WP/06/67

IMF Working Paper

Primary Surplus Behavior and Risks to Fiscal Sustainability in Emerging Market Countries: A "Fan-Chart" Approach

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INTERNATIONAL MONETARY FUND

 WP/06/67

IMF Working Paper

Research and Fiscal Affairs Departments

Primary Surplus Behavior and Risks to Fiscal Sustainability in Emerging Market Countries: A "Fan-Chart" Approach

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March 2006

Abstract

This paper proposes a probabilistic approach to public debt sustainability analysis (DSA) using "fan charts." These depict the magnitude of risks—upside and downside—surrounding public debt projections as a result of uncertain economic conditions and policies. We propose a simulation algorithm for the path of public debt under realistic shock configurations, combining pure economic disturbances (to growth, interest rates, and exchange rates), the endogenous policy response to these, and the possible shocks arising from fiscal policy itself. The paper emphasizes the role of fiscal behavior, as well as the structure of disturbances facing the economy and due to fiscal policy, in shaping the risk profile of public debt. Fan charts for debt are derived from the "marriage" between the pattern of shocks on the one hand and the endogenous response of fiscal policy on the other. Applications to Argentina, Brazil, Mexico, South Africa, and Turkey are used to illustrate the approach and its limitations.

JEL Classification Numbers: H62, H63, C15, E62

Keywords: Public debt sustainability, fiscal reaction function, risk analysis, fan charts.

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[1] Celasun and Ostry, Research Department, International Monetary Fund; Debrun, Fiscal Affairs Department, International Monetary Fund. We thank Laura Alfaro, Stephen Bond, Luis Catão, Mark De Broeck, Bob Flood, Simon Johnson, Mohan Kumar, Saul Lizondo, Davide Lombardo, Enrique Mendoza, Raghu Rajan, Luca Ricci, Marta Ruiz, Alex Swoboda, CharlesWyplosz, and IMF colleagues for comments, and Marcos Souto for superb research assistance.

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I. INTRODUCTION

In the public debt context, a sustainable position is often viewed as one where the government (or public sector) is solvent. To be deemed solvent, a government must be expected to honor current and future financial obligations, including the implicit commitment to continue providing certain public goods, services, and transfers in the future. Solvency thus implies that the present value of government disbursements (including inherited debt amortization, interest payments, and non-interest expenditure) should not exceed the present value of revenues, or equivalently that, the present value of future revenues *net of* non-interest spending (the primary balances) should at least cover the existing public debt. The intertemporal budget constraint and the relationship between the primary balance and the public debt have therefore been at the center of the literature on debt sustainability.

In practice, the notion of sustainability is less straightforward than it appears. First, at a conceptual level, it always implies a judgment as to what constitutes an acceptable strategy for the government to ultimately satisfy its intertemporal budget constraint (Mendoza and Oviedo, 2004). By definition, solvency excludes outright default or forced restructuring.[2] Yet most analysts would also exclude the inflation tax from the set of acceptable strategies, limiting the latter to adjustments in the primary balance. Hence, solvency is only a necessary condition for sustainability, and defining sufficient conditions involves judgment. Second, at a technical level, the forward-looking nature of solvency makes it difficult to assess. No one knows for sure the primary surplus a government will be able (or willing) to generate in 5, 10, or 20 years, nor the future path of interest rates, inflation, and productivity growth over that period. Absent uncertainty, of course, assessing solvency would be a mere arithmetical exercise. In reality, however, it requires making judgments under uncertainty, as well as the recognition that such judgments are subject to risk.

In practice, assessments of debt sustainability—including those performed by IMF country teams—rely on medium-term simulations of the debt-to-GDP ratio given specific macroeconomic forecasts and fiscal policy assumptions. In the absence of reliable "sustainability thresholds," however, such estimates *per se* do not allow one to determine the sustainability of a particular public debt position. Instead, the expected *dynamics* of the debt-to-GDP ratio over the medium term (generally 5 to 10 years) are interpreted as a signal as to whether underlying policies can be sustained under plausible macroeconomic conditions without endangering solvency. Specifically, a declining trend in the debt ratio signals that government policies are unlikely to jeopardize sustainability, whereas a positive trend or even stabilization at a high level may motivate concerns about sustainability, especially if other factors—such as the fiscal adjustment needed to stabilize or reduce the debt ratio—point to likely difficulties in keeping debt under control.

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[2] Solvency is generally defined as the ability to meet one's financial obligations on time.

Uncertainty about future macroeconomic conditions and fiscal policy inevitably weakens the basis for drawing compelling policy conclusions using such analyses. This paper proposes a methodology that improves our understanding of the risks surrounding debt dynamics, and explicitly acknowledges the inherently probabilistic nature of debt sustainability analysis (DSA) exercises. A more nuanced and more credible assessment of long-term sustainability results.

Of course, standard DSA does recognize the importance of uncertainty, with risks to the baseline debt projection appraised through simulating alternative debt paths under less favorable conditions—so-called "bound tests." This approach to risk assessment is entirely deterministic, however, involving a set of scenarios in which one key variable at a time is hit by an adverse shock—including lower growth, higher interest rates, a lower primary balance, and exogenous debt increases, such as those resulting from exchange rate depreciation or the recognition of off-budget obligations. The calibration of the shocks generally uses a multiple or fraction of the unconditional variance of the underlying series.

Although the bound tests approach gives a broad sense of the sensitivity of the sustainability assessment to adverse developments, significant methodological limitations undermine its credibility and operational relevance. First, both the correlations among shocks and the *joint* dynamic response of the variables relevant for debt dynamics are ignored. Indeed, simulated deterministic disturbances can realistically be of only two types: large and transitory, or small and permanent.[3] Second, fiscal policy is assumed not to react to the simulated economic developments, as if a deterministic policy process could reasonably prevail in an intrinsically stochastic environment. Third, in an uncertain world of course, each individual bound test formally has a near-zero probability of occurrence, making any meaningful quantification of risk impossible.

Measuring risk to debt dynamics requires a stochastic simulation apparatus whereby many bound tests covering a range of likely shock combinations could be generated. With a framework capable of randomly generating a large sample of bound tests, frequency distributions of the debt ratio can be derived for each year of a projection, permitting an explicitly probabilistic assessment of debt sustainability.

Our paper proposes a stochastic DSA algorithm along these lines. The algorithm preserves a certain degree of standardization (to ease cross-country comparisons) while allowing for country-specific risk factors to be reflected. To illustrate its versatility, it is applied to five emerging market countries with fairly different risk profiles, namely Argentina, Brazil, Mexico, South Africa, and Turkey.

The algorithm consists of three building blocks. First, for each country, the joint distribution of shocks is calibrated to fit the statistical properties of historical data. These

[3] Recent adjustments to the IMF's DSA template placed greater emphasis on small and permanent shocks, leaving only exogenous debt increases as large and temporary disturbances.

properties are captured in unrestricted VAR models which (i) describe comovements among the determinants of debt dynamics (essentially GDP growth, interest rates and exchange rates); (ii) provide estimates of the conditional variances and covariances of the shocks; and (iii) generate a consistent set of projections for the determinants of debt dynamics. The estimation of the VAR model requires quarterly data.

A second block characterizes fiscal behavior through an explicit fiscal reaction function. The latter is calibrated using panel estimates obtained for a sample of 34 emerging market economies during 1990-2004, and can be adjusted to reflect country-specific information on future policies. Allowing for endogenous fiscal policy improves the risk analysis by taking into account the plausible policy response of the primary balance to economic shocks and public debt developments. The use of panel techniques to estimate fiscal policy responses provides a common benchmark for all countries represented in the sample, though if sufficient time series data are available to estimate a country-specific reaction function, this could replace the use of panel techniques.[4]

The third block combines simulated economic scenarios (first block) with the estimated fiscal policy process (second block) to produce annual public debt paths.[5] Hence, the latter not only reflect plausible constellations of shocks, but also consistent projections for growth, interest rates, exchange rates, and fiscal policy. Through repeated simulations of random shocks, we construct a large sample of public debt projections for each year of the forecasting horizon. The corresponding frequency distributions yield a probabilistic assessment of debt dynamics. Specifically, we use "fan charts" to depict confidence bands for varying degrees of uncertainty around the median projection, which helps refine the usual debt sustainability assessment—based solely on the trend in the central projection for debt.

The paper also draws on earlier work looking at public debt sustainability in relation to primary surplus behavior. IMF (2003) focuses on determining debt thresholds beyond which sustainability could be considered at risk given average fiscal behavior. The same study introduces the concept of "overborrowing," defined as the excess of current public debt over the annuity value of future primary surpluses. Using an expanded version of the dataset in IMF (2003), Abiad and Ostry (2005) refine the estimations of fiscal reaction functions (including a richer set of political and institutional variables) and of the determinants of overborrowing, and calculate the impact on sustainable debt levels of a variety of fiscal and institutional reforms. The present paper marries the approach to fiscal policy reaction functions in Abiad and Ostry (2005) and the stochastic analysis of debt issues in Garcia and Rigobon (2005) and Penalver and Thwaites (2004). These latter papers, by focusing on

[4] We are dubious about the utility of using quarterly fiscal data in country-specific VARs, given the observation that these data have a very high noise-to-signal ratio.

[5] Quarterly projections generated by VARs thus need to be annualized and fed into the conventional stock-flow identity of the public debt-to-GDP ratio. Simulated primary balances and public debt are determined recursively to account for the fiscal policy response to public debt developments.

higher frequency macroeconomic data, pay insufficient attention to the constraints on the evolution of public debt created by the endogenous response of fiscal policy to debt shocks.

The remainder of this paper is organized as follows. Section II discusses some building blocks of deterministic DSA, and compares the latter with newer, stochastic approaches. In Section III, we present the estimates of primary surplus behavior, and how we overcome a number of pitfalls (endogeneity problems) in estimation. Section IV describes the simulation algorithm for public debt and presents fan charts for five major emerging market economies. Concluding remarks and policy implications are provided in Section V.

II. Debt Sustainability Analysis and Risk

This section compares deterministic DSA with stochastic approaches, highlighting the value added of the latter over the former. As previously mentioned, debt sustainability is directly related to the notion of solvency, which requires that, in present value terms, future revenues be at least as large as the stock of current obligations and future commitments. As a consequence, a given debt position is sustainable as long as it does not exceed the present value of all future primary surpluses. The path of primary surpluses over the indefinite future being essentially unknown, however, implementing this definition is a tremendous challenge, calling for operational alternatives.

A. DSA Frameworks

Since the time horizon for macroeconomic projections rarely extends beyond a few years, the solvency concept is typically operationalized by asserting that sustainability is *not* in jeopardy if the expected path of primary surpluses causes the debt-to-GDP ratio to decline over the simulation horizon. Concerns about sustainability may arise if the debt ratio trends upwards or if it stabilizes at a high level relative to peer countries with similar fundamentals, or relative to its own historical track record; sustainability could also be called into question if the magnitude of fiscal adjustment required to stabilize the debt ratio were deemed to be excessive. In the standard DSA setup, the assessment does not relate to the sustainability of a particular debt position but rather to whether given policies lead to particular trends in the debt-to-GDP ratio which may in turn motivate calls for policy corrections.

A key issue with the DSA as just described is the omission of uncertainty—be it for example about future income growth, interest rates, fiscal policies, exchange rate movements, or even the recognition of contingent liabilities. A natural response is to subject the DSA's baseline projection to a series of shocks ("bound tests") that deteriorate the outlook. These include lower GDP growth, higher interest rates, a weaker primary balance, a depreciation of the exchange rate, and the recognition of off-budget obligations. The standard bound-testing approach is deterministic, however, and limited to either isolated shocks or ad-hoc combinations of them. While the *unconditional* variance of the underlying series determines the magnitude of the simulated disturbances, actual covariances—especially between fiscal and nonfiscal variables—are ignored. This may lead one to severely underestimate risks to debt sustainability if adverse shocks—say to growth, interest rates, and exchange rates—combine in an explosive cocktail for debt dynamics.[6]

Calibrating deterministic bound tests to reflect economic and policy patterns observed in a given economy thus constitutes another challenge for the standard DSA framework. One possibility is to devise a small number of standardized scenarios—where isolated shocks are expressed as a proportion of the historical standard deviations of the variables—such that both the shock itself *and* the resulting debt path appear plausible in probabilistic terms (IMF, 2003). By its nature, this approach lends itself to the construction of standardized bound tests applicable to many different countries, and requires only a fairly parsimonious dataset.

For the sake of illustration, the outcome of the IMF's deterministic bound-testing exercise is presented in Figure 1 for South Africa over the 2005-2010 time horizon.[7] The DSA template provides debt paths corresponding to several standardized scenarios: the baseline (reflecting macroeconomic projections and policy assumptions); small but permanent adverse shocks (half a standard deviation) to real GDP growth, the real interest rate and the primary balance; a combination of these three shocks (this time assuming a quarter of a standard deviation); and two large temporary disturbances, namely a 30 percent real depreciation and a shock to the debt stock (mimicking the recognition of contingent liabilities) equivalent to 10 percent of GDP. In line with South Africa's relatively stable economic environment and low external indebtedness, the selected bound tests suggest fairly low risks to public debt sustainability over the medium term (Debrun, 2005a).

B. Benefits of an Explicit Risk Assessment

One drawback of the deterministic bound-testing approach depicted in Figure 1 is that the underlying scenarios hardly ever follow shock patterns observed in the economy. Specifically, the method ignores co-movements among the determinants of debt dynamics whereas such comovements are central in the stability of the debt ratio (see Goldfajn, 2005). Furthermore, the standardization of those tests implies fairly different degrees of realism across countries. Since the likelihood of the resulting debt paths cannot be precisely assessed, observers have no choice but to judge the plausibility of these bound tests on the basis of their core assumptions—e.g., a growth slowdown without repercussions on interest rates or fiscal policy—rather than on their probabilistic merits in terms of debt outcomes.

A legitimate question is thus to ask whether a diagnostic based on a few highly stylized scenarios is robust to more realistic constellations of shocks. If a joint distribution of relevant economic disturbances can be estimated for the country under review, stochastic simulations reflecting actual co-movements of shocks in the economy can produce a large sample of

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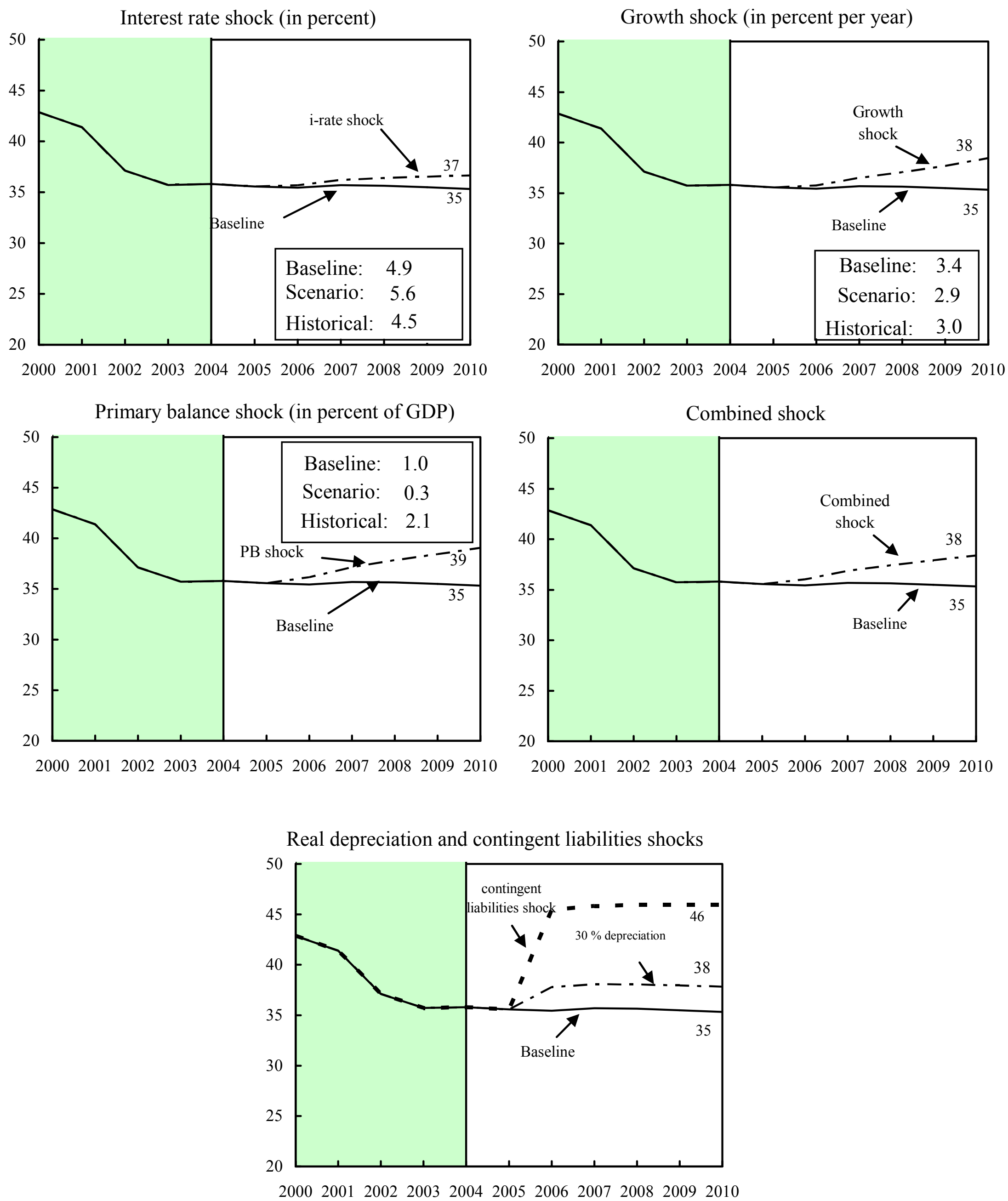
[6] That would be the case if a slowdown in economic activity were typically followed by a depreciation of the currency, rising interest rates, and deteriorating primary balance.

