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Welfare Cost of (Low) Inflation: A General Equilibrium Perspective

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Abstract

This paper provides general equilibrium estimates of the steady-state welfare gains of lowering inflation from a low level to close to price stability, using an overlapping-generations growth model. Money demand is modeled on the basis that real money balances are a factor of production. Assuming a standard Fisher equation modified by the presence of an income tax, it is found that inflation unambiguously reduces capital intensity, drives up the before-tax real rate of return to capital, and unambiguously imposes a life-time welfare cost. This welfare cost is, however, quantitatively very modest (under 0.2 percent of GDP annually) within reasonable ranges of all parameter values.

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SUMMARY

This paper addresses the fundamental question, from a welfare perspective in a general equilibrium framework, whether it would still be desirable for policymakers to pursue vigorously, or target, complete price stability if and when inflation has already reached a relatively low level.

The theoretical framework used for the assessment is an overlapping-generations growth model with real money balances treated as a factor of production and with a standard Fisher equation modified by the presence of an income tax. The model is constructed to capture the four most commonly cited effects of inflation: (1) the impact on money demand stemming from the inflation tax; (2) the impact on investment via the cost of capital; (3) the impact on savings owing to the change in the intertemporal price of consumption; and (4) the budgetary impact of seigniorage.

Numerical estimates, based on a range of realistic parameter values, indicate that the case for further lowering inflation from an already low level (say, 4 percent) to a level approximating price stability (say, 2 percent) is not compelling from a welfare perspective. While inflation unambiguously imposes a welfare cost, the steady-state welfare gains from eliminating low inflation are very modest—less than 0.2 percent of output annually under all cases considered.

I. INTRODUCTION

In view of the widely recognized costs of inflation, in terms of both its propensity to endanger macroeconomic stability in the short run and its possible harmful effects on growth in the longer term, reducing inflation has long been a top priority of most policy makers. Looking at recent global inflation experiences, it can arguably be said that, on the whole, substantial headway on this front has been made. In most industrialized countries, as well as in a large number of Asian countries (prior to the current crisis), inflation has dropped to below 5 percent. Even in many other developing economies, including Latin American countries, inflation has been brought down to low double-digit levels in recent years. If and when inflation in a country has reached a relatively low level, would it still be desirable for policy makers to vigorously pursue, or target, complete price stability as an ultimate policy goal?

It seems that this question could be addressed from three different perspectives. First, from the perspective of short-run macroeconomic management, Fischer (1996) has argued that targeting a positive inflation rate (in the range of 1 percent to 3 percent) could provide more flexibility than targeting price stability in conducting counter-cyclical policies. Second, from a growth perspective, any negative impact of inflation on the growth rate, however small, would have significant cumulative effects over time. This would argue, therefore, for a target of zero inflation. There is, however, little compelling evidence that *low* inflation has statistically significant growth effects.² Finally, from a welfare perspective, whether the argument for eliminating inflation completely is compelling depends on the significance of the welfare gains that can be had from moving from low inflation to price stability.

The traditional, partial equilibrium approach to measuring the welfare costs of inflation, as pioneered by Bailey (1956), focuses on inflation's role as a tax on the demand for real money balances and, therefore, on the inflation-induced loss of the consumer surplus under the money demand curve. This approach often produces a very low estimate of the welfare cost of inflation.³ Phelps (1973), recognizing the revenue implications of seignorage, was the first to analyze the welfare effects of inflation in the context of overall budgetary finance, and stimulated a large body of literature that integrated inflation into the optimal taxation framework.⁴ At a minimum, this public finance approach points to a potential role for inflation as a source of budgetary revenue.

²See Briault (1995), Thornton (1996), and Tanzi and Zee (1997) for reviews of the empirical evidence.

³See, for example, Fischer (1981). Lucas (1994), however, has recently revisited the Bailey approach and obtained results that suggest that the welfare cost, even within this partial equilibrium framework, may not be insignificant.

⁴For a recent illuminating review of this literature, see Chari, Christiano, and Kehoe (1996).

Recently, Feldstein (1997) has emphasized the distortionary impact of inflation on the intertemporal allocation of consumption. Adopting Harberger's deadweight loss approach, he argues that the interaction between inflation and a preexisting income tax results in a loss of consumer surplus under the demand curve for future consumption that is represented by a first-order trapezoidal area, rather than by a second-order triangular area. Feldstein's numerical calculations indicate that the welfare gains of reducing inflation from 4 percent to 2 percent could be substantial—as high as 1 percentage point of GDP per year based on likely parameter values. The bulk of this welfare gain stems from the intertemporal distortion on consumption, rather than from the other effects of inflation (including the revenue and money demand effects).

