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Pricing and Hedging of Contingent Credit Lines

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Abstract

Contingent credit lines (CCLs) are widely used in bank lending and also play an important role in the functioning of short-term capital markets. Yet, their pricing and hedging has not received much attention in the finance literature. Using a financial engineering approach, the paper analyzes the structure of simple CCLs, examines methods for their pricing, and discusses the problems faced in hedging CCL portfolios.

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I. Introduction

Contingent credit lines (CCLs) are widely used by banks for commercial and industrial lending and also play an important role in the functioning of short-term capital markets. A recent survey by the U.S. Federal Reserve shows that over three-quarters of bank lending is done using commitment contracts, and as of June 2005, the outstanding (unused) CCLs of U.S. firms were close to $1.72 trillion, more than double that in 1990.[2] Yet, the pricing and hedging of CCLs has not received much attention in the finance literature.[3] Recent advances in financial modeling and the emergence of new liquid option contracts have begun to permit a more detailed analysis of pricing and hedging of CCLs. Using a financial engineering approach, this paper examines the structure of simple CCLs and develops a method for their pricing.

A typical credit line contract has the following characteristics. First, it specifies a maximum amount that a financial institution (henceforth a *bank*) is committed to lending a *client* over a given period; this amount is called the *commitment.* Typically, the client has the right to draw any amount up to the maximum that is committed by the lender. Second, the contract specifies an interest rate that will apply to the amount borrowed or the *drawdown.* This is specified as a fixed interest rate or, more commonly, as a *spread* over some reference rate such as the London interbank offered rate (LIBOR). Third, the contract specifies the various fees charged by the lender—an upfront commitment fee, an annual fee levied on the total amount committed, and a usage fee levied annually on the undrawn portion of the commitment.[4] Fourth, contingent credit lines contain an escape clause, sometimes called a *material adverse change* (MAC) clause. Such a clause allows the bank to deny credit if the client's financial condition changes in a substantive way—for example, if the borrower is downgraded.[5]

Companies use lines of credit for three general reasons. First, many credit lines are issued as backstop facilities that give flexibility to issuers in capital markets. For example, companies with unused credit lines will avoid borrowing during periods when *commercial paper* (CP) rates have temporarily spiked in the market due to unforeseen events. Second, having a credit line in place signals that the company has the ability to pay for specific transactions and hence reduces *credit risk* on short term borrowing. In the commercial paper market, for example, back-up credit lines provide "insurance" to the investors for their short-term unsecured lending. Third, opening a credit line with a highly reputable bank usually sends a positive signal to other financial market participants. It reduces information asymmetries between the company management and the market about the company's financial condition (Fama (1985)). Normally, credit lines are rolled over after examining the financial health of the company. Renewal is important since it validates a companies credit-worthiness; termination, on the other hand, could be a negative signal about a clients financial health

[2] See, Federal Reserve Board (September 2005) and Federal Deposit Insurance Corporation (June 2005).

[3] See the recent survey by Ergungor (2002). An early exception is Thakor, Hong and Greebaum (1981) who utilize an option pricing approach to obtain the value of loan commitments and assess the sensitivity of these values to changes in interest rates.

[4] Few credit lines, however, carry all three types of fees. Most of them usually have two types of fees: a usage fee combined with an upfront or annual fee.

[5] See, for example, Shockley and Thakor (1997).

or business opportunities. Hence, clients have an incentive to roll over their existing credit lines with the same banks.[6]

Mozebach (1999) suggests that, in most cases, banks have considerable access to corporate information, and they are generally in the unique position of having little information asymmetry vis-à-vis the firms they lend to. Hence, the granting of a line of credit by a bank is a *signal* of a company's financial viability, and helps reduce the information asymmetry for other market players. Mozebach's paper also shows that there is a positive and significant market reaction on the protection buyer's stock to the announcement that a credit line has been granted. In addition, the bank implicitly sends a signal about itself, since it enters into a contract to provide funds on demand in the future. Therefore, lines of credit are an important signaling device for both firms (protection buyers) and banks (protection sellers).

Calomiris (1989) provides the framework that is closest to our treatment of contingent credit lines. He discusses evidence that bank loan commitments backing commercial paper provide insurance against systemic liquidity risk. The main function of commercial paper back-stops is to be another source of funds during periods of extreme CP market volatility. They are infrequently used and become automatically void if the underlying credit changes significantly. This paper is an attempt to replicate and price such CCLs.

The rest of the paper is organized as follows. Section II discusses the market for CCLs. Section III deals with modeling a canonical credit line. Section IV provides a replication strategy for the basic credit line. Section V proposes two methods for pricing CCLs and, using Monte Carlo simulations, examines how the CCL price is affected by contract and market parameters. The last section concludes.

II. Market Practice

In short-term capital markets, there are at least three standard types of financing for investment grade borrowers. The first is the *syndicated loan* market for relatively large issuers. The second is the commercial paper (CP) market where high grade issuers can borrow to finance their daily operations. And third, there are *CCL facilities or CP backstop loans* that are mainly associated with CP markets and other operations.

In this paper, we focus on this third category. In 2001, total volume of CP backstop facilities in the United States was around \$290 billion.[7] Such credit lines are generally opened by prime borrowers and carry various types of fees. Before we formally model such contracts it is useful to describe current market practices. Reviewing some of these practices will motivate certain features of CCLs:

[6] It should be pointed out that large, highly visible clients do not benefit much from such signals, because they are also monitored by rating agencies, the financial press, and bank analysts. Smaller and less visible clients benefit much more from the signals sent to the market by the approval of credit lines. The same is true for banks of different size. Large clients tend to use big highly reputable banks for large credit lines, since big banks are able to signal more reliable information to the market. Large lines of credit are usually underwritten by a syndicate of lenders, with a big financial institution being the lead manager.

[7] See IFR November 2001.

- *Participation fees.* Large contingent credit lines are issued by bank syndicates. The banks that participate in these syndicates are typically paid 1 basis point for every $10 million of the loan they underwrite. Obviously, this is a relatively small portion of the total cost of a CCL.

- *Undrawn or facility fees.* Once the CCL is opened, the borrower pays an annual fee even when the lines are not used. During the year 2001 the facility fees were in the 8-13 basis point range depending on the creditworthiness of the borrower.

- *Drawn fees.* This fee is the major component of the cost of a CCL. It is the spread over LIBOR on the portion of the CCL that is eventually drawn. The standard market practice is to express the price of the CCL as an *all-in drawn fee, i.e.,* the *total* funding cost over LIBOR paid by a borrower who avails of the CCL. During 2001 this fee averaged 73 basis points and in 2002 the average was 53 basis points[8]. This is a significant amount given that 1-year LIBOR averaged 3.8 percent in 2001 and 2.2 percent in 2002.

- *Term-out fees*. Before the year 2000 most CCLs were multi-year facilities and went up to five years. Then, with the collapse of the tech bubble, the bankruptcy of companies like Enron and WorldCom, and the revelation of accounting improprieties at many U.S. corporations, the time-length of CCLs was adversely affected. During 2002-2003, 60 percent of CCLs were marketed as one year (364 day) facilities. However, to lengthen the term of the CCL, the buyer can pay a *term-out fee* to buy an option to increase the term by another year. Such term-out fees were 12.5 basis points in 2001, but increased to 25 basis points in 2002.

- Some of the CCLs have embedded currency options. The options give the borrower the right to draw the loan in more than one currency. Such options make the product more complex but not necessarily more difficult to price. In this paper we will ignore such options.

Large CCLs are sold through syndications. The syndicates often consist of banks with which the borrower has a close relationship and the CCL *facility* is routinely extended. From the point of view of the banks, as long as the facility is not drawn, a commitment with original maturity of less than one year carries a zero risk-capital weight. Hence, the preference for maturities of less than a year. For commitments with original maturity greater than one year banks are required to hold half the capital required for regular loans.

III. Modeling a CCL

To fix ideas, suppose a corporation would like to transfer the risk associated with a low probability event $\mathcal{A}$ that may negatively affect its funding opportunities. In our case, this event could be a temporary closing of the CP market to the firm or a temporary spike in CP rates due to some outside shock. If the event $\mathcal{A}$ occurs the firm may need to secure some alternative financing and it decides to hedge this risk through a contingent credit line contract.

[8]IFR, 2001.

A CCL is a contract between a bank and a borrower, signed at time t_0, that specifies the time T till which the contract is valid, the maximum loan amount ($\$N$) that is available and the term of the loan. Any amount, up to the maximum available under the contract, can be drawn at an interest rate that is either fixed at the time the contract is written, or more generally specified as a spread, s_{t_0}, over some benchmark rate such as LIBOR, L_t. At the borrower's discretion, this amount can be drawn at any time in the interval $[t_0, T]$. The contract also contains a material adverse clause that could lead to the contract being voided under certain circumstances, for example, a deterioration of the borrower's credit rating.[9]

At regular times $\{t_i, i = 1, 2, .., n\}$ the firm has to roll over a funding requirement of $\$N$ in the money market. It pays the LIBOR rate L_{t_i} plus the credit spread c_{t_i} at the settlement date t_{i+1}. The CCL contract designed as a CP backstop has the following structure.

