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## Social Welfare and Cost Recovery in Two-Sided Markets

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## **IMF Working Paper**

Monetary and Financial Systems Department

### **Social Welfare and Cost Recovery in Two-Sided Markets**

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#### **Abstract**

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Using a simple model of two-sided markets, we show that, in the social optimum, platform pricing leads to an inherent cost recovery problem. This result is driven by the positive externality of participation that users on either side of the market exert on the opposite side. The contribution of this positive externality to social welfare leads the social planner to increase users' participation by setting prices at both sides of the market such that the total price is below marginal cost. This causes operational losses for the platform. Our result holds for both interior pricing and skewed pricing in two-sided markets.

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## I. Introduction

Credit card and debit card payment systems, computer operating systems, shopping malls, television networks, and nightclubs represent at first glance an odd shortlist of very different industries. Indeed, these industries serve different types of consumers, use very different technologies, and maintain quite dissimilar business arrangements and standards. Yet, firms in these so-called two-sided industries have adopted similar pricing strategies for solving the common problem they face – getting and keeping two sides of the market on board. These pricing strategies tend to entail heavily “skewed” prices towards one side of the market in the sense that the price markup is much larger on one side than the other.<sup>2</sup>

Two-sided markets involve two distinct types of end users, each of whom obtains value from “transacting” or “interacting” with end users of the opposite type. In these markets, one or several platforms (sometimes also referred to as central switches) enable these transactions by appropriately charging both sides of the market. As Rochet and Tirole (2004) say, platforms “court” each side of the market while attempting to make money overall. In view of our shortlist, credit and debit card payment schemes court cardholders and retailers, computer operating systems court users and application developers, shopping malls court buyers and sellers, television networks court viewers and advertisers, and nightclubs court men and women.

The recognition that many markets are two-sided (or multisided) has recently triggered a surge in the theoretic literature on the economics of two-sided markets. A key aspect of research focuses on price determination in two-sided markets. Under two-sidedness, platforms need not only choose a total price for their services but must also choose an optimal pricing structure, referring to the division of the total price between the two sides of the market. As Evans (2003) states, in two-sided industries the product may not exist at all if the business does not get the pricing structure right. The need for both a pricing level and a pricing structure is one of the defining characteristics that distinguishes two-sided markets from industries ordinarily studied by economists.<sup>3</sup> In an elegant article, Rochet and Tirole (2003) unveil the determinants of price allocation and end-user surplus for different market structures and compare the outcomes with those of a monopolist and a Ramsey planner. In a similar model, Bolt and Tieman (2003) focus on corner solutions and show that widely observed skewed pricing strategies in two-sided markets can be rationalized in terms of profit maximization and social welfare optimization.

This paper examines socially optimal pricing in two-sided markets. For two-sided markets, we show that, in a setting with constant marginal costs and without fixed costs, socially optimal pricing induces losses for the monopoly platform. Hence socially optimal behavior creates an inherent cost recovery problem. This finding is in striking contrast with the standard (one-sided) economic result that charging socially optimal prices results in zero profits. The intuition behind our result is straightforward though.

Our result is driven by the positive externality of participation that users on one side of the market exert on users of the opposite side. The contribution of this positive externality to social welfare leads the social planner to increase users’ participation by setting prices on both sides of the market

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<sup>2</sup>See Evans (2003) for a list of examples of two-sided markets and corresponding skewed pricing strategies; see also Rochet and Tirole (2003) for existing business models in two-sided markets.

<sup>3</sup>Rochet and Tirole (2004) provide a formal definition of two-sidedness and discuss conditions that make a market two-sided.

below marginal cost. The end result is an operational loss for the platform. In our analysis, we distinguish the case of an interior solution and a skewed pricing (corner) solution to the social optimization problem. Yet, both cases yield the same result: an inherent cost recovery problem in social optimum in two-sided markets.

This result has interesting implications for competition policy with respect to two-sided markets. In the field of payment systems, it might for instance be optimal to allow for cross-subsidization. At the very least, it implies that antitrust authorities should take careful account of the two-sided nature of the market when judging whether anticompetitive policies have been used (see, e.g., Evans, 2003; Wright, 2004; Armstrong, 2004; and Bolt and Tieman, 2003).

