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## A Solution to Two Paradoxes of International Capital Flows

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**IMF Working Paper**

Research Department

**A Solution to Two Paradoxes of International Capital Flows**

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**Abstract**

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International capital flows from rich to poor countries can be regarded as either too low (the Lucas paradox in a one-sector model) or too high (when compared with the logic of factor price equalization in a two-sector model). To resolve the paradoxes, we introduce a non-neo-classical model which features financial contracts and firm heterogeneity. In our model, free patterns of gross capital flow emerge as a function of the quality of the financial system and the level of protection for property rights (i.e., the risk of expropriation). A poor country with an inefficient financial system but a low expropriation risk may simultaneously experience an outflow of financial capital but an inflow of foreign direct investment (FDI), resulting in a small net flow.

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## I. INTRODUCTION

While cross-border capital flows worldwide have risen substantially, reaching nearly \$6 trillion in 2004, less than 10 percent of them go to developing countries. Lucas (1990) famously pointed out that, relative to the implied difference in the marginal returns to capital between rich and poor countries in a one-sector model, it is a paradox that not more capital goes from rich to poor countries (the paradox of too small flows). The Lucas paradox could be turned on its head in a two-sector, two-factor, neoclassical trade model. A well-known result in such a model is factor price equalization (FPE): with free trade in goods, returns to factors are equalized between countries even without factor mobility. Given this, any amount of observed capital flows is excessive (the paradox of too large capital flows).

A number of solutions to the Lucas paradox have been proposed in the literature: (i) thinking of a worker in a rich country as effectively equivalent to multiple workers in a poor country, (ii) adding human capital as a new factor of production, (iii) allowing for sovereign risk, and (iv) adding costs of goods trade. We will argue in this paper that none of these explanations can escape from the tyranny of the FPE. Similarly, while a number of reasons have been proposed to explain why FPE doesn't hold, we argue that very few of them imply a pattern of capital flow that resolves the Lucas paradox.

We argue that it is useful to think outside the neoclassical box and propose a new micro-founded theory to understand goods trade and factor mobility. We introduce a financial contract model following Holmstrom and Tirole (1998) and heterogeneous firms into the Heckscher-Ohlin-Samuelson framework. A key feature of the new theory is that return to financial investment is generally not the same as return to physical investment. Financial investors (or savers) obtain only a slice of the return to physical capital, as they have to share the return to capital with entrepreneurs. The more developed a financial system is, the greater the slice that goes to the investors. An important implication is that countries with a low capital-to-labor ratio and an inefficient financial sector may experience a large gross outflow of financial capital, together with inward foreign direct investment (FDI), resulting in a small net inflow (or even outflow). Besides financial development, our model also incorporates property rights protection as another institution. In spite of a low capital-to-labor ratio, countries with poor property rights protection (high expropriation risk) may very well experience an outflow of financial capital without a compensating inflow of FDI.

To break FPE, one needs to show that factor prices are determined by variables in addition to product prices. One way to do it is to assume that the production function is decreasing return to scale (DRS) (e.g., Kraay and others, 2004; and Wynne, 2005). While this assumption may be appropriate in the short run, it is hard to explain why firms cannot adjust their factor usage in the long run. In our model, we retain constant returns to scale at the firm level but endogenously generate decreasing returns to scale at a sector level. Specifically, entrepreneurs are assumed to be heterogeneous in their ability to manage capital. As a sector expands, more entrepreneurs enter and the ability of the marginal entrepreneur declines and so does the return to investment at the sector level. Although

free trade in goods equalizes product prices, factor returns remain different across countries. Other things equal, the interest rate is lower and the wage rate is higher in the capital-abundant country. In other words, our two-sector model restores these results from a typical one-sector model (but still predicts a small net capital flow between rich and poor countries).

While many papers in the literature have emphasized risk sharing as a motive for international capital flow, our model deliberately avoids this by assuming all entrepreneurs and financial investors are risk-neutral. Adding risk-sharing in the future will further enrich patterns of capital flow but will not likely undo the basic mechanisms in this model. Even without a risk sharing motive, our model can generate two-way gross capital flows. In addition to solving the two paradoxes, the paper also aims to provide a general equilibrium framework in which financial and property rights institutions play a crucial role in determining patterns of gross and net capital flows. For example, considering the case in which the expropriation risk is identical across countries, entrepreneurs are perfectly mobile, but financial sector efficiency is uneven across countries, the paper shows that the unique equilibrium in the world capital market is one in which the less developed financial system is completely bypassed by financial investors and entrepreneurs. The country with the less developed financial system may experience a complete exodus of its savings in the form of financial capital outflow to the country with a better financial system, but see inflow of the FDI from the other country.

While the literature sometimes lumps together various types of institutions, financial and property rights institutions play very different roles in this model. Whereas an inefficient financial system can be bypassed, high expropriation risk cannot be. Indeed, if risk of expropriation differs across countries, there may not be a complete bypass of the inefficient financial system either. Financial capital still leaves the country with an inefficient financial system, but FDI may be deterred by a high expropriation risk in spite of a low labor cost in the country. In equilibrium, we show that the wage rate is always higher in the country with better financial or property rights institutions, irrespective of the country's initial endowment.

This paper is related to the theoretical literature that investigates the effects on capital flow of financial market imperfection. Gertler and Rogoff (1990) show that a moral hazard problem between foreign investors and domestic entrepreneurs may cause capital flow from poor to rich. Gordon and Bovenberg (1996) develop a model with asymmetric information between countries that explains possible differences in the real interest rates. Shleifer and Wolfenzon (2002) show that the country with better investor protection has a higher interest rate. Matsuyama (2004, 2005) and Aoki, Benigno, and Koyotaki (2006) study the effect of credit market constraint on capital flow. Stulz (2005) develops a model of agency problems of government and entrepreneurs that limit financial globalization. Caballero, Farhi, and Gourinchas (2005) show that lower capacity to generate financial assets reduces the interest rate. Our theory differs from these papers in three ways. First, all of the above papers use a one-sector model, whose prediction on capital flow does not generally survive an extension to a two-sector, two-factor model. Second, our model endogenously generates

two-way gross capital flow with a small net flow.<sup>2</sup> Third, our model is the first in the literature that studies possible contrasting effects of financial development and expropriation risk on capital flow.

Obstfeld and Rogoff (1997, p 438) and Ventura (1997) have already pointed out that the sensitivity of the interest rate to the capital-labor ratio is a special feature of the one-sector model. They do not, however, develop a new two-sector model that breaks the factor price equalization, and therefore, do not explain why some capital would flow internationally in a multisector model.

Our model features heterogeneous entrepreneurs, which is somewhat related to the models of heterogeneous firms in the international trade literature. Melitz (2003) develops a monopolistic competition model with heterogeneous firms. Bernard, Redding, and Schott (2005) incorporate firm heterogeneity, and product variety into HO framework and maintain the factor price equalization. To the best of our knowledge, our model is the first that studies the effect of firm (entrepreneur) heterogeneity on international capital flow in a two-sector, two-factor framework.

The rest of paper is organized as follows. Section II reviews the two paradoxes of capital flow in the neoclassical theory. Section III sets up our model. Sections IV and V study the aggregation and equilibrium conditions, and some key comparative statics, respectively. Section VI analyzes different forms of international capital flow under free trade in goods. Section VII concludes. An appendix provides the formal proofs for the propositions in the text and a table of the notations. A set of figures illustrate the model.

## II. PARADOXES OF INTERNATIONAL CAPITAL FLOWS

In this section we examine return to capital in standard neoclassical models. The production functions generate constant returns to scale, and firms are perfectly competitive. We start with a one-sector model and then move to a two-sector model.

### A. The Lucas Paradox of Too Small Capital Flows

Using a one-sector model, Lucas (1990) suggested that it was a paradox that more capital does not flow from rich to poor countries. His reasoning goes as follows. Let  $y = f(L, K)$  be the production function where  $y$  is the output produced using labor  $L$  and capital  $K$ . Let  $p$  be the price of goods, and  $w$  and  $r$  be returns to labor and capital, respectively. Firm's profit maximization problem gives

$$r = p \partial f(L, K) / \partial K = p \partial f(1, K/L) / \partial K \quad (1)$$

With free trade, the price of goods is equalized across countries. The Law of Diminishing Marginal Product implies that  $r$  is higher in the country with lower per capita capital. As

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<sup>2</sup>Caballero, Farhi, and Gourinchas (2005) have as an extension of the model that includes a multiple sectors. Their purpose is to study the effect of exchange rate adjustment. Factor allocation across sectors and therefore possible factor price equalization across countries are not studied in their paper. While they also allow for two-way gross flows, its micro-foundation, however, is not developed.

an illustration, Lucas calculated that the return to capital in India should be 58 times as high as that in the United States. Facing a return differential of this magnitude, Lucas argued, we should observe massive capital flows from rich to poor countries. That it does not happen has come to be known as the Lucas paradox.

## B. The Opposite Paradox of Too Large Capital Flows

The logic of the Lucas paradox can be turned on its head in a multisector model. Specifically, in a standard Heckscher-Ohlin-Samuelson two goods, two factors, and two countries model, firms earn zero profit. So we must have:

$$p_1 = c_1(w, r) \text{ and } p_2 = c_2(w, r) \quad (2)$$

where  $c(\cdot)$  is the unit cost function and subscripts represent sectors. Comparing to the one-sector model, now

$$r = p_i \partial f_i(1, K_i/L_i) / \partial K = p_i \partial f_i(1, a_{iK}/a_{iL}) / \partial K, \text{ for } i = 1, 2 \quad (3)$$

where  $a_{iK}/a_{iL} = \frac{\partial c_i(w, r) / \partial r}{\partial c_i(w, r) / \partial w}$  is capital-labor ratio per unit of production. For given product prices,  $w$  and  $r$ , and therefore  $a_{iK}/a_{iL}$ , are determined and independent from factor endowments  $L$  and  $K$ —the well-known “factor price insensitivity” (Leamer, 1995). Increases in  $K$  change the composition of outputs: more capital-intensive goods and less labor-intensive goods, will be produced, but the marginal return to physical capital in each sector stays unchanged. Free trade equalizes product prices, and therefore equalizes the return to factors across countries, even in the absence of international factor movements. This result was first proved by Samuelson (1948, 1949) and has come to be known as the “Factor Price Equalization Theorem (FPE).” Countries indirectly export their abundant factors through trade in goods. The capital flow is completely substituted by goods trade. There is no incentive for any amount of capital to flow between countries once there is free trade in goods.

One might think that the assumptions needed for FPE are surely too restrictive to be realistic and are not likely to hold once one goes beyond the  $2 \times 2 \times 2$  model. Deardoff (1994) derives a necessary condition of the FPE, known as the “lens condition” in a  $n$  goods,  $m$  factors, and  $H$  countries model. The condition has also been proved to be sufficient in a model of  $n$  goods, 2 factors, and  $H$  countries by Xiang (2001).<sup>1</sup> We offer an intuitive version of sufficient condition of FPE below. As we will see, such a condition is relatively weak.