1. The contract is *written at time* t_0 *for* n *periods* (say, months or years). The maturity date is denoted by T, with $T = t_n$.

2. The credit line has a maximum size $\$N$. Normally, the borrower can draw any amount up to $N. We assume that when the facility is used the *full line is drawn.*[10] This simplifies the replication process and can be relaxed by assuming other rules for drawing the line.[11]

3. The unused portion of the CCL is subject to a *facility fee* denoted by u_{t_0}. In this paper, u_{t_0} is interpreted as the CCL premium that is paid up front at contract initiation. This is different than market practice, where u_{t_0} is paid over the contract period as an annual fee. In addition, the drawn portion is subject to a drawn-fee, s_{t_0}, expressed as a spread over LIBOR. These fees have subscript t_0 since they are decided when the contract is negotiated and remain constant throughout the contract period.

4. The CCL contract is of the *Bermudan-type* and the CCL can be tapped only at the predetermined times $t_1, t_2, .., t_{n-1}$. This is a simplification since in general the CCL lines can be accessed at any time during the life of the contract. However, if $t_i - t_{i-1}$ is small, this is an innocuous assumption that greatly simplifies the modeling and replication of the CCL.

5. When the credit line is drawn at time t_i, $i = 1, 2, ..., n-1$, the borrower pays the *annual all-in interest cost*:
$$r_{t_i} = L_{t_i} + s_{t_0}. \tag{1}$$
For simplicity, we ignore the participation and the term-out fees.

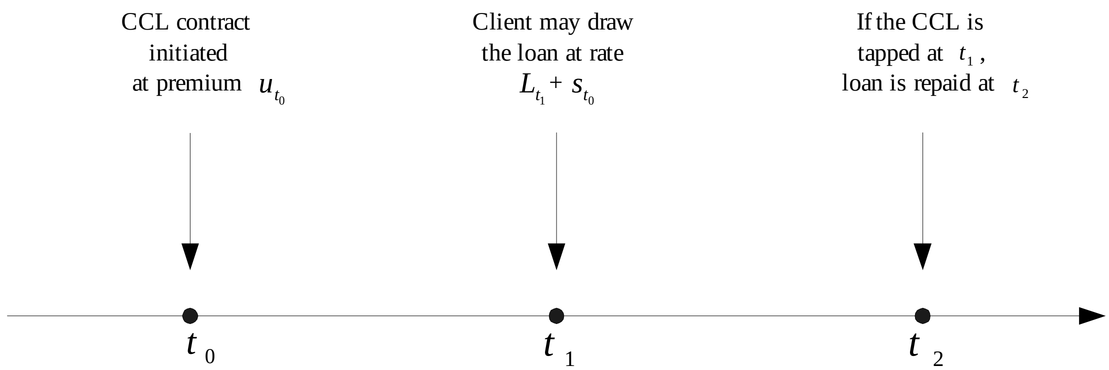
6. At times t_i, the borrower has the *option to prepay* the amount drawn from the CCL. Thus, if the firm's credit spread in the CP market falls below s_{t_0} the firm will choose to repay the bank.

[9] In some cases, the contract may specify that the credit line can be opened only under special conditions or for special purposes.

[10] Reliable empirical data on what proportion of the amount committed under CCLs is actually drawn is not available. However, casual evidence suggests that in a substantial majority of cases CCLs are not drawn at all.

[11] For a discussion on partial takedown of CCLs see Thakor, Hong and Greenbaum (1981).

Figure 1. CCL Structure



7. If an event $\mathcal{B}$ occurs at some date t, $t_0 < t < T$, the CCL contract is *automatically voided*. For the CCL being considered, event $\mathcal{B}$ is defined to be a rating downgrade of the borrower. This is assumed to occur if the time t credit spread observed in the market, c_t, exceeds a level c^*:
$$c_t > c^*, \tag{2}$$
where, obviously, we need to have
$$c^* > s_{t_0}. \tag{3}$$

The structure described above captures many important aspects of CCLs. A simplified version is provided in Figure 1. CCLs are tools that essentially decrease a company's cost of funding by lowering the credit risk for short-term lenders and by providing (partial) insurance against unforeseen market events. A large issue of CP is likely to be better received by the market if the investors know that the firm has access to a CCL and hence will be in a position to redeem its CP even under potentially adverse market conditions. It is worth noting that in offering a backstop loan or CCL facility, it is not the intention of the bank to lend (possibly) large sums. Rather, a CCL facility should be seen as a liquidity and credit enhancement, and not as a loan. And any loan that originates from drawing the CCL should be regarded as *temporary* in nature.

The CCL structure described above accomplishes this in the following way. If the corporation borrows from the short-term money markets at time t_i, the all-in interest paid will be:
$$r_{t_i} = L_{t_i} + c_{t_i} \tag{4}$$
where L_{t_i} is again the appropriate LIBOR. Under these conditions, a firm with a CCL facility will draw on it at time t_i only if the credit spread in the market exceeds the all-in drawn fee on the credit line:
$$c_{t_i} > s_{t_0} \qquad t_0 < t_i. \tag{5}$$
Once the market spread c_{t_i} goes below s_{t_0} the loan is repaid, ensuring the *temporary* nature of the instrument. In contrast, when c_{t_i} exceeds the level c^*, the borrower is downgraded and the CCL becomes void.

IV. Replicating Portfolio

In this section we show that the CCL contract can be replicated by using two well known instruments written on the credit spread c_t — a *cap* on the credit spread c_t with exercise dates t_i and a reverse knock-out option on c_t.

The first instrument is the *caplet* and its associated *cap*. A plain vanilla caplet is like a call option written on an interest rate, say x, such that at settlement time t_{i+1} the caplet pays

$$\text{Max}\ [N\delta(x_{t_i} - \kappa), 0] \tag{6}$$

where κ is the cap rate and the δ is the period of the loan expressed in years. In other words, if at the expiration date t_i, the interest rate x_{t_i} exceeds the specified cap rate κ, the buyer of the caplet gets compensated for the difference.[12]

In our case, instead of the interest rate x_{t_i} we assume that the caplet is written on the market credit spread c_{t_i}, and that the cap rate κ is the all-in drawn fee s_{t_0}. Thus the time t_{i+1} payoff of the caplet is assumed to be,

$$\text{Max}\ [N\delta(c_{t_i} - s_{t_0}), 0]. \tag{7}$$

Hence, if c_{t_i} exceeds the pre-specified level s_{t_0}, the CP market becomes less attractive than the CCL and the credit line is used (fully).[13] Putting together n such "caplets" with exercise dates t_i, $i = 1, 2, ..., n$, in a single contract, we obtain a cap written on the credit spread c_{t_i}.

The second instrument we use in engineering a CCL is a *reverse barrier* option. In particular, we consider a *knock-out* option that is contingent on the "credit event":

$$c_t > c^*, \quad t_0 \leq t \leq t_n = T. \tag{8}$$

If this event occurs, the firm is downgraded and the CCL contract is voided and the bank is under no obligation to provide the loan.

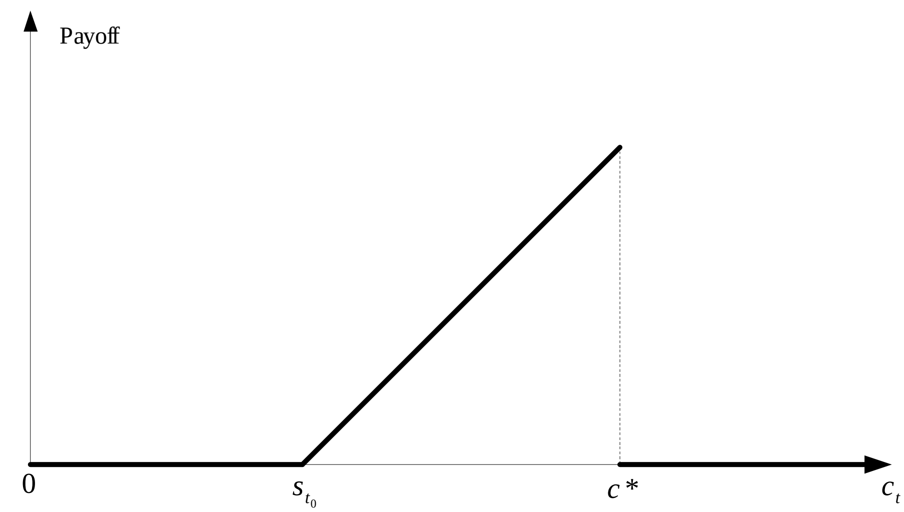
The underlying risk is c_t, the credit spread at time t, $t_0 \leq t \leq t_n = T$. The replicating portfolio for the CCL structure described above consists of a cap written on the credit spread c_t, combined with a barrier option that leads to the cap being knocked-out if the event $\{c_t > c_t^*\}$ occurs during the life of the contract. Since the CCL knocks-out when the cap is in-the-money, we are dealing with a *reverse* barrier option. The payoff structure is shown in Figure 2

We assume that at exercise times t_i of the caplets, the firm could get a loan of size $\$N$ from the bank. This loan has an interest cost made up of the floating rate L_{t_i} and the all-in-fee, s_{t_0}. The fair market value at time t of the i^{th} (knock-out) caplet that expires at time t_{i+1} can be written as:

[12] In this paper we use the terms *cap* and *caplet* to discuss the structure of the CCL contract. In fact, these are also spread options and we could formulate CCLs as a basket of spread options.