The remainder of the paper is organized as follows. Section II describes the model. In section III we present our main results and show that socially optimal platform prices yield losses. Two examples that illustrate our findings are analyzed in section IV. Section V concludes.

## II. Model Setup

Our model setup is similar to Schmalensee (2002) and Rochet and Tirole (2003). Potential gains from trade are created by transactions between two end users, whom we will call buyers (subscript  $b$ ) and sellers (subscript  $s$ ). Such transactions are mediated and processed by the monopoly platform. To provide these (network) services, the platform charges buyers and sellers a transaction fee, denoted by  $t_b \geq 0$  and  $t_s \geq 0$ . For simplicity, we abstract from any fixed periodic fees for end users to connect to the platform. In performing its tasks, the platform incurs joint marginal costs, which are set at  $c \geq 0$  per transaction. It is convenient to introduce a distinction between the pricing level, defined as the total price  $t_T = t_b + t_s$  set by the platform, and the pricing structure, referring to the allocation of the total price over  $t_b$  and  $t_s$ .

Buyers and sellers enjoy benefits when transacting on the platform. We assume that buyers are heterogeneous in the benefits  $b_b$ ,  $b_b \in [\underline{b}_b, \bar{b}_b]$ ,  $0 \leq \underline{b}_b \leq \bar{b}_b \leq \infty$ , they receive from a transaction. The probability density function of these benefits is labelled  $h_b(\cdot)$  with cumulative density  $H_b(\cdot)$ . Similarly, sellers differ in the benefits  $b_s$  associated with transacting on the platform,  $b_s \in [\underline{b}_s, \bar{b}_s]$ ,  $0 \leq \underline{b}_s \leq \bar{b}_s \leq \infty$ , with probability density function  $h_s(\cdot)$  and cumulative density  $H_s(\cdot)$ . We assume that platform services potentially generate a social surplus:

$$\bar{b}_b + \bar{b}_s > c. \tag{1}$$

Only buyers with benefits  $b_b$  larger than incurred fees  $t_b$  will transact on the platform. Formally, the fraction of buyers connecting to the platform is given by

$$q_b = D_b(t_b) = \Pr(b_b \geq t_b) = 1 - H_b(t_b). \tag{2}$$

Note that, as  $h_b(\cdot) \geq 0$ ,  $D_b(t_b)$  is a decreasing function of  $t_b$  and as such a regular demand function. Analogously, the fraction of sellers which connects to the platform is equal to

$$q_s = D_s(t_s) = \Pr(b_s \geq t_s) = 1 - H_s(t_s). \tag{3}$$

Assuming independence between  $b_b$  and  $b_s$ , the total expected fraction of transactions processed by the platform amounts to

$$q = D(t_b, t_s) = D_b(t_b)D_s(t_s). \quad (4)$$

Additionally, we assume that the platform operates in a price region such that the price elasticities of (quasi-)demand exceed 1 for both sides of the market. That is, we define

$$\epsilon_i(t) = -\frac{\partial D_i}{\partial t} \frac{t}{D_i} \quad (5)$$

and assume that  $\epsilon_i(t) > 1$ ,  $i = b, s$ , for every feasible fee  $t \geq 0$ .

For simplicity, we exogenously fix the total number of transactions, both on and off the platform, at  $N$ . So, total demand for network services on the platform is given by  $N \cdot D(t_b, t_s)$ . Then, platform profits are given by

$$\pi(t_b, t_s, c) = (t_b + t_s - c)ND(t_b, t_s). \quad (6)$$

Further, in our model, total (expected) social welfare that is generated from platform services is equal to buyer plus seller (expected) benefits minus costs, conditional upon buyers' and sellers' participation in the network. Formally, total social welfare is described by

$$W(t_b, t_s, c) = (\beta_b(t_b) + \beta_s(t_s) - c)ND(t_b, t_s), \quad (7)$$

where  $\beta_b(t_b)$  denotes the (conditional) expected benefit to a buyer, defined by

