FOR AGENDA

SM/08/5
Correction 1

January 9, 2008

To: Members of the Executive Board

From: The Secretary

Subject: **Dominican Republic—Selected Issues**

Chapter I (pages 3–26) of SM/08/5 (1/7/08) is being reissued to reflect corrections that have been provided by the staff. The corrections are due to formatting errors that affected most of the mathematical expressions in Chapter I. Other typographical errors are listed below.

Typographical Errors

Page 3, bullet 1, line 7: for "This is done in Section II."
read "This is done in Sections B–D."

Page 3, bullet 2, line 6: for "This is done in Section III."
read "Sections E–G analyze these issues."

Page 4, lines 4 and 5: for "Section IV deals"
read "Sections H–J deal"

Page 4, bullet 1: for "Section V"
read "Section K"

Page 4, para. 2, line 1: for "This Section develops"
read "Sections B to D develop"

Page 7, footnote 7, line 1: for "Note that in (7)"
read "Note that in (6)"

Page 8, para. 5, last line: for "expressions (8) and (9)"
read "expressions (7) and (8)"

Page 9, lines 3 and 4: for "This Section draws on (and at times adapts)"
read "Sections E through G draw on (and at times adapt)"

Page 13, para. 12, line 2: for "(Section II), debt threshold (Section III),"
read "(Section D), debt threshold (Section G),"

Page 13, bullet 1, line 1, and para. 13, line 1: for "in Section II"
read "in Section D"

Page 13, para. 12, line 2: for "(Section D)"
read "(Sections B - D)"

Page 13, para. 12, line 2: for "(Section G)"
read "(Sections E - G)"

Page 13, para. 12, line 4: for "achieving such a debt target."
read "achieving such a debt target. This is done in Sections H through J."

Page 15, bullet 1, line 5; page 16, para. 14, lines 3 and 8: for "Section II"
read "Section D"

Page 16, footnote 14, line 1: for "target."
read "target, which is about the ratio that marks the boundary between the Dominican Republic belonging to Club C or B (II). See Table 3."

Page 17, para. 16, line 8; page 20, para. 17, line 7; page 23, line 19; **page 24, lines 2 and 12**; **page 25, line 7; and page 26, line 2:** for "Section II"
read "Section D"

Questions may be referred to Mr. Wolfe (ext. 38620) and Mr. Di Bella (ext. 37483) in WHD.

This document will shortly be posted on the extranet, a secure website for Executive Directors and member country authorities.

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I. Consolidated Public Debt Sustainability Analysis (DSA) for the Dominican Republic: An Alternative Framework[1]

A. Introduction

1. **This paper analyzes the sustainability of the Dominican Republic's consolidated public sector (CPS) debt, using a framework complementary to the Fund's standard template**. This DSA has three components: (i) an integrated and consistent accounting framework; (ii) the estimation of an appropriate and country-specific debt threshold for the Dominican Republic; and (iii) a method for the calculation of the CPS primary balance to achieve the desired debt targets, without resorting to ad-hoc assumptions for the values of the macroeconomic variables during the planning horizon.

- Regarding the first component, standard DSAs do not explicitly consider the effects on public finances of quasi-fiscal operations of the central bank; i.e., the costs associated with the implementation of monetary policy, usually linked to the sterilization of excess liquidity, are not explicitly incorporated into the analysis.[2] In the case of the Dominican Republic, the relatively large level of central bank debt suggests that the DSA framework needs to incorporate the central bank accounts in a manner as detailed as possible. This is done in Sections B–D.

- Regarding the second component, standard DSAs do not explicitly take into account whether the initial level of public debt exceeds (or is close to exceeding) what history suggests is that country's tolerable debt burden (Reinhart, Rogoff and Savastano, 2003). In this regard, standard DSAs focus more on debt trends and not on the level of debt compared with what the market is prepared to absorb, taking into consideration the country's historical experience. Sections E–G analyze these issues.

- Regarding the third component, standard DSAs usually associate a primary surplus of a given size with a targeted debt-to-GDP ratio; however, the likelihood of the baseline scenario coming to fruition is not assessed. While standard DSAs include a set of stress tests that assume shocks to one or more variables, they do not consider any covariance among the variables. The framework in this paper links the size of the primary surplus with the (cumulative) probability of achieving a targeted debt-to-GDP ratio. This in turn, enables policymakers to assess the risk in terms of the

[1] Prepared by Gabriel Di Bella.

[2] Excess liquidity could be the consequence, for instance, of large capital inflows and/or the result of large monetary expansions of quasi-fiscal (or fiscal) origin (such as those resulting from bailouts of private depositors during an economic crisis).

increased vulnerability that comes with a larger debt ratio behind any decision in terms of the size of the primary surplus target. Moreover, it allows for stress tests that consider the covariance among macroeconomic variables, an essential feature of the economy in times of stress (as suggested by Garcia and Rigobon, 2004). Sections H-J deal with these issues.

- Section K summarizes and concludes.

B. The Accounting Framework

2. **Sections B to D develop an integrated framework for analyzing debt sustainability of the consolidated public sector (CPS), where most flows/stock variables of both the non-financial public sector (NFPS) and the central bank (BCRD) balance sheets are considered explicitly**.

C. The budget constraint for the Consolidated Public Sector

Non-Financial Public Sector

$$T_t + V_t^{CB} - S_t^{G,CB} - S_t^{G,H} + i_{t-1}^{Q} \cdot Q_{1t-1}^{d,G} - i_{t-1}^{F} \cdot F_{t-1}^{s,G} - i_{t-1}^{H} \cdot H_{t-1}^{s,G} - i_{t-1}^{U} \cdot U_{t-1}^{s,G} - i_{t-1}^{E} \cdot E_{t-1}^{s,G} - p_t \cdot Z_t^{d,G} - w_t \cdot N_t^{d,G} = \\ = \Delta Q_{1t}^{d,G} + \Delta Q_{2t}^{d,G} - \Delta H_t^{s,G} - \Delta F_t^{s,G} - \Delta E_t^{s,G} - \Delta U_t^{s,G} \quad (1)$$

Equation (1) shows the flow budget constraint for the NFPS (or G as denoted in all expressions).[3] "Above the line" flow variables (the left hand side–LHS-of (1)) include: taxes (T_t), dividends from the BCRD(V_t^{CB}), interest from deposits (domestic-currency denominated) in the domestic banking system ($i_{t-1}^{Q} \cdot Q_{1t-1}^{d,G}$), wages ($w_t \cdot N_t^{d,G}$), goods and services ($p_t \cdot Z_t^{d,G}$), transfers to the BCRD ($S_t^{G,CB}$), transfers to the households ($S_t^{G,H}$), interest payments on domestic-currency denominated bonds ($i_{t-1}^{F} \cdot F_{t-1}^{s,G}$), interest payments on domestic-currency denominated BCRD recapitalization bonds ($i_{t-1}^{U} \cdot U_{t-1}^{s,G}$), interest payments on domestic-currency denominated BCRD domestic credit ($i_{t-1}^{H} \cdot H_{t-1}^{s,G}$), and interest payments

[3] For a given stock variable ($X_j^{h,i}$) and its respective flow ($\Delta X_j^{h,i}$), $h = \{s, d\}$ indicates whether the variable refers to a "demand"or a supply, $i = \{G, CB, CPS\}$ indicates the economic agent that is demanding/supplying, while j denotes the time period to which the associated stock/flow corresponds; when necessary, in order to separate between subcomponents of a given variable, a subscript 1 is used with the time period to denote that the corresponding variable is originally denominated in domestic currency, while a subscript 2 is used to denote that the corresponding variable is originally denominated in foreign currency. All variables are expressed in nominal terms, in domestic currency units. Finally, an asterisk ($*$) denotes that the associated variable is measured in foreign currency. It is assumed throughout the analysis that there is only one foreign currency.

on foreign-currency denominated external debt ($i^{E}_{t-1} \cdot E^{s,G}_{t-1}$).[4,5] "Below the line" flows (the right hand side–RHS-of (1)), includes changes in the NFPS' financial assets, including domestic-currency denominated deposits in the domestic banking sector ($Q^{d,G}_{1t}$) and foreign-currency denominated deposits in the BCRD ($Q^{d,G}_{2t}$); it also includes changes in the NFPS' liabilities including domestic credit from the BCRD ($H^{s,G}_{t}$), domestic-currency denominated bonds ($F^{s,G}_{t}$), domestic-currency denominated BCRD recapitalization bonds ($U^{s,G}_{t}$), and foreign-currency denominated external debt ($E^{s,G}_{t}$).