[7] This is based on the new DSA template introduced in July 2005.

Figure 1. South Africa: IMF Standard Debt Sustainability Analysis, 2004-2010



Sources: South African National Treasury and IMF staff estimates.

more realistic bound tests from which country-specific frequency distributions of debt can be derived. These frequency distributions provide a quantitative assessment of the risks to the baseline debt projections that may ultimately help refine fiscal policy recommendations.

Another issue is the extent to which the sustainability diagnostic reflects plausible fiscal policy behavior and properly accounts for the fact that fiscal policy itself is a source of uncertainty. Commonly used DSA scenarios assume that fiscal policy is invariant to the stylized shocks. While this assumption contributes to the policy dialogue by highlighting the consequences of policy inaction, there are strong a priori grounds as well as ample evidence that the primary balance tends to systematically respond to variations in public debt and to the business cycle, among other factors. Ignoring these feedbacks may thus distort the risk assessment. [8] In particular, assuming a constant primary surplus in the face of business cycle fluctuations reduces the estimated risk to debt dynamics because slowdowns are generally accompanied by fiscal easing while expansions often fail to improve the primary balance (see Section III). Also, the estimated residuals of the reaction functions provide information on the stability of fiscal behavior, with erratic policy impulses being another independent source of uncertainty. Conversely, the passive policy assumption fails to capture the stabilizing response of the primary surplus to the debt itself, thereby increasing estimated risks to debt.

In addition to improved reliability of the risk analysis, accounting for systematic features of the policy process should lead to more focus of policy advice on fiscal effort (measured as the departure from the estimated reaction function) rather than on the unconditional change in the fiscal balance. Deviations of actual policies from the benchmark provided by the estimated reaction function may prove useful in assessing such effort, and thereby the room for fiscal adjustment (Abiad and Ostry, 2005 and Debrun, 2005b).

Table 1 summarizes some key difference between the usual DSA framework and the extended DSA proposed in this paper.

C. Overview of the Simulation Algorithm

Our simulation algorithm seeks to satisfy three main objectives: (*i*) provide a sensible way to account for fiscal behavior and simulate changes to it; (*ii*) provide a tool easily applicable to different emerging market countries while referring to a common benchmark for policy assessment; and (*iii*) keep data requirements close to those in the standard DSA.

The first objective puts the fiscal reaction function as an essential building block. The second objective points to panel data techniques to estimate an average behavior across a group of countries that could serve as a "positive" benchmark for each individual member of the group though, as suggested earlier, if sufficient country-specific data are available, it would in principle be possible to estimate a country-specific fiscal reaction function. The third objective suggests relying as much as possible on annual data.

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[8] This being said, our simulation tool can accommodate any normative policy scenario, including the constant policy assumption.

Table 1. DSA and Risk Assessment

	Deterministic bound-testing (commonly used DSA)	Probabilistic approach (in this paper)
Diagnostic based on...	...a few stylized, isolated shocks; exogenous policies.	...a large number of random shock constellations drawn from an estimated joint distribution; endogenous fiscal policy.
Calibration of shocks	Fraction or multiple of historical standard deviations of underlying variables. Calibration based on the likelihood of the resulting debt path for a sample of emerging market economies.	Directly based on the estimated joint distribution of disturbances (country specific).
Output	Large temporary shocks provide a probabilistic upper bound to the debt ratio; small permanent shocks delineate interval of most probable outcomes.	Frequency distributions of the debt ratio over time, "fan charts."
Main advantages	Amenable to standardized bound tests across countries; low data requirement.	Better reflection of country specificity (in terms of shocks and fiscal policy behavior); explicitly probabilistic output.

Sources: IMF, and the authors.

These objectives impose a significant departure from existing algorithms.[9] Specifically, there is a need to initially separate the estimation of the fiscal reaction function from that of the other economic relationships before merging them again when simulating the behavior of the public debt ratio. There are (at least) two compelling reasons for doing so. First, the estimation of the variance-covariance matrix of shocks inevitably relies on time-series techniques (an unrestricted VAR model) that demand higher-than-annual frequency data. However, in contrast to financial variables and GDP, budgetary data at such frequency are often either unavailable or unreliable for the purpose of policy evaluation.[10] The second

[9] IMF (2003) develops a tractable stochastic simulation tool that shares many features with subsequent research, including ours. However, the IMF algorithm relies on annual data for all relevant variables, placing a premium on the availability of long and stable time series. In contrast, the simulation tools developed by Garcia and Rigobon (2005), Penalver and Thwaites (2004) and Tanner and Samake (2005) require high-frequency data, including for fiscal variables. Such data requirements limit the number of countries to which these algorithms can be applied. Our approach tries to better internalize data constraints, focusing on a shorter time span (during which regime shifts and structural changes are less likely) while keeping annual fiscal data at the center of the analysis. This potentially increases the number of countries to which our algorithm could be applied.

[10] Higher frequency budgetary data (quarterly or monthly) are typically quite noisy, and are meant to serve cash management purposes rather than policy assessment. While countries with Fund-supported programs or greater leeway to vote supplementals during the budget year may well undertake policy corrections on a quarterly basis in response to shocks or slippages, the overall policy stance still tends to be reflected in the annual figures (see Wyplosz, 2005, who finds standard reaction functions to fit very poorly using monthly data for Brazil).

reason is that the VAR framework restricts the specification of the reaction function in undesirable ways: e.g., the primary surplus responds to contemporaneous variations in the output gap, not lagged ones, as imposed by the VAR framework.

Accordingly, our algorithm comprises three building blocks, the first one being discussed more fully in Section III below, and the other two, in the Appendix.

1. A fiscal *reaction function*—in the *positive* sense of a *description* of average fiscal policy patterns—is estimated for a panel of emerging market economies with annual data. In line with the literature, the general specification is given by:

$$p_{i,t} = \alpha_0 + \rho d_{i,t-1} + \gamma\, ygap_{i,t} + X_{i,t}\beta + \eta_i + \varepsilon_{i,t}, \qquad t = 1,...T, i = 1,..., N \tag{1}$$

   where $p_{i,t}$ is the ratio of the primary balance to GDP in country *i* and year *t*, $d_{i,t-1}$ is the public debt to GDP ratio observed at the end of period *t*-1, $ygap_{i,t}$ is the output gap, η_i is an unobserved, country fixed-effect, and $X_{i,t}$ is a vector of control variables.

2. For each country, we estimate an unrestricted VAR model comprising the non-fiscal determinants of public debt dynamics. Formally, we have $Y_t = \gamma_0 + \sum_{k=1}^{p} \gamma_k Y_{t-k} + \xi_t$ where $Y_t = \left(r_t^{us}, r_t, g_t, z_t\right)$, and γ_k is a vector of coefficients, r^{us} denotes the real foreign interest rate, r, the real domestic interest rate, g, the real GDP growth rate, z, the (log of the) real effective exchange rate, and ξ is a vector of well-behaved error terms: $\xi_t \sim N(0,\Omega)$. This model serves two purposes. First, the variance-covariance matrix of residuals Ω characterizes the joint statistical properties of the contemporaneous, non-fiscal disturbances affecting debt dynamics. Specifically, the simulations use a sequence of random vectors $\hat{\xi}_{t+1},...,\hat{\xi}_T$ such that $\forall \tau \in [t+1,T]$, $\hat{\xi}_\tau = W\upsilon_\tau$, where $\upsilon_\tau \sim N(0,1)$, and W is such that $\Omega = W'W$ (W is the Choleski factorization of Ω). Second, the VAR generates forecasts of Y consistent with the simulated shocks. As shocks occur each period, the VAR produces joint dynamic responses of all elements in Y. It should be noted that the method is not sensitive to the ordering of variables in the VAR. Ordering matters only to the extent that one tries to capture causal relationships between innovations and the other variables (e.g., for impulse response functions). In the present context, stochastic simulation results are shaped by the variance-covariance matrix of reduced-form errors Ω, which is unique (see also Garcia and Rigobon, 2005).

3. For each simulated constellation of shocks, quarterly VAR projections are annualized, and the simulated annual output gap results from the growth differential between predicted GDP growth and the (annualized) steady-state growth rate produced by the VAR (to ensure that shocks to the output gap are zero on average). The corresponding debt path can then be calculated recursively using equation (1) and the conventional stock-flow identity: $d_t \equiv (1+g_t)^{-1}\left[(1+r_t^{us})(1+\Delta z_t)d_{t-1}^* + (1+r_t)\tilde{d}_{t-1}\right] - p_t + s_t$, where d_t^*

captures the foreign-currency-denominated debt, $\widetilde{d}_t$ designates the public debt in domestic currency, and s_t, stock-flow adjustments, for instance due to the recognition of contingent liabilities or the realization of assets. Notice that in each simulation, the primary surplus incorporates a fiscal policy shock $\varphi_{i,t} \sim N\left(0,\sigma^2_{\varphi_i}\right)$, where $\sigma^2_{\varphi_i}$ is the country-specific variance of the reaction function's residuals.

This algorithm can produce an arbitrarily large number of debt paths (say 1000 or 10,000) corresponding to different shock constellations. Frequency distributions of the debt ratio can then be obtained for each year of projection, and used to draw "fan charts" and probabilistic sustainability indicators discussed below in Section IV.

Results from using this algorithm are still subject to three limitations. First, in many emerging market economies, a lack of long time series combined with ongoing structural change and frequent shifts in policy regime reduce the reliability of econometric estimates for predicting future developments. Second, extreme situations—such as crises—are bound to remain low-probability events in this framework. Fan charts can at best detect the risk of undesirable "non-crisis" situations.[11] Third, the combination of annual and quarterly data shuts off any feedback effect of fiscal policy on economic variables (the causation runs only in the other direction), and in particular interest rates (through credibility effects) and growth (through the quality of fiscal policy). This may be an important issue if fiscal reforms, for example, are likely to produce significant changes in the future course of growth and interest rate spreads. While our proposed methodology will not pick up such effects (with future growth and interest rates being driven by the steady state values of these variables from the VAR), our approach can nonetheless accommodate imposing such effects on the results (overriding the steady state values from the VAR used in the algorithm) if these are believed to be important in coming to appropriate judgments about the risks to future debt dynamics.

The next section turns to the first building block of our model, and provides a benchmark fiscal reaction function for a group of emerging market economies.

III. Debt Dynamics and the Conduct of Fiscal Policy

As shown by Bohn (1998), governments concerned with solvency would be expected to raise the primary balance in response to an increase in the public debt-to-GDP ratio. If all other determinants of fiscal policy are stationary, the positive correlation between debt and the primary surplus is sufficient to guarantee that the debt ratio will revert to some finite steady-state value. This section describes the estimation of fiscal reaction functions for a group of emerging market economies over 1990-2004, where the regressors include debt and a range of other economic, policy, and institutional variables of interest.

[11] Conversely, fan charts will inevitably tend to exaggerate the risks faced by countries coming out of troubled times. More generally, it is critical to bear in mind that the *simulated* frequency distributions of debt are not the "true" probability distributions at a point in time.

A. Primary Surplus Behavior and Public Debt Sustainability

A growing number of studies have recently estimated fiscal reaction functions (Mélitz, 1997; Galí and Perotti, 2003; IMF, 2003, 2004; Wyplosz, 2005, among others). The aim of this literature is to identify a stable relationship between fiscal policy (measured by the primary balance) and various potential determinants. Such an exercise does not attempt to establish causality; the idea is rather to extract information on the key considerations that may shape and be correlated with policy decisions. Debt sustainability is expected to be one of those considerations, along with cyclical developments, and institutions affecting a government's incentives. Accordingly, a version of equation (1) is commonly estimated.

One difficulty with estimating fiscal rules is the scarcity of relevant budgetary data, which has led researchers to use panel data methods. The most meaningful data from the perspective of policy evaluation is available annually, in line with the budget procedure of most countries. Although fiscal policy adjustments may occur at lower (say quarterly frequency), these signals are blurred by the intrinsically noisy nature of high frequency budgetary data, which are generally used for cash management purposes rather than policy evaluation.[12] Wyplosz's (2005) attempt to fit a fiscal rule to high-frequency data for Brazil shows how difficult it is to capture policy behavior at higher-than-annual frequency.

Panel estimation assumes similar fiscal behavior across countries. To account for possible heterogeneity, we control for a large number of potential determinants of fiscal policy (that may differ across countries), and also explore the possibility of non-linear relationships between the primary balance and some of its determinants.

B. Estimating Fiscal Reaction Functions

In line with the literature, the reaction function we estimate, given in equation (1), relates the annual primary fiscal balance to the outstanding level of public debt, the gap between actual and trend output, and a number of potentially important drivers of the primary balance in emerging market economies.[13] These include real oil prices in oil exporters, an index of institutional quality, and two indicator variables accounting for whether the country is in a status of sovereign default and whether it is committed to an IMF-supported program.

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[12] One notable exception is in the case of an IMF-supported program, where quarterly reviews of policy implementation take place.

[13] Fiscal balances react to economic fluctuations both through the discretionary attempts of policymakers to stabilize output fluctuations and the tendency for primary balances to "automatically" decline (increase) as a share of GDP during cyclical downturns (expansions). The literature on policy cyclicality typically focuses on the first channel (see Kaminsky, Reinhart and Vegh, 2004). Given our interest in debt sustainability, we focus on the evolution of actual primary balances (rather than the cyclically adjusted balances which better reflect discretionary behavior), implying that our specification captures both the automatic and discretionary responses of fiscal policy to the business cycle.

The estimation of equation (1) needs to take into account three sources of endogeneity bias. The first is simply the endogeneity of the output gap with respect to contemporaneous fiscal policy shocks, $\varepsilon_{i,t}$. The remaining two sources stem from the dependence of lagged debt on past values of the primary surplus. As to the second of the three sources, clearly the lagged debt level, $d_{i,t-1}$, will necessarily be correlated with the country-specific and time-invariant determinants of primary surpluses, η_i: countries able to generate higher surpluses on average—captured by higher values of the fixed-effects η_i—will tend to have a lower level of public debt; and if this is not properly accounted for, the negative correlation between debt levels and the unobserved country fixed-effects would exert a downward bias on the estimated primary surplus response to debt. As to the third source of endogeneity, to the extent that there is persistence in the idiosyncratic error term, $\varepsilon_{i,t}$, the dependence of lagged debt on past surpluses will render lagged debt in equation (1) endogenous.[14]

Ideally, one could address these potential endogeneity problems with adequate instrumentation. To tackle the first source of endogeneity, the output gap needs to be instrumented with variables that are exogenous to the idiosyncratic primary surplus shocks. The second source of endogeneity—the endogeneity of debt to the primary surplus fixed-effects—can in principle be addressed by the inclusion of country indicator variables among the regressors, but this method would still be subject to the third endogeneity problem if there is strong serial correlation in the idiosyncratic errors.[15] Moreover, the use of country-dummies can potentially introduce an additional bias, commonly referred to as the small-sample bias of the fixed effects estimator.[16] By contrast, instrumenting both the output gap and lagged debt (with variables that are exogenous to the primary surplus fixed effects and to the idiosyncratic errors) would simultaneously address all three potential endogeneity problems. That said, reliable instrumental variables (IV) based estimations require the use of suitable exogenous instruments that are strongly correlated with the endogenous regressors.