We assume  $m \leq n$ , and that countries can always be ranked in a way so that at least  $m$  products are commonly produced by a pair of neighboring countries. For example, for countries  $h$  and  $h + 1$ , they both produce products  $n_1^{hh+1}, n_2^{hh+1}, \dots, n_m^{hh+1}$ . Neighboring countries may specialize in the rest of  $n - m$  products. Note that we only require

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<sup>1</sup>For a recent discussion on the lens condition and additional literature review, readers are guided to Bernard, Robertson and Schott (2004).



neighboring countries to produce a common set of  $m$  products. They do not have to trade directly with each other. Furthermore, non-neighboring countries may specialize in different sets of products. For countries  $h$  and  $h + 1$ , zero profit conditions in sectors  $n_1^{hh+1}, n_2^{hh+1}, \dots, n_m^{hh+1}$  give

$$p_i^{hh+1} = c_i^{hh+1}(w_1^h, \dots, w_m^h) \text{ for } i = 1, \dots, m \text{ and} \quad (4)$$

$$p_i^{hh+1} = c_i^{hh+1}(w_1^{h+1}, \dots, w_m^{h+1}) \text{ for } i = 1, \dots, m \quad (5)$$

$m$  equations in (4) determine  $m$  factor prices  $w_1^h, \dots, w_m^h$  for country  $h$ , while  $m$  equations in (5) determine  $m$  factor prices  $w_1^{h+1}, \dots, w_m^{h+1}$  for country  $h + 1$ . Because the technology is assumed to be identical across countries, and product prices are equalized under free trade, factor prices in these two countries must be the same. By the same logic, factor prices in countries  $h + 1$  and  $h + 2$  must be equal. Extending the logic, factor prices in all countries are equalized. As an illustration, consider a world with two factors, labor and land. Factor returns in the United States and India can be equalized even if the two countries do not trade each other, and do not produce any product in common. All that is needed is for the United States and India to be linked by a sequence of country pairs, with enough common products within each pair. For example, the United States and Greece may both produce apples and apricots, Greece and Thailand may both produce beer and bottles, and Thailand and India may both produce cotton and carriages. Free trade in goods would ensure factor price equalization between the United States and India. We summarize the result by the following chain rule of factor price equalization.

**Lemma 1** *Let the number of factors be  $m$  in a standard neoclassical model. For any two countries, if they can be linked by a sequence of country pairs, and the countries in each pair produce a common set of  $m$  products, then the factor prices are equalized among all these countries in a free-trade world, even in the absence of international factor movement.*

Lucas (1990) himself provided three explanations for the puzzle of too small capital flows. The first is an *effective labor* differentiation: if each U.S. worker is five times as productive as an Indian, holding other things constant, then the predicted return to capital in India became 5 rather than 58 times that in the United States. We can show that this intuition does not survive a generalization from a one-sector to a two-sector model. Let production function be  $y_i = f_i(EL_i, K_i)$  where  $E$  represents labor productivity. It can be shown that the zero profit conditions in a two goods, two factors model become

$$p_1 = c_1\left(\frac{w}{E}, r\right) \text{ and } p_2 = c_2\left(\frac{w}{E}, r\right)$$

which gives rise to a unique solution  $\left[\left(\frac{w}{E}\right), r\right]$ . Note that  $\left(\frac{w}{E}\right)$  and  $r$  are determined by  $(p_1, p_2)$ . For given product prices, the increase in labor productivity  $E$  will increase the wage rate  $w$  proportionally so as to keep  $\left(\frac{w}{E}\right)$  constant. The return to capital,  $r$ , is not affected by the increase in  $E$ . That is, in the two-sector model, if the U.S. worker is 5 times more productive than the Indian workers, then the wage rate in the United States is exactly 5 times higher than in India. The return to capital, however, is still the same between the countries.

Lucas' second explanation is *missing factor(s)*. If human capital is to be included as another factor, then the predicted return to capital in India would be further reduced from 5 to 1.04 times that in the United States. This argument, once again, does not survive a generalization to a two-sector model. Using our chain rule of factor price equalization, the returns to the three factors (capital, labor, and human capital) are equalized across countries as long as at least three common products are produced by a sequence of country pairs. Free trade in goods substitutes factor flow. The abundance of human capital in the United States does not affect the return to capital, but simply changes the composition of outputs.

Lucas' third explanation, downplayed by him but emphasized by Reinhart and Rogoff (2004), is *sovereign risk*. The risk of sovereign default prevents capital from flowing from rich to poor countries. In a two-factor, two-sector model, free trade in goods has already led to equal return to capital across countries. There is no room for sovereign risk to further affect the return to capital.

What about the various reasons for why FPE does not hold? Consider first the costs of goods trade (see, for example, Obstfeld and Rogoff, 2000). Trading costs do break FPE, and so it is a valid explanation in a static sense. However, it does not work in a dynamic sense. As tariffs and transportation costs have declined over the last four decades, goods prices should have converged across countries. By the logic of FPE, factor returns should have converged as well. A two-sector, two-factor model would predict a decline in international capital flows, which obviously is contradicted by the data.

A popular explanation for both paradoxes is cross-country differential in total factor productivity (TFP), of which difference in legal institutions is a special case. If TFP is different, the return to factors is, of course, different across countries. The TFP explanation, however, may not predict, the direction of capital flows. Let the TFP in foreign country be higher in the two-goods, two-factors, and two-countries model. That is,

$$p_1 = B_1 c_1(w^*, r^*) \text{ and } p_2 = B_2 c_2(w^*, r^*) \quad (6)$$

and  $B_i < 1$ .<sup>2</sup> Let sector 1 be labor-intensive. Using the Stolper-Samuelson theorem, higher TFP ( $B_1 < 1$ ) in sector 1 increases  $w^*$  but reduces  $r^*$ , while higher TFP in sector 2 increases  $r^*$  but reduces  $w^*$ . Unless we know exactly the magnitudes of TFP in all sectors, the return to capital in the more technologically advanced country can be either higher or lower. Differences in institutions may have asymmetric effects on productivity for different sectors. Unless a structural model of institution is developed, as we do in the next section, reduced form TFP may be too general to predict the direction of capital flow.

Furthermore, it is worth pointing out that no equilibrium exists in the HOS model once a difference in technology and free capital flows are allowed, unless in knife-edge or specialization cases. We can prove this by contradiction. If free trade in goods leads to  $p_i = p_i^*$ , then (2) and (6) imply that  $r \neq r^*$  in most cases, and so capital must flow. But if

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<sup>2</sup>A superscript “\*” is used to denote variables in the foreign country.

free capital flows lead to  $r = r^*$ , then (2) and (6) imply that  $p_i \neq p_i^*$ ; and so the goods market would be out of equilibrium. Thus, no equilibrium exists in general.<sup>3</sup>

It is useful to note that we are not claiming that factor price equalization is realistic. However, we point out that it is perhaps more difficult to escape from the tyranny of TFP than the existing literature may indicate. Both the Lucas paradox and FPE rely on the assumption that marginal product of physical capital determines capital flow.<sup>4</sup> In general, the return to financial investment and the return to physical capital do not have to be the same. Our model will make this point.

### III. THE MODEL

We introduce financial contracts between investors and entrepreneurs a la Holmstrom and Tirole (1998), and entrepreneurs' heterogeneity into an otherwise two-good, two-factor, and two-country HOS framework.

#### A. Basic Setup

Focusing for the moment on a single country, we assume that the production process takes two periods and each firm has a stochastic technology. The first period production function of industry  $i$  is  $y_i^1 = G_i(L_i^1, K_i^1)$  ( $i = 1, 2$ ), where the superscript 1 denotes date 1. The labor-capital ratio,  $L_i^1/K_i^1$ , is assumed to be fixed and denoted by  $a_i$ .

The timing of events is described in Figure 1. An initial investment  $K_i^1$  is injected to the firm at date 1. Correspondingly,  $a_i K_i^1$  amount of labor is hired. At the beginning of date 2, a liquidity shock occurs. An additional and uncertain amount  $\rho_i K_i^1 > 0$  of financing is needed to cover operating expenditures and other needs. The liquidity shock per unit of capital,  $\rho_i$ , is distributed according to a cumulative distribution function  $F_i(\rho)$  with a density function  $f_i(\rho)$ . The firm (entrepreneur) then makes a decision on whether to continue or abandon the project. If  $\rho_i K_i^1$  is paid, the project continues and output  $G_i(a_i, 1)K_i^1$  will be produced at the end of date 2. In the process,  $a_i \rho_i K_i^1$  amount of additional labor is hired and paid for as a part of additional financing. If  $\rho_i K_i^1$  is not paid, however, the project is terminated and yields no output.

Investment in the firm is subject to a moral hazard problem. The utility for entrepreneur  $n \geq 1$  of managing one unit of capital in sector  $i$  is defined as

$$V_{ni}(e) = \lambda_i(e)R_{ni}^E - c_{ni}(e) \quad (7)$$

where  $e$  denotes the level of effort which takes a binary value of either high,  $e^H$  (work), or low,  $e^L$  (shirk).  $R_{ni}^E$  is what the entrepreneur gets from managing one unit of capital if the

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<sup>3</sup>We thank Arvind Panagariya for pointing this out. Another textbook explanation for the failure of FPE is the existence more factors than goods. The simplest case would be a one-product and two-factor model: two identical goods, each produced by one country, with immobile labor and mobile capital across countries, which would be the same model used by Lucas and would lead to the Lucas paradox.

<sup>4</sup>This view is common in the literature. In models without risk, Ventura (2003, p. 488) states that the rule is: "invest your wealth in domestic capital until its marginal product equals the world interest rate."

project succeeds. If the entrepreneur works, the probability of success is  $\lambda_i(e^H)$ ; if she shirks, the probability of success is  $\lambda_i(e^L)$ . For simplicity, the probability of success is assumed to be identical across all entrepreneurs. However, the cost of “work”,  $c_{ni}(e^H)$ , is heterogeneous across entrepreneurs. We normalize the cost of “shirk” to zero. Furthermore, in subsequent discussion, we assume  $\lambda_1(e^H) = \lambda_2(e^H) = \lambda$  and normalize  $\lambda_i(e^L) = 0$ .

The firm is run by the entrepreneur who owns a part of the firm. In the absence of proper incentives, the entrepreneur may deliberately reduce the effort level in order to reduce the effort cost. The entrepreneur makes a decision on the effort level after the continuation decision is made at date 2. The labor is paid at  $w$  in the second period if the project succeeds and zero if it fails. Consumption takes place at the end of the second period.

The total return to one unit of initial capital if the project succeeds,  $R_i$ , is determined by firm’s zero profit condition

$$p_i y_i^1 - w L_i^1 = [p_i G_i(a_i, 1) - w a_i] K_i^1 = R_i K_i^1 \quad (8)$$

On date 2, the first period investment  $K_i^1$  is sunk. The net present value of the investment is maximized by continuing the project whenever the expected return from continuation,  $\lambda R_i$ , exceeds the cost  $\rho_i$ , that is,  $\lambda R_i - \rho_i \geq 0$ . Let

$$\rho_i^1 = \lambda R_i \quad (9)$$

be the *first-best cutoff value* of  $\rho_i$ . As in Holmstrom and Tirole (1998), we assume that the project’s net present value is positive if the entrepreneur works; but negative if she shirks. Therefore, we only need to consider those contracts that implement a high level of effort.

One institutional feature emphasized in this model is property rights protection, or control of the risk of expropriation. An emerging literature has suggested that cross-country differences in property rights protection are a major determinant of cross-country differences in long-run economic growth and patterns of international capital flow (see, for example, Acemoglu, Johnson, and Robinson, 2001; and Alfaro, Kalemli-Ozcan, and Volosovych, 2005). One could conveniently think of a higher value of  $\lambda$  in our model as representing better property rights protection (or lower expropriation risk). Equivalently, a higher value of  $\lambda$  also represents a lower tax rate on capital return.

## B. Financial Contracts

There are  $K$  number of capitalists in the country. Each capitalist is assumed to be born with 1 unit of capital and an index  $n$ , which determines her cost of effort and is observable. She can choose to become either an entrepreneur or a financial investor at the beginning of date 1. If she chooses to be an entrepreneur, she would manage one project, investing her 1 unit of capital (labeled as internal capital) and raising  $K_{ni}^{X1}$  amount of external capital from financial investors. The total initial investment in the firm is the sum of internal and external capital, or  $K_{ni}^1 = 1 + K_{ni}^{X1}$ . She and financial investors sign a contract at the beginning of date 1, which specifies the total amount of investment, her

plan on whether to continue or terminate the project for every realization of the liquidity shock, and how the final project return is going to be divided between the financial investors and herself. More precisely, let  $C_{ni} = \{K_{ni}^1, \mu_{ni}(\rho_i), R_{ni}^E(\rho_i)\}$  be the financial contract, where  $\mu_{ni}(\rho_i)$  is a state-contingent policy on project continuation ( $1 = \text{continue}$ ,  $0 = \text{stop}$ ), and  $R_{ni}^E(\rho_i)$  is the entrepreneur's portion of the revenue per unit of investment. For every dollar of investment, investors are left with  $R_i - R_{ni}^E(\rho_i)$ . If the project is terminated, both sides are assumed to receive zero.