In addition to being a partial equilibrium analysis, Feldstein's framework omits the possible impact of inflation on the nominal return to savings via the Fisher equation. If through this channel savers are largely compensated for the consequences of inflation, the intertemporal distortion on consumption would largely disappear. The basic question of interest is then whether the welfare effects stemming from other inflationary distortions would remain significant in a *general equilibrium* framework that takes full account of their interactions. This is the primary focus of the present paper.

The theoretical framework used in this paper to assess the welfare cost of inflation is a one-sector, two-period overlapping generations growth model of the form employed in Zee (1988), extended here to include real money balances. The two-period construct is preferred over an infinite-horizon one⁵ because it would allow for a clearer articulation of saving and investment behavior. The model is constructed to capture the four most commonly cited effects of inflation: (1) the impact on money demand stemming from the inflation tax; (2) the impact on investment via the cost of capital; (3) the impact on savings due to the change in the intertemporal price of consumption; and (4) the budgetary impact of seignorage.

The basic motivation for adopting a general equilibrium model as a framework of analysis is, of course, to ensure that the interdependent nature of the various inflationary distortions is fully taken into account in a consistent and unified manner. In so doing, a direct derivation of the general equilibrium impact of inflation on the real rate of return to capital is a necessary intermediate step in the overall assessment of the welfare cost of inflation. This issue has been the subject of much attention in the literature;⁶ it is also of relevance in the present context because general equilibrium changes in the real rate of return to capital have clear welfare consequences that are distinct from, and additional to, the four direct inflationary effects identified above. Hence, only a general equilibrium model would allow for a fully integrated

⁵Abel's (1997) is a recent example of the infinite-horizon approach.

⁶See, for example, the analyses by Mundell (1963) in a wealth-savings framework, by Tobin (1965) in a portfolio choice framework, and by Feldstein (1976) and Summers (1981) in a growth framework. See also discussions below in connection with the Fisher equation.

welfare assessment of the inflationary impact on the real rate of return to capital and of the other inflationary effects.⁷

II. THEORETICAL FRAMEWORK

A. General Considerations

Two crucial issues deserve special attention in any discussion of the welfare cost of inflation. The first is the demand function for real money balances. There are three broad approaches to modeling the micro-theoretic basis for holding real money balances, on each of which there is now a voluminous literature: (1) they yield direct utility in consumption,⁸ (2) they are a direct factor of production,⁹ and (3) they reduce transaction costs.¹⁰ As is well known, conceptual linkages exist among the three approaches. Fischer (1974), for example, has shown that, with appropriate reinterpretation, the first (second) approach is equivalent to a model in which real money balances reduce the transaction costs of consumers (producers). Irrespective of the approach adopted, however, obtaining a money demand function with sufficient specificity from which interesting implications could be drawn would invariably require an explicit formulation of the manner by which real money balances enter the relevant utility, production, or transaction technology function.¹¹ To avoid complicating the interpretation of savings dynamics and the necessity of specifying an additional technological relationship, the present model generates a money demand function on the basis that real money balances are a direct productive input.¹²

⁷Dotsey and Ireland (1996) have recently shown that, based on a general equilibrium model containing a transaction technology as a key feature of the model, while none of the many inflationary distortions is significant individually, these distortions could yield a substantial welfare cost when taken together.

⁸Sidrauski (1967) is the seminal contribution. Abel (1997) follows the same modeling strategy.

⁹This is a central consideration, for example, in Friedman's (1969) argument for establishing the optimal quantity of money.

¹⁰Dornbusch and Frenkel (1973) is a notable early example.

¹¹For an extensive discussion of the dependence of the properties of the money demand function on the nature of the transaction technology and the associated recent literature, see Correia and Teles (1996).

¹²Fischer (1974) contains detailed discussions on the conditions under which this approach is valid, as well as its limitations.

The second crucial issue concerns the form of the Fisher equation in the presence of taxation. In the absence of a tax on interest income, it is a widely accepted behavioral proposition that savers would demand some adjustment in the nominal interest rate (i) in response to the (expected) rate of inflation (π) as given by the standard Fisher equation: $i = r + \pi$, where r is the *before-tax* real interest rate.¹³ It then follows that $di/d\pi = 1 + dr/d\pi$. On this basis, both Mundell (1963) and Tobin (1965) have argued that $di/d\pi < 1$, because $dr/d\pi < 0$ (although for different reasons¹⁴). Clearly, this result would not necessarily follow if nominal interest is taxed (even if $dr/d\pi < 0$ is accepted), since the Fisher equation must be modified to take account of the effect of taxation.

