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Unconditional IMF Financial Support and Investor Moral Hazard

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Abstract

This Working Paper should not be reported as representing the views of the IMF.

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This paper develops a simple model of international lending, and calibrates it to assess quantitatively the effects of contingent IMF financial support on the risk premiums and the crisis probability. In the model, the country borrows in both short and long term; market (coordination) failure triggers a liquidity run and inefficient default; and the IMF lends *unconditionally* under a preferred creditor status. The model shows that IMF financial support can help prevent a liquidity crisis without causing investor moral hazard by helping to remove a distortion—effectively subsidizing *ex post* short-term investors (who run for the exit) at the expense of long-term investors (who are locked in). The resulting equilibrium is welfare enhancing as both the country's borrowing costs and the likelihood of a crisis are lower. The calibration exercises suggest that IMF-induced investor moral hazard—which occurs if the IMF lends at a subsidized rate—is unlikely to be a concern in practice, particularly if the country's economic fundamentals are strong and short-term debt is small.

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I. INTRODUCTION

The issue of IMF-induced moral hazard has received considerable attention since the IMF assembled a large bailout package for Mexico in 1995. It has often been conjectured that the bailout expectations caused the surge in capital flows to emerging market countries in the mid-1990s, planting seeds for subsequent sudden stops and financial crises. Such speculation have spurred policy discussions on the design of IMF-supported programs and the need for private sector involvement in resolving financial crises.

In the insurance literature where the term has roots, moral hazard is defined as a situation where the provision of insurance increases the probability of the event being insured against, due to diminished incentives for the insured party to take preventive actions. A necessary condition for moral hazard is asymmetric information or some other reason which prevents the insurer from responding fully (by adjusting terms or cancelling coverage) to the behavior that leads to an increase in the event's probability.

By analogy, the IMF could induce *debtor* moral hazard whereby emerging market countries pursue excessively risky policies, expecting a bailout from the IMF should a crisis occur. Similarly, it could encourage *creditor* moral hazard—which is the focus of this paper—whereby private creditors underprice lending risks to emerging market countries in the expectation of an IMF bailout if a crisis occurs. However, the analogy is not exact. Emerging market countries do not receive compensation in the event of a crisis but a loan that must be repaid with interest, while private creditors do not purchase insurance from the IMF at all.

Emphasizing these critical differences, Lane and Phillips (2000) and Jeanne and Zettelmeyer (2001) argue that IMF resources or subsidies in IMF lending are not large enough to create serious moral hazard, and financial losses of creditors are far greater than the potential size of IMF loans. Mussa (1999, 2004) argues that if the IMF does not make expected losses on its lending and the debtor government maximizes national welfare, then there can be no moral hazard. Intuitively, if the IMF does not make expected losses, there is no expected transfer from the IMF either to the borrowing country or to private investors. Without any expected transfer, ex ante incentives of both creditors and borrowers would not change, so there can be no moral hazard. Conversely, if there is IMF-induced moral hazard, the IMF must expect losses on its lending.

This intuition is formalized as the so-called Mussa theorem by Jeanne and Zettelmeyer (2005). In their model, the IMF can lend in rollover crises without incurring a loss on its lending when private investors cannot because the IMF has a better enforcement technology (e.g., through conditionality) than private investors. By helping to avoid a crisis and subsequent inefficient default, the IMF can make international lending less risky. In equilibrium, private investors' lending rate is lower and emerging market countries borrow more with the IMF than otherwise. Since the IMF lends at an actuarially fair rate (i.e., no expected transfer from the IMF), however, this does not mean moral hazard but rather the optimal response of investors and debtors to the reduced lending risks because of the IMF.

Despite well-articulated theoretical hypotheses, it is very difficult to establish empirically whether there is moral hazard associated with IMF lending. Most empirical studies

investigate the behavior of emerging market bond spreads or private capital flows following crisis events that could be associated with changes in moral hazard.² As noted by Jeanne and Zettelmeyer (2005), however, the effect of moral hazard on spreads or capital flows would be observationally equivalent to an optimal response to reduced real hazard of a crisis effected by an IMF bailout. Consequently, the validity of tests of moral hazard based on spreads or capital flows is in question.

An exception in the empirical literature is Zettelmeyer and Joshi (2005) who directly test whether the necessary conditions for the Mussa theorem are fulfilled. They estimate implicit transfers in IMF lending from historical data, and find that implicit transfers in IMF lending to emerging market countries is trivial. Their finding suggests—by the Mussa theorem—that IMF-induced moral hazard may not have been real possibility. But their finding is an *ex post* average result for IMF lending to emerging market countries. Therefore, they may not coincide with the *ex ante* expectations of private investors, nor hold in every individual cases.

As an alternative approach to investigate IMF-induced moral hazard, this paper develops a simple model of international lending and calibrates it to assess quantitatively the effect of IMF-induced investor moral hazard on emerging market risk premiums, in comparison to the intended effects of an IMF bailout. For simplicity, the model focuses on investor moral hazard abstracting from issues of debtor moral hazard. To that end, the borrowing country's behavior is assumed exogenous: the country simply borrows a fixed amount from international investors to finance a given amount of investment and repays debt within its debt-servicing capacity.³

The model is similar in its basic structure to that of Jeanne and Zettelmeyer (2005) but nonetheless departs from it in several important respects. First, the IMF is assumed to lend *unconditionally*. The focus on unconditional IMF lending enables us to disentangle the role of liquidity support by the IMF in crisis prevention from that of conditionality. Moreover, investor moral hazard is more likely if the IMF lends unconditionally. Given the focus on unconditional lending the model highlights the seniority of IMF credit as the critical difference between IMF lending and private lending, other than the fact that the IMF is not subject to any coordination failure while private investors are. Second, two classes of debt—short-term debt and long-term debt—are considered to better differentiate between liquidity and solvency risks. Finally, the model incorporates informational uncertainty in the determination of the risk premiums.

² See Lane and Phillips (2000), Zhang (1999), Kamin (2004), Dell'Arricia, Schnabel and Zettelmeyer (2002, 2006). Dreher (2004) surveys the empirical literature on IMF-induced moral hazard.

³ Kim (2006) discusses in greater detail debtor moral hazard associated with unconditional IMF lending. In his model, the government does not maximize the national welfare as it cares about political costs of policy adjustment. As a result, weaker policy adjustment is an optimal response of the government to unconditional IMF lending, but not necessarily optimal for the country.

The model is calibrated for the risk premiums and the crisis probability under three scenarios of international lending: i) *laissez-faire* lending in the absence of the IMF, ii) lending with the possibility of an IMF bailout but without investor moral hazard (that is, no expected losses on IMF lending), and iii) lending with IMF-induced investor moral hazard (i.e., expected losses on IMF lending). The IMF is assumed to lend at an actuarially fair rate in the second scenario while at a subsidized rate in the third. The net effect of IMF-induced investor moral hazard is identified by comparing the calibration results under the second and third scenarios. The intended welfare-enhancing effect of an IMF bailout in the absence of investor moral hazard is identified by comparing the results of the first and second scenarios.

The model's key results may be summarized as follows:

First, the IMF can play a role in preventing a liquidity crisis and inefficient default without causing any investor moral hazard. Intuitively, an IMF financial support helps to reduce a distortion arising from the creditor coordination failure by effectively subsidizing *ex post* short-term investors (who run for the exit)—possibly at the expense of long-term investors (who are locked in). Higher *ex post* return on short-term debt under an IMF bailout is optimally priced into lower *ex ante* risk premium which, in turn, leads to lower short-term debt service than otherwise. Since the country's economic fundamentals remains unaffected by unconditional IMF lending, lower short-term debt service improves the country's liquidity, reducing the likelihood of a liquidity crisis (and inefficient default).

Second, the prospect of possible IMF financial support can help lower borrowing costs of emerging market countries. While the possibility of an IMF bailout always leads to lower short-term premium, the net effect of an IMF bailout on the long-term premium is ambiguous. On the one hand, long-term investors benefit from the reduced likelihood of a crisis and inefficient default. On the other hand, they suffer from their claims being subordinated to IMF credit if a crisis occurs. Depending on which effect dominates, the long-term premium could be higher or lower. Nevertheless, the country's borrowing costs (averaged over short-term and long-term debt) are lower with the IMF as the short-term risk premium falls by more than fully offset the effect of higher long-term premium on borrowing costs. Since private investors are risk neutral and lend at an actuarially fair rate in equilibrium, this result implies that the insurance benefit of IMF financial support accrues entirely to the borrowing country.⁴

Third, the prospect of an IMF bailout may encourage international borrowing in general and short-term borrowing in particular by emerging market countries. The spread between short- and long-term interest rates increases with the IMF even if both decline in absolute terms. Faced with reduced borrowing costs and larger interest rate differential, emerging market countries may be encouraged to borrow more and shorter term. In the absence of investor

⁴ More generally, the incidence of the insurance benefit of the IMF, as well as the benefit of moral hazard, depends on the elasticities of supply and demand of private capital flows. If the supply of private capital is perfectly inelastic with respect to the expected return, the insurance benefit would accrue entirely to international investors.

moral hazard, however, this is an optimal response to the reduced riskiness of short-term borrowing.

Fourth, IMF-induced investor moral hazard is unlikely to be a concern in reality. The calibration results suggest that the net effect of IMF-induced moral hazard on the risk premiums would be far smaller in magnitude than that of reduced real hazard. Moreover, it would be smaller the stronger the country's economic fundamentals and the smaller its short-term debt. These results could be usefully taken into consideration in the design of new lending facilities of the IMF—such as the Reserve Augmentation Line (RAL)—which involve ex ante eligibility requirements but no ex post conditionality. Specifically, if the ex ante qualification standards are appropriately chosen to ensure access to the RAL is restricted to members with relatively strong fundamentals and sustainable debt, IMF-induced investor moral hazard would be quantitatively insignificant if extant at all.

Finally, the model implies that if the IMF bails out short-term investors only partially, the prospect of a larger-scale IMF lending may not necessarily be more effective for crisis prevention than a smaller-scale one. This implication—which resembles a Laffer-curve type relationship between the size of prospective IMF lending and the likelihood of a crisis and the risk premiums—is particularly relevant for countries with relatively large short-term debt. This result may also contribute to the discussion on the eligibility criteria and appropriate access levels for IMF financial support.

The remainder of the paper is organized as follows. Section II presents the basic setup of the model. Sections III derives the equilibrium solutions of the model with and without the IMF while Section IV discusses comparative statics results and several conjectures about the model's implications for the role of the IMF in crisis prevention. Section V presents the results of the model calibration. Section VI concludes the paper.

II. BASIC SETUP OF THE MODEL

There are three periods, $t = 0, 1, 2$. The representative emerging market country invests k in period 0 that yields an output in period 2. The investment must be financed by international borrowing. We assume that δk is financed by short-term debt maturing in period 1 and the remaining $(1 - \delta)k$ financed by long-term debt maturing in period 2. Since the investment yields an output only in period 2, the country must roll over its short-term debt in period 1 by issuing new debt maturing in period 2. Long-term debt, once contracted, is locked in until period 2.

The investment could be liquidated in period 1 as each short-term investor has a right to liquidate her share of investment. If partially liquidated, the remaining investment still yields an output but at a loss greater than proportional to the extent of liquidation. Specifically, denoting by k_1 the investment at the end of period 1, the output is characterized by

$$y(k_1, \theta) = \omega \exp(\theta) k_1$$

where $\omega = 1$ if $k_1 = k$ (no liquidation) and $\omega = \rho < 1$ if $k_1 < k$ (liquidation). θ is stochastic productivity which is normally distributed with mean μ and variance σ^2 , and realized in period 2. Since $\rho < 1$, liquidation is never efficient.

