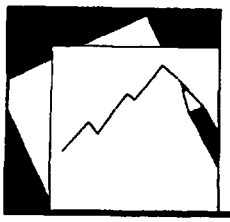

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Bad Dreams Under Alternative Anchors: Are the Consequences Different?

Leonardo Auernheimer and Susan M. George

INTERNATIONAL MONETARY FUND

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Bad Dreams Under Alternative Anchors: Are the Consequences Different?

Prepared by Leonardo Auernheimer and Susan M. George[1]

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February 2000

Abstract

Using a simple model, this paper shows how a strict monetary rule exhibits characteristics similar to those of an exchange rate anchor, in terms of a lack of robustness in the presence of adverse expectations ("bad dreams"). More specifically, as an anticipated devaluation under an exchange rate rule leads to well-known contractionary effects, an anticipated increase in the money stock under a monetary rule, though initially expansionary, becomes contractionary when these expectations are not validated. This suggests that much of the criticism of an exchange rate anchor implicitly considers not another rule but rather, discretion as the alternative.

JEL Classification Numbers: F41

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I. INTRODUCTION

The discussion of a fixed exchange rate system (often referred to as an "exchange rate anchor," or an "exchange rate rule"), which has a long tradition, has been revived in the last 15 years. It is natural this happened for at least two reasons. At the theoretical level, many previous conclusions needed to be reviewed, and sometimes revised, in light of the rational expectations approach. At the policy level, many countries attempting to stabilize have adopted such a rule, some in its most binding form, i.e., a currency board.[2] In the context of this discussion, it has become clear (at least in our interpretation) that the controversy among policy makers is not so much about the comparative merits of an exchange rate *versus* another exogenous rule (for example, a "monetary rule" à la Friedman), but rather about exogenous rules *versus* discretion.[3] The issue goes beyond one of semantics. Our conjecture is that some of the negative characteristics attributed to an exchange rate rule are actually shared by the alternative strict monetary rule, despite the conventional contention to the effect that price rigidity introduces important differences among regimes. If this is the case, the focus of the discussion needs to be shifted either towards clarifying the features of other exogenous rules, or towards the consequences of adopting discretionary policies.[4] [5] This paper intends to contribute to such a discussion, by showing how a strict monetary rule, in which the path of a monetary aggregate is exogenously set in advance, involves some of the very same undesirable features of the strict exchange rate anchor. A simple but yet complete

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[2] A currency board is nothing else than a central bank that abides by the rules of the game of a fixed (exogenous) exchange rate, letting the classical adjustment mechanism operate.

[3] A recent piece by Levy Yeyati and Sturzenegger (1999) attempts to classify exchange rate regimes beyond reported IMF classifications.

[4] It should be mentioned that the rational expectations paradigm has (or should have) changed the terms of the old discussion between "rules" and "discretion." If "discretion" involves a response rule of behavior or algorithm adopted by the policy maker, no matter how complicated—the exception being, of course, a pure random response—the public eventually learns such an algorithm. Under rational expectations, there is no ostensible role for discretional policy as defined since such policy eventually becomes a rule. We propose to restrict the characterization of a "rule" to only non–feedback rules of policy behavior. In other words, the path of the policy variable (the exchange rate, the money supply, or the nominal interest rate) is independent of events—i.e., the policy maker who follows a rule "does not read the newspapers." Therefore, we characterize feedback rules, such as the Taylor rule, as discretion.

[5] Mishkin and Savastano (1999) also argue for a change in the terms of the monetary policy debate. In their view, given the success of Latin American countries in bringing inflation down, the discussion should now become more about how discretionary policy can effectively be constrained.

model of a small open economy is used, derived from rigorous micro foundations. The model includes the particular assumption of price sluggishness in the home goods sector, which seems to correspond to the implicit framework that many economists have in mind while discussing the relative merits of these rules.

Comparison of exogenous rules, for the most part, emphasize the differential response to external shocks, either real or monetary. An alternative approach, which is used in this paper, concentrates on the "robustness" of the rule, i.e., the extent to which shocks may force the monetary authority to give up the rule. More specifically, we consider the case in which the shock is a sudden lack of credibility—i.e., the anticipation that at some time in the future government will violate the rule, despite the "fundamentals" being correct. This is the problem of multiple equilibria in which "dreams become reality."[6] The pressure brought about on the monetary authority to validate the public's expectations can take many forms, from a deterioration of the government's fiscal position to changes in the real exchange rate, output and real interest rates. Although the former is most discussed, in this paper the latter is our focus, mostly because the pitfalls of an exchange rate anchor often centers on this discussion.

The preliminary results confirm our conjecture that even with price rigidity both rules show remarkably similar responses to unanticipated changes in the policy rule variable (the quantity of money in the case of the monetary rule and the exchange rate in the exchange rate rule). More importantly, the consequences of the monetary authority not validating the public's anticipation are equally devastating.

The plan of the paper is as follows. The basic model in an abbreviated form is presented, relegating a more detailed discussion to the Appendix. The adjustment mechanism under both the exchange rate and monetary rules with the assumption of perfect price flexibility is briefly discussed. Next, the assumption of sluggishness in home goods prices is introduced, and adjustment under the same two policy rule regimes is examined. The final section presents the conclusions and suggests the lines of further work.

II. The Model

Consider a "small" economy, in which there is perfect foresight and individuals live forever. The economy produces two commodities, traded and home goods, in quantities that initially are assumed to be fixed. Individuals derive utility from their consumption of both commodities and from the services of fiat money. There is perfect capital mobility, and individuals hold foreign assets that yield a fixed world real interest rate. Since there are no restrictions to trade and the country is small, by assuming the world price equal to unity, then the domestic price of traded goods is equal to the nominal exchange rate. Government also holds foreign assets, issues fiat money and may impose head taxes and spend on home goods.

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[6] See, for example, Calvo (1988) and the extensive literature on "sunspot equilibria."

