WP/06/17

IMF Working Paper

The Net Worth Approach to Fiscal Analysis: Dynamics and Rules

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INTERNATIONAL MONETARY FUND

 WP/06/17

IMF Working Paper

IMF Institute

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Authorized for distribution by Enzo Croce

January 2006

Abstract

The net worth approach to fiscal analysis is cast in a simple model able to capture the dynamics and steady-state equilibria of public sector's debt, nonfinancial and financial assets, and net worth under alternative fiscal rules, including the golden rule and the golden rule cum debt stabilization fund. The paper also presents an adaptation of the model to the case of economies with depletable resources that have introduced investment oil funds, and illustrates the fiscal conditions required for the solvency of the associated fiscal rules. The model brings to the forefront the rate of return of public assets, highlighting the need for policymakers to decide on the appropriate level of assets and debt ratios. Finally, the model's potential for use in a range of contexts is demonstrated with a simple numerical simulation.

JEL Classification Numbers: H5, H6

Keywords: Net worth approach, first-order difference equation system, fiscal rules, debt stabilization fund, and oil fund

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[1] We would like to thank Adolfo Barajas, Enzo Croce, Andrew Feltenstein, Etibar Jafarov, Evan Tanner, Ethan Weisman, and IMF Institute Seminar participants for their helpful comments and suggestions.

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I. INTRODUCTION

The analysis of the public sector level of indebtedness and its evolution over time is traditionally based on the public debt to GDP ratio and its main determinants: the nominal, or real, interest and growth rates, and the primary balance ratio. This analysis does not explicitly take into consideration the role of financial and nonfinancial assets of the public sector. By contrast, the net worth approach to fiscal analysis does take into account these types of public assets, introduces important accounting changes, and shifts the emphasis of the analysis from one type of liability (the public debt) towards the public sector's net worth, defined as total assets minus total liabilities.

This paper encapsulates the net worth approach to fiscal analysis into a simple model and studies the dynamics and the steady-state equilibria values (as GDP ratios) of debt, nonfinancial and financial assets, and net worth of the nonfinancial public sector under alternative fiscal-investment fund rules. The model can also be used to study fiscal sustainability, conduct numerical simulations, and analyze the implications of debt stabilization funds.

The model brings to the forefront the importance of the average rates of return of both financial and nonfinancial public assets, and the related issue of productive and nonproductive investments. It highlights the need for policymakers to decide on the appropriate level of financial and nonfinancial assets (and the related issue of the size of the government) and public debt ratios, and the corresponding fiscal policy, reflected in current and future values of gross operating primary balance ratio, nonfinancial and financial investments ratios.

The analysis shows that under the golden rule, the constancy of net worth ratio might have different underlying situations. For example, under this rule, a surge in relatively unproductive investments that deteriorates the average rate of return of assets would require an additional fiscal effort to maintain the net worth ratio constant. Even in cases where the rates of return are not affected and the net worth ratio remains constant, the debt ratio could reach levels considered too high. This possibility has motivated some governments to complement the golden rule with a rule that caps the debt ratio at a prudent level. This implies that once the cap has been reached, investments in nonfinancial assets have to be financed with budgetary surpluses. Alternatively, countries may choose to complement the golden rule with a debt stabilization fund, which reduces the *net* debt ratio.

The above considerations alert us to focus not only on the level of the net worth ratio and its evolution over time in fiscal analysis, but also on the whole set of indicators related to the net worth approach. From the viewpoint of fiscal sustainability, the net worth approach should be seen as a more comprehensive tool, complementary to the debt approach.

We also use the model to analyze economies with depletable resources, a related financial asset fund (oil fund), and a fiscal rule. In this regard, we analyze the Norwegian oil fund. One of its main features is that it facilitates the decoupling of the oil revenue inflows from oil revenue use. The model is helpful in establishing guidelines on the size of the trend structural gross operating primary balance in order to keep constant the oil fund ratio at a desired level. In addition, the model could be used to project key variables such as the debt and financial assets ratios.

Section II presents the essentials of the net worth approach to fiscal analysis, explained in detail in the IMF's *Government Finance Statistics Manual* (2001). Section III derives a first-order difference equation system that governs the dynamics of financial and nonfinancial assets, debt and net worth (*Prototype Model*), and discusses the stability conditions for the system and steady-state equilibria. This section also presents a numerical simulation of the prototype model and briefly discusses the dynamics of debt under a feedback fiscal rule when the real interest rate is greater than the growth rate. Section IV uses the prototype model to analyze alternative fiscal rules. Finally, Section V offers concluding remarks and suggests possible extensions of the model.

II. Fundamentals of the Net Worth Approach

This section draws upon the IMF's *Government Finance Statistics Manual* (2001), henceforth referred to as *GFSM 2001*. The basic structure of the net worth approach to fiscal analysis is captured in Figure 1, and in the following identities, where all variables are measured in nominal terms and in local currency:

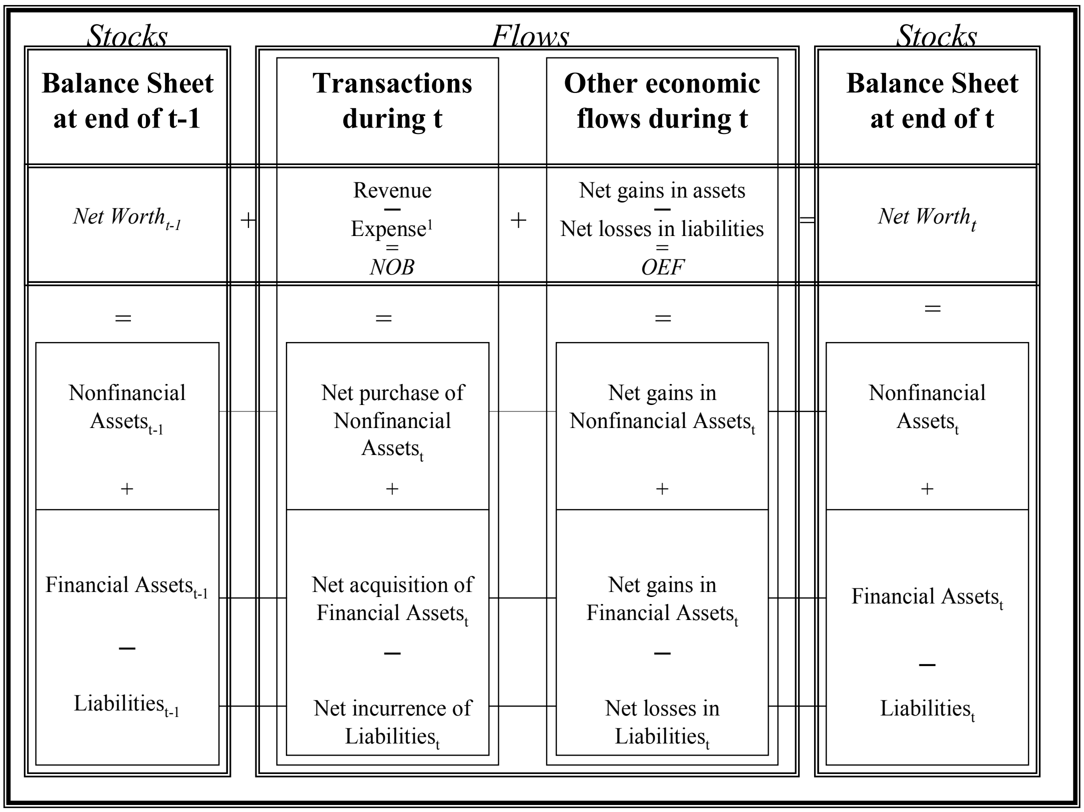
$$
\begin{aligned}
NW_t &= NW_{t-1} + NOB_t + OEF_t;\\
NW_t &= (A_F + A_{NF})_t - D_t - OL_t;\\
GOB_t &= T_t - G_t;\\
NOB_t &= GOB_t - DEP_t;\\
OEF_t &= NGPA_t - NLPD_t + NGQA_t - NLQD_t.
\end{aligned}
$$

Where,

NW:	Net worth.
GOB, NOB:	Gross and net operating balance, respectively.
A_J:	End-of-period stock of assets (where j = NF, F; nonfinancial and financial, respectively).
D:	End-of-period stock of debt.
OL:	End-of-period stock of nondebt, noncontingent liabilities, namely, financial derivatives, shares and equity of publicly controlled enterprises, held by the private sector.
T:	Revenue.
G:	Expense.
DEP:	Consumption of fixed capital.
OEF:	Other economic flows.
NGPA, NLPD:	Net gains (gains–losses) and net losses (losses–gains) in the value of assets and liabilities, respectively, due to changes in their market price; later referred to as *valuation adjustment*.

NGQA, NLQD: Net gains (gains–losses) and net losses (losses–gains) in the value of assets and liabilities, respectively, due to changes in their quantity, not related to any transaction.

Figure 1. The Structure of the New Accounting Framework



[1] Includes depreciation.

The concepts of revenue, expense and assets require further clarification, particularly, when compared to the traditional framework (presented in *GFSM 1986)*.

Revenue/Expense can be any transaction that increases/reduces the net worth. When resources are obtained from the sale of an asset (financial or nonfinancial), or the incurrence of a liability, the transaction is *not* classified as revenue. Similarly, when resources are employed in the purchase of an asset or in the repayment of a debt the transaction is not classified as expense. Accordingly, an investment in physical capital (including infrastructure) is classified as an acquisition of a nonfinancial asset. A property is classified as an *asset* if: (i) property rights can be enforced; and (ii) either it has a market value or generates, directly or indirectly, now or in the future, an economic benefit to its owner (including its use in the production of a public service).

All items in the balance sheet, as well as all transactions are recorded at market values.[2] An increase (decrease) in the price of an asset from one period to the next is recorded as a gain (loss); similarly, an increase (decrease) in the price of a liability is recorded as a loss (gain). The value of an asset/liability may also be affected by changes in volume derived from events different from transactions. For example, increases in assets due to discovery of new sources of natural resources, reductions of assets due to natural disasters (flood, earthquake, fire, etc), depletion of nonrenewable resources and unilateral reductions of debt (for example due to bankruptcy, but not due to restructuring). All these flows are presented in the statement of o*ther economic flows*.

The new accounting framework can be summarized in a few identities, which provide: (i) a simple way to present the principles of the net worth approach to fiscal analysis; and (ii) building blocks for the prototype model derived in Section III. Let us first present and define some useful additional variables. Unless otherwise specified, all variables refer to the nonfinancial public sector and are measured in nominal, domestic currency terms, at market value. Variables pertaining to the general government and nonfinancial public enterprises are identified by the superscript *GG* and *PE*, respectively.

GOPB:	Gross operating primary balance. Defined as *GOB* plus net accrued interest obligations (*due–earned*).
PS:	Primary surplus. Defined as *GOPB* minus investment in nonfinancial assets.
INV:	Investment. Equals net purchase of nonfinancial assets (*purchase – sale*).
A_j^i:	End-of-period stock of assets at market value; $i = GG, PE$, and $j = NF, F$.
E:	Nominal exchange rate (units of local currency per unit of foreign currency).
D^i:	End-of-period outstanding domestic and external debt (*) in local currency at market value ($i = GG, PE$). Thus, $D = (D^{GG}+ D^{PE})+E(D^{GG}+ D^{PE})^*$. This aggregation is possible after excluding intra-governmental debt.