$$\beta_b(t_b) = \mathbb{E}(b_b | b_b \geq t_b) = \frac{\int_{t_b}^{\bar{b}_b} x h_b(x) dx}{1 - H_b(t_b)}, \quad (8)$$

and, similarly for a seller,

$$\beta_s(t_s) = \mathbb{E}(b_s | b_s \geq t_s) = \frac{\int_{t_s}^{\bar{b}_s} x h_s(x) dx}{1 - H_s(t_s)}. \quad (9)$$

Note that  $\beta_i(\cdot)$ ,  $i = b, s$ , is increasing and bounded between  $\mathbb{E}(b_i)$  and  $\bar{b}_i$ ,  $i = b, s$ .

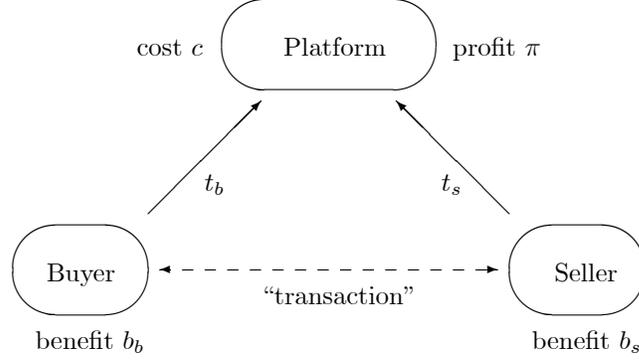
Figure 1 schematically depicts the model.

### III. Main Result

In this section we show that in the model above, with fixed marginal costs and without fixed costs, socially optimal pricing leads to underrecovery of costs for the platform. By definition, the socially optimal prices maximize social welfare. As in any maximization program, the global maximum is characterized by either an interior solution (“interior pricing”) or a corner solution (“skewed pricing”). Which solution occurs is governed by the second order conditions of the maximization program. These, in turn, depend on the choice of the probability density functions  $h_i(\cdot)$ ,  $i = b, s$ , describing the heterogeneity among both groups of end users and determining the curvature of the induced demand functions  $D_i(t_i)$ ,  $i = b, s$ . Yet, we will show that both types of solutions yield qualitatively the same result: negative profits in the social optimum.

Examples of the different types of solutions abound. Clearly, standard log-concave demand functions will result in an interior solution, as the second order conditions for a maximum are automatically satisfied. Hence, the first order conditions are necessary and sufficient for a (unique)

Figure 1: The monopoly platform



social optimum. In contrast, a skewed pricing solution derives from, for instance, demand functions defined by constant-elasticity distributions. As shown in Bolt and Tieman (2003), such demand functions yield a (unique) saddle point solution and hence maximum social welfare is found at one of the corner solutions.

The conditions on the global maximum, and hence the proof of our main result, differ for both cases. Therefore, in the remainder of this section, we discuss both cases separately. While in the first case the solution follows from solving the first order conditions of the problem, in the latter case the corner solutions have to be compared to find the global maximum.

### A. Interior platform pricing

Consider the following social welfare maximization program:

$$\max_{t_b, t_s} (\beta_b(t_b) + \beta_s(t_s) - c)ND(t_b, t_s). \quad (10)$$

Taking the derivative with respect to  $t_i$  and equating to zero gives

$$Nh_i(t_i) \left( (1 - H_j(t_j)) t_i + \int_{t_j}^{\bar{b}_j} x h_j(x) dx - c(1 - H_j(t_j)) \right) = 0, \quad (11)$$

yielding

$$t_i + \beta_j(t_j) = c, \quad i, j = b, s, \quad i \neq j. \quad (12)$$

These two equations determine the socially optimal prices, which we denote by  $(t_b^*, t_s^*)$ . Further, it is easy to see that  $t_j \leq \beta_j(t_j)$ , because

$$t_j = t_j \frac{\int_{t_j}^{\bar{b}_j} h_j(x) dx}{1 - H_j(t_j)} < \frac{\int_{t_j}^{\bar{b}_j} x h_j(x) dx}{1 - H_j(t_j)} = \beta_j(t_j). \quad (13)$$

Combining (12) and (13) it follows that

$$t_T^* = t_b^* + t_s^* < t_b^* + \beta_s(t_s^*) = c. \quad (14)$$

Hence, we find the striking result that the total platform price is lower than joint marginal costs, even in absence of any fixed cost. In other words, under socially optimal interior prices, costs cannot be fully recovered by the platform and operational profits are negative. Thus, we have established the following proposition.