[13] When a firm taps its CCL, the bank acquires a loan at a pre-determined spread s_{t_0} on its balance sheet. The bank's loan credit portfolio is affected and this raises questions regarding the management of CCL portfolios that are beyond the scope of the current paper.

Figure 2. Reverse Barrier Option



$$(cpl)_t^i = B(t, t_{i+1})\ E_t^P \left[N\delta\ \mathrm{Max}\left\{(c_{t_i} - s_{t_0}), 0\right\}\ \mathbf{1}_{\{c_t<c^*,\ t_0\le t\le t_i\}}\right] \tag{9}$$

where $\mathbf{1}_{\{c_t<c^*,\ t_0\ \le t\le t_i\}}$ is the indicator function for the event that the firm is not downgraded before the caplet expires. The term $B(t, t_{i+1})$ is the *risky* discount at time t on \$1 to be paid at time t_{i+1} and is calculated from the appropriate risky zero-coupon bonds. The valuation equation written above uses the discount bonds with default risk for normalizing securities prices—the t_{i+1} risky forward-measure is used in evaluating the expectation. This normalization requires that during default the recovery rate is bounded away from zero with probability one. The cap, formed by the collection of caplets, has a value at time t of:

$$(cap)_t = \sum_{i=1}^{n} (cpl)_t^i. \tag{10}$$

We can now complete the replication process. Suppose the client buys the knock-out cap from the bank at the all-in cost s_{t_0} and, at the same time, obtains a floating rate loan in the money markets that has rates $L_{t_i} + c_{t_i}$ at reset dates t_i. This portfolio will be equivalent to a market loan backed by a contingent credit line written by the bank.

The contract can be described in heuristic terms as follows. The corporation has to make a decision at the dates $t_1, ..., t_n$ on whether to tap the CCL or not. If the CCL is tapped at the date t_i, (by assumption in the full amount) then the corporation pays the drawn fee s_{t_0} over the L_{t_i} rate observed at that time. The loan is for the period $t_{i+1} - t_i = \delta$.[14] At the next date t_{i+1}, the corporation has two choices. The corporation may decide to pay back the CCL loan and draw it again (i.e. roll it over), or it may want to pay the bank and borrow in the CP market rather than access the CCL. The latter will be the case if $c_{t_{i+1}} < s_{t_0}$.

[14] Thus, in this characterization there is no pre-payment that takes place.

In this setup, the CCL amounts to a sequence of options on a series of floating rate loan contracts struck at the credit spread s_{t_0}. At t_i, the option to draw on the CCL will be exercised if the observed credit spread c_{t_i} exceeds s_{t_0}. This option is "knocked out" if this excess is too much and the corporation is downgraded. The price of the CCL depends on the underlying L_{t_i}, c_{t_i}, their volatilities, and other parameters like s_{t_0}, c^*, and whether there are options to prepay or extend the maturity of the CCL. Having the prepayment and term-out privileges, would involve adding two more options to the replicating portfolio.

V. Pricing

The pricing of the CCL contract cannot use the standard approach for foreign exchange and equity options. Normally, a reverse knock-out option can be priced with reasonable accuracy by combining an American digital option with a regular knock-out option. However, in our case the standard approach to pricing knock-out options is not appropriate because we are dealing with interest rates and credit spreads. The standard Black-Scholes methodology has one factor and assumes constant interest rates. In our case there may be *two* factors and, depending on the term of the CCL contract, constant interest rates may not be a realistic assumption.

Suppose for simplicity that the cost of borrowing in the CP market for a firm with *no* default-risk is L_t.[15] The firm buying the CCL has a credit risk represented by the spread c_t, and the firm has raised $\$N$ in the commercial paper market as of time t.

Let $F(t, t_i, t_{i+1})$ be the forward LIBOR rate at time t for a loan made at time t_i that matures at time t_{i+1}, where $t < t_i < t_{i+1}$. We assume that this rate applies to loans with no default risk. We use the notation $f(t, t_i, t_{i+1})$ for the forward rate that applies to loans made to a specific firm. Pricing of the CCL contract requires modeling the dynamics of: (i) n risk-free *forward rates,* $F(t, t_i, t_{i+1})$, $i = 1, ..., n-1$ and (ii) n risky *forward rates*, $f(t, t_i, t_{i+1})$, that apply to loans made to a specific firm at times t_i and maturing at times t_{i+1}, $i = 1, ..., n-1$. The resulting dynamics for $c(t, t_i, t_{i+1})$, the (forward) corporate credit spread, is derived from

$$c(t, t_i, t_{i+1}) = f(t, t_i, t_{i+1}) - F(t, t_i, t_{i+1}). \tag{11}$$

We model the dynamics for $F(t, t_i, t_{i+1})$, $f(t, t_i, t_{i+1})$ and $c(t, t_i, t_{i+1})$ using two methods.

A. Method 1

The first method uses a framework provided by Schönbucher's extension of the forward-LIBOR model.[16] For the *forward-LIBOR process* we have:

$$F(t, t_i, t_{i+1}) = E_t^{P^{t_{i+1}}} [L_{t_i}] \tag{12}$$

[15] Normally, LIBOR is the funding cost for a AA-rated firm. But the LIBOR market model makes the assumption of zero credit risk.

[16] See, Schönbucher (2000) and (2004).

where $P^{t_{i+1}}$ represents the t_{i+1}-risk-neutral forward measure. This measure is obtained by a normalization using the time t_{i+1} default-free pure discount bond price $P(t, t_{i+1})$ and yields a $F(t, t_i, t_{i+1})$ process that has no drift.[17]

For expositional clarity, we consider the pricing of a single caplet in the CCL structure described in the previous section. Assuming that the caplet applies to the period $[t_1, t_2]$, considerably simplifies the notation. Dropping the i subscripts, we redefine the forward rates as $F_t \equiv F(t, t_1, t_2)$ and $f_t \equiv f(t, t_1, t_2)$.[18] It is well known that under the t_2-forward measure the F_t process has no drift:

$$dF_t = F_t \ \sigma_t^F \ d\omega(t) \tag{13}$$

where σ_t^F is the volatility parameter for F_t. and $\omega(t)$ is the Brownian motion under the t_2-forward measure. However, under the t_2-forward measure the f_t process has an unknown drift that depends on the market prices for interest rate and credit risks, and it does *not* have martingale dynamics:

$$df_t = \gamma(f_t, \lambda(t)) \, dt + f_t \ \sigma_t^f \ d\omega(t) \tag{14}$$

where the $\lambda(t)$ is a vector of market prices and σ_t^f is the volatility parameter for f_t.

One approach is to use Schönbucher's extension of the Forward LIBOR model. To this end, define $\tilde{P}$ as the t_2-*survival* measure. This measure is obtained by scaling the time t_2 state-price vector using the value of the time-t_2 maturity defaultable bond issued by the borrower. Schönbucher(2004) shows that when this measure is used f_t has martingale dynamics:

$$df_t = f_t \ \sigma_t^f \ d\overline{\omega}(t) \tag{15}$$

where $\overline{\omega}(t)$ is the Brownian motion under the t_2-*survival* measure.

These two measures can be connected by using a spot-martingale measure, which is obtained by normalizing asset prices using a properly defined savings account.[19] After changing the probability under the spot-martingale measure to that under the t_2-forward measure, $d\omega(t)$ can be written as

$$d\omega(t) = d\omega_Q(t) + \alpha_{t_2}(t) \ dt, \tag{16}$$

[17] See, for example, Rebonato (2002).

[18] Note that the risky forward rate dynamics does *not* include a separate jump component to account for default by the underlying credit. This is a convenient approximation that can be justified in our set up because: (i) back stop facilities for highly rated clients have very small default probabilities; (ii) as in Calomiris(1989), CCLs are viewed as liquidity enhancers and not as tools for default protection; (iii) in very short periods of time the probability of credit deterioration from AAA to full default is likely to be very small; (iv) the existence of a MAC clause limits the credit exposure, and in the case of a big jump in credit spreads the option knocks out.

[19] The spot martingale measure is also the t_1-forward measure t_0. See Musiela and Rutkowski (1998) for further details.

where $\omega_Q(t)$ is the Brownian motion under the spot-martingale measure, and $\alpha_{t_2}(t)$ can be recursively expressed as

$$\alpha_{t_2}(t) = \alpha_{t_1}(t) + \frac{\delta F_t}{1+\delta F_t}\ \sigma_t^F. \tag{17}$$

Here α_{t_1} is defined as minus the volatility of the default free bond $P(t,t_1)$. Changing measure from spot-martingale measure to t_2-survival measure, we get

$$d\overline{\omega}(t) = d\omega_Q(t) + \overline{\alpha}_{t_2}(t)\ dt, \tag{18}$$

$$\overline{\alpha}_{t_2}(t) = \overline{\alpha}_{t_1}(t) + \frac{\delta f_t}{1+\delta f_t}\ \sigma_t^f, \tag{19}$$

where $\overline{\alpha}_{t_1}$ is minus the volatility of the defaultable bond $B(t,t_1)$. These connections between the Wiener process increments are used in writing the dynamics of the underling processes under a *single* measure. This is needed since pseudo-random numbers need to be drawn from one probability distribution in the Monte Carlo approach.