Central Bank

$$(2)\quad \begin{aligned} & S^{G,CB}_{t} - V^{CB}_{t} + i^{R}_{t-1} \cdot R^{d,CB}_{t-1} + i^{F}_{t-1} \cdot F^{d,CB}_{t-1} + i^{H}_{t-1} \cdot H^{d,CB}_{t-1} + i^{U}_{t-1} \cdot U^{d,CB}_{t-1} - i^{E}_{t-1} \cdot E^{s,CB}_{t-1} - i^{D}_{t-1} \cdot D^{s,CB}_{t-1} - w_t \cdot N^{d,CB}_{t} = \\ & = \Delta R^{d,CB}_{t} + \Delta H^{d,CB}_{t} + \Delta F^{d,CB}_{t} + \Delta U^{d,CB}_{t} - \Delta B^{s}_{1t} - \Delta B^{s}_{2t} - \Delta D^{s,CB}_{t} - \Delta E^{s,CB}_{t} - \Delta Q^{s,CB}_{2t} \end{aligned}$$

Equation (2) shows the flow budget constraint for the BCRD (or CB as denoted in all expressions). Many of the terms in this equation are merely the counterparts of the respective terms in (1) namely, V^{CB}_{t}, $S^{G,CB}_{t}$, $i^{U}_{t-1} \cdot U^{d,CB}_{t-1}$, $Q^{s,CB}_{2t}$, and $U^{d,CB}_{t}$. There are other terms in (1) that have a "partial" counterpart in (2): $i^{F}_{t-1} \cdot F^{d,CB}_{t-1}$, $F^{d,CB}_{t}$ (as the domestic-currency denominated bonds supplied by the NFPS are held both by the BCRD and the private sector), and $i^{H}_{t-1} \cdot H^{d,CB}_{t-1}$, $H^{d,CB}_{t}$ (as the BCRD lends both to the NFPS and the banking sector). In addition, there are other variables that do not have a counterpart in (1): Foreign-currency denominated gross international reserves ($R^{d,CB}_{t}$), interest income derived from such international reserves ($i^{R}_{t-1} \cdot R^{d,CB}_{t-1}$), foreign-currency denominated external debt ($E^{s,CB}_{t}$), and interest paid on such debt ($i^{E}_{t-1} \cdot E^{s,CB}_{t-1}$), domestic-currency denominated BCRD debt ($D^{s,CB}_{t}$), interest paid on such debt ($i^{D}_{t-1} \cdot D^{s,CB}_{t-1}$), domestic-currency denominated monetary base ($B^{s,CB}_{1t}$), foreign-currency denominated monetary base ($B^{s,CB}_{2t}$), and wages paid to BCRD employees ($w_t \cdot N^{d,CB}_{t}$).

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[4] Note that the NFPS holds non-remunerated foreign-currency denominated deposits in the BCRD ($Q^{d,G}_{2t-1}$).

[5] In (1)-(3), w_j is the wage rate per labor unit (which, to simplify, is assumed equal for all economic agents) in period j, $N^{h,i}_{j}$ denotes de amount of labor units demanded/supplied by sector i in period j, p_j is the price for the only good in the economy in period j, $Z^{h,i}_{j}$ denotes de amount of units of goods demanded/supplied by sector i in period j, while i^{x}_{j} denotes the nominal interest rate for financial instrument x in period j.

Consolidated Public Sector

Solving for V_t^{CB} in (2), replacing in (1), aggregating/consolidating BCRD and NFPS flows when appropriate, and taking into consideration that both (1) and (2) are *ex-post* expressions, yields the consolidated public sector (CPS) budget constraint (3):

$$(3)\quad \begin{aligned} &T_t + i_{t-1}^R \cdot R_{t-1}^{s,CB} + i_{t-1}^Q \cdot Q_{1t}^{d,G} + i_{t-1}^H \cdot H_{t-1}^{d,CPS} - i_{t-1}^F \cdot F_{t-1}^{s,CPS} - i_{t-1}^E \cdot E_{t-1}^{s,CPS} - i_{t-1}^D \cdot D_{t-1}^{s,CB} - p_t \cdot Z_t^{d,G} - w_t \cdot N_t^{d,CPS} = \\ &= \Delta R_t^{d,CB} + \Delta H_t^{d,CPS} + \Delta Q_{1t}^{d,G} - \Delta B_{1t}^s - \Delta B_{2t}^s - \Delta D_t^{d,CB} - \Delta F_t^{s,CPS} - \Delta E_t^{s,CPS} \end{aligned}$$

3. **Equation 3 relates the flow and stock variable changes for the CPS**. The intuition behind equation (3) is that: (i) the overall CPS balance (LHS) should be equal to the increase in the CPS (net) assets (RHS); (ii) increases in the overall CPS debt may be explained by a CPS overall deficit, but also by increases in CPS financial assets, most notably, by increases in international reserves; (iii) an overall CPS deficit may be financed by increases in debt, seignorage, and/or decreases in financial assets; (iv) increases in international reserves can be financed by monetary base expansion, increases in CPS debt, and/or an overall CPS surplus; (v) increases of international reserves in excess of increases in the monetary base may be sterilized by increases in BCRD debt, NFPS debt, and/or an overall CPS surplus, which underscores the desirability for the coordination of monetary and fiscal policies; (vi) sterilization has a cost when it is done by means different than an overall CPS surplus and the effective interest rate paid on CPS debt (in peso terms) is larger than the effective interest rate (in peso terms) obtained from the placement of international reserves; and (vii) the recapitalization of the BCRD does not have any impact for the CPS as a whole, as it only implies flows between the NFPS and the BCRD that cancel out after consolidation.[6]

D. Target Public Debt Ratios and the Size of the Primary Balance

4. **Algebraic manipulation of (3) reveals the factors contributing to CPS debt accumulation**. In this regard, expression (4) aggregates, on the LHS, all CPS liabilities (in stock terms, Φ_t), while the RHS includes all variables explaining CPS debt levels.

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[6] In (3), consolidation implies that: $F_t^{s,CPS} = F_t^{s,G} - F_t^{d,CB}$, $H_t^{d,CPS} = H_t^{d,CB} - H_t^{s,G}$ and $i_{t-1}^U \cdot U_{t-1}^{d,CB} = i_{t-1}^U \cdot U_{t-1}^{s,G}$; in turn, aggregation implies that, $E_t^{s,CPS} = E_t^{s,G} + E_t^{s,CB}$ and $N_t^{d,CPS} = N_t^{d,G} + N_t^{d,CB}$; finally consolidation plus the ex-post condition imply that $Q_{2t}^{s,CB} = Q_{2t}^{d,G}$, and $U_t^{d,CB} = U_t^{s,G}$.

$$(4)\ \Phi_t = \Phi_{t-1}\cdot\left(1+\widetilde{i_{t-1}^{\Phi}}\right)+R_{t-1}\cdot\left(1+\widetilde{\beta_t}\right)+H_{t-1}\cdot\left(1+\widetilde{h_t}\right)-B_{t-1}\cdot\left(1+\widetilde{y_t}\right)-\overline{PB_t^{CB}}-\overline{PB_t^{G}}$$

In (4), $\widetilde{\beta_t}$ is a function of the rate differential (in domestic-currency terms) between the rate of growth of international reserves and the interest rate on international reserves (the risk-free rate). Analogously, $\widetilde{h_t}$ is a function of the rate differential between the rate of growth of BCRD domestic credit and interest charged on such credit. In turn, $\widetilde{y_t}$ is a function of nominal GDP growth, while $\overline{PB_t^{G}}$, and $\overline{PB_t^{CB}}$ are the primary balances of the NFPS and BCRD respectively, after consolidating the crossed terms.