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[14] For instance, a positive shock to the primary surplus in period *t*-1, i.e. a positive realization of $\varepsilon_{i,t-1}$, would reduce the debt stock, $d_{i,t-1}$. Thus, persistence in the idiosyncratic policy shocks (serial correlation between $\varepsilon_{i,t-1}$ and $\varepsilon_{i,t}$) would result in a negative correlation between $d_{i,t-1}$ and $\varepsilon_{i,t}$.

[15] The inclusion of country indicator variables addresses the endogeneity of debt to η_i by transforming the equation to eliminate η_i. Specifically, when country dummies are included, the mean values of the dependent and explanatory variables across all time periods for each country are obtained and the regression is performed on the variables in deviations from their country means.

[16] A large literature analyzes the bias of the least squares with dummy variables estimator in dynamic models that include the lagged dependent variable as a regressor. The bias of this estimator decreases with the time dimension of the sample and the variance of the lagged dependent variable that is attributable to factors other than the disturbance terms (see e.g. Kiviet, 1995, or Judson and Owen, 1999). Our model falls into the category of dynamic panel models given the presence of lagged debt among the regressors.

Such ideal instruments are difficult to find in our context. Moreover, IV-based regressions are generally not very efficient, yielding estimates with relatively large standard errors.

Against this background, our strategy is to estimate five possible specifications. The first two, a LIML regression and a system GMM specification, respectively, instrument for the output gap and lagged debt, and exclude country dummies. A third uses instruments for the output gap only and includes country dummies to account for the fixed effects. This specification eliminates the first two sources of endogeneity, but not the endogeneity from the persistence in idiosyncratic policy shocks; it should yield similar results to the first two methods if the serial correlation in the errors is weak and if the small-sample bias associated with the use of country dummies is small.[17] Beyond this, specifications 4 and 5 below incorporate nonlinearities to better capture heterogeneities in fiscal behavior across countries and circumstances. A detailed discussion of the estimation strategy is given in the Appendix.

Estimation Results

Our panel comprises 34 countries and a maximum of 15 years (1990-2004); data on primary balances and public debt levels were obtained from IMF desk economists for the largest available coverage of the fiscal sector (see appendix).

For the linear reaction function, we present in columns 1-3 of Table 2 three specifications. The first eliminates the country effects by using first differences, and instruments for the lagged change in debt and contemporaneous change in the output gap (see equation B.1 in the appendix), using as instruments lags of one-year U.S. bond rates, changes in real oil prices, lagged fiscal costs of banking crises, and import demand in industrial-country trading partners.[18] We run a LIML regression, which is preferable to GMM if the instruments are not very strong. In the second regression, we implement Blundell-Bond (1998) system-GMM (SGMM), which jointly estimates the level and differenced forms of the equation, using lagged differences and levels of the endogenous regressors as instruments in addition to the exogenous instruments used in the LIML regression. Third, we estimate a version with country dummies, instrumenting only the output gap with import demand in industrialized trade partners (GMM-DV).[19]

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[17] Analytical derivations show that the expected small sample bias of the fixed effects estimator would be positive for ρ, the coefficient of lagged debt. Monte Carlo simulations suggest that the magnitude of the bias on ρ is typically less than 15-20 percent of the true coefficient (Celasun and Kang, 2005).

[18] The fiscal costs of banking crises typically take the form of below-the-line expenditures, thereby increasing the public debt burden without affecting recorded primary surpluses (see the Appendix for further details).

[19] Including time dummies in this equation slightly increases the estimated standard error of the coefficient on the output gap (from 0.11 to 0.17), while the estimated coefficients on the time dummies are not statistically significant and the coefficients on all other regressors remain the same. This suggests that any common time trend in fiscal policy behavior is due to, and fully captured by, comovements in the output gap across countries.

Table 2. Estimates of the Fiscal Reaction Function, 1990-2004

Dependent Variable: Level or Change in the Primary Fiscal Balance

	(1) LIML (Difference)	(2) System GMM	(3) GMM with DV	(4) LIML (Difference)	(5) GMM with DV
Lagged Debt	0.039	0.030***	0.046***	0.121	0.097***
	[0.032]	[0.007]	[0.008]	[0.172]	[0.036]
Output Gap	0.104	0.217***	0.328***		
	[0.109]	[0.072]	[0.113]		
Real Oil Price	0.481***	0.084**	0.354***	0.487***	0.361***
	[0.072]	[0.030]	[0.082]	[0.112]	[0.086]
Institutions	0.374	-0.219	-0.675***	0.463	-0.380
	[0.484]	[0.322]	[0.258]	[0.445]	[0.256]
IMF Program	0.765**	1.121	1.108***	0.777**	0.939**
	[0.347]	[0.689]	[0.328]	[0.344]	[0.328]
Default	0.870**	0.884	1.190***	0.749***	1.077***
	[0.351]	[0.813]	[0.401]	[0.297]	[0.368]
Debt Spline				-0.108	-0.062*
(50 percent)				[0.194]	[0.037]
Positive output gap				-0.092	0.181
				[0.358]	[0.631]
Negative output gap				0.258	0.268
				[0.246]	[0.225]
Constant		-0.684	-0.963		-3.628
		[1.479]	[1.138]		[2.892]
Country dummies	No	No	Yes	No	Yes
Observations	349	399	418	368	418
Hansen test (P-value)	0.84	1.00	0.45	-	0.03
AR(1) test (P-value)		0.05			
AR(2) test (P-value)		0.09			
Cragg-Donald Stat.	7.23		19.63	1.96	

Notes: Standard errors in brackets, * denotes significance at 10%; ** at 5%; *** at 1%. P-values of the test statistics are reported for the tests of overidentifying restrictions and tests of serial correlation in the residuals of the difference equation in the System GMM regressions (AR tests). In the LIML regression in the first column, the second and third lags of U.S. one-year bond rates, second and third lags of the changes in real oil prices, lagged fiscal banking crisis costs, the contemporaneous value of trade partners import demand were used as instruments for lagged debt and the output gap. The Blundell and Bond (1998) system-GMM regression in Column 2 uses the second lags of the output gap and debt, in addition to the banking fiscal cost measure and the trade partners import demand. The third column presents a GMM regression with country dummies, where the output gap is instrumented with the contemporaneous and lagged values of trade partners import demand. The equation in Column 4 is exactly identified, hence there is no test of overidentifying restrictions. The instruments include lagged fiscal banking crisis costs, the contemporaneous value of trade partners import demand, and the interaction of these variables with a dummy that indicates whether debt exceeds 50 percent, and a dummy that indicates whether the output gap is positive. The estimation in column 5 instruments only for the positive and negative output gap terms, using the interactions of the trade partners import demand measure.

All three estimations suggest a positive response of primary surpluses to public debt. The LIML and SGMM estimates of ρ (Columns 1 and 2) imply an only slightly weaker response (0.030-0.039) than the GMM-DV regression (0.046). With a positive estimated coefficient on the output gap, primary balances are estimated to be countercyclical. However, this effect is likely to be driven mostly by the worsening of the balance during recessions rather than improvements during booms, as discussed below in the context of the nonlinear specification. Our estimates confirm that countries with IMF-supported programs run higher surpluses. And countries in default—that is, not current on their sovereign obligations—run larger primary balances, reflecting their restricted market access.[20]

The estimated coefficient of the index of institutional quality is mostly negative, but significant only in the GMM-DV regression that explicitly controls for country fixed effects.[21] An interesting observation is that the estimated country fixed effects in the GMM-DV regression are positively correlated with the average institutional quality over the sample period (Figure 2). This suggests that countries with stronger institutions run larger primary balances on average, holding all other factors constant. The negative estimated impact of institutions may stem from the fact that improvements in institutional quality are typically associated with decreases in borrowing costs, implying that countries would need to run smaller primary balances to service a given level of debt as their institutions improve. Thus, once debt levels and the country fixed effects are controlled for, an improvement in institutional quality is estimated to be associated with lower fiscal effort.

Columns 4-5 in Table 2 present estimates of the nonlinear fiscal reaction function, which allows for a "kinked" response to debt at 50 percent of GDP, and a different response to the output gap depending on the latter's sign.[22] The specification takes the form:

$$p_{i,t} = \alpha_0 + \rho d_{i,t-1} + \overline{\rho} D_{i,t-1}\left(d_{i,t-1} - 50\right) + \gamma^p P_{i,t} ygap_{i,t} + \gamma^n N_{i,t} ygap_{i,t} + X_{i,t}\beta + \eta_i + \varepsilon_{i,t} ,$$

where $D_{i,t-1}$ is a dummy variable that equals one if debt in period *t*-1 exceeds 50 percent of GDP, $P_{i,t}$ is a dummy variable that equals one if the output gap is positive (a boom), and $N_{i,t}$ is a dummy variable that equals one if the output gap is negative (a downturn).

The larger number of parameters and required instruments limits the choice of estimation methods.[23] In particular, SGMM is known to be unreliable when the total number of instruments becomes large relative to the number of cross-section units. Thus, for this.

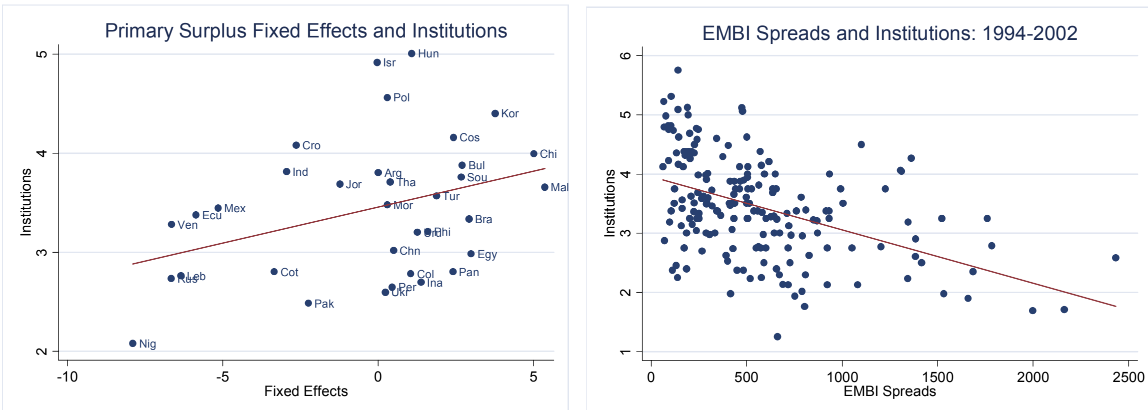
[20] The size of the estimated coefficient on the default dummy is robust to dropping Argentina from the sample.

[21] When we replace the country fixed effects with the time average of institutional quality as a time-invariant regressor for each country, it is estimated to have a positive coefficient.

[22] Previous work suggests a structural shift in the primary surplus equation when debt reaches 50 percent of GDP and a different response of the surplus depending on the sign of the output gap (Abiad and Ostry, 2005).

[23] Given the interaction terms in the specification, any instrument used for debt needs to be interacted with $D_{i,t-1}$ as an instrument for the splined term, and any instrument for the output gap needs to be interacted with $P_{i,t}$ and $N_{i,t}$ to serve as instruments for the positive and negative gap measures. Thus, the number of excluded instruments needed for the nonlinear specification is double that needed for the linear specification.

Figure 2. Institutional Quality, Primary Surplus Behavior, and EMBI Spreads



specification, we present two regressions : a LIML regression that excludes country dummies and uses instruments for lagged debt and the output gap,[24] and a GMM regression that includes country dummies and instruments only for the (negative and positive) gap terms.

Although the findings are less precise than those obtained for equation (1) (due to the lower degrees of freedom given the two extra parameters that are estimated), they qualitatively confirm the hypotheses that the fiscal response to debt is stronger when debt is below 50 percent of GDP, and that the response to booms and recessions is asymmetric. Once debt exceeds 50 percent of GDP, ρ declines to 0.01-0.03—slightly lower than the response indicated by the linear specifications. Moreover, both regressions suggest that the worsening in primary balances during economic contractions exceeds the improvements attained during economic booms, qualitatively confirming the hypothesis proposed in discussing the linear specification.

For our baseline calibration exercise in the next section, we use the GMM-DV parameter estimates in Column 3. This regression provides us with estimated country fixed effects which are useful for gauging heterogeneity across countries, and we also note that the parameter estimates in column 3 are statistically more significant than those using other methods. As a robustness check, however, we also present simulations based on the SGMM method in Column 2, with a somewhat weaker response to debt accumulation. For simulations using the nonlinear specification, we use the GMM-DV estimates in Column 5.

[24] In the LIML estimation in column 4, the banking-crisis fiscal cost measure and its interaction with $D_{i,t-1}$ were used to instrument for lagged debt and splined debt. The interactions of the import demand measure with $P_{i,t}$ and $N_{i,t}$ were used as instruments for the gap measures, respectively. This equation was thus exactly identified.

IV. Risks to Debt Sustainability in Five Emerging Market Economies

This section proposes various prospective and retrospective risk analyses of public debt in five emerging market economies with fairly different risk profiles: Argentina, Brazil, Mexico, South Africa, and Turkey. After a discussion of the calibration, we apply the simulation algorithm outlined in Section II to generate a sample of 1000 simulations from which we derive frequency distributions of public debt. We then discuss possible tools of analysis—fan charts and a summary indicator of debt sustainability—under two possible "baseline scenarios." Finally, we examine the sensitivity of the risk analysis to variations in the underlying assumptions.

A. Calibrating the Simulations

For a given country, all simulations assume the same joint distribution of disturbances and comovements among the variables; the VAR estimates are given in the Appendix.

The fiscal reaction function combines both standardized and country-specific features as follows—an upper "hat" designates parameter estimates obtained in Section III as well as the corresponding predictions:

$$\hat{p}_{i,t+\tau} = \Lambda_{i,t+\tau} + \hat{\rho} d_{i,t+\tau-1} + \hat{\gamma}\ ygap_{i,t+\tau} + \varphi_{i,t+\tau}, \text{ for } \tau = 1,...,5, \qquad (3)$$

with $\Lambda_{i,t+\tau} = \hat{p}_{i,t} - \hat{\rho} d_{i,t-1} - \hat{\gamma}\ ygap_{i,t} + \kappa_{i,t+\tau}$, and $\varphi_{i,t+\tau}$, a policy shock drawn from a mean-zero normal distribution with variance equal to the country-specific variance of residuals.[25]

Equation (3) splits fiscal policy into an automatic, a pre-determined, and a random part. The automatic component follows the average response of the primary balance to the public debt and to the output gap, and is considered identical for all countries. The pre-determined part, summarized by $\Lambda_{i,t+\tau}$, captures the impact of all other determinants of the primary surplus, including institutional quality, the existence of an IMF-supported economic program, a default/restructuring option and, if relevant, the budgetary effect of oil-price fluctuations. By default, $\kappa_{i,t+\tau} = 0$ so that $\Lambda_{i,t+\tau}$ is a country-specific constant providing an anchor to simulated primary balance paths. We also allow for non-zero, time-varying values for $\kappa_{i,t+\tau}$ to account for specific information about future policy changes, such as those related to institutional reforms likely to affect fiscal performance, the adoption of an IMF-supported adjustment program, or alternatively, discretionary impulses envisaged in medium-term budget plans. The estimated reaction functions discussed in Section III may provide some guidance for the calibration of $\kappa_{i,t+\tau}$.

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[25] Recall that the fiscal disturbance is assumed to be orthogonal to economic developments.

Although this paper emphasizes the usefulness of estimated fiscal reaction functions, the simulation framework is flexible enough to accommodate a range of policy behavior, including under normative scenarios (e.g. the constant primary surplus assumption or program targets). Where sufficiently long data series are available, parameters corresponding to country-specific estimates of the fiscal reaction function can also be used. Finally, it is worth noting that deterministic stress tests representing shocks ignored in the empirical model, such as the materialization of contingent liabilities, are straightforward to perform (see Debrun, 2005a).

