$ , and is repaid over two periods of  $t+k$  and  $t+j$ , where  $k, j \in \{2, 3, 4, \dots\}$  and  $k \neq j$ , such that:

- The foreign insurer breaks even in expected value at each period that a contract is offered:  $\pi^h \tau^h + \pi^l \tau^l = 0$ .
- Both old and young voters maximize their expected utility.

$f$  is interpreted as a penalty for current citizens because the country defaulted in the past.

**Lemma 2.** If  $w_t = w^h$  and the country does not have a time  $t$  contract due to prior defaults, then the young vote for payment of  $f < -\tau^h$  if payment entitles the country to an insurance contract  $(\tau^h, \tau^l + \tau^h + f)$  and if  $Z(w^h, w^l, \tau^h, \tau^l + \tau^h + f) \geq \frac{u(w^h) - u(w^h - f)}{u(w^h) - u(w^h + \tau^h)}$ .

**Proposition 3.** For  $0 < \gamma < 1$ ,  $(\tau^h, \tau^l)$  is a default-renegotiation contract if

$$Z(w^h, w^l, \tau^h, \tau^l + \tau^h + f) \geq \frac{u(w^h) - u(w^h - f)}{u(w^h) - u(w^h + \tau^h)} \text{ for } f < -\tau^h, \text{ and } \pi^h \tau^h + \pi^l \tau^l = 0.$$

While it is very difficult to exactly characterize a default-renegotiation (contract even in the case of quadratic preferences), the existence of equilibrium is fairly easy to establish given

the existence of a default-repayment equilibrium and continuity, which indicates that for  $f$  smaller but very close to  $\tau^h$  a default-renegotiation equilibrium is the same as a default-repayment equilibrium.

Define:  $\hat{Z}(w^h, w^l, \tau^h, \tau^l, f) = Z(w^h, w^l, \tau^h, \tau^l + \tau^h + f) - \frac{u(w^h) - u(w^h - f)}{u(w^h) - u(w^h + \tau^h)}$ .  $\hat{Z}$  is

interpreted like  $Z$ . It is a measure of desire for insurance for a country that has defaulted in the previous period.

Showing that  $\frac{\partial \hat{Z}}{\partial f} < 0$  says that a country's desire for insurance decreases because the punishment for default on the previous contracts ( $f$ ) increases.

It should be noted that the default-repayment contract is a special case of default-renegotiation contract for  $f = -\tau^h$ .

## G. Income Heterogeneity and Changes in Population Mixture

In this section we introduce heterogeneity in income and changes in population mix (e.g., aging). For simplicity of analysis we look only at a default-free equilibrium (with  $\gamma = 1$ ). In each period  $t$ , the economy is populated with a continuum of individuals uniformly distributed over unit interval  $[0, 1]$ . Fraction  $\alpha_t$ , ( $0.5 \leq \alpha_t < 1$ ) of population is young and the rest are old.<sup>8</sup> In the beginning of each period  $t$ , each young individual  $i$  is endowed with  $w^i$  units of the consumption good;  $w^i$  has a standard uniform distribution. This endowment then experiences an exogenous shock,  $\varepsilon_i \in \{\varepsilon_i^h, \varepsilon_i^l\}$ . We assume  $\varepsilon_i^l < \varepsilon_i^h$  and that the net endowment ( $w^i + \varepsilon_i$ ) is positive for all  $i$ . This last assumption is based on the common perception that countries default because they do not want to repay their debt even if they can afford to. At each period,  $\varepsilon_i^h$  and  $\varepsilon_i^l$  happen with respective probabilities of  $\pi_t^h$  and  $\pi_t^l$  (such that  $\pi_t^h + \pi_t^l = 1$  and  $0 < \pi_t^h, \pi_t^l < 1$ ). In period  $t + 1$ , individual  $i$  (who was young in period  $t$ ) becomes old and receives an endowment of  $w^i$ , which will then experience the shock of  $\varepsilon_{i+1}$ . Apart from this modification, the model is the same as the original model in previous sections.

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<sup>8</sup> The case of  $\alpha_t < 0.5$  refers to when the old are in the majority. With a simple majority rule the model's result for this case is trivial: the country always votes for default after a high endowment shock. For this case there is no default-free contract. It is interesting to note (and easy to verify) that this result is robust even if one allows for bargaining between coexisting generations.

$Z$  is modified as follows:

$$Z(w^i, \varepsilon^h, \varepsilon^l, \tau^h, \tau^l) = \beta[\pi^l \frac{u(w^i + \varepsilon^l + \tau^l) - u(w^i)}{u(w^i + \varepsilon^h) - u(w^i + \varepsilon^h + \tau^h)} - \pi^h]$$

We consider only two cases:  $\frac{\partial Z}{\partial w} > 0$  and  $\frac{\partial Z}{\partial w} < 0$ .

$\frac{\partial Z}{\partial w} > 0$  ( $\frac{\partial Z}{\partial w} < 0$ ): In this case, as an individual becomes richer, her desire for insurance increases. This in turn can be a result of increasing or decreasing risk aversion.