Equation (4) can be expressed in terms of GDP as follows:

$$(5)\qquad \phi_t = \phi_{t-1}\cdot\left(1+\overline{i_{t-1}^{\Phi}}\right)+r_{t-1}\cdot\left(1+\overline{\beta_t}\right)+h_{t-1}\cdot\left(1+\overline{h_t}\right)-b_{t-1}\cdot\left(1+\overline{y_t}\right)-\overline{pb_t^{CB}}-\overline{pb_t^{G}}$$

In (5), lowercase letters denote corresponding variables expressed in terms of GDP, while $\overline{i_{t-1}^{\Phi}}$, $\overline{\beta_t}$, $\overline{h_t}$, and $\overline{y_t}$ are functions of the rate differential between the corresponding variables and the (nominal) GDP growth.

Recursive substitution in (5) yields an expression relating the NFPS primary balance with a target debt ratio to be reached in J periods, $\phi_{t+J}=\bar{\bar{\phi}}$:

$$(6)\ \overline{pb^{G}}=\frac{\overline{i^{\Phi}}}{\left(1+\overline{i^{\Phi}}\right)^{J}-1}\cdot\left[\phi_t\cdot\left(1+\overline{i^{\Phi}}\right)^{J}+r_t\cdot\left(1+\overline{\beta}\right)\cdot\frac{\left[\left(1+\overline{i^{\Phi}}\right)^{J}-\left(1+\overline{\beta^{*}}\right)^{J}\right]}{\left(\overline{i^{\Phi}}-\overline{\beta^{*}}\right)}+h_t\cdot\left(1+\bar{h}\right)\cdot\frac{\left[\left(1+\overline{i^{\Phi}}\right)^{J}-\left(1+\hat{\bar{h}}\right)^{J}\right]}{\left(\overline{i^{\Phi}}-\hat{\bar{h}}\right)}-\bar{\bar{\phi}}\right]-$$
$$-b_t\cdot\left(1+\bar{y}\right)-\overline{pb^{CB}}$$

In (6), it is assumed that (i) $\overline{pb_t^{CB}}$ is exogenous and constant, and (ii) that all relevant macroeconomic variables remain constant through time. The CPS primary balance needed to reach $\bar{\bar{\phi}}$ would simply result from adding (the assumed constant) $\overline{pb_t^{CB}}$ plus the NFPS primary balance resulting from (6).[7,8]

[7] Note that in (6) $\hat{\bar{h}}_t$ and $\overline{\beta_t^{*}}$ are also functions of the differentials between the corresponding variables and nominal GDP growth.

In contrast, if sequences of macroeconomic variables are allowed to change through time, expression (6) turns into:

$$(7) \qquad \overline{pb^G} = \left[\Theta(J)\right]^{-1} \cdot \left[\phi(J) + r(J) + h(J) - b(J) - \overline{pb^{CB}}(J) - \bar{\bar{\phi}}\right]$$

In turn, if $\overline{pb_t^G}$ is allowed to change with the cycle, $\overline{pb_t^G} = \overline{\overline{pb^G}} \cdot \left[1 + \eta \cdot \left(\gamma_t - \bar{\gamma}\right)\right]$, where $\overline{\overline{pb^G}}$ is the (constant) structural primary balance, $\bar{\eta}$ is the elasticity of the primary balance with respect to the output gap $\left(\gamma_t - \bar{\gamma}\right)$, and $\bar{\gamma}$ is the long-term GDP growth rate, then (7) turns into:

$$(8) \qquad \overline{\overline{pb^G}} = \left[\bar{\Theta}(J)\right]^{-1} \cdot \left[\phi(J) + r(J) + h(J) - b(J) - \overline{pb^{CB}}(J) - \bar{\bar{\phi}}\right]$$

5. **The intuition behind (6)-(8) is that, to reach a given debt ratio target, the CPS primary surplus will need to be larger**: (i) the larger the initial debt ratio; (ii) the larger the size of any realized contingent liability shocks during the J periods; (iii) the larger the expected increase in CPS gross assets (most notably, international reserves); (iv) the lower the expected seignorage (including that coming from legal reserve requirements on bank deposits); (v) the lower the target for the debt ratio; (vi) the faster the speed at which the debt ratio is to be reached (i.e., the lower is J); (vii) the larger the differential between the (real) interest rate on CPS debt and real GDP growth; (viii) the larger the difference between the expected rate of increase in international reserves and the risk-free interest rate; and (ix) the larger the difference between the expected rate of increase in BCRD domestic assets and their rate of return. Finally, expressions (7) and (8) only differ in their denominator.[9]

E. Establishing a debt ratio target for the Dominican Republic's CPS

6. **Reinhart, Rogoff and Savastano (RRS, 2003), argue that a country's track record at meeting its debt obligations (measured by the country's default history) and managing its macro economy (measured by the country's historical inflation rate) have an influence on its ability to access voluntary debt markets**. In this regard, RRS introduce the concept of "debt intolerance" to describe the problems that some economies experience when reaching debt levels that would seem manageable by the standards of more advanced

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[8] Burnside (2005, Chapter 3) derives a similar, though less general, formula.

[9] See the Mathematical Appendix for a detailed explanation as to how expressions (4)–(8) are derived, as well as for definitions of each of their terms, including the differences between expressions (7) and (8).

economies. Even though their analysis of debt intolerance is mainly focused on external debt, they argue that overall debt intolerance (i.e., including domestic and external debt) may be viewed as linked to a common set of factors. Sections E through G draw on (and at times adapt) the concepts, sources of data and terminology used by RRS, but attempt to extend their analysis to the consideration of domestic debt for a sample of 38 countries for the period 1989–2005. The objective of this analysis is to determine a "safe threshold" for the Dominican Republic's CPS debt.

F. Clubs and Regions of Debt intolerance

7. **Following RRS, this paper uses the country ratings (IIR) published bi-annually by "Institutional Investors" magazine to organize countries in "Clubs"**. The IIR for a given country is used as a proxy to measure that country's creditworthiness, or conversely, 100-IIR, would proxy its sovereign risk. To define such Clubs, the IIR mean (51.7) and standard deviation (23.0) were calculated for 38 industrial and developing countries included in a sample over the period 1989–2005. As shown in Table 1 (below), Club A includes those countries whose average IIR is larger than the mean plus one standard deviation; Club C includes those countries whose average IIR is lower than the mean less one standard deviation; Club B includes all countries in the intermediate range, which in turn is divided in two sub-ranges, Club B(I) includes countries whose average IIR is larger than the mean but lower than the mean plus one standard deviation, while Club B(II) includes those countries whose average IIR is lower than the mean but higher than the mean less one standard deviation.

Table 1. "Institutional Investors' Country Credit Survey"
(Annual ratings average, 1989-2005)

A (IIR >=74.8)	B		C (IIR <28.7)
	I (51.7=< IIR < 74.8)	II (28.7=< IIR < 51.7)	
Canada (85.0)	Chile (56.4)	Argentina (30.4)	Bolivia (23.2)
Denmark (82.1)	Czech Republic (57.7)	Brazil (35.5)	**Dominican Republic (25.4)**
Finland (80.2)	Greece (58.4)	Colombia (41.8)	Kenya (25.7)
Ireland (77.4)	Hungary (52.5)	Indonesia (40.4)	Nigeria (18.4)
Italy (79.0)	Korea (66.5)	India (45.8)	Pakistan (26.0)
Japan (89.1)	Malaysia (61.5)	Sri Lanka (30.4)	Tanzania (17.5)
Norway (84.9)	Thailand (57.2)	Mexico (47.2)	Zimbabwe (23.5)
Singapur (82.5)		Philippines (37.3)	Ghana (28.4)
USA (91.0)		Poland (44.4)	
		South Africa (45.5)	
		Turkey (40.3)	

Source: Fund Staff calculations using data from *Institutional Investors magazine*

8. **With an IIR average of 25.4, the Dominican Republic is well within Club C**. RRS argue that members of Club A have continuous access to voluntary debt markets, while members of Club B only enjoy intermittent access to voluntary debt markets. In contrast, members of Club C would be able to access voluntary markets only rarely, mainly resorting to bilateral and multilateral financing. Thus, members of Club A are the least debt intolerant, while countries in Club C are the most debt intolerant. Note that RRS argue that even though graduation to higher clubs is possible, it is not easy, as it would require many years of uninterrupted debt repayment, good macroeconomic management as measured by continuously low inflation rates, and relatively low public debt levels.[10]

G. A Country-Specific Debt Threshold for the Dominican Republic

9. **This section establishes a link between a country's sovereign risk (as proxied by the IIR) and its history of default and inflation**. Table 2 (below) shows three different specifications, whose results are similar to those in RRS: higher default rates and higher