B. Baseline Scenarios

Our baseline scenarios only allow for automatic responses of the primary balance to real output shocks and public debt developments. One issue however is that $\Lambda_{i,t+\tau}$ incorporates the residual of the reaction function in year t. The discrepancy between the predicted primary surplus and actual fiscal behavior may result from discretionary slippages or adjustment efforts. Depending on whether that deviation is expected to be permanent or temporary, two baseline scenarios can be envisaged.

In the first one (the "constant" policy scenario), any deviation from the predicted primary surplus in t is assumed to persist over the entire simulation horizon, as if the most recent stance signaled a sustained departure from past primary surplus behavior (in that case, $\kappa_{i,t+\tau} = 0$ over the entire simulation horizon). That scenario is reminiscent of the "constant policy scenario" commonly found in deterministic DSAs although our framework only freezes the (non-debt related) discretionary part of policy. Alternatively (the "predicted" policy scenario below), recent deviations from the predicted primary surplus may result from purely temporary factors, in which case it is more appropriate to assume that the simulated surplus paths follow those predicted by the reaction function (so that $\kappa_{i,t+\tau} = -\hat{\varepsilon}_{i,t}$ over the simulation horizon). In both cases, we calibrate surplus behavior using a fairly common specification of the reaction function—linear relationship with country fixed-effects—where only the output gap is instrumented (see Section III). Simulations are performed for $t = 2004$ and $\tau = 5$, except for Argentina where we used $t = 2005$ to account for the debt exchange operation. In line with the DSA analysis presented in the latest staff country report (No 05/236), for Argentina we applied our framework to federal-level fiscal data rather than the more comprehensive data used for the other countries in the sample.

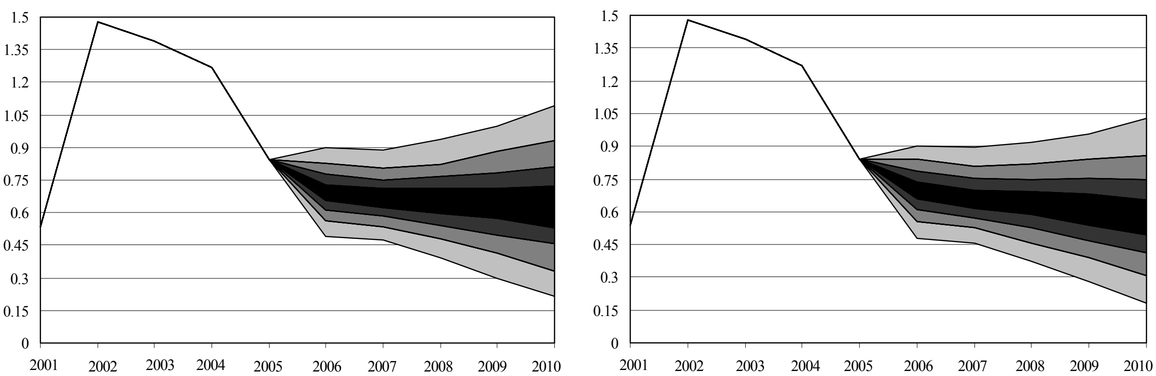
Fan charts summarize risks to debt dynamics by representing the frequency distribution of a large sample of debt paths generated by means of stochastic simulations (Figure 3). Different colors delineate deciles in the distributions of debt ratios, with the zone in dark grey representing a 20 percent confidence interval around the median projection and the overall colored cone, a confidence interval of 80 percent. A number of results flow from the charts.

First, compared with the outcome of simple bound tests (reported in Figure 1 for the example of South Africa), it appears that these fall within the 40 percent confidence interval only, confirming that deterministic bound tests very imperfectly account for the overall.

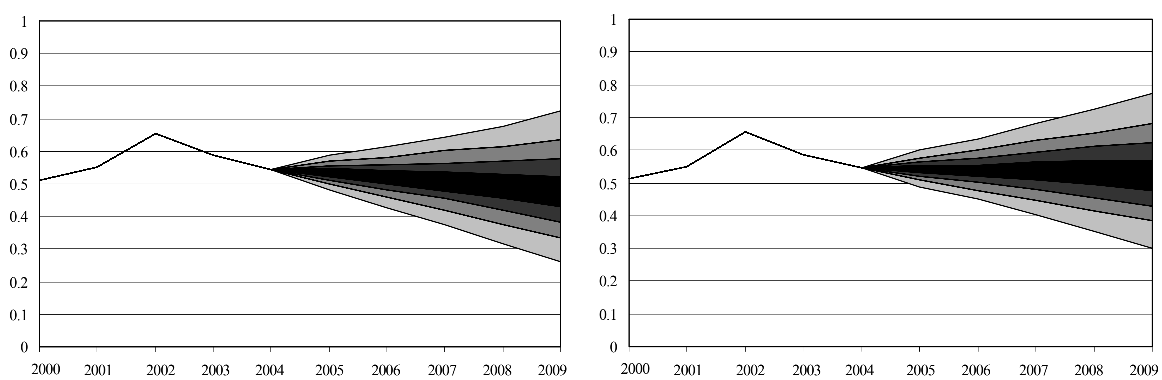
Figure 3. Fan Charts for Public Debt-to-GDP Ratios in 5 Emerging Market Economies (2005-2009)

"Constant" Policy *"Predicted" Policy*

Argentina



Brazil



Mexico

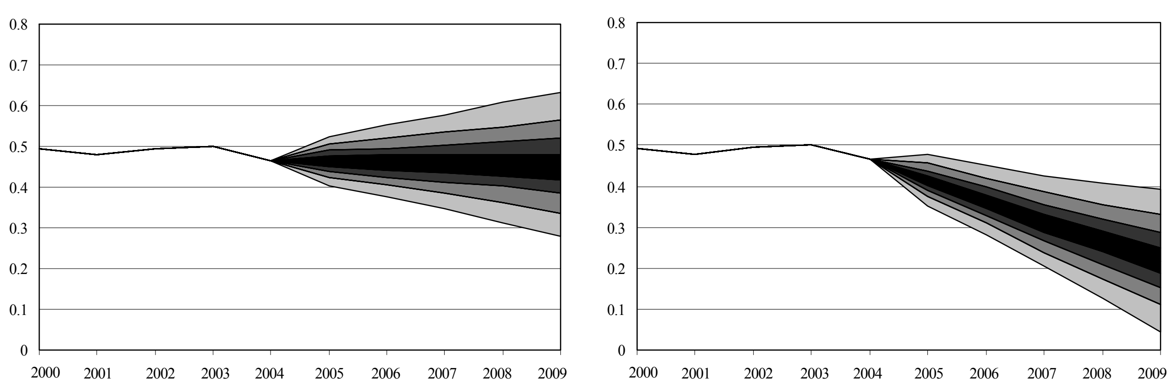
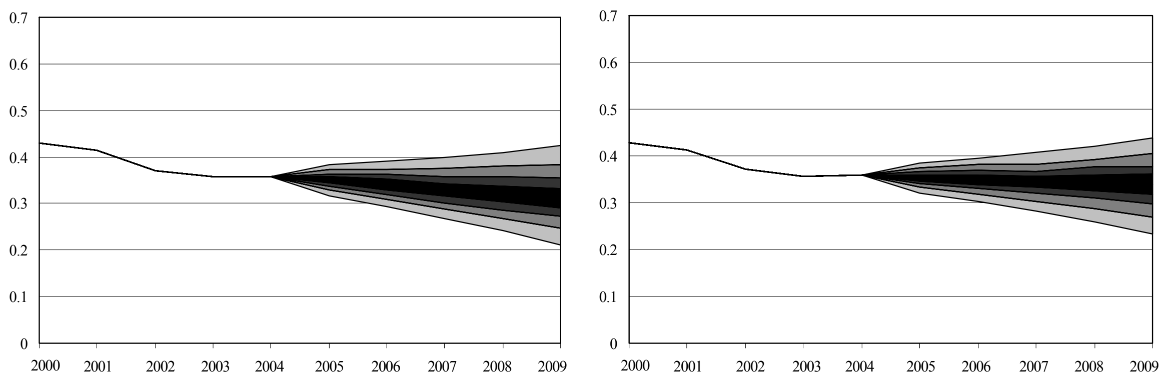


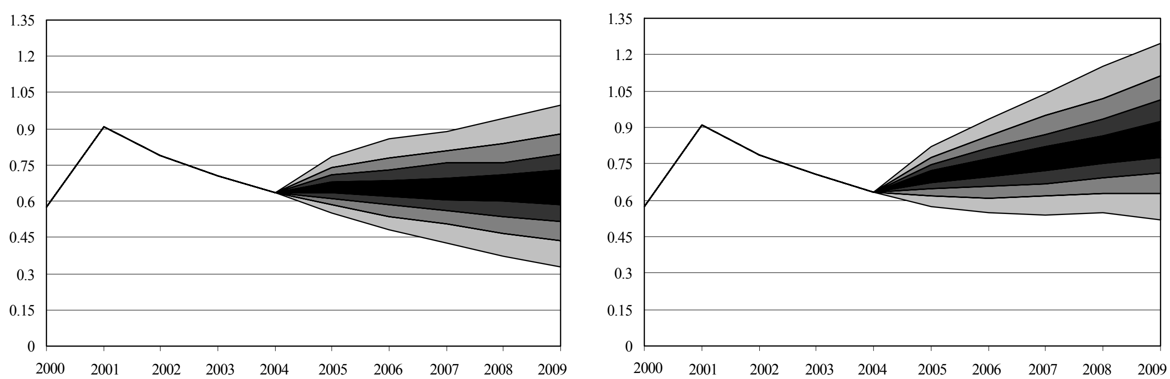
Figure 3 (Cont'd). Fan Charts for Public Debt-to-GDP Ratios in 5 Emerging Market Economies (2005-2009)

"Constant" Policy *"Predicted" Policy*

South Africa



Turkey



magnitude of risks to public debt sustainability. Fan charts provide a more comprehensive, and therefore reliable picture of the uncertainty surrounding public debt simulations.

Second, in the five countries under review, the fiscal reaction function generally appears sufficiently responsive to the public debt to ensure that the median debt path is sustainable (in the sense that the debt ratio is stable or declining over the simulation horizon). However, in most instances, the response proves too weak to prevent growing debt ratios in the two upper bands of the charts, representing the second and third deciles of the debt ratio.

Hence, in those cases, there is at least a 30 percent chance that combinations of adverse economic and policy shocks may lead to concerns over debt sustainability.[26]

Third, the overall risk profile obtained for the different countries reflects the intrinsic volatility of their respective economies, with less volatile economies—e.g., South Africa—exhibiting narrower confidence intervals than others. Incidentally, such cross-country differences cast further doubt on the merits of standardized bound tests. Of course, our assessment is contingent on the relatively short period of time over which the VARs are estimated. In particular, wide confidence intervals inevitably reflect past crises, and may thus overestimate the true magnitude of risks. This is especially evident in the simulations for Argentina and Turkey.

Fourth, the fan charts for Mexico and Turkey illustrate the issue of large residuals in the last year of observation. In Mexico's case, a highly negative residual (close to 5 percent of GDP) was observed in 2004, reflecting the lesser sensitivity of Mexico's primary balance to oil prices in comparison to other oil producers in our panel. The very large increase in real oil prices in 2004 thus showed up in the residual. A more detailed analysis of Mexico could usefully discuss alternative scenarios, including a reaction function excluding oil prices as a determinant of the primary balance. In Turkey, large positive residuals were observed in 2003 and 2004, reflecting significant fiscal adjustment efforts (Debrun, 2005b). Here, the charts clearly show that sustaining those high-primary surpluses (as reflected in the "constant" policy scenario) is key to containing the risks to debt sustainability.

Finally, fan charts give a good sense of the symmetry of the risks to debt. In Argentina and Turkey for instance, the distribution of debt ratios is skewed towards the upside. As the joint distribution of simulated shocks is symmetric, this outcome indicates that the standard response of the primary balance to debt proves insufficiently stabilizing in these cases. Hence, given the shock configurations typically observed in these countries, a more aggressive response of the surplus to debt increases than in the "average" emerging market country would seem warranted to contain upside risks to debt dynamics.

C. Sustainability Assessments

So far, the value added in our approach has been to produce probability distributions of debt at different horizons, rather than a deterministic path for debt.[27] But what is the policymaker to make of all these distributions? Presumably, while recognizing that there is

[26] Recall, as previously mentioned, that the simulations use the steady state values of growth and interest rates from the VAR, which may differ—in some cases substantially—from those used in the deterministic DSAs undertaken by country desks (see the Appendix).

[27] Although not undertaken as an explicit exercise in this paper (but see Abiad and Ostry, 2005), the approach can also be used to assess how the distribution of debt shifts in response to changes in policy fundamentals that are captured in the estimated reaction function (such as institutional reform, an IMF-supported program, etc.).

more information in being able to say "There is a 30 percent chance that debt will be below a given ratio to GDP in three years' time, and a 20 percent chance that it will be above another threshold over the same horizon" than in simply asserting "Under present policies, the debt ratio will decline over time or rise over time," the policymaker is ultimately likely to be interested in whether the country's debt profile/distribution is a problem or not. This is the issue that we try to get at in this section, while recognizing that what is an acceptable risk for one policymaker may be an unacceptable risk for another—risk aversion being in the end in the eye of the beholder, rather than something that can be objectively justified.

At the most basic level, the prospect of a downward trend in the debt ratio in a deterministic setup corresponds to the probability that the debt ratio falls below its initial value in our stochastic setting. How large this probability is—10 percent, 50 percent, 90 percent—which can be read off the fan charts, is clearly more informative from the point of view of generating useful sustainability indicators, than what is available to the policymaker from a deterministic exercise. But the policymaker may be interested in more than simply an assessment of whether debt is likely to decline or not, particularly if he/she is very concerned about the possibility that the debt ratio could rise (i.e., is very averse to upside risk).

Clearly, two countries with the same probability of a declining debt ratio may face quite different upside risks for debt. To account for the overall risk profile, one can posit a sustainability indicator that combines the probability that debt declines over time with the risk that debt *not* rise beyond some specific (or tolerable) amount over the same horizon. A plausible such indicator could be calculated as $\Pr(d_{t+\tau} < d_t) \times [1 - \Pr(d_{t+\tau} > (d_t + x))]$, where x is a positive mark-up over the initial public debt d_t. The *value* of the sustainability indicator increases with the likelihood of desirable outcomes, that is a non-increasing trend, and well-contained upside risks. That value has no intrinsic meaning, and it is up to the policymaker to set a *critical threshold* below which the debt situation would be a cause for concern. Such a threshold essentially depends on the degree of risk aversion and on the perceived need for a reduction in debt. These factors are reflected in the markup, x, the level of upside risk deemed acceptable (i.e., how small should $\Pr(d_{t+\tau} > (d_t + x))$ be in order to consider that upside risks are well contained), and the desired probability that debt declines in the future. The sustainability indicator being the product of these two probabilities—the probability of a declining debt ratio times the probability that the debt ratio will *not* rise by more than the markup—once these probabilities are set, a critical threshold is established.[28] The analyst or the policymaker can then focus on the problematic cases where the indicator's value falls below the threshold value of the sustainability indicator, and not worry the rest of the time.

In Table 3, we calculate the sustainability indicator for $x = 0.1$ (10 percent of GDP). Which entries in the table are worrisome, and which are benign? To illustrate, we pick a threshold value of 0.4. Many combinations of $\Pr(d_{t+\tau} < d_t)$ and $[1 - \Pr(d_{t+\tau} > (d_t + x))]$ can

[28] The arithmetic properties of the sustainability indicator are described in the Appendix.