Under these conditions, and assuming that the utility function is separable in all its three arguments, the solution of the model results in a "double dichotomy." The "foreign" sector yields a system of two equations (one of them being the balance of payments identity) which contain only the consumption of traded goods and the aggregate of private and government net foreign assets. By assuming that the level of head taxes (or subsidies) endogenously adjusts such that the appropriate transversality conditions for both the private and public sectors are satisfied,[7] this system can be solved independently of the "domestic" and the "monetary" sectors. In particular, it can be solved for a constant level of the consumption of traded goods.

As a result, the domestic and the monetary sectors can be characterized for a given exogenous level of the consumption of traded goods. As discussed in the Appendix, such characterization is given by the two fundamental expressions:

$$\frac{U_T(c_T)}{\varepsilon} = U_H(c_H) \tag{1}$$

$$\frac{U_m(m)}{(i^* + \hat{E})} = U_H(c_H) \tag{2}$$

where c_T, c_H, $m \equiv M/P_H$, i^* and E are the consumption of traded goods and home goods, the real money stock (deflated by the price of the home good), the world interest rate and the nominal exchange rate, respectively; $\hat{E} \equiv (dE/dt)(1/E)$ denotes the proportional rate at which the nominal exchange rate is changing; and U_i denotes the marginal utility of each of the three arguments in the utility function. The "real" exchange rate, defined as $\varepsilon \equiv E/P_H$, is the relative price of traded goods in terms of home goods.[8]

The intuition of the above two expressions is straightforward. For a given level of consumption of traded goods, expression (1) yields the demand for home goods as a function of its relative price, which is the real exchange rate derived from using the small country assumption. Expression (2) yields the demand for money as a function of the nominal interest rate which measures the cost of holding money.

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[7] By introducing this assumption, we ignore the effects of credibility shocks on the government's fiscal position.

[8] A similar framework was used in Auernheimer (1987).

A. Exchange Rate and Monetary Rules with Perfect Price Flexibility

With perfect price flexibility in the home goods sector, the model yields the standard results of instantaneous adjustment to both an unanticipated devaluation in the case of an exchange rate rule and an unanticipated change in the nominal money stock in the case of a monetary rule. Following an unanticipated devaluation there is an immediate rise in the nominal money stock, and the same proportional change in the level of home goods prices; a rise in the nominal money stock, for the case of the monetary rule, brings about the same proportional change in the nominal exchange rate and in home goods prices. As mentioned before, our assumption is that "fiscal" effects are compensated by endogenous adjustments in head taxes or subsidies.

The anticipation of a future devaluation or a change in the nominal money stock in each of the two regimes can be analyzed without difficulty, since in both cases the real exchange rate does not change during the adjustment. For the case of a monetary rule, the adjustment following the anticipation of a change in the nominal money stock can be analyzed in the context of the demand for money given by equation (2). The asset price continuity principle assures that the nominal exchange rate will not change discretely at the time when the change takes place. Since the real exchange rate remains constant throughout, the same will be true of the price of home goods. The result is the same as in the case of a closed economy: a discrete increase in both the nominal exchange rate and the price of home goods, and hence an initial fall in the real money stock, followed by an increasing path of the rates of domestic inflation and devaluation and a decreasing path of the real money stock. At the time of the increase in nominal money there is a discrete increase in the real money back to its initial level, with the nominal exchange rate and home goods prices having risen by the same proportion as the nominal money stock.

The analysis of an anticipated devaluation requires additional assumptions, and the explicit modeling of how anticipations are formed. To this effect, the following mechanism is adopted. Assume, for simplicity, that the exchange rule implies a zero rate of devaluation (i.e., the classical "fixed" exchange rate case). Call $\Delta lnE_{0,\tau}$ the anticipation arising at the time $t = 0$, that ***at or before*** a future time $t = \tau$ (the natural log of the) the nominal exchange rate will increase by the absolute magnitude Δ. Assume further the probability that such an event takes place between current time t and future time τ ($t \in [0, \tau]$) is perceived to be constant, with a cumulative distribution equal to unity (i.e., there is certainty that the event will take place before $t = \tau$. At each point in time between $t = 0$ and time $t = \tau$, then, $(\Delta E_{t,\tau}/E_0)/(\tau - t)$ is the certainty equivalent of the *rate of devaluation* $\hat{E} = \hat{E}^a$ expected for the next instant. Obviously, as time gets closer to τ, if the devaluation has not yet occurred, $\hat{E}^a$ becomes higher, and it approaches infinity as $t \rightarrow \tau$. The graph in Figure 1 helps to visualize the process.[9]

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[9] The natural outcome at the time of the change is that the real money stock goes to zero for an instant. Notice that this does not necessarily pose a problem for the level of utility which

(continued...)

Again, as in the previous case, the solution can be found in the demand for money from equation (2)—in which the rate of devaluation is the anticipated rate—with a similar behavior of the real money stock: an initial fall, at $t = 0$, followed by a descending path as the anticipated rate of devaluation increases. As before, the real exchange rate remains constant throughout.

B. The Effects of Price Sluggishness

When discussing differences between exchange rate and monetary rules, a common conception is that differences between the two, which may not be important when prices are perfectly flexible, become substantial when some prices exhibit short-run rigidity. In order to evaluate this proposition, the basic model is modified by assuming slow adjustment (sluggishness) in the price of home goods.

Short-run price rigidity in the following form is introduced. First, and only for purposes of simplicity, an exchange rate rule is assumed as a zero rate of devaluation and, correspondingly, a monetary rule is a constant preannounced path of the nominal money stock, with a zero rate of monetary growth.[10] Second, the domestic money price of home goods (P_H) is assumed to be a "state variable" (i.e., given by past history at every point in time), which adjusts according to:

$$\left(\frac{dP_H}{dt}\right)\left(\frac{1}{P_H}\right) = \left(\frac{dE}{dt}\cdot\frac{1}{E}\right)^a + \gamma\left(\varepsilon - \varepsilon^*\right) \tag{3}$$

Notice that for the case of a monetary rule, $(\hat{E})^a = \hat{E}$, where $\hat{E}$ is the proportional rate of change of the nominal exchange rate.