A common modification found in the literature¹⁵ involves rewriting the standard Fisher equation in the form of $i = r + \pi/(1 - \tau)$, where $1 > \tau > 0$ is the tax rate. Hence, $di/d\pi = 1/(1 - \tau) + dr/d\pi$. Feldstein (1976) has argued on this basis that $di/d\pi > 1$ is likely if $dr/d\pi$ is small even if negative. An implicit assumption in this modification is, however, that savers demand an inflation adjustment in the nominal interest rate on the basis of the *after-tax* real interest rate, or, equivalently, that they have no money illusion. In other words, savers demand compensation for inflation but not for taxation. For a complete compensation of *both* inflation and taxation, the Fisher equation would have to be modified in the following manner: $i = (r + \pi)/(1 - \tau)$, that is, the inflation adjustment to the nominal interest rate is carried out on the basis of the *before-tax* real interest rate. The two modifications clearly could lead to very different welfare consequences of inflation. It is, of course, an empirical question as to which modification to the standard Fisher equation is the more relevant one.¹⁶ While as a modeling strategy this paper adopts the conventional modification to the Fisher equation in the presence of taxation, its implications for the analysis that follows will be explicitly noted.

¹³For simplicity, the term on the interaction between r and π has been omitted in the present discussion (but not in the model developed below).

¹⁴The real interest rate is lowered in the Mundell model because an increase in the expected inflation rate reduces the desired level of real money balances. This effect lowers wealth, which discourages consumption and increases savings. In contrast, the real interest rate effect in the Tobin model operates through a portfolio switch from real money balances to capital following a rise in expected inflation. The present paper shows just the reverse, i.e., $dr/d\pi > 0$, however, as inflation would increase the cost of capital and reduce investment.

¹⁵See, for example, Darby (1975), Feldstein (1976), and Tanzi (1976).

¹⁶Tanzi's (1980) investigation of interest rate behavior in the United States for the period from the 1950s through the mid-1970s suggests that the nominal interest compensation tended only to cover the effects of inflation but not of taxation, an outcome he termed fiscal illusion.

B. Model Structure

Production technology and factor accumulation

In period t , let L_t be the inelastically supplied labor force; M_t be the nominal money stock; p_t be the price level; $m_t \equiv M_t / (p_t L_t)$ be the per capita real money balances; k_t be the physical capital-labor ratio; and y_t be the per capita output. Assume that the technology of producing y_t requires both k_t and m_t as inputs in the form of

$$(1) \quad y_t = f(k_t, m_t),$$

where f satisfies the usual Inada conditions with respect to both k_t and m_t . Competitive capital markets ensure that capital's real rate of return, r_t , is equated with its marginal product:

$$(2) \quad r_t = \partial y_t / \partial k_t.$$

The stock of physical capital available in any period t is determined by *gross* savings in equity shares in the preceding period. In per capita terms, this can be written as

$$(3) \quad (1 + n) \cdot k_t = s_{t-1},$$

where s_{t-1} denotes the amount of per capita equity savings undertaken in period $t-1$, and n is the constant exogenous rate of growth of the labor force:

$$(4) \quad L_t = (1 + n) \cdot L_{t-1}.$$

The nominal money stock in each period is assumed to grow at the constant rate μ :

$$(5) \quad M_t = (1 + \mu) \cdot M_{t-1},$$

where μ is taken to be a policy variable.

Taxation and the Fisher equation

As noted earlier, this paper adopts the conventional modification to the standard Fisher equation to take into account of the effects of taxation on the nominal interest rate, i_t . In a discrete-period set-up, it would be of the form of

$$(6) \quad 1 + i_t(1 - \tau) = [1 + r_t(1 - \tau)] \cdot (1 + \pi_t),$$

where $\pi_t \equiv (p_t - p_{t-1})/p_{t-1}$ is the rate of inflation and τ is the proportional income tax rate. An important consequence of equation (6) is that savers are fully compensated for the *direct* effects of inflation (but not of taxation), i.e., there would be no inflationary distortion on the real rate of return to savings, except through the inflation's general equilibrium impact on r_t .

This is tantamount to assuming, as shown below, that the *direct* burden of inflation falls entirely on investors via its impact on the cost of capital.¹⁷

Consumer budget constraints

Each (identical) consumer lives for two periods, and there is no labor-leisure choice. When young, the individual is an entrepreneur, has access to the production technology as specified in equation (1), and produces output by holding real money balances as working capital and hiring physical capital, the latter being paid the prevailing nominal interest rate. When old, the individual is retired and lives on savings from the previous period. Savings carried forward by the young into the next period for consumption during retirement consist only of equity shares (real output not consumed and not taxed, and which becomes physical capital as given by equation (3)). Equity shares earn the prevailing nominal interest rate in the period in which they are redeemed. Real money balances in each period are entirely left for the young to be used in the production process; they are not available for consumption by the old.