When any of the above variables is expressed in lower case, it denotes a ratio with respect to *GDP* (later denoted by *Y*). Other useful definitions include:

i:	Ratio of interest payments to end-of previous period nominal debt.
ρ_j :	Ratio of current period return to end-of-previous period asset, where $j = NF, F$
δ:	Average depreciation rate of nonfinancial assets.
r:	Average real interest rate on debt; $r_t = r^{dom}{}_t\, (1 - \sigma)_{t-1} + r^{ext}{}_t\, (\sigma)_{t-1}$, where σ is the share of external debt in total debt.
g:	Rate of growth of real *GDP.*
π:	Inflation, measured by a general price index.
π^{NF}:	Percentage change in a price index for nonfinancial assets.

[2] In the case of stocks, if market values were not available, an alternative is to use the net present value of the expected stream of net flows associated with the asset or liability. In the case of loans however, the *GFSM 2001* recommends the recording at nominal value.

π^F:	Percentage change in a price index for financial assets.
π^D:	Percentage change in a price index for total debt.
π_t^{dom}	Percentage change in a price index for domestic debt.
π_t^{ext}	Percentage change in a price index for external debt.
E:	Price of one U.S. dollar in terms of local currency.
$\acute{\hat{E}}$	Rate of devaluation of the local currency.
VAA_{NFt}:	Valuation adjustment of nonfinancial assets = $(1-\delta)\pi_t^{NF} A_{NFt-1}$.
VAA_{Ft} :	Valuation adjustment of financial assets = $\pi_t^F A_{Ft-1}$.
VAD_t:	Valuation adjustment of debt = $[(1\text{-}\sigma)\ \pi_t^{dom} D_{t-1} + \sigma\ (\pi_t^{ext} + \acute{\hat{E}}_t)]\ D_{t-1} = \pi_t^D D_{t-1}$.

The set of identities below captures the essentials of the net worth approach to fiscal analysis:

$$(A_{NF_t} - A_{NF_{t-1}}) + (A_{F_t} - A_{F_{t-1}}) - (D_t - D_{t-1}) = GOPB^{GG}{}_t - \delta A_{NF_{t-1}} - iD_{t-1} + \rho_{NF_t} A_{NF_t} + \rho_{F_t} A_{F_t} + \pi_t{}^{NF}(1-\delta)A_{NF_{t-1}} - \pi_t{}^D D_{t-1} + \pi_t^F A_{F_{t-1}}; \tag{1}$$

$$A_{NF_t} = (1-\delta)A_{NF_{t-1}} + (1\text{-}\delta)\pi^{NF}{}_t A_{NF_{t-1}} + INV_{NF_t}\ ; \tag{2}$$

$$A_{F_t} = A_{F_{t-1}} + \pi^F{}_t A_{F_{t-1}} + INV_{F_t}. \tag{3}$$

Equation (1) describes the components of the change in the nonfinancial public sector net worth. For convenience, we redefine the *gross operating primary balance of the general government, $GOPB^{GG}$*, as revenue minus expense excluding interest due on debt and earned returns of both financial and nonfinancial assets. The return on nonfinancial assets includes the returns of real estate and infrastructure assets, and the gross operating primary balance of public enterprises, and is captured by $\rho_{NF_t} A_{NF_t}$.[3] On the right hand side of Equation (1), the sum of the first five terms equals the nonfinancial public sector savings—or net operating balance, *NOB*, that is revenue minus expense, and minus consumption of fixed capital—while the sum of last three terms equals other economic flows, *OEF*. To simplify the analysis, we assume that changes in volume due to nontransaction operations are equal to zero. Although included here, valuation adjustments due to price changes could be difficult to determine because secondary markets do not exist for all assets and debts.

Equation (2) describes the *general* motion law of nonfinancial assets: The current end-of-period stock of nonfinancial assets is determined by the end-of-previous period stock net of depreciation, plus investments and valuation adjustments (see Appendix I). Equation (3) describes the *general* motion law of financial assets: The current end-of-period stock of financial assets is determined by the end-of-previous period stock, plus investments and valuation adjustments.

[3] Therefore, ρ_{NF} is a weighted average rate of return: $\rho_{NF} = \rho_{rent}^{GG} \frac{A_{rent}^{GG}}{A_{NF}} + \rho_{NF}^{PE} \frac{A_{NF}^{PE}}{A_{NF}} + \rho_{inf}^{GG} \frac{A_{inf}^{GG}}{A_{NF}}$, where A_{rent} and A_{inf} refer to the stock of real state and infrastructure assets, respectively.

III. The Prototype Model

The prototype model is derived from Equations (1)–(3) plus some simplifying assumptions. The latter include: (i) nondebt liabilities as well as other economic flows (valuation adjustments and nontransaction changes in volume for assets and liabilities) are assumed to be zero; (ii) nonfinancial and financial assets are governed by the *general* motion law presented above, wherein investment ratios in both assets are exogenous.

Expressing Equations (1)–(3) in terms of GDP, and performing some algebraic manipulation, we obtain the first-order difference equation system that governs the dynamics of the debt ratio, nonfinancial and financial assets ratios.[4] The motion law of the net worth ratio could be derived from this system (see Appendix I).

$$\begin{pmatrix} d_t \\ a_{NF_t} \\ a_{F_t} \end{pmatrix} = \begin{pmatrix} \dfrac{1+i_t}{1+\hat{Y}_t} & -\dfrac{\rho_{NF}}{1+\hat{Y}_t} & -\dfrac{\rho_F}{1+\hat{Y}_t} \\ 0 & \dfrac{(1-\delta)}{1+\hat{Y}_t} & 0 \\ 0 & 0 & \dfrac{1}{1+\hat{Y}_t} \end{pmatrix} \begin{pmatrix} d_{t-1} \\ a_{NF_{t-1}} \\ a_{F_{t-1}} \end{pmatrix} + \begin{pmatrix} -gopb_t^{GG} + inv_{NF_t} + inv_{F_t} \\ inv_{NF_t} \\ inv_{F_t} \end{pmatrix}. \tag{4}$$

This prototype model highlights the point that, given the relevant parameters (interest and growth rates, the rates of return of both assets and the depreciation rate of the nonfinancial asset), the authorities have three degrees of freedom in formulating fiscal policy: they can set independently the values of the policy-determined variables—inv_{NF}, inv_F, and $gopb^{GG}$—to obtain specific values of the endogenous variables—d, a_{NF} and a_F (and consequently nw). As shown later, fiscal rules result in fewer degrees of freedom.

As a general principle, public investments should complement private investments and be consistent with the desired role and size of the government. Once these considerations are taken into account, the *quantity* and *quality* of public investments are other dimension of the

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[4] The prototype model does not consider the case when a fraction of the outstanding nominal debt is indexed to inflation and yields a real interest rate. However, introducing this feature is quite simple. Assume that a fraction α of the total debt is indexed to the inflation rate, π, and yields a real interest rate of r, while a fraction $(1-\alpha)$ is not indexed to inflation and yields a nominal interest rate of i. In this case, the coefficient to d_{t-1} (the γ_{11} element in the prototype model's main matrix) becomes $[1+\alpha\, r + (1-\alpha)\, i + \alpha\, \pi]/(1+\hat{Y})$.

public policy. In the prototype model, inv_{NF} and inv_F capture the *quantity*, while all the ρ parameters and, to some extent, $gopb^{GG}$ capture the *quality* of public investments.

A. Steady-State Equilibria and Stability Conditions

Keeping constant the three policy-determined variables (a sort of strict fiscal rule) and solving the system of first-order difference equations (4), the steady-state equilibria for the debt, the nonfinancial and financial assets, and the net worth ratios are:

$$d_{ss} = \frac{(1+\hat{Y})}{(\hat{Y}-i)}\left[-gopb^{GG} + \left(\frac{\hat{Y}+\delta-\rho_{NF}}{(\hat{Y}+\delta)}\right)inv_{NF} + \left(\frac{\hat{Y}-\rho_F}{\hat{Y}}\right)inv_F\right];$$

$$a_{NF_{ss}} = \frac{1+\hat{Y}}{(\hat{Y}+\delta)}inv_{NF};$$

$$a_{F_{ss}} = \frac{1+\hat{Y}}{\hat{Y}}inv_F;$$

$$nw_{ss} = \left(\frac{1+\hat{Y}}{\hat{Y}-i}\right)\left[\left(\frac{\rho_{NF}-\delta-i}{\hat{Y}+\delta}\right)inv_{NF} + \left(\frac{\rho_F-i}{\hat{Y}}\right)inv_F + gopb^{GG}\right].$$

Note that only d_{ss} (and therefore nw_{ss}) depends on the net rates of return of both assets relative to the interest rates. This is a consequence of setting both investments (inv_{NF} and inv_F) as exogenous variables (thus, the return of assets affects debt but not investment). Furthermore, the steady-state value of nw may be positive or negative. For example if stability conditions are met, $\hat{Y} > i$, the steady-state value of nw will be unequivocally positive if $gopb^{GG} > 0,\ \rho_{NF} \geq i+\delta,$ and $\rho_F \geq i$.

The equation for the steady-state net worth ratio also highlights the point that while it is true that in the short run the purchase of an asset financed with debt might not affect the net worth, it may however reduce (increase) the net worth ratio in the long run (steady state) if the higher investments negatively (positively) affect the average rates of return of both assets. This is because in the prototype model $gopb^{GG}$ is set to be constant. In particular, as we will see below, under the golden rule $gopb^{GG}$ would change to offset a deterioration of the quality of assets (reflected in a lower average rate of return).

As shown in Appendix I, the stability of the system depends on the nominal interest rate i, the rate of growth of nominal output $\hat{Y}$, the rate of depreciation of nonfinancial assets, δ, and the valuation adjustments of debt, and nonfinancial and financial assets (π^D, π^{NF} and π^F, respectively, here assumed to be zero). Ruling out both, permanent overvaluations and undervaluations, and considering that $0 < \delta < 1$, stability requires that the real interest rate be smaller than the rate of growth. Interestingly, stability does not depend on the rates of return of assets, ρ_{NF} and ρ_F. This is because those returns do not play any role in the *general* motion laws of the assets; however, they will play a role under some fiscal rules.

The steady-state solutions above do not necessarily correspond to the *desired* values for those variables. For example, sustainability and vulnerability considerations might determine a *desired* steady-state value for the debt ratio; political economy and efficiency consideration might dictate a *desired* steady-state value for the nonfinancial assets ratio (as a proxy for the size of government); and the need for a debt stabilization fund might determine a *desired* steady-state value for the financial assets ratio. Simply inverting the above system will determine the constant values of $gopb^{GG}$, inv_{NF}, and inv_F that lead to the *desired* steady-state levels for the debt ratio, nonfinancial, and financial ratios.

