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Applied General Equilibrium Tax Modeling

by

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1. Introduction

The general equilibrium model of an economy is the product of nearly two centuries of conceptual innovation and continued intellectual refinement. Its roots may be found in Adam Smith's description of the behavior of capitalists motivated by considerations of profitability in the selection of economic activities. The elements of demand theory appear in John Stuart Mill's treatment of international trade and in his analysis of the response of economic agents to changes in taxes and import duties. The model reaches its mature form later in the nineteenth century in the work of Leon Walras, who provided a general description of the functioning of a complex economic system based on the interaction of a number of interdependent economic units. Today, the general equilibrium model is the centerpiece of microeconomic theory.

The fundamental themes of the general equilibrium model are extremely simple and lie at the heart of economic theory. The production side of the economy, engaged in the transformation of certain commodities into other commodities, is distinguished from the consumption side, whose goals are the acquisition and eventual consumption of goods and services. Stocks of commodities, which may be consumed directly or offered as factors of production, are owned by households in their physical form or by means of a variety of financial instruments. Each consumer's income, or wealth, is determined by evaluating his stock of commodities in terms of those prices at which the commodities can be sold. Income and a knowledge of relative prices permit the consumer to express his demands for goods and services and his offerings of labor and other stocks that are made available for the productive side of the economy.

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In the general equilibrium model, producers are assumed to be informed of the prices of all inputs and the prices at which outputs can be sold. These prices are assumed to be independent of the scale and composition of productive activity; each producer then selects from among the choices that are technologically available to him the production plan which maximizes profits. Production technology is assumed to exhibit constant or decreasing returns to scale.

The decisions of the production and consumption sides of the economy need not be consistent with each other if they are based on an arbitrary list of prices. If the price of a desired commodity is too low, consumers may be motivated to demand large quantities of this commodity, and producers may be averse to supplying a commodity whose sales generate insufficient revenue to cover the costs of manufacture. Equilibrium prices are those which equate demand and supply in all markets. Once they are known, in the context of a particular model of an economy, the entire range of economic decisions based on them is determined.

Despite the appeal of this general model of all markets simultaneously clearing, progress in this field of economic research has proved to be slow. A proof of the existence of a market-clearing set of prices remained unsolved for more than half a century after Walras had formalized the model. It was not until the work of Arrow, Debreu, Gale, Kuhn, McKenzie, and Nikaido in the 1950s that such a proof was formulated by using fixed-point methods from mathematical topology. The next hurdle, which also proved formidable, was to compute the equilibrium set of prices. The proofs of existence seemed to offer little guidance since they were fundamentally nonconstructive. However, in 1967 both Harold Kuhn and Herb Scarf developed ingenious computer-based algorithms (based on almost identical logic) for the numerical determination of the equilibrium set of prices for Walrasian models. It could be shown that the methods always found an equilibrium if a fixed-point proof of existence was available. In fact, since the algorithms could be used to constructively establish the fundamental fixed-point theorems on which the original proofs of existence were based, they were just as general as the analytic proofs of existence.

The ability to compute equilibria of relatively complicated general equilibrium models opened the door to what may be referred to as applied general equilibrium modeling. It seemed natural to be able to add to these models some features of the real world, such as governments, taxes, tariffs, and transfer payments, specifying them so that they resembled actual economies and doing policy evaluation with them. Before the development of the algorithms, general equilibrium analysis was limited to the two by two analytic or graphic models associated with Johnson, Meade, and Harberger. Now, larger and more realistic models were feasible. 1/ The purpose of this paper is to describe the models and techniques that have been developed for using applied general equilibrium analysis for the analysis of tax policy. The paper also mentions some of the results to date, the shortcomings of current models, and the expected directions of further developments.

2. Fundamental structure of applied general equilibrium models

General equilibrium models have four essential ingredients. There must be a specification of (1) the endowments of consumers, (2) their preferences, (3) the production technology, and (4) the conditions of equilibrium.