[10] Looking at sub-periods within the 1989-2005 range, the Dominican Republic alternated between Club C (1989–1998 and again in 2005 in the aftermath of the banking crisis of 2003) and Club B(II) (1999–2004). In 2006–2007, as the economy recovered from the crisis, the IRR has been consistent with Club B(II).

inflation rates both result in lower IIRs (or higher country risk). The public debt ratio enters with a negative and significant coefficient for all countries in Clubs B and C, while the coefficient is positive for countries in Club A. The dummy variable for the Dominican Republic enters specifications 2 and 3 with a negative (and significant) coefficient; the rationale for including a Dominican Republic dummy was to "catch" the additional country-risk premium that the IIR seems to include in the case of the Dominican Republic. RRS argue that to identify countries that may be plausible candidates to graduate from a lower to a higher Club, one should look at those countries in which actual IIRs are consistently higher than those predicted by models as such included in Table 2 (RRS use the examples of Greece, Portugal, Thailand, Malaysia and Chile). In the case of the Dominican Republic, all specifications that exclude a Dominican Republic dummy (i.e., specification 1 in Table 2 and others not presented in this paper) result in IIR predictions that are consistently larger than actual IIRs. This would suggest that despite the progress in stabilizing the economy during the last years, the Dominican Republic would still be relatively far from graduation from Club C to Club B(II).

Table 2. The Role of History and Clubs: Cross Section Results

Following RRS the regression is:

$Y_i = \alpha + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + \beta_5 X_{5i} + \beta_6 X_{6i} + \beta_7 X_{7i} + u_i$

Y = IIR, 1989-2005 average
X_1 = DR Dummy
X_2 = Percent of 12-month periods of inflation at or above 40 percent since 1989
X_3 = Percent of years in a state of default or restructuring since 1946
X_4 = Public Debt / GDP (1989-2005 average) x Club A Dummy
X_5 = Public Debt / GDP (1989-2005 average) x Club B Dummy
X_6 = Public Debt / GDP (1989-2005 average) x Club C Dummy
X_7 = Public Debt / GDP (1989-2005 average) x Club Not A Dummy

38 observations

Regression Number	X_1	X_2	X_3	X_4	X_5	X_6	X_7	Adjusted R^2
				Least Squares Estimates, Robust errors				
1		-0.28	-0.21	0.21	-0.23	-0.44		0.78
		(-2.21)	(-1.27)	(2.32)	(-2.99)	(-6.07)		
2	-22.43	-0.27	-0.17	0.17	-0.27	-0.46		0.80
	(-2.05)	(-2.40)	(-1.18)	(1.95)	(-3.83)	(-6.12)		
3	-25.86	-0.25	-0.31	0.14			-0.33	0.75
	(-2.14)	(-1.97)	(-1.97)	(1.48)			(-4.77)	

Notes: t-statistics in parenthesis. Clubs are defined above and broadly follow the criteria established by RRS.

Sources: Reinhart, Rogoff and Savastano (2003), *Institutional Investor* , IMF's International Financial Statistics

10. **Given the Dominican Republic's historical performance with inflation and default, a target debt ratio of 25 percent of GDP would be appropriate**. The estimated coefficients from the third specification in Table 2, together with the actual values of the regressers, are used to predict the IIR for the Dominican Republic for varying ratios of CPS public debt. This exercise, shown in Table 3 (below), suggests that given the Dominican Republic's historical performance with inflation and default, as well as the additional risk premium that investors seem to have placed on the country during the period under consideration, a CPS debt level of 25 percent of GDP marks a sound "focal point", as this debt ratio: (i) would allow accommodation of short-term shocks without compromising the "country's membership" to Club B; and (ii) was the public debt ratio prevalent before the financial/economic crisis. [11]

Table 3: Predicted IIR for the Dominican Republic

Public Debt / GDP	Predicted IIR	Club
0	37.8	B II
5	36.3	B II
10	34.8	B II
15	33.2	B II
20	31.7	B II
25	30.2	B II
30	**28.6**	**C**
35	27.1	C
40	25.6	C
50	22.5	C

Note: Fund staff calculations based on the results of specification 3 in Table 2

11. **Graduating from Club C to Club B is best ensured by a relatively low debt ratio (of around 25 percent of GDP) and maintenance of the sound macroeconomic management of recent years and timely servicing of debt**. Such a profile would result in progressively larger IIRs, as well as (when extending the sample period) lower values for the regressors associated with default and inflation rates. Graduation from Club C would allow

[11] Debt ratios for the Dominican Republic are calculated using the GDP, base 1970, series, in order to ensure consistency with figures in SBA documents. However, the same analysis was performed with debt ratios that were calculated using the new GDP series (base 1991). As the new GDP levels are 14 percent larger than those in the 1970 series, the country-specific debt threshold for the Dominican Republic is 27 percent of GDP (base 1991).

the Dominican Republic to have a larger public debt threshold, reflecting a larger "appetite" of investors for exposure on Dominican Republic's paper.

H. Assessing the Appropriate CPS Primary Balance for the Dominican Republic [12]

12. **Having defined the conceptual framework that will be used for the DSA calculations (Sections B-D), as well as a country-specific debt threshold (Sections E-G), what remains is to determine the size of the CPS primary balance that would result in achieving such a debt target**. **This is done in Sections H through J.**

- Values for the macroeconomic variables in the expressions included in Section D were obtained using a data-generating process that reflects the observed historical behavior of the economy. As noted earlier, this captures the covariance among the macroeconomic variables, which is an essential feature of an economy in times of stress. Recommendations arising from this framework link the size of the primary surplus with the (cumulative) probability of achieving a targeted debt-to-GDP ratio. This allows policymakers to assess the risk, in terms of the increased vulnerability that comes with a larger debt ratio, behind any decision in terms of the size of the primary surplus target.

- In addition, the analysis assumes: (i) a decrease of 5 percentage points in the legal reserve requirements on commercial bank deposits (one percentage point per year beginning in 2010) as recommended by successive Fund TA missions to the Dominican Republic during 2006 and 2007; (ii) that the economy is subject (on average) to a shock costing the government the equivalent of 5 percent of GDP once every ten years, reflecting the experience of the Dominican economy during the last 30 years; and (iii) for 2008, the CPS primary balance will reach 1.2 percent of GDP, consistent with the authorities' budget.

I. The Data Generating Process

13. **The macroeconomic data used to calculate the value of expressions in Section D were calculated using a three-stage process**.

- *First* a VAR with 2 lags was computed with (yearly) data for the period 1961–2007; the variables (all in log first differences) included in the VAR were international

[12] The results presented in this section use the 1970 GDP series, but calculations were also performed using the 1991 GDP series. At the DSA seminar held in the context of Article IV consultation discussions, the results using the 1991 GDP series were presented. The main consequence of using the 1991 GDP series is one of scale, as cumulative GDP growth rates since 1991 do not differ significantly between the two series.

reserves (measured in foreign currency, $\widehat{\beta_t^*}$), the nominal exchange rate ($\widehat{\varepsilon_t}$), the price level (π_t), and real GDP (γ_t), in that order. After computing the (reduced form) VAR, the coefficients of the corresponding structural VAR were calculated. This allows the recovery of structural innovations that, in turn, allow the recovery of the variance-covariance matrix of the structural innovations (Table 4, below).

Table 4: Variance-Covariance Matrix VAR(2)

	International Reserves	Exchange Rate	Inflation	Real GDP Growth
International Reserves	0.2156	-0.0324	-0.0073	0.0043
Exchange Rate		0.0183	0.0026	-0.0011
Inflation			0.0022	0.0000
GDP Growth				0.0017

Source: Fund Staff calculations

- The signs of the covariances are as expected: increases in international reserves result in exchange rate appreciations, decreases in the inflation rate, and increases in GDP growth. Shocks resulting in exchange rate depreciations result in increases in the inflation rate and decreases in the rate of GDP growth.

- *Second*, two separate regressions were estimated linking foreign capital flows (proxied by the log first difference of international reserves), with the sovereign risk premium, and the exchange rate risk with the level of the sovereign risk premium (Table 5 below). As expected, capital outflows result in increases in sovereign risk that, in turn, result in increases in exchange rate risk.