**Example 3.** For quadratic preferences, case  $\frac{\partial Z}{\partial w} > 0$  ( $\frac{\partial Z}{\partial w} < 0$ ) occurs if the difference in shock sizes relative to insurance payments is big or small enough, such that:

$$\frac{\varepsilon^h - \varepsilon^l}{\tau^l} > \frac{1 + \frac{\pi^l}{\pi^h}}{2} \quad (\frac{\varepsilon^h - \varepsilon^l}{\tau^l} < \frac{1 + \frac{\pi^l}{\pi^h}}{2}).$$

**Proposition 4.**

(i) Suppose  $\frac{\partial Z}{\partial w} > 0$ . If there is an individual  $i$ , with endowment  $w^i = \bar{w}$  such that

$$Z(\bar{w}, \varepsilon^h, \varepsilon^l, \tau^h, \tau^l) = 1, \text{ there then exists a default-free contract only if } \bar{w} < 1 - \frac{1}{2\alpha_{t+1}}.$$

(ii) Likewise, for  $\frac{\partial Z}{\partial w} < 0$ , if there is an individual  $i$ , with endowment  $w^i = \bar{w}$  such that

$$Z(\bar{w}, \varepsilon^h, \varepsilon^l, \tau^h, \tau^l) = 1, \text{ there then exists a default-free contract only if, } \bar{w} > \frac{1}{2\alpha_{t+1}}.$$

(iii) If  $Z(w^i, \varepsilon^h, \varepsilon^l, \tau^h, \tau^l) < 1$  for all  $i$ , there then is no default-free contract.

(iv) If  $Z(w^i, \varepsilon^h, \varepsilon^l, \tau^h, \tau^l) > 1$  for all  $i$ , then  $(\tau^h, \tau^l)$  is a default-free contract.

**Corollary 3.** As the country ages a default-free contract becomes less likely.

This is intuitively clear. The old have less taste for insurance than the young. As a result, foreign countries are less likely to insure older economies.

#### IV. EMPIRICAL RESULTS

Regression results for nine different models are reported in Table 2. Default on sovereign debt is the dependent variable in all of them.

Models 2 and 8 link default to macroeconomic variables alone. Not surprisingly these models show that higher indebtedness (debt/GDP ratio) or unemployment will lead to a higher probability of default. However, there is no significant relationship between how rich a country is (GDP/capita ratio) and the probability of default on its sovereign debt.

Model 1 is our preferred model. It essentially adds political economy explanatory variables, representing heterogeneities in the economy, to Model 2. Estimation results indicate that democracies are more likely to default on sovereign debt. A word of caution here is that the variable “democracy” is not significant in some of the other models reported in Table 2. Therefore, we will need to study alternative measures of democracy to further test the robustness of our result about the influence of democracy on default. Model 1 also suggests that younger economies (more precisely, economies with a higher percentage of people aged 15–59 in total population) are less likely to default. This is consistent with the intuition that younger people care more than older ones about the reputation of their country in international capital markets.

An interesting observation from Table 2 is that GDP per capita does not significantly explain default in any of the models with all-significant regressors. This is evidence in favor of the findings of previous studies that a country’s income is not a determinant of default; countries default not because they cannot service debt, but simply because they do not want to.

**Table 2. Regression Results**

Dependent Variable: Default

	Debt/GDP	Unemployment	GDP/Capita	Democracy	Population 15-59 (%)	Constant	Pseudo R2
Model 1	1.156* (0.141) z=8.21	0.014* (0.006) z=2.31		0.259* (0.135) z=1.92	-0.052* (0.011) z=-4.74	1.598* (0.644) z=2.48	0.150
Model 2	1.326* (0.127) z=10.45	0.013* (0.005) z=2.30				-1.164* (0.106) z=-11.02	0.128
Model 3	1.272* (0.149) z=8.54	0.014* (0.006) z=2.24	0.000067* (0.000024) z=2.77	0.216 (0.136) z=1.58	-0.063* (0.012) z=-5.37	2.020* (0.665) z=3.04	0.156
Model 4	0.4138* (0.0449) z=9.21		0.000025 (0.000019) z=1.31	0.061* (0.070) z=0.88	-0.0235* (0.008) z=-2.89	0.554 (0.419) z=1.32	0.049
Model 5	1.2348* (0.1461) z=8.45	0.0123 (0.0057) z=2.16	0.000076* (0.000024) z=3.18		-0.065* (0.012) z=-5.67	2.335* (0.650) z=3.59	0.156
Model 6	1.4159* (0.1443) z=9.81	0.0157* (0.0059) z=2.67	0.000021 (0.000022) z=0.95	0.158 (0.134) z=1.18		-1.432* (0.182) z=-7.88	0.132
Model 7		0.0194* (0.0056) z=3.43	0.0000039 (0.000023) z=0.17	0.036 (0.128) z=0.28	-0.089* (0.011) z=-7.93	4.497* (0.601) z=7.48	0.083
Model 8	1.4001* (0.1408) z=9.94	0.0126* (0.0055) z=2.3	0.000028 (0.000022) z=1.26			-1.278* (0.140) z=-9.15	0.129
Model 9	0.3995* (0.0434) z=9.2			0.074 (0.069) z=1.07	-0.018* (0.007) z=-2.59	0.330 (0.382) z=0.86	0.049

Annual data for 1975-2003.