An optimal contract can be found by choosing  $\{K_{ni}^1, \mu_{ni}(\rho_i), R_{ni}^E(\rho_i)\}$  to solve the following entrepreneur's optimization problem.

$$\max U_{ni} = \left( \frac{1}{1+r} \right) K_{ni}^1 \int \lambda R_{ni}^E(\rho_i) \mu_{ni}(\rho_i) f_i(\rho_i) d\rho_i - 1 \quad (10)$$

subject to

$$\left( \frac{1}{1+r} \right) K_{ni}^1 \int \{\lambda[R_i - R_{ni}^E(\rho_i)] - \rho_i\} \mu_{ni}(\rho_i) f_i(\rho_i) d\rho_i \geq K_{ni}^{X1} \quad (11)$$

and

$$\lambda R_{ni}^E(\rho_i) - c_{ni}(e^H) \geq 0 \quad (12)$$

Expression (10) is the present value of the firm's *net return to internal capital*. (11) is the participation constraint for outside investors, while (12) is the entrepreneur's incentive compatibility constraint.

Solving the above problem, the optimal continuation policy  $\mu_{ni}(\hat{\rho}_{ni})$  takes the form of a cutoff rule so that the project continues, or  $\mu_{ni}(\rho_i) = 1$  if  $\rho_i \leq \hat{\rho}_{ni}$ , and it terminates, or  $\mu_{ni}(\rho_i) = 0$  if  $\rho_i > \hat{\rho}_{ni}$ . Note that the most that the firm (the entrepreneur) can promise to outside investors at date 2 is  $\rho_{ni}^{\max} = \lambda[R_i - R_{ni}^E(\rho_i)]$ . As is shown in Holmstrom and Tirole (1998), the incentive compatibility constraint (12) must be binding, which gives

$$R_{ni}^E = \frac{c_{ni}(e^H)}{\lambda} \text{ and } \rho_{ni}^{\max} = \lambda R_i - c_{ni}(e^H) \quad (13)$$

The participation constraint (11) is also binding, which implies that the firm's initial investment is

$$K_{ni}^1(.) = \frac{1+r}{(1+r) - \int_0^{\hat{\rho}_{ni}} (\rho_{ni}^{\max} - \rho_i) f_i(\rho_i) d\rho_i} \quad (14)$$

Substituting binding constraints (11) into (10), the firm's *net return to internal capital* becomes

$$U_{ni}(\hat{\rho}_{ni}) = \frac{\lambda R_i - h(\hat{\rho}_{ni})}{h(\hat{\rho}_{ni}) - \rho_{ni}^{\max}} \quad (15)$$

where

$$h_i(\hat{\rho}_{ni}) = \frac{(1+r) + \int_0^{\hat{\rho}_{ni}} \rho_i f_i(\rho_i) d\rho_i}{F_i(\hat{\rho}_{ni})} \quad (16)$$

$h_i(\widehat{\rho}_{ni})$ , in the terminology of Holmstrom and Tirole (1998), is called *expected unit cost of total investment*, which is the opportunity cost of initial investment at date 1,  $(1+r)$ , plus the expected financing for the liquidity shock at date 2,  $\int_0^{\widehat{\rho}_{ni}} \rho_i f_i(\rho_i) d\rho_i$ , under the condition that the project continues. Maximizing  $U_{ni}(\widehat{\rho}_{ni})$  is equivalent to minimizing  $h(\widehat{\rho}_{ni})$ . The first order condition then gives

$$\int_0^{\rho_{ni}^{opt}} F_i(\rho_i) d\rho_i = 1 + r \quad (17)$$

$\rho_{ni}^{opt}$  gives the second-best solution to the project cutoff point in response to liquidity shocks. Note that equation (17) implies that  $\rho_{ni}^{opt}$  is independent of  $n$ . Thus all entrepreneurs in sector  $i$  have the same optimal cutoff,  $\rho_{ni}^{opt} = \rho_i^{opt}$ . Equation (17) shows  $\rho_i^{opt}$  increases as  $r$  increases. Intuitively, as the interest rate increases, the opportunity cost of the investment becomes higher. To attract investors to the project, the firm needs to promise a higher probability that the project will continue in the face of a liquidity shock, which implies higher optimal cutoff point  $\rho_i^{opt}$ .

We will assume that  $f_1(\rho_1) = f_2(\rho_2) = f(\rho)$  has a uniform distribution in  $[0, \bar{\rho}]$  thereafter. Then equation (17) gives the solution of  $\rho_i^{opt} = \rho^{opt}$  as

$$\rho^{opt} = [2(1+r)\bar{\rho}]^{\frac{1}{2}} \quad (18)$$

As indicated by Holmstrom and Tirole (1998), the optimal cutoff of the liquidity shock  $\rho_i^{opt}$  is between the most that the entrepreneur can promise to outside investors at date 2,  $\rho_{ni}^{\max}$ , and the *first-best cutoff*,  $\rho_i$ . That is,  $\rho_{ni}^{\max} < \rho_i^{opt} \leq \rho_i^1$ . Ex post, investors would rather write the project off if  $\rho_i > \rho_{ni}^{\max}$ . Therefore, to implement the second-best liquidity solution  $\rho_i^{opt}$ , the financial system must be sufficiently developed for investors to commit funds in advance.

We now introduce financial development into our model. We use  $\theta$  to represent *the level of financial development* of a country. More precisely, we assume only liquidity shocks  $\widehat{\rho}_{ni} \leq \theta \rho_i^{opt}$  can be met by the financial system. Higher  $\theta$  represents a more developed financial system. Two interpretations are possible: either each firm is financed up to the liquidity shock  $\widehat{\rho}_{ni} = \theta \rho_i^{opt}$ , or  $\theta$  portion of firms are financed up to  $\rho_i^{opt}$  and  $1 - \theta$  portion of firms are not financed for any shock.  $\min\{\rho_{ni}^{\max}/\rho_i^{opt} \text{ for all } n \text{ and } i\} \leq \theta \leq 1$ .<sup>5</sup>

Let  $\widehat{\rho}_{ni} = \theta \rho_i^{opt}$ . Expression (16) now becomes

$$h_i(\widehat{\rho}_{ni}) = h(r, \theta) = \left( \frac{1 + \theta^2}{\sqrt{2}\theta} \right) [(1+r)\bar{\rho}]^{\frac{1}{2}} \quad (19)$$

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<sup>5</sup>Ju and Wei (2005) use a country's capacity of external capital to represent the level of financial development. Financial system consists of both the financial market and public supply of liquidity. In an economy where both individual and aggregate uncertainties exist, Holmstrom and Tirole (1998) show that the financial market alone cannot provide enough funds to meet firms' liquidity demands at the optimal policy  $\rho_i^{opt}$ .

It is easy to see that  $\partial h/\partial r > 0$  and  $\partial h/\partial \theta < 0$ .

Let there be a continuum of entrepreneurs (firms) in type  $n$  with a unit mass.  $F_i(\rho_i)$  denotes both the ex ante probability of a firm facing a liquidity shock below  $\rho_i$ , and a realized fraction of firms with liquidity shock below  $\rho_i$  in sector  $i$ . The total capital usage by type  $n$  entrepreneur is the sum of initial investment  $K_{ni}^1(\cdot)$  and expected liquidity shocks being paid. Denoting the total capital usage by  $K_{ni}$ ,

$$\begin{aligned} K_{ni}(\cdot) &= \left[ 1 + \left( \frac{1}{1+r} \right) \int_0^{\theta \rho^{opt}} \rho_i f_i(\rho_i) d\rho_i \right] K_{ni}^1(\cdot) \\ &= \frac{(1+r) + \int_0^{\theta \rho^{opt}} \rho_i f_i(\rho_i) d\rho_i}{(1+r) - \int_0^{\theta \rho^{opt}} (\rho_{ni}^{\max} - \rho_i) f_i(\rho_i) d\rho_i} \end{aligned} \quad (20)$$

The labor-capital ratio for firm  $n$  in the entire production process is

$$a_{ni} = \frac{L_{ni}}{K_{ni}} = a_i \quad (21)$$

which is identical for all entrepreneurs in sector  $i$ .

### C. Allocation of Capital and Market for Entrepreneurs

There are two sectors in the economy. Sector 1 is assumed to be one in which entrepreneurs' cost of "work" differs. We rank entrepreneurs by their costs of work from low to high, and index them by  $n$  directly. Entrepreneur  $n$  has lower cost of work than that of the entrepreneur  $n'$  if  $n < n'$ . In other words, the cost of work by entrepreneur  $n$  in sector 1,  $c_{n1} = c_{n1}(e^H)$ , is an increasing function in  $n$ . We will assume  $c_{n1} = c_1 n$  for simplicity. Expression (13) gives  $\rho_{n1}^{\max} = \lambda R_1 - c_1 n$ , which is decreasing in  $n$ . Expression (15) then implies that the firm's *net return to internal capital* in sector 1,  $U_{n1}(\cdot)$  is decreasing in  $n$ .

In Sector 2, all entrepreneurs are assumed to have the same cost of work. That is,  $c_{n2}(e^H) = c_2$ . Expression (13) indicates that  $\rho_{n2}^{\max} = \lambda R_2 - c_2$ , which is identical for all entrepreneurs. Thus, all entrepreneurs have the same profit,  $U_2(\cdot)$ , in sector 2. Let  $N_1$  be the number of firms in Sector 1.  $N_1$  solves for

$$U_{N_1 1} = \frac{\lambda R_1 - h(r, \theta)}{h(r, \theta) - [\lambda R_1 - c_1 N_1]} = U_2 \quad (22)$$

As Figure 2 illustrates, entrepreneurs in the interval of  $[1, N_1]$  enter Sector 1 and earn the *net return to internal capital*  $U_{n1} \geq U_2$ . Entrepreneurs of  $n > N_1$  enter Sector 2 and earn the *net return to internal capital*  $U_2$ .

We assume that a capitalist (a potential entrepreneur) needs to pay a fixed entry cost of  $f$  units of the numeraire good at the beginning of the first period to become an entrepreneur. The *net return to internal capital* in Sector 2,  $U_2$ , should be equal to  $f$ . On the other hand, the marginal entrepreneur in Sector 1,  $N_1$ , should have the same *net return to internal*

*capital* as  $f$ , while all other entrepreneurs in Sector 1 earn higher net returns. Using equation (22), the conditions can be stated as

$$\begin{aligned} U_{N_1} &= \frac{\lambda R_1 - h(r, \theta)}{h(r, \theta) - [\lambda R_1 - c_1 N_1]} = f \\ U_2 &= \frac{\lambda R_2 - h(r, \theta)}{h(r, \theta) - [\lambda R_2 - c_2]} = f \end{aligned} \quad (23)$$

The career choice of a capitalist (between being an entrepreneur and a financial investor) is determined by the interest rate  $r$ . If the return to investment  $r$  increases, the *net return to internal capital* in sector 2,  $U_2$ , declines. Thus, some entrepreneurs in Sector 2 would exit and become financial investors.

It is clear from (11) that investors' expected revenue from the project is larger as entrepreneur's pay to work,  $R_{ni}^E$ , becomes smaller. For a given interest rate  $r$ , that means date 1 investment  $K_{ni}^1$  is larger as indicated by (14). Expression (20) then implies that total capital managed by the entrepreneur in Sector 1 is larger for more productive managers (smaller  $n$ ). We summarize our results by the following lemma.