Table 3. Probabilistic Debt Sustainability Assessment
("Constant" Policy Scenario)

	t+1	t+2	t+3	t+4	t+5
Debt ratio lower than in t					
Argentina (t=2005)	0.83	0.85	0.82	0.76	0.73
Brazil (t=2004)	0.59	0.63	0.64	0.63	0.64
Mexico (t=2004)	0.51	0.54	0.53	0.55	0.56
South Africa (t=2004)	0.60	0.64	0.69	0.70	0.71
Turkey (t=2004)	0.40	0.45	0.46	0.47	0.47
Debt ratio more than 10 percent of GDP higher than in t					
Argentina (t=2005)	0.06	0.06	0.10	0.15	0.20
Brazil (t=2004)	0.00	0.05	0.10	0.15	0.19
Mexico (t=2004)	0.03	0.08	0.11	0.16	0.20
South Africa (t=2004)	0.00	0.00	0.00	0.03	0.04
Turkey (t=2004)	0.21	0.28	0.34	0.35	0.38
Sustainability index 1/					
Argentina (t=2005)	0.78	0.79	0.74	0.65	0.59
Brazil (t=2004)	0.59	0.60	0.57	0.54	0.52
Mexico (t=2004)	0.50	0.50	0.47	0.46	0.45
South Africa (t=2004)	0.60	0.64	0.69	0.68	0.68
Turkey (t=2004)	0.32	0.32	0.30	0.30	0.29

1/ The sustainability index is defined as [Pr(debt < t level)*(1-Pr(debt > (t level + 10 percent)]

deliver such a value. For example, the level of comfort associated with that threshold corresponds to a probability that the debt ratio declines of at least 50 percent and a probability that debt rises by more than 10 percent of GDP of less than 20 percent: 0.40 [= 0.50 X (1 - 0.20)]; alternatively, of course, probabilities of decline of less (more) than 50 percent could be offset by lower (higher) probabilities that the debt ratio will rise by more than 10 percent of GDP. In Table 3, entries where the sustainability index is less than 0.4 are shaded.[29] The indicator underscores vulnerability particularly in the case of Turkey, which stems mainly from the upside risks to debt driven by the shocks—notably to domestic real intersest rates and the exchange rate—identified through the VAR. Mitigating this vulnerability requires, as shown by a comparison of the constant policy and predicted policy fan charts, persisting with Turkey's strong fiscal effort of recent years.

Probabilistic indicators of debt sustainability can prove especially useful for policymakers if they convey a credible signal that fiscal policy needs to be changed in order to reduce the likelihood of adverse outcomes to acceptable levels. A reasonable test of our approach to sustainability is thus whether it gives appropriate warnings of trouble on the eve of a crisis. Given data constraints, we investigate this issue—namely the track record of our

[29] More cells would be shaded if the policymaker wanted to reduce the probability of upside risk to say 10 percent from 20 percent: in that case the problematic entries would be all those where the sustainability indicator was lower than 0.45. In the limit, if the policymaker was extremely averse to upside risk, all cells below 0.50 would be shaded, signaling more worrisome cases than those shaded in Table 3.

sustainability indicator—in three "eve-of-crisis" cases: Argentina and Brazil at end-2000; and Turkey at end-1999, using only information available to policymakers at the time.[30] As a control, we also introduce a retrospective analysis of South Africa (at end-2000) to check whether our model would have properly differentiated the risks faced by these countries.

The fan charts in Figure 4 point to clear upside risks in all three "crisis" countries, with Argentina looking particularly vulnerable to explosive debt outcomes. Turkey's public debt dynamics also looked unmanageable under most circumstances, while Brazil's situation appeared broadly under control albeit with significant upside risks. This indicates that the endogenous debt-stabilizing response typically observed in emerging market economies was insufficient to prevent explosive debt dynamics, calling for significant fiscal adjustment efforts in Argentina and Turkey, and prudent fiscal management in Brazil. In contrast, our model nicely predicts the declining debt path that was effectively observed in South Africa.

This impression is confirmed by our sustainability indicator (Table 4), which exhibits low and declining values for Argentina and Turkey over the simulation horizon. The situation in Brazil at end-2000 looked less alarming than that in Argentina and Turkey, although the dramatic decline in the indicator over the simulation horizon signaled a rapid deterioration in the sustainability outlook. In sum, however, the proposed sustainability indicator would have been shining red for Argentina and Turkey in 2000 and 1999, respectively, and yellow in the case of Brazil in 2000, providing a signal of impending troubles.[31] As expected, South Africa exhibits a very high value of the sustainability indicator.

D. Alternative Calibrations and Policy Scenarios

The outcome of simulation exercises is potentially highly sensitive to initial assumptions. The impact of plausible variations in our baseline assumptions on the sustainability analysis thus needs to be assessed. For the sake of brevity, we only show and discuss the results for Argentina, Brazil and Turkey.

First, we consider the impact of a change in the specification of the fiscal reaction function along the lines suggested in Section III. In particular, we use parameters stemming from the system-GMM estimates of the linear reaction function (to account for a possible upward bias in the estimated responsiveness of the primary surplus to the debt). We also envisage a nonlinear reaction function, including a debt spline and differentiated responses to the output gap depending on the sign of the latter.

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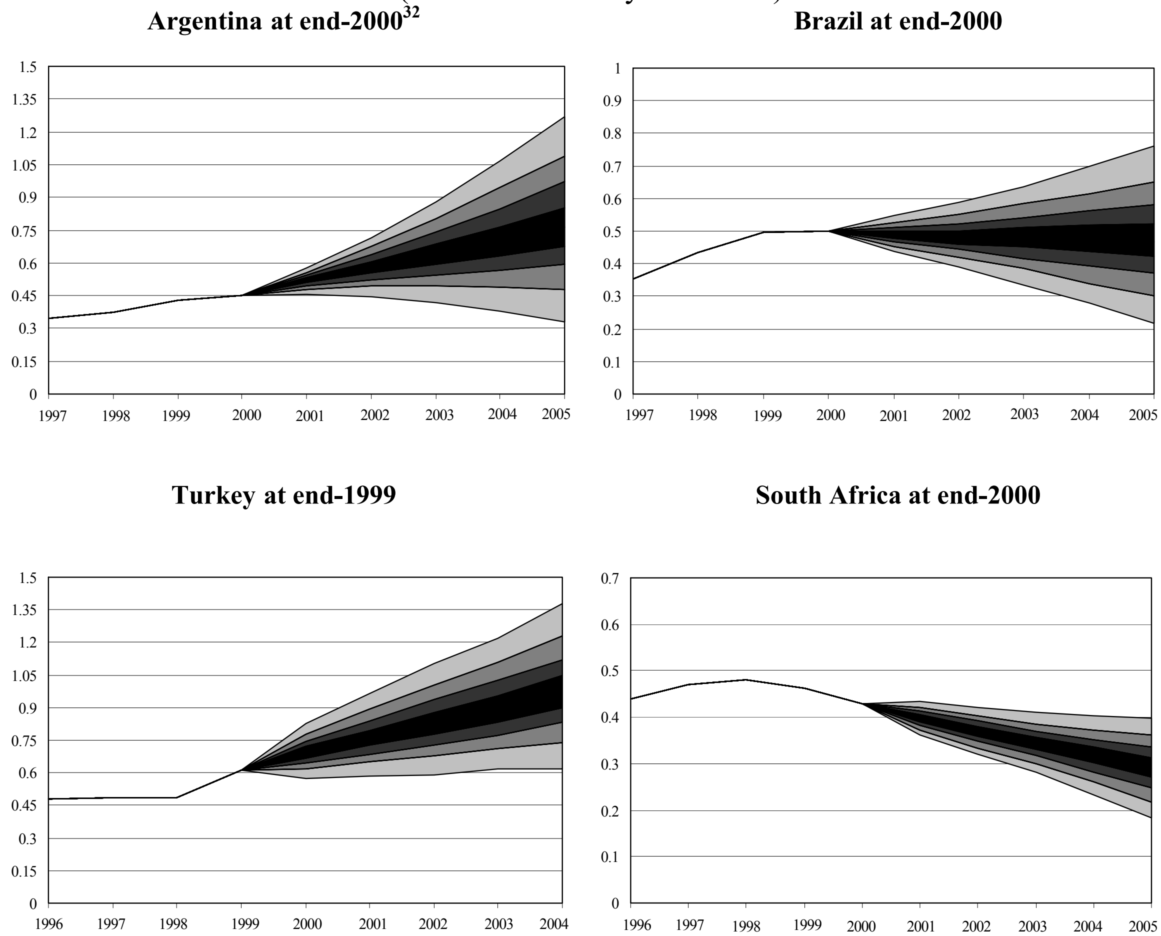
[30] The dating of crises is problematic for Brazil and Turkey, since neither country defaulted. While it is straightforward to extend the exercise to other dates, the dates chosen correspond to instances where sovereign spreads rose to very high levels (above, say, 1,000 basis points) at some point during the following year.

[31] As already mentioned, it would be worth repeating the exercise for Brazil for 1998 and 2001, which are arguably more plausible dates ahead of crises the following years.

Figure 4. Fan Charts on the Eve of Troubled Times
("Constant" Policy Scenarios)



Those changes in the policy reaction function do not materially affect the risk analysis (see Figure 5 and Table 5). In a sense, this should not be too surprising since all parameter estimates are based on the same information concerning fiscal behavior. Specifically, the weaker responsiveness to the public debt associated with system-GMM estimates is compensated by a higher pre-determined component of fiscal policy ($\Lambda_{i,t+\tau}$).

The apparent robustness of the risk analysis to different specifications of the fiscal reaction function immediately raises the question of the importance of accounting for policy endogeneity in this type of analysis. We therefore produced a set of simulations under the

[32] This considers the "U.S. junk bond" rate as the relevant foreign interest rate. Other specifications of the VAR proved unstable.

assumption of deterministic paths for the primary surpluses (using publicly available data from IMF staff reports). The corresponding fan charts reported in Figure 6 indicate that a

Table 4. Probabilistic Debt Sustainability Assessment: Looking Back at the Eve of Troubled Times
("Constant" Policy Scenario)

	2000	2001	2002	2003	2004	2005
	Debt ratio lower than in 2000 (1999 for Turkey)					
Argentina	n.a.	0.07	0.11	0.13	0.17	0.18
Brazil	n.a.	0.71	0.66	0.61	0.59	0.58
South Africa	n.a.	0.87	0.93	0.96	0.95	0.95
Turkey	0.18	0.14	0.12	0.09	0.10	n.a
	Debt ratio more than 10 percent of GDP higher than in 2000 (1999 for Turkey)					
Argentina	n.a.	0.24	0.60	0.68	0.72	0.75
Brazil	n.a.	0.01	0.06	0.14	0.20	0.25
South Africa	n.a.	0.00	0.00	0.00	0.00	0.00
Turkey	0.19	0.64	0.73	0.80	0.83	n.a
	Sustainability index 1/					
Argentina	n.a.	0.06	0.04	0.04	0.05	0.05
Brazil	n.a.	0.71	0.63	0.53	0.47	0.44
South Africa	n.a.	0.87	0.93	0.96	0.95	0.94
Turkey	0.14	0.05	0.03	0.02	0.02	n.a

1/ The sustainability index is defined as [Pr(debt < 2000 level)*(1-Pr(debt > (2000 level + 10 percent)], except for Turkey where we used [Pr(debt < 1999 level)*(1-Pr(debt > (1999 level + 10 percent)].

deterministic path for the primary surplus can substantially reduce the overall perception of risk. This is due to the fact that the deterministic assumption ignores the fiscal response to output gap variations as well as the volatility in the fiscal policy process itself. Although the deterministic assumption also overlooks the *stabilizing* response of the primary balance to debt, primary balance variations related to output gap and fiscal shocks appear to dominate. Overall, the deterministic policy assumption leads one to underestimate the risks to public debt sustainability, which in turn entails an upward bias in the sustainability index (Table 5).

Although the reaction function plays an important role in the analysis, the overall risk profile of public debt is essentially shaped by the statistical properties of economic shocks as identified by the VAR. This makes the method particularly sensitive to the choice of data entering the VAR.[33] While the selection of statistical series for GDP and the real exchange rate is relatively uncontroversial, choosing the relevant interest rates raises important questions, notably because actual interest payments do not respond one-for-one to market interest rates. To illustrate that sensitivity, we performed simulations using the U.S. junk bond rate (instead of the 10-year treasury bond) as the relevant foreign interest rate. Although the latter still ignores country-specific shocks to risk premia, it provides a reasonable approximation of changes in market sentiment regarding high-risk borrowers. Unsurprisingly, fan charts reveal much wider confidence intervals (Figure 6) while the sustainability index falls dramatically for all 3 countries (Table 5), confirming vulnerability to higher and more volatile interest rates.

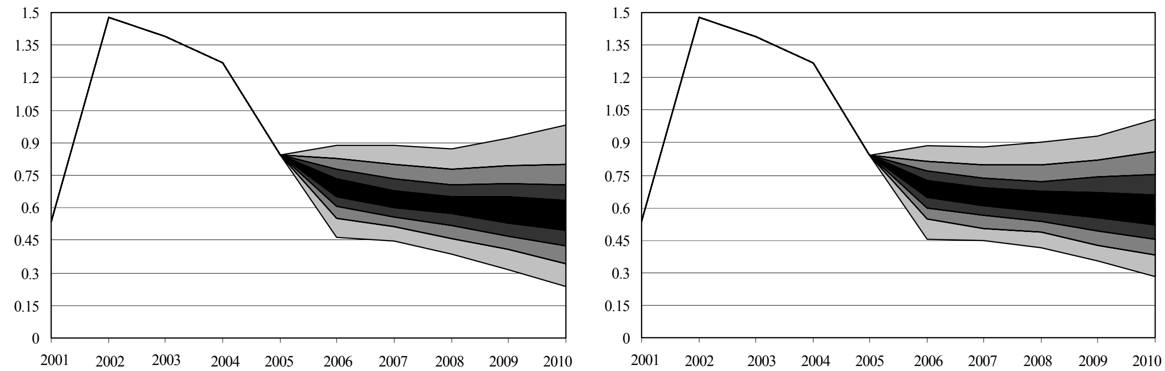
[33] Data availability also dictates the use of only one lag in the VAR.

Figure 5. Fan Charts for Public Debt-to-GDP Ratios in Argentina, Brazil and Turkey Based on Alternative Specifications of the Reaction Function

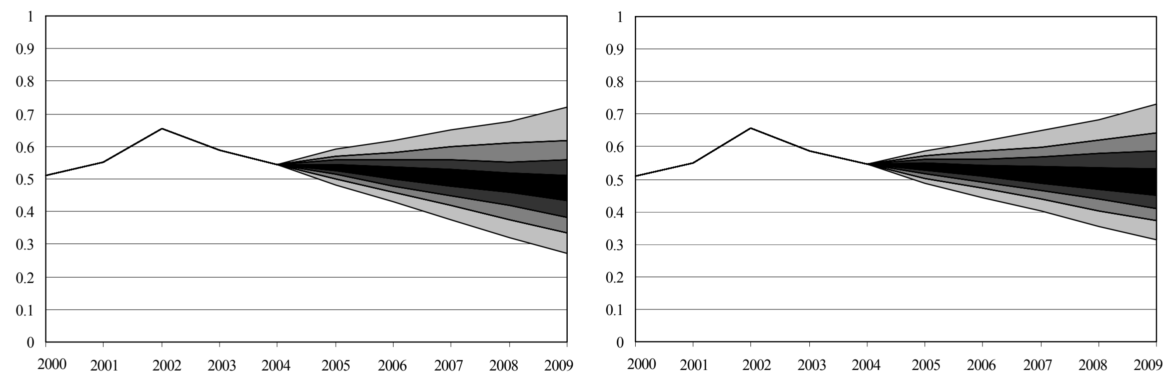
System-GMM estimates of the linear specification

Non-linear reaction function (debt spline and differentiated response to negative and positive output gaps

Argentina



Brazil



Turkey

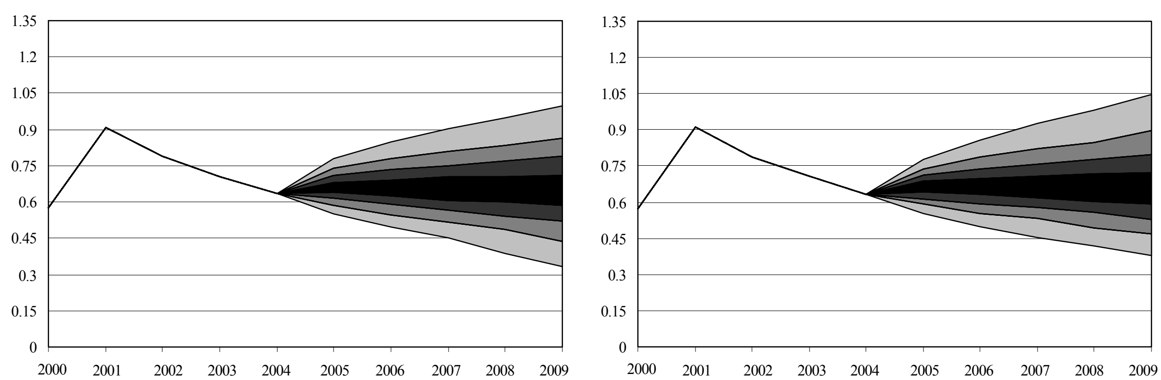


Table 5. Probabilistic Debt Sustainability Assessment: Alternative Calibrations
("Constant" Policy Scenario unless otherwise indicated, values for 2009 1/)