There is a continuum of private investors whose mass is normalized to 1. We assume that private investors are *near* risk neutral in the sense that they are neutral to default risk up to a certain level, beyond which they resort to credit rationing. Specifically, we assume that there is a maximum risk premium, denoted by \bar{r} , that private investors can take as an actuarially fair premium. Thus, private investors lend as if they are risk neutral if the actuarially fair risk premium is no greater than \bar{r} ; otherwise, they do not lend. As discussed below, this assumption does not affect the analysis in any essential way but nonetheless proves useful to rule out an infinite rollover interest rate in equilibrium.

We assume that coordination failure could trigger a run by short-term investors in period 1. If a run occurs, the IMF may provide crisis lending to the country. Short-term investors exit by liquidating their share of investment if the IMF does not bail them out. With contingent IMF lending, some or all short-term investors can exit at no cost without resorting to liquidation as the country repays them in full with an IMF loan, while the remaining short-term investors must liquidate their investment for the exit. The liquidation value of each unit of the investment is $\lambda < 1$. The country can credibly pledge a fraction α of output in total to the creditors, including to the IMF if it lends, who have claims in period 2.

Both private investors and the IMF are subject to informational uncertainty about the productivity shock θ . Specifically, they receive in period 1 a noisy signal $q = \theta + \varepsilon$ where ε is independent of θ , and normally distributed with mean 0 and variance $\tau\sigma^2$, $\tau > 0$. The IMF and short-term investors decide to lend or roll over after they receive the signal.⁵ For comparison, Appendix I discusses the model assuming no informational uncertainty.

In order to abstract from debtor moral hazard issues, we assume that the IMF lends unconditionally, and that the country's investment level (k) and the maturity composition of initial borrowing (δ) are exogenously given. Finally, we assume that IMF credit is senior to private claims, and is subject to the same interest rate ceiling \bar{r} as private credit.

III. EQUILIBRIUM SOLUTIONS OF THE MODEL

We begin by solving the model assuming a world in which the IMF does not exist, and then introduce the IMF as an official institution that provides crisis lending to countries facing a rollover crisis.

⁵ Introducing informational uncertainty, while complicating the analysis, is more realistic and provides better insight about how transparency in economic data affects the determination of the price of emerging market debt.

Let us denote by r_0^S the short-term interest rate contracted in period 0. Similarly, the long-term interest rate that covers two periods is denoted by r_0^L so that the annualized long-term interest rate is given by $(1 + r_0^L)^{1/2} - 1$. The amount of short-term debt falling due in period 1 and long-term debt maturing in period 2 is respectively given by

$$(1) \quad d_1^S = (1 + r_0^S)\delta k \quad \text{and} \quad d_2^L = (1 + r_0^L)(1 - \delta)k$$

For later purposes, we define the ratio $\psi = d_2^L / d_1^S$. Note that ψ is predetermined in period 1, and uniquely determines the long-term risk premium r_0^L for given δ and r_0^S .

Without loss of generality, the risk-free interest rate is normalized to zero. Given this normalization, we use the term “interest rate” and “risk premium” interchangeably in what follows.

A. Without the IMF

Given that the signal is noisy, the output in period 2 and thus debt repayment is uncertain in period 1. Therefore, short-term investors would demand a risk premium if they were to roll over their debt. For ease of exposition, we map the short-term debt maturing in period 1 into a certain productivity level. Specifically, we define $\bar{\theta}$ as the level of productivity that satisfies $d_1^S = \alpha y(k, \bar{\theta}) = \alpha \exp(\bar{\theta})k$. For given α and k , $\bar{\theta}$ is predetermined in period 1, and uniquely determines d_1^S and thus r_0^S .

We also define two productivity thresholds, θ^* and θ^{**} as follows:

$$(2) \quad R^S d_1^S + d_2^L = \alpha y(k, \theta^*) \quad \text{and} \quad d_2^L = \alpha y((1 - \delta)k, \theta^{**})$$

where $R^S = 1 + r_1^S \geq 1$ denotes the (gross) rollover interest rate for short-term debt. θ^* is the minimum level of productivity that ensures full repayment of private debt in period 2 conditional on a rollover of short-term debt in period 1. Note that the investment is preserved at the initial level k if short-term debt is rolled over. Similarly, θ^{**} is the minimum level of productivity that ensures full repayment of long-term debt in period 2 conditional on a rollover crisis in period 1. In this case, the investment is reduced to $(1 - \delta)k$ due to early liquidation by short-term investors. Combining (1) and (2) suggests that

$$(2a) \quad \theta^* = \bar{\theta} + \ln[R^S + \psi] \quad \text{and} \quad \theta^{**} = \bar{\theta} + \ln \psi - \ln[\rho(1 - \delta)]$$

For given signal q , the (zero-profit) condition for the rollover interest rate R^S to be actuarially fair is characterized by

$$(3) \quad d_1^S = E_1[R^S d_1^S \mid \theta \geq \theta^*] + E_1[\{S \cdot \alpha y(k, \theta) \mid \theta < \theta^*\}]$$

where E_1 refers to the expectation taken in period 1 based on the posterior distribution of θ , and $S = R^S d_1^S / (R^S d_1^S + d_2^L) = R^S / (R^S + \psi)$ is the short-term investors' share in total private claims falling due in period 2. If equation (3) has a solution, denoted by $R^{SNO}(q | \bar{\theta}, \psi)$ where superscript *NO* refers to equilibrium solutions without the IMF, it is unique and finite (see Appendix II). In addition, the following comparative statics results are obtained:

$$\partial R^{SNO} / \partial q < 0, \quad \partial R^{SNO} / \partial \bar{\theta} > 0, \quad \partial R^{SNO} / \partial \psi > 0, \quad \text{and} \quad \lim_{q \rightarrow \infty} R^{SNO} = 1$$

The existence of an equilibrium solution for equation (3), however, depends on the observed signal. Since the rollover interest rate cannot exceed \bar{r} by assumption, there must be a threshold of q such that no actuarially fair rollover interest rate less than \bar{r} can be found for a weaker signal than the threshold. Such a threshold, denoted by $\bar{q}^{NO}(\bar{\theta}, \psi)$, can be uncovered from the equality $R^{SNO}(\bar{q}^{NO}) = 1 + \bar{r}$.⁶ By using the properties of R^{SNO} , it is straightforward to show that $\partial \bar{q}^{NO} / \partial \bar{\theta} > 0$ and $\partial \bar{q}^{NO} / \partial \psi > 0$.

We assume that short-term investors rollover whenever an actuarially fair interest rate can be found at or below \bar{r} , but otherwise run for the exit because of coordination failure.⁷ This assumption rules out multiple equilibria in the model by excluding, for example, an equilibrium in which short-term investors collectively roll over their debt even at a rate less than the actuarially fair rate if it yields higher return than available from a run.

Since short-term investors liquidate their investment in order to exit, the crisis threshold \bar{q}^{NO} is also the default threshold. Therefore, the probability of a crisis, which equals the probability of a default, perceived in period 0 is given by

$$p^{CNO} = p^{DNO} = \Pr(q < \bar{q}^{NO}) = \int_{-\infty}^{\bar{q}^{NO}} z(q) dq$$

⁶ A finite threshold of q can be found even if $\bar{r} = \infty$ by using the condition that the expected total debt repayment cannot exceed the expected output. Suppose that $R^S = \infty$ is an actuarially fair rollover interest rate. Since long-term investors would recover nothing in this case, the expected total debt repayment would simply equal d_1^S . Thus, a finite threshold of q can be found from the condition that $d_1^S \leq E_1 y(k, \theta)$. The resulting threshold constitutes the minimum possible value of \bar{q} .

⁷ In a similar context, Flood and Marion (2006) show that an emerging market borrower who might default can be shut out of international capital markets without warning even for a modest haircut on obligations.

where p^{CNO} and p^{DNO} denote the crisis and default probability, respectively, and $z(q)$ is the normal density function of the signal q .

The ex ante zero-profit condition for short-term investors in period 0 would be given by

$$(4) \quad \delta k = d_1^S (1 - p^{DNO}) + \lambda \delta k p^{DNO}$$

Bearing in mind that p^{DNO} is a function of $\bar{\theta}$ and ψ , we assume that equation (4) has a solution denoted by $\bar{\theta}^{NO}(\psi)$. In case of multiple solutions, the lowest one would be considered as the economically relevant one. It is easy to show that $\partial \bar{\theta}^{NO} / \partial \psi > 0$.

Now we turn to the equilibrium risk premium for long-term debt. The country's repayment on long-term debt depends on whether short-term debt is rolled over in period 1 or not. If it is rolled over, no output loss is incurred and thus long-term investors are more likely to be repaid. But they have to compete with short-term investors for debt service in period 2 in case of low productivity. If a run occurs in period 1, an inefficient default follows with output falling by more than proportional to the extent of liquidation, although long-term investors no longer compete for debt service as they are sole creditors in period 2. Specifically, long-term debt repayment, DS_2^L , is characterized as follows:

$$q \geq \bar{q}^{NO} : DS_2^L = \begin{cases} d_2^L & \text{if } \theta \geq \theta^* \\ (1-S)\alpha y(k, \theta) & \text{otherwise} \end{cases}$$

$$q < \bar{q}^{NO} : DS_2^L = \begin{cases} d_2^L & \text{if } \theta \geq \theta^{**} \\ \alpha y(k_1, \theta) & \text{otherwise} \end{cases}$$

where $k_1 = (1-\delta)k$, and θ^* , θ^{**} , and \bar{q}^{NO} are all evaluated at $\bar{\theta} = \bar{\theta}^{NO}(\psi)$. The ex ante zero-profit condition for long-term investors in period 0 would thus be given by

$$(5) \quad (1-\delta)k = E_0 \left[E_1(DS_2^L | q \geq \bar{q}^{NO}) + E_1(DS_2^L | q < \bar{q}^{NO}) \right]$$

where E_0 refers to the expectation taken in period 0 based on the distribution of signal q . We assume that equation (5) has a unique solution denoted by ψ^{NO} (see Appendix II). Once ψ^{NO} is determined, $\bar{\theta}^{NO}$ and \bar{q}^{NO} are uniquely determined.

Accordingly, the equilibrium solution without the IMF is characterized as follows:

$$(6) \quad 1 + r_0^{SNO} = (\alpha / \delta) \exp(\bar{\theta}^{NO}), \quad 1 + r_0^{LNO} = (\alpha / (1-\delta)) \psi^{NO} \exp(\bar{\theta}^{NO}),$$

$$p^{CNO} = p^{DNO} = \int_{-\infty}^{\bar{q}^{NO}} z(q) dq$$

In equilibrium, the country borrows δk in short term at an interest rate r_0^{SNO} , and $(1 - \delta)k$ in long term at an interest rate r_0^{LNO} in period 0. Short-term investors roll over their debt in period 1 at an actuarially fair (gross) risk premium, $R^{SNO}(q)$, if the observed signal q exceeds \bar{q}^{NO} . Otherwise, short-term investors run for the exit liquidating their share of the investment and, as a result, the country defaults partially. Long-term investors whose investment is locked in until period 2 suffer from an output loss associated with an inefficient liquidation.