**Lemma 2** *As the interest rate increases, fewer capitalists choose to become entrepreneurs at the beginning of date 1. Among the entrepreneurs, the more productive ones enter the heterogeneous sector, while the less productive ones enter the homogeneous sector. In the heterogeneous sector, the more productive entrepreneurs manage more capital.*

Note that part of the lemma resembles the results in Shleifer and Wolfenzon's (2002) one-factor model. In particular, in their model, it is also the case that fewer capitalists become entrepreneurs when the interest rate increases, and more productive entrepreneurs manage more capital.

#### IV. AGGREGATION AND EQUILIBRIUM CONDITIONS

The first set of equilibrium conditions are free entry conditions which are summarized by equations (23). Rewrite them as

$$\begin{aligned} \lambda R_1 &= h(r, \theta) + \frac{f c_1 N_1}{1 + f} \\ \lambda R_2 &= h(r, \theta) + \frac{f c_2}{1 + f} \end{aligned} \quad (24)$$

which we label as *capital revenue-sharing conditions*. The lefthand sides of equations (24) are expected marginal products of physical capital in two sectors, respectively. Each is a sum of an expected unit cost of total investment,  $h(r, \theta)$ , and a payment to the entrepreneurs' efforts. Using (19), it is clear that  $R_i$  is uniquely determined by the interest rate  $r$ .



The second set of equilibrium conditions comprises full employment conditions. Each entrepreneur in Sector 2 manages  $K_2(\cdot)$  amount of capital. Entrepreneur  $n$  in Sector 1 manages  $K_{n1}(\cdot)$  amount of capital. (21) implies that the labor-capital ratio is identical for all entrepreneurs within a sector. Let the number of entrepreneurs in Sector 2 be  $N_2$ . Let  $L$  and  $K$  be the country's labor and capital endowments, respectively. The full employment conditions are

$$a_1 \int_1^{N_1} K_{n1}(\cdot) dn + a_2 K_2(\cdot) N_2 = L \quad (25)$$

$$\int_1^{N_1} K_{n1}(\cdot) dn + K_2(\cdot) N_2 = K \quad (26)$$

Substituting (13), (16), and (24) into (20), we obtain

$$K_{n1}(\cdot) = \frac{h(r, \theta)}{c_1 [n - (fN_1) / (1 + f)]} \text{ and } K_2(\cdot) = \frac{h(r, \theta) (1 + f)}{c_2} \quad (27)$$

Applying expressions (27) to (25) and (26), we can rewrite the full employment conditions as follows:

$$a_{1L} \ln \left[ \frac{N_1}{1 + f - fN_1} \right] + a_{2L} N_2 = L \quad (28)$$

$$a_{1K} \ln \left[ \frac{N_1}{1 + f - fN_1} \right] + a_{2K} N_2 = K \quad (29)$$

where

$$\begin{aligned} a_{1L} &= \frac{a_1 h(r, \theta)}{c_1}, \quad a_{1K} = \frac{h(r, \theta)}{c_1} \\ a_{2L} &= \frac{a_2 (1 + f) h(r, \theta)}{c_2}, \text{ and } a_{2K} = \frac{(1 + f) h(r, \theta)}{c_2} \end{aligned} \quad (30)$$

We close this section with the market-clearing conditions in product markets. The firms' expected output (or the realized industry output) in Sector 1 is

$$\begin{aligned} y_1 &= F_1(\theta \rho^{opt}) \lambda G_1(a_1, 1) \int_0^{N_1} K_{n1}^1(\cdot) dn \\ &= \frac{G_1(a_1, 1) \lambda (1 + r)}{c_1} \ln \left[ \frac{N_1}{1 + f - fN_1} \right] \end{aligned} \quad (31)$$

where we have used (14), (16) and (24) to derive the second equality. The expected output

in Sector 2 is

$$\begin{aligned} y_2 &= F_2(\theta \rho^{opt}) \lambda G_2(a_2, 1) K_2^1(\cdot) N_2 \\ &= \frac{G_2(a_2, 1) \lambda (1+r) (1+f) N_2}{c_2} \end{aligned} \quad (32)$$

We assume that the representative consumer's preference is homothetic. Thus, the ratio of the quantities consumed in the country depends only upon the relative goods price ratio and can be represented by  $D(\frac{p_1}{p_2})$ . In equilibrium, the relative supply equals the relative demand. The condition is stated as

$$\frac{y_1}{y_2} = \left[ \frac{G_1(a_1, 1) c_2}{G_2(a_2, 1) (1+f) c_1} \right] \frac{\ln \left[ \frac{N_1}{1+f-fN_1} \right]}{N_2} = D(p) \quad (33)$$

where  $p = p_1/p_2$ . Let good 2 be the numeraire good whose price is normalized to 1 in subsequent sections.

## V. COMPARATIVE STATICS

Substituting (9), (13), and (19) into (24), the free entry conditions can be written as

$$\lambda a_1 w + \frac{1+\theta^2}{2\theta} [2(1+r)\bar{\rho}]^{\frac{1}{2}} = \lambda p G_1(a_1, 1) - \frac{f c_1 N_1}{1+f} \quad (34)$$

$$\lambda a_2 w + \frac{1+\theta^2}{2\theta} [2(1+r)\bar{\rho}]^{\frac{1}{2}} = \lambda G_2(a_2, 1) - \frac{f c_2}{1+f} \quad (35)$$

The endogenous variables,  $w$ ,  $r$ ,  $p$ ,  $N_1$  and  $N_2$  are determined by equations (28), (29), (33), (34), and (35). The outputs  $y_1$  and  $y_2$  are then derived from expressions (31) and (32). We study the effects of changes in endowments, the level of financial development, and expropriation risk on equilibrium prices and quantities.

### A. Determination of Factor Prices

The free entry conditions (34) and (35) are represented by curves  $z_i z_i (i = 1, 2)$  in Figure 3. They are convex toward origin and downward sloping in  $(w, r)$  space. The slopes of the curves for given  $p$ ,  $N_1$  and  $\theta$  are

$$\frac{dr}{dw} = - \left[ \frac{\lambda 2^{\frac{3}{2}} (1+r)^{\frac{1}{2}} \theta}{\bar{\rho}^{\frac{1}{2}} (1+\theta^2)} \right] a_i \text{ for } i = 1, 2 \quad (36)$$

Assume that  $a_1 < a_2$ , and so Sector 2 is more labor intensive than Sector 1. As indicated in Figure 3,  $z_2 z_2$  is steeper than  $z_1 z_1$ . Let the initial factor price equilibrium be given by point  $M$ . A decrease in the relative price of good 1, or an increase in  $N_1$ , will shift  $z_1 z_1$  inward to  $z_1^k z_1^k$ , and move the equilibrium to point  $A$ . It is clear that the wage goes up and the interest rate declines. When  $\theta$  is increased, both  $z_1^k z_1^k$  and  $z_2 z_2$  shift out to  $z_1^\theta z_1^\theta$  and  $z_2^\theta z_2^\theta$ .

The equilibrium moves from point  $A$  to point  $B$  which is vertically above  $A$ . The wage rate stays at exactly the same level, while the interest rate increases. A better financial system reduces the expected unit cost of total investment,  $h(r, \theta)$ , and therefore increases the return to investment. The return to labor, however, is unaffected by the financial development due to the Leontif technology assumed in our model. When  $\lambda$  is increased, both  $z_1^k z_1^k$  and  $z_2 z_2$  shift out to  $z_1^\lambda z_1^\lambda$  (represented by  $z_1^\theta z_1^\theta$  for simplification) and  $z_2^\lambda z_2^\lambda$ . The equilibrium moves from point  $A$  to point  $C$ . As we formally prove in the Appendix, under the condition that the highest cost of entrepreneurs' effort in the heterogeneous sector is more than that in the homogeneous sector, the interest rate increases but the wage rate declines as  $\lambda$  increases. Our analysis is similar to classical Stolper-Samuelson (1941) theorem, augmented by effects of entrepreneurs' heterogeneity, financial development and expropriation risk on factor prices. We summarize the above results by a "Stolper-Samuelson plus" theorem and relegate the formal proof to the Appendix.

**Proposition 1** (*Stolper-Samuelson plus*) *Ceteris paribus, a decrease in the price of a good decreases the return to the factor used intensively in that good, and increases the return to the other factor. Furthermore, an increase in the number of entrepreneurs in the heterogeneous sector decreases the return to the factor used intensively in that sector and increases the return to the other factor. An improvement in the level of financial development increases the interest rate but has no effect on the wage rate. If the highest cost of entrepreneurs' effort in the heterogeneous sector is more than that in the homogeneous sector, a lower expropriation risk increases the interest rate but reduces the wage rate.*

Note that factor price equalization does not hold in our model, making it different from the textbook version of the Heckscher-Ohlin model. Differences in  $\theta$  and  $\lambda$  make factor prices differ. Even if  $\theta$  and  $\lambda$  are the same across countries, as we show next, more entrepreneurs enter the heterogeneous sector in the capital-abundant country. Then the proposition above indicates that a larger  $N_1$  results in a lower interest rate  $r$  and a higher wage rate  $w$  at the capital-abundant country.

## B. Changes in Endowment and Institutions

We turn now to the response of outputs (represented by  $N_1$  and  $N_2$ ) to changes in exogenous variables: the Rybczynski (1955) effect of endowment, augmented by effects of financial development and expropriation risk. Let equations (28) and (29) be denoted as  $LL$  curve and  $KK$  curve, respectively. The numbers of entrepreneurs (or amounts of internal capital) in equilibrium,  $E = (N_1, N_2)$  are determined by the intersection of the  $LL$  and the  $KK$  curves, as indicated in Figure 4.  $KK$  curve is steeper than  $LL$  curve because Sector 1 is capital-intensive. Totally differentiating equations (28) and (29) and using Jones algebra (Jones, 1965), we obtain

$$\begin{aligned}\xi_{1L}\hat{N}_1 + \phi_{2L}\hat{N}_2 &= \hat{L} - [\phi_{1L}\hat{a}_{1L} + \phi_{2L}\hat{a}_{2L}] \\ \xi_{1K}\hat{N}_1 + \phi_{2K}\hat{N}_2 &= \hat{K} - [\phi_{1K}\hat{a}_{1K} + \phi_{2K}\hat{a}_{2K}]\end{aligned}\tag{37}$$

We define  $dN_1/N_1 = \widehat{N}_1$ , and likewise for all other variables. In addition, we define the *fraction of labor used in industry  $i$  by*,

$$\begin{aligned} \phi_{1L} &= \frac{a_{1L} \ln [N_1 / (1 + f - fN_1)]}{L}, \phi_{2L} = \frac{a_{2L} N_2}{L} \\ \text{and } \xi_{1L} &= \frac{a_{1L} (1 + f)}{L (1 + f - fN_1)} \end{aligned} \quad (38)$$

where  $\phi_{1L} + \phi_{2L} = 1$ . We define  $\phi_{iK}$  and  $\xi_{1K}$  in an analogous manner.

Let the initial equilibrium output be at point  $E$ . The effect of a change in endowment is similar to the standard HOS model.  $\widehat{L}$  and  $\widehat{K}$  represent the *direct effect* of a change in endowment at given product prices, while the second terms on the righthand side of equations (37) represent the *feedback effect* of induced factor price changes on the factor usage per unit of production. For given factor prices, as depicted in Figure 4, the *direct effect* of an increase in the capital endowment shifts  $KK$  out to  $K'K'$  and moves the equilibrium to point  $E'$ . It is clear that  $N_1$  goes up, whereas  $N_2$  declines. The increase in  $N_1$  raises  $y_1$ , while the decrease in  $N_2$  reduces  $y_2$ . Thus, the relative price of good 1,  $p$ , decreases. By Proposition 1, both the decrease in  $p$  and the increase in  $N_1$  reduces  $r$  while increasing  $w$ . Using (30), we know that both labor and capital usages per unit of production decrease. Thus, the *feedback effect* shifts the  $K'K'$  curve out further to  $K''K''$  and shifts the  $LL$  curve out to  $L''L''$ , which moves the equilibrium from  $E'$  to  $E''$ . The shifting out of  $KK$  curve further increases  $N_1$  and reduces  $N_2$ , while the shifting out of  $LL$  curve reduces  $N_1$  and increases  $N_2$ . As we formally prove in the Appendix, if a *modified condition for nonreversal of factor intensity* is satisfied, the overall effect of an increase in  $K/L$  is to increase  $N_1$ , while the overall effect on  $N_2$  is ambiguous. However, the relative price  $p$  declines, and as a result, the relative output  $y_1$  to  $y_2$  increases.