	Baseline	System GMM calibration	Non-linear reaction function	Deterministic fiscal policy	US junk bond interest rates
			Debt ratio lower than in 2004		
Argentina	0.73	0.83	0.79	0.98	0.64
Brazil	0.64	0.67	0.62	0.72	0.45
Turkey	0.47	0.43	0.44	0.51	0.36
			Debt ratio more than 10 percent of GDP higher than in 2004		
Argentina	0.20	0.12	0.14	0.01	0.27
Brazil	0.19	0.17	0.20	0.10	0.37
Turkey	0.38	0.37	0.37	0.28	0.51
			Sustainability index 2/		
Argentina	0.59	0.73	0.68	0.97	0.47
Brazil	0.52	0.56	0.50	0.65	0.28
Turkey	0.29	0.27	0.28	0.37	0.18

1/ Values for 2010 in the case of Argentina.
2/ The sustainability index is defined as [Pr(debt < 2004 level)*(1-Pr(debt > (2004 level + 10 percent)]

E. Implications of the Analysis

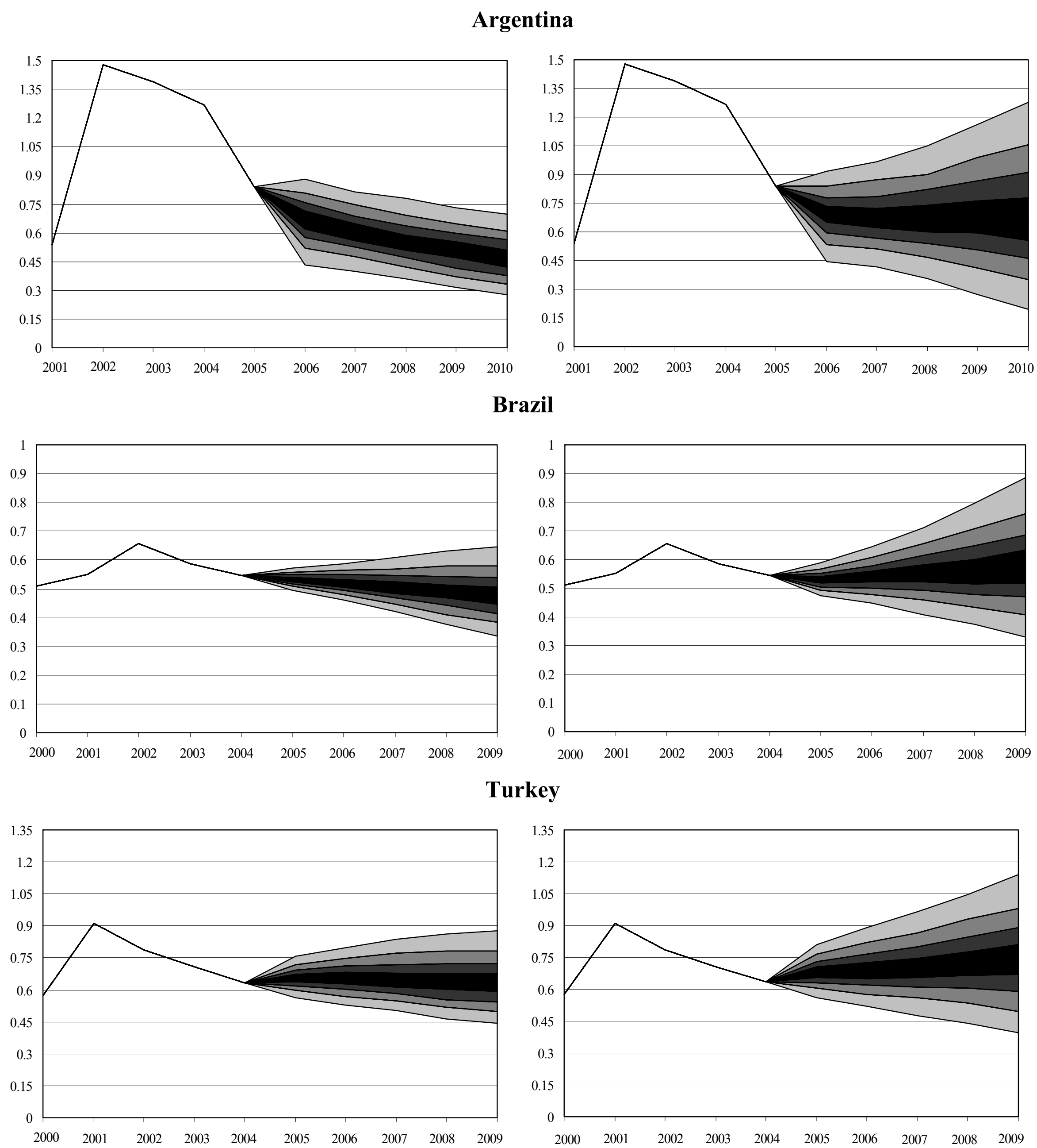
The analysis in this section has shown that our simulation algorithm could be easily applied to various emerging market economies in order to help develop a "risk-management" approach to public debt sustainability. The central idea was to provide policymakers with graphical illustrations (in the form of fan charts) that summarize the distribution of public debt, as well as numerical sustainability indicators that take into account characteristics of this distribution, notably the probability that debt will decline over some horizon and the probability that upside risks to the debt ratio will be limited.

The algorithm we proposed was also used to generate fan charts and numerical sustainability indicators on the eve of crises in order to check whether our approach would have provided credible early warning to policymakers: the indicator we developed indeed was flashing bright red on the eve of crises, and the fan charts give an equally supportive visual impression of the prospect of troubled times ahead. The analysis also illustrates the extent to which fiscal policy—summarized in the estimated fiscal reaction function—provides adequate insurance to stabilize the debt ratio in the face of shocks (on the eve of the above crises, it clearly did not). The algorithm is also flexible enough to study alternative policy scenarios, and assess their implications for risks to debt dynamics.

Sensitivity analysis showed that the method was robust to various specifications and estimation methods of the fiscal reaction function. We also showed that assuming a deterministic path for the primary balance—besides being highly unrealistic under changing economic conditions—led one to significantly underestimate risks to debt dynamics by ignoring the effect of the business cycle on the primary balance and the volatility of fiscal policy itself. For these reasons in our view, the probabilistic approach to debt sustainability represents a considerable improvement over bound tests used in deterministic DSAs.

Figure 6. Fan Charts for Public Debt-to-GDP Ratios in Argentina, Brazil and Turkey
(Alternative scenarios—2005-2009)

Deterministic policy (from IMF staff reports) *Using U.S. junk bonds interest rates*



V. Conclusion

This paper has developed an algorithm for generating an explicit risk analysis of debt dynamics, and applied it to five emerging market economies. The method builds on the standard approach to debt sustainability used at the IMFand elsewhere by accounting for country-specific risk factors regarding the economy and the fiscal policy process. Overall, we obtain a more complete, objective, and realistic assessment of risks than is possible with nonstochastic DSA templates, which rely on a few deterministic and standardized bound tests.

Our approach introduces greater realism in three critical dimensions. First, it uses estimates of joint probability distributions of economic shocks to construct a large number of scenarios that capture covariances among disturbances as well as the dynamic response of the economy. Second, it allows for fiscal policy to adjust to these shocks (to debt and growth, for example) according to the pattern commonly observed in emerging market economies, as given by our estimated fiscal reaction functions; the latter also appropriately shift the focus of policy analysis from adjustments in the nominal primary balance to measures of fiscal effort (defined as the difference between the predicted and actual or projected surplus). Third, we allow for fiscal policy itself to be a source of risk.

The debt sustainability assessment proposed here is explicitly probabilistic and can prove useful for policymakers in a variety of ways. First, the method offers a flexible tool allowing policymakers to capture country-specific features relevant for debt dynamics, and to have clearer signals of the risks involved in delaying fiscal adjustment or undertaking fiscal expansions. Second, more complete information on the debt risk profile should in turn improve medium-term budgetary planning. Indeed, one would expect greater awareness of the risks to public debt to promote caution in the conduct of fiscal policy. For example, this could imply less reliance on debt to finance new expenditure programs, thereby reducing the likelihood that the dynamics spin out of control due to the realization of macroeconomic risks such as lower growth, or higher interest rates. Also, an explicit quantification of risks could help in the design of consolidation strategies, as governments could evaluate the merits of alternative adjustment plans not only in terms of their impact on future trends of debt, but also on the upside risks to the debt path itself. More generally, governments with low credibility and operating in a volatile environment could better internalize the costs of policies implying higher debt ratios, while governments with greater credibility and facing a more stable environment could avoid taking excessive comfort in a benign baseline outlook.

Applications to five emerging market economies with different risk profiles illustrate the merits of our approach. In particular, we show how the fan charts and sustainability indicators can be used to guide policymakers in making judgments about whether the present course of fiscal policy will, or will not, lead to problems (a crisis) down the road. As important, the estimated fiscal reaction functions—which connect fiscal behavior to various economic and institutional fundamentals—should also be of use in guiding policymakers on how to forestall problems by pursuing reforms that will shift the distribution of public debt paths that the economy faces. On both scores, the approach advocated here yields significant value added relative to deterministic approaches that are still widely used to assess debt sustainability issues in emerging market countries.

Appendix

A. Debt Sustainability and Fiscal Behavior

The extent to which governments raise primary balances in response to increases in public debt—captured by the magnitude of ρ in equation (1)—is a key parameter in public debt sustainability assessments. This can be seen by deriving the path of public debt as a function of its determinants. The government's flow budget identity implies that the stock of debt at the end of period *t*, denoted by D_t, equals debt outstanding at the end of the previous period, D_{t-1}, minus the overall government budget surplus in period *t*. The overall surplus, in turn, is the difference between the primary balance P_t, and the sum of any "below the line" expenditures, B_t, and nominal interest payments on debt, $i_t D_{t-1}$, where i_t is the nominal interest rate paid on public debt in period *t*. Thus, D_t is given by $D_t = (1+i_t)D_{t-1} + B_t - P_t$. To account for the effect of output growth, the elements of this identity can be expressed in ratios to GDP. Letting Y_t denote nominal GDP, y_t the growth rate of nominal GDP, and b_t, d_t, and p_t the ratios of B_t, D_t and P_t to GDP, respectively, we obtain:

$$\frac{D_t}{Y_t} = \frac{Y_{t-1}}{Y_t}\frac{D_{t-1}}{Y_{t-1}}(1+i_t) + \frac{B_t}{Y_t} - \frac{P_t}{Y_t},$$

$$d_t = \frac{(1+i_t)}{(1+y_t)} d_{t-1} + b_t - p_t.$$

Deflating both the nominal interest factor and growth factor by the inflation rate, the flow budget identity can be expressed in terms of real interest and growth rates:

$$d_t = \left(\frac{1+r_t}{1+g_t}\right) d_{t-1} + b_t - p_t, \tag{A.1}$$

where r_t is the real interest rate on government debt and g_t is the real GDP growth rate. Combining the expression for the primary surplus given in equation (1) with the government's flow budget identity in (A.1), assuming that the real interest rate and the real growth rate are constant at *r* and *g*, respectively, and setting β to zero for simplicity, we obtain a first order difference equation for d_t, the public debt-to-GDP ratio:

$$d_t = \left(\frac{1+r}{1+g} - \rho\right) d_{t-1} + b_t - \gamma\, ygap_t - \eta_i - \varepsilon_t.$$

The backward-solution of this difference equation gives the stock of debt d_t as the weighted sum of past shocks to the primary balance, below the line expenditures and the output gap:[34]

$$d_t = \sum_{j=0}^{\infty}\left(\frac{1+r}{1+g}-\rho\right)^j (b_t - \gamma\, ygap_{t-j} - \eta_i - \varepsilon_{t-j}). \tag{A.2}$$

Assuming that the output gap and the shocks to the primary balance are stationary, equation (A.2) implies that a necessary condition for the path of debt to be non explosive—that is, for the debt to GDP ratio to converge to a finite level—is that $\left(\frac{1+r}{1+g}-\rho\right)<1$. In other words, if the government does not react to increases in the public debt strongly enough to ensure that ρ is greater than $\frac{r-g}{1+g}$ on average, the stock of public debt can grow without bound.

The manner in which primary surpluses respond to the output gap is also an important determinant of the path of public debt. If the response is asymmetric, in the sense that the primary deficit is allowed to worsen during recessions whereas no improvement is attained during booms, the path of debt is likely to have a sustained upward trend unless the response of primary surpluses to debt is strong enough to offset this "deficit bias."

B. Econometric Issues involved in Estimating Fiscal Reaction Functions

We now discuss some econometric issues involved in estimating fiscal reaction functions using panel data techniques, in particular the need to address the potential endogeneity of lagged debt and the output gap to the unobserved disturbances to the primary surplus. Since endogeneity of the output gap can be dealt with by straightforward instrumentation, most of the discussion focuses on ways to address the endogeneity of lagged debt. In order to evaluate the appropriate estimation techniques, this appendix also presents estimates of a simple baseline specification obtained using alternative estimation methods.

Potential Sources of Endogeneity Bias

In estimating equation (1), an obvious source of bias arises from the endogeneity of the output gap $ygap_{i,t}$ to contemporaneous policy shocks, $\varepsilon_{i,t}$. Specifically, fiscal contractions are often associated with a reduction in the output gap, and if not addressed, this negative correlation would tend to weaken the estimated impact of the output gap on the surplus.

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[34] When the real interest rate and growth are allowed to vary over time, the solution is:

$$d_t = \sum_{j=0}^{\infty}\left(\prod_{k=0}^{j}\left(\frac{1+r_t}{1+g_t}-\rho\right)\right)\left(b_t - \gamma\, ygap_{t-j} - \eta_i - \varepsilon_{t-j}\right).$$

Two other sources of endogeneity stem from the dependence of lagged debt on the past values of the primary balance. First, given that public debt represents the accumulation of primary deficits, the country-specific, time-invariant factors determining primary surplus generation capacity, η_i, will necessarily be negatively correlated with debt levels, $d_{i,t-1}$, giving rise to a second source of endogeneity. The third source of endogeneity exists only if there is persistence in the policy shock process, $\varepsilon_{i,t}$. If idiosyncratic policy shocks are correlated over time, the lagged debt term, $d_{i,t-1}$, which is endogenous to the fiscal policy shock in period *t*-1, will also be correlated with the period-*t* shock, $\varepsilon_{i,t}$. Like the endogeneity of debt to the country fixed-effects, this source of endogeneity would exert a downward bias on the least squares estimate of ρ, the coefficient on lagged debt, since positive realizations of the idiosyncratic shocks would tend to reduce the stock of public debt.

Possible Estimation Methods

Given the sources of endogeneity, the OLS estimator results in biased estimates of equation (1). The biases on ρ and γ would both be expected to be negative given the expected correlations among the variables. To deal with this, instrumentation for the output gap and use of country dummies to control for fixed effects is the usual approach. This method may still result in biased estimates for two reasons, however. First, serial correlation in the idiosyncratic errors would result in a negative correlation between lagged debt and the shock $\varepsilon_{i,t}$, imparting a downward bias to ρ. Second, the inclusion of country dummies is equivalent to running a regression on demeaned data; and while this eliminates the bias due to correlation between lagged debt and the fixed effects, it potentially introduces a positive correlation between demeaned-debt and demeaned-error terms in short samples, exerting a positive bias on ρ.[35] If the serial correlation in the error term is not strong, the positive

[35] Note that the demeaned lagged-debt term is $d_{i,t-1} - (d_{i,1} + ... + d_{i,T-1})/T-1$ and the demeaned error term is $\varepsilon_{i,t} - (\varepsilon_{i,2} + ... + \varepsilon_{i,T})/T-1$ in the fixed-effects estimation framework. These two terms are not orthogonal, since given equation (A.2), $d_{i,t-1}$ is correlated positively with each term in the sum $-(\varepsilon_{i,2} + ... + \varepsilon_{i,t-1})/T-1$, $\varepsilon_{i,t}$ is positively correlated with each term in $-(d_{i,t} + ... + d_{i,T-1})/T-1$, and $-\varepsilon_{i,s}/T-1$ is negatively correlated with each term in $-(d_{i,s} + ... + d_{i,T-1})/T-1$, for $s = 2,...,T-1$. The magnitudes of these correlations decline with T, the time-dimension of the sample. Celasun and Kang (2005) show the expected bias to be positive in fiscal reaction function estimations, as long as the serial correlation in the disturbance term $\varepsilon_{i,t}$ is not excessively large. They also show, using Monte Carlo simulations, the size of the bias on ρ to be small—typically less than 0.015 and mostly involving the third decimal—as long as the gap between growth-adjusted real interest rates and ρ, given by $\frac{1+r_t}{1+g_t} - \rho$, is in the range of 0.8–1. This condition is satisfied in the data since real interest rates typically exceed real growth rates by a small margin and ρ is typically estimated to be less than 0.10. Under this condition, the correlation between debt and any past shock to the primary balance is relatively small in magnitude, implying that the correlations that underlie the LSDV bias are small.

small-sample bias would tend to exceed the negative serial-correlation bias, so the estimates of ρ obtained from regressions with country dummies should have a small upward bias.