Short-term investors have an option to exit whenever the country's economic fundamental is expected to be weak. At the same time, however, they are susceptible to the risk of costly coordination failure. In contrast, long-term investors are locked in and suffer from an output loss if a run occurs. Upon a run, short-term investors recover λ per unit of investment while the recovery value of long-term investors depends on ρ . Thus, it cannot be ruled out in equilibrium that the long-term risk premium is lower than the short-term premium, particularly if ρ is large relative to λ .

B. With the IMF

Now we introduce the IMF into the model. Before proceeding, it is useful to emphasize that what matters for the risk premium and the crisis/default probability is not IMF financial support (or bailout) *per se* but rather the prospect of possible IMF financial support—as perceived by private investors in period 0. In what follows, we use the expression “with the IMF” to stand for the prospect of possible IMF financial support (or bailout).

Let us denote the amount of IMF lending by $L = \beta d_1^S$ where $0 < \beta \leq 1$. If $\beta = 1$, all short-term investors are bailed out and thus no default occurs when the IMF lends. Otherwise, only a fraction β of short-term investors are bailed out while the remaining short-term investors liquidate their investment for the exit, in which case output falls by more than proportional to the reduction in the investment. Consequently, the equilibrium solutions of the model with the IMF would not be continuous at $\beta = 1$. We assume that if a run occurs under a partial bailout ($\beta < 1$), short-term investors are bailed out randomly with an equal probability which simply equals β .

We begin by noting that the determination of the rollover interest rate for short-term debt and the rollover threshold continues to be characterized by (3), although their equilibrium levels would in general differ from those without the IMF. Thus, the rollover interest rate and threshold with the IMF, denoted by $R^{S IMF}$ and \bar{q}^{IMF} respectively, share the same properties with $R^{S NO}$ and \bar{q}^{NO} as discussed in the previous section. We use superscript *IMF* to denote equilibrium solutions with the IMF. The crisis probability is correspondingly defined as

$$p^{C IMF} = \Pr(q < \bar{q}^{IMF}) = \int_{-\infty}^{\bar{q}^{IMF}} z(q) dq$$

Let us now turn to the determination of the IMF lending rate. Since a fraction $1 - \beta$ of short-term investors liquidate their investment upon a run, the post-run level of investment and output are given by $k_1 = [1 - (1 - \beta)\delta]k$ and $y = \omega \exp(\theta)k_1$. Since $k_1 = k$ and $\omega = 1$ if $\beta = 1$, these expressions are valid for all $\beta > 0$.

We define two productivity thresholds, denoted by $\bar{\theta}^F$ and θ^{*F} respectively, as follows:

$$(7) \quad R^F \beta d_1^S = \alpha y(k_1, \bar{\theta}^F) \quad \text{and} \quad R^F \beta d_1^S + d_2^L = \alpha y(k_1, \theta^{*F})$$

where $R^F = 1 + r_1^F \geq 1$ is the (gross) lending rate of the IMF. $\bar{\theta}^F$ is the minimum level of productivity that ensures full repayment to the IMF. Since IMF credit is senior to private claims, long-term investors recover nothing if $\theta \leq \bar{\theta}^F$. θ^{*F} is a threshold at which the output pledged by the country after a run by short-term investors is just sufficient to repay in full both the IMF and long-term investors. By using the definition of $\bar{\theta}$, those two thresholds can be expressed as follows:

$$(7a) \quad \bar{\theta}^F = \bar{\theta} + \ln R^F + \ln \phi \quad \text{and} \quad \theta^{*F} = \bar{\theta} + \ln[R^F + \psi / \beta] + \ln \phi$$

where $\phi = \beta / \{\omega[1 - (1 - \beta)\delta]\}$. Note that ϕ could be larger or smaller than unity under a partial bailout ($\beta < 1$) while $\phi = 1$ under a full bailout ($\beta = 1$). Also note that ϕ is not continuous at $\beta = 1$.

Given the assumption that IMF credit is senior to private claims, the ex ante zero-profit condition for the IMF is given by

$$(8) \quad \beta d_1^S = E_1[R^F \beta d_1^S \mid \theta \geq \bar{\theta}^F] + E_1[\alpha y(k_1, \theta) \mid \theta < \bar{\theta}^F]$$

Note that the expectations are valid only for a signal below \bar{q}^{IMF} . If equation (8) has a solution, it is unique and independent of ψ (see Appendix II). Denoting such solution by $R^F(q \mid \bar{\theta}, \beta)$, it is straightforward to show that

$$\partial R^F / \partial q < 0, \quad \partial R^F / \partial \bar{\theta} > 0, \quad \lim_{q \rightarrow \infty} R^F = 0, \quad \text{and} \quad \partial R^F / \partial \beta > 0 \quad \text{if} \quad \beta < 1$$

Since, by assumption, IMF lending is subject to the same interest rate ceiling \bar{r} as private lending, equation (8) would have no solution if the signal q falls below a threshold $\bar{q}^F(\bar{\theta}, \beta)$, which is defined by $R^F(\bar{q}^F \mid \bar{\theta}, \beta) = 1 + \bar{r}$. It is straightforward to show that $\partial \bar{q}^F / \partial \beta > 0$ for $\beta < 1$, and that \bar{q}^F is discontinuous at $\beta = 1$.

The existence of long-term debt is crucial for the equilibrium solutions with the IMF. Without long-term debt ($\delta = 1$ or $\psi = 0$), the IMF is no different from short-term investors

under a full bailout ($\beta = 1$) because the seniority of IMF credit has no relevance. More specifically, it can be shown that $R^F = R^{S\text{IMF}}$ and $\bar{q}^F = \bar{q}^{\text{IMF}}$ if $\delta = \beta = 1$, suggesting that the IMF has no role to play.⁸

Since $\partial \bar{q}^{\text{IMF}} / \partial \psi > 0$, \bar{q}^F is strictly smaller than \bar{q}^{IMF} for all $\psi > 0$ under a full bailout. Thus, as long as long-term debt is not zero, there always exists a range of the signal over which short-term investors would not roll over but the IMF can lend at an actuarially fair rate. In contrast, there is no guarantee under a partial bailout that the inequality $\bar{q}^F < \bar{q}^{\text{IMF}}$ holds for all $\psi > 0$.⁹ This is because the IMF lending rate must cover the solvency risk associated with output disruptions caused by liquidation, which is absent in the determination of the rollover interest rate $R^{S\text{IMF}}$. If the solvency risk is large enough, \bar{q}^F could be higher than \bar{q}^{IMF} , in which case no equilibrium solutions exist under a partial bailout by the IMF.

Intuitively, the larger the amount of IMF lending relative to the country's debt servicing capacity in period 2 the higher would be the solvency risk faced by the IMF. Specifically, the ratio of IMF lending to the expected output that the country can pledge after a run occurs is given by

$$\beta d_1^S / E_1[\alpha y(k_1, \theta) | q < \bar{q}^{\text{IMF}}] = \phi \delta [\exp(\bar{\theta}) / E_1[\exp(\theta) | q < \bar{q}^{\text{IMF}}]]$$

For given expectation about productivity, $\phi \delta$ would be a good, albeit not perfect, measure of the solvency risk faced by the IMF. If $\phi \delta$ is too large, the inequality $\bar{q}^F < \bar{q}^{\text{IMF}}$ is likely to be violated and thus no equilibrium solutions would be found under a partial bailout. Consequently, the size of IMF lending under a partial bailout would have to be restricted not to exceed a certain level denoted by $\bar{\beta}(\delta, \rho) < 1$. Intuition suggests that $\partial \bar{\beta} / \partial \delta < 0$ and $\partial \bar{\beta} / \partial \rho > 0$. In Section V, we report the value of $\phi \delta$ together with the results of the model calibration. In what follows, we assume that the inequality $\bar{q}^F < \bar{q}^{\text{IMF}}$ always holds under a partial bailout on the ground that the IMF has full discretion to set β at less than $\bar{\beta}$ if necessary.

Under a full bailout, the country defaults only if the signal is weak enough to make IMF lending unwarranted: no default occurs as long as the IMF lends. In contrast, the country defaults whenever there is a rollover crisis under a partial bailout since a fraction $1 - \beta$ of

⁸ This can be seen directly from the fact that equation (8) collapses into equation (3) if $\psi = 0$ and $\beta = 1$.

⁹ The inequality holds under a partial bailout if $\phi \leq 1$ or, equivalently, if $\beta \leq \rho(1 - \delta) / (1 - \rho\delta)$. Otherwise, it cannot be guaranteed.

short-term investors liquidate their share of investment. Accordingly, we define the ex ante default probability as follows:

$$p^{D IMF} = \begin{cases} p^{D NB} = \Pr(q < \bar{q}^F) = \int_{-\infty}^{\bar{q}^F} z(q) dq & \text{if } \beta = 1 \\ p^{C IMF} & \text{otherwise} \end{cases}$$

where superscript *DNB* stands for “default without any bailout by the IMF”. In any case, $p^{C IMF} - p^{D NB}$ represents the probability that the IMF lends.

By using the default probability defined above, the ex ante zero-profit condition for short-term investors would be characterized by

$$(9) \quad \delta k = d_1^S (1 - p^{D IMF}) + [\beta d_1^S + (1 - \beta) \lambda \delta k] (p^{D IMF} - p^{D NB}) + \lambda \delta k p^{D NB}$$

The first term on the right hand side of (9) represents the expected repayment for short-term debt conditional on a rollover. The second term reflects the expected return for short-term investors conditional on IMF lending while the last term is the expected return from liquidation if the IMF does not lend. Bearing in mind that both $p^{C IMF}$ and $p^{D NB}$ depend on $\bar{\theta}$, we assume that equation (9) has a solution denoted by $\bar{\theta}^{IMF}(\psi, \beta)$. It can be shown that $\bar{\theta}^{IMF}$ is independent of ψ under a full bailout ($\beta = 1$) because equation (9) collapses into $\delta k = d_1^S (1 - p^{D NB}) + \lambda \delta k p^{D NB}$ and $p^{D NB}$ does not depend on ψ .

The expected return for long-term investors depends not only on whether short-term debt is rolled over but also on whether the IMF lends. If short-term debt is rolled over, long-term investors must compete with short-term investors for the country’s debt service in period 2. Otherwise, they compete—under disadvantages—with the IMF for debt services if the IMF lends; they are sole creditors in period 2 if the IMF does not lend. Specifically, long-term debt repayment in period 2 with the IMF is characterized as follows:

$$\begin{aligned} q \geq \bar{q}^{IMF} : \quad DS_2^L &= \begin{cases} d_2^L & \text{if } \theta \geq \theta^* \\ (1 - S) \cdot \alpha y(k, \theta) & \text{otherwise} \end{cases} \\ \bar{q}^F \leq q < \bar{q}^{IMF} : \quad DS_2^L &= \begin{cases} d_2^L & \text{if } \theta \geq \theta^{*F} \\ \alpha y(k_1, \theta) - R^F \beta d_1^S & \text{if } \bar{\theta}^F \leq \theta < \theta^{*F} \\ 0 & \text{otherwise} \end{cases} \\ q < \bar{q}^F : \quad DS_2^L &= \begin{cases} d_2^L & \text{if } \theta \geq \theta^{**} \\ \alpha y(k_1, \theta) & \text{otherwise} \end{cases} \end{aligned}$$

where all thresholds of θ and q are evaluated at $\bar{\theta} = \bar{\theta}^{IMF}(\psi, \beta)$.

Accordingly, the ex ante zero-profit condition for long-term investors is given by

$$(10) \quad (1-\delta)k = E_0 \left[E_1(DS_2^L | q \geq \bar{q}^{IMF}) + E_1(DS_2^L | q < \bar{q}^F) + E_1(DS_2^L | \bar{q}^F \leq q < \bar{q}^{IMF}) \right]$$

We assume that equation (10) has a solution denoted by $\psi^{IMF}(\beta)$, which uniquely determines $\bar{\theta}^{IMF}$, \bar{q}^F , and \bar{q}^{IMF} (see Appendix II).