The adjustment equation (3) implies a proportional rate of change of the real exchange rate of the form

$$\left(\frac{d\varepsilon}{dt}\right)\left(\frac{1}{\varepsilon}\right) \equiv \hat{\varepsilon} = \hat{E} - (\hat{E})^a + \gamma\left(\varepsilon^* - \varepsilon\right) \tag{3'}$$

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is evaluated as an integral. Furthermore, it approximates some real world situations in which the real base falls to negligible levels towards the end of an exchange rate crises. Nevertheless, whether it is to validate or not (validate) the public's anticipation, we assume that the government response takes place shortly before $t = \tau$

[10] We initially specify the model without either restricting it to a zero rate of monetary growth or a zero rate of devaluation.

In equations (3) and (3′), ε* is the "equilibrium real exchange rate," which corresponds to a level of consumption of the home good equal to a constant exogenous level of "long run production," which call x_H^*, net of government purchases. Notice that this equilibrium real exchange rate depends also on the level of consumption of traded goods. Assuming for simplicity that government purchases of home goods are zero, the long run real exchange rate is then obtained by solving equation (1) after substituting $c_H = x_H^*$, as

$$\varepsilon^* = \frac{U_H(c_T)}{U_H(x_H^*)} \tag{4}$$

This equation implies that at any point in time, for a given level of consumption of the traded good and a given level of the real exchange rate, production of the home good is given by the level of consumption of that good, so as to satisfy equation (1). Adjustment of the home goods price assures that eventually production will adjust to the long-run level x_H^*.

Notice that equation (3) contains two components: the anticipated rate of change of the nominal exchange rate (the price of traded goods), and a term that is proportional to the difference between current and steady state production of the home good. As in the conventional text book analysis of aggregate demand and short-run aggregate supply, production adjusts in the short-run, rather than prices. In this case, the "demand" function, for a given level of c_T, can be obtained with the inverse of the real exchange rate as the (relative) price of the home good (Figure 2). At "too high" a real exchange rate, so that $(1/\varepsilon) < (1/\varepsilon^*)$, as for the case of ε_0 in Figure 2, consumption and production will be at a level $c_{H_0} > x_H^*$, and the price of home goods will be rising, with the real exchange rate falling.[11]

Under these assumptions, for a given level of consumption of the traded good and government expenditures in home goods (kept here at zero for notational convenience, with no consequences for the analysis), at every point in time consumption of home goods is given by:

$$c_H = g(c_T, \varepsilon) \qquad g_\varepsilon > 0 \tag{5}$$

which is just another formulation of equation (1).

Consider now the adjustment characteristics of an exchange rate rule (more specifically, a fixed exchange rate) and a monetary rule (more specifically, a fixed quantity

[11] Equation (3) is reminiscent of a similar form often used in macroeconomics, in which the rate of inflation is determined by the expected inflation rate and the difference between actual and full-employment output.

of nominal money), in that order. For purposes of added simplicity, assume that the utility function is of log form[12]

$$U = \alpha \ln c_T + \beta \ln c_H + \sigma \ln m. \tag{6}$$

C. The Case of an Exchange Rate Rule

Under an exchange rate rule, the term $\hat{E}$ is given by policy. The demand for money (2), nevertheless, depends on the anticipated, and ***not*** on the actual rate of devaluation. Using (1) and (2), we obtain the following association between the real exchange rate and the real money stock that holds at all times:

$$\left(\frac{U_m(m)}{U_H(g(\varepsilon))}\right) = i^* + \hat{E}^a. \tag{7}$$

Using the particular case of a logarithmic utility function as in (6), obtain:

$$\varepsilon = \frac{\alpha m\left(i^* + \hat{E}^a\right)}{\sigma\, c_T} \tag{7'}$$

In turn, the adjustment equation (3′) repeated here for convenience as (7″), dictates the behavior of the real exchange rate:

$$\left(\frac{d\varepsilon}{dt}\right)\left(\frac{1}{\varepsilon}\right) = -\left(\hat{E}\right)^a + \gamma\left(\varepsilon^* - \varepsilon\right) \tag{7''}$$

where the term $\left(\hat{E}\right)^a$ is exogenously given by the mechanism described before.[13]

The dynamics is as portrayed in Figure 3, where the function $F\left(m, \varepsilon; \hat{E}^a\right)$ is the representation of equation (7′), and the horizontal line $(d\,\varepsilon/dt)(1/\varepsilon) = 0$ corresponds to (7″). Notice that (7′) holds at all times, and that both lines are conditional on the anticipated rate of devaluation. Higher anticipated rates of devaluation imply a shift of both lines, where (7′)

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[12] For as long as the utility function is separable in money the dichotomy between the real and the monetary sector is preserved. In the case of perfect price flexibility, the dichotomy between the traded and the home goods sectors is also obtained, simply because the consumption and production of home goods is constant.

[13] Recall that the actual rate of devaluation is, for simplicity, assumed to be zero.

shifts up and (7″) shifts down. Notice also that in this case the real exchange becomes a state variable, since the nominal exchange rate is determined by government policy and the price of home goods is a state variable by assumption. The "control" variable is now the real money stock, which can change discretely at any point in time via changes in the nominal stock due to perfect capital mobility.