- *Third*, using the estimated variance-covariance matrix for the VAR, 1,000 sequences were generated for each of its structural innovations covering the period 2008–2015; each of the sequences generated were fed into the VAR, resulting in 1,000 different sequences for the log first differences of the macroeconomic variables included in the VAR (i.e., international reserves, the nominal exchange rate, the price level and the real GDP), for the period 2008–2015. In turn, using the sequences for the (log first differences) of international reserves, and the regression coefficients of the sovereign and exchange rate risk premia, 1,000 sequences were generated for such premia. Finally, using a covered interest rate parity model (like in Furman and Stiglitz, 1998), 1,000 sequences for interest rates on NFPS foreign-currency debt, BCRD peso-

denominated debt and NFPS peso-denominated debt covering the period 2008–2015 debt were generated.[13]

Table 5: Sovereign and Exchange Rate Risk Regressions

The regression (1) for the sovereign risk is:

$Y_i = \alpha + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$

Y = sovereign risk premium (DR US$ debt yield - LIBOR)
X_1 = First Log difference of international reserves (lagged one period)
X_2 = First Log difference of international reserves (lagged two periods)

The regression (2) for the exchange rate risk is:

$Z_i = \alpha + \beta 3\ X_{3i} + v_i$

Z = exchange rate risk (DR peso debt yield- DR US$ debt yield)
X_3 = sovereign risk

12 observations after adjustments

Regression Number	X_1	X_2	X_3	Adjusted R^2
	Least Squares Estimates, Robust errors			
1	-0.03 (-4.32)	-0.03 (-4.97)		0.78
2			0.80 (2.56)	0.33

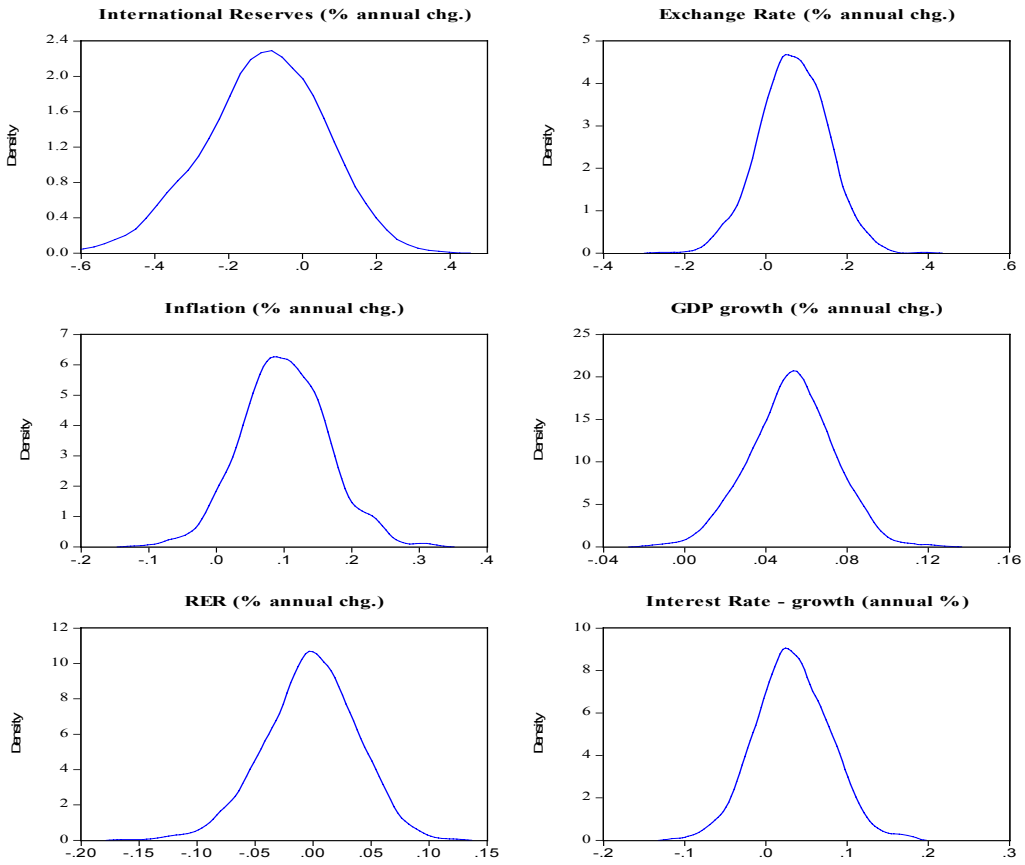
Notes: t-statistics in parenthesis.

Sources: Fund staff calculations on BCRD data.

- Figure 1 shows the (kernel density) distribution of the macroeconomic variables included in the VAR (yearly averages for the period 2008–2015 resulting from the 1,000 sequences generated), plus the resulting distribution of REER changes and $\overline{i_t^{\Phi}}$ (the differential between the weighted average interest rate and GDP growth, see Section D and Mathematical Appendix). The relative large volatility of the Dominican economy during the last three decades is reflected in distributions for the generated macroeconomic variables that have relatively large variances; this is particularly true in the case of international reserves, where the (kernel) distribution is the flattest of all the macroeconomic variables considered.

[13] Ideally, interest rates should have formed part of the VAR, which would have eliminated one step of the data generation process. The problem with incorporating interest rates into the VAR was one of data availability: relevant interest rate data is only available since the mid 1990s, while data on other macroeconomic variables is available since early 1960s.

Figure 1. Dominican Republic: (Generated) Macroeconomic Data through 2012
(Kernel Densites)



Source: Fund Staff calculations

J. The Results

14. **This section uses the sequences of generated data referred to in the previous section to calculate the CPS primary balance needed to achieve the 25-percent-of-GDP debt ratio suggested in Section D.**[14] In the calculations it was assumed that the target is to be achieved in 5 years (by end-2012) or 8 years (by end-2015). As the primary balance for 2008 is already set in the budget, the calculations were made assuming that the primary balances suggested by the framework start in 2009. As the sequences of macroeconomic variables described in the previous section are not constant through time, the CPS primary balance was calculated using expression (7) in Section D.[15]

[14] For reference, calculations were also made for a 30-percent-of-GDP target, which is about the ratio that marks the boundary between the Dominican Republic belonging to Club C or B(II). See Table 3.

[15] As the BCRD primary balance (its operating expenses) is assumed constant and exogenous, what is calculated in reality are 1,000 values for the NFPS primary balance, each one mapped to one of the generated sequences for the macroeconomic variables.

Table 6: CGS Primary balance needed to decrease the Debt/GDP ratio to:

Probality (cummulative)	Debt / GDP Target			
	25%		30%	
	end- 2012	end- 2015	end- 2012	end- 2015
50	3.1%	1.7%	1.7%	1.2%
60	3.5%	2.1%	2.3%	1.6%
70	4.0%	2.6%	2.9%	2.1%
75	4.3%	2.8%	3.3%	2.3%

