WP/06/224

IMF Working Paper

A Debt Overhang Model for Low-Income Countries: Implications for Debt Relief

Junko Koeda

INTERNATIONAL MONETARY FUND

 WP/06/224

IMF Working Paper

Finance Department

A Debt Overhang Model for Low-Income Countries: Implications for Debt Relief

Prepared by Junko Koeda[1]

Authorized for distribution by Jianhai Lin

October 2006

Abstract

The paper presents a theoretical model to explain how debt overhang is generated in low-income countries and discusses its implications for debt relief. The paper indicates that the extent of debt overhang, and the effectiveness of debt relief, would depend on a recipient country's initial economic conditions and level of total factor productivity.

JEL Classification Numbers: E21, F34, F35, F43, O16, O21

Keywords: debt overhang, debt relief, participation constraint, low-income countries, dynamic programming

Author's E-Mail Address: jkoeda@imf.org

[1] The model was developed during my PhD program at UCLA and thus I would like to thank Carlos Vegh and other UCLA faculty and colleagues for helpful comments. I thank Atish Rex Ghosh for his introduction to the issues of debt sustainability in low-income countries during my internship at the IMF. I am grateful to Jianhai Lin, Andrew Tweedie, the reviewers in the African, Policy Development and Review, and Research Departments, Yasemin Bal Gunduz, and participants in an informal seminar at the Finance Department for their useful discussion and comments. All remaining errors are my own.

Contents Page

I. Introduction

Debt overhang—the relationship between heavy debt and low growth—has been a fundamental concept in the literature that supports debt relief. Unfortunately, the majority of existing theoretical models are designed for middle-income countries that suffer from heavy debt burdens under non-concessional private debt. These models may not apply to low-income country (LIC) environments where external loans are highly concessional and comprise a large share of debt.

This paper formulates Cohen and Sachs's (1986) sovereign debt model as a concessional lending problem and numerically demonstrates how a link between large debt and low growth may be *generated* in LICs—in the existing debt overhang models, no explanation is provided as to why the debtor country has excess debt in the first place.[2] The model focuses on an incentive problem of a cutoff, an income level above which the country loses its eligibility for aid assistance. Such a cutoff exists, for example, with multilateral concessional lending such as loans from the World Bank's International Development Association (IDA) and IMF loans under the Poverty Reduction and Growth Facility (PRGF). [3]

The paper shows that an LIC—in the absence of effective measures to raise the country's total factor productivity (TFP)—may have an incentive to accumulate a significant amount of concessional debt and allocate resources to consumption rather than investment. The country manages its large debt at a very low cost by stagnating around the cutoff and becoming permanently aid-dependent. More than just a theoretical possibility, the paper provides empirical evidence of growth stagnation around the cutoff.

There are two types of agents in the model: an official creditor and an LIC debtor. The creditor lends at a fixed subsidized interest rate if the debtor country lies below or at the cutoff. Above the cutoff, the creditor lends at the world interest rate. The creditor can commit itself to the contracts but the debtor country cannot. The creditor thus imposes a participation constraint to prevent the debtor country from defaulting. Imposing a participation constraint is equivalent to imposing an endogenous debt ceiling constraint. The LIC debtor maximizes the representative agent's welfare subject to this lending rule. Some researchers argue that the focus should be on "bad" governments who care about

---

[2] For studies examining causes for debt accumulation in low-income countries, see for example, Easterly (1999) and Helbling, Mody, and Sahay (2004).

[3] Note, however, in practice there are other criteria, such as the country's creditworthiness and performance, that affect the decision of IDA loan eligibility, in addition to per capita income eligibility criterion. This means that there may be some countries below the cutoff that are disqualified for IDA loans or some which are above the cutoff but are qualified for IDA loans or both.

their own welfare, rather than that of households. An important point of this paper, however, is that even with a benevolent government, a debt overhang problem may occur.

Lastly, the paper discusses the model's implications for debt relief. In particular, the model shows that a one-time stock treatment can be effective depending on the country's initial conditions and TFP. Elaborating on this result can provide useful insight for the recent one-time debt relief stock treatment, known as the Multilateral Debt Relief Initiative (MDRI)—a 100 percent debt stock cancellation by the IMF, the IDA, and the African Development Bank for a group of low-income countries—which aims to free up resources to help countries achieve the United Nation's Millennium Development Goals.

The structure of the paper is as follows. Section II discusses the applicability of the existing theoretical literature to LIC environments. Section III documents the empirical motivation. Section IV presents the model and Section V discusses its implications for the effectiveness of debt relief. And Section VI concludes.

II. Theoretical Literature

This paper is related to the sovereign debt and debt overhang literature. First, the model endogenizes debt sustainability[4] by incorporating enforcement mechanisms—an important topic in the sovereign debt literature. There are two main types of models that explain enforcement mechanisms in the literature: reputation and sanction models. In reputation models, debtors find it painful to be excluded from future credit markets. One classic reputation model is that of Eaton and Gersovitz (1981). They assume a concave utility function so that the country has an incentive to smooth consumption over time. The output path takes two values in turn, high and low. In this environment, the country does not want to be excluded from the international capital markets, since in financial autarky it cannot smooth consumption. Bulow and Rogoff (1989), on the other hand, show conditions under which reputation does not provide sufficient repayment incentives. Other aspects of reputation have been studied by, for example, Atkeson (1991) and Cole and Kehoe (1998).

In sanction models, debtors are penalized upon default. A common way of introducing the default penalty is to assume a loss of a fraction of output upon default. This can be, for example, the loss of access to short-term trade credits. Some researchers argue that sanction models fail to consider possible renegotiation processes and analyze the processes in the context of dynamic bargaining games.[5] Yet, debt renegotiation itself can be costly—Rose

[4] For more details on the debt sustainability framework for low-income countries, see IMF and the World Bank (2003), (2004), (2005), and (2006).