Using the steady-state equations for the debt ratio, nonfinancial and financial assets ratios, as well as the restricted concept of primary surplus ratio, $ps = gopb^{GG} - inv_{NF}$, and the concepts of *net* debt, $nd_{ss} \equiv d_{ss} - a_{F_{ss}}$, and *net* interest rate $i_{nd} \equiv i\left[d/\left(d - a_F\right)\right] - \rho_F\left[a_F/\left(d - a_F\right)\right]$, yields:

$$ps_{ss} = \frac{i_{nd} - \hat{Y}}{1 + \hat{Y}} nd_{ss} - \frac{\rho_{NF}}{1 + \hat{Y}} a_{NF_{ss}}.$$

This is the primary surplus required to maintain constant the net debt ratio, given the steady-state value of the nonfinancial assets ratio and key parameters. Furthermore, redefining ps_{ss} to include the second term on the right hand side yields:

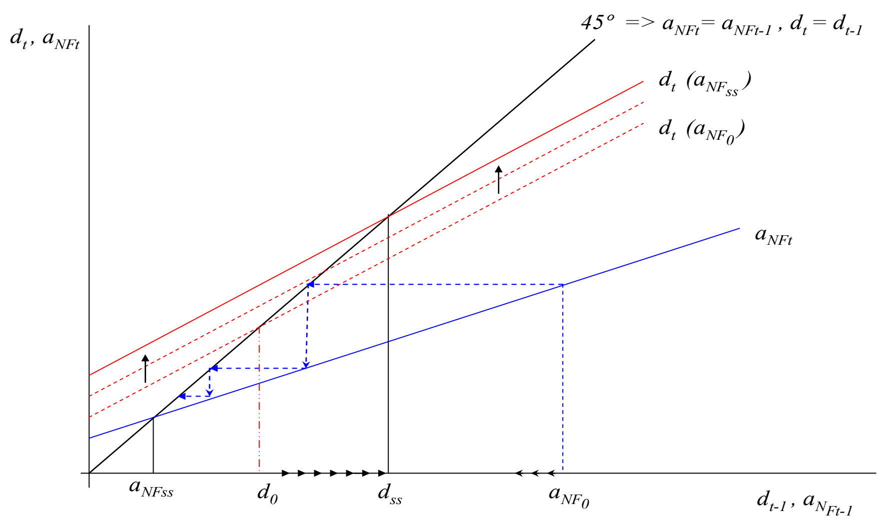
$$ps'_{ss} = \frac{r_{nd} - g}{1 + g} nd_{ss},$$

where r_{nd} and g are the real interest rate on net debt and the rate of growth, respectively. Note the similarity of this equation with that from the traditional analysis of debt sustainability.

B. The Geometry of Stability Conditions, Steady-State Equilibria, and Comparative Statics

The phase diagram of the dynamic system, shown in Figure 2 below, illustrates the geometry of the stability conditions, the steady-state equilibria, and the movements of the debt and assets ratios to their steady-state values. To simplify the phase diagram, we assume that the government does not hold financial assets, thus only the difference equations for d and a_{NF} are drawn. All stability conditions are met (the d_t and a_{NF_t} lines cut the 45^{o} line from above). Assume that the initial steady-state values for debt ratio and nonfinancial assets ratios are such that $d_0 < d_{ss}$, and $a_{NF_0} > a_{NF_{ss}}$. Given a_{NF_0}, $gopb^{GG}$, and inv_{NF}, the relevant difference equation for d is plotted as the dotted lower line. From its initial value, d increases as the nonfinancial assets ratio decreases towards its steady-state equilibrium, which causes the dotted line for the d difference equation to shift upward until the nonfinancial assets ratio reaches its steady-state value.

Figure 2. Dynamics of Debt and Assets Ratios



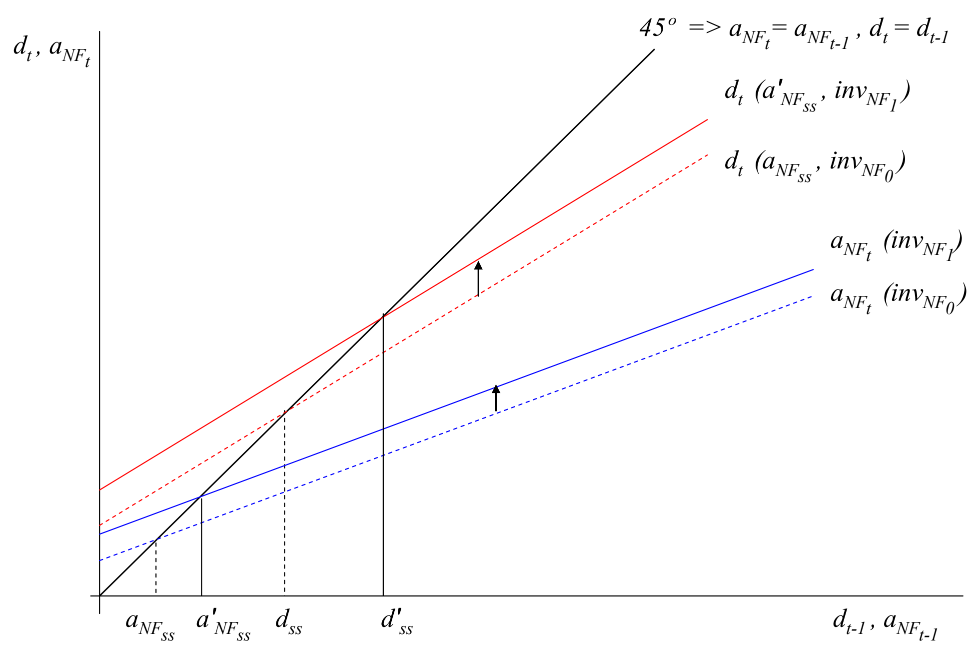
The change from one steady-state equilibrium to another due to an increase in a policy variable, inv_{NF} , is shown in Figure 3. The higher nonfinancial investment ratio shifts upward the d_t and the a_{NF_t} lines. A more precise comparative static arises from totally differentiating the system of equations describing the steady-state values of the debt ratio, and the nonfinancial and financial assets ratios. As shown in Appendix I, the sign and magnitude of those derivatives depend on the parameters $\hat{Y}$, δ, i, ρ_F, and ρ_{NF}.

If the stability condition were not met, that is, $i > \hat{Y}$, the system in the prototype model would be unstable (the line d would cut the 45° line from below). In this case, any perturbation to the steady state would indefinitely either increase or decrease the debt ratio, unless $gopb^{GG}$ is changed to a level compatible with keeping the debt ratio constant at a certain desired level. For developing countries, which usually have relative capital scarcity and high levels of debt, most likely the interest rate would be larger than the growth rate.[5] Having an interest rate larger than the growth rate implies that the prototype model looses one degree of freedom: the authorities no longer can set independently the values of the policy-determined variables—inv_{NF}, inv_F, and $gopb^{GG}$—because, any perturbation, would lead the debt ratio to diverge from the steady-state value. Thus, $gopb^{GG}$ has to be set at a particular level compatible with the debt ratio that the government wants to keep constant.

[5] Another argument that applies to both developed and developing countries, advanced by Barro (1976) is as follows: $i < \hat{Y}$ would lead, in the steady state, to inefficient capital over accumulation. He also demonstrated that a competitive equilibrium would have to be in the efficient region where $i > \hat{Y}$ in the steady state.

Alternatively, the fiscal policy might follow a rule designed to force convergence of the debt ratio to a target level (see Croce and Juan-Ramón (2003, 2005). We show this case in Appendix II.

Figure 3. An Increase in the Nonfinancial Investment Ratio



C. Numerical Simulations

We assigned numbers to the parameters in the prototype model and performed numerical simulations under stability conditions (see Appendix I). We compare a benchmark scenario with new steady-state values for the debt ratio, financial and nonfinancial assets ratios, and the net worth ratio, obtained after changing policy variables (for example, an increase in the investment in nonfinancial assets ratio) and/or parameters (for example, changes in the nonfinancial assets return). We found that the changes in the ratios are exacerbated when considering positive inflation and valuation adjustments. For example, assuming zero inflation and valuation adjustment, and positive values for the rest of the parameters, the public debt ratio increases from 20 percent to about 83 percent in response to a permanent increase in the investment (in nonfinancial assets) ratio from 2 percent to 3 percent. And if, in addition, inflation and valuation adjustment are introduced, even at very low rates, the debt ratio increases to 239 percent.

The results discussed above could also be obtained by totally differentiating the steady-state solutions of the prototype model. For example, the change in d_{ss} in response to an increase in the nonfinancial investment ratio, other things equal (see bloc 3 in Table 1, Appendix I), is given by the following expression (assuming zero inflation and valuation adjustments):

$$\Delta d_{ss} = \frac{(1+\hat{Y})}{(\hat{Y}-i)} \frac{\left(\hat{Y}+\delta-\rho_{NF}\right)}{(\hat{Y}+\delta)} \Delta inv_{NF} .$$

According to the equation above, the increase in d_{ss} in response to a one percentage point change in the nonfinancial investment ratio would be higher, the lower is the average rate of return of the nonfinancial assets, the closer the difference between the growth rate and the interest rate, and the lower the growth rate.

IV. Fiscal Rules

Many countries have already adopted, or are planning to adopt, rule-based fiscal policy aiming at conferring credibility on policy actions (by minimizing discretionary intervention), and restraining the deficit and the debt levels.[6] In lieu of the growing attention that economists and policy makers give to fiscal rules, we extend our prototype model to analyze the dynamics of the net worth ratio, nonfinancial and financial assets ratios, and the debt ratio under the golden rule, the golden rule cum debt stabilization fund, and oil-fund related rules. The case of a "Constant Net Worth Rule" is presented in Appendix III.

A. The Golden Rule

The basic form of the golden rule states that any investment in fixed capital should be financed with debt. Behind this formulation are two simple principles: (i) the acquisition of an asset with debt should not affect net worth; it is just an increase in a liability compensated by an increase in an asset; and (ii) intergenerational fairness; as future generations will enjoy the benefits of higher capital stocks, it would be unfair to charge the total cost to the present generation. We specify the golden rule as follows:

$$D_t - D_{t-1} = INV_{NF_t} - \delta A_{NF_{t-1}} + \pi_t^D D_{t-1} . \tag{5}$$

This rule states that debt will increase to finance net-of-depreciation investment in nonfinancial assets, where investment is defined as *purchases* minus *sales* of those assets.

[6] Of course fiscal rules are not necessarily always a panacea; for example, Kopits (2001) notes that sometimes rules reduce budgetary flexibility and invite abuse. He concludes that rules are most useful if applied in a consistent and transparent manner, at all levels of government, and in countries that lack a firm reputation for fiscal prudence.

Debt also changes in response to valuation adjustments. In terms of GDP, and assuming again that valuation adjustments are equal to zero, the above equation becomes:

$$d_t = \frac{1}{1+\hat{Y}_t} d_{t-1} - \frac{\delta}{1+\hat{Y}_t} a_{NF_{t-1}} + inv_{NF_t} . \tag{5'}$$

This rule-determined motion law for the debt ratio replaces the first equation in the prototype model, so that the system becomes:

$$\begin{pmatrix} d_t \\ a_{NF_t} \\ a_{F_t} \end{pmatrix} = \begin{pmatrix} \frac{1}{1+\hat{Y}_t} & -\frac{\delta}{1+\hat{Y}_t} & 0 \\ 0 & \frac{1-\delta}{1+\hat{Y}_t} & 0 \\ 0 & 0 & \frac{1}{1+\hat{Y}_t} \end{pmatrix} \begin{pmatrix} d_{t-1} \\ a_{NF_{t-1}} \\ a_{F_{t-1}} \end{pmatrix} + \begin{pmatrix} inv_{NF_t} \\ inv_{NF_t} \\ inv_{F_t} \end{pmatrix} . \tag{6}$$

In this case, the model could be solved for the steady state by imposing values for inv_{NF} and inv_F. The golden rule implies that now $gopb^{GG}$ is endogenous; that is, it must adjust to finance interest due net of assets returns, depreciation, and financial investment. The endogenous $gopb^{GG}$, which is consistent with the law of motion of the debt ratio under both the prototype model and the golden rule, is shown below.

$$gopb_t^{GG} = \frac{i_t}{1+\hat{Y}_t} d_{t-1} + \frac{\delta - \rho_{NF_t}}{1+\hat{Y}_t} a_{NF_{t-1}} - \frac{\rho_{F_t}}{1+\hat{Y}_t} a_{F_{t-1}} + inv_{F_t} .$$

Thus, for the golden rule to hold each period, the government has to adjust $gopb^{GG}$ in response to temporary and permanent shocks to the parameters in the above formula. Note that $gopb^{GG}$ also finances investments in financial assets, which might be zero as the golden rule does not necessarily imply the accumulation of financial assets. Adjusting $gopb^{GG}$ each period might be difficult and inefficient because of lags in fiscal policy and the principle of tax smoothing. Alternatively, the government could adhere to the golden rule on an intertemporal basis. In this case, $gopb^{GG}$ would be set *ex-ante* at a level given by the above formula but using trend (rather than current) values of the relevant parameters. Therefore, temporary shocks to those parameters should be absorbed by d, while permanent shocks should be absorbed by $gopb^{GG}$.