\* Represents significance at 95% confidence level.

Sources : Database of Political Institutions, Population Database,  
Standard and Poor's, World Economic Outlook, and World Bank.

## V. Conclusion

**We showed that the incentive for a sovereign government to honor external liabilities is sensitive to the national economy and heterogeneities within it,** in addition to the environment provided by the international financial system. The literature has mostly ignored the importance of interactions within borrowing countries in sovereign debt decisions.

**The novel features of our contribution were twofold.** First, we explained a borrowing country's decisions on whether to respect repayment of sovereign debt by solving the basic utility maximization problems of individuals. Second, we replaced the traditional infinitely lived representative agent setup with an economy populated with heterogeneous selfish individuals with finite lives.

**We provided an explanation for episodes of default and repayment of sovereign debt.** We characterized conditions in which a debtor country sometimes defaults on its debt and other times repays outstanding debt. We also characterized situations in which international reputational risk-sharing is impossible. In such cases, losses to the current finitely lived voters from payment of the country's international commitments dominate the benefits of having insurance. Therefore, even if it is beneficial for the country as a whole to have a good reputation in international debt markets, the selfish voters might vote for default.

**An alternative intuition to what we explained throughout, about our results, is as follows:** Even if a sovereign debtor has an infinite horizon objective function, its effective reputational horizon in international capital markets is significantly shortened by the decisions of its finitely lived selfish voters.<sup>9</sup> Risk-neutral international insurers might not find it profitable to provide insurance to governments that are domestically constrained from making repayments.

**Our results do not imply that democratic economies should not be granted international loans.** Depending upon the model parameters, reputational risk-sharing may be possible. Democratic countries might be less creditworthy than non-democratic ones, depending on such parameters as distribution of income shocks, population structure, risk-aversion of voters, voter income heterogeneity, and richness of the alternative asset markets available. This result is in line with, but stronger than that of Bulow and Rogoff (1989), which ignores political uncertainty.

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<sup>9</sup> We assumed at the outset that altruism does not hold at the micro level. This assumption relates to the fact that the bulk of empirical evidence is against altruism. For an example, see Altonji, Hayashi, and Kotlikoff (1997).

**The main drawback of our paper is that, in the model, default takes place merely because of the voters' willingness to do so.** Therefore, our model is not capable of capturing episodes of default that have been due to a country's lack of foreign reserves to service its debt.

**Our work can be enriched on several fronts:**

- **Alternative political structures are worth studying.** We adopted a democratic structure with a simple majority decision rule. An interesting line of further research would be to study the effects of alternative democratic structures that include elements such as parties and interest groups. We conjecture that irrespective of the democratic political structure, equilibriums with default cannot be ruled out as long as voters are finitely lived and selfish. Each person votes for default at least once, e.g., the last period of their lives. If such voters form a strong enough political group, they will force the economy to default.
- **Including alternative distributions of income shocks would enrich the model.** We assumed a simple *i.i.d.* binary output shock process for simplicity of analysis. This assumption is not very accurate for a world where expansions last longer than recessions, or for countries with persistent natural disasters like droughts.
- **Domestic insurance markets could be added.** These markets have been ruled out in most of the paper by assuming that the young. When we introduced income heterogeneity among the young, the structure of the model permitted domestic insurance markets. However, even with heterogeneity, we assumed that there were no domestic insurance markets. This allowed us to focus solely on the international market.
- **The assumption of symmetric information and beliefs among all players could be relaxed.** This is another simplifying assumption that can be relaxed in a more elaborate model. It is certainly arguable that borrowing countries are more aware of their economies than foreign lenders. However, the recent surge of information technology and credit rating companies has been helpful in reducing information asymmetries.



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## VI. Appendix

### A. List of Countries

Albania, Algeria, Angola, Argentina, Bolivia, Bosnia-Herzegovina, Brazil, Bulgaria, Burkina Faso, Cape Verde, Cameroon, Central African Republic, Chile, Congo (Republic of), Costa Rica, Cote d'Ivoire, Croatia, Cuba, Democratic Republic of Congo, Dominican Republic, Ecuador, Egypt, Ethiopia, Gabon, Gambia, Ghana, Guatemala, Guinea, Guinea-Bissau, Guyana, Haiti, Honduras, Indonesia, Iran, Iraq, Jamaica, Jordan, Kenya, Liberia, Macedonia, Madagascar, Malawi, Mauritania, Mexico, Moldova, Morocco, Mozambique, Myanmar, Nicaragua, Niger, Nigeria, Pakistan, Panama, Paraguay, Peru, Philippines, Poland, Romania, South Africa, Senegal, Sierra Leone, Slovenia, Russia, Sudan, Tanzania, Togo, Trinidad-Tobago, Turkey, Uganda, Ukraine, Uruguay, Venezuela, Vietnam, Yemen, Yugoslavia, Zambia, and Zimbabwe.