Using (16) and (18), the Wiener processes under the t_2-forward measure and the t_2-survival measure are related by

$$d\overline{\omega}(t) = d\omega(t) + \alpha_{t_2}^D(t)\ dt \tag{20}$$

where, $\alpha_{t_2}^D(t)$, defined as $\overline{\alpha}_{t_2}(t) - \alpha_{t_2}(t)$, has the recursion formula

$$\alpha_{t_2}^D(t) = \alpha_{t_1}^D(t) + \frac{\delta H(t)}{1+\delta H(t)}\ \sigma_t^H. \tag{21}$$

Here $H(t)$ is interpreted as the *hazard rate* at time t. Using the equality in (20) we can now express the two martingale dynamics for F_t and f_t under *one single* measure. This introduces a drift term in one of the martingale dynamics, but this can be calculated since $\alpha_{t_2}^D(t)$ is known.

Using the t_2-survival measure, we write the dynamics of F_t, f_t, $c(t)$, and $H(t)$ under a single measure as

$$dF_t = F_t\ \sigma_t^F\ \left(d\overline{\omega}(t) - \alpha_{t_2}^D(t)\ dt\right), \tag{22}$$

$$df_t = f_t\ \sigma_t^f\ d\overline{\omega}(t), \tag{23}$$

$$dc(t) = F_t\ \sigma_t^F\ \alpha_{t_2}^D(t)\ dt + \left[f_t\ \sigma_t^f\ - F_t\ \sigma_t^F\right]\ d\overline{\omega}(t), \tag{24}$$

$$dH(t) = \frac{\delta F_t}{1+\delta F_t}\ \left[(1+\delta H(t))\ \alpha_{t_2}^D(t) - \delta H(t)\ \sigma_t^H\right]\ dt + H(t)\ \sigma_t^H\ d\overline{\omega}(t). \tag{25}$$

Note that the dynamics for F_t now has a non-zero drift. Note also that this drift is known at time t. Discretizing these equations using an Euler scheme, we obtain Monte Carlo trajectories for $c(t)$, which we use to price the CCL contract.

The CCL price is the discounted payoff from the replicating portfolio consisting of the caplet and the associated knock-out option. In the introduction, this price is also denoted by the symbol u_{t_0} :

$$(CCL)_{t_0} = B(t_0,t_2)\ N\ \delta\ E\left[Max(c_{t_1} - s_{t_0}, 0)\ \mathbf{1}_{\{c_t<c^*,\ t_0\le t\le t_1\}}\right] \tag{26}$$

where $B(t_0,t_2)$ is the risky discount factor and $\mathbf{1}_{\{c_t<c^*,\ t_0\le t\le t_1\}}$is the indicator function representing the requirement that the firm is not downgraded before time t_1.

The pricing approach used above is built on Schönbucher's extension of the Forward LIBOR Model. Our discussion of synthetically creating a CCL was based on the dynamics of the credit spread and did not directly model the event of default. Schönbucher (2004) provides the rationale for our procedure. Note that the synthetic CCL and its pricing is done using the *survival measure.* This eliminates the need to directly model the default event. The key to this is modeling the dynamics *conditional* on the fact that default has not occurred—that is, using a normalization that depends on the value of the defaultable bond, but under the condition that default has not yet occurred. Schönbucher(2004) shows how a model that is conditional on no default having occurred, can be used to price defaultable securities.

The structure of the replicating portfolio for the CCL makes two tendencies clear. The higher the probability that the borrower's credit spread (c_t) will be greater than the negotiated spread in the CCL contract (s_{t_0}), the larger is the payoff to the CCL and hence the higher is its price. However, higher credit spreads also increase the probability of breaching the knock-out barrier c^*, and the increase in this probability tends to dampen the payoff to the CCL.

The simulations are designed to show the effects of σ^f, s_{t_0} and c^* on the price of the contingent credit line. We assume that the CCL is for a notional sum of \$1 and the values for the other parameters are as follows: $\sigma^F = 15\%$, $t_1 = 1$, $t_2 = 2$. For the simulation exercise, we take the initial values to be $F_{t_0} = 6.7\%$, $f_{t_0} = 8.3\%$, $c_{t_0} = 1.6\%$. Figures 3 - 5 show the expected payoff[20] at time t_2 under different parameter specifications. Figure 3 depicts the effect of σ^f and s_{t_0} on the CCL price, holding c^* constant. It shows that the payoff is a decreasing function of s_{t_0}, for given levels of σ^f, and c^*. The higher is s_{t_0}, the lower the probability that the CCL will be drawn, and hence the lower is its price. For fixed c^*, the higher is s_{t_0} ($< c^*$), the smaller is the range over which the CCL has positive value. For fixed s_{t_0}, the CCL price initially rises with volatility σ^f and then declines, since eventually high values for σ^f increase the probability of the knock-out threshold being crossed.

Figure 4 looks at the effect of σ^f and c^* on the CCL price, keeping s_{t_0} constant. The relationship between σ^f and the CCL price is similar to the one seen in Figure 3. What is interesting here is that the σ^f corresponding to the peak CCL price for a given c^* increases with c^*. Intuitively, given s_{t_0}, for a higher c^* the "knock-out" effect begins to dominate at a higher σ^f. The positive relationship between c^* and the CCL price is also in line with our intuition. c^* is the knock-out threshold; if c_t exceeds c^*, the CCL terminates. The higher the c^*, the lower is the knock-out probability, and hence the higher the payoff.

Figure 5 shows how the CCL price varies with s_{t_0} and c^*, for fixed values of σ^f. The results are intuitive and clear-cut. For fixed c^*, raising s_{t_0} lowers the price of the CCL. For fixed s_{t_0}, raising c^* increases the CCL price. As one would expect, for given values of σ^f, the price curve flattens out after c^* has reached a certain level.

[20] The discount factor for the CCL payoff is obtained under the assumption that the yield curve is flat. Since CCLs are generally short-term facilities, or are "reset" frequently, this is a reasonable assumption.