Finally, the equilibrium solutions with the IMF are characterized as follows:

$$(11) \quad \begin{aligned} 1 + r_0^{S\,IMF} &= (\alpha / \delta) \exp(\bar{\theta}^{IMF}), \quad 1 + r_0^{L\,IMF} = (\alpha / (1 - \delta)) \psi^{IMF} \exp(\bar{\theta}^{IMF}), \\ p^{C\,IMF} &= \int_{-\infty}^{\bar{q}^{IMF}} z(q) dq, \quad p^{DNB} = \int_{-\infty}^{\bar{q}^F} z(q) dq \end{aligned}$$

In equilibrium, the country borrows δk in short term at an interest rate $r_0^{S\,IMF}$, and $(1 - \delta)k$ in long term at an interest rate $r_0^{L\,IMF}$ in period 0. If the signal q exceeds \bar{q}^{IMF} in period 1, short-term investors voluntarily roll over at an actuarially fair (gross) interest rate $R^{S\,IMF}(q)$. For an intermediate signal between \bar{q}^F and \bar{q}^{IMF} , a liquidity run occurs and the IMF lends $\beta d_1^S = \beta(1 + r_0^S)\delta k$ at an actuarially fair (gross) rate $R^F(q)$. The run results in no default by the country under a full bailout ($\beta = 1$) but leads to an inefficient liquidation by a fraction $1 - \beta$ of short-term investors under a partial bailout ($\beta < 1$). Finally, short-term investors run for the exit by liquidating their investment with no bailout by the IMF if $q < \bar{q}^F$.

By the Mussa theorem, the equilibrium solutions characterized by (11) involve no IMF-induced investor moral hazard since the IMF lends at an actuarially fair rate. Thus, any reduction in the risk premiums or the crisis/default probabilities should be attributed solely to the reduction in real hazard of a crisis. A simple modification of the model, however, can generate equilibrium solutions with IMF-induced investor moral hazard. Specifically, the zero-profit condition for the IMF shown in (8) can be replaced by

$$(8a) \quad (1 - \gamma)\beta d_1^S = E_1 \left[R^F \beta d_1^S | \theta \geq \bar{\theta}^F \right] + E_1 \left[\alpha y(k_1, \theta) | \theta < \bar{\theta}^F \right]$$

where $\gamma \geq 0$ denotes implicit transfers in IMF lending. If $\gamma > 0$, the IMF lends at a subsidized rate, expecting to make losses on its lending (the expected rate of return on IMF lending is negative at $-\gamma$). It is straightforward to show that the larger the implicit transfers, the lower are the risk premiums and the crisis probability, a result which holds for both full and partial bailouts.

For later purposes, we denote the equilibrium solutions with IMF-induced investor moral hazard ($\gamma > 0$) by using superscript *MH*. The net effect of investor moral hazard on the risk premiums and the crisis/default probabilities would then be easily identified by comparing the moral hazard equilibrium solutions with those characterized by (11). More specifically,

the difference in the risk premiums and the crisis/default probabilities between equilibrium solutions without the IMF and with IMF-induced investor moral hazard can be decomposed into two parts:

$$\begin{aligned} r_o^{NO} - r_o^{MH} &= (r_o^{NO} - r_o^{IMF}) + (r_o^{IMF} - r_o^{MH}) \\ p^{NO} - p^{MH} &= (p^{NO} - p^{IMF}) + (p^{IMF} - p^{MH}) \end{aligned}$$

In each decomposition, the first component reflects the welfare-enhancing effect of an IMF bailout that results from reduced real hazard of a crisis while the second captures the net effect of IMF-induced investor moral hazard.

IV. COMPARATIVE STATICS

Because of the complexity of the equilibrium solutions, only a limited set of comparative statics results can be obtained analytically. For this reason, we also present several critical conjectures as to the relationship between the risk premiums, the country's economic fundamentals, and debt structure. Since investor moral hazard always lower the risk premiums as well as the crisis/default probabilities, we focus in what follows on the comparative statics results obtained by assuming no investor moral hazard.

First, the short-term premium and the default probability are always lower with the IMF than without the IMF if the IMF bails out all short-term investors, simply because no inefficient default occurs under a full bailout. Reduced default risk translates into lower short-term risk premiums.

Second, the IMF lending rate could be lower than the short-term rollover interest rate in the neighborhood of the crisis threshold \bar{q}^{IMF} , although they are not directly comparable as the former is defined only for a signal below the threshold while the latter would prevail only for a signal above the threshold. More specifically, there exists $\bar{e} > 0$ such that for all $e \in (0, \bar{e}]$, $R^F(\bar{q}^{IMF} - e) < R^{S\ IMF}(\bar{q}^{IMF} + e)$. The assumed seniority of IMF credit matters for this result.

Third, the larger the liquidation value λ the smaller are the risk premiums and the crisis probability. The liquidation value plays a direct role in the determination of the short-term premium by changing the recovery value of short-term investors, but has no direct bearing on the determination of the rollover interest rate or the crisis probability. Nonetheless, it affects the crisis probability indirectly through its impact on the short-term risk premium.¹⁰

¹⁰ If the model is further extended to incorporate strategic uncertainty under private information as in Morris and Shin (2004), the crisis probability would be directly influenced by the liquidation value. In a Morris-Shin type model, larger liquidation value could increase, rather than decrease, the crisis probability by making the exit less costly relative to a rollover.

In addition to these analytical results, intuition suggests several critical conjectures regarding the role of the IMF in crisis prevention and the interaction among risk premiums, country characteristics, and the size of IMF lending.

We conjecture first that even under a partial bailout by the IMF, the short-term premium and the default probability would be lower than without the IMF. Intuitively, the country defaults on a smaller scale under a partial bailout than in the case without the IMF.

Our second conjecture is that the possibility of an IMF bailout would reduce the country's borrowing costs while increasing the spread between the short-term and long-term interest rates even if they both decline in absolute terms. Specifically,

$$r_0^{A\text{ IMF}} < r_0^{A\text{ NO}} \quad \text{and} \quad (r_0^{L\text{ IMF}} - r_0^{S\text{ IMF}}) > (r_0^{L\text{ NO}} - r_0^{S\text{ NO}})$$

where $r_0^A = \delta r_0^S + (1 - \delta)r_0^L$ is the (weighted) average risk premium. The long-term premium could be higher with the IMF because of the seniority of IMF credit. However, the higher long-term premium would be more than fully offset by the lower short-term premium. As a result, the country's borrowing costs—averaged over short- and long-term debt—would likely be lower with the IMF than otherwise.

At the center of these results is the role played by the IMF as a public institution in helping to reduce a distortion arising from the market (coordination) failure. In the absence of the IMF, the market failure might trigger liquidity crises and inefficient defaults too often and the country would pay too high risk premiums. By subsidizing ex post short-term investors (who run for the exit) through contingent lending at times of a liquidity crisis—possibly at the expense of long-term investors (who are locked in)—the IMF can help reduce the market failure at no economic cost.¹¹ If the gain from avoiding inefficient default is large enough, long-term investors could also benefit from an official bailout by the IMF. However, the insurance benefit of IMF financial support accrues ultimately to the borrowing country in terms of lower borrowing costs and reduced crisis/default probabilities because private investors are risk neutral and lend at an actuarially fair rate.

The third conjecture is related to the relationship between the risk premiums and the country's economic fundamentals and debt structure:

$$\partial r_0^j / \partial \delta > 0, \quad \partial r_0^j / \partial \rho \leq 0, \quad \partial r_0^j / \partial \mu < 0, \quad \text{and} \quad \partial r_0^j / \partial \sigma > 0, \quad j = S, L$$

which imply that the risk premiums are lower, the smaller the share of short-term debt and the output cost of default, and the stronger the country's economic fundamentals.

¹¹ The IMF plays the same role as a social planner who taxes, ex post, long-term investors and use the revenue to subsidize short-term investors. I am grateful to Jaewoo Lee for this intuition.

The fourth conjecture is related to the role that the IMF can play in crisis prevention, which can be summarized as follows:

$$\Delta p^C \geq 0, \quad \partial(\Delta p^C)/\partial\delta > 0, \quad \partial(\Delta p^C)/\partial\rho < 0, \quad \partial(\Delta p^C)/\partial\mu < 0, \quad \text{and} \quad \partial(\Delta p^C)/\partial\sigma < 0$$

where $\Delta p^C = p^{C\,NO} - p^{C\,IMF}$ denotes the change in the crisis probability effected by the possibility of an IMF bailout. The first inequality implies that the IMF can play a role in crisis prevention even with unconditional lending while the remaining inequalities suggest that the IMF can play a better role in crisis prevention the larger the short-term debt and the weaker the country's economic fundamentals.

In the model, the probability of a rollover crisis depends ultimately on the amount of total debt maturing in period 2 relative to the expected output. Given the conjecture that the prospect of an IMF bailout would reduce the country's borrowing costs, the amount of debt maturing in period 2 would be smaller than without the IMF. Since the country's economic fundamentals remain unaffected by unconditional IMF lending, smaller debt services tend to improve the country's liquidity position and, hence, lower the likelihood of a crisis.¹²

Moreover, the effect of an IMF bailout on the crisis probability is likely to depend on the characteristics of a borrowing country. The risk premiums and the crisis probability in the absence of the IMF would be high in the first place if the country's economic fundamental is weak and/or the share of short-term debt is large. Starting from already high levels, the prospect of an IMF bailout would have greater impact on the risk premiums and the crisis probability.

Finally, we conjecture that the relationship between the risk premiums and the size of IMF lending could be non-monotonic and complex under a partial bailout. The complex relationship arises because of the tradeoff between liquidity and solvency risks, which could vary discretely due to discontinuity in the expected debt repayment. Intuitively, a larger-scale partial bailout would reduce the liquidity risk but at the same time increase the solvency risk faced by long-term investors given the assumed seniority of IMF credit. Therefore, the response of risk premiums to changes in the size of IMF lending would differ depending on which risk dominates at the margin. Indeed, it cannot be ruled out that the short-term premium rises as the size of IMF lending increases under a partial bailout.¹³

¹² This reasoning is not inconsistent with the theoretical prediction of Kim (2006) that unconditional IMF lending, unless very large, would not be effective in reducing the likelihood of a liquidity crisis. In his model, the country's economic fundamental is affected by unconditional IMF lending because it is broadly defined to include policy adjustment. In fact, the country's fundamental deteriorates with unconditional IMF lending in that model because policy adjustment and IMF financing are (perfect) substitutes in equilibrium.

¹³ Since $p^{D\,IMF} = p^{C\,IMF}$ under a partial bailout, the condition (9) can be rearranged to yield

(continued...)

The relationship between the long-term risk premium and the size of IMF lending could also be non-monotonic. As shown in Appendix II, the expected return for long-term investors is discontinuous at $q = \bar{q}^F$. Because of such discontinuity, a larger-scale IMF lending may or may not lead to a lower long-term premium.

The non-monotonic relationship between the risk premiums and the size of IMF lending suggests that under a partial bailout a larger-scale IMF lending may not necessarily be more effective for crisis prevention than a smaller-scale lending as it could raise the country's borrowing costs with relatively little effect on the crisis probability. This implication could provide useful guidance for the design of new lending instruments of the IMF that involve no ex-post conditionality, particularly with regard to the appropriate access levels.