**Proposition III..1.**

*When the social-welfare-maximizing program yields an interior solution:*

*i) The socially optimal interior platform prices are characterized by the fixed point of the two equations*

$$t_b^* + \beta_s(t_s^*) = c, \quad (15)$$

$$t_s^* + \beta_b(t_b^*) = c. \quad (16)$$

*ii) In the social optimum, interior platform prices do not recover full costs and hence induce losses for the platform. That is,*

$$t_T^* < c \quad \Leftrightarrow \quad \pi(t_b^*, t_s^*, c) < 0. \quad (17)$$

**Proof**

The first order conditions of the social welfare program (10) yield (12), which proves the first claim. Combining (12) with (13) shows that total fees are smaller than total costs in (14), directly resulting in negative profits, which proves the second claim.  $\square$

The intuition is straightforward. In a two-sided market, participation on either side of the market exerts a positive externality on users on the opposite side. This social benefit, which is measured by the conditional expected benefit, pops up in the standard “marginal revenue is equal to marginal cost” equation, seen in (12). The contribution of this positive externality to social welfare leads the social planner to increase end users’ participation by setting prices below marginal cost. This follows from the fact that these social benefits for both sides of the market are larger than their own price (equation (13)), which induces less-than-full recovery of marginal costs in (14). Hence, the platform accrues operational losses.

Put differently, as also noted by, e.g., Evans and Schmalensee (2005) and Roson (2005), the positive network externalities operate as economies of scale on demand, similar to the case of a natural monopoly. Akin to a natural monopoly, welfare maximization is associated with negative profits for the firm. In fact, our result thus holds more generally in settings with positive network externalities, be they in two-sided or one-sided networks.<sup>4</sup>

**B. Skewed platform pricing**

Again, let us consider the social welfare maximization program (10). If demand functions are such that the interior solution to this program exhibits a saddle point or induces a local maximum rather

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<sup>4</sup>In contrast to a two-sided network, where two distinct groups of users can clearly be distinguished, a one-sided network is characterized by a single group of users. In a one-sided network the network externality arises from increased network participation among this single group of users. An example would be telephony services, which become more valuable to the individual subscriber the more other potential users have subscribed.

than a global maximum, maximal social welfare is achieved at one of the corner solutions. In a corner solution, participation of one side of the market is complete, while the pricing structure is completely “skewed” towards the other side of the market.<sup>5</sup> Formally, a *corner solution* is characterized by player  $i$  being tied down to its minimum benefit level  $\underline{b}_i$  and player  $j$  being charged an optimal “residual” price  $t_j^{**}$ , where

$$t_j^{**} = \arg \max_{t_j} W(\underline{b}_i, t_j, c), \quad i, j = b, s, \quad i \neq j. \quad (18)$$

First, observe that in the low-price market  $i$ , participation is complete and demand is at its maximum, that is,  $D_i(\underline{b}_i) = 1$ . Second, note that at this minimum price, conditional expected benefits equal the unconditional expectation, that is,  $\beta(\underline{b}_i) = \mathbb{E}(b_i)$ .