Solving the system with the golden rule, the steady-state values for d, a_{NF}, a_F and nw are:

$$d_{ss}^{GR} = \frac{1+\hat{Y}}{\hat{Y}+\delta} inv_{NF};$$

$$a_{NF_{ss}}^{GR} = \frac{1+\hat{Y}}{\hat{Y}+\delta} inv_{NF};$$

$$a_{F_{ss}}^{GR} = \frac{1+\hat{Y}}{\hat{Y}} inv_{F};$$

$$nw_{ss}^{GR} = \frac{1+\hat{Y}}{\hat{Y}} inv_{F}.$$

The steady-state value for d would basically depend on the value set for inv_{NF} (as well as on the values of the parameters). As expected, the solutions for a_{NF}, and a_F are not affected by the rule. The above equations reveal that in steady state (i) the debt ratio equals the nonfinancial assets ratio, $d = a_{NF}$, and (ii) the net worth ratio equals the financial assets ratio, $nw = a_F$; thus, the net worth ratio would be zero if the public sector does not maintain a stock of financial assets. This implies that higher investment in nonfinancial assets that leads to a higher steady-state nonfinancial assets ratio does not affect the steady-state level of the net worth ratio, regardless of the impact on the average rate of return of those assets.

Under the golden rule, and unlike the case of the prototype model, the deterioration of the average return on nonfinancial assets (for example, as a result of new, less profitable investments) is offset by an upward adjustment in the $gopb^{GG}$, maintaining constant the net worth ratio. This case highlights the fact that the constancy of the net worth ratio can be misleading, as it may hide the underlying requirement of greater fiscal efforts to compensate for worsening in the quality of assets. It also shows the importance of ex-ante appraisals of all investment decisions undertaken by the public sector.

B. The Golden Rule Cum Debt Stabilization Fund

Debt Stabilization Fund Based on Nonfinancial Assets Return

Under the golden rule cum debt stabilization fund, investments in nonfinancial assets are financed with debt and, also, a fraction of the nonfinancial asset returns are parked in a debt stabilization fund (financial assets) designed to back up the gross debt. This fund is a temporary abode of purchasing power to repay public debt when debt management considerations so warrant. As an alternative scheme, the fund could be maintained permanently without ever using it for debt repayments. Suppose the government would like to achieve a target debt ratio of zero but recognizes that some stock of public debt is a public good with positive externalities for the financial market. This tradeoff is reconciled by maintaining equal amounts of gross public debt and financial assets; that is, zero *net* public debt. A debt stabilization fund requires good governance to prevent the diversion of the fund to other uses, and an additional fiscal effort measured by the gap between present values of future stream of interest payments and fund returns.

Under a debt stabilization fund, a fraction α of the nonfinancial assets return is parked in this fund (financial asset). Thus, the end-of-period stock of financial assets is given by:

$$A_{F_t} = \left(1+\rho_{F_t}\right) A_{F_{t-1}} + \alpha\ \rho_{NF_t} A_{NF_{t-1}}\ , \tag{7}$$

where $0 \le \alpha \le 1$. Scaling the variables by GDP, the rule can be written as:

$$a_{F_t} = \left(\frac{1+\rho_{F_t}}{1+\hat{Y}_t}\right) a_{F_{t-1}} + \alpha \left(\frac{\rho_{NF_t}}{1+\hat{Y}_t}\right) a_{NF_{t-1}}\ . \tag{7'}$$

Under the golden rule cum debt stabilization fund, the steady-state values for a_F and for nw are shown below. The steady-states values for d and a_{NF} are the same as under the golden rule shown in the previous section.

$$a_{F_{ss}}^{GR-c-D} = \frac{\alpha\rho_{NF}(1+\hat{Y})}{(\hat{Y}-\rho_F)(\hat{Y}+\delta)} inv_{NF} = nw_{ss}^{GR\text{-}c\text{-}D}\ .$$

To achieve a desired level of nonfinancial assets in the steady state, a_{NF}^{*}, we need an appropriate investment ratio. Thus,

$$inv_{NF}^{*} = \frac{\hat{Y}+\delta}{1+\hat{Y}} a_{NF}^{*} \qquad \Rightarrow \qquad a_{F_{ss}}^{GR-c-D} = \frac{\alpha\rho_{NF}}{\hat{Y}-\rho_F} a_{NF}^{*}\ .$$

The steady-state net debt ratio, nd_{ss}, is given by:

$$nd_{ss} \equiv d_{ss} - a_{F_{ss}} = \left(\frac{\hat{Y}-\rho_F-\alpha\rho_{NF}}{\hat{Y}-\rho_F}\right) a_{NF}^{*}\ .$$

Note that assuming a positive rate of return of the nonfinancial asset, the expression in brackets above is less than one; therefore, the steady-state net debt is lower than the desired steady-state nonfinancial assets ratio. Furthermore, since the golden rule implies that the gross debt ratio equals the nonfinancial assets ratio, then the net debt ratio is lower than gross debt ratio. Concretely, $nd_{ss} - d_{ss} = -\left[\alpha\rho_{NF} /(\hat{Y}-\rho_F)\right] d_{ss}$. Therefore, assuming that the stability condition of $\hat{Y} > \rho_F$ holds, the higher are both rates of return and the closer to one is the policy parameter, the lower is the net debt ratio vis-à-vis the gross debt ratio.

Debt Stabilization Fund Based on the Gross Operating Primary Balance

Let us now assume that a fraction α of the public sector's gross operating primary balance is parked in a debt stabilization fund (financial asset). Thus, Equation (7′) now becomes:[7]

$$a_{F_t} = \left(\frac{1+\rho_{F_t}}{1+\hat{Y}_t}\right) a_{F_{t-1}} + \underbrace{\alpha gopb_t^{GG}}_{inv_{F_t}}. \qquad (7'')$$

As the government also follows the golden rule, the steady-state financial assets and net public debt ratios, respectively, are (see Appendix IV):

$$a_{F_{ss}} = \frac{\theta\left(\delta + i - \rho_{NF}\right)}{\hat{Y} - \rho_F} a_{NF_{ss}}; \quad nd_{ss} = \left[\frac{\hat{Y} - \rho_F - \theta\left(\delta + i - \rho_{NF}\right)}{\hat{Y} - \rho_F}\right] a_{NF_{ss}}.$$

Where $\theta = \alpha / (1 - \alpha)$ and the rest of the variables as already defined. Under stability condition ($\hat{Y} > \rho_F$), these results are basically driven by the sign and magnitude of $\delta + i - \rho_{NF}$. For example, if $\rho_{NF} = \delta + i$, the debt stabilization fund is zero, implying that only the nonfinancial assets ratio might be used as collateral. Alternatively, if $\rho_{NF} < \delta + i$, the financial assets ratio is positive. In this case, both the nonfinancial and financial assets ratios might be used as collateral, and the net public debt ratio is lower than the gross public debt ratio (which equals the nonfinancial assets ratio).

C. Norwegian Type Oil Fund: Decoupling Oil Revenue Inflow from Oil Revenue Use

This section analyzes the operation of an oil fund cum fiscal rule based on the one adopted by Norway in 1990. The net proceeds from the sale of oil are accumulated in an oil fund (financial asset), and the return of that fund is used to finance (up to a predetermined cap) the structural, non-oil operating deficit.[8] Intergenerational fairness is one of the motivations for building and maintaining such a financial asset, as it accumulates proceeds from extracting a depletable resource (oil, a nonfinancial asset). In addition, an oil fund facilitates the

[7] This is akin to the view that a government's collateral for the stock of public debt may be the stock of some *tangible* nonfinancial assets and *intangible* assets. The latter could be measured by the present value of either the government's gross operating primary balance or its future taxing capacity.

[8] An oil fund is different from an oil "stabilization" fund designed solely to smooth out the impact of fluctuating oil prices on fiscal revenues. Unlike the oil fund, the trend value of an oil "stabilization" fund could be set to zero, assuming symmetric shocks to oil prices.

decoupling of oil revenue inflow from oil revenue use, reducing the undesirable "Dutch disease" effects by investing the fund's resources in foreign financial assets.[9]

To preserve the oil fund, the fiscal rule entails that only the return of the oil fund can be used to finance the structural, non-oil operating deficit and up to a predetermined cap. The excess return over the cap is accumulated into the oil fund regardless of the size of the deficit. In addition, any excess of deficit over the cap has to be financed with public debt. Such a strategy implies that to keep the public debt ratio on check (fiscal sustainability), the structural, non-oil operational deficit should be determined by permanent income considerations, thus, independently of short-term fluctuations in either the oil price or the rate of return on the oil fund.[10]

The prototype model can be used to study this Norwegian type oil fund cum fiscal rule in any of the natural resource phases.[11] For the sake of simplicity, we assume that all financial assets correspond to the oil fund, which on period *t* yields an *actual* rate of return of ρ_{Ft} , and that non-oil, nonfinancial assets are zero. Also, for modeling purposes it is convenient to define a hypothetical rate of return on financial assets that would be *required* to totally finance *any* size of the structural, non-oil operating primary balance plus interest payments on the debt in period *t*. This *required* rate, $\rho_{F_t}^{REQ}$, is obtained from the "financing" equation below:

$$-\overline{gopb}_t^{GG} + \frac{i_t}{1+\hat{Y}_t} d_{t-1} = \frac{\rho_{F_t}^{REQ}}{1+\hat{Y}_t} a_{F_{t-1}} . \qquad (8)$$

According to the equation above, the *required* rate in period *t* is determined by the sizes of the total structural, non-oil operating primary deficit during that period, and the stock of

[9] The success of the oil fund in mitigating the Dutch disease problem depends on whether consumers are Ricardians or not. Jafarov and Moriyama's (2005) econometric analysis suggests that Norwegian households are partly but not fully Ricardian, implying that the oil fund can indeed mitigate the Dutch disease.

[10] The short-term price and rate of return fluctuations create uncertainty in the *cash flow* rather than uncertainty in *net worth*. The latter, ignored here, includes uncertainty regarding long-term trend in prices, cost of extraction of new fields, total stock of reserves and the corresponding production potential.

[11] Analysts distinguish three phases for economies endowed with natural resources: An initial phase of exploration and investment; a mature phase with a stable production level; and a terminal phase when production and revenue decline. Maintaining the government's total assets requires saving early oil proceeds to replace the nonfinancial asset with a financial asset. In this way present generations do not crowd out future generations' consumption given a constant across-generations tax burden.

financial assets (oil fund) at the end of period *t-1*. Thus, the larger the deficit and the smaller the oil fund, the larger the *required* rate. However, the *required* rate is just an analytical device as the government will finance its deficit with the proceeds from the *actual* rate of return of the oil fund, ρ_F, but up to a cap. This cap, predetermined by policy, could be thought of as an amount equivalent to the proceeds from the oil fund given by a *cap* rate of return; that is, $(\rho_F^{CAP} a_{F_{t-1}})/(1+\hat{Y}_t)$.

The motion laws of the debt and financial assets ratios depend, among other things, on the relative ranking among the *required* rate of return (determined by the sizes of the deficit and the oil fund), the *cap* rate of return (determined by the fiscal rule), and the *actual* rate of return (determined by the market and the risk composition of the financial assets). All possible six ranking combinations of these rates are shown below. Each case is associated with two ancillary parameters (λ and γ), which can take the value of either zero or one.