Figure 3. CCL price: $c^* = 0.055$

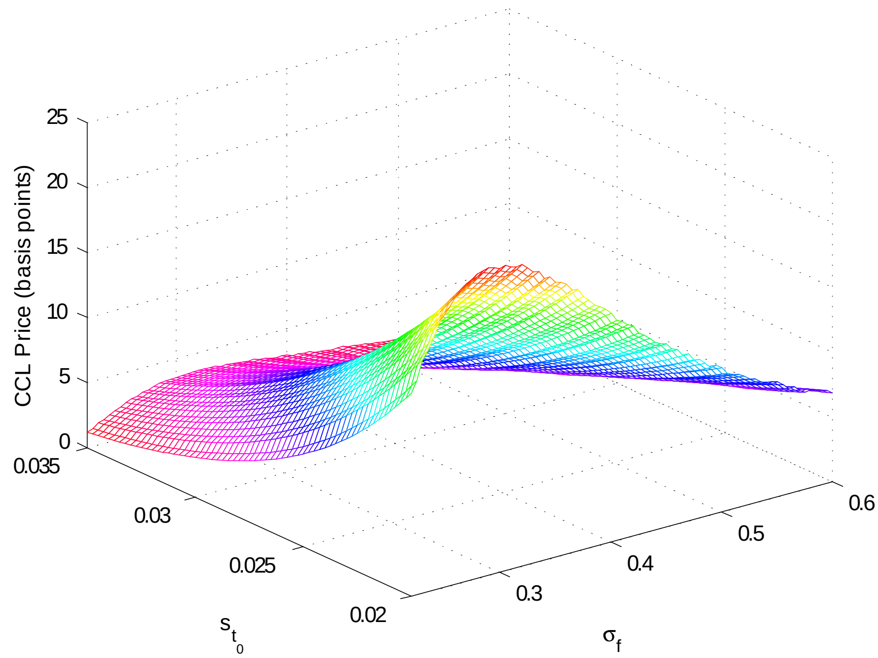


Figure 4. CCL Price: $s_{t_0} = 0.025$

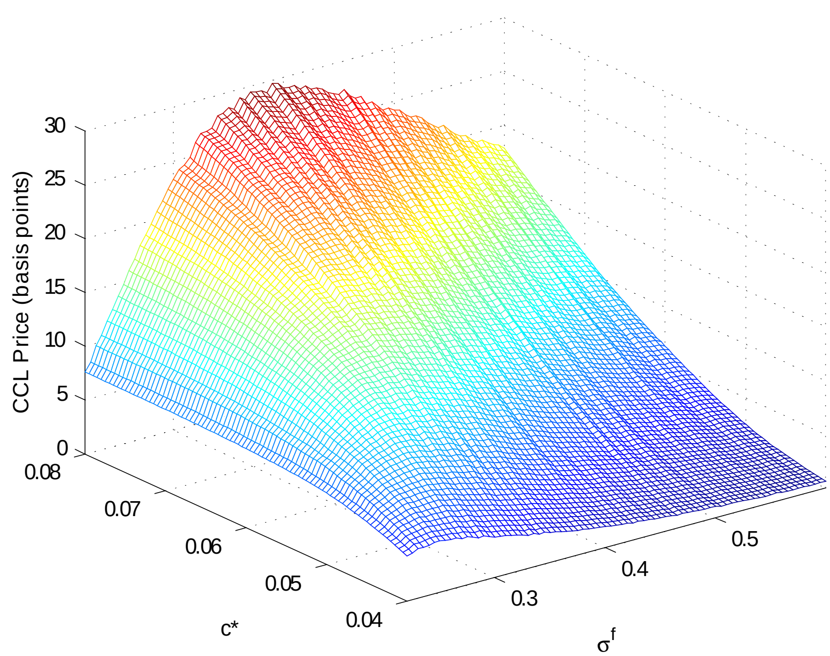
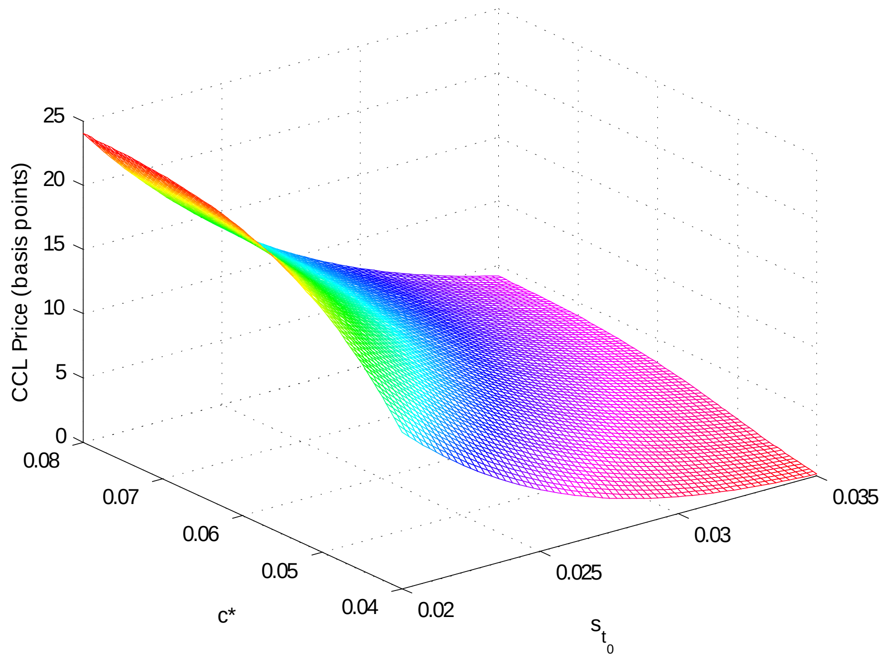


Figure 5. CCL Price: $\sigma^f = 0.025$



B. Method 2

An alternative way of proceeding in modeling the dynamics of the forward rates is to assume that when a firm defaults the recovery rate is bounded away from zero, and normalize using the price of the risky discount bond, $B(t, t_{i+1})$. Under this forward measure $\tilde{P}^{t_{i+1}}$, the forward rate $f(t, t_i, t_{i+1})$ is a martingale without drift. Hence, using this approach, only the drift of the risk-free forward rate $F(t, t_i, t_{i+1})$ under the forward measure $\tilde{P}^{t_{i+1}}$ has to be specified.

Using the same notation as in Method 1, the real-world dynamics of F_t, f_t and c_t are specified as

$$\begin{aligned} dF_t = &\; a(F_t, t)\; dt + \sigma_t^F\; F_t\; d\omega_t^{(1)}, \\ df_t = &\; b(F_t, f_t, t)\; dt + \sigma_t^f\; f_t\; d\omega_t^{(2)}, \\ dc_t = &\; df_t - dF_t, \end{aligned} \tag{27}$$

where the $d\omega_t^{(1)}$ and $d\omega_t^{(2)}$ are standard, possibly correlated, Wiener processes. Under the forward measure $\tilde{P}^{t_2}$ obtained through normalization using $B(t, t_2)$, the forward rate f_t is a martingale without drift

$$df_t = \sigma^f\; f_t\; d\tilde{\omega}_t^{(2)}, \tag{28}$$

where $d\tilde{\omega}_t^{(2)}$ is a standard Wiener process and volatility is assumed to be constant.

To specify the drift of the risk-free forward rate F_t, we proceed as follows. We assume that F_t follows a mean-reversion model:

$$dF_t = \beta(\mu - F_t)dt + \sigma^F F_t d\omega_t^{(1)}, \tag{29}$$

where μ is the long-term average rate and β is the speed of reversion. Under the $B(t,t_2)$ normalization, $E[d\omega_t^{(1)}] = \lambda_t dt \neq 0$. By adding and subtracting the term $\lambda_t\, \sigma^F\, F_t\, dt$ in (29), we get

$$dF_t = [\beta(\mu - F_t) + \lambda_t\ \sigma^F\ F_t]\ dt + \sigma^F\ F_t\ d\tilde{\omega}_t^{(1)} \tag{30}$$

where $d\tilde{\omega}_t^{(1)}$ is a Wiener process, such that $d\tilde{\omega}_t^{(1)} \equiv d\omega_t^{(1)} - \lambda_t\ dt$ and $E[d\tilde{\omega}_t^{(1)}] = 0$.

We assume λ_t is a constant and calibrate it by making use of the fact that the caplet price can be calculated in two ways. Black's formula gives a closed-form analytical solution.[21] The other is a λ_t-dependent simulation based method that makes use of the interest rate dynamics specified in (30). We choose λ_t to equalize the two calculations of the caplet price.[22]

Suppose t_0 ($t_0 < t_1 < t_2$) is the contract initiation time and the caplet contract applies to the future period $[t_1, t_2]$. The payoff at time t_2 of the caplet with a fixed strike rate of K is

$$N\ \delta\ (L_{t_1} - K)^+, \tag{31}$$

where N is the notional amount decided when entering the contract, δ is the days adjustment factor, and L_{t_1} is the LIBOR rate observed at time t_1. Black's formula for a (at-the-money) caplet price with strike rate F_{t_0} is (see, for example, Hull (2003))

$$\mathrm{cpl}(t_0, t_1, t_2, F_{t_0}) = P(t_0, t_2)\ N\ \delta\ F_{t_0}\left[2\Phi\left(\frac{\sigma^{\mathrm{cpl}}\sqrt{t_1 - t_0}}{2}\right) - 1\right], \tag{32}$$

where, $P(t_0, t_2)$ is the time t_0 value of the relevant default-free bond, the $\Phi(\cdot)$ is the standard normal distribution, and σ^{cpl} is the average realized annual caplet volatility. The simulated caplet price is the expected payoff:

$$\mathrm{cpl}(t_0, t_1, t_2, F_{t_0}) = P(t_0, t_2)\ N\ \delta\ E[Max(L_{t_1} - F_{t_0}, 0)]. \tag{33}$$

We simulate F_t using the Euler discretization,

$$F_{t+1} = F_t + [\beta\ (\mu - F_t) + \lambda\ \sigma^F\ F_t]\ \Delta t + \sigma^F\ F_t\ \Delta\tilde{\omega}_t^{(1)},\ \ \Delta\tilde{\omega}_t^{(1)} \sim N(0, \sqrt{\Delta t}). \tag{34}$$

The paths are simulated from time t to time t_1 to obtain $F(t, t_1, t_2) = E_t^{P_{t_2}}[L_{t_1}]$. By simulating p (say, 3000) paths of F_t, we get the expected payoff of the caplet. The desired λ^* is then the λ that makes (32) equal to (33).

Having calculated λ^*, the dynamics of the forward rates are given by

$$\begin{aligned} df_t =&\ \ \sigma^f\ f_t\ d\tilde{\omega}_t^{(2)}, \\ dF_t =&\ \ [\beta\ (\mu - F_t) + \lambda^*\sigma^F F_t]\ dt + \sigma^F F_t\ d\tilde{\omega}_t^{(1)}. \end{aligned} \tag{35}$$

We assume that the Wiener processes driving the two forward rates, f_t and F_t, have an instantaneous correlation ρ :

$$d\tilde{\omega}_t^{(1)} d\tilde{\omega}_t^{(2)} = \rho\ dt, \tag{36}$$

[21] See, Black (1976).

[22] Note that here we are modeling the risk-free forward rate drift instead of calculating it explicitly under the risky bond normalization. However, this drift is then calibrated to arbitrage-free bond prices.