Let c_t and e_t be the consumption of the consumer born in period t (henceforth consumer(t)) when young and when old, respectively (e_t takes place, therefore, in period $t+1$). The nominal budget constraint of consumer(t) when young is, therefore,

$$(7) \quad (1 - \tau) \cdot (p_t y_t - i_t p_{t-1} k_t) = p_t (c_t + s_t) + (M_t - M_{t-1}) / L_t$$

Note that consumer(t)'s total investment expenditure, i.e., the amount $p_{t-1} k_t$ is equal to the value (in terms of the price prevailing in period $t-1$) of the stock of physical capital owned by consumer($t-1$), who is currently old in period t . After deflating equation (7) by p_t and using equations (5) and (6), the real budget constraint of the young can be written as

$$(8) \quad \Omega_t = c_t + s_t$$

where

¹⁷To take account of alternative assumptions about the incidence of inflation, one could generalize the form of equation (6) to $[1 + i_t(1 - \tau_t\gamma)] = (1 + r_t)(1 + \pi_t)$, where $1 \geq \gamma \geq 0$ is a constant. In this form, the response of the nominal interest rate to taxation ranges from nil ($\gamma = 0$, in which case the *after-tax* real rate of return to savings is, for given r_t and π_t , reduced by the full effect of the tax) to complete ($\gamma = 1$, in which case i_t would rise to fully compensate for the tax burden, leaving the *after-tax* real rate of return to savings unaffected). In all cases where $1 > \gamma$, it can be shown that savers bear at least some of the *direct* burden of inflation, as a result of the *interaction* between the inflation and the tax rates (see Ebrill and Zee (1997) for a more detailed discussion). Equation (6) can be recovered from this generalized formulation by setting $\gamma_t = \pi_t / \{[1 + r_t(1 - \tau)](1 + \pi_t) - 1\}$, where γ_t is now a variable rather than a constant.

$$(9) \quad \Omega_t \equiv (1 - \tau)(y_t - r_t k_t) - \Pi_t k_t - \Psi m_t$$

is the present value of consumer(t)'s after-tax life-time real wealth *net of the expenditure on holding real money balances*; $\Psi \equiv \mu/(1 + \mu)$ is the price of m_t ; and $\Pi_t \equiv \pi_t/(1 + \pi_t)$. Hence, inflation distorts Ω_t by raising the cost of capital for the young. Ω_t is also distorted by the growth in the nominal money stock, as it affects the price of holding real money balances.

The nominal budget constraint of consumer(t) when old is

$$(10) \quad [1 + i_{t+1}(1 - \tau)]p_t s_t = p_{t+1} e_t$$

After deflating equation (10) by p_{t+1} and using equation (6), the real budget constraint of the old can be written as

$$(11) \quad s_t [1 + r_{t+1}(1 - \tau)] = e_t$$

Equation (11) confirms the point made earlier that the real rate of return to savings is not directly affected by inflation, on account of the inflationary compensation provided to savers under the assumed form of the Fisher equation (6). Substituting equation (11) into equation (8), the life-time real budget constraint of consumer(t) takes the familiar form of

$$(12) \quad \Omega_t = c_t + e_t / [1 + r_{t+1}(1 - \tau)].$$

Government budget constraints

Total nominal government outlay in period t comprises government consumption expenditure, while total nominal government receipts in the same period consists of income taxes paid by the currently young and the currently old, and newly printed nominal money stock. Hence, the nominal government budget constraint in per capita terms can be written as

$$(13) \quad p_t g_t = \tau(p_t y_t - i_t p_{t-1} k_t) + \tau i_t p_{t-1} s_{t-1} / (1 + n) + \Psi M_t / L_t$$

where g_t is per capita real government consumption that is assumed to be neither productive nor valued by the consumer. Deflating equation (13) by p_t and using equation (6), the government budget constraint as a share of output can be stated, after some rearranging, as

$$(14) \quad g^* = \tau + \Psi m_t / y_t$$

where $g^* \equiv g_t / y_t < 1$ is taken to be a policy variable.

Consumption and money demand functions

Consumer(t)'s life-time utility, u_t , is represented by a strictly-quasiconcave, twice-differentiable, increasing real-valued function v :

$$(15) \quad u_t = v(c_t, e_t).$$

Maximizing equation (15) with respect to c_t , e_t , and m_t subject to the life-time budget constraint as given by equation (12) and taking k_t , r_t , r_{t+1} , Π_t , Ψ , and τ as given, yields the following familiar first-order condition for an interior maximum:

$$(16) \quad v_c = v_e \cdot [1 + r_{t+1} \cdot (1 - \tau)],$$

where the subscripts on v denote partial derivatives. Together with equation (12), equation (16) yields consumer(t)'s consumption function when young as

$$(17) \quad c_t(\cdot) = c_t[\Omega_t, r_{t+1} \cdot (1 - \tau)].$$

Since m_t does not directly enter the utility function, the consumer's demand for real money balances is entirely governed by the usual profit-maximizing condition of a producer: m_t would be held up to the point where its (after-tax) marginal product is equated with its price. From the definition of Ω_t in identity (9), the first-order condition for profit maximization with respect to m_t is

$$(18) \quad \Psi = (1 - \tau) \cdot (\partial y_t / \partial m_t),$$

which gives the implicit function for the demand for real balances.