CASES		PARAMETER ASSOCIATED TO EACH CASE
A-i:	$\rho^{REQ} \le \rho^{CAP} \le \rho_F$	$\lambda = 0$; $\gamma = 1$
A-ii:	$\rho^{REQ} \le \rho_F \le \rho^{CAP}$	$\lambda = 0$; $\gamma = 1$
B-i:	$\rho^{CAP} \le \rho^{REQ} \le \rho_F$	$\lambda = 1$; $\gamma = 1$
B-ii:	$\rho^{CAP} \le \rho_F \le \rho^{REQ}$	$\lambda = 1$; $\gamma = 1$
C-i:	$\rho_F \le \rho^{REQ} \le \rho^{CAP}$	$\lambda = 1$; $\gamma = 0$
C-ii:	$\rho_F \le \rho^{CAP} \le \rho^{REQ}$	$\lambda = 1$; $\gamma = 0$

Using the ancillary parameters, λ and γ, and the Norwegian type oil fund together with the fiscal rule just described, the general motion laws of d and a_F are modeled as follows:

$$d_t = \left(\frac{1}{1+\hat{Y}_t}\right) d_{t-1} - (gopb_t^{GG} - \overline{gopb}^{GG}) + \lambda \left(\frac{\rho_{F_t}^{REQ} - (1-\gamma)\rho_{F_t} - \gamma\rho^{CAP}}{1+\hat{Y}_t}\right) a_{F_{t-1}} + inv_{NF_t}^{O}; \qquad (9)$$

$$a_{F_t} = \left(\frac{1}{1+\hat{Y}}\right) a_{F_{t-1}} + \left(\frac{\rho_{NF_t}^{O}}{1+\hat{Y}_t}\right) a_{NF_{t-1}}^{O} + \gamma \left(\frac{\rho_{F_t} - (1-\lambda)\rho_{F_t}^{REQ} - \lambda\rho_{F_t}^{CAP}}{1+\hat{Y}_t}\right) a_{F_{t-1}}. \qquad (10)$$

These equations could represent any of the six combinations presented above by assigning the appropriate values to the parameters λ and γ. The parameter λ determines whether debt accumulates ($\lambda = 1$), or not ($\lambda = 0$); while γ determines whether financial assets accumulates ($\gamma = 1$), or not ($\gamma = 0$). For example, if realization of the *actual*, *cap* and *required* rates in period *t* yields a relative ranking as in any of the cases B-i or B-ii (each represented by the ancillary parameters $\lambda = 1$ and $\gamma = 1$), the above equations show that d increases due to the excess of the *required* over the *cap* rates, and a_F also increases due to the excess of *actual* rate over the *cap* rate. Solving for the *required* rate in Equation (8), inserting it into the above equations, and rearranging terms yields the general model:

$$\begin{pmatrix} d_t \\ a^O_{NF_t} \\ a_{F_t} \end{pmatrix} = \begin{pmatrix} \frac{1+\lambda i_t}{1+\hat{Y}_t} & 0 & -\lambda\frac{(1-\gamma)\rho_{F_t}+\gamma\rho^{CAP}_{F_t}}{1+\hat{Y}_t} \\ 0 & \frac{1-\delta^O}{1+\hat{Y}_t} & 0 \\ -\frac{\gamma(1-\lambda)i_t}{1+\hat{Y}_t} & \frac{\rho^O_{NF_t}}{1+\hat{Y}_t} & \frac{1+\gamma(\rho_F-\lambda\rho^{CAP}_{F_t})}{1+\hat{Y}_t} \end{pmatrix} \begin{pmatrix} d_{t-1} \\ a^O_{NF_{t-1}} \\ a_{F_{t-1}} \end{pmatrix} + \begin{pmatrix} -gopb^{GG}_t + (1-\lambda)\overline{gopb}^{GG}_t + inv^O_{NF_t} \\ inv^O_{NF_t} \\ \gamma(1-\lambda)\overline{gopb}^{GG}_t \end{pmatrix}. \quad (11)$$

This general model could be simulated either in a stochastic or nonstochastic context. In either case one should take into account the phase that characterizes the economy. For example, in an initial phase of exploration and investment, the required rate will most likely be high (because of the still small size of the oil fund and probably, a high operating deficit). Case C better characterizes this initial phase where the debt ratio increases but the oil fund ratio does not.

In a mature phase, with a stable oil production level, the required rate will be lower than before because of the now larger size of the oil fund and possibly a smaller operating deficit. In this phase, the cap rate might also be set at a lower rate than before due to an improved fiscal balance and a larger oil fund. Thus, case B better characterizes this mature phase where both the debt and financial asset (oil fund) ratios increases.

In a terminal phase, with declining oil production and revenues, the size of the oil fund is large enough to finance the structural, non-oil operating deficit (which has been reduced to a level compatible with the oil fund's permanent income). Case A better characterized this phase. In the terminal-phase steady state, the natural resource (oil nonfinancial asset) has been depleted and replaced with the oil fund (financial asset).

D. The Norwegian Oil Fund: Strict Adherence to a Rule that Targets the Size of the Structural Non-Oil Deficit

The general model of the previous section could be used to analyze the specific oil fund cum fiscal rule established by the Norwegian authorities according to which, the structural, non-oil operational deficit is set at a level such that it could be financed by the oil fund's yield given by its *long-term trend* rate of return, $\overline{\rho}_F$, estimated at 4 percent, inflation adjusted (IMF, 2005). Thus, assuming zero inflation and valuation adjustments, the financing equation becomes:

$$-\overline{gopb}_t^{GG} + \frac{i_t}{1+\hat{Y}_t} d_{t-1} = \frac{\bar{\rho}_F}{1+\hat{Y}_t} a_{F_{t-1}} . \qquad (8')$$

Unlike the financing equation of the previous section (Equation (8)), now is the *structural*, operational primary balance which must adjust for this equation to hold. This implies that the *required* and *cap* rates must be identical to the *long-term trend* rate of return of the oil fund. Therefore, the relevant comparison now is between the oil fund's *actual* and *long-term trend* rates of return, ρ_F and $\bar{\rho}_F$, respectively:

CASES	PARAMETER ASSOCIATED TO EACH CASE
i. $\rho_{F_t} \geq \bar{\rho}_F$	$\gamma = 1$; $\lambda = 1$
ii. $\rho_{F_t} < \bar{\rho}_F$	$\gamma = 0$; $\lambda = 1$

Maintaining the assumption of zero inflation and valuation adjustments and, as this strategy entails that the *required* and *cap* rates must be identical to the *long-term trend* rate of return of the oil fund, the motion law of the debt and financial assets ratios, respectively, becomes:

$$d_t = \left(\frac{1}{1+\hat{Y}_t}\right) d_{t-1} - (gopb_t^{GG} - \overline{gopb}^{GG}) + (1-\gamma)\left(\frac{\bar{\rho}_F - \rho_{F_t}}{1+\hat{Y}_t}\right) a_{F_{t-1}} + inv_{NF_t}^O ; \qquad (12)$$

$$a_{F_t} = \left(\frac{1}{1+\hat{Y}}\right) a_{F_{t-1}} + \left(\frac{\rho_{NF_t}^O}{1+\hat{Y}_t}\right) a_{NF_{t-1}}^O + \gamma \left(\frac{\rho_{F_t} - \bar{\rho}_F}{1+\hat{Y}_t}\right) a_{F_{t-1}} . \qquad (13)$$

As by construction, on average, $\bar{\rho}_F - \rho_{F_t} = 0$, and $gopb_t^{GG} - \overline{gopb}^{GG} = 0$, the motion law of the debt ratio *on average* depends on the investment ratio of oil nonfinancial assets, and the debt ratio always converges to a steady-state value. And the motion law of the financial asset (oil fund) *on average* depends on the stock of oil nonfinancial asset and its rate of return, and the financial assets ratio always converges to a steady-state value. Solving for $\overline{gopb}^{GG}$ in Equation (8′), inserting it into the motion law for the debt ratio, and rearranging terms obtains:

$$\begin{pmatrix} d_t \\ a_{NF_t}^O \\ a_{F_t} \end{pmatrix} = \begin{pmatrix} \frac{1+i_t}{1+\hat{Y}_t} & 0 & \frac{\gamma(\rho_{F_t} - \bar{\rho}_{F_t}) - \rho_{F_t}}{1+\hat{Y}_t} \\ 0 & \frac{1-\delta^O}{1+\hat{Y}_t} & 0 \\ 0 & \frac{\rho_{NF_t}^O}{1+\hat{Y}_t} & \frac{1+\gamma(\rho_F - \bar{\rho}_{F_t})}{1+\hat{Y}_t} \end{pmatrix} \begin{pmatrix} d_{t-1} \\ a_{NF_{t-1}}^O \\ a_{F_{t-1}} \end{pmatrix} + \begin{pmatrix} -gopb_t^{GG} + inv_{NF_t}^O \\ inv_{NF_t}^O \\ 0 \end{pmatrix} . \qquad (14)$$

Mature and Terminal Phases Steady State

In a mature phase, the solutions of the above system for the debt and financial assets ratios ("steady-state" values) are: $d_{ss} = \left[\left(1+\hat{Y}\right)/\hat{Y}\right]\left(inv_{NF}^{O} + inv_{NF}^{NO}\right)$[12], and $a_{F_{ss}} = \rho_{NF}^{O} a_{NF_{t-1}}^{O} / \hat{Y}$, respectively. Note that the "steady-state" value of the financial assets ratio decreases as the oil nonfinancial assets is being depleted. If the depletion is accompanied by a reduction of investment in oil (no new technologically-derived additions to the stock of oil reserves) the steady-state value of the debt ratio also decreases (assuming the investment in non-oil nonfinancial assets ratio remains the same). Recall that the structural, operating primary balance must adjust for the financing equation to hold in response to the lower stock of financial asset.

In the steady-state terminal phase, by definition, the oil nonfinancial asset has been depleted, that is, $a_{NF}^{O} = 0$ and $inv_{NF}^{O} = 0$. This implies that the financial assets ratio is zero, the debt ratio is $d_{ss} = \left[\left(1+\hat{Y}\right)/\hat{Y}\right]\left(inv_{NF}^{NO}\right)$, and the right-hand side of Equation (8′) is zero. Consequently, the total structural operating budget must be balanced. In this terminal phase, the distinction between oil and non-oil structural balances becomes irrelevant.

Restrictions Imposed on the Structural Operational Balance

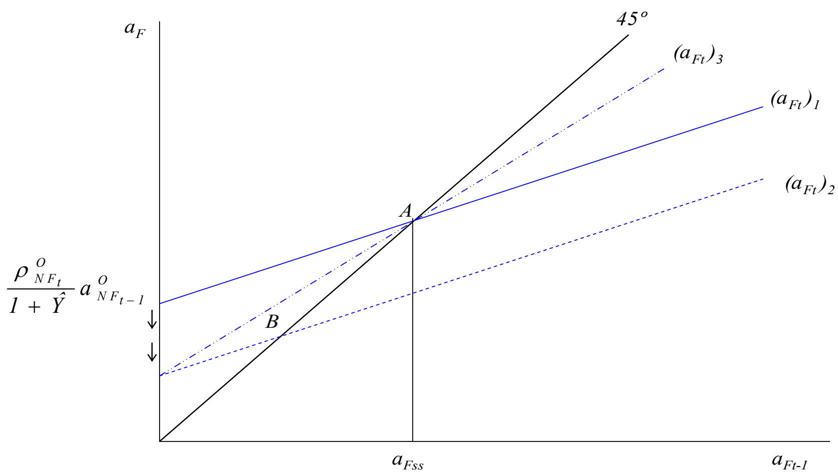
Suppose now that at the steady-state terminal phase, the government would like to maintain the financial assets ratio at a positive constant value. This implies that the non-oil, structural operating deficit has to be adjusted as the oil stock is being depleted. In the terminal phase steady state, this deficit should be financed by the stream of income given by $(\rho_{F_t} - \hat{Y}_t) a_{F_{ss}} / (1+\hat{Y})$. That is, in Equation (8′), substitute $(\rho_{F_t} - \hat{Y}_t)$ for $\bar{\rho}_F$.