The Cholesky decomposition of the covariance matrix of ($\tilde{\omega}_t^{(1)}$, $\tilde{\omega}_t^{(2)}$) allows us to express the dynamics of the forward rate processes in terms of two independent Wiener processes $W_t^{(1)}$ and $W_t^{(2)}$ where (see, for example, Brigo and Mercurio (2001))

$$\begin{aligned} d\tilde{\omega}_t^{(1)} &= dW_t^{(1)} \\ d\tilde{\omega}_t^{(2)} &= \rho\, dW_t^{(1)} + \sqrt{1-\rho^2}\, dW_t^{(2)}. \end{aligned} \tag{37}$$

The dynamics of f_t and F_t can be written in terms of $W_t^{(1)}$ and $W_t^{(2)}$ as

$$\begin{aligned} df_t &= \sigma^f\ f_t\ [\rho\ dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)}], \\ dF_t &= [\beta\ (\mu - F_t) + \lambda^*\ \sigma^F\ F_t]\ dt + \sigma^F\ F_t\ dW_t^{(1)}, \end{aligned} \tag{38}$$

Since $c_t = f_t - F_t$, the simulated paths of f_t and F_t, also yield a corresponding path for the credit spread. One can interpret $dW_t^{(1)}$as a shock to the macroeconomic environment that affects all firms and $dW_t^{(2)}$ as an "idiosyncratic" shock that only affects the specific company.

The simulations show the effects of σ^f, ρ, s_{t_0} and c^* on the price of the contingent credit line. Again, we assume that the CCL is for a notional sum of \$1 and the values for the other parameters are as follows: $\sigma^F = \sigma^{\text{cpl}} = 15\%$, $\beta = 0.05$, $\mu = 6.5\%$, $t_1 = 1$, $t_2 = 2$, $F_{t_0} = 6.7\%$, $f_{t_0} = 8.3\%$, $c_{t_0} = 1.6\%$. Figures 6–11 show the expected payoff at time t_2 under different parameter specifications. Notice that the graphs in Figures 6–8 are similar to those in Figures 3-5, except that they are shifted up by a few basis points. In this alternative approach we also examine the sensitivity of the CCL price to the correlation parameter ρ.

Figure 9 shows how the CCL price varies with σ^f and ρ when s_{t_0} and c^* are held constant at the specified levels. For a fixed ρ, the graph of the CCL price against σ^f first increases and then decreases. Initially, as volatility σ^f increases it is more likely that c_t will exceed s_{t_0} and the CCL will be drawn. However, as volatility keeps increasing, eventually the knock-out effect dominates and the CCL price falls at still higher volatilities.

The effect of varying ρ while keeping σ^f constant is more complex. A higher correlation ρ between f_t and F_t implies that c_t is less volatile and the chance of c_t exceeding s_{t_0} and the CCL being drawn, is low. Hence, the CCL price is negatively related to the correlation ρ. For low levels of σ^f, this holds true. But for relatively high levels of σ^f, the story is different. A high ρ now makes it less likely that c_t will exceed c^*, therefore attenuating the knock-out effect. Thus, at high σ^f volatility, the CCL price and the correlation ρ can be positively related.

Figure 10 plots the CCL price against ρ and s_{t_0}, for given values of σ^f and c^*. As before, it is clear that the relationship with s_{t_0} is negative. With respect to ρ, the curve is relatively flat; an examination of cuts for given values of s_{t_0}, shows that for high values of ρ, the CCL price is a decreasing function of ρ. This can be explained as follows: the higher the ρ, the lower is the volatility of c_t and hence the lower is the probability of drawing on the CCL. For low values of ρ, the graph is almost flat.

Figure 11 illustrates the effect of varying ρ and c^* on the CCL price, for given values of σ^f and s_{t_0}. As expected, for given ρ, the CCL price increases with c^*. The effect of ρ on the