[5] For a summary of debt renegotiation literature, see for example, Eaton and Fernandez (1995) and Yue (2005).

(2005) finds that debt renegotiation is associated with an economically and statistically significant decline in bilateral trade between a debtor and its creditors.

Cohen and Sachs (1986) incorporate components of both reputation and sanction models into their enforcement mechanisms. Their model is a neoclassical growth model where the initial capital stock lies below the steady state. The country can borrow from abroad at the given world interest rate. As long as the country's capital remains scarce, the world interest rate is lower than the initial marginal product of capital so the country finds it painful to be excluded from external borrowing. The marginal product of capital decreases as the country accumulates capital eventually converging to the world interest rate. In the steady state, the country's default cost is merely the one that comes from sanctions. An important contribution of Cohen and Sachs is that they analyze sovereign debt dynamics in the context of growth. Thus, their model may be useful when thinking about development problems of an LIC with good growth prospects.

The theoretical literature on debt overhang that explains the relationship between large debt and low growth in LICs lags behind the empirical literature. The existing debt overhang models typically consider the case where initial debt is so large that the country would be insolvent unless it receives some form of debt relief (Krugman (1988) and Sachs (1989)). In these models, excess debt reduces the supply of new loans by scaring off creditors; it also reduces the demand for new investment by acting like a distortionary tax where a fraction of future output is assumed to be used for repayments of the initial debt. This discourages domestic investment, resulting in low growth.

However, as some key features of LICs are not incorporated into these models, their applicability to this context is questionable. In particular, the majority of loans to LICs is highly concessional and is provided by official creditors who are neither profit maximizers nor risk-neutral. This may generate a unique lending pattern—for example, contrary to the existing models, large debt may not discourage new official lending.

The model presented below formulates Cohen and Sachs's (1986) model as a concessional lending problem. It takes into account some specific LIC characteristics by considering the case where an LIC debtor has no access to foreign private loans but has access to subsidized loans provided by a benevolent creditor.

III. Empirical Motivation

The motivation of this work begins with empirical documentation that shows that there is some economic stagnation around the cutoff. I run growth regressions using an unbalanced panel of 94 countries, of which 33 are low-income countries.[6] The data set are taken from the Summers-Heston data set (version 6.1), the World Bank's World Development Indicators, and the Barro-Lee data set.

As for data on the cutoff, I use the operational cutoff, which was formally recognized by IDA donors in IDA8 in 1987. Prior to this date, a higher cutoff, known as the historical cutoff which was initially set equal to $250 in 1964, was used for the IDA cutoff. The operational cutoff was introduced in the early 1980s due to the limited availability of IDA resources and the attention to poor performance in low-income countries. Both cutoffs are updated annually according to the world inflation rate using the SDR deflator.[7]

The dependent variable is the percentage annual growth rate of real GDP per capita. The explanatory variables are those typically included in a standard growth regression: the percentage of population growth (GPO), the percentage investment share of real GDP per capita (INV), the initial secondary schooling attained as the percentage of the total population in 1985[8] (INIT_EDU), and the initial level of real GDP per capita in 1988 (INIT_RGDP). In addition to these variables, I include the variable of interest, a measure of proximity to the cutoff in the form of a Bartlett kernel:

$$PROX_{it} = \begin{cases} 1 - |z_{it}| & for \quad |z_{it}| \le 1 \\ 0 & for \quad |z_{it}| > 1 \end{cases} \qquad where \quad z_{it} = \frac{\ln y_{it} - \ln \bar{y}_t}{\ln(1+b)}$$

where y_{it} is country i's GNI per capita in year t, $\bar{y}_t$ is the cutoff in year t, and b is a scaling factor which controls the bandwidth of the kernel. Note that a negative coefficient for PROX implies that there is a negative relationship between the country's growth rate and its proximity to the cutoff. The scaling factor, b, is set equal to 1/2, but I obtain similar results in the cases where b = 1/3 and b = 1/4. Appendix B-1 reports the distribution of PROX and partial regression of the growth rate on PROX.

Table 1 shows that the estimation results based on a pooled sample for 1988-2000. The OLS coefficient for PROX is negative and is statistically significant at the 5 percent level. To take into account endogeneity of PROX, INV, and INIT_RGDP, I also run two-stage least square (2SLS) regressions. Here one-year lags of these variables are used as the instruments for PROX and INV and the log of real GDP per capita in 1985 is used as the instrument for INIT_RGDP. The corresponding first-stage regressions are reported in

---

[6] I exclude observations on Rwanda for 1994 from the sample where Cook's distance, leverage, and studentized residuals exceed the conventional cutoffs. For conventional cutoffs and methodology, see UCLA Academic Technology Services' Stata web books, Regression with Stata, Chapter 2 , 2.1 at http://www.ats.ucla.edu/stat/.

[7] See World Bank (2001) for a detailed description of IDA eligibility criteria.

[8] The corresponding 1988 data are not available, thus I use the 1985 data.

Appendix B-2. The coefficient of PROX remains negative and significant at 5 percent.[9] When the sample is restricted to only those countries that lie below the cutoff, the significance level improves. This empirical evidence is consistent with the paper's theoretical result—the existence of a cutoff could result in economic stagnation at or around it[10].