That point is illustrated in Figure 4 below. Suppose that the "steady-state" value of the financial assets ratio at the mature phase, given by point A, is the level that the government would like to keep constant in the steady-state terminal phase. Line $(a_{F_t})_1$ cuts the 45° line from above as its *average* slope is $1/(1+\hat{Y})$. However, as the stock of oil nonfinancial asset is being depleted, the line $(a_{F_t})_1$ shifts downward to line $(a_{F_t})_2$ (parallel to line $(a_{F_t})_1$). The new steady state would be point B if there were no changes in the average slope. However, suppose that to offset the reduction in the oil nonfinancial assets ratio, the government limits the financing of the non-oil, structural operating deficit to an amount given by a rate lower than the long-term trend rate. This will change the slope of line $(a_{F_t})_2$ obtaining line $(a_{F_t})_3$ which implies keeping the steady-state value of the financial assets at the same level (given

[12] For simplicity, so far in this section we have assumed that $a_{NF}^{NO} = inv_{NF}^{NO} = 0$. Now it is convenient to bring them back.

by point A). When the oil nonfinancial assets have been totally depleted, the government limits the financing of the structural operational deficit to an amount given by $(\rho_{F_t} - \hat{Y}_t)a_{F_{ss}}/(1+\hat{Y})$. This implies that the slope of line $(a_{F_t})_3$ becomes $(1+\hat{Y})/(1+\hat{Y})$, coinciding with the 45° line.

Figure 4. An Objective Financial Assets Ratio



The previous analysis implies that if on average the actual rate of return of the oil fund equals the growth rate, the non-oil structural operating deficit would be balanced on average. Assuming that the government does accumulate non-oil nonfinancial assets financed with debt (golden rule), then the steady-state debt ratio would be equal to the desired steady-state non-oil nonfinancial assets ratio, and the steady-state net worth ratio would be equal to the steady-state financial assets ratio.

To sum up, an oil fund has to be complemented with additional fiscal rules over all the oil phases to be effective in pursuing its objectives (decoupling oil inflows from their use, achieving intergenerational fairness, and serving as a precautionary cushion). In the mature and terminal phases, the possibility of financing the non-oil structural operating balance with the return of financial assets depends on how the real rate of return of the latter compares to the rate of growth of real GDP. If the long-term average of these two rates were about the same, the non-oil structural operating budget has to be balanced on average to keep the financial assets ratio constant at a policy-determined level. If the government follows a golden rule (financing investments in nonfinancial assets with debt), the net worth ratio would be equal to the financial assets ratio in the steady-state terminal phase. Once the country has become *non-oil*, a constant financial assets ratio might be rationalized as a debt stabilization fund or, alternative, as a temporary abode for future investments in non-oil nonfinancial assets yielding higher rate of returns.

V. Concluding Remarks

The paper encapsulates the net worth approach to fiscal analysis into a simple model of first-order difference equations, which could be used to study the dynamics and steady-state equilibria values (as GDP ratios) of debt, nonfinancial and financial assets, and net worth of the nonfinancial public sector. This model is useful to study alternative fiscal-investment fund rules and fiscal sustainability.

The model brings to the forefront the importance of the average rate of return of nonfinancial public assets, and the related issue of productive and nonproductive investments. It highlights the need for policymakers to decide on the appropriate levels of the nonfinancial assets ratio (and the related issue of the size of the government) and the debt ratio, as well as on the purpose for accumulating financial assets.

By treating separately each component of the net worth (nonfinancial and financial assets, and liabilities), the model allows for a more complete analysis of fiscal sustainability. This is an important feature because focusing solely on the level of the net worth ratio and its evolution over time might be misleading. This is illustrated in the case of the golden rule, which allows borrowing to finance investments: in the steady state, the debt ratio equals the nonfinancial assets ratio. Thus, if left unconstrained, the debt ratio could reach levels considered too high, regardless of the quality of investments. This possibility has motivated some governments to complement the golden rule with an additional rule that limits the debt ratio to a range below a “danger” threshold. Alternatively, others may choose to complement the golden rule with a debt stabilization fund.

Under the golden rule cum debt stabilization fund, the steady-state values of the debt stabilization fund and the net debt depend basically on the origin of the inflows that accrue to the fund (in addition to the fund’s own return). We analyzed the cases when the inflows are a fraction of the nonfinancial asset returns and when they are a fraction of the gross operating primary surplus. The latter is akin to the view that a government’s collateral for its stock of public debt may also be the present value of either the government’s gross operating primary balance or its future taxing capacity.

The model makes explicit that under most fiscal rules, the government needs to adjust each period its gross operating balance for the rule to hold. This could be difficult to do in practice due to lags, political economy and efficiency considerations. Applying the rules to a multiyear horizon may be one solution.

The model could be applied to analyze economies with depletable resources and a related financial assets fund. The Norwegian case—of an oil producer country following fiscal rules and accumulating an oil fund—is quite interesting. One of the main features of the Norwegian oil fund is that it facilitates the decoupling of the oil revenue inflows from oil revenue use. The prototype model could also be applied to project future values of the debt and the fund under a Norwegian type strategy. The analysis of the dynamics and steady-state solutions of the model under a Norwegian type strategy suggests that in the absence of additional rules, both the oil nonfinancial assets and the oil fund ratios would become zero in the terminal phase of the oil industry. Thus, for an oil fund strategy to be effective in pursuing its objectives in the long run, the authorities have to impose a constant positive value for the financial assets ratio, which implies maintaining the structural operating budget balanced. Keeping a constant financial assets ratio once the country has become *non-oil*, might be rationalized as a debt stabilization fund or, alternative, as a temporary abode for future investments in non-oil nonfinancial assets yielding higher rates of return.

Possible extensions to this paper include taking into account the possibility of default and extreme shocks. The model could incorporate the stochastic processes governing key variables and parameters and their comovements, and use this set up to establish confidence bands around the projected values of key ratios.

APPENDICES

NONFINANCIAL ASSETS ACCUMULATION AND THE PROTOTYPE MODEL

Derivation of the Motion Law of Nonfinancial Assets

In real terms:

$$K_t = (1 - \delta)K_{t-1} + FCF_t .$$

Where:

K Fixed capital

FCF Fixed capital formation (net investment in real terms)

Multiplying both terms by P_{k_t}, and defining $\pi_t^{NF} = [(P_{k_t} / P_{k_{t-1}}) - 1]$, the above equation in nominal terms becomes:

$$A_{NFt} = (1+\pi^{NF}_t)(1 - \delta)A_{NFt-1} + INV_{NFt} .$$

The Prototype Model

The general model

The prototype model presented in the text is a simplified version of a more complete model which considers the possibility of valuation adjustments different from zero, and expands the definition of rates of returns by separating the return of assets into an average rate applied to the stock at end of *t-1* and a marginal return on investments during period *t*, as presented below:

$$\rho_{NF_t} A_{NF_t} = \rho^M_{NF_t} INV_{NF_t} + \rho^{AVG}_{NF_t} A_{NF_{t-1}} ;$$
$$\rho_{F_t} A_{F_t} = \rho^M_{F_t} INV_{F_t} + \rho^{AVG}_{F_t} A_{F_{t-1}} .$$

Where,

ρ_j^M : Ratio of return to current period investment, also referred to as the marginal return on assets; where $j = NF, F$ (nonfinancial and financial);

ρ_j^{AVG} : Ratio of return to end-of-previous period asset, also referred to as average return on assets; where $j = NF, F$.

From Equations (1)–(3) in the text, and the definitions for the rates of return above, we can derive the following system of difference equations:

$$D_t = (1+i_t+\pi_t^D)D_{t-1} - \rho_{NF_t}^{AVG}A_{NF_{t-1}} - \rho_{F_t}^{AVG}A_{F_{t-1}} - GOPB_t^{GG} + (1-\rho_{NF_t}^M)INV_{NF_t} + (1-\rho_{F_t}^M)INV_{F_t}; \quad (15)$$

$$A_{NF_t} = (1-\delta)(1+\pi_t^{NF})A_{NF_{t-1}} + INV_{NF_t}; \quad (16)$$

$$A_{F_t} = (1+\pi_t^F)A_{F_{t-1}} + INV_{F_t}. \quad (17)$$

Expressing all variables as ratios to GDP, this system of difference equations can also be presented in matrix notation as follows:

$$\begin{pmatrix} d_t \\ a_{NF_t} \\ a_{F_t} \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{1+i_t+\pi_t^D}{1+\hat{Y}_t} & -\frac{\rho_{NF_t}^{AVG}}{1+\hat{Y}_t} & -\frac{\rho_{F_t}^{AVG}}{1+\hat{Y}_t} \\ 0 & \frac{(1-\delta)(1+\pi_t^{NF})}{1+\hat{Y}_t} & 0 \\ 0 & 0 & \frac{(1+\pi_t^F)}{1+\hat{Y}_t} \end{pmatrix}}_{\Omega} \begin{pmatrix} d_{t-1} \\ a_{NF_{t-1}} \\ a_{F_{t-1}} \end{pmatrix} + \begin{pmatrix} -gopb_t^{GG} + (1-\rho_{NF_t}^M)inv_{NF_t} + (1-\rho_{F_t}^M)inv_{F_t} \\ inv_{NF_t} \\ inv_{F_t} \end{pmatrix}.$$

Stability Conditions

The stability conditions of the dynamic system described above are given by the values of the main matrix's characteristic roots. These roots are obtained by solving the characteristic equation. The polynomial characteristic for $\mathbf{\Omega}$, f(λ), is given by

$$f(\boldsymbol{\lambda}) = |\Omega - \boldsymbol{\lambda I}| = \boldsymbol{\lambda}^3 + \boldsymbol{\lambda}^2(\gamma_{11}+\gamma_{22}+\gamma_{33}) - \boldsymbol{\lambda}(\gamma_{11}\gamma_{22}+\gamma_{11}\gamma_{33}+\gamma_{11}\gamma_{33}) + \gamma_{11}\gamma_{22}\gamma_{33} = 0$$

where γ_{ij} represents the element of $\mathbf{\Omega}$ located in row *i* and column *j*. The well-known relation between the characteristic roots (λ_1, λ_2, λ_3) and the coefficients of the characteristic polynomial is given by:

$$\gamma_{11}\gamma_{22}\gamma_{33} = \lambda_1\lambda_2\lambda_3;$$
$$\gamma_{11}\gamma_{22} + \gamma_{11}\gamma_{33} + \gamma_{22}\gamma_{33} = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3;$$
$$\gamma_{11} + \gamma_{22} + \gamma_{33} = \lambda_1 + \lambda_2 + \lambda_3.$$

Thus, the solution for this system is straightforward:

$$\lambda_1 = \gamma_{11} = \frac{1+i+\pi^D}{1+\hat{Y}};$$
$$\lambda_2 = \gamma_{22} = \frac{(1-\delta)(1+\pi^{NF})}{1+\hat{Y}};$$
$$\lambda_3 = \gamma_{33} = \frac{1+\pi^F}{1+\hat{Y}}.$$

The stability condition for the system requires that $|\lambda_{ij}| < 1, \forall\ i = j$. Thus,

$$\left|\frac{1+i+\pi^D}{1+\hat{Y}}\right|,\ \left|\frac{(1-\delta)(1+\pi^{NF})}{1+\hat{Y}}\right|,\ \left|\frac{1+\pi^F}{1+\hat{Y}}\right|, \text{ all} < 1.$$