Intertemporal equilibrium

The intertemporal equilibrium for the entire economy is established when the capital market is cleared in each period. By substituting the capital accumulation equation (3) and the consumption function (17) into the real budget constraint of the young (equation (8)), this equilibrium path is given by

$$(19) \quad (1 + n) \cdot k_{t+1} = \Omega_t - c_t(\cdot).$$

Along this path, the government budget constraint (equation (14)) must, of course, also be satisfied to achieve a general equilibrium.

C. Steady State Analytics

The remaining analysis is focused on the steady state, in which all time-subscripted variables remain constant over time (except for the labor force and the nominal money stock, both of which grow at constant rates). The time subscript t can, therefore, be dropped without leading to confusion. As the ultimate objective of the analysis is to provide numerical estimates of the welfare cost of inflation, both the production function (1) and utility function (15) will now be assigned specific functional forms.

Specific production and consumption technologies

Assume that the production function (1) is of the Cobb-Douglas form:

$$(20) \quad y = k^\alpha \cdot m^\beta, \quad \alpha > 0, \beta > 0, 1 > \alpha + \beta > 0.$$

It then follows that the real rate of return to capital can be written as

$$(21) \quad r = \alpha \cdot y/k,$$

and the implicit demand for real balances takes the form of

$$(22) \quad m = (1 - \tau) \cdot \beta \cdot y / \Psi,$$

which has an unitary income elasticity (as is common). Note that equation (22) also provides that, for a given k , the partial elasticity of money demand with respect to Ψ is $1/(\beta - 1) < 0$, which has an absolute magnitude that varies directly with β .¹⁸

Let the utility function (15) be of the form

$$(23) \quad \begin{aligned} u &= \theta \cdot c^{1-1/\sigma} + (1 - \theta) \cdot e^{1-1/\sigma}, & \sigma > 1, 1 > \theta > 0, \\ &= \theta \cdot \ln c + (1 - \theta) \cdot \ln e, & \sigma = 1, 1 > \theta > 0, \end{aligned}$$

where σ is the intertemporal elasticity of substitution between current and future consumption. The sign restrictions on σ ensure that the interest elasticity of savings is nonnegative.¹⁹ With this specification of the utility function, the consumption function of the young is simply

$$(24) \quad c(\cdot) = \Omega / (1 + \delta \cdot \rho^{\sigma-1}),$$

where $\delta \equiv [(1 - \theta)/\theta]^\sigma$ and $\rho \equiv [1 + r \cdot (1 - \tau)]$.

Budgetary finance

It has been noted earlier that the rate of growth of the nominal money stock, μ , is a policy variable which determines in part the amount of seignorage for the budget by setting the price

¹⁸Utilizing the definitions of Π and Ψ , and anticipating equation (25) below, it can easily be shown that the partial elasticity of money demand with respect to the inflation rate, ϵ_π^m , is simply $\epsilon_\pi^m = \Pi / \{(\beta - 1) \cdot [n + \pi \cdot (1 + n)]\} < 0$, whose absolute magnitude varies with π itself.

¹⁹It can be shown that σ and the elasticity of savings with respect to the after-tax real rate of return, ϵ_{p-1}^s , are related according to $\epsilon_{p-1}^s = (\rho - 1) \cdot (\sigma - 1) / (\rho + \delta \cdot \rho^\sigma) \geq 0$, with ρ and δ defined immediately below. Hence, if $\sigma = 1$, then $\epsilon_{p-1}^s = 0$.

of holding real money balances. In the steady state, however, with per capita real money balances remaining constant but the nominal money stock growing at the constant rate μ , it must follow that the rate of inflation is determined by

$$(25) \quad (1 + \pi) = (1 + \mu)/(1 + n).$$

Hence, controlling μ is equivalent to controlling π , and the latter will henceforth be treated as a policy variable.

Substituting the money demand function (22) into the government budget constraint (14), the tax rate that would be needed to finance the exogenously given government expenditure as a share of output is

$$(26) \quad \tau = (g^* - \beta)/(1 - \beta),$$

which is seen to be dependent on the parameter β of the money demand function. A tax rate $\tau > 0$ is necessary only when $g^* > \beta$. If $g^* = \beta$, the entire budgetary outlay could be financed from seignorage and no income tax would be required. Equation (26) indicates that the amount of seignorage is a constant share of output, an outcome that follows directly from the unitary income elasticity property of the money demand function. This constancy suggests that the welfare cost of inflation in the present model is possibly overstated, since a higher inflation would not confer revenue benefits on the budget to allow a lowering of other distortionary taxes²⁰. From equation (22), this share is simply $(1 - \tau)\beta$, which is independent of the inflation rate. Hence, in this model the government budget plays a role in determining the welfare cost of inflation, not in terms of a *direct* trade-off between seignorage and the income tax, but in terms of the required tax rate to finance budgetary expenditure that *exceeds* seignorage as a share of output.