Table 1: Growth Regressions 1988-2000

Variables	OLS	2SLS	
	All Countries	All Countries	Countries Below the Cutoff
PROX	-1.73**	-1.38*	-3.97**
	(0.70)	(0.76)	(1.96)
GPO	-0.90***	-0.91***	-1.13***
	(0.14)	(0.14)	(0.29)
INV	0.11***	0.14***	0.17***
	(0.02)	(0.02)	(0.06)
INIT_EDU	-5.32e-03	-1.54e-03	0.05
	(0.01)	(0.01)	(0.06)
INIT_RGDP	-0.64***	-0.88***	-1.23
	(0.24)	(0.25)	(1.04)

*** significantly different from 0 at the 1 percent level
** significantly different from 0 at the 5 percent level
* significantly different from 0 at the 10 percent level
Note: standard errors are in parentheses

IV. THE MODEL

Official creditors typically fix their concessional interest rates; for example, the rates of the World Bank's IDA and the IMF's PRGF are 0.75 percent[11] and 0.5 percent, respectively. I

[9] This significance level corresponds to the one-sided test.

[10] I also carried out regressions using specifications similar to the basic regression discussed in Chapter 12 of Barro and Sala-i-Martin (2004). The data I used here contains 79 countries, 13 of which are low-income countries. The dependent variable is the average real GDP per capita growth for 1990-2000. The explanatory variables are similar to those of Barro and Sala-i-Martin (2004) plus the average of PROX for 1990-2000. The 2SLS coefficient for the average of PROX becomes negative and insignificant but the t-values of all other regressors are considerably lower than in the Barro and Sala-i-Martin (2004, Table 12.3). This is probably because the sample size is only 79—1/3 less than that of Barro and Sala-i-Martin (2004) as the period covered (1990-2000) is much shorter.

[11] More precisely, this is the service charge that the World Bank currently imposes on the credits.

thus consider the following concessional lending rule: the lender who has full access to the world financial markets loans out funds at a fixed subsidized interest rate $(\bar{r})$ as long as the country's GNI per capita (y) is below the cutoff $(\bar{y})$. The interest rates for concessional lending are thus set according to the following rule:

$$\tilde{r}_{t+1} = \begin{cases} \bar{r} & if\ y_t \leq \bar{y} \\ r & otherwise, \end{cases} \tag{1}$$

where r is the world interest rate. The borrower country is assumed to have no access to foreign private financing—in practice, the majority of loans to LICs are offered by official lenders. In addition, a participation constraint is imposed in order to motivate the borrower to adhere to the contract.[12] With the constraint, the value function under repayment is required to be greater than or equal to the value function under default. The borrower country maximizes the following problem:

$$\max_{\{c_t, k_{t+1}, D_{t+1}\}} \sum_{t=1}^{\infty} \beta^{t-1} u(c_t) \tag{2}$$

subject to: (3)

$$v^D(k_t) \leq u(c_t) + \beta \sum_{j=1}^{\infty} \beta^{j-1} u(c_{t+j}), \quad \forall t$$

$$c_t = f(k_t) - x_t + D_{t+1} - (1+\tilde{r}_t) D_t, \tag{4}$$

$$k_{t+1} = (1-\delta) k_t + x_t, \tag{5}$$

k_1 and D_1 are given, (6)

$\tilde{r}_t$ follows the rule given by (1), (7)

where c, x, k, and D denote consumption, investment, capital, and concessional debt respectively. β is the discount factor where $r \equiv 1/\beta - 1$ is assumed in order for consumption in the steady state to be flat. The participation constraint is given by (3). The LIC's flow budget constraint is given by (4). The production function is given by $y_t = f(k_t)$. The transition equation for capital is given by (5), where δ is the rate of capital depreciation. The value function under default, $v^D(k)$, is the value function in autarky with penalties for violating the participation constraint:

$$v^D(k) = \max_{k'} \left\{ u\big((1-\lambda) f(k) - k' + (1-\delta)k\big) + \beta v^D(k') \right\}, \tag{8}$$

where and λ is the fraction of output lost. I assume that such a violation incurs two types of costs: the exclusion of the violator from future concessional lending and the loss of a fraction of the violator's output. I also assume that when the participation constraint is binding, the LIC adheres to the borrowing contract.

[12] The cases with no participation constraint are discussed in Appendix A.

In order to solve this sequential problem, i.e., eq. (2) subject to eqs. (3) -(7), define X_t as the repayment obligation in period t where $X_t = (1+\tilde{r}_t)D_t$. In each period, the borrower country compares the value function under repayment, $v^R(k, X)$, with that under default, $v^D(k)$. When $v^R(k, X) \geq v^D(k)$ the country repays, otherwise it defaults. The value function under repayment, $v^R(.,.)$, is given by:

$$v^R(k, X) = \max_{k', X'} \left\{ u\left(f(k) - k' + (1-\delta)k + \frac{X'}{(1+\tilde{r}(k))} - X \right) + \beta v^R(k', X') \right\}, \tag{9}$$

subject to $v^R(k', X') \geq v^D(k')$, (10)

where $v^R(.,.)$ is increasing in k and is decreasing in X. I express the recursive equation with X rather than D, otherwise the interest rate rule $\tilde{r}(.)$, would be a function of the previous period's capital (call this k_{-1}) and thus k_{-1} would needs to be treated as an additional state variable.

To make a connection with Cohen and Sachs's model (1986), the participation constraint can be replaced with a debt capacity function $h(k)$, which is defined implicitly by $v^R(k, h) = v^D(k)$, where $\partial v^R(k, X)/\partial X$ is strictly negative. In other words, given k, $h(k)$ is uniquely determined and thus the case where $h(k)$ is backward bending in k can be excluded. Thus the debt capacity function is well-defined. The original value function under repayment can be rewritten as:

$$v^R(k, X) = \max_{k', X'} \left\{ u\left(f(k) - k' + (1-\delta)k + \min\left\{ \frac{X'}{(1+\tilde{r}(k))}, \frac{h(k')}{(1+\tilde{r}(k))} \right\} - X \right) + \beta v^R(k', X') \right\}. \tag{11}$$

This formulation is the same as that of Cohen and Sachs (1986)[13] except that in this paper I numerically derive the value functions and the implied debt capacity function using the value function iteration method. I also extend their model to analyze the dynamics of concessional loans to low-income countries.