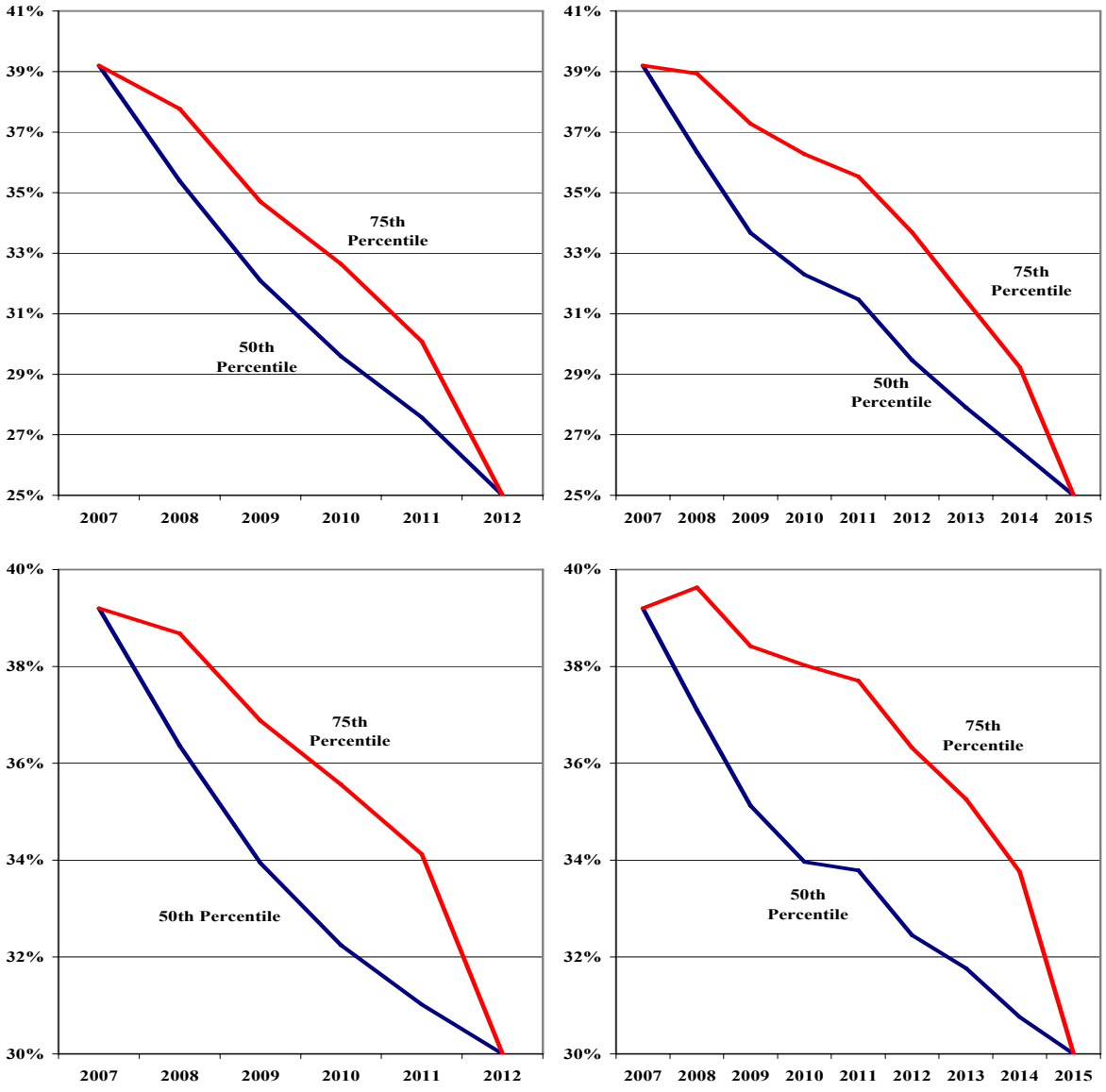
Source: Fund staff calculations

15. **To achieve a debt ratio of 25 percent of GDP by 2015 with a 70 percent probability would require a CPS primary balance of 2.6 percent of GDP** (Table 6). In the case that a CPS primary surplus of this size was chosen for 2009, the analysis suggests that this would result in a larger-than-expected decrease in debt ratios in 70 percent of the cases, i.e., in all those cases in which the effective realization of the macroeconomic variables is better than what was planned. Thus, if authorities choose such a level for the CPS primary balance, this would likely result, beginning in 2010, in a lower CPS primary balance needed to achieve the same debt targets.[16]

16. **The level of the primary surplus to reduce the debt-to-GDP ratio is sensitive to the degree of macroeconomic stress**. Figure 2 shows the paths for the CPS debt ratios for each combination of target debt ratios and periods to achieve them. Note that each of the charts in the panel include the paths for the 50th and 75th percentiles for the debt ratios, with the 75th percentile reflecting greater macroeconomic stress than implicit in the 50th percentile. Figure 3 shows the path for the debt ratios for a number of different CPS primary balances (0.5 percent of GDP, 1.0 percent of GDP and 1.5 percent of GDP). This was done by calculating expression (6) in Section D using each of the 1,000 generated sequences for the macroeconomic variables. Note that even though in the median (or 50th percentile) scenario, CPS primary balances of these sizes result in a decrease of debt ratios, they do not result in significant declines of debt ratios in stress situations (reflected by the 75th percentile). Moreover, they result in increases in debt ratios for CPS primary surpluses of 0.5 percent of GDP and 1 percent of GDP.

[16] These calculations were shared with the Dominican authorities and are available upon request.

Figure 2. Dominican Republic: Evolution of the CGS Debt Ratio
(for different debt ratio targets and time horizons)



Source: Fund Staff calculations

Figure 3. Dominican Republic: Evolution of the CGS Debt Ratio
(for different CGS primary balances)

Source: Fund Staff Calculations

17. **The CPS structural primary balance ($\overline{\overline{pb^G}}$) to achieve the target debt ratios can also be calculated**. Data for the period 1980-2007 suggests that when GDP growth exceeds long term growth, fiscal revenue (in GDP terms) would increase by a factor of about 0.2 times the output gap;[17] in cases when primary spending remains constant (in GDP terms), this would result in an improvement in the primary balance (in GDP terms) in times of positive output gaps. This is consistent with evidence for other emerging markets, as pointed out in IMF (2003). Using expression (8) in Section D for each of the 1,000 sequences of macroeconomic variables, yields the associated structural primary balances for given debt reduction targets (Table 7). The actual CPS primary balance would be equal to the CPS structural primary balance plus a term that will be positive when the output gap is expected to be positive and negative when the output gap is expected to be negative. The application of such a rule provides space for macroeconomic stabilization, without ignoring the objective of CPS debt reduction.

Table 7: CGS Primary balance to decrease the debt ratio, but allowing for automatic stabilization

Probality (cummulative)	Debt / GDP target: 25% by end- 2015				
	Output Gap				
	-2	-1	0	1	2
50	0.6%	0.8%	1.6%	1.2%	1.4%
60	1.0%	1.2%	2.0%	1.6%	1.8%
70	1.6%	1.8%	2.5%	2.2%	2.4%
75	1.9%	2.1%	2.7%	2.5%	2.7%

Source: Fund staff calculations

K. Summary and Conclusions

18. **This paper develops an alternative (and complementary) framework for debt sustainability analysis for the Dominican Republic, explicitly incorporating central bank accounts and operations to account for the expected cost of conducting monetary policy**. DSA recommendations regarding the size of the CPS primary surplus can be also adapted to take into consideration the need for macroeconomic stabilization. Both factors are especially relevant in the case of the Dominican Republic, given the relatively large quasi-fiscal losses, as well as the relatively large variability in aggregate demand observed during the last decade.

[17] Results available upon request.

19. **A public debt threshold of around 25 percent of GDP would be appropriate for the Dominican Republic** This conclusion stems from the arguments and extension of the analysis by Reinhart, Rogoff and Savastano (2003). Should the Dominican Republic maintain its good macroeconomic management and timely servicing of its public debt, the country's debt threshold could increase in the future.

20. **The size of the CPS primary balance needed to achieve a 25 percent of GDP debt threshold by 2015, would be about 2.5 percent of GDP (beginning in 2009), with about 70 percent probability**. In turn, if automatic stabilization is allowed, the CPS structural primary balance needed to achieve such target would also be about 2.5 percent of GDP that would increase/decrease by 0.2 percentage points of GDP for each positive/negative percentage point of output gap.

21. **Consolidated public sector primary balances of lower magnitudes would still decrease debt ratios in the median scenario, but would result in increases in this ratio if the macro situation were subject to sustained stress**. The magnitude of the additional fiscal effort required to increase the probability that the debt decreases to a given target ratio reflects the relatively large historical variability of the macroeconomic variables. As the economy stabilizes, and thus, such variability decreases, the "cost" in terms of the additional fiscal effort to increase the probability of achieving any targeted debt ratio will decrease.

22. **This analysis underscores the importance of choosing an appropriate baseline scenario**. If the economy remains vulnerable to policy and/or market shocks, choosing a relatively optimistic baseline may result in a CPS primary surplus that is not large enough to accommodate such shocks. In these circumstances, it is more prudent to pick a baseline scenario associated with a primary surplus of a size that is large enough to achieve the target debt ratio with a larger probability (say 70 percent), rather than choosing the "median" baseline scenario (i.e., that linked with a primary surplus that results in achieving the debt target 50 percent of the time).