### B. Proofs

**Lemma 1.** Suppose  $w_t = w^h$ . The young vote for repayment only if  $Z \geq 1$ .

**Proof.** The young vote for repayment following  $w_t = w^h$  only if:

$$u(w^h + \tau^h) + \beta[\gamma\pi^h u(w^h + \tau^h) + (1 - \gamma)\pi^h u(w^h) + \pi^l u(w^l + \tau^l)] \geq u(w^h) + \beta[\pi^h u(w^h) + \pi^l u(w^l)]$$

The left and right hand side of the above inequality are a young person's lifetime expected utility without and with insurance, respectively.

This inequality is simplified in the following steps:

$$u(w^h + \tau^h) + \beta[\gamma\pi^h u(w^h + \tau^h) - \gamma\pi^h u(w^h) + \pi^l u(w^l + \tau^l)] \geq u(w^h) + \beta\pi^l u(w^l)$$

or

$$\beta\pi^l [\pi^l u(w^l + \tau^l) - u(w^l)] \geq (1 + \beta\gamma\pi^h)[u(w^h) - u(w^h + \tau^h)]$$

or

$$\beta[\pi^l \frac{u(w^l + \tau^l) - u(w^l)}{u(w^h) - u(w^h + \tau^h)} - \gamma\pi^h] \geq 1$$

or

$$Z \geq 1$$

**Lemma 2.** If  $w_t = w^h$  and the country does not have a time  $t$  contract due to prior defaults, then the young vote for payment of  $f < -\tau^h$  if it entitles the country to insurance contract

$$(\tau^h, \tau^l + \tau^h + f) \text{ and if } Z(w^h, w^l, \tau^h, \tau^l + \tau^h + f) \geq \frac{u(w^h) - u(w^h - f)}{u(w^h) - u(w^h + \tau^h)} .$$

**Proof.** The young vote for repayment following  $w_t = w^h$  only if:

$$\begin{aligned} & u(w^h - f) + \beta[\gamma\pi^h u(w^h + \tau^h) + (1 - \gamma)\pi^h u(w^h) + \pi^l u(w^l + \tau^l + \tau^h + f)] \\ & \geq u(w^h) + \beta[\pi^h u(w^h) + \pi^l u(w^l)] \end{aligned}$$

The left- and right-hand sides of the above inequality are a young person's lifetime expected utility without and with insurance, respectively. This inequality is simplified in the following steps:

$$\begin{aligned} & u(w^h - f) - u(w^h + \tau^h) + u(w^h + \tau^h) + \beta[\gamma\pi^h u(w^h + \tau^h) - \gamma\pi^h u(w^h) + \pi^l u(w^l + \tau^l + \tau^h + f)] \\ & \geq u(w^h) + \beta\pi^l u(w^l) \end{aligned}$$

or

$$\begin{aligned} & u(w^h - f) - u(w^h + \tau^h) + \beta\pi^l [\pi^l u(w^l + \tau^l + \tau^h + f) - u(w^l)] \\ & \geq (1 + \beta\gamma\pi^h)[u(w^h) - u(w^h + \tau^h)] \end{aligned}$$

or

$$\beta[\pi^l \frac{u(w^l + \tau^l + \tau^h + f) - u(w^l)}{u(w^h) - u(w^h + \tau^h)} - \gamma\pi^h] \geq 1 - \frac{u(w^h - f) - u(w^h + \tau^h)}{u(w^h) - u(w^h + \tau^h)}$$

or

$$Z(w^h, w^l, \tau^h, \tau^l + \tau^h + f) \geq \frac{u(w^h) - u(w^h - f)}{u(w^h) - u(w^h + \tau^h)} .$$

**Corollary 1.** Given  $\beta, \pi^h, \pi^l, w^h, w^l$ , the existence and uniqueness of a default-free contract depends only on the shape of the utility function ( $\gamma = 1$ ).

**Proof.** The proof is trivial, noting the fact that for  $\gamma = 1$ , a default-free contract is the solution of the following two (in)equalities:

$$Z(w^h, w^l, \tau^h, \tau^l) = \beta[\pi^l \frac{u(w^l + \tau^l) - u(w^l)}{u(w^h) - u(w^h + \tau^h)} - \pi^h] \geq 1 \text{ and } \pi^h \tau^h + \pi^l \tau^l = 0, \text{ for } (\tau^h, \tau^l) .$$