V. MODEL CALIBRATION

The model is calibrated to examine how the model fares with the conjectures discussed in the previous section and related empirical findings. Specifically, four sets of the calibration exercises are undertaken. The first exercise focuses on the effect on the risk premium and the crisis probability of the size of short-term debt and the output cost of default. The second aims to identify the relationship between the crisis probability and the country's economic fundamentals. We consider two types of the economic fundamentals: mean and volatility of productivity. In the third, we focus on the role of informational and fundamental uncertainty in the determination of the risk premium and the crisis probability. This exercise sheds light on how informational uncertainty interacts with fundamental economic uncertainty. All these exercises assume that the IMF lends at an actuarially fair rate so that no investor moral hazard occurs in equilibrium.

In contrast, the final exercise is geared toward quantifying the net effect of IMF-induced investor moral hazard on the risk premium and the crisis probability by assuming an implicit transfer of 10 percent in IMF lending ($\gamma = 0.1$), which would be considered large relative to the estimates by Zettelmeyer and Joshi (2005).¹⁴ As noted in the previous section, comparing

$$1 + r_0^S = \frac{1 - \lambda p^{DNB}}{1 - p^{C IMF} + \beta(p^{C IMF} - p^{DNB})}$$

where use is made of $d_1^S = (1 + r_0^S)\delta k$. The numerator on the right hand side is decreasing in β since $\partial p^{DNB} / \partial \beta > 0$. However, the denominator could be either increasing or decreasing in β because the probability of IMF lending, $(p^{C IMF} - p^{DNB})$, is decreasing in β .

¹⁴ They estimate that IMF lending to high and middle income countries during 1973-2003 were, on average, 30–150 basis points lower than comparable lending rates paid by industrial countries on their debt, which corresponds to less than a 2 percent transfer in the context of our model. They also find that standard IMF lending through non-concessional facilities has been essentially subsidy free since 1987.

the calibration results for $\gamma > 0$ with those for $\gamma = 0$ allows us to quantify the net effect of IMF-induced investor moral hazard separately from the welfare-enhancing effect (i.e., reduced real hazard) of the prospect of possible IMF support.

In each exercise, the parameters of interest are varied while other parameters are fixed at their respective benchmark values. The benchmark values of the model parameters are chosen as follows:

$$\alpha = 0.8, \quad \mu = \sigma = 1, \quad \delta = \rho = \lambda = \tau = 0.5, \quad \gamma = 0, \quad \text{and} \quad \bar{r} = 0.6$$

In the first exercise, δ and ρ are varied between 0.25 and 0.75 while μ and σ are varied between 0.75 and 1.25 in the second. In the third, τ is varied between 0.25 and 0.75 while the same variation is considered for σ as in the second exercise. The final exercises uses the same set of parameter values as used in the first and the second.

Regarding the size of IMF lending, we consider five interim values of β ranging from 0.05 to 0.75 to account for partial bailouts while setting $\beta = 0$ for the case without the IMF, and $\beta = 1$ for a full bailout. The third and fourth exercises focus only on a full bailout on the grounds that the interplay between informational and fundamental uncertainty would be little different between partial and full bailouts, and that it is most likely under a full bailout if the IMF induces investor moral hazard.

Table 1 reports the results of the first calibration exercise. The results are consistent with the predictions of the model and the conjectures discussed in the previous section. The short-term premiums and the crisis probabilities are lower with the IMF ($\beta > 0$) than without the IMF ($\beta = 0$), reaching the lowest levels under a full bailout.¹⁵ The reduction in the short-term premium under a full bailout is quite substantial and greater than that under a partial bailout. In contrast, the effect on the crisis probability appears rather limited with less than two percentage point reduction at most even under a full bailout. As discussed below, however, the effect on the crisis probability is significantly larger if the country's economic fundamental is weaker than assumed for Table 1.

The default probabilities—which correspond to p^{DNB} in the model—also turn out to be lower with the IMF than otherwise. Unlike the crisis probabilities, however, they are often lower under smaller-scale partial bailouts with $\beta \leq 0.1$ than under a full bailout while the opposite holds more often for larger-scale partial bailouts with $\beta \geq 0.5$. This is because the repayment risk faced by the IMF depends on the size of IMF lending relative to the country's debt-servicing capacity in period 2—the latter of which is smaller under a partial bailout because

¹⁵ Although the calibrated crisis probabilities are marginally higher with the IMF than without the IMF in several cases associated with $\rho = 0.25$, it is because of unavoidable errors in numerical approximation of theoretical probabilities and expected values.

of inefficient liquidation. Therefore, the IMF may face smaller repayment risk and thus can lend for a weaker signal than under a full bailout if β is small enough, while the opposite would hold otherwise.

The long-term premiums are typically higher under a partial bailout and even under a full bailout if δ and ρ are both large. As a result, the differential between long- and short-term premiums is always positive and larger with the IMF than otherwise, suggesting a steeper yield curve. Despite the opposite movements in the short- and long-term premium, however, the country's borrowing costs always turn out to be lower with the IMF than otherwise, implying a positive welfare gain for a debtor country.

Also consistent with our conjectures and empirical evidence on the critical role of short-term debt in triggering a crisis, the calibrated risk premiums and crisis probabilities are rising steeply as δ is increased in both cases with and without the IMF. The output cost of a default (ρ) also matters but far less than the share of short-term debt under a partial bailout. Under a full bailout, it does not matter at all for the short-term premium and the default probability because no liquidation occurs.

A notable result from Table 1 is the non-monotonic relationship between the risk premiums and the size of IMF lending under a partial bailout. The non-monotonicity appears in two forms: the nonexistence of equilibrium solutions for $\beta > \bar{\beta}$ and the nonlinearity of the risk premiums with respect to the size of IMF lending for $\beta \leq \bar{\beta}$ where $\bar{\beta}$ is a ceiling for the size of IMF lending discussed in Section III.B. Consistent with the model's predictions, the incidence of no equilibrium solutions is more frequent for larger δ 's and smaller ρ 's. As conjectured, it is also associated with larger values of the solvency risk measure $\phi\delta$ (bottom, Table 1).

The nonlinearity with respect to the size of IMF lending is seen in both short- and long-term premiums if $\delta \geq 0.5$. The nonlinear relationship is typically of U-shape for the short-term premium: as β increases from 0, it falls initially to reach the lowest level (highlighted in bold figures) before rising. Interestingly, the relationship is of the opposite shape for the long-term premium as it tends to move in the opposite direction with the short-term premium. The (weighted) average risk premium or the country's borrowing cost, however, moves in tandem with the short-term premium in all but one cases, exhibiting the same U-shape pattern. These nonlinearity results suggest that if the country's short-term debt is large, the prospect of a larger-scale partial bailout may not necessarily be more effective for crisis prevention than a smaller-scale partial bailout—a result that resembles a Laffer curve relationship in the tax literature.

The results of the second exercise are displayed in Table 2. The results are well conforming to the predictions of the model. The risk premiums, as well as the crisis/default probabilities, are all higher the weaker the country's economic fundamentals. Moreover, the risk premiums

turn out to be less sensitive to the country's economic fundamentals with the IMF than without the IMF.¹⁶

The effect of an IMF bailout on the crisis probability, albeit small in general, turns out to be stronger the weaker the country's economic fundamentals. Further calibration results (not reported) show, however, that for $\sigma = 1.0$, the reduction in the crisis probability under a full bailout increases sharply to 10 percentage points from less than 3 percentage points if μ is reduced from 0.75 to 0.5. This result implies that the IMF can play a more effective role in crisis prevention if the country's economic fundamentals are weaker, an implication broadly in line with the theoretical predictions of Kim (2006) and the empirical findings of Bordo, Mody and Oomes (2004) and Ramakrishnan and Zaldendo (2006).

Table 3 reports the results of the third exercise. The calibrated risk premiums are more sensitive to output volatility (σ) than to the noisiness of the signal (τ), suggesting that fundamental uncertainty plays a greater role than informational uncertainty in the determination of the risk premium and the crisis probability. Surprisingly, however, risk premiums and crisis probabilities are both negatively related to informational uncertainty, a counterintuitive result at first glance that suggests that improved transparency in data on economic fundamentals could increase, rather than decrease, the borrowing costs of emerging market countries.¹⁷

Intuitively, such a negative relationship between the risk premium and informational uncertainty is closely related to the nature of creditor coordination failure assumed in the model. As discussed in Appendix I, the rollover interest rate for short-term debt always equals the risk free interest rate if there were no informational uncertainty and thus plays no role in pricing lending risks. As such, private investors have no other alternative than resorting to credit rationing based on a binary decision to exit or roll over. Consequently, coordination failure is more likely to occur in the absence of informational uncertainty, resulting in a higher risk premium than otherwise. These results regarding the role of informational uncertainty, however, should be interpreted with caution because it is specific to the nature of creditor coordination failure assumed in the model.

Finally, Table 4 presents the calibration results obtained by assuming a full bailout with a 10 percent implicit transfer in IMF lending. According to the Mussa theorem, IMF-induced

¹⁶ This result suggests that increased cross-country variance in bond spreads after crisis events may not necessarily be considered as evidence of investor moral hazard prior to the events. For further discussion, see Dell'Ariccia, Schnabel, and Zettelmeyer (2002, 2006).

¹⁷ A similar result obtains in the global games model of Morris and Shin (2004). In their model, the risk of coordination failure reaches the highest when the private signal received by private investors is arbitrarily precise. Such counterintuitive result arises as a result of the complex interplay between two types of uncertainty—fundamental uncertainty about the state of nature that determines the payoff and strategic uncertainty concerning the actions of other investors.

investor moral hazard would arise in this case, and the resulting risk premium and the crisis probability would be even lower. The first three columns report the total reduction in the risk premium and the crisis probability (in percentage points) achieved under a full bailout with a 10 percent transfer in lending. The middle and last three columns present the reduction accounted for by the welfare-enhancing effect (i.e., reduced real hazard) of IMF financial support and by the net effect of IMF-induced investor moral hazard, respectively.

Apparently, IMF-induced moral hazard accounts for only a small portion of the total reduction in the risk premium, except for when the share of short-term debt is high (Panel A). In addition, the effect of investor moral hazard on the risk premium remains relatively insensitive to changes in economic fundamentals (Panel B). These results—together with the findings of Zettelmeyer and Joshi (2005) that implicit transfers in IMF lending have been far smaller than assumed for the calibration—suggest that IMF-induced investor moral hazard is unlikely to be a real concern, particularly if the country’s economic fundamentals are strong and short-term debt is small.

VI. CONCLUSION

This paper shows that even if it lends unconditionally, the IMF can play a useful role in crisis prevention without causing investor moral hazard. In the model, the prospect of a bailout by the IMF not only reduces real hazard of a crisis but also lowers the likelihood of a crisis itself. As a result, both the risk premium and the crisis probability are lower with the IMF than otherwise, implying a positive welfare gain for a debtor country. If the IMF lends at an actuarially fair rate, no investor moral hazard occurs and, therefore, the reduction in the risk premiums is solely attributed to the intended welfare-enhancing effects of an IMF bailout.

Although the model abstracts from issues of debtor moral hazard, it has an implication regarding debtor moral hazard. Specifically, the possibility of IMF financial support could help allow borrowing by emerging market countries in general, including short-term borrowing: it reduces borrowing costs and increases the spread between short- and long-term interest rates. If emerging market governments are maximizing national welfare, this should not be a problem by the Mussa theorem: borrowing more and shorter term would not be an outcome of debtor moral hazard but rather an optimal response to reduced riskiness of external borrowing that results from the prospect of possible IMF financial support.