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Mathematical Appendix

Algebraic manipulation of (3) allows to establish the factors contributing to CPS debt accumulation. In this regard, expression (9) separates, on the LHS, all CPS liabilities (in stock terms), noting whether such liabilities belong to the NFPS or to the BCRD, while the RHS includes all variables explaining CPS debt levels.

$$E_t^G + E_t^{CB} + F_t^G + D_t^{CB} = E_{t-1}^G \cdot \left(1+\widetilde{i_{t-1}^E}\right) + E_{t-1}^{CB} \cdot \left(1+\widetilde{i_{t-1}^E}\right) + F_{t-1}^G \cdot \left(1+i_{t-1}^F\right) + D_{t-1}^{CB} \cdot \left(1+i_{t-1}^D\right) + R_{t-1} \cdot \left(1+\widetilde{\beta_t}\right) + \quad (9)$$
$$+H_{t-1} \cdot \left(1+\widetilde{h_t}\right) - B_{t-1} \cdot \left(1+\widetilde{y_t}\right) - \overline{PB_t^{CB}} - \overline{PB_t^G}$$

In (9), note that, $B_t = B_{t-1} \cdot \left(1+\widehat{y_t}\right)$, $R_t^* = R_{t-1}^* \cdot \left(1+\widehat{\beta_t^*}\right)$ and $H_t = H_{t-1} \cdot \left(1+\widehat{h_t}\right)$; thus, the monetary base is assumed to remain constant in GDP terms (i.e. it grows at a rate equal to the nominal GDP, $\widehat{y_t} = \pi_t + \gamma_t + \pi_t \cdot \gamma_t$, where π_t is the inflation rate and γ_t is the real GDP growth rate).[18] Also, $\widehat{\beta_t^*}$ is the rate growth of international reserves (measured in foreign currency) and $\widehat{h_t}$ is the (policy determined) rate of growth of BCRD's domestic credit. In addition, $\widetilde{i_{t-1}^E} = i_{t-1}^E + \widehat{\varepsilon_t}$, where $i_{t-1}^E = \left(1+\widehat{\varepsilon_t}\right) \cdot i_{t-1}^{E*}$, $\widehat{\varepsilon_t}$ is the depreciation of the exchange rate; and, $\widetilde{\beta_t} = \left(1+\widehat{\varepsilon_t}\right) \cdot \left(\widehat{\beta_t^*} - i_{t-1}^{R*}\right) - 1$, i.e., the rate differential (in domestic currency terms) between the rate of growth of international reserves and the interest rate on international reserves (the risk free rate). Analogously, $\widetilde{h_t} = \widehat{h_t} - i_{t-1}^H - 1$ is the rate differential between the rate of growth of BCRD domestic credit and interest charged on such credit. Finally, $\widetilde{y_t} = \widehat{y_t} - 1$, while $\overline{PB_t^G} = T_t - S_t^{G,H} - p_t \cdot Z_t^{d,G} - w_t \cdot N_t^{d,G}$, and $\overline{PB_t^{CB}} = -w_t \cdot N_t^{d,CB}$, i.e., the primary balances of the NFPS and BCRD respectively, after consolidating the crossed terms.

Further simplifying (9) results in expression (10) (which is equivalent to (4) in Section D), where $E_t = E_t^G + E_t^{CB}$, $\Phi_t = E_t + D_t^{CB} + F_t^G$ and $\widetilde{i_{t-1}^\Phi} = \theta_{t-1}^E \cdot \widetilde{i_{t-1}^E} + \theta_{t-1}^D \cdot i_{t-1}^D + \theta_{t-1}^F \cdot i_{t-1}^F$, i.e., a weighted average for CPS debt, where $\theta_{t-1}^z = \dfrac{z_{t-1}}{\Phi_{t-1}}$ and $z = \{E, D, F\}$:

$$\Phi_t = \Phi_{t-1} \cdot \left(1+\widetilde{i_{t-1}^\Phi}\right) + R_{t-1} \cdot \left(1+\widetilde{\beta_t}\right) + H_{t-1} \cdot \left(1+\widetilde{h_t}\right) - B_{t-1} \cdot \left(1+\widetilde{y_t}\right) - \overline{PB_t^{CB}} - \overline{PB_t^G} \quad (10)$$

Equation (10) can be expressed in terms of GDP as follows:

[18] Note that for simplification purposes the monetary base is assumed to be denominated in domestic currency only, and that the interest rate on external debt is assumed equal for the NFPS and the BCRD.

$$(11)\qquad \phi_t = \phi_{t-1}\cdot\left(1+\overline{i^{\Phi}_{t-1}}\right)+r_{t-1}\cdot\left(1+\overline{\beta_t}\right)+h_{t-1}\cdot\left(1+\overline{h_t}\right)-b_{t-1}\cdot\left(1+\overline{y_t}\right)-\overline{pb^{CB}_t}-\overline{pb^{G}_t}$$

In (11) (which is equivalent to (5) in Section D), lowercase letters denote corresponding variables expressed in terms of GDP. In addition, note that, $\frac{1+\widetilde{i^{\Phi}_{t-1}}}{1+\widehat{y_t}}=\frac{1}{1+\gamma_t}\cdot\frac{1+\widetilde{i^{\Phi}_{t-1}}}{1+\pi_t}$, so if $\widetilde{\widetilde{i^{\Phi}_{t-1}}}=\frac{\widetilde{i^{\Phi}_{t-1}}-\pi_t}{1+\pi_t}$, then $\frac{1+\widetilde{i^{\Phi}_{t-1}}}{1+\widehat{y_t}}=\frac{1+\widetilde{\widetilde{i^{\Phi}_{t-1}}}}{1+\gamma_t}$, or, $\overline{i^{\Phi}_{t-1}}=\frac{\widetilde{i^{\Phi}_{t-1}}-\widehat{y_t}}{1+\widehat{y_t}}=\frac{\widetilde{\widetilde{i^{\Phi}_{t-1}}}-\gamma_t}{1+\gamma_t}$; moreover, $\overline{\beta_t}=\frac{\widetilde{\beta_t}-\widehat{y_t}}{1+\widehat{y_t}}$, $\overline{h_t}=\frac{\widetilde{h_t}-\widehat{y_t}}{1+\widehat{y_t}}$, and $\overline{y_t}=\frac{\widetilde{y_t}-\widehat{y_t}}{1+\widehat{y_t}}$.

Now, if the authorities want to reach a target debt ratio in J periods, $\phi_{t+J}=\overline{\overline{\phi}}$, and assuming that (i) $\overline{pb^{CB}_t}$ is exogenous, and (ii) that all relevant macroeconomic variables remain constant through time, recursive substitution allows to transform (11) into (12):

$$(12)\qquad \begin{aligned}\phi_{t+J} = {} & \phi_t\cdot\left(1+\overline{i^{\Phi}}\right)^J + r_t\cdot\left(1+\overline{\beta}\right)\cdot\sum_{j=0}^{J-1}\left(\frac{1+\widehat{\beta^*}}{1+\widetilde{i^{\Phi}}}\right)^j + h_{t-1}\cdot\left(1+\overline{h}\right)\cdot\sum_{j=0}^{J-1}\left(\frac{1+\hat{h}}{1+\widetilde{i^{\Phi}}}\right)^j - \\ & -b_{t-1}\cdot\left(1+\overline{y}\right)\cdot\sum_{j=0}^{J-1}\left(\frac{1+\hat{y}}{1+\widetilde{i^{\Phi}}}\right)^j - \overline{pb^{CB}}\cdot\sum_{j=0}^{J-1}\left(1+\overline{i^{\Phi}}\right)^j - \overline{pb^{G}}\cdot\sum_{j=0}^{J-1}\left(1+\overline{i^{\Phi}}\right)^j\end{aligned}$$

Then the (constant) NFPS primary balance, in terms of GDP, needed to achieve such debt target is given by (13), which is the same as (6) in Section D:

$$(13)\qquad \begin{aligned}\overline{pb^{G}} = {} & \frac{\overline{i^{\Phi}}}{\left(1+\overline{i^{\Phi}}\right)^J-1}\cdot\left[\phi_t\cdot\left(1+\overline{i^{\Phi}}\right)^J + r_t\cdot\left(1+\overline{\beta}\right)\cdot\frac{\left[\left(1+\overline{i^{\Phi}}\right)^J-\left(1+\overline{\beta^*}\right)^J\right]}{\left(\overline{i^{\Phi}}-\overline{\beta^*}\right)} + h_t\cdot\left(1+\overline{h}\right)\cdot\frac{\left[\left(1+\overline{i^{\Phi}}\right)^J-\left(1+\hat{\overline{h}}\right)^J\right]}{\left(\overline{i^{\Phi}}-\hat{\overline{h}}\right)} - \overline{\overline{\phi}}\right] - \\ & -b_t\cdot\left(1+\overline{y}\right)-\overline{pb^{CB}}\end{aligned}$$

Note that in (13) $\hat{\overline{h_t}}=\frac{\widehat{h_t}-\widehat{y_t}}{1+\widehat{y_t}}$ and $\overline{\beta_t^*}=\frac{\widehat{\beta_t^*}-\widehat{y_t}}{1+\widehat{y_t}}$.