Steady-state Equilibria and Some Comparative Statics

Solving the first-order difference equation system above, the steady-state equilibria for the debt and assets ratios are:

$$d_{ss} = \left[\frac{(1+\hat{Y})}{(\hat{Y}-i-\pi^D)}\right]\left\{-gopb^{GG} + \left[(1-\rho_{NF}^M) - \frac{\rho_{NF}^{AVG}}{(1+\hat{Y})-(1-\delta)(1+\pi^{NF})}\right]inv_{NF} + \left[(1-\rho_F^M) - \frac{\rho_F^{AVG}}{(\hat{Y}-\pi^F)}\right]inv_F\right\};$$
$$a_{NF_{ss}} = \frac{1+\hat{Y}}{(1+\hat{Y})-(1-\delta)(1+\pi^{NF})}inv_{NF};$$
$$a_{F_{ss}} = \frac{1+\hat{Y}}{(\hat{Y}-\pi^F)}inv_F.$$

Given that $nw_{ss} \equiv a_{NFss} + a_{Fss} - d_{ss}$, the steady-state net worth ratio could be easily derived from the above equations. Furthermore, assuming stability, different cases of comparative statics could be analyzed by totally differentiating the above system. Some of those cases, assuming zero marginal rate of return on assets and valuation adjustments, include:

$$\frac{\partial a_{NF\,ss}}{\partial inv_{NF}} = \frac{1+\hat{Y}}{1+\delta} > 0;\quad \frac{\partial d_{ss}}{\partial inv_{NF}} = \left(\frac{1+\hat{Y}}{\hat{Y}+\delta}\right)\left(\frac{\hat{Y}+\delta-\rho_{NF}^{AVG}}{\hat{Y}-i}\right);\quad \frac{\partial nw_{ss}}{\partial inv_{NF}} = \left(\frac{1+\hat{Y}}{\hat{Y}+\delta}\right)\left(\frac{\rho_{NF}^{AVG}-i-\delta}{\hat{Y}-i}\right);$$

$$\frac{\partial gopb^{GG}}{\partial inv_{NF}} = \frac{\hat{Y}+\delta-\rho_{NF}^{AVG}}{\hat{Y}+\delta}, \text{ for a constant } d_{ss}.$$

An Alternative Derivation of the Steady-state Net Worth Ratio

From Equations (1)–(3) in the text, the definition of the rates of return presented above, and using the fact that $\Delta NW_t = \Delta A_{NF_t} + \Delta A_{F_t} - \Delta D_t$, we can derive the following first-order difference equation system for nw, a_{NF} and a_F:

$$\begin{pmatrix} nw_t \\ a_{NF_t} \\ a_{F_t} \end{pmatrix} = \begin{pmatrix} \dfrac{1+i_t+\pi_t^D}{1+\hat{Y}_t} & \dfrac{\rho_{NF_t}^{AVG}-\delta-i-\pi_t^D+(1-\delta)\pi_t^{NF}}{1+\hat{Y}_t} & \dfrac{\rho_{F_t}^{AVG}-i-\pi_t^D+\pi_t^F}{1+\hat{Y}_t} \\ 0 & \dfrac{(1-\delta)(1+\pi_t^{NF})}{1+\hat{Y}_t} & 0 \\ 0 & 0 & \dfrac{(1+\pi_t^F)}{1+\hat{Y}_t} \end{pmatrix} \begin{pmatrix} nw_{t-1} \\ a_{NF_{t-1}} \\ a_{F_{t-1}} \end{pmatrix} + \begin{pmatrix} gopb_t^{GG}+\rho_{NF_t}^{M} inv_{NF_t}+\rho_{F_t}^{M} inv_{F_t} \\ inv_{NF_t} \\ inv_{F_t} \end{pmatrix}.$$

As before, the stability conditions do not depend on the returns on the assets. Solving this system we can obtain the steady-state solution for net worth. Note that the rates of return of both assets affect the motion law of the net worth ratio but not that of those assets.

$$nw_{ss} = \left(\frac{1+\hat{Y}}{\hat{Y}-i-\pi^D}\right)\left[\left(\frac{\rho_{NF}^{AVG}+\hat{Y}-i-\pi^D}{(1+\hat{Y})-(1-\delta)(1+\pi^{NF})}-1+\rho_{NF}^{M}\right)inv_{NF}+\left(\frac{\rho_{F}^{AVG}+\hat{Y}-i-\pi^D}{\hat{Y}-\pi^F}-1+\rho_{F}^{M}\right)inv_F+gopb^{GG}\right].$$

Numerical Simulations

We assigned numbers to the parameters of the prototype model and performed numerical simulations; in all cases stability conditions hold. Table 1 contains the steady-state values of debt, assets, and net worth ratios under different fiscal policy variables (investments, gross operating primary balance, and primary balance ratios), rates of return of nonfinancial and financial assets, and zero and nonzero valuation adjustments and inflation.

Table 1 has four blocs. Each bloc has a benchmark case (first column), in which the policy variables, $gopb^{GG}$, inv_{NF} and inv_F are such that the debt and assets ratios remain constant at their initial values. In blocs 1 and 3 the simulations assume zero inflation and valuation adjustments, whereas in blocs 2 and 4, those parameters have positive values. The simulations considered two alternative cases: i) a symmetric negative and positive change in the rate of return of nonfinancial and financial assets (second and third columns of blocs 1 and 2), and ii) an increase in the investment ratio for nonfinancial assets assuming that the return of nonfinancial assets is either the same as in the benchmark case (second column of blocs 3 and 4), or higher (third column of blocs 3 and 4).[13]

As shown in Table 1 (blocs 1 and 2), compared to the benchmark, the debt ratio increases (decreases) with a lower (higher) rate of return on assets. Inflation and valuation adjustments exacerbate those differences. The assets ratios do not change because in the prototype case the rates of return on assets do not affect investment policy (a constant investment ratio). Therefore, the net worth ratio changes in the same magnitude as the debt ratio, but with the opposite sign. A higher nonfinancial investment ratio (blocs 3 and 4) causes an increase in the debt ratio; as expected, the increase is smaller under the assumption of a higher rate of return of the nonfinancial asset.

The model could be calibrated with parameters and variables values relevant for a particular country. Furthermore, some of those parameters and variables might be better represented by a stochastic process rather than a single value (representing a trend or expected value). Although we have simulated the prototype model, a more complex model that incorporates a specific fiscal rule can also be simulated if the country under consideration follows such rule.

[13] Recall that the gross operating primary balance refers only to the general government, and any improvement of the net operating balance of public enterprises is captured in the model as an increase in the average rate of return of nonfinancial assets.

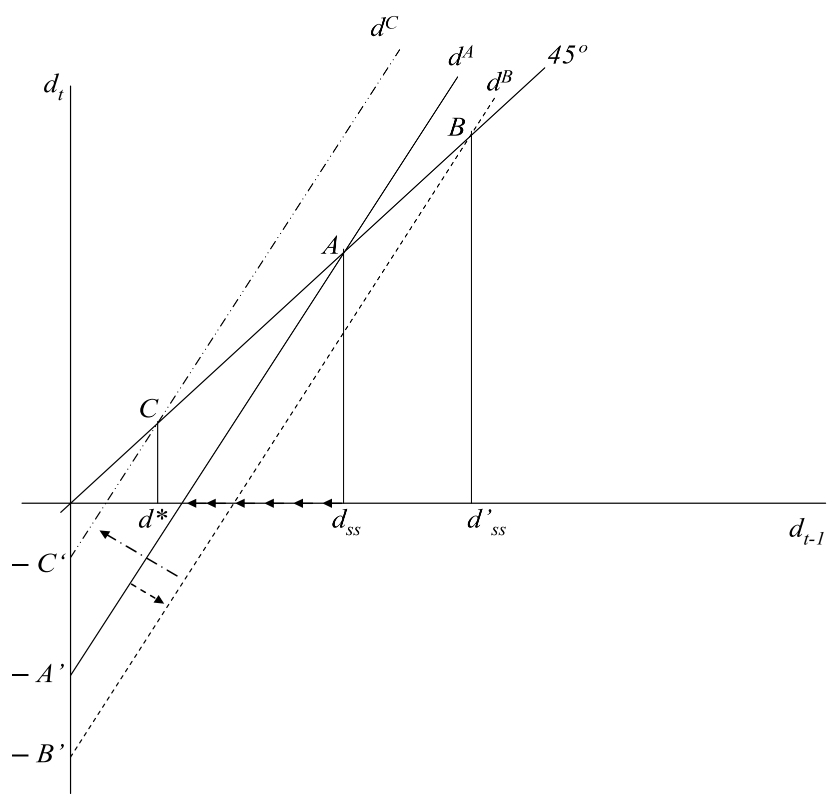
Table. Comparative Static; Impact of Higher Rates of Return on Assets and Investment Ratios
Under Zero and Non-Zero Valuation Adjustments

Benchmark Steady State	Scenarios / Steady State	Bloc 1: $\pi = 0,\ \forall i$			Bloc 2: $\pi \neq 0,\ \forall i$			Bloc 3: $\pi = 0,\ \forall i$			Bloc 4: $\pi \neq 0,\ \forall i$		
		Prototype Model			Prototype Model			Prototype Model			Prototype Model		
			$gopb = gopb^*$			$gopb = gopb^*$			$inv_{NF} > inv_{NF}^*$			$inv_{NF} > inv_{NF}^*$	
		$d^*=d_0$	lower ρ	higher ρ	$d^*=d_0$	lower ρ	higher ρ	$d^*=d_0$	same ρ_{NF}	higher ρ_{NF}	$d^*=d_0$	same ρ_{NF}	higher ρ_{NF}
Ratios	**Ratios**												
d_0 = 20.0%	dss	20.0%	22.1%	17.9%	20.0%	28.8%	11.2%	20.0%	82.5%	39.9%	20.0%	239.2%	61.9%
a_{NF0} = 15.0%	a_{NFss}	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	15.0%	22.7%	22.7%	15.0%	22.2%	22.2%
a_{F0} = 7.0%	a_{Fss}	7.0%	7.0%	7.0%	7.0%	7.0%	7.0%	7.0%	7.0%	7.0%	7.0%	7.0%	7.0%
nw_0 = 2.0%	nw_{ss}	2.0%	-0.1%	4.1%	2.0%	-6.8%	10.8%	2.0%	-52.8%	-10.2%	2.0%	-210.0%	-32.7%
	Policy variables												
	$gopb^{GG}$	1.8%	1.8%	1.8%	1.7%	1.7%	1.7%	1.8%	1.8%	1.8%	1.7%	1.7%	1.7%
	inv_{NF}	2.0%	2.0%	2.0%	2.1%	2.1%	2.1%	2.0%	3.0%	3.0%	2.1%	3.1%	3.1%
	inv_F	0.2%	0.2%	0.2%	0.3%	0.3%	0.3%	0.2%	0.2%	0.2%	0.3%	0.3%	0.3%
	$ps\ (= gopb^{GG} - inv_{NF})$	-0.2%	-0.2%	-0.2%	-0.4%	-0.4%	-0.4%	-0.2%	-1.2%	-1.2%	-0.4%	-1.4%	-1.4%
	Other variables												
	i (nominal interest rate on debt)	1.90%	1.90%	1.90%	4.45%	4.45%	4.45%	1.90%	1.90%	1.90%	4.45%	4.45%	4.45%
	$\hat{Y}$ (nominal GDP growth)	3.50%	3.50%	3.50%	6.09%	6.09%	6.09%	3.50%	3.50%	3.50%	6.09%	6.09%	6.09%
	ρ (rho)average non-financial assets	0.50%	0.35%	0.65%	3.01%	2.86%	3.17%	0.50%	0.50%	3.50%	3.01%	3.01%	6.09%
	ρ (rho)average financial assets	0.75%	0.60%	0.90%	3.27%	3.12%	3.42%	0.75%	0.75%	0.75%	3.27%	3.27%	3.27%
	δ (delta) depreciation rate	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%
	π^{NF} (change non-financial assests valuation)	0.00%	0.00%	0.00%	1.50%	1.50%	1.50%	0.00%	0.00%	0.00%	1.50%	1.50%	1.50%
	π^{F} (change financial assests valuation)	0.00%	0.00%	0.00%	1.25%	1.25%	1.25%	0.00%	0.00%	0.00%	1.25%	1.25%	1.25%
	π^{D} (change debt valuation)	0.00%	0.00%	0.00%	1.25%	1.25%	1.25%	0.00%	0.00%	0.00%	1.25%	1.25%	1.25%
	π (change in general price level)	0.00%	0.00%	0.00%	2.50%	2.50%	2.50%	0.00%	0.00%	0.00%	2.50%	2.50%	2.50%