The model also has useful implications for the design of new lending instruments of the IMF that involve no ex post conditionality—such as the RAL. In particular, the model suggests that IMF-induced investor moral hazard is unlikely to be a real concern, and that the prospect of a larger-scale bailout would not necessarily be more effective for crisis prevention than a smaller-scale bailout, particularly if the borrowing country’s economic fundamentals are relatively weak and short-term debt is large. Accordingly, an exceptional and upfront access stipulated in the RAL may well be justified if the eligibility for the instrument is restricted only to members with relatively strong fundamentals and sustainable debt.

Table 1. Risk Premium and Crisis Probability: Effect of Debt Structure and Cost of Default
($\mu = \sigma = 1$, $\lambda = \tau = 0.5$; in percent)

	β	$\rho = 0.25$			$\rho = 0.5$			$\rho = 0.75$		
		δ			δ			δ		
		0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75
1. Short-term premium 1/	0.00	5.78	7.52	9.52	5.58	7.31	9.36	5.42	7.14	9.23
	0.05	5.46	7.10	9.11	5.27	6.89	8.81	5.12	6.72	8.66
	0.10	5.15	6.77	9.51	4.97	6.48	8.45	4.82	6.31	8.15
	0.25	4.34	7.27	...	4.08	5.56	8.98	3.95	5.20	7.38
	0.50	4.16	2.81	5.61	...	2.59	4.00	7.82
	0.75	1.98	1.42	3.62	...
	1.00	0.17	1.71	5.38	0.17	1.71	5.38	0.17	1.71	5.38
2. Long-term premium 1/	0.00	8.33	9.44	10.64	7.23	7.99	8.78	6.30	6.75	7.21
	0.05	8.42	9.78	11.50	7.30	8.24	9.69	6.35	6.95	7.92
	0.10	8.51	10.03	10.65	7.37	8.49	10.19	6.41	7.14	8.49
	0.25	8.73	9.62	...	7.57	9.00	9.25	6.56	7.64	9.00
	0.50	8.76	7.85	8.86	...	6.79	8.05	8.20
	0.75	8.00	6.99	8.04	...
	1.00	6.26	7.81	9.81	6.25	7.59	8.90	6.24	7.38	8.05
3. Average premium 1/ 2/	0.00	7.69	8.48	9.80	6.82	7.65	9.22	6.08	6.95	8.73
	0.05	7.68	8.44	9.71	6.80	7.57	9.03	6.05	6.84	8.47
	0.10	7.67	8.40	9.80	6.77	7.49	8.88	6.01	6.73	8.24
	0.25	7.63	8.44	...	6.70	7.28	9.05	5.91	6.42	7.79
	0.50	7.61	6.59	7.23	...	5.74	6.03	7.92
	0.75	6.50	5.59	5.83	...
	1.00	4.74	4.76	6.48	4.73	4.65	6.26	4.72	4.54	6.04
4. Crisis Probability 3/	0.00	10.36	13.07	15.99	10.05	12.76	15.77	9.79	12.50	15.59
	0.05	10.36	13.07	15.99	10.04	12.75	15.74	9.78	12.48	15.53
	0.10	10.36	13.08	15.99	10.04	12.74	15.71	9.77	12.46	15.48
	0.25	10.37	13.07	...	10.04	12.71	15.73	9.76	12.39	15.36
	0.50	10.37	10.03	12.68	...	9.73	12.30	15.37
	0.75	10.01	9.70	12.25	...
	1.00	9.43	11.93	15.00	9.42	11.89	14.91	9.42	11.85	14.82
5. Default Probability 4/	0.00	10.36	13.07	15.99	10.05	12.76	15.77	9.79	12.50	15.59
	0.05	0.00	0.16	4.45	0.00	0.01	0.53	0.00	0.00	0.11
	0.10	0.03	1.50	15.94	0.00	0.13	3.16	0.00	0.02	0.92
	0.25	0.82	11.55	...	0.06	1.94	13.71	0.01	0.51	5.39
	0.50	4.99	0.60	7.49	...	0.13	2.53	11.68
	0.75	1.75	0.45	4.93	...
	1.00	0.34	3.30	9.71	0.34	3.30	9.71	0.34	3.30	9.71
6. Solvency risk ($\phi\delta$)	0.05	0.07	0.19	0.52	0.03	0.10	0.26	0.02	0.06	0.17
	0.10	0.13	0.36	0.92	0.06	0.18	0.46	0.04	0.12	0.31
	0.25	0.31	0.80	1.71	0.15	0.40	0.86	0.10	0.27	0.57
	0.50	0.57	1.33	2.40	0.29	0.67	1.20	0.19	0.44	0.80
	0.75	0.80	1.71	2.77	0.40	0.86	1.38	0.27	0.57	0.92

1/ Annualized.

2/ Weighted by the maturity share of total debt.

3/ Also refers to the probability of a default p^D under a partial bailout by the IMF.

4/ Probability of a default with no bailout by the IMF denoted by p^{DNB} in the text.

Table 2. Risk Premium and Crisis Probability: Effect of Economic Fundamentals
($\delta = \rho = \lambda = \tau = 0.5$; in percent)

	β	$\sigma = 0.75$			$\sigma = 1.00$			$\sigma = 1.25$		
		μ			μ			μ		
		0.75	1.00	1.25	0.75	1.00	1.25	0.75	1.00	1.25
1. Short-term premium 1/	0.00	7.93	2.85	1.00	15.38	7.31	3.56	22.20	12.43	7.16
	0.05	7.46	2.70	0.95	14.36	6.89	3.37	20.59	11.65	6.75
	0.10	7.00	2.55	0.90	13.40	6.48	3.18	19.22	10.95	6.37
	0.25	5.83	2.14	0.76	11.44	5.56	2.73	16.91	9.67	5.62
	0.50	5.59	2.03	0.70	11.56	5.61	2.74	17.45	9.97	5.79
	0.75
	1.00	1.02	0.34	0.10	3.50	1.71	0.79	6.90	4.05	2.33
2. Long-term premium 1/	0.00	8.55	3.84	1.72	14.35	7.99	4.52	19.10	12.12	7.82
	0.05	8.82	3.94	1.76	14.91	8.24	4.65	19.93	12.57	8.07
	0.10	9.09	4.03	1.79	15.43	8.49	4.77	20.60	12.97	8.30
	0.25	9.77	4.29	1.88	16.37	9.00	5.03	21.48	13.56	8.69
	0.50	9.77	4.32	1.90	16.01	8.86	4.98	20.84	13.20	8.48
	0.75
	1.00	7.65	3.68	1.69	12.83	7.59	4.43	17.32	11.52	7.63
3. Average premium 1/ 2/	0.00	8.24	3.34	1.36	14.86	7.65	4.04	20.65	12.28	7.49
	0.05	8.14	3.32	1.35	14.63	7.57	4.01	20.26	12.11	7.41
	0.10	8.05	3.29	1.34	14.42	7.49	3.97	19.91	11.96	7.34
	0.25	7.80	3.21	1.32	13.90	7.28	3.88	19.19	11.61	7.15
	0.50	7.68	3.17	1.30	13.79	7.23	3.86	19.14	11.58	7.13
	0.75
	1.00	4.34	2.01	0.90	8.16	4.65	2.61	12.11	7.79	4.98
4. Crisis Probability 3/	0.00	13.68	5.39	1.97	23.52	12.76	6.65	30.75	19.91	12.53
	0.05	13.67	5.39	1.97	23.48	12.75	6.65	30.69	19.89	12.52
	0.10	13.65	5.39	1.97	23.44	12.74	6.65	30.63	19.87	12.51
	0.25	13.60	5.38	1.96	23.32	12.71	6.64	30.47	19.80	12.49
	0.50	13.55	5.37	1.96	23.25	12.68	6.63	30.40	19.77	12.47
	0.75
	1.00	12.11	5.11	1.92	20.83	11.89	6.39	27.76	18.58	11.96
5. Default Probability 4/	0.00	13.68	5.39	1.97	23.52	12.76	6.65	30.75	19.91	12.53
	0.05	0.00	0.00	0.00	0.03	0.01	0.00	0.37	0.14	0.05
	0.10	0.02	0.00	0.00	0.42	0.13	0.04	2.00	0.89	0.40
	0.25	0.96	0.25	0.06	4.52	1.94	0.81	9.69	5.40	2.94
	0.50	6.55	2.42	0.81	14.31	7.49	3.76	21.34	13.48	8.27
	0.75
	1.00	2.01	0.67	0.20	6.54	3.30	1.56	12.13	7.49	4.44

1/ Annualized.

2/ Weighted by the maturity share of total debt.

3/ Also refers to the probability of a default p^D under a partial bailout by the IMF.

4/ Probability of a default with no bailout by the IMF denoted by p^{DNB} in the text.

Table 3. Risk Premium and Crisis Probability: Effect of Informational Uncertainty
($\mu = \beta = 1$, $\delta = \rho = \lambda = 0.5$; in percent unless otherwise indicated)

	τ	Without the IMF (A)			Full Bailout by the IMF (B)			Difference (A-B) 1/		
		σ			σ			σ		
		0.75	1.00	1.25	0.75	1.00	1.25	0.75	1.00	1.25
1. Short-term premium 2/	0.00	11.86	19.70	25.79	1.34	4.14	7.60	10.52	15.56	18.20
	0.25	4.04	9.04	14.20	0.63	2.50	5.23	3.40	6.54	8.97
	0.50	2.85	7.31	12.43	0.34	1.71	4.05	2.51	5.61	8.38
	0.75	2.08	6.05	11.02	0.19	1.20	3.20	1.89	4.85	7.82
2. Long-term premium 2/	0.00	6.95	11.80	15.84	4.52	8.58	12.38	2.43	3.22	3.46
	0.25	4.24	8.49	12.57	4.00	8.00	11.91	0.24	0.48	0.66
	0.50	3.84	7.99	12.12	3.68	7.59	11.52	0.16	0.39	0.60
	0.75	3.56	7.59	11.74	3.44	7.26	11.18	0.12	0.33	0.56
3. Average premium 2/ 3/	0.00	9.40	15.75	20.82	2.93	6.36	9.99	6.48	9.39	10.83
	0.25	4.14	8.76	13.39	2.32	5.25	8.57	1.82	3.51	4.82
	0.50	3.34	7.65	12.28	2.01	4.65	7.79	1.34	3.00	4.49
	0.75	2.82	6.82	11.38	1.82	4.23	7.19	1.00	2.59	4.19
4. Crisis probability	0.00	19.18	28.26	34.03	16.68	25.07	30.99	2.50	3.19	3.04
	0.25	7.47	15.31	22.12	7.02	14.27	20.75	0.45	1.04	1.37
	0.50	5.39	12.76	19.91	5.11	11.89	18.58	0.28	0.86	1.33
	0.75	3.99	10.79	18.06	3.81	10.07	16.79	0.18	0.72	1.27
5. Default probability	0.00	19.18	28.26	34.03	2.61	7.64	13.19	16.57	20.61	20.84
	0.25	7.47	15.31	22.12	1.25	4.76	9.47	6.22	10.55	12.65
	0.50	5.39	12.76	19.91	0.67	3.30	7.49	4.72	9.46	12.42
	0.75	3.99	10.79	18.06	0.37	2.34	6.01	3.62	8.45	12.05

1/ In percentage points.

2/ Annualized.

3/ Weighted by the maturity share of total debt.