**Corollary 2.** A default-free contract exists if preferences are sufficiently risk-averse.

**Proof.** We need to show that for sufficiently risk-averse preferences there exists a bundle  $(\tau^h, \tau^l)$ , for which the following holds if  $\gamma = 1$ :

$$Z(w^h, w^l, \tau^h, \tau^l) = \beta[\pi^l \frac{u(w^h + \tau^l) - u(w^l)}{u(w^h) - u(w^h + \tau^h)} - \pi^h] \geq 1 \quad \text{and} \quad \pi^h \tau^h + \pi^l \tau^l = 0.$$

$$\begin{aligned} Z(w^h, w^l, \tau^h, \tau^l) &= \beta[\pi^l \frac{u(w^h + \tau^l) - u(w^l)}{u(w^h) - u(w^h + \tau^h)} - \pi^h] = \beta[\pi^l \frac{[u(w^h + \tau^l) - u(w^l)]/\tau^l}{[u(w^h) - u(w^h + \tau^h)]/\tau^l} - \pi^h] \\ &= \beta\pi^h \left[ \frac{[u(w^h + \tau^l) - u(w^l)]/\tau^l}{[u(w^h) - u(w^h + \tau^h)]/(-\tau^h)} - 1 \right] \end{aligned}$$

Choose  $\tau^l \rightarrow 0$ . As a result,  $-\tau^h \rightarrow 0$  and

$$Z \rightarrow \beta\pi^h \left[ \frac{u'(w^l)}{u'(w^h)} - 1 \right] = \beta(1 - \pi^l) \left( -\frac{u'(w^h) - u'(w^l)}{u'(w^h)} \right), \text{ which is bigger than 1 for sufficiently}$$

risk-averse preferences (high  $-\frac{u'(w^h) - u'(w^l)}{u'(w^h)}$ ) and low  $\pi^l$ .

**Corollary 3.** As the country ages, a default-free contract becomes less likely.

**Proof.** The proof is a direct result of the previous theorem: as  $\alpha_{t+1}$  decreases, it becomes less likely for the theorem's inequalities ( $\bar{w} < 1 - \frac{1}{2\alpha_{t+1}}$  and  $\bar{w} > \frac{1}{2\alpha_{t+1}}$ ) to hold, and therefore default becomes more likely. Hence, the existence of a default-free contract becomes less likely.

**Proposition 1.**  $(\tau^h, \tau^l)$  is a default-free contract only if  $\gamma = 1$ ,

$$Z(w^h, w^l, \tau^h, \tau^l) = \beta[\pi^l \frac{u(w^l + \tau^l) - u(w^l)}{u(w^h) - u(w^h + \tau^h)} - \pi^h] \geq 1, \text{ and } \pi^h \tau^h + \pi^l \tau^l = 0.$$

In other words,  $(\tau^h, \tau^l)$  is a default-free contract only if the young are the majority ( $\gamma = 1$ ) and have sufficient desire for insurance ( $Z(w^h, w^l, \tau^h, \tau^l) \geq 1$ ), and the foreign insurer breaks even ( $\pi^h \tau^h + \pi^l \tau^l = 0$ ).

**Proof.** If  $\gamma = 1$ , the young form the majority and are pivotal. By Lemma 1, assumption  $Z \geq 1$ , implies that the young vote for repayment. Finally,  $\pi^h \tau^h + \pi^l \tau^l = 0$  ensures that the foreign insurer breaks even, and therefore insurance contracts are offered every period. Therefore, if all of these conditions hold,  $(\tau^h, \tau^l)$  is a default-free contract.

It remains to demonstrate that if any one of these conditions is violated,  $(\tau^h, \tau^l)$  cannot form a default-free contract. If  $Z < 1$ , then, the young vote for default. Since the old always vote for default,  $(\tau^h, \tau^l)$  cannot be a default-free contract. If  $Z < 1$ , in some period the old will have the majority and the realization of the endowment will be  $w^h$ . In that period the majority will be in favor of default and again  $(\tau^h, \tau^l)$  cannot form a default-free contract. Finally, by definition any contract in which  $\pi^h \tau^h + \pi^l \tau^l \neq 0$  is not a default-free contract.

**Proposition 2.**

For  $0 < \gamma < 1$ ,  $(\tau^h, \tau^l)$  is a default-repayment contract if

$$Z(w^h, w^l, \tau^h, \tau^l) = \beta[\pi^l \frac{u(w^l + \tau^l) - u(w^l)}{u(w^h) - u(w^h + \tau^h)} - \gamma \pi^h] \geq 1 \text{ and } \pi^h \tau^h + \pi^l \tau^l = 0.$$

**Proof.**

Start with a period at which the country has no outstanding debt. At some period later with  $w_t = w^h$ , the young will have the majority because  $0 < \gamma$ . By Lemma 1,  $Z \geq 1$  ensures that in this period the country buys insurance. The country will not default and is given new insurance contracts in subsequent periods so long as the young have the majority. Since  $\gamma < 1$  at some subsequent period, the old will be pivotal and the country defaults. However, at some period later the young again form the majority and since  $Z \geq 1$ , the country repays the outstanding debt and signs a new contract. This is a default-repayment contract.