We now discuss the effect of a change in  $\theta$ . As Proposition 1 shows, the increase in  $\theta$  raises the interest rate  $r$  but has no effect on the wage  $w$ . That is, the impact of changing in  $\theta$  is completely absorbed by the increase of  $r$ , while leaving  $w$  unaffected. Expression (8) and (24) then indicate that the change in  $\theta$  must be offset by the change in  $r$  so that  $h(r, \theta)$  stays constant. Using (30), we know that  $a_{ij}$  must remain constant as  $\theta$  changes. As a result,  $N_1$ ,  $N_2$ , and  $p$  are not affected by the increase in  $\theta$ . Note that although the increase in  $\theta$  does not affect the number of entrepreneurs, it raises  $y_1$  and  $y_2$  by the same proportion as indicated by expressions (31) and (32).

The increase in  $\lambda$  raises the interest rate so that  $h(r, \theta)$  is higher. Expression (30) indicates that factor usages per unit of production increases. Thus both,  $LL$  and  $KK$  shift back, and equilibrium moves from  $E$  to  $E'''$  in Figure 4.  $N_1$  and  $N_2$  both decline. As we formally prove in the Appendix, under the condition that the highest cost of entrepreneur's effort in the heterogeneous sector is more than that in the homogeneous sector,  $N_1$  and  $N_2$  decrease proportionally in the way that relative price  $p$  does not change. Lower expropriation risk reduces the number of firms. Each firm, however, becomes larger and produces more. As we show in the Appendix, the positive effect of  $\lambda$  on interest rate  $r$  dominates the negative effect on  $N_i$ . Using (31) and (32), industry outputs  $y_1$  and  $y_2$  are larger as  $\lambda$  increases. We

summarize the above results by a “Rybczynski plus” theorem.

**Proposition 2** (*Rybczynski plus*) *Suppose a modified condition for nonreversal of factor intensity is satisfied, so that sector 1 is always capital-intensive. An increase in capital endowment will increase the number of entrepreneurs in Sector 1, and decrease the relative price of good 1. Furthermore, an improvement in the level of financial development will raise the output in both sectors proportionally, leaving the number of entrepreneurs and the relative product price unchanged. If the highest cost of entrepreneurs’ effort in the heterogeneous sector is more than that in the homogeneous sector, a lower expropriation risk will raise the output but reduce the number of entrepreneurs in both sectors proportionally and have no effect on the relative product price.*

Propositions 1 and 2 together give rise to predictions on how a change in endowment (or financial and property rights institution) on factor prices. In particular, an increase in capital endowment increases  $N_1$  and reduces  $p$  by Proposition 2. Both effects reduce  $r$  but increase  $w$  by Proposition 1. We can work out in a similar way the effects of an increase in  $\theta$  or  $\lambda$ . For convenience, these results can be summarized by the following corollary.

**Corollary 1** *In equilibrium, an increase in the capital-labor ratio reduces the interest rate but raises the wage rate. An improvement in the financial system raises the interest rate but leaves the wage rate unchanged. A reduction in the expropriation risk raises the interest rate but reduces the wage rate.*

Note that in this two-sector, two-factor model, the corollary implies that some of the intuition from a typical one-sector model—in particular, a rich country may have a lower return to financial capital—is resurrected. However, as we will show later, this does not resurrect the Lucas paradox as the cross-country difference in return to capital is much smaller in this model than in a typical one-sector model.

## VI. FREE TRADE AND CAPITAL FLOWS

Using the comparative statics results derived above, we are now ready to describe patterns of goods trade and capital flows. Consider two countries with identical and homothetic tastes, identical technologies, identical liquidity shocks and managers’ behavior, but different factor endowments, levels of financial development, and expropriation risks. Labor is immobile across countries. After studying free trade in goods without international capital flow, we move sequentially by allowing only financial capital flow at first, only foreign direct investment (FDI) next, and both types of capital flows simultaneously in the end.

### A. Free Trade in Goods

Let the equilibrium autarky prices at home and abroad be  $p$  and  $p^*$ , respectively.  $p^*$  may differ from  $p$  if  $L^*$ ,  $K^*$ ,  $\theta^*$  and  $\lambda^*$  are different from corresponding domestic variables. Comparing  $p^*$  with  $p$  is equivalent to the exercise of comparative statics in the last section

that changes  $K/L$ ,  $\theta$ , and  $\lambda$  to  $K^*/L^*$ ,  $\theta^*$  and  $\lambda^*$ , respectively. Let  $\hat{p} = (p^* - p)/p$  be the percentage difference in the autarky prices. Ignoring a second order effect and using equation (65) in the Appendix., we have

$$A_p \hat{p} = \hat{L} - \hat{K} \quad (39)$$

where  $A_p = -|\phi| \sigma_D / \sigma_N > 0$ .  $\hat{L}$ ,  $\hat{K}$ ,  $\hat{\theta}$  and  $\hat{\lambda}$  are now percentage differences in the labor and capital endowments, financial development, and risk expropriation between two countries. Noting that  $\hat{\theta}$  and  $\hat{\lambda}$  have no effect on relative product prices, our analysis of goods trade is essentially a generalized Heckscher-Ohlin model in an environment of imperfect capital market and heterogeneous entrepreneurs. The usual Heckscher-Ohlin result holds here: a labor-abundant country has a higher relative price of the capital-intensive good than the other country. Thus, it exports the labor-intensive product and imports the capital-intensive product.

**Proposition 3** *Suppose capital flows are prohibited. In this model with financial market imperfection and heterogeneous entrepreneurs, the Heckscher-Ohlin result on trade patterns still holds: each country produces and exports the good that uses its relatively abundant factor intensively.*

## B. Financial Capital Flows

We now turn to capital flow under the equilibrium of free trade in goods. There are two types of international capital flow: financial capital flow decided by investors and FDI decided by entrepreneurs. International financial flow occurs when the investor invests her endowment in a foreign financial market (or directly in a foreign entrepreneur's project). On the other hand, FDI occurs when the entrepreneur takes her project to the foreign country and produces there. Investors will invest in the country with a higher interest rate (return to financial investment), while entrepreneurs will locate their projects in the country with a lower production cost. In the rest of this subsection, we discuss the special case in which only financial capital flow is permitted, but no FDI.

The direction of *financial flow* is determined by  $\hat{r} = (r^* - r)/r$ . If  $\hat{r} > 0$ , financial capital will flow from home to the foreign country. Otherwise, it will flow in the reverse direction. As we have shown in Corollary 1, if the country is either relatively abundant in labor, more financially developed, or has less risk of expropriation, its interest rate in the absence of international capital flow is higher.

In the equilibrium with free trade in goods, the endogenous variables in each country are determined by equations (28), (29), (34), and (35), and their foreign-country counterparts. The product market clearing condition now becomes  $(y_1 + y_1^*) / (y_2 + y_2^*) = D(p)$ . The equation (59) in the Appendix no longer holds but is not needed since  $\hat{p} = 0$  in the free trade equilibrium. All other proofs in the Appendix go through. We again ignore the second order effect. Slightly abusing notations and substituting (64) into (55), we obtain

$$\hat{r} = A_L \hat{L} - A_K \hat{K} + A_\theta \hat{\theta} + A_\lambda \hat{\lambda} \quad (40)$$

where  $A_L$ ,  $A_K$ ,  $A_\theta$ ,  $A_\lambda$  are all positive.<sup>6</sup> We can summarize three polar cases with the following proposition.

**Proposition 4** *Let there be free trade in goods, no barrier to international financial capital flow, but no FDI is permitted. If two countries are the same in terms of financial development and expropriation risk but different in endowment, then financial capital will flow from capital-abundant country into labor-abundant country. If the two countries have the same capital-labor ratio and identical expropriation risk but different levels of financial development, financial capital will flow from the country with a less developed financial system into the other one. If the two countries have the same capital-labor ratio and levels of financial development, financial capital will flow from the country with a higher expropriation risk into the one with lower expropriation risk.*

### C. Foreign Direct Investment

We now allow projects and entrepreneurs to move freely across countries. Rewrite expression (15) of entrepreneur's *net return to internal capital* as

$$U_{ni}(w, r, \theta, \lambda) = \frac{\lambda [p_i^T G_i(a_i, 1) - wa_i] - \left(\frac{1+\theta^2}{\sqrt{2\theta}}\right) [(1+r)\bar{\rho}]^{\frac{1}{2}}}{\left(\frac{1+\theta^2}{\sqrt{2\theta}}\right) [(1+r)\bar{\rho}]^{\frac{1}{2}} - \lambda [p_i^T G_i(a_i, 1) - wa_i] + c_{ni}} \quad (41)$$

where  $p_i^T$  represents the product price in free trade. It is easy to see  $\partial U_{ni}/\partial w < 0$ ,  $\partial U_{ni}/\partial r < 0$ ,  $\partial U_{ni}/\partial \theta > 0$ , and  $\partial U_{ni}/\partial \lambda > 0$ . We assume that entrepreneurs collect the capital at home and utilize their home financial system even if they produce abroad. We first consider the case of  $\hat{\lambda} = (\lambda^* - \lambda)/\lambda = 0$ . In this case domestic entrepreneurs will have an outbound *FDI* if and only if  $w > w^*$ . Substituting (64) into (54), we obtain

$$\hat{w} = -B_L \hat{L} + B_K \hat{K} - B_\lambda \hat{\lambda} \quad (42)$$

where  $B_L$ ,  $B_K$ , and  $B_\lambda$  are all positive. Thus  $w > w^*$  if and only if the home country is capital-abundant. That is, entrepreneurs from a capital-abundant country will engage in outbound FDI to take the advantage of lower labor cost abroad.

**Proposition 5** *With free trade in goods, identical expropriation risk but prohibition of international financial capital flow, FDI will go from the capital-abundant country to the labor-abundant one.*

### D. Complete Bypass of the Inefficient Financial System

We now allow for both types of capital flows. Let both countries be diversified. We start with the simplest case in which expropriation risk is identical across countries and entrepreneurs are perfectly mobile. The unique equilibrium in this case is a complete *capital bypass circulation* in which all capital owned by financial investors (households) in

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<sup>6</sup>A detailed proof of equations (40) and (42) is available from authors upon request.

the country with a less developed financial system leaves the country in the form of *financial capital outflow*, but physical capital (and projects) reenters the country in the form of *FDI*. The less developed financial system serves no capital at all in the equilibrium.

The proof is straightforward: In the equilibrium, the interest rates and wage rates must be equalized across two countries. Since entrepreneurs are perfectly mobile, if entrepreneur  $n$  in a low  $\theta$  country could be hired to manage a factory (project) in a high  $\theta$  country, she would like to move to the high  $\theta$  country since  $\partial U_{ni}/\partial \theta > 0$ . If some managers had used the financial system of low  $\theta$  country in the equilibrium, the most efficient manager among them would like to bring her one unit of capital and move to high  $\theta$  country. That would reduce the wage rate in the low  $\theta$  country (hence making the low  $\theta$  country more attractive to FDI from the high  $\theta$  country), and crowd out the less efficient managers in the high  $\theta$  country whom would bring her project to low  $\theta$  country (hence raising the interest rate in the high  $\theta$  country in the process and making it more attractive to financial capital from the low  $\theta$  country). Another wave of *capital bypass circulation* would occur until all financial capital leaves the low  $\theta$  country and enough FDI comes into the low  $\theta$  country so that factor prices are equalized between two countries in the equilibrium.