Since one cannot solve this problem analytically unless the participation constraint is absent, I solve it numerically using the value function iteration method. I specify the functional forms of the utility and production functions as $u(c) = c^{1-1/\sigma}/(1-1/\sigma)$ and $f(k) = Ak^{\eta}$. I set η = 1/3 and use Ostry and Reinhart's (1992) calibration results for African countries for the values of the elasticity of intertemporal substitution (σ = 0.451) and the discount factor (β = 0.945). I set the rate of capital depreciation (δ) at 0.1, the fraction of output lost upon

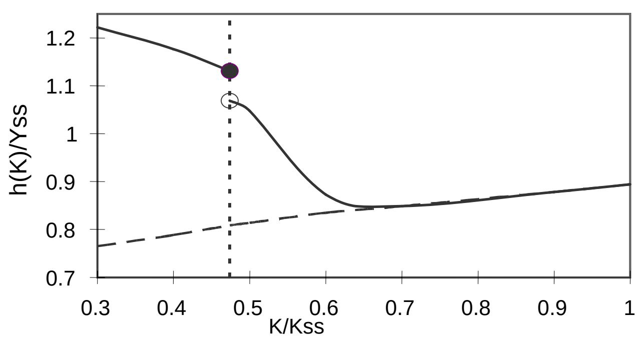
[13] An extension of Cohen and Sachs (1986) can be seen in Borensztein and Ghosh (1989).

default (λ) at 0.05, and the world interest rate (r) at $1/\beta - 1 = 0.0582$, and normalize the level of TFP (A) at 10. I set the concessional interest rate ($\bar{r}$) at 1 percent and fix the cutoff level ($\bar{y}$) at 0.7796 of steady state output (y_{ss}).[14] Note that the numerical value for the donor's budget or the implicit cost of the concessional lending (α) is given by:

$$\alpha \equiv \sum_{t=2}^{\infty} \left(\frac{1}{1+r} \right)^{t-1} (r - \tilde{r}_t) D_t .$$

Consider the benchmark economy with initial income (y_1) equal to 90 percent of the cutoff, or 70 percent (precisely 70.47 percent) of steady state output (y_{ss}), and an initial debt-output ratio (D_1 / y_1) of 1. The results from the numerical solution are summarized in Figures 1 and 2. Here, regardless of the starting function, I obtain the same fixed point in the functional space. To better understand the dynamics of concessional lending, these results are displayed along with those for *non-concessional* lending. The only difference between these two loans is the interest rate level: for all t under non-concessional loans $\tilde{r} = r$.

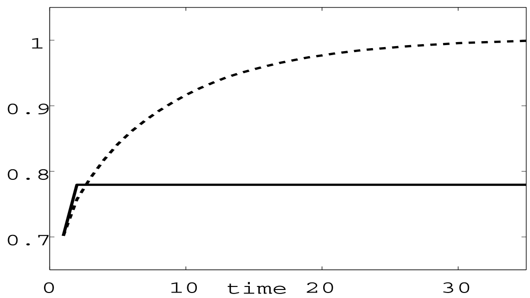
Figure 1: Endogenous Debt Ceilings (the benchmark case)



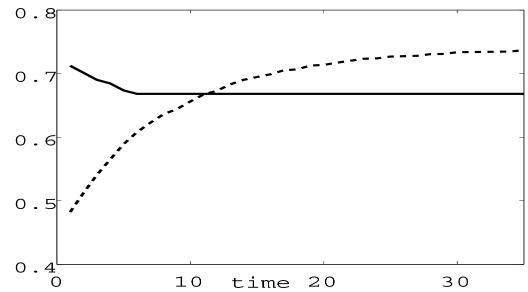
[14] This number (0.7796) is based on two additional assumptions. First, I introduce a TFP difference between the United States and LICs. A cursory glance at the TFP ratios of forty LICs relative to the United States between 1960 and 2000 shows that about one-fourth of LICs have TFP levels that are stable and are less than one-third of the U.S. level. I thus assume that the TFP ratio of LIC to the Unite States (A / A_{US}) is one-third. Second, I believe it is reasonable to set y as a percentage of the U.S. steady state output (y_{US}). I set this percentage at 15 percent because in the data the purchasing-power-parity-adjusted real outputs per capita in most lower-middle income countries are above this level. In this way, aid schemes used here can be interpreted as those that restrict eligibility for concessional loans to LICs only. Given these assumptions and the parameter values (σ, η, β, r and δ), it is easily shown that the steady state output ratio of the LIC to the United States is 0.1924. Therefore y is 0.7796 of y_{ss}.

Figure 2: The Benchmark Economy

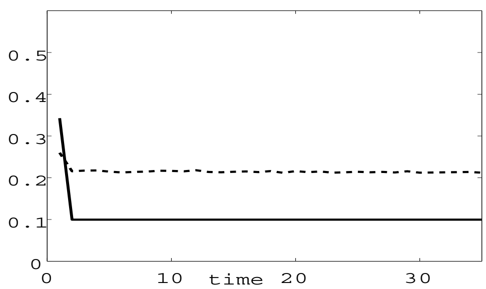
(a) output (fraction of y_{ss})



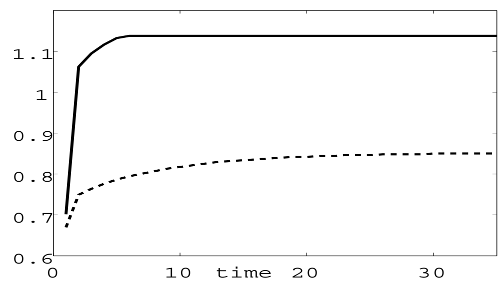
(b) consumption (fraction of y_{ss})