**Proposition 3.** For  $0 < \gamma < 1$ ,  $(\tau^h, \tau^l)$  is a default-renegotiation contract if

$$Z(w^h, w^l, \tau^h, \tau^l + \tau^h + f) \geq \frac{u(w^h) - u(w^h - f)}{u(w^h) - u(w^h + \tau^h)} \text{ for } f < -\tau^h, \text{ and } \pi^h \tau^h + \pi^l \tau^l = 0.$$

**Proof.** The proof is very similar to that of the previous proposition for a default-free contract. By Lemma 2, if the country does not have a time  $t$  contract and  $w_t = w^h$ , the young vote for payment of  $f < -\tau^h$  (partial repayment of debt) only if

$$Z(w^h, w^l, \tau^h, \tau^l + \tau^h + f) \geq \frac{u(w^h) - u(w^h - f)}{u(w^h) - u(w^h + \tau^h)}.$$

Start with a period  $t$  in which the country does not have any outstanding debt. In some period later the young will have the majority because  $0 < \gamma < 1$ . By Lemma 2, the assumption on  $Z$  ensures that in this period the country buys insurance  $(\tau^h, \tau^l + \tau^h + f)$ . If the young have the majority in period  $t+1$  as well, they will vote for repayment of the remainder of the country's debt and the country will start fresh with no debt at period  $t+2$ . However, if the old have the majority, the country defaults with debt outstanding of  $-\tau^h + f$ . Since  $0 < \gamma < 1$ , at some period after default the young again form the majority and vote for paying the debt off. Notice that the country's total debt will never exceed  $-\tau^h$ . This is a default-renegotiation contract

**Proposition 4.** Suppose  $\frac{\partial Z}{\partial w} > 0$ . If there exists an individual  $i$  with endowment  $w^i = \bar{w}$  such

that  $Z(\bar{w}, \varepsilon^h, \varepsilon^l, \tau^h, \tau^l) = 1$ , there then exists a default-free contract only if  $\bar{w} < 1 - \frac{1}{2\alpha_{t+1}}$ .

Likewise, for  $\frac{\partial Z}{\partial w} < 0$ , if there is an individual  $i$  with endowment  $w^i = \bar{w}$  such that

$Z(\bar{w}, \varepsilon^h, \varepsilon^l, \tau^h, \tau^l) = 1$ , there then exists a default-free contract only if  $\bar{w} > \frac{1}{2\alpha_{t+1}}$ . If

$Z(w^i, \varepsilon^h, \varepsilon^l, \tau^h, \tau^l) < 1$  for all  $i$ , there then exists no default-free contract. If

$Z(w^i, \varepsilon^h, \varepsilon^l, \tau^h, \tau^l) > 1$  for all  $i$ , then  $(\tau^h, \tau^l)$  is a default-free contract.

**Proof.** Individual  $i$  with  $w^i = \bar{w}$  is indifferent between default and not default.  $\frac{\partial Z}{\partial w} > 0$

implies that each young individual  $j$  with  $w^j < \bar{w}$  votes for default, and each individual  $s$  with  $w^s > \bar{w}$  votes for repayment. A default-free contract occurs if the population of the old plus the population of the type  $j$  young is less than the population of the type  $s$  young, i.e.,

when  $(1 - \alpha_{t+1}) + \alpha_{t+1}\bar{w} < \alpha_{t+1}(1 - \bar{w})$  or  $\bar{w} < 1 - \frac{1}{2\alpha_{t+1}}$ . For the case of  $\frac{\partial Z}{\partial w} < 0$ , the condition

for repayment will be  $(1 - \alpha_{t+1}) + \alpha_{t+1}(1 - \bar{w}) < \alpha_{t+1}\bar{w}$  or  $\bar{w} > \frac{1}{2\alpha_{t+1}}$ . If  $Z(w^i, \varepsilon^h, \varepsilon^l, \tau^h, \tau^l) \leq 1$

for all  $i$ , the young vote for default. The old (as always) vote for default. Therefore, the country defaults. If  $Z(w^i, \varepsilon^h, \varepsilon^l, \tau^h, \tau^l) \geq 1$  for all  $i$ , then all the young vote for repayment.

Since the young have the majority ( $0.5 \leq \alpha_{t+1} < 1$ ), the country will not default.

**Example 1.** For a country in which the young are pivotal and have quadratic preferences

$u(x) = ax - bx^2$ , a default-free contract exists if risk aversion is sufficiently high (small  $\frac{a}{b}$ )

and  $\pi^l$  is small.



**Proof.** We need to show that for quadratic preferences there exists a bundle  $(\tau^h, \tau^l)$  for which the following holds if  $\gamma = 1$ :

$$Z(w^h, w^l, \tau^h, \tau^l) = \beta[\pi^l \frac{u(w^l + \tau^l) - u(w^l)}{u(w^h) - u(w^h + \tau^h)} - \pi^h] \geq 1 \text{ and } \pi^h \tau^h + \pi^l \tau^l = 0.$$