It is useful at this time to define the "real" interest rate for the case of home and traded goods (each sector's "own" interest rate) as

$$r_H = i^* + \hat{E}^a - \hat{P}_H \tag{8}$$

$$r_T = i^* + \hat{E}^a - \hat{E} \tag{8'}$$

Consider now the results of an unanticipated devaluation, which is the analogue of an unanticipated once-and-for-all increase in the nominal money stock for the alternative case of the monetary rule. These are comparable experiments because in both cases the final result is a proportionally equal increase of both the nominal exchange rate and the nominal money stock. The government engages in these policy options so as to appropriate assets from the private sector: through the purchase of real assets (foreign exchange) from an increase in the quantity of money under the monetary rule scenario; and through the building up of foreign assets when the private sector rebuilds its cash balances after a devaluation under the exchange rate rule.[14]

The instantaneous result of an unanticipated devaluation is to raise the real exchange rate, what generates an immediate rise in the real money stock, as the private sector rebuilds its real money stock, initially to a level ***higher*** than the initial equilibrium level, due to the rise in the consumption of home goods—the rise in its production having been generated by the increase in the real exchange rate. From then on, the adjustment takes the form of a slow return back to the initial equilibrium along the $F(m, \varepsilon)$ long run equilibrium line corresponding to a zero anticipated rate of devaluation. During the transition the level of production and consumption of the home good is higher than in the steady state, and since the real exchange rate is falling, the real interest rate for the home goods sector is lower. Notice that this is the case because after the unanticipated devaluation occurs, both the actual and anticipated rates of devaluation are equal (and zero). Therefore, expression (8) reduces to $r_h = i^* + \hat{\varepsilon}$. The devaluation has an expansionary effect of exactly the same form as in the case of an unanticipated rise in the quantity of money for the case of the monetary rule. The response of all the variables (the real exchange rate, the real money stock, output, consumption and the real interest rate) depict the same qualitative behavior. Although this

[14] Notice the difference between this analogy and the one suggested by classical analysis, which emphasizes the initial impact effect, and draws the parallel between a devaluation and a ***fall*** in the nominal money stock (Hume's annihilation experiment).

result is evident and expected, it is important to emphasize it because it means that in the case of either rule there is a temptation for government to "cheat," i.e., to depart from the preannounced path of the policy variable, and, with it, the potential for the same "time inconsistency" problem.

Consider, next, the more interesting experiment of a "bad dream", which in this case takes the form of an anticipation, at time $t = 0$, that at any time between "now" and a future time $t = \tau$, a devaluation by the absolute amount ΔE will take place. Using the same mechanism discussed previously, for $0 \leq t \leq \tau$, the expected rate of devaluation will be given by:

$$\left(\hat{E}\right)^a(t) = \left(\frac{\Delta E_{t,\tau}}{E_0}\right)\left(\frac{1}{\tau - t}\right) \tag{9}$$

In Figure 3 the result is that at the onset of the new anticipation *F (m, ε)* shifts up to the left, and the steady-state real exchange rate line shifts downward, and as time goes by, and the anticipated rate of devaluation keeps increasing, the two lines continue to shift gradually (and increasingly), *F (m, ε)* shifting up and the steady-state real exchange rate line shifting downward. To avoid clutter in Figure 3 only one of each of those new two lines are shown. Assume that at time $t = \tau$ the system is at a point such as B. At that time, or immediately before, the situation is resolved either with the public suddenly totally reversing its expansionary expectations or with the monetary authority validated them via a devaluation. In both cases, the anticipated rate of devaluation reverses back to zero. In the first, the public moves into real cash balances (to a point like C), and from then on gradually towards the long run equilibrium point A. The transitional effects of the "bad dream" (lower real exchange rate and hence lower production and consumption of the home good, and a higher real interest rate in both sectors) start to be reversed, but during the whole adjustment back to equilibrium these variables are on the "negative" side of their long run equilibrium.

Compare these results with the effects of a validation of the public's expectations, i.e. a devaluation at some time before $t = \tau$. Depending on the magnitude of the devaluation, the real exchange rate "jumps" to a higher level (corresponding to either the points F or D, shown as examples in Figure 3), and the public immediately acquires real cash balances so as to "jump" to the function *F (m, ε)*. Although a formal proof is not provided here, various simulations suggest that if the devaluation is exactly by the amount initially anticipated, the real exchange rate will rise to a level ***below*** its long run equilibrium level (to a point such as F), so that the final adjustment occurs with the real exchange rate and the level of home goods production still at a level below their long run equilibrium, and with the real interest rate for the home goods sector still higher than its long run equilibrium. If the equilibrium levels of the real exchange rate and production are to be restored immediately (or positioned at a higher level, such as a point D), this would then mean that the monetary authority needs to devaluate by ***more*** than was initially anticipated—i.e., the monetary authority would need to "overvalidate" the adverse expectations. The various graphs in Panel A show a representative time profile of the main variables, for both the cases when the government response is "validate" and "not validate".

Consider now the features of the same economy under a monetary rule, where the form of the rule is a constant predetermined level of the nominal money stock.

D. The Case of the Monetary Rule

The case in which government fixes the nominal money stock, the system that governs the determination of the real exchange rate and the real money stock (and therefore consumption and production of the home good) is given by expressions:

$$\frac{d\varepsilon}{dt}\frac{1}{\varepsilon} = \gamma(\varepsilon^* - \varepsilon) \tag{10}$$

$$\frac{dm}{dt}\frac{1}{m} = \mu + i^* + \gamma(\varepsilon^* - \varepsilon) - \left(\frac{\sigma c_T \varepsilon}{\alpha m}\right) \tag{10'}$$

where μ is the rate of monetary expansion, that from now on is assumed to be zero.

Equation (10) is derived from the adjustment equation (3') and the demand for money given by (2). Equation (10') is derived from (3) and the definition of the real money stock as $m \equiv M / P_H$. Assume the simple utility equation (6).

Linearization of (10) and (10') yields a solution with a unique equilibrium and two real roots of opposite sign assuring a unique saddle path. Figure 4 illustrates the system. The real money stock is the "state" variable, given by past history (the nominal money stock is exogenous, and its deflator, the price of home goods, is a state variable by assumption). The real exchange rate can take any value at every point in time (i.e., is the "control" variable),via changes in the nominal exchange rate. An initial situation of "too high" ("too low") a real money stock, in the absence of future anticipated changes, brings about an adjustment along the saddle path, for which both the real money stock and the real exchange rate are falling (rising).

Before proceeding, observe that under a monetary rule and with rational expectations, the actual and the anticipated rates devaluation coincide. As a result, the real interest rate relevant for the home goods sector will be higher (lower) when the real exchange rate is rising (falling), while the rate in the traded goods sector is constant, and equal to the world rate. It follows that expressions (8) and (8') become $r_h = i^* + \hat{\varepsilon}$ and $r_T = i^*$.