Table 4. Risk Premium and Crisis Probability: Real Hazard versus Investor Moral Hazard

Panel A. Effect of Short-term Debt and Cost of Default

($\mu = \sigma = \beta = 1$, $\lambda = \tau = 0.5$; in percentage points)

	δ	Total Reduction (A+B)			Reduced Real Hazard (A)			Moral Hazard (B)		
		ρ			ρ			ρ		
		0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75
1. Short-term premium 1/	0.25	5.68	5.49	5.32	5.61	5.41	5.25	0.07	0.07	0.07
	0.50	6.41	6.20	6.03	5.81	5.61	5.44	0.60	0.60	0.60
	0.75	5.89	5.73	5.60	4.14	3.99	3.86	1.74	1.74	1.74
2. Long-term premium 1/	0.25	2.23	1.14	0.21	2.07	0.98	0.06	0.16	0.16	0.15
	0.50	2.06	0.72	-0.39	1.64	0.39	-0.63	0.42	0.33	0.24
	0.75	1.49	0.17	-0.87	0.83	-0.12	-0.84	0.66	0.29	-0.03
3. Average premium 1/ 2/	0.25	3.09	2.23	1.49	2.95	2.09	1.36	0.14	0.14	0.13
	0.50	4.23	3.46	2.82	3.72	3.00	2.40	0.51	0.46	0.42
	0.75	4.79	4.34	3.98	3.32	2.96	2.68	1.47	1.38	1.30
4. Crisis probability	0.25	0.98	0.67	0.41	0.93	0.62	0.36	0.05	0.05	0.04
	0.50	1.30	1.01	0.77	1.14	0.86	0.65	0.16	0.15	0.13
	0.75	1.44	1.27	1.14	0.99	0.86	0.76	0.45	0.41	0.38
5. Default probability	0.25	10.16	9.85	9.59	10.02	9.71	9.44	0.14	0.14	0.14
	0.50	10.90	10.59	10.33	9.77	9.46	9.20	1.13	1.13	1.13
	0.75	9.22	9.00	8.82	6.28	6.06	5.88	2.94	2.94	2.94

Panel B. Effect of Economic Fundamentals

($\beta = 1$, $\delta = \rho = \lambda = \tau = 0.5$; in percentage points)

	μ	Total Reduction (A+B)			Reduced Real Hazard (A)			Moral Hazard (B)		
		σ			σ			σ		
		0.75	1.00	1.25	0.75	1.00	1.25	0.75	1.00	1.25
1. Short-term premium 1/	0.75	7.33	13.04	17.30	6.90	11.88	15.30	0.43	1.16	2.00
	1.00	2.66	6.20	9.59	2.51	5.61	8.38	0.15	0.60	1.21
	1.25	0.95	3.07	5.56	0.90	2.77	4.84	0.05	0.29	0.73
2. Long-term premium 1/	0.75	1.38	2.21	2.57	0.90	1.52	1.78	0.48	0.69	0.79
	1.00	0.34	0.72	1.03	0.16	0.39	0.60	0.18	0.33	0.43
	1.25	0.09	0.25	0.42	0.03	0.09	0.19	0.06	0.16	0.24
3. Average premium 1/ 2/	0.75	4.35	7.63	9.94	3.90	6.70	8.54	0.45	0.93	1.40
	1.00	1.50	3.46	5.31	1.34	3.00	4.49	0.17	0.46	0.82
	1.25	0.52	1.66	2.99	0.47	1.43	2.51	0.06	0.23	0.48
4. Crisis probability	0.75	1.77	3.10	3.53	1.57	2.69	2.99	0.20	0.41	0.54
	1.00	0.32	1.01	1.59	0.28	0.86	1.33	0.04	0.15	0.26
	1.25	0.05	0.31	0.69	0.04	0.26	0.57	0.01	0.05	0.12
5. Default probability	0.75	12.50	19.05	21.82	11.67	16.98	18.62	0.82	2.07	3.21
	1.00	5.02	10.59	14.54	4.72	9.46	12.42	0.30	1.13	2.12
	1.25	1.87	5.67	9.43	1.77	5.10	8.09	0.10	0.57	1.35

1/ Annualized.

2/ Weighted by the maturity share of total debt.

APPENDIX I. EQUILIBRIUM SOLUTIONS WITH NO INFORMATIONAL UNCERTAINTY

This appendix derives the equilibrium solutions of the model assuming that the signal is perfectly informative (i.e., $\tau = 0$).

A. Without the IMF

As private investors face no uncertainty in period 1 about the country's debt servicing capacity, the rollover interest rate for short-term debt (r_1^S) should equal the zero risk-free interest rate. Since $R^S \equiv 1 + r_1^S = 1$, productivity threshold θ^* defined in (2a) reduces to

$$(A.I.1) \quad \theta^* = \bar{\theta} + \ln(1 + \psi)$$

If $\theta < \theta^*$, short-term investors would run for the exit because the country's output in period 2—which is known in period 1 by assumption—falls short of repaying all private investors in full. Otherwise, they would roll over their debt at the risk-free interest rate. Therefore, the probability of a rollover crisis, p^C , is simply given by

$$(A.I.2) \quad p^C = \int_{-\infty}^{\theta^*} g(\theta) d\theta$$

where $g(\theta)$ is the (unconditional) normal density of θ . Since the country always defaults upon a rollover crisis without the IMF, the default probability, p^D , simply equals the crisis probability so that $p^D = p^C$.

The ex ante zero-profit condition for short-term investors in period 0 would be given by

$$(A.I.3a) \quad \delta k = d_1^S (1 - p^D) + \lambda \delta k p^D$$

which can be reduced to

$$(A.I.3b) \quad \delta = \alpha \exp(\bar{\theta}) \int_{\theta^*}^{\infty} g(\theta) d\theta + \lambda \delta \int_{-\infty}^{\theta^*} g(\theta) d\theta$$

Long-term debt repayment in period 2, DS_2^L , also depends on the level of θ relative to θ^* and θ^{**} where θ^{**} is as defined in (2a). Long-term investors are fully repaid if either short-term debt is rolled over ($\theta \geq \theta^*$), or short-term debt is not rolled over but nonetheless $\theta \geq \theta^{**}$; otherwise, they are repaid partially. Denoting by $\theta_{\min} = \min[\theta^*, \theta^{**}]$ the smaller of the two thresholds, long-term debt repayment in period 2 is characterized as follows:

$$DS_2^L = \begin{cases} d_2^L & \text{if } \theta \geq \theta_{\min} \\ \alpha y(k_1, \theta) & \text{otherwise} \end{cases}$$

where $k_1 = [1 - (1 - \beta)\delta]k$. The expected long-term debt repayment is therefore given by

$$\begin{aligned} E_0(DS_2^L) &= E_0(DS_2^L \mid \theta \geq \theta_{\min}) + E_0(DS_2^L \mid \theta < \theta_{\min}) \\ &= d_2^L \int_{\theta_{\min}}^{\infty} g(\theta) d\theta + \rho(1 - \delta) \alpha k \int_{-\infty}^{\theta_{\min}} \exp(\theta) g(\theta) d\theta \end{aligned}$$

The ex ante zero-profit condition for long-term investors requires that $(1 - \delta)k = E_0(DS_2^L)$, which can be reformulated to yield

$$(A.I.4) \quad (1 - \delta) = \alpha \psi \exp(\bar{\theta}) \int_{\theta_{\min}}^{\infty} g(\theta) d\theta + \alpha \rho(1 - \delta) \int_{-\infty}^{\theta_{\min}} \exp(\theta) g(\theta) d\theta$$

Equilibrium solutions can be found by solving (A.I.3b) and (A.I.4) simultaneously for $\bar{\theta}$ and ψ : in case of multiple solutions, the lowest pair would be considered as the economically relevant one. To be specific, the equilibrium solutions without the IMF, denoted by superscript *NO*, are characterized as follows:

$$(A.I.5) \quad \begin{aligned} 1 + r_0^{SNO} &= (\alpha / \delta) \exp(\bar{\theta}^{NO}), \quad 1 + r_0^{LNO} = (\alpha / (1 - \delta)) \exp(\bar{\theta}^{NO}) \psi^{NO}, \\ p^{DNO} &= p^{CNO} = \int_{-\infty}^{\theta^*} g(\theta) d\theta \end{aligned}$$

where θ^* is evaluated at $\bar{\theta} = \bar{\theta}^{NO}$ and $\psi = \psi^{NO}$.

B. With the IMF

Since the country's economic fundamentals remain unaffected by unconditional IMF lending, the same threshold θ^* as defined in (A.I.1) would continue to characterize the crisis threshold. Thus, the crisis probability is correspondingly characterized as defined by (A.I.2).

With no informational uncertainty, the lending rate of the IMF (r_1^F) should also equal the zero risk-free interest rate so that $R^F \equiv 1 + r_1^F = 1$. By using this, the productivity thresholds defined in (7a) reduce to

$$(A.I.6) \quad \bar{\theta}^F = \bar{\theta} + \ln \phi \quad \text{and} \quad \theta^{*F} = \bar{\theta} + \ln(1 + \psi / \beta) + \ln \phi$$

Since the IMF lends contingently upon a crisis, it would lend only if $\theta < \theta^*$. Also, it would lend only if $\theta \geq \bar{\theta}^F$ because the IMF lending rate—which equals the risk-free interest rate—cannot be actuarially fair for $\theta < \bar{\theta}^F$. Therefore, the inequality $\bar{\theta}^F < \theta^*$ must hold in equilibrium for the IMF to play any role. Under a full bailout, the inequality holds as long as long-term debt is not zero ($\psi > 0$). Under a partial bailout, it can be shown that there exists a

threshold $\hat{\beta} < 1$ such that the inequality holds if $\beta \leq \hat{\beta}$ but not otherwise, suggesting that the size of IMF lending (relative to short-term debt) cannot be too large under a partial bailout.

In what follows, we assume without loss of generality that the inequality $\bar{\theta}^F < \theta^*$ always holds on the grounds that the IMF can flexibly choose the size of its lending to ensure the inequality. Given this assumption, we define the default probability as follows:

$$(A.I.7) \quad p^D = \begin{cases} p^{DNB} = \int_{-\infty}^{\bar{\theta}^F} g(\theta) d\theta & \text{if } \beta = 1 \\ p^C & \text{otherwise} \end{cases}$$

where p^{DNB} is the probability that a default occurs while the IMF does not lend. The probability of IMF lending is accordingly characterized by $p^C - p^{DNB}$.

The ex ante zero-profit condition for short-term investors is given by

$$\delta k = d_1^S (1 - p^D) + [\beta d_1^S + (1 - \beta) \lambda \delta k] (p^D - p^{DNB}) + \lambda \delta k p^{DNB}$$

which can be transformed to yield

$$(A.I.8) \quad \delta = \alpha \exp(\bar{\theta}) \left[\int_{\theta^*}^{\infty} g(\theta) d\theta + \beta \int_{\bar{\theta}^F}^{\theta^*} g(\theta) d\theta \right] + \lambda \delta \left[(1 - \beta) \int_{\bar{\theta}^F}^{\theta^*} g(\theta) d\theta + \int_{-\infty}^{\bar{\theta}^F} g(\theta) d\theta \right]$$

We assume that (A.I.8) has solutions denoted by $\bar{\theta}^{IMF} = \bar{\theta}^{IMF}(\psi, \beta)$, where superscript *IMF* stands for the equilibrium solutions with the IMF. It is easy to show $\bar{\theta}^{IMF} < \bar{\theta}^{NO}$ for all ψ .