(c) investment (fraction of y_{ss})



(d) debt (fraction of y_{ss})



Concessional loans (solid) and non-concessional loans (dashed)

This discontinuity of $h(k)$, which is due to the interest rule (eq. (1)), implies that the recipient country must drastically reduce its external debt precisely when it surpasses the cutoff. Such debt reduction is possible only through a steep decline in consumption which the country may find too painful. For the benchmark case, this one-time cost of consumption reduction outweighs the long-run benefit of achieving the steady-state where output is much higher than $\bar{y}$ (call this “high" steady state.) Thus the country decides not to cross the line and converges to the cutoff (call this “low" steady state.) On the other hand, under non-concessional lending, this perverse incentive is absent, and the country steadily grows to reach the high steady state, yet consumption in the short-run is lower than in the case of concessional lending (Figure 2).

This result captures the paper’s debt overhang mechanism. Because the cost of servicing debt is kept artificially low, the country is motivated to carry a large amount of debt by consuming excessively and thus does not grow.

Whether or not the country is trapped in a debt overhang depends on the country’s initial conditions: initial debt and capital. The intuition is as follows. First, the higher the initial debt level, the more likely the country is to converge to the low steady state since this allows the country to manage heavier debt at a low interest rate. Second, the lower the country’s initial capital, the larger the impact of short-run growth. As a result, the country tries to borrow a larger quantity of concessional loans to raise both investment and consumption in the short-run; and thus it is more likely to be trapped in the low steady state. In short, the country converges to the high steady state only if initial debt is low enough, initial income is high enough, or both conditions hold.

Figure 3: Initial Conditions and the Debt Overhang Threshold

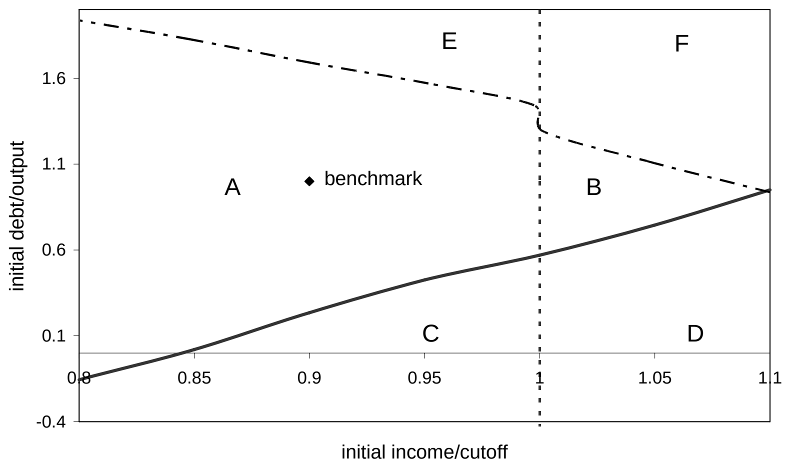


Figure 4: Debt Overhang Thresholds with Different TFPs

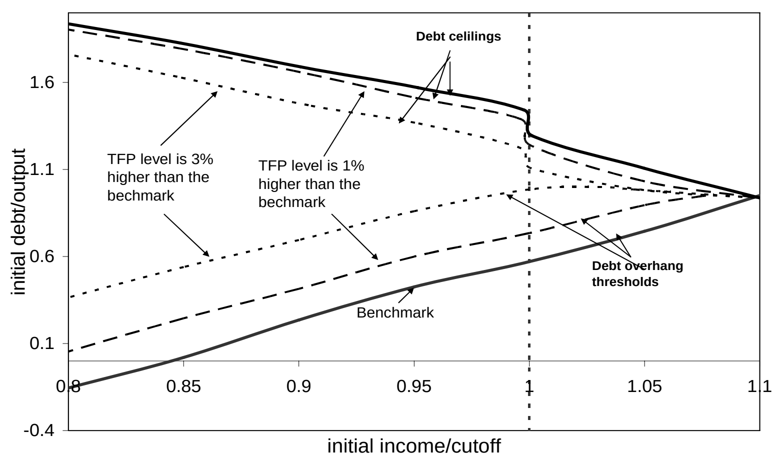


Figure 3 shows that there exists a debt overhang threshold (solid) above which the country is trapped with large debt and no growth. The dash-dotted line is the endogenous debt ceilings. Arrears countries that lie above these ceilings (E and F) are outside the scope of this paper. The vertical dotted line is the cutoff. The figure shows that if the country lies in regions A or B, it converges the low steady state, whereas if it lies in C or D (below the solid line), it achieves the high steady state.

Whether or not the country is trapped in a debt overhang is also conditional on the country's TFP level. The higher the TFP, the higher the steady state output level relative to the cutoff, and therefore the greater the long-run benefit of achieving the high steady state. With a higher TFP, the country thus finds it more costly to be trapped in the low steady state. Figure 4 shows that the debt overhang threshold shifts up with a higher TFP level. With a higher TFP level, the country is more likely to lie below the debt overhang threshold. Note that here the initial income level is kept the same across different TFP levels; initial capital levels are adjusted accordingly.

V. Implications for Debt Relief

The model implies that a one-time debt relief stock treatment may be effective in helping the country get out of the poverty trap and achieve growth. For example, suppose that the benchmark economy receives a one-time relief which enables the country to move below the debt overhang threshold (solid line in Figure 3). The country now converges to the high steady state.