Figure 6. CCL Price: $c^* = 0.055$, $\rho = 0.5$

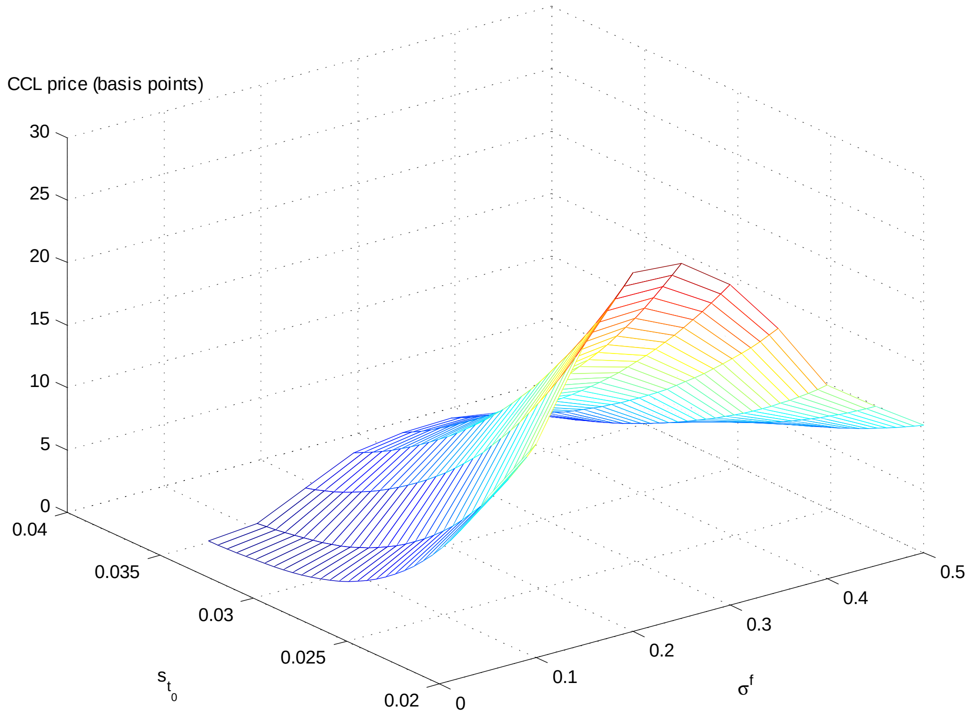


Figure 7. CCL Price: $s_{t_0} = 0.025$, $\rho = 0.5$

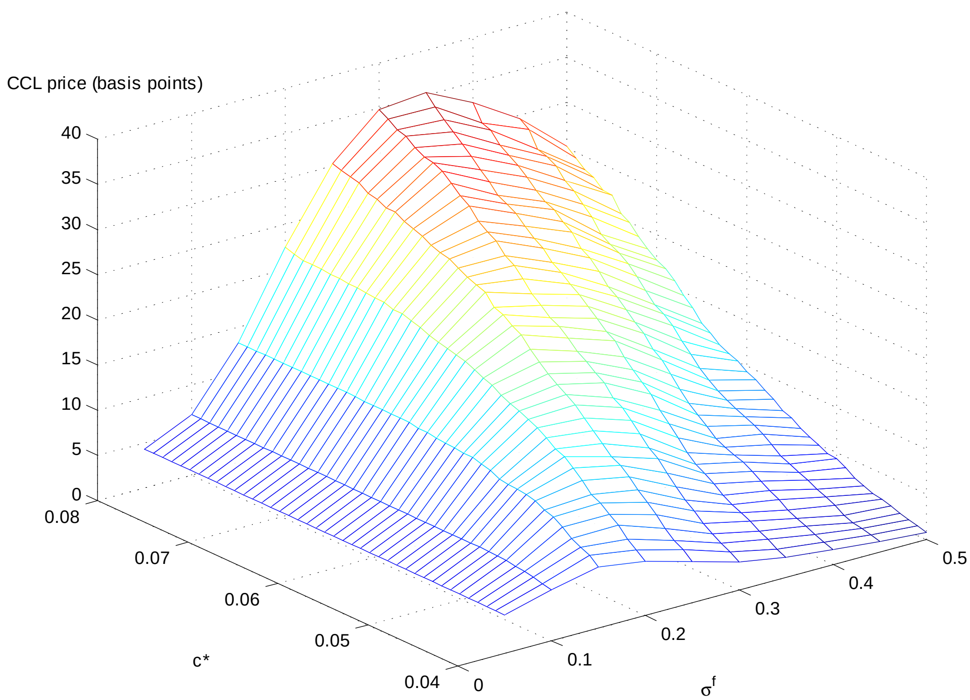


Figure 8. CCL Price: $\sigma^f = 0.25$, $\rho = 0.5$

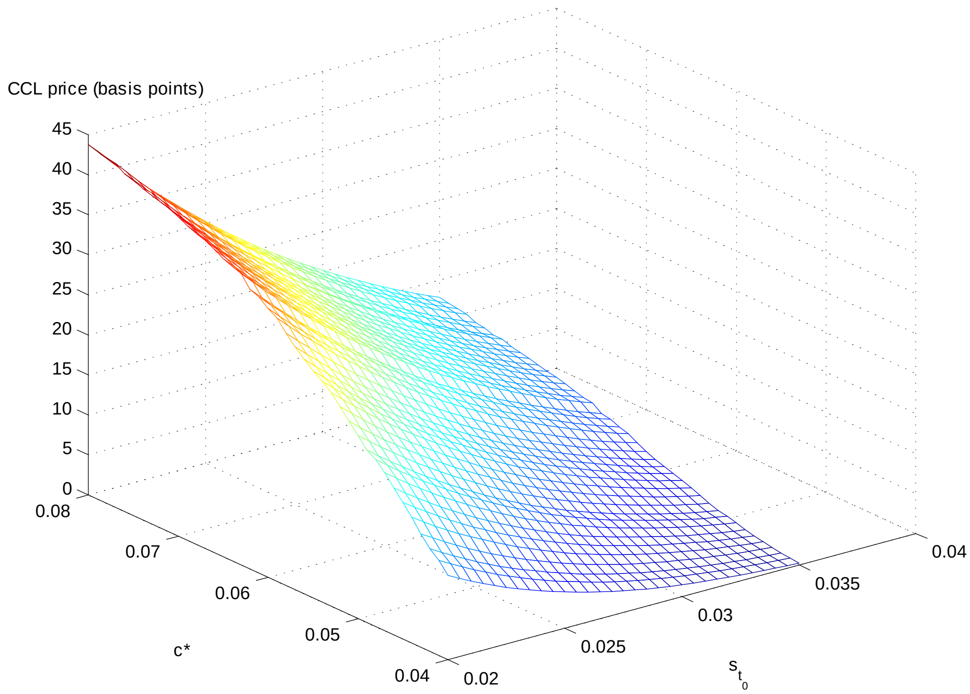


Figure 9. CCL Price: $s_{t_0} = 0.025$, $c^* = 0.055$

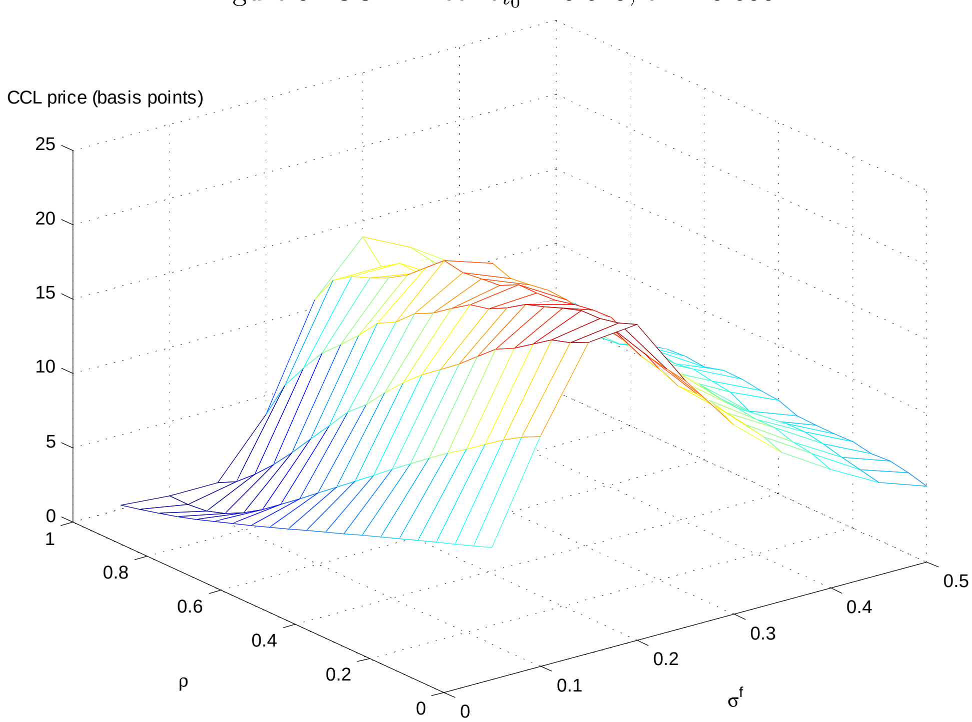


Figure 10. CCL Price: $c^* = 0.055$, $\sigma^f = 0.25$

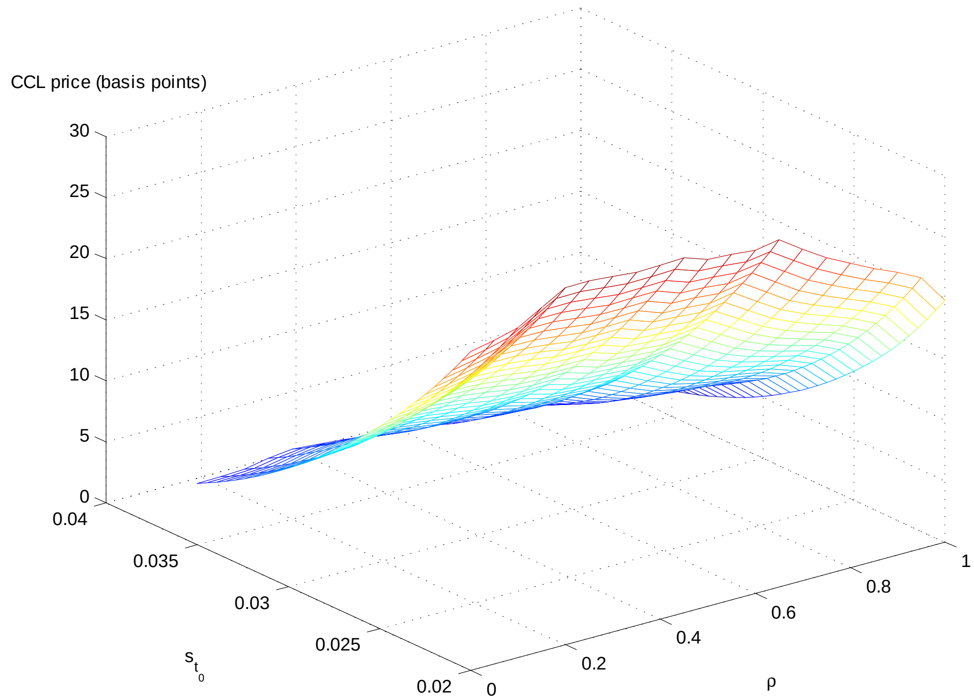


Figure 11. CCL Price: $s_{t_0} = 0.025$, $\sigma^f = 0.25$

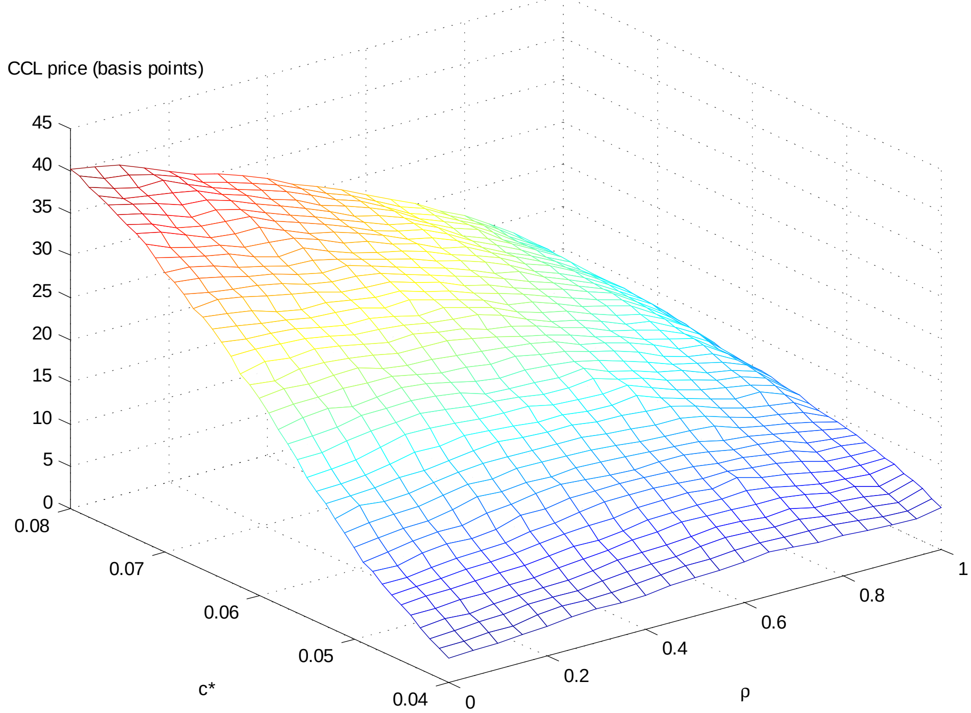
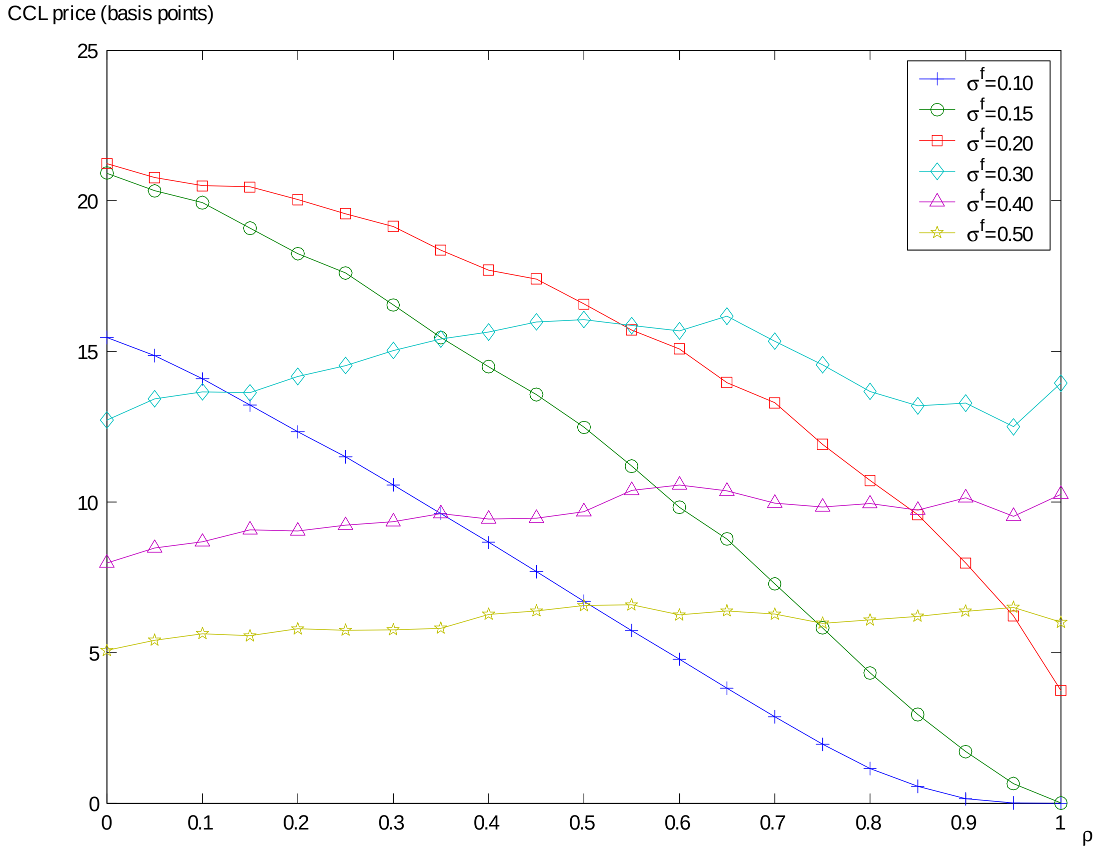