General equilibrium impact of inflation

Given the consumption function (24), the intertemporal equilibrium condition (19) can be stated, after a slight rearrangement, as

$$(27) \quad \Omega/k = (1 + n)/\phi,$$

where

$$(28) \quad \phi \equiv \delta \cdot p^{\sigma-1} / (1 + \delta \cdot p^{\sigma-1}) < 1,$$

²⁰See Helpman and Sadka (1979) for a detailed discussion of budgetary finance where such revenue substitutions are possible.

which simplifies to a constant $\phi \equiv (1 - \theta) < 1$ if $\sigma = 1$. Substituting capital's marginal product condition (equation (21)) and the money demand function (22) into the definition of Ω as given by identity (9), it can be shown that the ratio Ω/k is alternatively given by

$$(29) \quad \Omega/k = (1 - \tau) \cdot (1 - \alpha - \beta) \cdot r/\alpha - \Pi.$$

Hence, by equating equation (27) with equation (29), a general equilibrium relationship between the inflation rate and the real rate of return to capital can be derived:

$$(30) \quad \Pi = (1 - \tau) \cdot (1 - \alpha - \beta) \cdot r/\alpha - (1 + n)/\phi.$$

It is straightforward to show that a higher rate of inflation raises capital's real rate of return:²¹

$$(31) \quad \frac{dr}{d\Pi} = \frac{\alpha \cdot \delta \cdot \rho^\sigma}{(1 - \tau) \cdot [(1 - \alpha - \beta) \cdot \delta \cdot \rho^\sigma + \alpha \cdot (1 + n) \cdot (\sigma - 1)]} > 0.$$

This result comes about in the present model because a higher inflation raises the cost of capital and reduces the consumer's life-time income, which in turn depresses capital accumulation and drives up capital's real rate of return—although at a decreasing rate, i.e., $d^2r/d\Pi^2 < 0$, as the higher real rate of return generates some stimulating effect on savings. As noted earlier, this is just the reverse of the outcomes obtained by Mundell (1963) and Tobin (1965). For the case of the Cobb-Douglas utility function (i.e., $\sigma = 1$), equation (31) simplifies substantially to $dr/d\Pi = \alpha/[(1 - \tau) \cdot (1 - \alpha - \beta)] > 0$, and the general equilibrium impact of inflation is no longer dependent on the consumption side of the model, as the interest elasticity of savings in this case is zero.

Welfare impact of inflation

An illuminating way to evaluate the welfare impact of inflation in the steady state is to substitute equations (12) and (17) into the utility function (15) to obtain

$$(32) \quad u = v\{c(\cdot), [\Omega - c(\cdot)] \cdot \rho\}.$$

Totally differentiating equation (22) yields

$$(33) \quad \begin{aligned} du &= v_c \cdot dc(\cdot) + v_\rho \cdot \{\rho \cdot d[\Omega - c(\cdot)] + [\Omega - c(\cdot)] \cdot (1 - \tau) \cdot dr\} \\ &= v_c \cdot d\Omega + v_c \cdot \{[\Omega - c(\cdot)] \cdot (1 - \tau) / \rho\} \cdot dr, \end{aligned}$$

where the second equality follows from equation (16). Equation (33) indicates that the welfare impact comprises the sum of two distinct terms: the first measures the change in welfare

²¹Note that $d\Pi = d\pi/(1 + \pi)^2$ and $dr = d\rho/(1 - \tau)$.

arising from a change in life-time income ($d\Omega$), and the second measures the change in welfare from a change in the real rate of return (dr) on a given amount of savings $[\Omega - c(\cdot)]$. It is instructive to view the former from the perspective of an investor and the latter from that of a saver. From equation (29), it can clearly be seen that a higher rate of inflation would lower Ω by raising the cost of capital, both directly and indirectly by raising r through general equilibrium effects as indicated by equation (31). Hence, higher inflation unambiguously lowers the investor's welfare. The saver's welfare, however, is increased by a rise in the inflation rate, as it raises the reward to a given amount of savings. Since the consumer in this model takes on the dual role of the investor and the saver, the overall welfare impact of inflation is thus ambiguous. It is shown in the Appendix, however, that for the assumed production function (20) and the utility function (23), the welfare gain to the saver is not sufficient to compensate for the welfare loss to the investor. Hence, in this model, *inflation imposes an unambiguous welfare cost*, i.e., $du/d\pi < 0$. A remarkable aspect of this result is that it is not dependent on whether or not the economy is dynamically efficient, i.e., whether the economy's equilibrium occurs to the left ($\rho - 1 > n$) or right ($n > \rho - 1$) of the golden rule.