DEBT DYNAMICS WITH A FISCAL REACTION FUNCTION AND REAL INTEREST RATE LARGER THAN GROWTH RATE: A GEOMETRIC REPRESENTATION

The analysis in this appendix is adapted from Croce and Juan-Ramón (2003, 2005). At point A, the steady-state equilibrium, d_{ss}, is unstable because the real interest rate is larger than the growth rate (or, geometrically, the d^A line cuts the 45° line from below). The debt ratio will remain at d_{ss} as long as the real interest rate, the growth rate, the rate of return of both assets, the rate of depreciation, the primary surplus, and the debt ratio itself do not change. The debt ratio might exogenously increase or decrease due to shocks. If any of these conditions changes without offsetting changes somewhere else, the debt ratio will either decrease or increase constantly without ever converging (the system is unstable).

Figure 5. Unstable Equilibrium, Debt Target, and Fiscal Reaction Function



Suppose the government considers d_{ss} (point A) too high and would like the debt ratio to converge to a lower level of indebtedness, say, d^* (point C). To achieve that target, the government increases the gross operating primary balance changing the intercept to –B' from –A' (the d^A line shifts immediately to the right, d^B, cutting the 45° line at point B). The higher gross operating primary surplus could sustain a higher debt ratio, d'_{ss} if, for example, the debt ratio were to jump exogenously to exactly that higher level. A higher surplus, all other things constant, implies that the debt ratio will steadily decrease reaching eventually negative values (the government becomes a net creditor). However, it is possible to force convergence to d^* by introducing the following fiscal reaction function: $gopb_t = gopb^* + \lambda\ (d_{t-1} - d^*)$. According to this rule, *gopb* shrinks as the gap between the debt ratio and the target debt is being reduced. This implies that the d^B line gradually shifts to the left until reaching d^C. Eventually, *gopb* converges to, say, $gopb^*$ which is the gross operating primary surplus necessary to maintain the debt ratio constant at d^* once it has reached that level.

Using the prototype model and assuming that both assets have reached their steady-state equilibrium (which, as shown in the text, both are stable equilibria), any of the intercepts of the *d* line say, *–A'*, is given by:

$$-A' = -\left[gopb^{GG} + \left(\frac{\rho_{NF} - \hat{Y} - \delta}{\hat{Y} + \delta} \right) inv_{NF} + \left(\frac{\rho_F - \hat{Y}}{\hat{Y}} \right) inv_F \right].$$

The above formula shows that once the desired steady-state equilibrium has been reached, a change in the rate of return of any of the assets will have to be offset by a change in *gopb* to maintain the debt ratio constant.

With a lower steady-state debt ratio (say, point C), it is conceivable that expectations and credibility effects would lead to close somewhat the gap between the real interest rate and the growth rate; which, in turn, would imply that a lower gross operating primary surplus is necessary to keep the debt ratio constant at d^*. In this case, the line d^C will be less steeper than the one shown in Figure 5, but still cutting the 45-degree line from below at point C.

CONSTANT NET WORTH RATIO RULE

The prototype model can be used to analyze the impact of stetting a rule of zero change in the net worth ratio: $nw_t - nw_{t-1} = 0$. This rule implies:

$$NW_t = (1+\hat{Y}_t)NW_{t-1} \Rightarrow NW_t - NW_{t-1} = \hat{Y}_t NW_{t-1}.$$

Thus, Equation (1) in the text becomes:

$$\hat{Y}_t NW_{t\text{-}1} = GOPB_t^{GG} - \delta A_{NF_{t-1}} - i_t D_{t\text{-}1} + \rho_{NF_t} A_{NF_t} + \rho_{F_t} A_{F_t}$$
$$+ \pi_t^{NF}(1\text{-}\delta)A_{NF_{t-1}} - \pi_t^D D_{t\text{-}1} + \pi_t^F A_{F_{t-1}}.$$

Solving for $GOPB^{GG}$,

$$GOPB_t^{GG} = A_{NF_{t-1}}\left(\hat{Y}_t + \delta - \rho_{NF_t} - \pi_t^{NF}(1-\delta)\right) + A_{F_{t-1}}\left(\hat{Y}_t - \rho_{F_t} - \pi_t^F\right) + D_{t-1}(i_t + \pi_t^D - \hat{Y}_t).$$

In terms of GDP, the above equation becomes:

$$gopb_t^{GG} = \frac{\hat{Y}_t + \delta - \rho_{NF_t} - \pi_t^{NF}(1-\delta)}{1+\hat{Y}_t} a_{NF_{t-1}} + \frac{\hat{Y}_t - \rho_{F_t} - \pi_t^F}{1+\hat{Y}_t} a_{F_{t-1}} + \frac{i_t + \pi_t^D - \hat{Y}_t}{1+\hat{Y}_t} d_{t-1}.$$

This $gopb^{GG}$ guarantees compliance with the constant net worth rule. Inserting the above equation in the difference equation for d in the prototype model, and again assuming for simplicity that $\pi^{NF} = \pi^F = \pi^D = 0$ obtains the motion law for d under the constant net worth ratio rule:

$$d_t = d_{t-1} - \frac{\hat{Y}_t + \delta}{1+\hat{Y}_t} a_{NF_{t-1}} - \frac{\hat{Y}_t}{1+\hat{Y}_t} a_{F_{t-1}} + inv_{NF_t} + inv_{F_t}. \tag{18}$$

Inserting the steady-state values of both assets: $a_{NF_{ss}} = \frac{1+\hat{Y}}{\hat{Y}+\delta} inv_{NF}$ and $a_{F_{ss}} = \frac{1+\hat{Y}}{\hat{Y}} inv_F$ in the above equation yields: $d_t = d_{t-1}$.

The constant net worth ratio rule does not guarantee the value (positive or negative) of the net worth ratio, it just maintains the steady-state net worth ratio over time regardless of its sign. The transition to the steady state under the constant net worth ratio rule could also be analyzed with a phase diagram similar to Figure 1 in the text. However, the slope of the difference equation for d will be one, coinciding with the slope of the 45° line. Then, as the asset ratio decreases towards its steady-state equilibrium (as in Figure 1), the dotted lines for d (now parallel to the 45° line) shifts upward, and it collapses with the 45° line when the asset ratio reaches the steady state. Thus, the steady-state value of the debt ratio is anchored by the steady-state value of the asset ratio.

Alternatively, the constant net worth ratio rule, $nw_t - nw_{t-1} = 0$, could be expressed as:

$$\left(a_{NF_t} + a_{F_t} - d_t\right) - \left(a_{NF_{t-1}} + a_{F_{t-1}} - d_{t-1}\right) = 0 \;\Rightarrow\; \left(d_t - d_{t-1}\right) = \left(a_{NF_t} + a_{F_t}\right) - \left(a_{NF_{t-1}} + a_{F_{t-1}}\right). \quad (19)$$

This formulation highlights the resemblance between the constant net worth ratio rule and the golden rule. While under the golden rule, changes in nonfinancial assets are financed with debt, under the constant net worth ratio rule, changes in the total assets ratios are financed with changes in the debt ratio. Inserting the steady-state values for both asset ratios (a_{NFss} and a_{Fss}) in the above equation, yields $d_t = d_{t-1}$. Alternatively, inserting in Equation (19) the difference equations for a_{NF_t} and a_{F_t} of the prototype model, yields the motion law of d under the constant net worth ratio given in Equation (18).

The Golden Rule cum Debt Stabilization Fund Based on the Gross Operating Primary Balance

The debt stabilization fund based on the *gopb* can be expressed as follows:

$$A_{F_t} = \left(1+\rho_{F_t}\right) A_{F_{t-1}} + \alpha\ GOPB_t; \tag{20}$$

$$a_{F_t} = \left(\frac{1+\rho_{F_t}}{1+\hat{Y}_t}\right) a_{F_{t-1}} + \underbrace{\alpha gopb_t}_{inv_{F_t}}. \tag{20′}$$

This scheme implies a particular public debt ratio dynamics and an endogenously determined gross operating primary balance ratio, given by Equations (21) and (22), respectively:

$$d_t = \frac{1}{1+\hat{Y}_t} d_{t-1} - \frac{\delta}{1+\hat{Y}_t} a_{NF_{t-1}} + inv_{NF_t}; \tag{21}$$

$$gopb_t^{GG} = \frac{i_t}{1+\hat{Y}_t} d_{t-1} + \frac{\delta - \rho_{NF_t}}{1+\hat{Y}_t} a_{NF_{t-1}} + \underbrace{inv_{F_t}}_{\alpha gopb_t}. \tag{22}$$

Given that $inv_{F_t} = \alpha gopb_t$,

$$gopb_t^{GG} = \frac{1}{(1-\alpha)}\left[\frac{i_t}{1+\hat{Y}_t} d_{t-1} + \frac{\left(\delta - \rho_{NF_t}\right)}{1+\hat{Y}_t} a_{NF_{t-1}}\right]. \tag{22′}$$

The endogenous $gopb_t^{GG}$ is positive and also finances the debt stabilization fund. Substituting Equation (22') into Equation (20') and defining θ = α / (1 – α), yields:

$$a_{F_t} = \left(\frac{1+\rho_{F_t}}{1+\hat{Y}_t}\right) a_{F_{t-1}} + \frac{\theta\left(\delta - \rho_{NF_t}\right)}{1+\hat{Y}_t} a_{NF_{t-1}} + \frac{\theta i}{(1+\hat{Y})} d_{t-1}. \tag{23}$$

Inserting the golden-rule, steady-state solutions for a_{NF} and d in Equation (23), obtains the steady-state solution for the financial assets ratio and consequently, the steady-state solution for the net debt:

$$a_{F_{ss}} = \left(\frac{1+\hat{Y}}{\hat{Y}-\rho_F}\right)\left[\frac{\theta\left(\delta - \rho_{NF} + i\right)}{\hat{Y}+\delta}\right] inv_{NF} \quad \Rightarrow \quad a_{F_{ss}} = \frac{\theta\left(\delta - \rho_{NF} + i\right)}{\hat{Y}-\rho_F} a_{NF_{ss}};$$

$$nd_{ss} \equiv d_{ss} - a_{F_{ss}} = \left(\frac{1+\hat{Y}}{\hat{Y}-\rho_F}\right)\left[\frac{\hat{Y}-\rho_F-\theta\left(\delta - \rho_{NF} + i\right)}{\hat{Y}+\delta}\right] inv_{NF} \quad \Rightarrow nd_{ss} = \left[\frac{\hat{Y}-\rho_F-\theta\left(\delta - \rho_{NF} + i\right)}{\hat{Y}-\rho_F}\right] a_{NF}.$$

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