Without loss of generality we will assume that the corner solution in which market side  $i$  is kept to its minimum benefit level and market side  $j$  is charged  $t_j^{**}$  yields maximum social welfare,  $i, j = b, s, i \neq j$ . Following this assumption, we substitute  $t_i = \underline{b}_i$ ,  $\beta_i(\underline{b}_i) = \mathbb{E}(b_i)$ , and  $D_i(\underline{b}_i) = 1$ , into social welfare function (10), which results in the the maximization program (18) for the corner solution

$$\max_{t_j} N \left( \mathbb{E}(b_i) D_j(t_j) + \int_{t_j}^{\infty} x h_j(x) dx - c D_j(t_j) \right). \quad (19)$$

Taking the derivative with respect to  $t_j$  and equating to zero gives

$$N h_j(t_j) (-\mathbb{E}(b_i) - t_j + c) = 0, \quad (20)$$

yielding

$$t_j + \mathbb{E}(b_i) = c. \quad (21)$$

Hence, equation (21) and  $t_i = \underline{b}_i$  determine the socially optimal skewed prices, denoted by  $(t_i^{**}, t_j^{**})$ . Since the unconditional expectation  $\mathbb{E}(b_i)$  is larger than the minimum benefit level  $\underline{b}_i$ , it holds that  $t_j^{**} + t_i^{**} < t_j^{**} + \mathbb{E}(b_i) = c$ , and hence marginal costs cannot be fully recovered. That is,

$$t_T^{**} = t_i^{**} + t_j^{**} = \underline{b}_i + c - \mathbb{E}(b_i) < c. \quad (22)$$

We have now arrived at the following proposition.

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<sup>5</sup>The other corner solutions in which both sides of the market either pay their minimum benefit or pay their maximum benefit can be shown to yield lower social welfare than the skewed corner solutions.

**Proposition III.2.**

When the social-welfare-maximizing program yields a corner solution:

i) The socially optimal skewed platform prices are characterized by the fixed point of the two equations

$$t_i^{**} = \underline{b}_i, \tag{23}$$

$$t_j^{**} + \mathbb{E}(b_i) = c. \tag{24}$$

ii) In the social optimum, skewed platform prices do not recover full costs and hence induce losses for the platform. That is,

$$t_T^{**} < c \iff \pi(t_i^{**}, t_j^{**}, c) < 0. \tag{25}$$

**Proof**

Equation (23) follows from the definition of a corner solution. Given (23), equation (24) follows as the first order condition of the social welfare program (19) for the corner solution. This proves the first claim. Combining (23) and (24) with the fact that, by definition,  $\mathbb{E}(b_i) > \underline{b}_i$ , shows that total fees are smaller than total costs (Equation (22)). In the absence of other income or costs, per transaction fees lower than marginal costs directly result in negative profits, proving the second claim.  $\square$

The intuition is the same as before. Again, in the case of a corner solution, the standard “price equals marginal cost equation” is augmented by a term which arises from the positive network externality. To put it differently, equating marginal costs and marginal revenues on side  $j$  of the market requires adjusting the socially optimal fee for side  $j$  to reflect the positive externality of complete participation of market side  $i$ , as measured in this case by its average benefit of platform services.

## IV. Two Examples

Our theoretic results are illustrated by the following two examples. The first example exhibits a uniform distribution, and hence log-concave demand and an interior solution. The second example yields skewed pricing strategies that correspond to a corner solution by applying a constant elasticity of demand distribution which induces demand functions that are not log-concave.

### A. Interior pricing: Uniform distribution

We impose the following uniform probability density function:

$$h_{\underline{b}_i, \bar{b}_i}^U(x) = \frac{1}{\bar{b}_i - \underline{b}_i}, \quad x \in [\underline{b}_i, \bar{b}_i], \quad i = b, s, \tag{26}$$

which yields a log-concave demand function, given by

$$D_i^U(t) = \frac{(\bar{b}_i - t)}{\bar{b}_i - \underline{b}_i}, \quad t \in [\underline{b}_i, \bar{b}_i], \quad i = b, s. \tag{27}$$

Table 1. Results for a uniform distribution: Social welfare

		Interior pricing		
		buyer	seller	total
Price:	$t_i^*$	0.17	0.12	0.29
Conditional expected benefit:	$\beta_i^U(t_i^*)$	0.20	0.15	0.35
Elasticity:	$\epsilon^U(t_i^*)$	3.00	2.20	
Demand (in %):	$D_i^U(t_i^*)$	45.1	62.6	28.2
Profit:				
per transaction	$t_T^* - c$			-0.03
total	$\pi^U(t_b^*, t_s^*, c)$			-0.79
Welfare:	$W^U(t_c^*, t_r^*, c)$			0.79

Parameters:  $c = 0.32$ ,  $\underline{b}_b = 0.10$ ,  $\bar{b}_b = 0.22$ ,  $\underline{b}_s = 0.09$ ,  $\bar{b}_s = 0.18$ ,  $N = 100$ .