A modified graphical representation of an integrated world economy (Dixit and Norman, 1980; Helpman and Krugman, 1985) can help to illustrate the equilibrium. In Figure 5,  $O$  and  $O^*$  represent the origins for home and foreign countries, respectively. Vectors  $OY_1$  and  $OY_2$  represent the world employment of capital and labor in Sectors 1 and 2 in the equilibrium of the integrated world economy, respectively. Let  $L = L^*$  for simplicity. Suppose  $\theta > \theta^*$ . Point  $H$  defines the distribution of factor endowments. Let home be capital-abundant so  $H$  is above the diagonal line of the parallelogram  $OY_2O^*Y_1$ .

International financial capital flow equalizes the interest rates, while FDI equalizes the wage rates across two countries. For  $(w, r)$  to be equal in the two countries, from equations (34) and (35),  $N_1$  and  $N_1^*$  must be the same since investors in both countries use the same financial system  $\theta$ . Thus the factor usages of production in the equilibrium must be in the middle line of the parallelogram,  $AA^*$ . That is, factor usages in Sector 1 represented by lengths of  $OA$  and  $O^*A^*$  must be the same for the two countries. Each country uses its own labor endowment. Therefore, the intersection between  $AA^*$  and  $LF$ , represented by point  $E$ , indicates factor usages of production in the equilibrium.  $E$  happens to be in the middle of the parallelogram since we assume  $L = L^*$ .  $OB$  and  $O^*B^*$  represent the factor usages in sector 2. All foreign capital flows into the home country in the form of financial capital flow since  $\theta > \theta^*$ , which is represented by  $FH$ . FDI, however, flows to the foreign country and is represented by  $EF$ . The circle  $FHEF$  represents the *capital bypass circulation*.  $HE$  indicates the net capital outflow of the home country. The consumption bundle is represented by point  $C$  in the diagonal line.  $C$  locates outside point  $E$  since the GNP also includes interest income from the net capital outflow. The home country experiences a current account surplus as the capital account account is negative<sup>7</sup>. To summarize, we have:

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<sup>7</sup>When entrepreneurs are not perfectly mobile and expropriation risk is not identical, a capital abundant country with developed financial system and low expropriation risk may experience a net capital inflow and therefore a current account deficit.



**Proposition 6** *Let the expropriation risk be identical and entrepreneurs be perfectly mobile across two countries with identical populations. In the unique equilibrium, the less developed financial system is completely bypassed. All capital owned by the country with the less developed financial system will leave the country in the form of financial capital flow. However, the country also experiences capital inflow in the form of FDI. In equilibrium, the capital-abundant country incurs a net capital outflow (and a trade surplus).*

The complete *capital bypass circulation* equilibrium predicts the same direction of net capital flow as a typical neoclassical one-sector model. The magnitude of the interest rate differential (return to financial investment), however, is different between this model and a typical one-sector model. To see this, let  $\hat{L} = 0$  for simplicity. Substituting (64) into (55), we obtain:

$$\hat{r} = -\pi_{1N} \left( \frac{\pi_{2w}(\phi_{2L} + \eta_2)}{|\pi| |\Phi|} \right) \hat{K}$$

where  $\pi_{1N} = f c_1 N_1 / (1 + f)$ .  $\hat{K}$  is no longer the only factor that determines interest rate differential as in Lucas paradox. The capital market imperfection which is measured by the entry cost of entrepreneurs  $f$ , the cost of effort in moral hazard problem  $c_1$ , as well as the level of entrepreneurs heterogeneity represented by the function form of  $c_{n1}(\cdot)$  all affect  $\hat{r}$ . In other words, even if the capital-labor ratio is very different between two countries, as long as  $f$  or  $c_1$  are sufficiently small, the difference in the returns to financial investment between the two countries can be small<sup>8</sup>. To put it another way, going back to the original Lucas (1990) example, while it may take an enormous friction equivalent to 5800% tax to stop capital to flow from the United States to India in a one-sector model, it may take only a small amount of friction, say 5% of tax, to stop the capital flow.

### E. The Role of Expropriation Risk

The above discussion focuses on the role of financial sector efficiency in determining gross and net capital flows. The result that an inefficient financial system would be completely bypassed may be somewhat surprising and is derived under the assumptions of identical expropriation risks across countries and perfectly mobile entrepreneurs. We relax these assumptions in this section: the risk of expropriation may be different, and entrepreneurs can only move along with projects (firms).

We also assume that there is a fixed cost,  $d$ , for an entrepreneur to locate abroad. In the equilibrium, interest rates are equalized across countries by financial capital flow. As before, we assume that entrepreneurs continue to use home financial services when they locate abroad. The entrepreneur's *net return to internal capital* when they produce at home is given by expression (41) at domestic wage rate and expropriation risk, and denoted by  $U_{ni}(w, r, \lambda, \theta)$ . It becomes  $U_{ni}^d = U_{ni}(w^*, r, \lambda^*, \theta) - d$  when they produce abroad. Entrepreneur  $n$  produces abroad if and only if  $U_{ni} \leq U_{ni}^d$ . A corner solution occurs in Sector 2. Suppressing the notations of  $r$  and  $\theta$  for convenience, all firms in Sector 2 produce at

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<sup>8</sup>Caselli and Freyrer (2005) computed that the financial rates of return from investing in physical capital are very similar across the 53 developing and rich countries for which they have the relevant data.

home if and only if  $U_2(w, \lambda) > U_2(w^*, \lambda^*) - d$ . We assume that this condition is satisfied so that home produces in both sectors (i.e., the countries are diversified in the equilibrium). Let the marginal entrepreneur in sector 1 be  $N_1^d$ . We have:

$$U_{N_1^d}(w, \lambda) = U_{N_1^d}(w^*, \lambda^*) - d \quad (43)$$

This implies that  $U_{N_1^d}(w, \lambda) < U_{N_1^d}(w^*, \lambda^*)$ , which, by expression (41), in turn implies that

$$\lambda [p^T G_1(a_1, 1) - w a_1] < \lambda^* [p^T G_1(a_1, 1) - w^* a_1]$$

Therefore,  $U_{n1}^d$  as a function of  $n$  must be steeper than  $U_{n1}$ . As illustrated in Figure 2,  $U_{n1}$  and  $U_{n1}^d$  intersect at  $N_1^d$ . Entrepreneurs in the interval of  $[1, N_1^d]$  choose outward FDI in the foreign country, while entrepreneurs in the interval of  $(N_1^d, N_1]$  choose to produce at home. In other words, the more efficient firms choose FDI and the less efficient ones produce at home. This result is similar to Helpman, Melitz, and Yeaple (2004). Given the identical fixed cost  $d$  for all firms, lower foreign labor cost generates more profit for larger firms than for smaller ones.

Similar to expression (20), we derive the capital usage for a FDI firm  $n$ , as  $k^d(n) = h(r, \theta) / [h(r, \theta) - \lambda^* R_1^* + c_1 n]$ . The capital usage for all FDI firms becomes

$$\kappa^d = \int_1^{N_1^d} k^d(n) dn = \frac{h(r, \theta)}{c_1} \ln \frac{h(r, \theta) - \lambda^* R_1^* + c_1 N_1^d}{h(r, \theta) - \lambda^* R_1^* + c_1} \quad (44)$$

The expected output of all FDI firms is

$$\begin{aligned} y_1^d &= F_1(\theta \rho_1^{opt}) \lambda^* G_1(a_1, 1) \int_1^{N_1} K_{n1}^1(w^*, \lambda^*) dn \\ &= \frac{G_1(a_1, 1) \lambda^* (1 + r)}{c_1} \ln \frac{h(r, \theta) - \lambda^* R_1^* + c_1 N_1^d}{h(r, \theta) - \lambda^* R_1^* + c_1} \end{aligned} \quad (45)$$

FDI firms employ source-country capital but host-country labor by assumption. Thus, the domestic full employment conditions now become:

$$a_1 \int_{N_1^d}^{N_1} K_{n1}(\cdot) dn + a_2 K_2(\cdot) N_2 = L \quad (46)$$

$$\kappa^d + \int_{N_1^d}^{N_1} K_{n1}(\cdot) dn + K_2(\cdot) N_2 = K + \kappa^f \quad (47)$$

where  $\kappa^f$  is the amount of financial capital flow.  $\kappa^f > 0$  represents financial capital inflow

while  $\kappa^f < 0$  represents outflow. The foreign full employment conditions are

$$a_1 \left[ \kappa^d + \int_1^{N_1^*} K_{n1}^*(.)dn \right] + a_2 K_2^*(.)N_2^* = L^* \quad (48)$$

$$\int_1^{N_1^*} K_{n1}^*(.)dn + K_2^*(.)N_2^* = K^* - \kappa^f \quad (49)$$

Similar to equations (34) and (35), the free entry conditions in the foreign country can be written as

$$\lambda^* a_1 w^* + \frac{1 + \theta^{*2}}{2\theta^*} [2(1+r)\bar{p}]^{\frac{1}{2}} = \lambda^* p^T G_1(a_1, 1) - \frac{f c_1 N_1^*}{1+f} \quad (50)$$

$$\lambda^* a_2 w^* + \frac{1 + \theta^{*2}}{2\theta^*} [2(1+r)\bar{p}]^{\frac{1}{2}} = \lambda^* G_2(a_2, 1) - \frac{f c_2}{1+f} \quad (51)$$

Finally, the condition for clearing the world product market is

$$\frac{y_1 + y_1^d + y_1^*}{y_2 + y_2^*} = D(p^T) \quad (52)$$

The equilibrium is characterized by 10 non-linear equations, (34), (35), (46), (47), (48), (49), (50), (51), (52), and (43) with ten endogenous variables,  $p^T$ ,  $w$ ,  $r(=r^*)$ ,  $N_1$ ,  $N_2$ ,  $w^*$ ,  $N_1^*$ ,  $N_2^*$ ,  $N_1^d$ , and  $\kappa^f$ . While a closed form solution is hard to obtain, it is possible to compare wage rates across the countries and analyze the effects of financial sector efficiency and expropriation risk on capital flow.

From (35), it can be verified that  $\partial w / \partial \theta > 0$  and  $\partial w / \partial \lambda > 0$ . Recall that (35) is derived from revenue sharing condition (24), and that the pay to an entrepreneur in Sector 2 is fixed as  $f c_2 / (1+f)$ . As a better financial system reduces the investment cost  $h(r, \theta)$ , it therefore raises the wage rate. A better property rights protection (a lower expropriation risk) increases the expected revenue and therefore raises the wage, too. Comparing (35) with (51), we have  $w > w^*$  if  $\theta > \theta^*$  or  $\lambda > \lambda^*$ . It is worth emphasizing that in equilibrium, the relative wage across countries is determined by the two institutional parameters,  $\theta$  and  $\lambda$ , but independent of the initial endowment. A country with a low initial capital-to-labor ratio but better property rights protection (higher  $\lambda$ ) or more efficient financial system (higher  $\theta$ ) can attract more capital in the world market so that its labor is paid at a higher wage in the equilibrium. The result can be illustrated by Figure 3 in which home has a lower expropriation risk ( $\lambda > \lambda^*$ ). Before capital flow is allowed, the home country is at point  $C$  and the foreign country is at point  $M$ . Once capital flow is allowed, financial capital (and possibly direct investment) leaves the foreign country to come to the home country. Therefore,  $N_1^*$  declines, shifting  $z_1 z_1$  curve up to  $z_1^{e*} z_1^{e*}$ . At the same time, capital inflow to the home country increases  $N_1$  so  $z_1^\theta z_1^\theta$  shifts down to  $z_1^e z_1^e$ . In the equilibrium,  $r$  is equalized across countries but  $w > w^*$ . Similar exercise can be done for the case of  $\theta > \theta^*$ . We summarize the discussion by the following proposition:

**Proposition 7** *Suppose the two countries are diversified in the equilibrium. With free mobility of capital, the wage rate is higher in the country with a more efficient financial system or better property rights protection, irrespective of the initial endowment.*

While financial development and expropriation risk have similar effects on equilibrium wage rate, they differ in their effects on patterns of gross and net international capital flows. A less efficient financial system, by depressing domestic return on financial investment, leads to an outflow of financial capital. As a result of this financial outflow, the wage rate becomes lower, which encourages inward FDI. In contrast, worse property rights protection, by depressing domestic financial returns, leads to an outflow of financial capital, and at the same time, by depressing firm profits, also discourages inward FDI. Therefore, poor property rights protection may result in financial outflow without a compensating inflow of FDI.