III. NUMERICAL ESTIMATES

The theoretical model laid out in the preceding section has five structural parameters: α , β , θ , σ , and n ; and two policy parameters: g^* and π . For given feasible values of these parameters, equation (30) provides the general equilibrium solution to r in the steady state. Once this solution is obtained, all other endogenous variables can be solved in a straightforward manner. The welfare cost of inflation can then be deduced by computing the change in the utility level as a result of a given change in the inflation rate, holding the values of all other parameters constant. The sensitivity of the welfare cost so computed to alternative parameter values can also be easily calculated.

A convenient way to translate a change in the utility level into an equivalent change in the level of income is to employ the Hicksian concept of compensating variation (CV), and is defined in the present context as follows.²² Given the young's consumption function (24) and noting that the old's consumption function is $e = c \cdot \delta \cdot p^\sigma$, the life-time level of utility can be expressed as a function of after-tax real rate of return and life-time income:

$$(34) \quad u = \theta \cdot (1 + \delta \cdot p^{\sigma-1})^{1/\sigma} \cdot \Omega^{1-1/\sigma}$$

²²Alternatively, one could employ the Hicksian concept of equivalent variation (EV) to translate the utility change into equivalent income change, with $EV \geq CV$. The quantitative difference between the EV and CV measures of the welfare cost of inflation in the present model turns out to be insignificant for all parameter values used in the numerical calculations described below. The EV measure is, therefore, not reported.

Using the superscript "0" and "1" on variables to denote, respectively, their values before and after the change in the inflation rate from π^0 to π^1 , it follows from earlier discussions (and shown in the Appendix) that $u^1 > u^0$ as $\pi^0 > \pi^1$ (and vice versa). For the case of a lowering of the inflation rate, the CV is defined by

$$(35) \quad [1 + \delta \cdot (\rho^1)^{\sigma-1}]^{1/\sigma} \cdot (\Omega^1 - CV)^{1-1/\sigma} = [1 + \delta \cdot (\rho^0)^{\sigma-1}]^{1/\sigma} \cdot (\Omega^0)^{1-1/\sigma}.$$

Hence, the CV in equation (35) measures, in equivalent income terms, the welfare gain from lowering the inflation rate from π^0 to π^1 . If $\sigma = 1$, then equation (35) simplifies to

$$(36) \quad (1 - \theta) \cdot \ln \rho^1 + \ln(\Omega^1 - CV) = (1 - \theta) \cdot \ln \rho^0 + \ln \Omega^0.$$

The Table reports numerical estimates of steady-state welfare gains, as measured by the CV and expressed as percentages of output per year, of lowering the inflation rate from 4 percent (a level that has been achieved or bettered in recent years by most industrialized countries and a number of Asian economies before the current crisis) to 2 percent (taken to represent approximately price stability, on account of the upwards bias in the CPI) under different combinations of values for β , σ , and g^* chosen to bracket their reasonable ranges. The elasticities of savings (with respect to the after-tax real rate of return) and of money demand (with respect to the rate of inflation) implied by these combinations are also indicated. All calculations are based on the following fixed values of the other parameters: $\alpha = 0.3$; $\theta = 0.7$; and $n = 0.02$ per year. Each period in the model is taken to represent 30 years.

The overall picture conveyed by the numerical estimates is that the welfare gains are very modest, ranging from 0.08 percent to 0.17 percent of output per year. As expected, the gains increase with β in all cases. The impact of σ on welfare is not monotonic and depends on g^* (and hence τ): the gains increase as σ is raised from 1 to 10 with $g^* = 0.3$ (implying $\tau = 0.22 \sim 0.29$), but decrease with $g^* = 0.5$ (implying $\tau = 0.44 \sim 0.49$). A further increase of σ from 10 to 25 produces no discernible welfare impact, however, indicating that the inflationary impact on welfare becomes rather insensitive to both β and g^* when the savings elasticity is relatively high.

IV. CONCLUDING REMARKS

Employing a general equilibrium framework with a well-articulated structure of savings and investment, of the demand for real money balances, and of government budgetary finance, this paper has shown that the case for further lowering inflation from a already low level (say, 4 percent) to a level approximating price stability (say, 2 percent) is not compelling from a welfare perspective. While inflation unambiguously imposes a welfare cost, under reasonable parameter values the welfare gains from eliminating low inflation are very modest—less than 0.2 percent of output per year under all cases considered. Even this is likely to be an over-estimate, since it is based on a model that does not incorporate a *direct* trade-off between seignorage and other distortionary taxes in the government budget.