As demand is log-concave, solving the first order conditions (15)-(16) yields the interior social optimum. This optimum is attained at prices

$$t_b^* = \frac{1}{3}(\bar{b}_b - 2\underline{b}_s + 2c), \quad t_s^* = \frac{1}{3}(\bar{b}_s - 2\underline{b}_b + 2c). \quad (28)$$

Table 1 numerically illustrates this result for the following parameter values:  $c = 0.32$ ,  $\underline{b}_b = 0.10$ ,  $\bar{b}_b = 0.22$ ,  $\underline{b}_s = 0.09$ ,  $\bar{b}_s = 0.18$ ,  $N = 100$ . With these parameters, it is socially optimal to use the platform for roughly 28 out of a total of 100 transactions. Although the platform loses money on every transaction, the positive externality of participation, as measured by the conditional expected benefit, is valued higher than the corresponding own price, i.e.,  $\beta(t_s^*) = 0.15 > t_s^* = 0.12$  and  $\beta(t_b^*) = 0.20 > t_b^* = 0.17$ .<sup>6</sup> As a result, positive total social welfare is generated,  $W^U = 0.79 > 0$ , but the platform is faced with a cost recovery problem,  $t_T^* = 0.29 < c = 0.32$ , resulting in an operational loss equal to total social welfare,  $\pi^U = -0.79$ .

## B. Skewed pricing: Constant elasticity of demand distribution

We illustrate skewed pricing in a two-sided market by imposing a density function that induces demand functions with constant elasticity. Let us assume the following probability density functions describing the benefits from a transaction:

$$h_{\underline{b}_i, \epsilon_i}(x) = \underline{b}_i^{\epsilon_i} \epsilon_i x^{-\epsilon_i - 1}, \quad x \in [\underline{b}_i, \infty], \quad \underline{b}_i > 0, \quad \epsilon_i > 1, \quad i = b, s. \quad (29)$$

These density functions feature constant elasticities of demand  $\epsilon_i$  and yield demand functions that are not log-concave:

$$D_i^C(t) = \underline{b}_i^{\epsilon_i} t^{-\epsilon_i}, \quad i = b, s. \quad (30)$$

<sup>6</sup>The conditional expected benefit can be derived to be  $\beta_i^U(t) = (\bar{b}_i + t)/2$ ,  $i = b, s$ .

Table 2. Results for constant elasticity of demand: Social welfare

		Skewed pricing		
		buyer	seller	total
Price:	$t_i^{**}$	0.10	0.17	0.27
Conditional expected benefit:	$\beta_i^C(t_i^{**})$	0.15	0.31	0.46
Elasticity:	$\epsilon_i$	3.00	2.20	
Demand (in %):	$D_i^C(t_i^{**})$	100	24.7	24.7
Profit:				
per transaction	$t_T^{**} - c$			-0.05
total	$\pi^C(t_b^{**}, t_s^{**}, c)$			-1.23
Welfare:	$W^C(t_c^{**}, t_r^{**}, c)$			3.50

Parameters:  $c = 0.32$ ,  $\epsilon_b = 3$ ,  $\epsilon_s = 2.2$ ,  $\underline{b}_b = 0.10$ ,  $\underline{b}_s = 0.09$ ,  $N = 100$ .