This discussion suggests that for some economic questions, one should not lump together different types of institutions. Is there any evidence that poor financial institutions and poor property rights protection give rise to different patterns of capital flows? Albuquerque (2003) and Wei (2005) examined the roles of these institutional features in determining patterns of capital flow. They found evidence that poor financial institutions are associated with a higher share of FDI in inward capital flow. In contrast, Wei (2000 and 2005) found that poor property rights protection or severe bureaucratic corruption clearly deters inward FDI. These pieces of evidence are consistent with the prediction of this model.

Due to space constraint, we leave a welfare analysis of international capital flow in the current model to an companion paper (Ju and Wei, 2006). We note here that the welfare implication of capital flow in our model differs from the literature. In most existing papers, removing barriers to capital flow improves welfare since it improves efficiency. Such a view relies on the assumption that the return to investment equals the marginal product of physical capital. In our model, however, financial investors often gain at the expense of entrepreneurs. If the loss of the entrepreneurs is large enough, financial capital outflow can reduce welfare.

## VII. Conclusions

This paper has two objectives. First, we aim to provide a solution to two opposing puzzles about international capital flows. Second, we provide a framework to discuss systematically the roles of financial and property rights institutions in determining patterns of gross and net capital flows.

Our model uses entrepreneur heterogeneity to partially restore the intuition of one-sector models in a two-sector setting that the interest rate is lower in a capital-abundant country. A revenue-sharing rule between financial investors and entrepreneurs, together with marginal product of capital, determine the interest rate. The quality of the financial system and expropriation risk play crucial roles in the model. The interest rate is higher in the country with a better financial system or a lower expropriation risk. Financial capital

flow and FDI can move in either the same or the opposite directions, and therefore form rich patterns of gross capital flows. The equilibrium in a frictionless world capital market and identical expropriation risks is unique: the less developed financial system of the two is completely bypassed.

A better financial system or better property rights protection in a country leads to a higher wage rate for the country in equilibrium. However, their effects on patterns of cross-border capital flow are different. A lower level of financial development results in a lower interest rate, which generates an outflow of financial capital. As a result, wages becomes lower, which attracts more FDI than otherwise. Higher expropriation risk, on the other hand, results in lower profit, leading to less FDI (and outflow of financial capital).

The current model is static; extending it to dynamic analysis will be a fruitful direction for future research. Taking the model to the data so that patterns of gross and net capital flow can be linked to different institutional variables is also high on our agenda.

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## Appendix: Proofs

### 1. Proof of Proposition 1

Totally differentiating equations (34) and (35), we obtain

$$\begin{aligned}\pi_{1w}\widehat{w} + \pi_{1r}\widehat{r} &= \pi_{1\theta}\widehat{\theta} + \pi_{1\lambda}\widehat{\lambda} + \pi_{1p}\widehat{p} - \pi_{1N}\widehat{N}_1 \\ \pi_{2w}\widehat{w} + \pi_{2r}\widehat{r} &= \pi_{2\theta}\widehat{\theta} + \pi_{2\lambda}\widehat{\lambda}\end{aligned}\tag{53}$$

where  $\widehat{w} = dw/w$  denotes the percentage change in wage rate and likewise for other variables. We define  $\pi_{iw} = \lambda a_i w$ ,  $\pi_{1r} = \pi_{2r} = \left[ \bar{\rho}^{\frac{1}{2}} r (1 + \theta^2) \right] / \left[ 2^{\frac{3}{2}} \theta (1 + r)^{\frac{1}{2}} \right]$ ,  $\pi_{1\theta} = \pi_{2\theta} = \left[ \bar{\rho}^{\frac{1}{2}} (1 + r)^{\frac{1}{2}} (1 - \theta^2) \right] / \left( 2^{\frac{1}{2}} \theta \right)$ , and  $\pi_{i\lambda} = \lambda R_i$ , while  $\pi_{1p} = p \lambda G_1(a_1, 1)$  and  $\pi_{1N} = f c_1 N_1 / (1 + f)$ . We can solve for the percentage change in factor prices from equations (53) as

$$\widehat{w} = \frac{\pi_{2r} (\pi_{1p}\widehat{p} - \pi_{1N}\widehat{N}_1)}{|\pi|} + \frac{\pi_{2r}\lambda (R_1 - R_2)\widehat{\lambda}}{|\pi|}\tag{54}$$

$$\begin{aligned}\widehat{r} &= \frac{\pi_{2w} (\pi_{1N}\widehat{N}_1 - \pi_{1p}\widehat{p})}{|\pi|} + \frac{2(1+r)(1-\theta^2)\widehat{\theta}}{r(1+\theta^2)} \\ &\quad + \frac{\lambda^2 w (a_1 R_2 - a_2 R_1)\widehat{\lambda}}{|\pi|}\end{aligned}\tag{55}$$

where  $|\pi| = \pi_{1w}\pi_{2r} - \pi_{1r}\pi_{2w} < 0$  if sector 1 is capital intensive than sector 2. Using (24), we have

$$\lambda (R_1 - R_2) = \frac{f}{1+f} (c_1 N_1 - c_2)$$

Thus,  $\lambda (R_1 - R_2) > 0$  if and only if  $c_1 N_1 - c_2 > 0$ , which also implies that  $a_1 R_2 - a_2 R_1 < a_1 (R_2 - R_1) < 0$ . Then results in Proposition 1 are immediately seen from expressions (54) and (55).

### 2. Proof of Proposition 2

Using (30), we have

$$\widehat{a}_{iL} = \widehat{a}_{iK} = \frac{r}{2(1+r)}\widehat{r} - \left( \frac{1-\theta^2}{1+\theta^2} \right)\widehat{\theta}\tag{56}$$

These solutions for  $\hat{a}_{ij} (j = L, K)$  can then be substituted into equation (37) to obtain

$$\xi_{1L}\hat{N}_1 + \phi_{2L}\hat{N}_2 = \hat{L} - \frac{r}{2(1+r)}\hat{r} + \left(\frac{1-\theta^2}{1+\theta^2}\right)\hat{\theta} \quad (57)$$

$$\xi_{1K}\hat{N}_1 + \phi_{2K}\hat{N}_2 = \hat{K} - \frac{r}{2(1+r)}\hat{r} + \left(\frac{1-\theta^2}{1+\theta^2}\right)\hat{\theta} \quad (58)$$

Let  $|\phi|$  denote the determinant of the  $2 \times 2$  matrix on the left hand side of the above system. It is immediately seen that  $|\phi| < 0$  if and only if  $a_1 < a_2$ .

Totally differentiating equation (33), we obtain

$$\sigma_N \hat{N}_1 - \hat{N}_2 = -\sigma_D \hat{p} \quad (59)$$

where  $\sigma_D > 0$  is the elasticity of substitution between goods on the demand side, and

$$\sigma_N = \frac{1+f}{(1+f-fN_1)\ln[N_1/(1+f-fN_1)]} \quad (60)$$

Now substituting (59) into (55), we have

$$\begin{aligned} \hat{r} = & \frac{\pi_{2w}}{|\pi|\sigma_D} \left[ (\pi_{1N}\sigma_D + \pi_{1p}\sigma_N) \hat{N}_1 - \pi_{1p}\hat{N}_2 \right] + \frac{2(1+r)(1-\theta^2)\hat{\theta}}{r(1+\theta^2)} \\ & + \frac{\lambda^2 w (a_1 R_2 - a_2 R_1) \hat{\lambda}}{|\pi|} \end{aligned} \quad (61)$$

Then substituting the above expression into equations (57) and (58), we obtain

$$\begin{aligned} (\xi_{1L} - \eta_1) \hat{N}_1 + (\phi_{2L} + \eta_2) \hat{N}_2 &= \hat{L} - \eta_\lambda \hat{\lambda} \\ (\xi_{1K} - \eta_1) \hat{N}_1 + (\phi_{2K} + \eta_2) \hat{N}_2 &= \hat{K} - \eta_\lambda \hat{\lambda} \end{aligned} \quad (62)$$

where

$$\begin{aligned} \eta_1 &= -\frac{r\pi_{2w}(\pi_{1N}\sigma_D + \pi_{1p}\sigma_N)}{2(1+r)|\pi|\sigma_D}, \eta_2 = -\frac{r\pi_{2w}\pi_{1p}}{2(1+r)|\pi|\sigma_D} \\ \eta_\lambda &= \frac{\lambda^2 w (a_1 R_2 - a_2 R_1) r}{|\pi| 2(1+r)} \end{aligned} \quad (63)$$

$\eta_1$ ,  $\eta_2$ , and  $\eta_\lambda$  are positive. Let  $|\Phi|$  denote the determinant of the  $2 \times 2$  matrix on the left hand side of (62). We assume a *modified condition for non-reversal of factor intensity* that  $|\phi|$  and  $|\Phi|$  have the same sign, which implies that  $|\Phi| < 0$ . The condition ensures that sector 1 is capital intensive both before and after changes in factor endowments, the level

of financial development, and expropriation risk. Solving for  $\widehat{N}_1$  gives

$$\begin{aligned}\widehat{N}_1 &= \frac{(\phi_{2K} + \eta_2) \widehat{L} - (\phi_{2L} + \eta_2) \widehat{K} + \eta_\lambda (\phi_{2L} - \phi_{2K}) \widehat{\lambda}}{|\Phi|} \\ \widehat{N}_2 &= \frac{(\xi_{1L} - \eta_1) \widehat{K} - (\xi_{1K} - \eta_1) \widehat{L} + \eta_\lambda (\xi_{1K} - \xi_{1L}) \widehat{\lambda}}{|\Phi|}\end{aligned}\quad (64)$$

Using the fact that  $\xi_{1L} - \xi_{1K} = \sigma_N (\phi_{2K} - \phi_{2L})$ , we have  $|\phi| = \sigma_N (\phi_{2K} - \phi_{2L})$ . Thus,  $\phi_{2L} - \phi_{2K} > 0$  and  $\xi_{1K} - \xi_{1L} > 0$ . So we have  $\widehat{N}_1 > 0$  if  $\widehat{K} > 0$ ,  $\widehat{L} = 0$ , and  $\widehat{N}_1 < 0$ ,  $\widehat{N}_2 < 0$  if  $\widehat{\lambda} > 0$ .

Subtracting (57) from (58), and using (59), we obtain

$$\left[ -\frac{|\phi| \sigma_D}{\sigma_N} \right] \widehat{p} = \widehat{L} - \widehat{K} \quad (65)$$

$\widehat{p} < 0$  when  $\widehat{K} - \widehat{L} > 0$ . Note that both  $\widehat{\theta}$  and  $\widehat{\lambda}$  have no effect on  $\widehat{p}$ .