$$\begin{aligned} Z(w^h, w^l, \tau^h, \tau^l) &= \beta[\pi^l \frac{u(w^l + \tau^l) - u(w^l)}{u(w^h) - u(w^h + \tau^h)} - \pi^h] \\ &= \beta[\pi^l \frac{a^*(w^l + \tau^l) - b^*(w^h + \tau^l)^2 - [a^*(w^l) - b^*(w^l)^2]}{a^*w^h - b^*(w^h)^2 - [a^*(w^h + \tau^h) - b^*(w^h + \tau^h)^2]} - \pi^h] \\ &= \beta[\pi^l \frac{a^*\tau^l - b^*\tau^l*(2w^l + \tau^l)}{-a^*\tau^h + b^*\tau^h*(2w^h + \tau^h)} - \pi^h] \\ &= \beta[\pi^l \frac{b^*\tau^l}{-b^*\tau^h} \frac{\frac{a}{b} - (2w^l + \tau^l)}{\frac{a}{b} - (2w^h + \tau^h)} - \pi^h] = \beta[\pi^h \frac{-\tau^h}{-\tau^h} \frac{\frac{a}{b} - (2w^l + \tau^l)}{\frac{a}{b} - (2w^h + \tau^h)} - \pi^h] \\ &= \beta\pi^h [\frac{\frac{a}{b} - (2w^l + \tau^l)}{\frac{a}{b} - (2w^h + \tau^h)} - 1] = \beta\pi^h [\frac{\frac{a}{b} - (2w^l + \tau^l) - \frac{a}{b} + (2w^h + \tau^h)}{\frac{a}{b} - (2w^h + \tau^h)}] \\ &= \beta\pi^h [\frac{2w^h + \tau^h - 2w^l - \tau^l}{\frac{a}{b} - (2w^h + \tau^h)}] \end{aligned}$$

$Z$  is positive for plausible values of the model parameters. (Since  $u'(w^h) > 0$ , the denominator of the above inequality is positive.) We can also plausibly assume the size of the insurance contract is less than double the size of the endowment, i.e.,  $\tau^l - \tau^h < 2(w^h - w^l)$

Set  $Z \geq 1$  and solve for  $\tau^l$  from the above inequality:

$$\beta\pi^h(2w^h + \tau^h - 2w^l - \tau^l) \geq \frac{a}{b} - (2w^h + \tau^h)$$

$$(-(1 + \beta\pi^h)\frac{\pi^l}{\pi^h} - \beta\pi^h) * \tau^l \geq \frac{a}{b} - 2(1 + \beta\pi^h) * w^h - \beta\pi^h w^l$$

$$(\beta + \frac{\pi^l}{\pi^h}) * \tau^l \leq -\frac{a}{b} + 2(1 + \beta\pi^h) * w^h + \beta\pi^h w^l$$

$$\tau^l \leq \frac{2(1 + \beta\pi^h) * w^h + \beta\pi^h w^l - \frac{a}{b}}{\beta + \frac{\pi^l}{\pi^h}}$$

For the right hand side to be positive we require:

$$2(1 + \beta\pi^h) * w^h + \beta\pi^h w^l - \frac{a}{b} \geq 0$$

This, combined with  $u'(w^h) > 0$  implies the following parameter restrictions:

$$2w^h \leq \frac{a}{b} \leq 2w^h + 2\beta\pi^h * (w^h + w^l) .$$

Now we find  $-\tau^h$ :

$$\tau^l = -\frac{\pi^h}{\pi^l} \tau^h$$

$$-\frac{\pi^h}{\pi^l} \tau^h \leq \frac{2(1 + \beta\pi^h) * w^h + \beta\pi^h w^l - \frac{a}{b}}{\beta + \frac{\pi^l}{\pi^h}}$$

$$-\tau^h \leq \frac{2(1 + \beta\pi^h) * w^h + \beta\pi^h w^l - \frac{a}{b}}{\beta \frac{\pi^h}{\pi^l} + 1}$$

It is obvious from the above equation that for small enough  $\pi^l$ ,  $-\tau^h$  is affordable, i.e.,  $-\tau^h \leq w^h$ .

In conclusion, if  $2w^h \leq \frac{a}{b} \leq 2w^h + 2\beta\pi^h * (w^h + w^l)$ , then a default-free contract exists.

**Example 2.** For quadratic preferences, there exists a default-repayment contract for any  $\gamma$  and sufficiently small  $\pi^l$ .

**Proof.** We need to show that for quadratic preferences there exists a bundle  $(\tau^h, \tau^l)$  for which the following holds if  $\gamma = 1$ :

$$\beta[\pi^l \frac{u(w^h + \tau^l) - u(w^l)}{u(w^h) - u(w^h + \tau^h)} - \gamma\pi^h] \geq 1 \text{ and } \pi^h \tau^h + \pi^l \tau^l = 0.$$

$$\begin{aligned} \beta[\pi^l \frac{u(w^h + \tau^l) - u(w^l)}{u(w^h) - u(w^h + \tau^h)} - \gamma\pi^h] &= \beta[\pi^l \frac{(w^h + \tau^l)^2 - (w^l)^2}{(w^h)^2 - (w^h + \tau^h)^2} - \gamma\pi^h] \\ &= \beta[\pi^l \frac{\tau^l}{-\tau^h} \frac{2w^l + \tau^l}{2w^h + \tau^h} - \gamma\pi^h] = \beta[\pi^h \frac{-\tau^h}{-\tau^h} \frac{2w^l + \tau^l}{2w^h + \tau^h} - \gamma\pi^h] = \beta\pi^h \frac{2(w^l - \gamma w^h) + \tau^l - \gamma\tau^h}{2w^h + \tau^h} \end{aligned}$$

$$\text{Choose } \tau^l > \max(0, \frac{2w^h - 2\beta\pi^h(w^l - 2w^h)}{\beta\pi^h + \beta\gamma\pi^l + \frac{\pi^l}{\pi^h}}) \text{ and } \tau^h = -\frac{\pi^h}{\pi^l} \tau^l$$