A one-time debt relief may also be effective even if initially the country lies above the cutoff. Consider a country that has relatively high initial debt and lies in region B in Figure 3. Note that this country has an incentive to go back to the cutoff because the benefit of raising the debt ceiling by reducing capital is greater than the cost of lowering output. The country is thus better-off reducing output until it eventually falls to the cutoff. Here an upfront debt relief that moves the country from B to D is effective in achieving growth.

Note that if such a stock treatment is accompanied by factors that can raise TFP, such as productivity growth and improvement in institutional quality, then the debt overhang threshold itself will shift upward (Figure 4) resulting in a larger number of countries that lie below the threshold. This means that if debt relief resources are used for development purposes that also directly raise TFP, more countries will be able to achieve growth given the same amount of debt relief. The above arguments provide some insight into the Multilateral Debt Relief Initiative (MDRI), especially the importance of utilizing freed-up resources for development purposes.

However, there are some caveats to the model. First, for this type of stock treatment to work, it is important that recipient countries view it as a one-time event. Otherwise the poverty trap may reemerge. Suppose the country receives repeated debt relief with a similar income per capita eligibility criterion. With a sufficiently large degree of debt relief, the country may have an incentive to stagnate around the cutoff thereby keeping its eligibility for future debt relief.

Second, the environment may be too efficient. The model assumes that the country can efficiently allocate freed resources from debt relief to productive uses. In reality though, it may be quite difficult to handle a sudden increase in freed resources in the presence of weak institutions.

Lastly, the model does not analyze whether debt relief is the most efficient form of aid that can be used to maximize the welfare of these countries. There may be an alternative aid program that effectively raises the country's TFP high enough such that the country no long needs debt relief to get out of the poverty trap.

VI. Conclusions

There is a lag in the development of the theoretical literature that explains the relationship between large debt and low growth in LICs. This paper presents a theoretical model that provides a possible explanation of how debt overhang is generated in LICs. The model focuses on an incentive problem of a cutoff, an income level above which the country loses its eligibility for aid assistance. Because the cost of servicing debt is kept artificially low, the country may be motivated to carry a large amount of debt by consuming excessively, resulting in low growth.

Whether or not the country is trapped in a debt overhang depends on its initial conditions and TFP. With larger initial debt, the country has stronger incentives to manage its debt at a low interest rate by becoming permanently aid-dependent. With lower initial income, the country tries to borrow a larger quantity of concessional loans to raise both investment and consumption in the short-run; and thus it becomes more likely to be trapped in the low steady state. Lastly, with a lower level of TFP, the benefit of remaining at the cutoff is more likely to exceed the long-run benefit of achieving the high steady state.

Finally, the paper discusses the model's implications for debt relief. It implies that a one-time stock treatment may help the country get out of the poverty trap, provided that the freed-up resources are used effectively. If the country's initial conditions are relatively poor, it is important to allocate the freed-up resources for development purposes that directly raise the country's TFP.

REFERENCES

Agenor, Pierre-Richard, and Peter J. Montiel, 1996, *Development Macroeconomics*, (Princeton: Princeton University Press).

Atkeson, Andrew, 1991, "International Lending with Moral Hazard and Risk of Repudiation," *Econometrica*, Vol. 59 (4), pp. 383-396.

Barro, J. Robert and Xavier Sala-i-Martin, 2004, *Economic Growth*, second edition (Cambridge, Massachusetts, the MIT Press).

Bulow, Jeremy and Kenneth Rogoff, 1989, "A Constant Reconstructing Model of Sovereign Debt," *Journal of Political Economy*, Vol. 97, pp. 155-178.

Borensztein, Eduado and Atish Ghosh, 1989, "Foreign Borrowing and Export Promotion Policies," *IMF Staff Papers*, Vo. 36 (December).

Claessens, Stijn, Enrica Detragiache, Ravi Kanbur and Peter Wickham, 1997, "HIPCs' Debt Review of the Issues," *Journal of African Economies*, Vol. 6, No. 2, pp. 231-54.

Clements, Benedict, Rina Bhattacharya, and Toan Quoc Nguyen, 2005, *Can Debt Relief Boost Growth in Poor Countries?*, Economic Issues, Vol. 34, (Washington: International Monetary Fund).

Cohen, Daniel, 1993, "Low Investment and Large LDC Debt in the 1980's," American Economic Review, Vol. 83 (3), pp. 437-449.

Cohen, Daniel and Jeffery Sachs, 1986, "Growth and External Debt under Risk of Debt Repudiation," *European Economic Review*, Vol. 30, pp. 529-550.

Cole, Harold L. and Patric J, Kehoe, 1998, "Models of Sovereign Debt: Partial versus General Reputations," *International Economic Review*, Vol. 39 (1) , pp. 55-70.

Cordella, Tito, Luca Antonio Ricci, and Marta Ruiz-Arranz, 2005, "Debt Overhang or Debt Irrelevance? Revisiting the Debt-Growth Link," IMF Working Paper 05/223 (Washington: International Monetary Fund).

Easterly, William, 1999, "How Did Highly Indebted Poor Countries Become Highly Indebted?," Policy Research Working Paper 2225, (Washington: The World Bank).

Easterly, William, 2002, *The Elusive Quest for Growth*, (Cambridge, Massachusetts: The MIT Press).

Eaton, Jonathan, and Raquel Fernandez, 1995, "Sovereign Debt," NBER Working Paper 5131.

Eaton, Jonathan and Gersovitz, Mark, 1981, "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies*, Vol. 48, pp. 289-309.

Helbling, Thomas, Ashoka Mody, and Ratna Sahay, 2004, "Debt Accumulation in the ICS-7 Countries: Bad Luck, Bad Policies, or Bad Advice?," IMF Working Paper 04/93, (Washington: International Monetary Fund).