Figure 12. CCL Price: $s_{t_0} = 0.025$, $c^* = 0.055$



CCL price, however, depends on how large c^* is relative to s_{t_0}. When c^* is relatively large compared to s_{t_0} ($c^* = 0.08$, $s_{t_0} = 0.025$), the relationship between ρ and the CCL price is clearly negative—the higher the ρ, the less volatile is c_t and hence the lower is the payoff. When c^* is small compared to s_{t_0} ($c^* = 0.04$, $s_{t_0} = 0.025$), the range of spreads for which the CCL has positive value is small, and this is true irrespective of ρ. In this case for low values of ρ, the knock-out effect is larger; as ρ increases, c_t becomes less volatile, lowering the probability of hitting c^*. Hence, the curve is almost flat at low values of ρ, and has a very small positive slope at high values of ρ.

The above simulations show that the nature of the relationship between the CCL price and σ^f, s_{t_0} and c^* is reasonably clear. However, the relationship between the CCL and the correlation parameter ρ can be quite different depending on the values taken by the other parameters—Figure 12 plots the CCL price against ρ for different values of σ^f. The implication for the CCL issuer is that, depending on the market environment and the CCL characteristics, if the assumption for ρ is off the mark, the CCL could be dramatically mispriced.

VI. Hedging Issues

We have shown that it may be possible to replicate a basic CCL by a cap that gets knocked out if the credit spread exceeds a pre-specified upper-limit c^* during the life of the contract. The question then arises: can the bank issuing the CCL hedge it by following a procedure similar to that used in constructing market hedges for reverse knock-out options?

There are two major difficulties. First, even under reasonably normal conditions reverse knock-out options are quite difficult to hedge and this is especially so close to their time of expiration. A reverse knock-out option becomes void when it is in the money. When the credit spread c_t increases and becomes larger than s_{t_0} the value of the option increases. But as the credit risk keeps increasing and approaches the c^* barrier, the value of the option decreases because it may get knocked-out. Thus, the delta of a reverse knock-out is first positive, and then it becomes negative. In fact, as its expiration time approaches this delta pattern becomes more pronounced, and close to expiration the delta can sharply turn from positive to negative for the option holder. Of course, the opposite is true for the option writer. Given such difficulties in hedging reverse knock-out options, some market makers are often forced to treat reverse knock-out option books the way insurers treat their insurance portfolios. They consider them as unhedgeable and use the principle of diversification to reduce the risk.

Second, a reverse knock out on a credit spread c_t could be even more difficult to hedge because constructing the hedge may require the buying and selling of a complex portfolio of default swaps. This is difficult for vanilla instruments and is likely to be even more difficult for credit spread knock-outs. Default swaps are expensive and the market may not be very liquid. Also, the cap that we use in this paper will require not only default swaps, but preferably, forward markets in these instruments. Forward markets in default swaps, even if they exist, are definitely not liquid yet.

Hence, the bank may have no recourse but to treat a portfolio of CCL's as an unhedgeable portfolio of risks and manage the risk through diversification. This is essentially how banks manage their loan portfolios and CCL's are no different in this sense.

Another issue is that although a CCL can be designed to reduce credit risk, it is also meant to be a temporary solution to a *market* risk faced by borrowers in the CP market. If the CCL contains ratings provisions (where the knock-out depends on a downgrading of the borrower) and other covenants, then it cannot be considered a *pure* credit instrument. Even on the credit risk aspect, ratings may not correctly represent the *true* credit risk associated with the borrower and the bank may have to hedge its exposure with a credit default swap.

As mentioned earlier, a large proportion of bank lending is done through CCLs. Yet, hedging CCLs and altering such loan portfolios is difficult through the *secondary market*. This stems from two factors. First, due to the nature of CCLs and the option to prepay the loans drawn, funding obligations are continual. Second, the secondary market is illiquid due to the fact that CCLs cannot be sold without the *consent of the borrower*. Often the borrowers are large corporations that have a good relationship with the bank and the contractual arrangements may prevent the sale of the drawn portion of the loan in the secondary market.

Given the non-existence of a secondary market, an alternative for hedging CCLs could be provided by the credit default swap (CDS) market. The problem in this market has been the cost of these instruments. CDS rates are normally higher than the price of the CCL, and would make the fashioning of a hedge too costly under most conditions. Also, the purchase of a CDS on a particular corporation by a bank issuing a CCL to the same corporation would be a negative signal on the underlying credit and may hurt the corporation. Hence, this alternative for hedging is currently not much in use.

VII. Concluding Remarks

In this paper we have examined the replication and pricing of a CCL facility to back up commercial paper issuance. It is shown that the CCL can be replicated by a cap written on the credit spread of the company, where the underlying caplets are reverse knock-out options. In general, pricing CCLs is difficult compared to pricing a simple option, because an option is exercised in full, or not exercised at all, but cannot be "partly exercised" as in the case of a credit line. Also, at present, as hedging can be costly, banks treat a portfolio of CCLs as being an unhedgeable portfolio of risks, and manage it through diversification.

The CCL structure considered in the paper can be extended in at least two ways. First, one could add a term-out option. Second, for longer-term CCLs one could model explicitly the pre-payment of loans drawn under a credit line.

References

Black, Fischer, 1976, “The Pricing of Commodity Contracts," Journal of Financial Economics, Vol. 3 (March), pp. 167-79.

Brigo, Damiano and Fabio Mercurio, 2001, *Interest Rate Models: Theory and Practice*, (Berlin: Springer).

Calomiris, Charles W, 1989, “The Motivations for Loan Commitments Backing Commercial Paper,” *Journal of Banking and Finance*, 13, 1989, pp. 271-277.

Ergungor, O. Emre, 2001, "Theories of Bank Loan Commitments," *Economic Review*, (Federal Reserve Bank of Cleveland), Vol. 37, No. 3, pp.2-19.

Fama Eugene F., (1985), "What’s Different about Banks?," *Journal of Monetary Economics*, vol. 15, No. 1, pp.29-39.

Federal Deposit Insurance Corporation, 2005, *Statistics on Banking*, (June).

Federal Reserve Board, 2005, "Survey of Terms of Business Lending," *Federal Reserve Board Statistical Releases*, E.2, (September 15, 2005).

Hull, John C., 2003, *Options, Futures, and Other Derivatives, 5th edition*, (Upper Saddle River, NJ: Prentice Hall).

International Financing Review, 2001-2004, (various issues).

Mosebach, Michael, 1999, “Market Response to Banks Granting Lines of Credit”, *Journal of Banking and Finance*, Vol. 23, pp. 1707-1723.

Musiela, Marek and Marek Rutkowski, 1998, *Martingale Methods in Financial Modelling*, (Berlin: Springer-Verlag).

Rebonato, Riccardo, 2002, *Modern Pricing of Interest Rate Derivatives*, (Princeton, NJ: Princeton University Press).

Schönbucher, Philipp J., A LIBOR Market Model with Default Risk, Working Paper, University of Bonn, 2000.

Schönbucher, Phillip J., A Measure of Survival, Risk, August 2004.

Shockley, Richard L., and Anjan V. Thakor, 1997, “Bank Loan Commitment Contracts: Data, Theory, and Tests”, *Journal of Money, Credit and Banking,* Vol. 29, No. 4, pp. 517-534.

Thakor, Anjan V., Hai Hong and Stuart I. Greenbaum, 1981, "Bank Loan Commitments and Interest Rate Volatility," *Journal of Banking and Finance*, Vol. 5, pp. 497-510.