In the face of the endogeneity problem due to dependence of the explanatory variable on past values of the dependent variable, the standard approach to consistent estimation involves running regressions on the differenced form of the equation and using instrumental variables for the lagged changes in the endogenous regressors.[36] First-differencing has the advantage of eliminating the fixed country effects η_i from the equation, thereby removing one source of bias, but it imparts persistence to the disturbance term—leading the change in lagged debt ($\Delta d_{i,t-1} = d_{i,t-1} - d_{i,t-2}$) to become endogenous to the differenced disturbance $\varepsilon_{i,t} - \varepsilon_{i,t-1}$, since lagged debt $d_{i,t-1}$ is endogenous to the primary surplus shock $\varepsilon_{i,t-1}$:

$$\Delta p_{i,t} = \rho \Delta d_{i,t-1} + \gamma \Delta ygap_{i,t} + \Delta X_{i,t}\beta + \varepsilon_{i,t} - \varepsilon_{i,t-1}, \quad t = 2,...T, i = 1,..., N \qquad \text{(B.1)}$$

Consistent estimation of equation (B.1) would, therefore, involve finding instruments that are correlated with the lagged change in the level of debt $\Delta d_{i,t-1}$, but not with $\varepsilon_{i,t-1}$ and $\varepsilon_{i,t}$.

A related method of consistent-estimation in dynamic panels is the GMM approach of Arellano and Bond (1991) and Blundell and Bond (1998), where identification of ρ depends on limited serial correlation in the error term, $\varepsilon_{i,t}$. The Arellano and Bond (1991) method would involve first-differencing equation (1)—to eliminate the country-fixed effects—and using the lagged levels of debt as instruments for the changes in debt in a GMM regression. One advantage of this approach is that it encompasses a test of serial correlation that guides the selection of instruments. In particular, to serve as valid instruments, the endogenous variables need to be lagged sufficiently to ensure that they are not correlated with the error term.[37] Blundell and Bond (1998) show that large gains in efficiency—and, if the lagged endogenous variable series is persistent, precision—can be achieved by estimating a system that includes both the first-difference and the level equations, using lagged levels and first-differences of the endogenous regressor as instruments for the respective equations.

Estimation strategy

Our estimation strategy is as follows. We report both the OLS and least squares with dummy variables (LSDV) estimates as benchmarks. We also present a GMM estimation that

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[36] This method was proposed originally by Anderson and Hsiao (1981).

[37] If the order of serial correlation in $\varepsilon_{i,t}$ is estimated to be one, for instance, the third lag of debt $d_{i,t-3}$ will serve as a valid instrument for $\Delta d_{i,t-1}$, since the transformed error term $\Delta\varepsilon_{i,t}$ in equation (4) will be correlated with $\varepsilon_{i,t-2}$ (and thus $d_{i,t-2}$) on account of the serial correlation, but not with the shocks dated *t*-3 and earlier.

includes country dummies and instruments for the output gap (GMM-DV). This third method would yield unbiased estimates if the serial correlation in the idiosyncratic errors is low and the small-sample bias associated with the use of country dummies is not large. We then estimate two groups of regressions that do not include country dummies but use instrumental variables to overcome the potential endogeneity of both lagged debt and the output gap to unobserved primary surplus errors. In the first group, we estimate equation (B.1) using exogenous instruments for the lagged change in debt and the change in the output gap. We perform GMM and LIML regressions, and report the Cragg and Donald (1993) test statistic of instrument relevance in the first stage regression and the Hansen test of the orthogonality of the instruments.[38] In the second group, we run Blundell and Bond (1998) system-GMM (SGMM) regressions using different sets of instruments.[39] In these SGMM regressions, all instrument sets include the lagged differences and levels of the two endogenous regressors.

The solution for the debt path given in equation (A.2) suggests a number of potential instruments for the lagged change in debt. Specifically, past values of below-the-line fiscal expenditures, real GDP growth, and real interest rates would seem appropriate choices. One instrument that we use for the change in debt is a proxy for the fiscal costs of banking crises—which typically take the form of below-the-line expenditures that increase the debt burden without affecting the recorded primary surplus.[40] We also use as instruments industrialized country interest rates and world real oil prices, as these variables are correlated with emerging market growth and interest rates, but would not be expected to affect primary surpluses other than through their impact on debt and the output gap.

As an instrument for the output gap, we use import demand in industrialized trade partners in deviation from a Hodrick-Prescott trend, computed as the weighted average of the non-oil imports of industrial trade-partners of a given country. Weights are given by the share of the country's exports to its various industrialized-country export destinations.

Results

Table B.1 presents the results from estimating a baseline equation that includes only the endogenous variables as regressors, notably lagged debt and the output gap. In each case, we use the same sample to ensure comparability. The table illustrates that the pattern of estimates are largely in line with what we would expect on the basis of the above discussion.

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[38] Limited information maximum likelihood (LIML) estimations are preferable when the instruments are weak.

[39] We chose the Blundell and Bond (1998) estimator over the Arellano and Bond (1991) estimator since we need to estimate the constant term of the primary surplus equation for simulation purposes. The Arellano and Bond (1991) difference-GMM estimator clearly does not provide such an estimate.

[40] Data on the fiscal cost of banking crises is from Honohan, Klingebiel, Laeven, and Noguera (2005). We divided the costs equally over the horizon over which the banking crises were reported to last.

In line with the negative expected bias, OLS gives the smallest estimate of ρ, which is nonetheless positive and significant. The LSDV regression produces a larger estimate of ρ and a statistically significant coefficient on the output gap. Column 3 presents the GMM-DV regression (which instruments for the output gap and includes country dummies as regressors). The GMM-DV estimates are similar to the LSDV estimates, but the estimated effect of the output gap on the primary surplus is stronger, thanks to instrumentation.

Columns 4 and 5 present the findings from GMM and LIML regressions on equation (B.1). In both cases, two and three years lagged one-year U.S. bond rates and real oil price changes, the one year lagged fiscal cost of banking crisis measure, and the contemporaneous value of trade partners import demand are used as instruments. In the GMM regression in Column 4, the estimate of ρ lies slightly above the LSDV estimate (but is within the 99 percent confidence interval around the LSDV estimate). The Hansen test of overidentifying restrictions rejects the joint hypothesis that the equation is misspecified and that the instruments are not valid. The first stage regressions indicate that most of the instruments are significant in the first stage regression, and the Stock and Yogo (2002) first-stage F-statistic of 7.58 based on six exogenous instruments (instruments that are excluded from the set of regressors) implies that any bias in the estimated coefficients would be expected to be between 10 and 20 percent of the OLS bias.[41] Column 5 presents the LIML estimates, which are more robust when instruments are not strong. The results, presented in Column 5, suggest a somewhat smaller estimate of ρ than the GMM estimate, but with a larger standard error band, implying significance at 87 percent confidence only.[42]

Columns 6-9 presents the results from Blundell and Bond (1998) SGMM estimations. In all cases, the estimate of ρ is statistically significant and between the pooled-OLS and LSDV estimates, but slightly smaller than the GMM and LIML estimates. The four different system-GMM estimates use different but overlapping sets of instruments. Column 6 uses the same set of instruments as in the GMM and LIML regressions in addition to the second lags of the output gap and lagged debt. Column 7 excludes the banking fiscal cost measure and the trade partners' import demand from the set of instruments. Column 8 includes these two variables but excludes the lags of U.S. interest rates and real oil prices from the set of instruments. Column 9 uses only the second lags of debt and the output gap as instruments. In all cases, these orders of lags for the endogenous variables are validated by the tests of serial correlation in the errors of the first-differenced equation, which suggest serial correlation of order one in $\varepsilon_{i,t} - \varepsilon_{i,t-1}$, but no significant serial correlation in $\varepsilon_{i,t}$, the error term in the level equation. The test of overidentifying restrictions confirms the validity of the instruments, but as the number of instruments are large, this test may have weak power.

[41] This largely stems from the relatively weak fit of the first-stage debt equation.

[42] The Anderson and Rubin (1949) test of the joint statistical significance of the endogenous regressors, which is robust to the presence of weak instruments, rejects the hypothesis of no significance at the 5 percent level.

Table B.1. Estimates of the Fiscal Reaction Function, Baseline Specification, 1990-2004

Dependent Variable: Level or Change in the Primary Fiscal Balance

	(1) OLS	(2) LSDV	(3) GMM with DV	(4) GMM (Difference)	(5) LIML (Difference)	(6) System GMM	(7) System GMM	(8) System GMM	(9) System GMM
Lagged Debt	0.023***	0.062***	0.069***	0.070*	0.055	0.025*	0.031*	0.034**	0.037**
	[0.005]	[0.008]	[0.008]	[0.042]	[0.037]	[0.014]	[0.017]	[0.014]	[0.016]
Output Gap	0.047	0.181***	0.3 4***	0.279**	0.312**	0.138*	0.155	0.243***	0.251***
	[0.058]	[0.050]	[0.133]	[0.109]	[0.121]	[0.070]	[0.101]	[0.059]	[0.078]
Constant	-0.447	-2.946***	-4.247***			-0.554	-0.921	-1.166	-1.408
	[0.366]	[0.477]	[0.720]			[0.828]	[0.919]	[0.801]	[0.873]
Country dummies?	No	Yes	Yes	No	No	No	No	No	No
Observations	330	33	330	330	330	330	330	330	330
P-value Hansen test			0.930	0.393	0.447	0.999	0.984	0.999	0.982
P-value AR(1) test						0.06	0.06	0.04	0.04
AR(2) test						0.12	0.13	0.13	0.14
Cragg-Donald Stat.			13.29	7.58	7.58				

Notes: Standard errors in brackets, * denotes significance at 10%; ** significance at 5%; *** significance at 1%. P-values of the test statistics are reported for the tests of overidentifying restrictions and AR tests. Column 3 instruments the output gap with the current and lagged values of trade partners' import demand. In the GMM and LIML regressions in the Columns 4 and 5, the second and third lags of U.S. one-year bond rates, second and third lags of the changes in real oil prices, lagged fiscal banking crisis costs, and the contemporaneous value of trade partners import demand were used as instruments. The Blundell and Bond (1998) system-GMM regression in Column 6 uses the same set of instruments as in Columns 4 and 5 in addition to the second lags of the output gap and debt. Column 7 excludes the banking fiscal cost measure and the trade partners import demand measure from the set of instruments. Column 8 includes these variables but excludes the lags of U.S. interest rates and real oil prices from the set of instruments. Column 9 uses only the second lags of debt and the output gap.

These baseline regressions largely confirm our priors on the biases of various estimators but also present some new findings. Most importantly, the upward bias on the estimates of ρ in the LSDV and GMM-DV estimations do not appear to be large, as the GMM and LIML regressions in Columns 4 and 5 yield very similar, if not slightly larger, estimates for ρ. This suggests that the serial correlation in the idiosyncratic errors is not strong and that the small sample bias associated with the use of country-dummies is not large. The true value of ρ would be expected to lie above a lower bound given by the SGMM estimates and below an upper bound given by the GMM-DV and LIML estimates.

In view of these conclusions, we present only the GMM-DV, LIML, and SGMM regression results for the fiscal reaction function specification given in the main text.

C. Data Sources and Descriptions

The dataset covers the following countries: Argentina, Brazil, Bulgaria, Chile, China, Colombia, Costa Rica, Côte d'Ivoire, Croatia, Ecuador, Egypt, Hungary, India, Indonesia, Israel, Jordan, Korea, Lebanon, Malaysia, Mexico, Morocco, Nigeria, Pakistan, Panama, Peru, the Philippines, Poland, Russia, South Africa, Thailand, Turkey, Ukraine, Uruguay, and Venezuela. Data descriptions and sources are listed below.

Primary fiscal surpluses and public debt**:** Data provided by IMF desk and fiscal economists for the most comprehensive coverage of the fiscal sector available (percent of GDP).

Output gap**:** The percent deviation of real GDP from its Hodrick-Prescott filtered trend. Real GDP series were obtained from the IMF's *International Financial Statistics* or the *World Economic Outlook* databases.

Institutional quality**:** Defined as the average of five indicators on the level of corruption, bureaucratic quality, democratic accountability, rule of law, and government stability from the *International Country Risk Guide*. The index takes values between the range of 0-6, with a higher value designating a higher level of institutional quality.

IMF program**:** An indicator variable that takes on a value of one if the country is participating in an IMF-supported program and zero otherwise. Source: IMF Survey, various issues.

Default Indicator**:** An indicator variable that equals one if the government is not current on any debt payments. Source: IMF desk economists.

Fiscal Costs of Banking Crises**:** The fiscal cost, in percent of GDP, of systemic banking crises. Source: Honohan, Klingebiel, Laeven, and Noguera (2005). The fiscal costs were distributed evenly over the crisis horizon if an annual breakdown was not available.

Interest rates on one-year U.S. government bonds**:** FRED database, the Federal Reserve Bank of St. Louis.

Real Oil Prices**:** The average of Brent, Dubai, and Texas crude oil price indices, deflated by the U.S. consumer price index. Sources: IMF's *International Financial Statistics* database.

Import Demand in Trading Partners**:** Weighted average of the non-oil import volume (2000=100) of goods, of all trading partners, in percentage deviation from a Hodrick-Prescott filtered trend. Weights correspond to the share of exports to all partner countries. Source: IMF's *World Economic Outlook* database.

Real GDP: real gross domestic product in billions of domestic currency. Source: IMF's *World Economic Outlook*.

Real interest rate: Calculated using domestic (nominal) T-Bills (short-term maturity)[43] and domestic CPI inflation through the expression, both at quarterly frequency:

$$\frac{(1+(\text{int. rate}_t/100))^{0.25}}{1+\text{CPI inflation}_{t+1}}-1, \tag{C.1}$$

where int. rate_t is the domestic nominal interest at quarter t and $\text{CPI inflation}_{t+1}$ is the CPI inflation for the quarter t+1. Source: *International Financial Statistics*.

Real effective exchange rate: *International Financial Statistics*.

Real foreign interest rate: Calculated through (C.1) above, using inflation from GDP deflator. For nominal interest rates, we considered two cases: (i) 10-year constant maturity U.S. T-bill rate (monthly data from Jan/1980 to Dec/2004); and (ii) U.S. junk bonds index (daily data from 1/31/1990 to 12/31/2004). To estimate quarterly values, we averaged corresponding period data. Sources: U.S. 10-year treasury data from Federal Reserve of Saint Louis web site, and high yield bond data came from Merrill Lynch.

D. Properties of the Sustainability Indicator

The table below describes the range of possible values taken by the sustainability indicator introduced in the text. Shaded values mark entries below 0.4. It is then straightforward to see the combinations of $[1-\Pr(d_{t+\tau} > (d_t + x))]$ and $\Pr(d_{t+\tau} < d_t)$ corresponding to problematic values of the indicator.

[43] For South Africa, we used 10-year government bond yields. Source: International Financial Statistics.