International Monetary Fund and the World Bank, 2003, "Debt Sustainability in Low-Income Countries—Toward a Forward-Looking Strategy," SM/03/185.

______, 2004, "Debt Sustainability in Low-Income Countries—Further Considerations on an Operational Framework and Policy Implications," http://www.imf.org/external/np/pdr/sustain/2004/091004.htm.

______, 2005, "Operational Framework for Debt Sustainability Assessments in Low-Income Countries—Further Considerations," http://www.imf.org/external/pp/longres.aspx?id=412.

______, 2006, "Review of Low-Income Country Debt Sustainability Framework and Implications of the MDRI," http://www.imf.org/external/pp/longres.aspx?id=557.

Krugman, Paul, 1988, "Financing vs. Forgiving a Debt Overhang, "*Journal of Development Economics*, No. 29, pp. 253-68.

Ostry, Jonathan D. and Carment M. Reinhart, 1992, "Private Saving and Terms of Trade Shocks, Evidence from Developing Countries," *IMF Staff Papers*, Vol. 39.

Pattilo, Catherine, Helene Poirson, and Luca Ricci, 2002, "External Debt and Growth," IMF Working paper 02/69 (Washington: International Monetary Fund).

Rajan, Raghuram, 2005, "Straight Talk: Debt Relief and Growth," *Finance & Development*, Vol. 42, No. 2 (June), pp. 46-49 (Washington: International Monetary Fund).

Rose, Andrew, 2005, "One reason countries pay their debt: renegotiation and international trade," *Journal of Development Economics*, Vol. 77, pp. 189-206.

Sachs, Jeffrey, 1989, "The Debt Overhang of Developing Countries," in *Debt Stabilization and Development: Essays in Memory of Carlos Diaz Alejandro*, ed. by Guillermo Calvo and others (Oxford: Basil Blackwell).

Sun, Yan, 2004, "External Debt Sustainability in HIPC Completion Point Countries," IMF Working Paper 04/160, (Washington: International Monetary Fund).

Yue, Vivian Z., 2005, "Sovereign Default and Debt Renegotiation," New York University (unpublished).

World Bank, 2001, "IDA Eligibility, Terms, and Graduation Policies" (Washington, World Bank).

Appendix A: No participation constraints

This appendix considers the environment in which the LIC fully precommits itself to honoring the conditions of the concessional lending scheme that is imposed by the creditor so that there is no need to impose a participation constraint. In practice though, this is an unrealistic assumption because it allows the LIC to have unlimited access to the donor's funds. Analyzing this non-participation constraint environment, however, is still useful to understand the role of a debt ceiling constraint.

In the absence of participation constraints, capital overshoots in period 1 due to the existence of subsidized interest rate. The problem is given by:

$$\max_{c_t, k_{t+1}} \sum_{t=1}^{\infty} \beta^{t-1} u(c_t)$$

Subject to the intertemporal budget constraint:

$$f(k_1) + (1-\delta)k_1 + \sum_{t=2}^{\infty}\left(\prod_{s=2}^{t}\left(\frac{1}{1+\tilde{r}_s}\right)\left(f(k_t)+(1-\delta)k_t\right)\right) = (1+\tilde{r}_1)D_1 + c_1 + k_2 + \sum_{t=2}^{\infty}\left(\prod_{s=2}^{t}\left(\frac{1}{1+\tilde{r}_s}\right)\left(c_t + k_{t+1}\right)\right),$$

where K_1, D_1, and $\tilde{r}_1$ are given. First order conditions are given by:

$$u'(c_t) = \mu, \qquad \text{for } t = 1, \qquad \text{(A-1)}$$

$$\beta^{t-1} u'(c_t) = \mu \prod_{s=2}^{t}\left(\frac{1}{1+\tilde{r}_s}\right), \qquad \text{for } t \geq 2, \qquad \text{(A-2)}$$

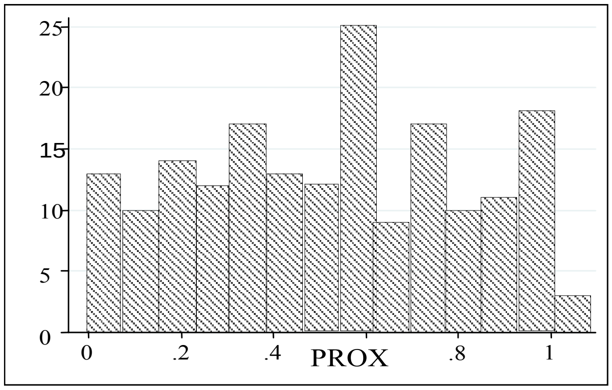
$$\tilde{r}_{t+1} = f'(k_{t+1}) - \delta, \qquad \text{for } t \geq 1, \qquad \text{(A-3)}$$

Where μ is the shadow price. Initially, the country can borrow at the concessional interest rate (i.e. $\tilde{r}_2 = \bar{r}$) because I assume that initial output lies below the cutoff. At any level of capital above the cutoff, the country can borrow only at the world interest rate. The capital levels in period 2 and in the steady state, k_2 and k_{ss}, are pinned down by $\bar{r} = f'(k_2) - \delta$ and $r = f'(k_{ss}) - \delta$ (by (A-3)). These equations imply that k_2 is greater than k_{ss}. because the concessional interest rate is lower than the world interest rate ($\bar{r} < r$.) Thus capital overshoots steady state in period 1. However, from period 3 onwards, capital is at its steady state level (i.e. $k_j = k_{ss}$. for $j \geq 3$) because as of period 2, the country no longer has access to concessional loans. Its capital level exceeds the cutoff ($k_2 > \theta k_{ss}$) and the capital level is k_{ss} (from (A-3)). Consumption, too, overshoots in period 1 ($c_1 > c_2 = c_{ss}$). This is implied by the following Euler equations: $u'(c_1) = \beta(1+\bar{r})u'(c_2)$ and $u'(c_2) = \beta(1+r)u'(c_3)$ because $\beta(1+\bar{r}) < 1$, $\beta(1+r) = 1$, and $u'(c)$ is decreasing in c. Once $\{k_2, k_{ss}\}$ and $\{c_1, c_{ss}\}$ are pinned down, the path of debt, $\{D_2, D_{ss}\}$, can be derived via the budget constraint. The dynamics of concessional loans without a participation constraint are thus characterized by the overshootings of capital and consumption in period 1 due to the low concessional interest rate. The donor's budget, α, is determined by $\alpha \equiv D_2(r-\bar{r})/(1+r)$.