In this system, consider first the effects of a discrete, unanticipated once-and-for-all increase in the nominal money stock. In Figure 4, this is shown as an instantaneous increase of the real money stock from its initial equilibrium level m^* to $m_0 > m^*$. Since by definition this is a once-and-for-all change and no further changes are anticipated, there is an immediate rise of the real exchange rate (entirely due to a rise of the nominal exchange rate) from ε^* to ε_0. From then on, the adjustment proceeds along the saddle path. By "plugging" these results

in some of the other equations, notice that the response implies an increase in the level of production and consumption of the home good, and during the adjustment, a lower level of the real interest rate. A quick inspection also reveals the "overshooting" of the nominal exchange rate first discussed by Dornbusch (1976).

Consider now the anticipation, at time $t = 0$, that a discrete once-and-for-all increase will take place at a future date $t = \tau$. The response to such an anticipation should be analyzed applying the asset price continuity principle: the optimal path of adjustment would not include a discrete change in the level of the nominal exchange rate in the future. In particular, such a jump will not occur at the time when the money stock is anticipated to rise. Since the price of the home good is a state variable, this implies that there is no discontinuity in the real exchange rate. Figure 5 shows the result of such an experiment. At time $t = 0$ when the anticipation occurs, the real exchange rate rises to the level ε_1 (point A), and moves on a path such that at the time of the anticipated increase in the money stock, the monetary injection will cause a movement to the saddle path without a discrete change in the nominal exchange rate (point B to point C).

It is interesting to follow what happens during the transition. First, an anticipation of expansionary monetary policy brings about a rise in home goods output through the higher real exchange rate (from point A to point B in the graph). As it is clear from equation (8), during this transition the real interest rate relevant for the home goods sector will be lower than its long run value.

If at the time at which it was anticipated the increase in the nominal money stock does take place, then from that time on the real exchange rate continues to fall back to its long run level, with output also falling, though remaining above its steady state value. The real interest rate relevant for the home goods sector, although increasing, is still below its final level.

It is important to consider the results for the case in which of the anticipation is ***not validated*** by the monetary authority. Suppose, in the extreme case, that at time $t = \tau$ the government does not increase the money supply as expected, and at which time the anticipation of expansionary policy is completely reversed. Again from Figure 5, at time τ the real exchange rate falls discretely from its level at point B to the saddle path level at point D, adjusting from then on along the saddle path where the real exchange rate and the real money stock rise until steady state levels are reached. As a consequence there is an important contraction in the home goods sector ***and,*** from then on, higher than equilibrium levels of the real interest rate.

The important point here is that the pressure to validate expansionary expectations arises also under a monetary rule. Interestingly, this scenario resembles an exchange rate crisis, when there is an anticipation of devaluation under the exchange rate rule.

Notice the following two similarities of the two rules. First, in the above scenario when there is an expectation that the nominal money stock will be raised, there is a reduction in the real money stock and an increase in the real exchange rate—the latter an often cited

feature of an exchange rate crises. Second, under both rules there is a high penalty to be paid by the policy maker for not validating the anticipation, once such an anticipation exists. Of course, it is precisely this evaluation that is well understood by the public that precipitates the crises. This is none other than the familiar time inconsistency problem.

Two conclusions can be drawn from the last experiment. First, the same anticipation brings about opposite transitional results from the case of an exchange rate rule (an expansion, rather than a contraction). Second, and more importantly for the purpose of our inquiry, the monetary rule brings about the same lack of robustness as the exchange rate rule.

III. Conclusions and Future Work

One proposition of conventional wisdom is that market imperfections (such as price rigidity) introduce important differences in the way the economy adjusts to shocks under an exchange rate anchor (an exchange rate rule) as opposed to other policy rules, presumably a monetary rule. Another is the widely held criticism of the exchange rate rule because of the consequences of perverse, unjustified expectations of a devaluation. We have used a simple neoclassical model, with the "Keynesian" twist of sluggishness in the adjustment of home goods prices, to examine these propositions. One of the motivations for the exercise is our belief that the criticism of an exchange rate rule fails to consider the possible shortcomings of alternative exogenous rules, in particular, a monetary rule. Indeed, we suspect that the policy maker's aversion to an "exchange rate rule" places the emphasis on "rule" rather than on "exchange rate". What our exercise shows is that some of the problems inherent in an exchange rule have a similar counterpart in a strict monetary rule, as far as the **robustness** of the rule is concerned-- the way in which the economy reacts to unjustified but adverse expectations, and the extent to which government may be forced to validate those expectations, and hence to violate the rule.

Further work is required. In our analysis, the assumption has been that an endogenous level of head taxes or subsidies adjusts so as to continuously assure the fulfillment of transversality conditions for both the private and the public sector, at a constant level of consumption of the traded good.[15] This is a valid approach when the focus is the pressure brought about on the monetary authority that takes the form of undesirable outcomes in variables such as production, consumption, the real exchange rate and real interest rates. As mentioned in the introduction, equally (or possibly more) important is the pressure generated by a deterioration of the government's fiscal stance. The obvious next step is to relax the assumption of automatically accommodating taxes.

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[15] This was possible due to the "dichotomy" between the external and domestic sectors allowed by our model.

A Detailed Exposition of the Model

Consider a "small" economy, in which there is perfect foresight. World commodity prices are constant, normalized at unity, and so is the world real rate of interest, i^*. There are two commodities (or "bundles of commodities"): traded and home goods. The private sector produces a fixed quantity of both the traded good (x_T) and the home good (x_H). Individuals, who are identical and live forever, consume the two commodities (c_T and c_H) and hold net foreign assets (a, denominated in terms of traded goods), that yield the world real interest rate, and domestic money.

Government also holds net foreign assets (A, denominated in terms of traded goods and also yielding the world interest rate), imposes per head taxes, spends in home goods (G_H) and issues fiat money.