Measuring Risks to Debt Sustainability by Combining the Probability of Decline with Contained Upside Risks
(Probability of decline in column and absence of upside risk in row 1/)

0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	**0.8**	0.85	0.9	0.95	1
0.05		0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.05	0.05	0.05
0.1			0.02	0.02	0.03	0.03	0.04	0.04	0.05	0.05	0.06	0.06	0.07	0.07	0.08	0.08	0.09	0.09	0.10	0.10
0.15				0.03	0.04	0.05	0.05	0.06	0.07	0.08	0.08	0.09	0.10	0.11	0.11	0.12	0.13	0.14	0.14	0.15
0.2					0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.20
0.25						0.08	0.09	0.10	0.11	0.13	0.14	0.15	0.16	0.18	0.19	0.20	0.21	0.23	0.24	0.25
0.3							0.11	0.12	0.14	0.15	0.17	0.18	0.20	0.21	0.23	0.24	0.26	0.27	0.29	0.30
0.35								0.14	0.16	0.18	0.19	0.21	0.23	0.25	0.26	0.28	0.30	0.32	0.33	0.35
0.4									0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36	0.38	0.40
0.45										0.23	0.25	0.27	0.29	0.32	0.34	0.36	0.38	0.41	0.43	0.45
0.5											0.28	0.30	0.33	0.35	0.38	**0.40**	**0.43**	**0.45**	**0.48**	**0.50**
0.55												0.33	0.36	0.39	0.41	**0.44**	**0.47**	**0.50**	**0.52**	**0.55**
0.6													0.39	0.42	0.45	**0.48**	**0.51**	**0.54**	**0.57**	**0.60**
0.65														0.46	0.49	**0.52**	**0.55**	**0.59**	**0.62**	**0.65**
0.7															0.53	**0.56**	**0.60**	**0.63**	**0.67**	**0.70**
0.75																**0.60**	**0.64**	**0.68**	**0.71**	**0.75**
0.8																	**0.68**	**0.72**	**0.76**	**0.80**
0.85																		**0.77**	**0.81**	**0.85**
0.9																			**0.86**	**0.90**
0.95																				**0.95**
1																				**1.00**

1/ Upside risk is measured by the probability that the debt-to-GDP ratio increases by less than a certain amount.

E. VAR models and Calibration

VAR models typically use up many degrees of freedom—with the result that, apart from South Africa where a VAR(3) model is estimated, we are restricted to estimating a first order process only. The tables below provide the estimated VAR coefficients, correlation matrices of shocks, and implied steady state values for real GDP growth and real interest rates for Argentina, Brazil and Turkey.

Argentina (foreign interest rate: 10-year US government bond yield)

VAR coefficients	Foreign interest rate	Domestic interest rate	Growth	log of REER
Foreign interest rate (-1)	0.46238	2.64413	1.12744	-3.17806
	[3.62040]	[1.65717]	[1.30866]	[-0.80966]
Domestic interest rate (-1)	-0.00480	0.36362	-0.20710	1.18078
	[-0.43735]	[2.65452]	[-2.80001]	[3.50394]
Growth (-1)	0.01715	0.08237	0.44585	2.76846
	[1.05281]	[0.40466]	[4.05639]	[5.52838]
Log of REER (-1)	0.00432	-0.01786	-0.02507	1.05379
	[3.26043]	[-1.07966]	[-2.80687]	[25.8971]
Constant	-0.01686	0.08037	0.12222	-0.30026
	[-2.85603]	[1.08942]	[3.06856]	[-1.65456]
R-squared	0.61730	0.19213	0.50761	0.96468
Adj. R-squared	0.58328	0.12031	0.46385	0.96154

Residuals correlation matrix	Foreign interest rate	Domestic interest rate	Growth	log of REER
Foreign interest rate	1	0.10582	0.01426	0.02499
Domestic interest rate	0.10582	1	-0.00447	0.44935
Growth	0.01426	-0.00447	1	0.32985
log of REER	0.02499	0.44935	0.32985	1

Implied steady state values (in percent, annualized)	
Real foreign interest rate	2.33456
Real domestic interest rate	9.76359
GDP growth	6.49988

Argentina (foreign interest rate: US junk bond yield)

VAR coefficients	Foreign interest rate	Domestic interest rate	Growth	log of REER
Foreign interest rate (-1)	0.71226	3.97492	0.23113	-8.25356
	[5.81206]	[3.17098]	[0.31247]	[-2.66226]
Domestic interest rate (-1)	0.00303	0.12057	-0.21514	1.69994
	[0.20492]	[0.79692]	[-2.40978]	[4.54319]
Growth (-1)	-0.01336	0.44174	0.47426	2.04010
	[-0.62879]	[2.03261]	[3.69810]	[3.79562]
Log of REER (-1)	0.00176	-0.01258	-0.01805	1.05876
	[1.47270]	[-1.03041]	[-2.50569]	[35.0727]
Constant	-0.00295	0.00146	0.09254	-0.19602
	[-0.52880]	[0.02559]	[2.74480]	[-1.38714]
R-squared	0.68461	0.29938	0.48998	0.96904
Adj. R-squared	0.65657	0.23710	0.44465	0.96629

Residuals correlation matrix

	Foreign interest rate	Domestic interest rate	Growth	log of REER
Foreign interest rate	1	0.36704	-0.19766	0.16626
Domestic interest rate	0.36704	1	0.02411	0.66879
Growth	-0.19766	0.02411	1	0.33995
log of REER	0.16626	0.66879	0.33995	1

Implied steady state values (in percent, annualized)

Real foreign interest rate	7.26897
Real domestic interest rate	9.34110
GDP growth	6.07628

Brazil (foreign interest rate: 10-year US government bond yield)

VAR coefficients				
	Foreign interest rate	Domestic interest rate	Growth	log of REER
Foreign interest rate (-1)	0.21244	-0.14324	0.68100	11.22975
	[1.14777]	[-0.13644]	[0.70431]	[1.54325]
Domestic interest rate (-1)	0.00601	0.22070	-0.07954	1.24833
	[0.22998]	[1.48917]	[-0.58276]	[1.21527]
Growth (-1)	0.00585	-0.46815	0.16764	1.04294
	[0.17592]	[-2.48424]	[0.96591]	[0.79849]
Log of REER (-1)	0.00617	0.03033	-0.00622	0.80083
	[2.60046]	[2.25449]	[-0.50199]	[8.58926]
Constant	-0.01977	-0.09831	0.02739	0.68533
	[-2.30356]	[-2.01967]	[0.61092]	[2.03138]
R-squared	0.53129	0.53893	0.06785	0.90484
Adj. R-squared	0.47772	0.48623	-0.03868	0.89396

Residuals correlation matrix				
	Foreign interest rate	Domestic interest rate	Growth	log of REER
Foreign interest rate	1	0.08430	-0.13856	0.35800
Domestic interest rate	0.08430	1	-0.30007	0.28921
Growth	-0.13856	-0.30007	1	0.01422
log of REER	0.35800	0.28921	0.01422	1

Implied steady state values (in percent, annualized)	
Real foreign interest rate	2.51465
Real domestic interest rate	9.79475
GDP growth	2.44727

Brazil (foreign interest rate: US junk bond yield)

VAR coefficients				
	Foreign interest rate	Domestic interest rate	Growth	log of REER
Foreign interest rate (-1)	0.89767	-0.77077	-0.41967	-2.55697
	[7.33181]	[-1.30010]	[-0.75163]	[-0.59118]
Domestic interest rate (-1)	-0.00154	0.16673	-0.11036	1.04718
	[-0.04949]	[1.10658]	[-0.77777]	[0.95267]
Growth (-1)	0.03720	-0.56451	0.09801	0.45857
	[0.90938]	[-2.85028]	[0.52545]	[0.31737]
Log of REER (-1)	0.00188	0.02961	-0.00007	0.89933
	[0.93657]	[3.04226]	[-0.00759]	[12.6669]
Constant	-0.00623	-0.07785	0.01753	0.42943
	[-0.75059]	[-1.93572]	[0.46287]	[1.46357]
R-squared	0.63401	0.55993	0.06966	0.89936
Adj. R-squared	0.59218	0.50964	-0.03667	0.88786

Residuals correlation matrix				
	Foreign interest rate	Domestic interest rate	Growth	log of REER
Foreign interest rate	1	-0.12848	-0.30009	-0.12758
Domestic interest rate	-0.12848	1	-0.33818	0.25993
Growth	-0.30009	-0.33818	1	0.03137
log of REER	-0.12758	0.25993	0.03137	1

Implied steady state values (in percent, annualized)	
Real foreign interest rate	7.42933
Real domestic interest rate	14.53428
GDP growth	2.61753

Turkey (foreign interest rate: 10-year US government bond yield)

VAR coefficients				
	Foreign interest rate	Domestic interest rate	Growth	log of REER
Foreign interest rate (-1)	0.43483	-4.35220	0.98611	-0.62498
	[3.53313]	[-1.57395]	[0.58295]	[-0.16144]
Domestic interest rate (-1)	-0.00015	-0.22916	0.17265	0.41789
	[-0.02473]	[-1.67808]	[2.06664]	[2.18576]
Growth (-1)	0.00981	-0.13410	-0.00175	0.38793
	[1.01598]	[-0.61806]	[-0.01320]	[1.27708]
Log of REER (-1)	-0.00952	-0.11675	-0.02069	0.84969
	[-3.16900]	[-1.73038]	[-0.50116]	[8.99551]
Constant	0.04935	0.63348	0.08868	0.68691
	[3.35679]	[1.91792]	[0.43889]	[1.48546]
R-squared	0.50466	0.08938	0.11289	0.71116
Adj. R-squared	0.46797	0.02192	0.04718	0.68976

Residuals correlation matrix				
	Foreign interest rate	Domestic interest rate	Growth	log of REER
Foreign interest rate	1	-0.06921	-0.16673	-0.20663
Domestic interest rate	-0.06921	1	-0.21936	-0.23997
Growth	-0.16673	-0.21936	1	0.56908
log of REER	-0.20663	-0.23997	0.56908	1

Implied steady state values (in percent, annualized)	
Real foreign interest rate	3.60884
Real domestic interest rate	16.97227
GDP growth	3.18259

Turkey (foreign interest rate: US junk bond yield)

VAR coefficients				
	Foreign interest rate	Domestic interest rate	Growth	log of REER
Foreign interest rate (-1)	0.87465	-2.27554	0.04956	2.92792
	[11.1757]	[-1.44025]	[0.05130]	[1.35032]
Domestic interest rate (-1)	-0.00209	-0.23260	0.16447	0.47997
	[-0.30562]	[-1.68551]	[1.94906]	[2.53424]
Growth (-1)	0.00395	-0.19912	0.00420	0.43993
	[0.36341]	[-0.90792]	[0.03131]	[1.46160]
Log of REER (-1)	-0.00285	-0.06284	-0.03464	0.86963
	[-1.04266]	[-1.14069]	[-1.02838]	[11.5016]
Constant	0.01576	0.39273	0.16201	0.52117
	[1.21103]	[1.49515]	[1.00870]	[1.44573]
R-squared	0.71680	0.08283	0.10736	0.72046
Adj. R-squared	0.69582	0.01489	0.04123	0.69975

Residuals correlation matrix				
	Foreign interest rate	Domestic interest rate	Growth	log of REER
Foreign interest rate	1	-0.09972	-0.15341	-0.18753
Domestic interest rate	-0.09972	1	-0.23341	-0.20228
Growth	-0.15341	-0.23341	1	0.57362
log of REER	-0.18753	-0.20228	0.57362	1

Implied steady state values (in percent, annualized)	
Real foreign interest rate	8.23170
Real domestic interest rate	18.65478
GDP growth	3.86027

The tables below provide the various calibrations of the fiscal reaction functions.

Baseline parameters used in simulations (Fixed effect-IV for output gap only)

	Argentina		Brazil		Mexico	South Africa		Turkey	
	Overall sample	end-2000	Overall sample	end-2000	Overall sample	Overall sample	end-2000	Overall sample	end-1999
Starting debt level (end-2004) 1/	0.84142	0.45000	0.54540	0.51100	0.46500	0.35700	0.42900	0.63500	0.61000
Fiscal behavior									
Constant	-0.00963	-0.00963	-0.00963	-0.00963	-0.00963	-0.00963	-0.00963	-0.00963	-0.00963
Output gap coefficient	0.32790	0.32790	0.32790	0.32790	0.32790	0.32790	0.32790	0.32790	0.32790
Debt coefficient	0.04614	0.04614	0.04614	0.04614	0.04614	0.04614	0.04614	0.04614	0.04614
Constant input in the simulator (matching predicted)	-0.02167	-0.01920	0.00967	0.01040	0.03466	-0.00318	0.00768	-0.00581	-0.01105
Constant input in the simulator (matching actual)	-0.02656	-0.01837	0.01804	0.00906	-0.01371	0.00108	0.02013	0.02893	-0.03426
Predicted primary surplus in 2004 2/	0.04239	0.00874	0.03783	0.03594	0.05637	0.01408	0.01816	0.03426	0.00321
Residual in 2004 2/	-0.00489	0.00083	0.00837	-0.00134	-0.04837	0.00426	0.01244	0.03474	-0.02321
Share of foreign debt	0.58312	0.96654	0.22940	0.39922	0.35054	0.14000	0.07680	0.32300	0.33000
Conditional variance of primary surplus	0.00013	0.00013	0.00029	0.00029	0.00076	0.00028	0.00028	0.00108	0.00108
Number of lags in VAR	1	1	1	1	1	3	3	1	1
Overall sample	93Q1-05Q4	93Q1-00Q4	94Q4-04Q4	94Q4-00Q4	93Q1-04Q4	80Q1-04Q4	80Q1-00Q4	90Q1-04Q4	90Q1-99Q4

1/ For Argentina, the starting debt level considers the federal government only at end-2005.
2/ For Argentina, the "residual" corresponds to the forecasting errors in 2005 and 2000, using federal government data.

Baseline parameters used in simulations (system GMM, no Fixed effect)

	Argentina	Brazil	Mexico	South Africa	Turkey
Starting debt level 1/	0.84142	0.54540	0.46500	0.35700	0.63500
Fiscal behavior					
Constant	-0.00068	-0.00068	-0.00068	-0.00068	-0.00068
Output gap coefficient	0.21656	0.21656	0.21656	0.21656	0.21656
Debt coefficient	0.03012	0.03012	0.03012	0.03012	0.03012
Constant input in the simulator (matching predicted)	0.00043	-0.00244	0.01355	-0.01316	-0.00405
Constant input in the simulator (matching actual)	-0.00428	0.02781	-0.00616	-0.00427	0.04279
Predicted primary surplus in 2004 2/	0.04221	0.01595	0.02771	-0.00189	0.02216
Residual in 2004 2/	-0.00471	0.03025	-0.01971	0.00889	0.04684
Share of foreign debt	0.58312	0.22940	0.35054	0.14000	0.32300
Conditional variance of primary surplus	0.00017	0.00056	0.00036	0.00058	0.00110
Number of lags in VAR	**1**	**1**	**1**	**3**	**1**
Sample	**1993Q1-2005Q4**	**1994Q4-2004Q4**	**1993Q1-2004Q4**	**1980Q1-2004Q4**	**1990Q1-2004Q4**

1/ For Argentina, the starting debt level considers the federal government only at end-2005.
2/ For Argentina, the "residual" corresponds to the forecasting errors in 2005 and 2000, using federal government data.

Baseline parameters used in simulations (non-linear model)

	Argentina	Brazil	Mexico	South Africa	Turkey
Starting debt level 1/	0.84142	0.54540	0.46500	0.35700	0.63500
Fiscal behavior					
Constant	-0.03628	-0.03628	-0.03628	-0.03628	-0.03628
Positive Output gap coefficient	0.18100	0.18100	0.18100	0.18100	0.18100
Negative Output gap coefficient	0.26800	0.26800	0.26800	0.26800	0.26800
Debt coefficient	0.09700	0.09700	0.09700	0.09700	0.09700
Spline coefficient	-0.06200	-0.06200	-0.06200	-0.06200	-0.06200
Constant input in the simulator (matching predicted)	-0.03892	-0.01208	0.01386	-0.02488	-0.02541
Constant input in the simulator (matching actual)	-0.04089	-0.00594	-0.03999	-0.02806	0.00916
Predicted primary surplus in 2004 2/	0.03947	0.04006	0.06184	0.01019	0.03443
Residual in 2004 2/	-0.00197	0.00614	-0.05384	-0.00319	0.03457
Share of foreign debt	0.58312	0.22940	0.35054	0.14000	0.32300
Starting output gap	0.01646	0.00328	-0.00554	0.00240	0.02301
Conditional variance of primary surplus	0.00010	0.00017	0.00090	0.00025	0.00094
Number of lags in VAR	**1**	**1**	**1**	**3**	**1**
Sample	**1993Q1-2005Q4**	**1994Q4-2004Q4**	**1993Q1-2004Q4**	**1980Q1-2004Q4**	**1990Q1-2004Q4**

1/ For Argentina, the starting debt level considers the federal government only at end-2005.
2/ For Argentina, the "residual" corresponds to the forecasting errors in 2005 and 2000, using federal government data.

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