In turn, if sequences of macroeconomic variables are not constant through time, expression (12) turns into (14):

$$
\begin{aligned}
&\phi_{t+J} = \phi_t \cdot \prod_{j=0}^{J-1}\left(1+\overline{i^{\Phi}_{t+j}}\right) + \\
&+r_t \cdot \left[\left(1+\overline{\beta_{t+1}}\right)\cdot\prod_{j=1}^{J-1}\left(1+\overline{i^{\Phi}_{t+j}}\right)+\sum_{j=1}^{J-2}\left(1+\overline{\beta_{t+j+1}}\right)\cdot\prod_{m=1}^{j}\left(1+\overline{\beta^{*}_{t+m}}\right)\cdot\prod_{n=j+1}^{J-1}\left(1+\overline{i^{\Phi}_{t+n}}\right)+\left(1+\overline{\beta_{t+J}}\right)\cdot\prod_{j=1}^{J-1}\left(1+\overline{\beta^{*}_{t+j}}\right)\right] + \\
&+h_t \cdot \left[\left(1+\overline{h_{t+1}}\right)\cdot\prod_{j=1}^{J-1}\left(1+\overline{i^{\Phi}_{t+j}}\right)+\sum_{j=1}^{J-2}\left(1+\overline{h_{t+j+1}}\right)\cdot\prod_{m=1}^{j}\left(1+\widehat{\overline{h_{t+m}}}\right)\cdot\prod_{n=j+1}^{J-1}\left(1+\overline{i^{\Phi}_{t+n}}\right)+\left(1+\overline{h_{t+J}}\right)\cdot\prod_{j=1}^{J-1}\left(1+\widehat{\overline{h_{t+j}}}\right)\right] - \\
&-b_t \cdot \left[\left(1+\overline{y_{t+1}}\right)\cdot\prod_{j=1}^{J-1}\left(1+\overline{i^{\Phi}_{t+j}}\right)+\sum_{j=1}^{J-2}\left(1+\overline{y_{t+j+1}}\right)\cdot\prod_{m=j+1}^{J-1}\left(1+\overline{i^{\Phi}_{t+m}}\right)+\left(1+\overline{y_{t+J}}\right)\right] - \\
&-\sum_{j=1}^{J-1}\overline{pb^{CB}_{t+j}}\cdot\prod_{m=j}^{J-1}\left(1+\overline{i^{\Phi}_{t+m}}\right)-\sum_{j=1}^{J-1}\overline{pb^{G}_{t+j}}\cdot\prod_{m=j}^{J-1}\left(1+\overline{i^{\Phi}_{t+m}}\right)-\overline{pb^{CB}_{t+J}}-\overline{pb^{G}_{t+J}}
\end{aligned}
\tag{14}
$$

Expression (14) can be simplified and presented as in (15), which is equivalent to (7) in Section D:

$$
\overline{pb^{G}} = \left[\Theta(J)\right]^{-1}\cdot\left[\phi(J)+r(J)+h(J)-b(J)-\overline{pb^{CB}}(J)-\overline{\overline{\phi}}\right] \tag{15}
$$

In (15):

$$
\phi(J) = \phi_t \cdot \prod_{j=0}^{J-1}\left(1+\overline{i^{\Phi}_{t+j}}\right),
$$

$$
r(J) = r_t \cdot \left[\left(1+\overline{\beta_{t+1}}\right)\cdot\prod_{j=1}^{J-1}\left(1+\overline{i^{\Phi}_{t+j}}\right)+\sum_{j=1}^{J-2}\left(1+\overline{\beta_{t+j+1}}\right)\cdot\prod_{m=1}^{j}\left(1+\overline{\beta^{*}_{t+m}}\right)\cdot\prod_{n=j+1}^{J-1}\left(1+\overline{i^{\Phi}_{t+n}}\right)+\left(1+\overline{\beta_{t+J}}\right)\cdot\prod_{j=1}^{J-1}\left(1+\overline{\beta^{*}_{t+j}}\right)\right],
$$

$$
h(J) = h_t \cdot \left[\left(1+\overline{h_{t+1}}\right)\cdot\prod_{j=1}^{J-1}\left(1+\overline{i^{\Phi}_{t+j}}\right)+\sum_{j=1}^{J-2}\left(1+\overline{h_{t+j+1}}\right)\cdot\prod_{m=1}^{j}\left(1+\widehat{\overline{h_{t+m}}}\right)\cdot\prod_{n=j+1}^{J-1}\left(1+\overline{i^{\Phi}_{t+n}}\right)+\left(1+\overline{h_{t+J}}\right)\cdot\prod_{j=1}^{J-1}\left(1+\widehat{\overline{h_{t+j}}}\right)\right],
$$

$$
b(J) = b_t \cdot \left[\left(1+\overline{y_{t+1}}\right)\cdot\prod_{j=1}^{J-1}\left(1+\overline{i^{\Phi}_{t+j}}\right)+\sum_{j=1}^{J-2}\left(1+\overline{y_{t+j+1}}\right)\cdot\prod_{m=j+1}^{J-1}\left(1+\overline{i^{\Phi}_{t+m}}\right)+\left(1+\overline{y_{t+J}}\right)\right],
$$

$$
\overline{pb^{CB}}(J) = \sum_{j=1}^{J-1}\overline{pb^{CB}_{t+j}}\cdot\prod_{m=j}^{J-1}\left(1+\overline{i^{\Phi}_{t+m}}\right)+\overline{pb^{CB}_{t+J}},
$$

and,

$$
\Theta(J) = \left[1+\sum_{j=1}^{J-1}\prod_{m=j}^{J-1}\left(1+\overline{i^{\Phi}_{t+m}}\right)\right]. \tag{16}
$$

In turn, if $\overline{pb^{G}_{t}}$ is allowed to change with the cycle, $\overline{pb^{G}_{t}} = \overline{\overline{pb^{G}}}\cdot\left[1+\eta\cdot\left(\gamma_t-\overline{\gamma}\right)\right]$, where $\overline{\overline{pb^{G}}}$ is the (constant) structural primary balance, $\overline{\eta} = \overline{\gamma}\cdot\eta$ is the elasticity of the primary balance with

respect to the output gap $\left(\gamma_t - \bar{\gamma}\right)$, and $\bar{\gamma}$ is the long-term GDP growth rate, then (15) turns into (17), which is equivalent to (8) in Section D:

$$\overline{\overline{pb^G}} = \left[\overline{\overline{\Theta}}(J)\right]^{-1} \cdot \left[\phi(J) + r(J) + h(J) - b(J) - \overline{pb^{CB}}(J) - \overline{\overline{\phi}}\right] \tag{17}$$

Note that expressions (15) and (17) only differ in their denominator: If automatic stabilization is to be allowed, the NFPS primary balance would be given by $\overline{pb^G}(J) = \overline{\overline{pb^G}} \cdot \left\{\sum_{j=1}^{J-1}\left[1+\eta\cdot\left(\gamma_t - \bar{\gamma}\right)\right]\cdot\prod_{m=j}^{J-1}\left(1+\overline{i_{t+m}^{\Phi}}\right)+\left[1+\eta\cdot\left(\gamma_{t+J}-\bar{\gamma}\right)\right]\right\}$ and thus, expression (16) needs to be replaced by (18) in expression (15), giving rise to (17).

$$\overline{\overline{\Theta}}(J) = \left\{\sum_{j=1}^{J-1}\left[1+\eta\cdot\left(\gamma_t - \bar{\gamma}\right)\right]\cdot\prod_{m=j}^{J-1}\left(1+\overline{i_{t+m}^{\Phi}}\right)+\left[1+\eta\cdot\left(\gamma_{t+J}-\bar{\gamma}\right)\right]\right\}\cdot \tag{18}$$