Appendix B-1: A Description of PROX

There are over 1300 observations on PROX between 1987 and 2000, of which about 1/7 takes positive values. (i.e. only 1/7 of the observations lie close to the cutoffs). The figure below shows the histogram of PROX excluding observations with PROX=0. When I carry out a simple OLS regression of the growth rate of real GDP per capita on PROX, the coefficient for PROX is negative (-2.22) and is statistically significant at 1 percent.

Frequency of PROX (where PROX>0)



Appendix B-2: The First-Stage Regressions

The following tables report the first-stage regression results for the 2SLS estimation discussed in Section III. Instruments for PROX, INV, and INIT_RGDP are the one-year lags of these variables are used as the instruments for PROX and INV (denoted as PROX(-1) and INV(-1)) and the log of real GDP per capita in 1985 (RGDP85) respectively.

First-stage regression: All countries

Variables	Dependent Variables		
	PROX	INV	INIT_RGDP
GPO	1.70e-03	0.03	-0.01***
	(2.45e-03)	(0.06)	(2.22e-03)
INIT_EDU	1.08e-04	4.12e-03	5.22e-04**
	(2.31e-04)	(5.73e-03)	(2.10e-04)
PROX(-1)	0.92***	0.59*	-0.01
	(0.01)	(0.31)	(0.01)
INV(-1)	-4.54e-04	0.94***	3.18e-03***
	(3.89e-04)	(0.01)	(3.54e-04)
RGDP85	-2.61e-03	0.34***	1.00***
	(4.13e-03)	(0.10)	(3.76e-03)
R-squared	0.80	0.93	0.99

First-stage regressions: Countries that lie below the cutoff

Variables	Dependent Variables		
	PROX	INV	INIT_RGDP
GPO	-3.98e-03	-5.30e-03	-8.64e-03**
	(3.98e-03)	(0.09)	(3.42e-03)
INIT_EDU	1.28e-03**	0.01	1.97e-03***
	(6.21e-04)	(0.01)	(5.34e-04)
PROX(-1)	0.90***	1.06**	1.10e-03
	(0.02)	(0.47)	(0.02)
INV(-1)	4.72e-04	0.91***	1.59e-03**
	(7.60e-04)	(0.02)	(6.53e-04)
RGDP85	-0.01	0.13	0.10***
	(0.01)	(0.21)	(0.01)
R-squared	0.81	0.88	0.99

*** significantly different from 0 at the 1 percent level
** significantly different from 0 at the 5 percent level
* significantly different from 0 at the 10 percent level

Note: standard errors are in parentheses

Appendix B-3: Data

Variables	Definition	Source
Growth Rate	The growth rate of real GDP per capita (in percentage)	Constructed from Real GDP per capita, Constant Prices: Laspeyres (RGDPL) in Summers-Heston data set, version 6.1
GNI	GNI per capita In current US$, Atlas methodology	World Development Indicators
$\overline{y}$	The operational IDA cutoff in terms of GNI per capita in US$ Atlas methodology	World Bank GNI/capita operational guidelines
GPO	Percentage of population growth (per annum)	Constructed from population (POP) in Summers-Heston data, version 6.1
INV	Investment share of Real GDP per capita (in percentage per annum)	Summers-Heston data set, version 6.1
INIT_RGDP	Real GDP per capita, Constant Prices: Laspeyres (in percentage) in 1985	Summers-Heston data set, version 6.1
INIT_EDU	Percentage of secondary schooling attained in the total population in 1985	Barro-Lee data set

Country or Regional Coverage of the Data Set

Low-Income	**Lower-Middle**	**Upper-Middle**	**High-Income**
Bangladesh	Algeria	Argentina	Australia
Benin	Bolivia	Barbados	Austria
Cameroon	Brazil	Botswana	Belgium
Central African Rep.	Dominican Rep.	Chile	Canada
Congo, Dem. Rep.	Ecuador	Costa Rica	Cyprus
Congo, Rep.	El Salvador	Hungary	Denmark
Gambia, The	Fiji	Malaysia	Finland
Ghana	Guatemala	Mauritius	France
Guinea-Bissau	Guyana	Mexico	Germany
Haiti	Honduras	Poland	Greece
India	Indonesia	Panama	Hong Kong SAR
Kenya	Iran, Islamic Rep.	South Africa	Iceland
Lesotho	Jamaica	Trinidad & Tobago	Ireland
Malawi	Jordan	Turkey	Israel
Mali	Paraguay	Uruguay	Italy
Mozambique	Peru	Venezuela, RB	Japan
Nepal	Philippines		Korea, Rep.
Nicaragua	Sri Lanka		Netherlands
Niger	Syrian Arab Rep.		New Zealand
Pakistan	Thailand		Norway
Papua New Guinea	Tunisia		Portugal
Rwanda			Singapore
Senegal			Spain
Sierra Leone			Switzerland
Tanzania			United Kingdom
Togo			United States
Uganda			
Zambia			
Zimbabwe			