Since the world price of traded goods is normalized at unity, assuming permanent arbitrage and no transportation costs or trade taxes, the domestic price of traded goods is the nominal exchange rate, i.e., $P_T = E$, where E is defined as the price of foreign in terms of domestic currency (say, pesos per dollar). The "real exchange rate" is defined as the ratio:

$$\varepsilon \equiv \left(\frac{E}{P_H}\right) \tag{A.1}$$

where P_H is the price of home goods in terms of domestic currency.

Given these assumptions, the budget constraint of the typical individual, expressed in terms of the traded good, is

$$x_T + \left(\frac{x_H}{\varepsilon}\right) + ai^* \equiv c_T + \left(\frac{C_H}{\varepsilon}\right) + T + \left(\frac{dm}{dt}\right)\left(\frac{1}{\varepsilon}\right) + m\pi\left(\frac{1}{\varepsilon}\right) + \left(\frac{da}{dt}\right) \tag{A.2}$$

where $m \equiv M/P_H$, and M is the nominal quantity of money. Notice that (A.2) is expressed in terms of traded goods, and that $\pi \equiv (dP_H/dt)(1/P_H)$ is the rate of inflation in home goods prices.

Likewise, the government's[16] budget constraint is

$$T + Ai^* + \left(\frac{dm}{dt}\right)\left(\frac{1}{\varepsilon}\right) + m\pi\left(\frac{1}{\varepsilon}\right) \equiv \left(\frac{G_H}{\varepsilon}\right) + \left(\frac{dA}{dt}\right) \tag{A.3}$$

[16] Notice that here we are defining "government" as the central government *cum* the central bank.

where T is a per head tax (or subsidy, if negative), and G_H is government expenditures in home goods. Assume, as customary, that tax proceeds are thrown into the ocean.

The typical individual, at every initial time t = 0, maximizes

$$\int_0^{\infty} U(c_H, c_T, m)\exp(-\rho t)dt$$

subject to the budget constraint (A.2), ρ is the rate of time preference, that is assumed to be equal to the world real interest rate, i^*[17], and $U(\cdot)$ is the utility function, which is assumed to be separable in all three components c_H, c_T, m, so that the marginal utility of each is independent of any other. This is an assumption that preserves reduced forms in which the two commodities are gross substitutes, and yields very plausible results as well as simplifies the analysis a great deal. Notice that the real money stock, which is an argument in the utility function, is obtained by deflating nominal money by the price of home goods, rather than a more general price index. This is not an innocuous assumption[18], and here it should be interpreted as assuming that the utility derived from the use of money is entirely due to the use of money in conducting transactions of the home good—perhaps because the "bundle" of home goods is made up of a large number of individual commodities.

Maximization of the utility functional yields the following marginal optimality conditions, that must be fulfilled at all times:

$$\frac{U_T(c_T)}{\varepsilon} = U_H(c_H) \tag{A.4}$$

$$\left(\frac{dc_T}{dt}\right) = \left(\frac{U_T}{-U_{TT}}\right)(i^* - \rho) = 0 \tag{A.4'}$$

$$\frac{U_m(m)}{(i^* + \hat{E})} = U_H(c_H), \tag{A.4''}$$

where $\hat{E} \equiv (dE/dt)(1/E)$ denotes the proportional rate at which the nominal exchange is changing.

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[17] This is another common assumption. Its rationale is as follows: if the rest of the world is in equilibrium, then in the rest of the world the real interest rate is equal to the rate of time preference of individuals. If individuals in the small country are equal to those of the rest of the world, then the assumption follows.

[18] See Auernheimer and Ellis (1995) for an analysis of this point.

Notice that(A.4) is the usual elementary condition that the ratio of marginal utility to prices is equalized for all consumed goods. (A.4′), in turn, is the direct result of access to a perfect capital market and the fact that the rate of time preference is equal to the real world interest rate. This expression implies that the consumption of the traded good is piecewise constant. Finally, (A.4″) results from (A.4′) and the assumption of separability of money in the utility function, and has the same interpretation as (A.4), with the term $i^* + \hat{E}$ (the nominal interest rate) measuring the cost of holding money. This last expression can be appropriately called the "demand for money."

The equilibrium condition in the market for home goods is assumed to be fulfilled at all times, which implies:

$$c_H = x_H + G_H. \tag{A.5}$$

Consolidation of the private sector's and the government's budget constraint yields the country's balance of payments identity, in which $w \equiv a + A$, i.e., the country's net foreign assets:

$$\frac{dw}{dt} \equiv wi^* + x_T - c_T. \tag{A.6}$$

Equations (A.4′) and (A.6), then, are sufficient to solve the overall aggregate system. These equations imply that, for a level of total net foreign assets, say, w_0, given by past history, ***and in the absence of future parameter changes***, the level of consumption of the traded good immediately adjusts so as to satisfy the condition $dw/dt = 0$, i.e., to the level $c_{T_0} = w_0 i^*$.[19] This is shown in Figure 6.

It should be pointed out, though, that fulfillment of the transversality condition in the aggregate system (A.4′)–(A.6) is a necessary but not a sufficient condition for the same to happen in both the private and government sectors. Since $w = a + A$, it is easy to think of situations in which $dw/dt = 0$ but $da/dt = -dA/dt \neq 0$. Notice that if head taxes (or subsidies) are assumed to endogenously accommodate to assure that both transversality conditions are met, then the real and monetary domestic sectors can be analyzed independently. Whether this procedure is appropriate depends on the purpose at hand.

Before proceeding, it is worthwhile to notice some of the features of this simple framework. The system (A.4′)–(A.6), for a given level of total net foreign assets and for exogenous levels of production, yields the values of private consumption of the traded good, c_T, and the eventual adjustment path of w for the case in which there is any anticipated future policy change. This level of c_T, via equation (A.4), determines the "real exchange rate," ε.

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[19] In more technical terms, the system exhibits a "saddle point" rather than a "saddle path."