As in the case of the default-free contract (Proposition 2), we can show that for sufficiently small  $\pi^l$  (big  $\pi^h$ ),  $\tau^h$  and  $\tau^l$  satisfy:

$$\beta\pi^h \frac{2(w^l - \gamma w^h) + \tau^l - \gamma\tau^h}{2w^h + \tau^h} \geq 1 \text{ and } \pi^h \tau^h + \pi^l \tau^l = 0$$

Therefore,  $(\tau^h, \tau^l)$  is a default-repayment contract.

**Example 3.** For quadratic preferences, case  $\frac{\partial Z}{\partial w} > 0$  ( $\frac{\partial Z}{\partial w} < 0$ ) occurs if the difference of

shock sizes, relative to insurance payments, is big (small) enough that:  $\frac{\varepsilon^h - \varepsilon^l}{\tau^l} > \frac{1 + \frac{\pi^l}{\pi^h}}{2}$

$$(\frac{\varepsilon^h - \varepsilon^l}{\tau^l} < \frac{1 + \frac{\pi^l}{\pi^h}}{2}).$$

**Proof:**

$$\begin{aligned}
Z(w, \varepsilon^h, \varepsilon^l, \tau^h, \tau^l) &= \beta[\pi^l \frac{u(w + \varepsilon^l + \tau^l) - u(w + \varepsilon^l)}{u(w + \varepsilon^h) - u(w + \varepsilon^h + \tau^h)} - \gamma\pi^h] \\
&= \beta[\pi^l \frac{a^*(w + \varepsilon^l + \tau^l) - b^*(w + \varepsilon^l + \tau^l)^2 - a^*(w + \varepsilon^l) + b^*(w + \varepsilon^l)^2}{a^*(w + \varepsilon^h) - b^*(w + \varepsilon^h)^2 - a^*(w + \varepsilon^h + \tau^h) + b^*(w + \varepsilon^h + \tau^h)^2} - \gamma\pi^h] \\
&= \beta[\pi^l \frac{a^*\tau^l - b^*\tau^l*(2w + 2\varepsilon^l + \tau^l)}{-a^*\tau^h + b^*\tau^h*(2w + 2\varepsilon^h + \tau^h)} - \gamma\pi^h]
\end{aligned}$$

Take the derivative of  $Z$  with respect to  $w$ :

$$\begin{aligned}
&\frac{\partial Z}{\partial w} \\
&= \beta\pi^l \frac{-2b^*\tau^l*(-a^*\tau^h + b^*\tau^h*(2w + 2\varepsilon^h + \tau^h)) - 2b^*\tau^h*(a^*\tau^l - b^*\tau^l*(2w + 2\varepsilon^l + \tau^l))}{(-a^*\tau^h + b^*\tau^h*(2w + 2\varepsilon^h + \tau^h))^2} \\
&= -2b\beta\pi^l \frac{\tau^l*(-a^*\tau^h + b^*\tau^h*(2w + 2\varepsilon^h + \tau^h)) + \tau^h*(a^*\tau^l - b^*\tau^l*(2w + 2\varepsilon^l + \tau^l))}{(-a^*\tau^h + b^*\tau^h*(2w + 2\varepsilon^h + \tau^h))^2} \\
&= -2b\beta\pi^l \frac{\tau^l*b^*\tau^h*(2w + 2\varepsilon^h + \tau^h) - \tau^h*b^*\tau^l*(2w + 2\varepsilon^l + \tau^l)}{(-a^*\tau^h + b^*\tau^h*(2w + 2\varepsilon^h + \tau^h))^2} \\
&= -2b^2\beta\pi^l*\tau^l*\tau^h \frac{(2w + 2\varepsilon^h + \tau^h) - (2w + 2\varepsilon^l + \tau^l)}{(-a^*\tau^h + b^*\tau^h*(2w + 2\varepsilon^h + \tau^h))^2} \\
&= -2b^2\beta\pi^l*\tau^l*\tau^h \frac{(2\varepsilon^h + \tau^h) - (2\varepsilon^l + \tau^l)}{(-a^*\tau^h + b^*\tau^h*(2w + 2\varepsilon^h + \tau^h))^2}
\end{aligned}$$

$\frac{\partial Z}{\partial w} < 0$  if and only if  $2\varepsilon^h + \tau^h - 2\varepsilon^l - \tau^l < 0$ . We can use the zero profit condition

( $\pi^h\tau^h + \pi^l\tau^l = 0$ ) to simplify the final equality as follows:  $\frac{\varepsilon^h - \varepsilon^l}{\tau^l} < \frac{1 + \frac{\pi^l}{\pi^h}}{2}$