Thus specified, the social welfare function exhibits an interior saddle point solution. In addition, we assume that the price elasticity of buyers' demand is sufficiently larger than the price elasticity of sellers' demand for global social optimum to be obtained at the corner solution where buyers are charged the lowest admissible fee.<sup>7</sup> Hence, all buyers want to use the platform services and the entire buyers' market is captured. In contrast, sellers, whose demand is less price-elastic, are confronted with higher socially optimal prices.

So, for sufficiently large  $\epsilon_b > \epsilon_s$ , solving the system of equations (23)-(24) yields

$$t_b^{**} = \underline{b}_b \text{ and } t_s^{**} = c - \frac{\underline{b}_b \epsilon_b}{\epsilon_b - 1}. \quad (31)$$

Table 2 summarizes these results for the following parameter values:  $c = 0.32$ ,  $\epsilon_b = 3$ ,  $\epsilon_s = 2.2$ ,  $\underline{b}_b = 0.10$ ,  $\underline{b}_s = 0.09$ ,  $N = 100$ . Under these parameters about 25 out of 100 transactions will be processed through the network, which is roughly the same as in the uniform case. While the price elasticities are the same in both examples, the pricing structure under constant elasticity of demand is however very different. Buyers are charged their minimum benefit such that there is complete participation on this side of the market. Sellers pay a higher price, but still too little to make the platform network profitable, i.e.,  $t_T^{**} = t_b^{**} + t_s^{**} = 0.10 + 0.17 = 0.27 < c = 0.32$ . Although the platform thus accrues operational losses  $\pi^C = -1.23$ , still, positive welfare is generated,  $W^C = 3.50 > 0$ . As before, this is due to the positive externality of complete buyers' participation, which generates social welfare in excess of the price buyers pay, i.e.,  $\mathbb{E}(b_b) = 0.15 > t_b^{**} = 0.10$ .<sup>8</sup>

<sup>7</sup>See Bolt and Tieman (2003) for a more in-depth analysis of skewed pricing with constant elasticity demand.

<sup>8</sup>The unconditional expectation is given by  $\mathbb{E}(b_i) = \beta_i^C(\underline{b}_i)$ , where  $\beta_i^C(t) = t\epsilon_i/(\epsilon_i - 1)$ .

## V. Conclusions

We have shown that socially optimal pricing in two-sided markets leads to an inherent cost recovery problem, inducing losses for the monopoly platform. The result is driven by the positive externality on users on one side of the market, which originates from network participation on the other side of the market. The contribution of this externality to social welfare is larger than the individual market side's price, which leads pricing below marginal cost to be socially optimal. The positive network externalities hence operate like economies of scale on demand, analogous to the case of a natural monopoly. This indicates that our result thus holds more generally in any setting with positive network externalities, be it in two-sided or one-sided networks.

To solve the cost recovery problem thus created, alternative pricing schemes might be considered. First, since the platform network generates positive social welfare, compensation through subsidies from the social planner or cross-subsidization from other sources of income could be warranted. A classic example would be credit or debit card payment operations which could be cross-subsidized by banks from other sources of income. However, such cross-subsidization is often unacceptable to antitrust authorities. Another way of overcoming the cost recovery problem would be to charge users a fixed periodic membership fee, as is seen often in reality, or to use more complex, two-part tariffs. Alternatively, the social planner might instruct the platform to implement Ramsey prices, that is, prices which optimize social welfare under a balanced budget constraint. However, these types of solutions have second-best distortionary side effects, which should be taken into account.

Interestingly, in payment systems there is often a formal requirement to fully recover costs. Such a requirement will for instance be imposed on the new European large-value payment system Target II. Our analysis indicates that a delicate balance should be struck between cost recovery on the one hand and generating prices close to the social optimum on the other. Too much attention to cost recovery reduces social welfare.

Moreover, payment systems have recently attracted antitrust scrutiny after merchants' complaints about the skewed pricing structure (see e.g. Evans, 2003; Wright, 2004; and Armstrong, 2004). Merchants felt put at a disadvantage by the high prices on their side of the market and claimed this pricing structure was the result of an abuse of the platform's monopoly power. Our analysis, however, shows that a skewed pricing structure might be socially beneficial rather than harmful.

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