The real exchange rate, then, is the relative price that results so as to assure individuals consume the available fixed flow of home goods. Finally, the value of consumption of the home good, c_H, together with the rate of devaluation ($\hat{E}$), feed into the demand for money (A.4″). This is not sufficient to determine the rate of devaluation and hence the real money stock: it becomes necessary to specify the behavior of monetary policy, i.e., the rate of devaluation (if the central bank pursues an exchange rate rule or the rate of monetary growth, $\mu \equiv (dM/dt)(1/M)$ if the central bank follows a monetary rule. In the latter case, the additional identity $dm/dt \equiv m\,\mu - m\,\pi$ is obtained.

The framework, then, exhibits a "double dichotomy": the "international sector" (equations (A.4′)–(A.7)) is determined independently of the rest; the "real domestic sector" (equations (A.4) and (A.5)) depends on the international sector, but is also independent of monetary considerations; finally, the "monetary sector" depends on the rest, but it does not influence any of it. Both the international and domestic real sectors can be determined, in principle, without a reference to the exchange rate and monetary policies of the central bank.

Figure 1. An Anticipated Devaluation

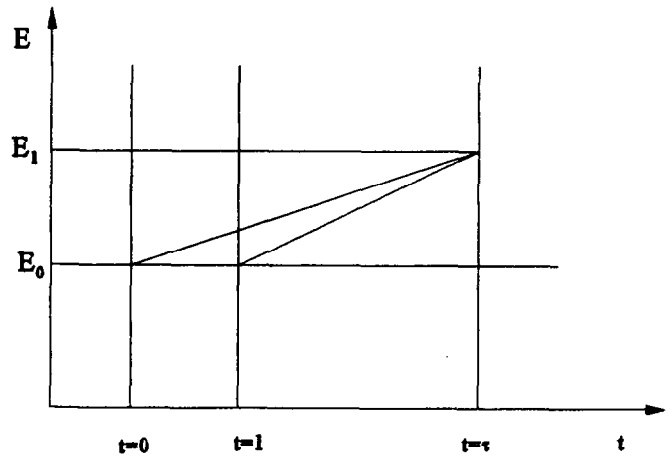


Figure 2. The Market for Home Goods

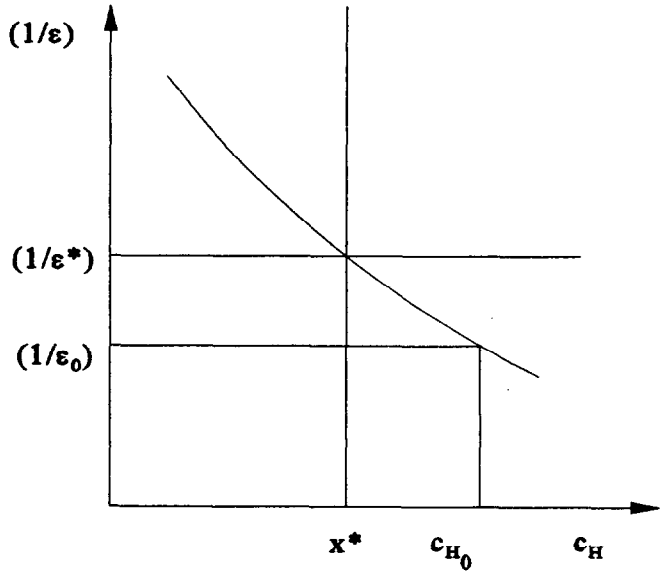


Figure 3. An Anticipated Devaluation Under an Exchange Rate Rule

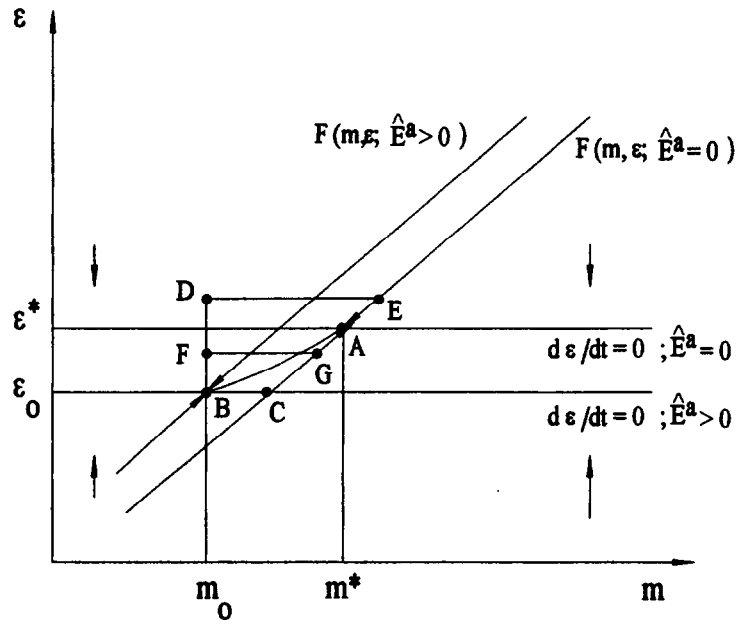


Figure 4. An Unanticipated Increase in the Money Stock Under a Monetary Rule

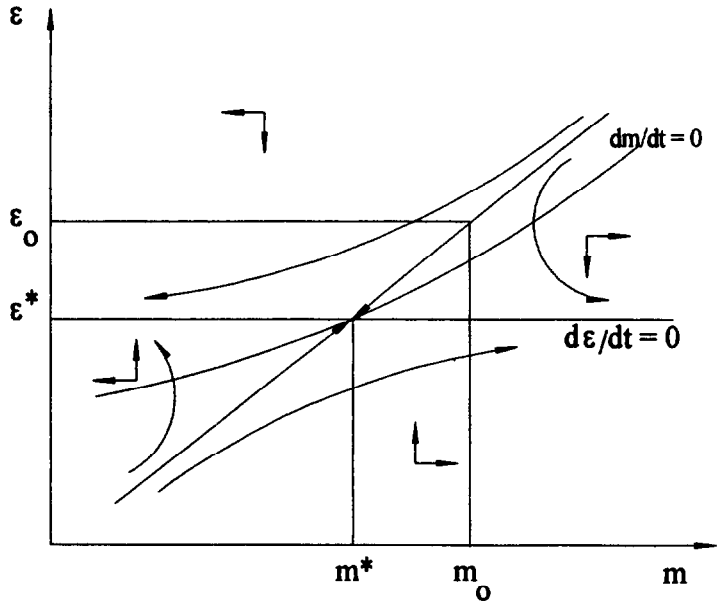


Figure 5. An Anticipated Increase in the Money Stock Under a Monetary Rule

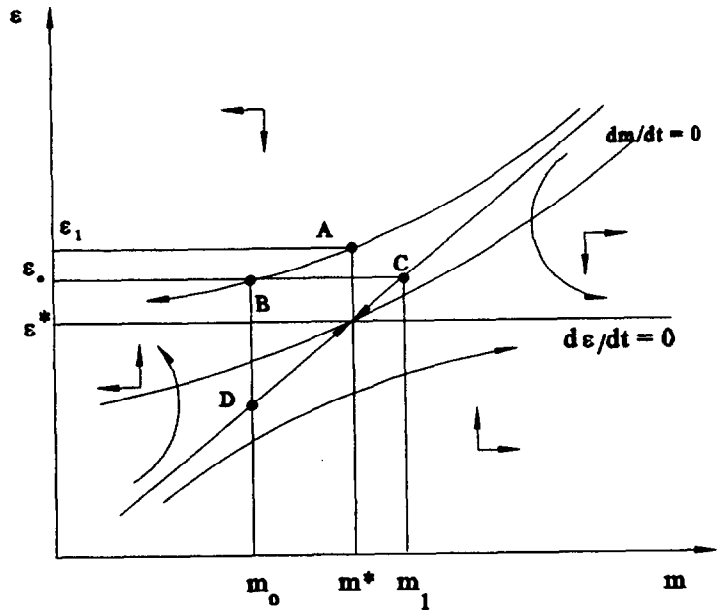
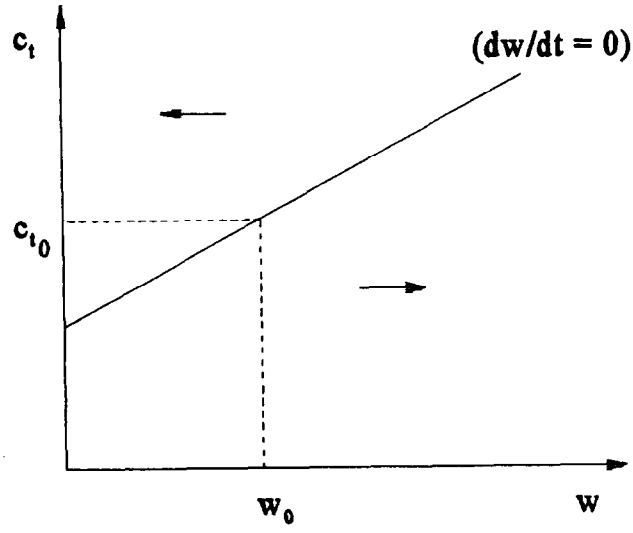
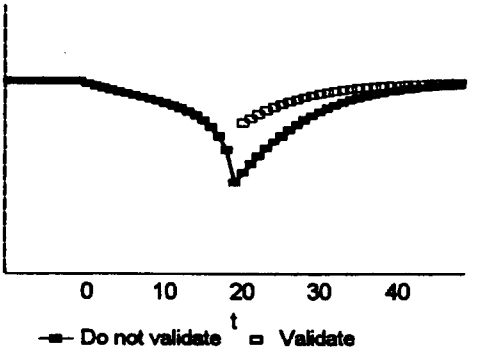


Figure 6. The Foreign Sector

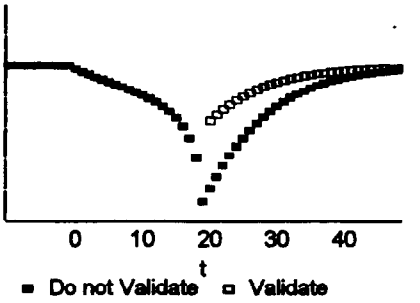


Panel A: An Anticipated Devaluation Under an Exchange Rate Rule, With and Without Validation

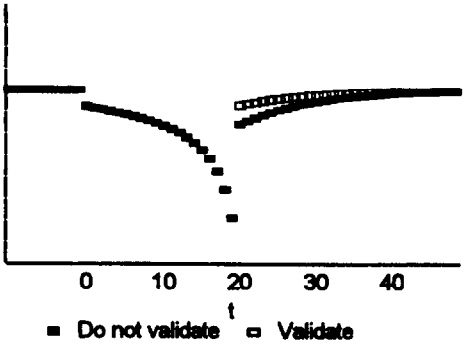
Real Exchange Rate



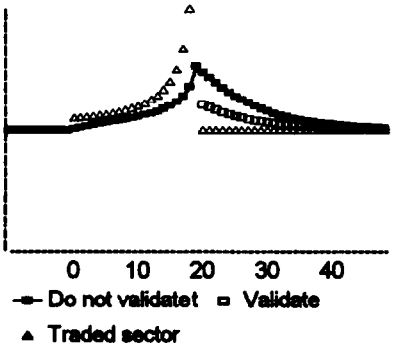
Output



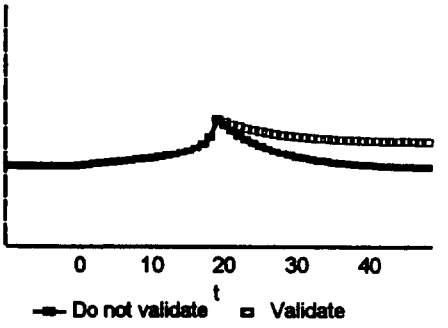
Real Money Stock



Real Interest Rates



Price of Home Goods



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