To study the effect of the increase in  $\lambda$  on  $y_1$ , we take the logarithm and total differentiate (31) and obtain

$$\widehat{y}_1 = \widehat{\lambda} + \frac{r}{1+r} \widehat{r} + \sigma_N \widehat{N}_1 \quad (66)$$

Substituting (55) and (64) into the above expression with some computations we have

$$\begin{aligned}\widehat{y}_1 &= \widehat{\lambda} - \frac{r \pi_{2w} \pi_{1N} \eta_\lambda}{(1+r) |\pi| [\sigma_N (1 + \eta_2) - \eta_1]} \widehat{\lambda} \\ &\quad - \frac{\eta_\lambda \sigma_N}{\sigma_N (1 + \eta_2) - \eta_1} \widehat{\lambda} + 2\eta_\lambda \widehat{\lambda} > \widehat{\lambda} > 0\end{aligned}$$

Similarly we have  $\widehat{y}_2 > \widehat{\lambda} > 0$ .

Table 1. Notations

Notations	Definitions
$p_i$	price of good $i$
$p^T$	equilibrium price under free trade
$w$	wage rate
$r$	interest rate
$\lambda$	control of expropriation risk
$\theta$	level of financial development
$R_i$	return to a unit of period 1 investment if the project succeeds
$R_{ni}^E$	entrepreneur's share in capital revenue if the project succeeds
$K$	capital endowment of the country
$L$	labor endowment of the country
$K_{ni}^1$	period 1 investment managed by entrepreneur $n$
$K_{ni}^{X1}$	external capital that entrepreneur raises at date 1
$K_{ni}$	the total amount of capital managed by entrepreneur $n$
$\kappa^d$	capital usage for all FDI firms
$\kappa^f$	amount of financial capital flow
$G_i(L_i^1, K_i^1)$	production function in sector $i$ at period 1
$a_i$	labor-capital ratio in sector $i$
$y_i^1$	output of the project in sector $i$ if the project succeeds
$y_i$	expected (or the realized) output in sector $i$
$y_i^d$	expected output of all FDI firms
$N_i$	number of entrepreneurs in sector $i$
$N_i^d$	number of FDI entrepreneurs in sector 1
$\hat{N}_i$	$= dN_i / N_i$ , and likewise for all other variables
$\rho_i$	liquidity shock in sector $i$
$F_i(\rho)$	distribution function of liquidity shock in sector $i$
$f_i(\rho)$	density function of liquidity shock in sector $i$
$\rho_i^1$	$= \lambda R_i$
$\rho_{ni}^{\max}$	$= \lambda (R_i - R_{ni}^E)$
$\rho^{opt}$	optimal cutoff of the liquidity shock
$h(\cdot)$	expected unit cost of total investment
$U_{ni}(\cdot)$	firm's net return to internal capital
$c_{ni}(\cdot)$	entrepreneur $n$ 's cost of effort
$f$	fixed cost to become an entrepreneur
$D(\cdot)$	relative demand
$\phi_{ji}$	fraction of factor $j$ used in sector $i$
$\mu_{ni}(\rho_i)$	state-contingent continuation policy

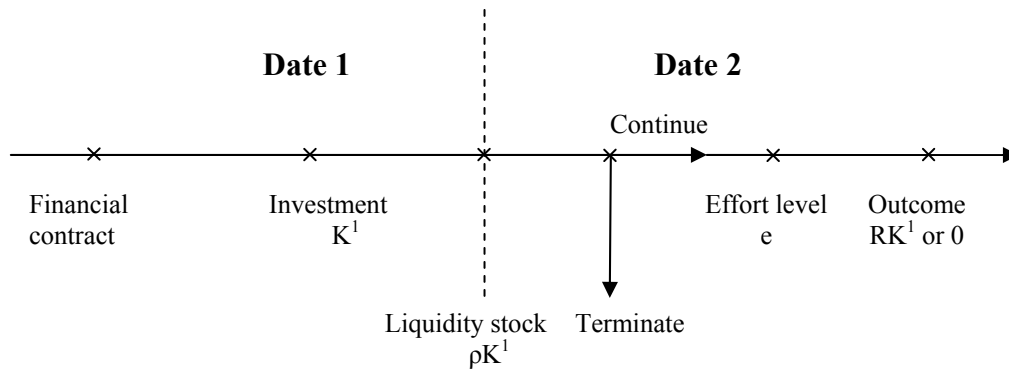


Figure 1: Time Line of the Model

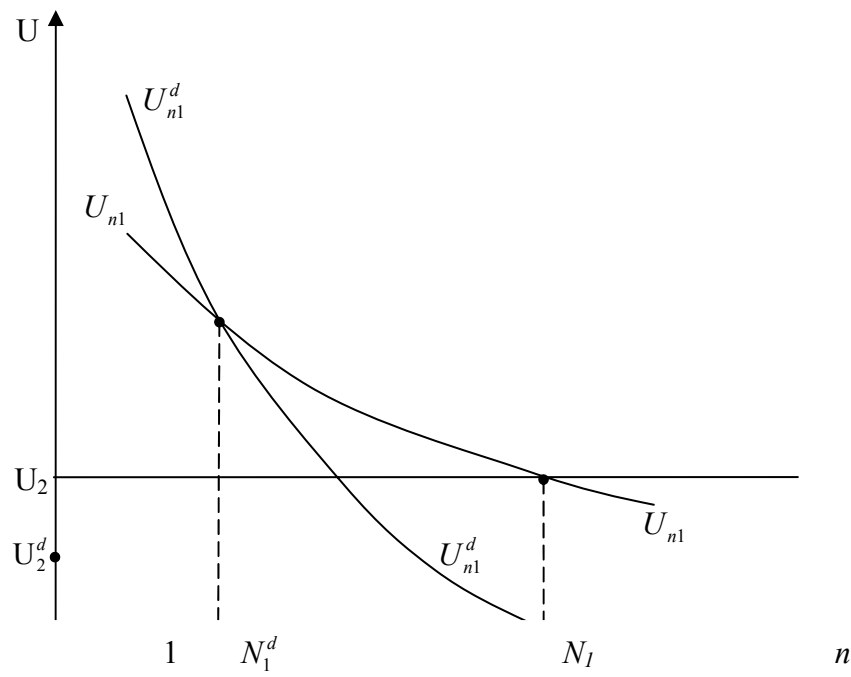


Figure 2: Allocation of Entrepreneurs Between the Sectors

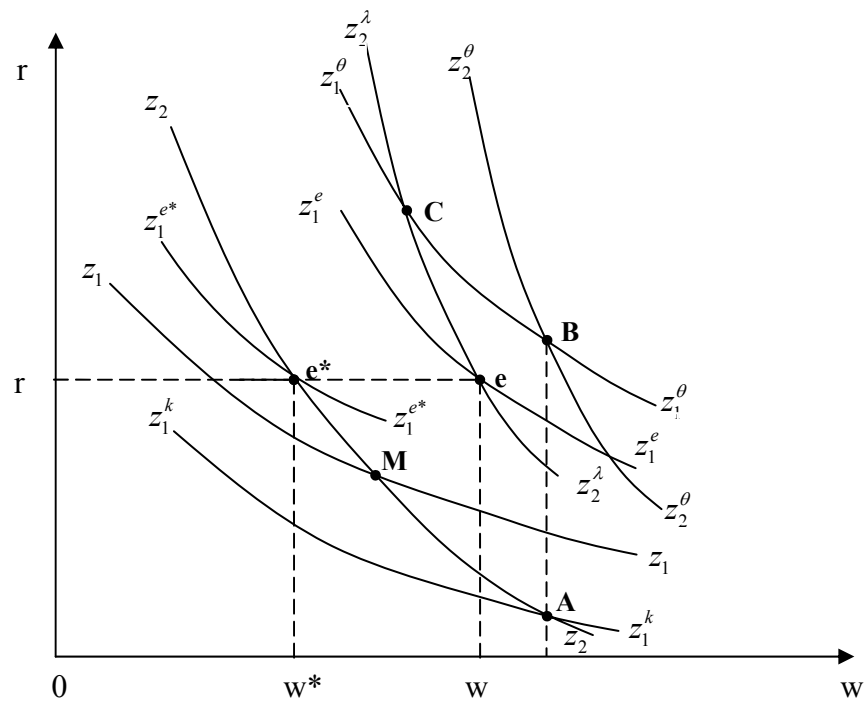


Figure 3: Determination of Factor Prices

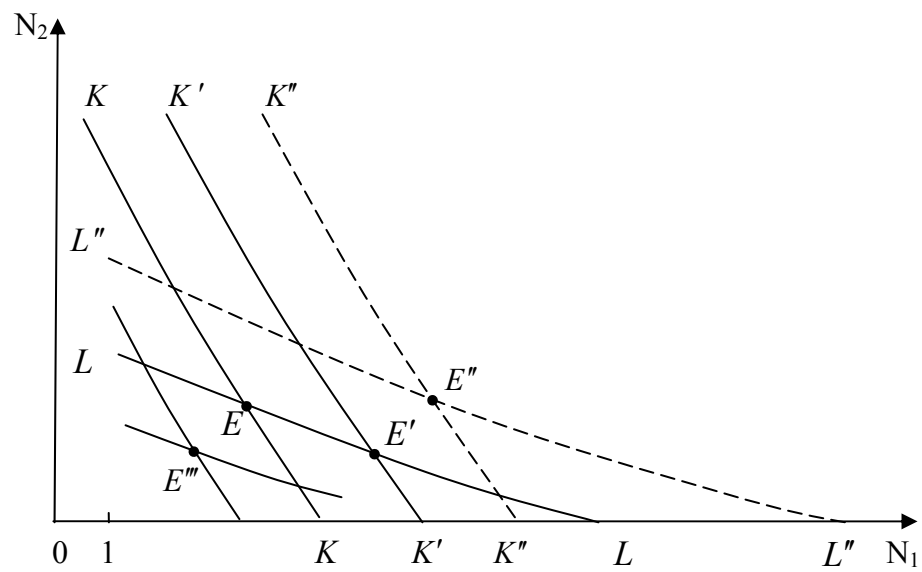


Figure 4: Changes in Endowment and Institutions

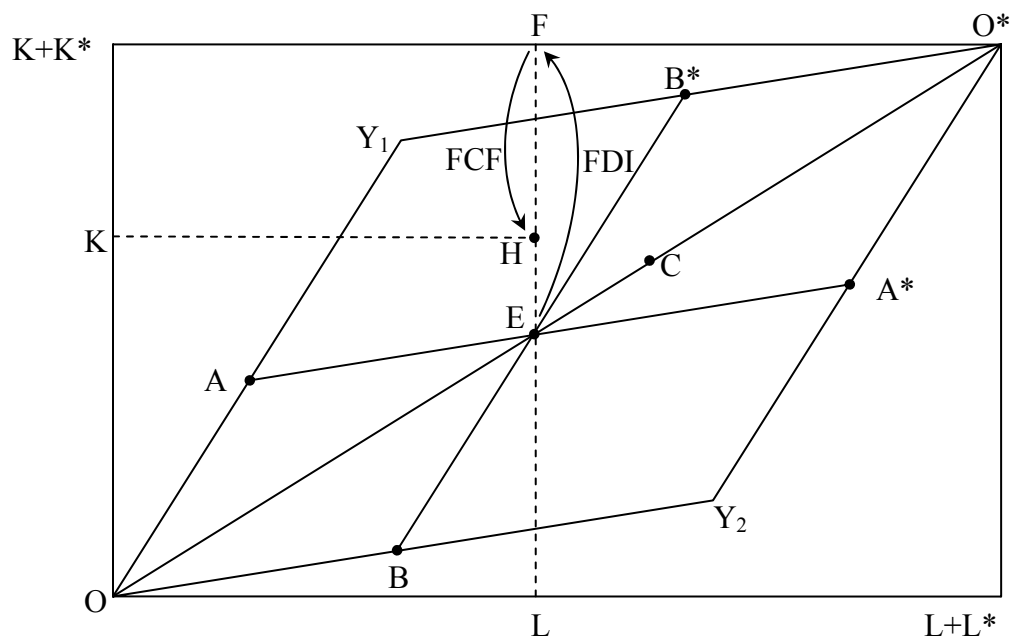


Figure 5: Capital Flows in an Integrated World Economy