Long-term debt repayment in period 2 is defined over the three regions of θ as follows:

$$DS_2^L = \begin{cases} d_2^L & \text{if } \theta \geq \hat{\theta}_{\min} \\ \alpha y(k_1, \theta) - \beta d_1^S & \text{if } \bar{\theta}^F \leq \theta < \hat{\theta}_{\min} \\ \alpha y(k_1, \theta) & \text{otherwise} \end{cases}$$

where $\hat{\theta}_{\min} = \min[\theta^*, \theta^{*F}]$ and $k_1 = [1 - (1 - \beta)\delta]k$. The expected long-term debt repayment is accordingly given by

$$E_0(DS_2^L) = E_0(DS_2^L \mid \theta \geq \hat{\theta}_{\min}) + E_0(DS_2^L \mid \bar{\theta}^F \leq \theta < \hat{\theta}_{\min}) + E_0(DS_2^L \mid \theta < \bar{\theta}^F)$$

The ex ante zero-profit condition for long-term investors requires $(1 - \delta)k = E_0(DS^L)$ which, after some algebra, can be rewritten as follows:

$$\begin{aligned}
(A.I.9) \quad (1 - \delta) = & \psi \exp(\bar{\theta}) \int_{\hat{\theta}_{\min}}^{\infty} g(\theta) d\theta + \beta \int_{\bar{\theta}^F}^{\hat{\theta}_{\min}} [\phi^{-1} \exp(\theta) - \exp(\bar{\theta})] g(\theta) d\theta \\
& + \rho(1 - \delta) \int_{-\infty}^{\bar{\theta}^F} \exp(\theta) g(\theta) d\theta
\end{aligned}$$

Substituting $\bar{\theta} = \bar{\theta}^{IMF}(\psi, \beta)$ into (A.I.9) and solving for ψ yields the equilibrium solution $\psi^{IMF}(\beta)$. Once $\bar{\theta}^{IMF}$ and ψ^{IMF} are identified, the equilibrium solutions with the IMF can be characterized as follows:

$$\begin{aligned}
(A.I.10) \quad 1 + r_0^{S IMF} = & (\alpha / \delta) \exp(\bar{\theta}^{IMF}), \quad 1 + r_0^{L IMF} = (\alpha / (1 - \delta)) \psi^{IMF} \exp(\bar{\theta}^{IMF}), \\
p^{C IMF} = & \int_{-\infty}^{\theta^*} g(\theta) d\theta, \quad p^{DNB IMF} = \int_{-\infty}^{\bar{\theta}^F} g(\theta) d\theta
\end{aligned}$$

where θ^* and $\bar{\theta}^F$ are both evaluated at $\bar{\theta} = \bar{\theta}^{IMF}$ and $\psi = \psi^{IMF}$.

APPENDIX II. EQUATIONS USED FOR MODEL CALIBRATION

This appendix summarizes specific equations used for model calibration.

A. Without the IMF

The zero-profit condition for the rollover interest rate in (3) can be rewritten as

$$d_1^S = R^S d_1^S \int_{\theta^*}^{\infty} v(\theta | q) d\theta + S \cdot \alpha k \int_{-\infty}^{\theta^*} \exp(\theta) v(\theta | q) d\theta$$

where $v(\theta | q)$ is the posterior distribution of θ given signal q which is normal with mean $\mu(q) = (q + \tau\mu)/(1 + \tau)$ and variance $\sigma(q)^2 = \tau\sigma^2/(1 + \tau)$. Substituting $\alpha k / d_1^S \equiv \exp(-\bar{\theta})$ into this equation and rearranging terms yield,

$$(A.II.1) \quad 1 = m(R^S | q, \psi, \bar{\theta}) = R^S \int_{\theta^*}^{\infty} v(\theta | q) d\theta + S \int_{-\infty}^{\theta^*} \exp(\theta - \bar{\theta}) v(\theta | q) d\theta$$

For given ψ and $\bar{\theta}$, (A.II.1) has a unique solution, $R^S(q)$, since $\partial m / \partial R^S > 0$ and $m(R^S = 1) < 1$ which, in turn, implies $\partial R^S / \partial q < 0$. The crisis threshold \bar{q} is also uniquely determined by

$$(A.II.2) \quad R^S(\bar{q}) = 1 + \bar{r}$$

The ex ante zero-profit condition for short-term investors in (4) can be reformulated to yield,

$$(A.II.3) \quad \delta = \alpha \exp(\bar{\theta}) \int_{\bar{q}}^{\infty} z(q) dq + \lambda \delta \int_{-\infty}^{\bar{q}} z(q) dq$$

By using $d_2^L \equiv \psi d_1^S$ and (A.II.1), the right hand side of (5) can be written as follows:

$$\begin{aligned} E_1(DS^L | q \geq \bar{q}) &= \psi d_1^S \int_{\theta^*}^{\infty} v(\theta | q) d\theta + (1 - S) \cdot \alpha k \int_{-\infty}^{\theta^*} \exp(\theta) v(\theta | q) d\theta \\ &= (\psi / R^S) d_1^S \left[R^S \int_{\theta^*}^{\infty} v(\theta | q) d\theta + S \int_{-\infty}^{\theta^*} \exp(\theta - \bar{\theta}) v(\theta | q) d\theta \right] \\ &= (\psi / R^S) d_1^S \end{aligned}$$

$$E_1(DS^L | q < \bar{q}) = \psi d_1^S \int_{\theta^{**}}^{\infty} v(\theta | q) d\theta + \rho(1 - \delta) \alpha k \int_{-\infty}^{\theta^{**}} \exp(\theta) v(\theta | q) d\theta = f(q) d_1^S$$

where $f(q) = \psi \int_{\theta^{**}}^{\infty} v(\theta | q) d\theta + \rho(1 - \delta) \int_{-\infty}^{\theta^{**}} \exp(\theta - \bar{\theta}) v(\theta | q) d\theta$. By using these expressions, the ex ante zero-profit condition for long-term investors in (5) reduces to

$$(A.II.4) \quad 1 - \delta = \alpha \exp(\bar{\theta}) \left[\int_{\bar{q}}^{\infty} [\psi / R^S] z(q) dq + \int_{-\infty}^{\bar{q}} f(q) z(q) dq \right]$$

In the model calibration, equations (A.II.1)-(A.II.4) are numerically solved to yield $\{R^{S\ NO}, \bar{q}^{NO}, \bar{\theta}^{NO}, \text{ and } \psi^{NO}\}$.

B. With the IMF: No Investor Moral Hazard

Equations (A.II.1) and (A.II.2) continue to characterize the equilibrium rollover interest rate for short-term debt $R^S(q)$ and the crisis threshold \bar{q} with the IMF, for given $\bar{\theta}$, ψ , and β .

The ex ante zero-profit condition for the IMF in (8) can be rewritten as

$$\beta d_1^S = R^F \beta d_1^S \int_{\bar{\theta}^F}^{\infty} v(\theta | q) d\theta + \alpha k \omega [1 - (1 - \beta)\delta] \int_{-\infty}^{\bar{\theta}^F} \exp(\theta) v(\theta | q) d\theta$$

which further reduces to

$$(A.II.5) \quad 1 = b(R^F | q, \bar{\theta}, \beta) = R^F \int_{\bar{\theta}^F}^{\infty} v(\theta | q) d\theta + \phi^{-1} \int_{-\infty}^{\bar{\theta}^F} \exp(\theta - \bar{\theta}) v(\theta | q) d\theta$$

where $\phi = \beta / \{\omega[1 - (1 - \beta)\delta]\}$. (A.II.5) has a unique solution, $R^F(q)$, since $\partial b / \partial R^F > 0$ and $b(R^F = 1) < 1$. Note that $R^F(q)$ is independent of ψ because the right hand side of (A.II.5) does not depend on ψ .

Since $\partial R^F / \partial q < 0$, the lower threshold for IMF lending, \bar{q}^F , is uniquely determined by

$$(A.II.6) \quad R^F(\bar{q}^F) = 1 + \bar{r}$$

Note that \bar{q}^F is also independent of ψ . Moreover, it is discontinuous at $\beta = 1$ (i.e., $\lim_{\beta \rightarrow 1} \bar{q}^F \neq \bar{q}^F|_{\beta=1}$) because ϕ is discontinuous at $\beta = 1$.

If $\psi = 0$ and $\beta = 1$, $m(R^S | q, \bar{\theta}, \psi)$ in (A.II.1) collapses to $b(R^F | q, \bar{\theta}, \beta)$ in (A.II.5). As a result, $R^F = R^S$ and $\bar{q} = \bar{q}^F$, implying that the IMF cannot lend for a weaker signal than \bar{q} unless it expects losses on its lending. This result suggests that under a full bailout, the existence of long-term debt is critical for unconditional IMF lending to play a role in crisis prevention.

The ex ante zero-profit condition for short-term investors in (9) is rewritten as

$$(A.II.7) \quad \delta = \alpha \exp(\bar{\theta}) \left[\int_{\bar{q}}^{\infty} z(q) dq + \beta \int_{\bar{q}^F}^{\bar{q}} z(q) dq \right] + \lambda \delta \left[(1 - \beta) \int_{\bar{q}^F}^{\bar{q}} z(q) dq + \int_{-\infty}^{\bar{q}^F} z(q) dq \right]$$

Under a full bailout ($\beta = 1$), $\bar{\theta}$ is independent of ψ because (A.II.7) does not depend on \bar{q} .

The IMF does not lend if either $q \geq \bar{q}$ or $q < \bar{q}^F$. In this case, the expected long-term debt repayment is characterized by the same expression as derived for the case without the IMF. Therefore,

$$E_1(DS^L | q \geq \bar{q}) = (\psi / R^S) d_1^S \quad \text{and} \quad E_1(DS^L | q < \bar{q}^F) = f(q) d_1^S$$

The expected long-term debt repayment contingent upon IMF lending is given by

$$E_1(DS^L | \bar{q}^F \leq q < \bar{q}) = \Omega(q) d_1^S$$

where

$$\Omega(q) = \psi \int_{\bar{\theta}^F}^{\infty} v(\theta | q) d\theta + \int_{\bar{\theta}^F}^{\bar{\theta}^*} [\exp(\theta - \bar{\theta}) \omega [1 - (1 - \beta)\delta] - \beta R^F(q)] v(\theta | q) d\theta.$$

Collecting these expressions for the expected long-term debt repayment, the ex ante zero-profit condition for long-term investors in (10) can be rewritten as follows:

$$(A.II.8) \quad 1 - \delta = \alpha \exp(\bar{\theta}) \int_{-\infty}^{\infty} H(q) z(q) dq$$

where

$$H(q) = \begin{cases} \psi / R^S(q) & \text{if } q \geq \bar{q} \\ \Omega(q) & \text{if } \bar{q}^F \leq q < \bar{q} \\ f(q) & \text{otherwise} \end{cases}$$

Equations (A.II.1)-(A.II.2) and (A.II.5)-(A.II.8) are numerically solved to yield $\{ R^{S IMF}, \bar{q}^{IMF}, R^F, \bar{q}^F, \bar{\theta}^{IMF}, \text{ and } \psi^{IMF} \}$.

C. With the IMF: IMF-induced Investor Moral Hazard

The same set of equations are used as in the case of IMF lending with no investor moral hazard, except for equation (A.II.5) being replaced by

$$(A.II.9) \quad 1 - \gamma = R^F \int_{\bar{\theta}^F}^{\infty} v(\theta | q) d\theta + \phi^{-1} \int_{-\infty}^{\bar{\theta}^F} \exp(\theta - \bar{\theta}) v(\theta | q) d\theta$$

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