Table 1. Steady-State Welfare Gain of Lowering Inflation from 4 Percent to 2 Percent 1/
(In percent of output per year)

	β	
	0.01 ($\epsilon_{\pi}^m = -0.56$) 2/	0.10 ($\epsilon_{\pi}^m = -0.62$) 2/
$\sigma = 1$ ($\epsilon_{p-1}^s = 0$)		
$g^* = 0.3$	0.08 ($\tau = 0.29$)	0.14 ($\tau = 0.22$)
$g^* = 0.5$	0.10 ($\tau = 0.49$)	0.17 ($\tau = 0.44$)
$\sigma = 10$ ($\epsilon_{p-1}^s = 0.24$) 3/		
$g^* = 0.3$	0.12 ($\tau = 0.29$)	0.16 ($\tau = 0.22$)
$g^* = 0.5$	0.08 ($\tau = 0.49$)	0.12 ($\tau = 0.44$)
$\sigma = 25$ ($\epsilon_{p-1}^s = 0.61$) 3/		
$g^* = 0.3$	0.12 ($\tau = 0.29$)	0.17 ($\tau = 0.22$)
$g^* = 0.5$	0.09 ($\tau = 0.49$)	0.12 ($\tau = 0.44$)

1/ Welfare gain is measured by the *CV* as defined. All calculations are based on the following parameter values: $\alpha = 0.3$; $\theta = 0.7$; and $n = 0.02$ per year. Each period in the theoretical model is assumed to be 30 years.

2/ Since the elasticity of money demand with respect to inflation depends on the inflation rate at which it is evaluated, the reported value of ϵ_{π}^m (for a given β) is its average value under inflation rates of 2 percent and 4 percent.

3/ Since the interest elasticity of savings depends on the point where it is evaluated, the reported value of ϵ_{p-1}^s (for a given σ) is its average value under the different combinations of β and g^* .

To demonstrate that the impact of inflation on welfare is unambiguously negative, divide equation (33) through by $(v_c \cdot k)$ to get, using equation (19) in the steady state,

$$(A.1) \quad du/(v_c \cdot k) = d\Omega/k + (1 + n) \cdot (1 - \tau) \cdot dr/\rho.$$

First, note that

$$(A.2) \quad d\Omega/k = d(\Omega/k) + (\Omega/k) \cdot dk/k,$$

and, from equation (27),

$$(A.3) \quad d(\Omega/k) = - (1 + n) \cdot (\sigma - 1) \cdot (1 - \tau) \cdot dr/(\delta \cdot \rho^\sigma).$$

Next, to evaluate dk/k in equation (A.2), obtain from the production function (20)

$$(A.4) \quad dy/y = \alpha \cdot dk/k + \beta \cdot dm/m,$$

where, from the money demand function (22),

$$(A.5) \quad dm/m = dy/y - d\Psi/\Psi.$$

Substituting equation (A.5) into equation (A.4) yields

$$(A.6) \quad dy/y = [\alpha/(1 - \beta)] \cdot dk/k - [\beta/(1 - \beta)] \cdot d\Psi/\Psi.$$

From equation (21), however,

$$(A.7) \quad dr/r = dy/y - dk/k.$$

Hence, substituting equation (A.6) into equation (A.7) finally gives

$$(A.8) \quad dk/k = - [\beta/(1 - \alpha - \beta)] \cdot d\Psi/\Psi - [(1 - \beta)/(1 - \alpha - \beta)] \cdot dr/r.$$

Equations (A.2), (A.3) and (A.8) can be substituted into equation (A.1) to get

$$(A.9) \quad du/(v_c \cdot k) = (1 + n) \cdot Z \cdot dr - \beta/[(1 - \alpha - \beta) \cdot \phi \cdot \Psi] \cdot d\Pi,$$

where use is made of equation (27) and of the fact that $d\Psi = d\Pi/(1 + n)$, and

$$(A.10) \quad \begin{aligned} Z &\equiv (1 - \tau)/\rho - (1 - \beta)/[(1 - \alpha - \beta) \cdot \phi \cdot r] - (1 - \tau) \cdot (\sigma - 1)/(\delta \cdot \rho^\sigma) \\ &\equiv Z_1 - Z_2 - Z_3. \end{aligned}$$

Since the coefficient of $d\Pi$ in equation (A.9) is negative, determining the sign of du requires determining the sign of Z . As given by identity (A.10), Z is comprised of three terms, each of which is positive by itself. Using the general equilibrium solution given by equation (30) to expand the denominators of Z_1 and Z_2 , it can be shown that, after some algebraic manipulation,

$$(A.11) \quad Z_1 - Z_2 = \frac{(1 - \tau) \cdot (1 - \alpha - \beta) \cdot \phi}{(1 - \alpha - \beta) \cdot \phi + \alpha \cdot (1 + n) + \alpha \cdot \phi \cdot \Pi} - \frac{(1 - \tau) \cdot (1 - \beta)}{\alpha \cdot (1 + n) + \alpha \cdot \phi \cdot \Pi}.$$

Since $1 > \phi$, the numerator of Z_1 is smaller than that of Z_2 . The denominator of Z_1 is, however, larger than that of Z_2 . Hence, $Z_1 < Z_2$, and, therefore, $Z < 0$. It immediately follows that $du/d\Pi < 0$, i.e., higher inflation leads to lower welfare.

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