In general, consumers may possess endowments of any or all of the commodities in the economy. Often, in practice, consumers are endowed only with factors of production (capital and labor). The preferences of consumers are specified with the demand function for each commodity. Commodity demands are nonnegative and depend on all prices in a continuous manner. They are homogeneous at degree zero in prices, meaning that only relative prices matter. Market demands are the sum of individual household demands, and they satisfy Walras's law. If some notation is introduced, the consumer side of the model can be specified. Let N be the number of commodities (including factors), Wi be the total endowments of commodity i, and $Di\ (\vec{P})$, $i = 1, \ldots, N$ be the market demand functions. With this notation, Walras's law now states

$$\sum_{i=1}^{N} Pi\ Di\ (\vec{P}) = \sum_{i=1}^{N} Pi\ Wi$$

The value of market demands must equal the value of market endowments at all prices. This condition automatically hold if market demands are simply the sum of individual demands when the individuals are subject to their budget constraints.

On the production side of a general equilibrium model, technology is usually described by a set of constant returns to scale activities or by production functions that exhibit nonincreasing returns to scale.

1/ It is perhaps ironic that, once the applied general equilibrium models were developed it was found that they could usually be solved with Newton-type methods that have long been available. Despite this, the expanding interest in computational general equilibrium models is clearly due to the work of Kuhn and Scarf. Improved versions of their algorithms are now competitive with Newton methods in terms of computational speed, even in cases where the Newton algorithms converge.

The advantage of the activity analysis approach is that the conditions for equilibrium are very simple when production is modeled in this way. On the other hand, production functions are more convenient to use in applied work. They are easily given parameters since most of the relevant econometric literature involves their estimation.

With the activity analysis approach, the J activities available to the economy can be listed in an (NxJ) matrix A, where the aij elements are negative for inputs and positive for outputs. The first N columns

$$A = \begin{bmatrix} -1 & \cdots & 0 & a_{1,N+1} & \cdots & a_{1,j} \\ 0 & \cdots & 0 & a_{2,N+1} & \cdots & a_{2,j} \\ \vdots & & & & & \\ 0 & & -1 & a_{N,N+1} & & a_{N,J} \end{bmatrix}$$

of this matrix are disposal activities. Joint products are possible; however, activities are restricted to satisfy the boundedness condition that $A\vec{x} + \vec{W}$ is bounded that at any nonnegative set of J activity levels $\vec{X}$. The interpretation of this condition is that the production possibility is finite in all dimensions. 1/

In the activity analysis modeling of production, equilibrium is characterized by a nonnegative vector of N prices and J activity levels (P*, X*) so that

(1) demand equals supplies for all commodities

$$D_i\ (P^*) = \sum_{j=i}^{J} a_{ij}\ X_j^* + W_i \text{ for } i = 1, \ldots, N$$

1/ Continuous constant returns to scale production functions are similar to an infinite listing of activities to produce each output. For any set of input prices, one can compute the cost-minimizing method of producing a unit output for each output. This is the technique or activity that will be used at those prices (if output prices are sufficient for production to take place).

and

(2) activities in use break even, with those not used having negative economic profits.

$$\sum_{i=1}^{N} P_i^* a_{ij} \leq 0 \; (= \text{ if } X_j^* > 0) \text{ for } j = 1, \ldots, J$$

A simplified numerical example may illustrate the general equilibrium structure. For expositional purposes, let us consider a model with two final goods (manufacturing and nonmanufacturing), two factors of production (capital and labor), and two classes of consumers. Consumers have initial endowments of factors but no initial endowments of goods. The "rich" consumer group owns capital, while the "poor" group owns labor. Production of each good takes place according to a constant elasticity of substitution (CES) production function, and each consumer class has demands derived from maximizing a CES utility function subject to its budget constraint.

The production functions are given by

$$(2.6) \quad Q_i = \phi_i \left[(\delta_i L_i^{\frac{\sigma_i - 1}{\sigma_i}} + (1-\delta_i) K_i^{\frac{\sigma_i - 1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i - 1}} \qquad i = 1,2,$$

where Q_i denotes output of the ith industry, ϕ_i is the scale or units parameter, δ_i is the distribution parameter, K_i and L_i are the factor inputs, and σ_i is the elasticity of factor substitution.

The CES utility functions are given by

$$U^q = \left[\sum_{i=1}^{2} \alpha_i^q X_i^{q \, \frac{\sigma_q - 1}{\sigma_q}} \right]^{\frac{\sigma_q}{\sigma_q - 1}} \qquad q = 1, 2$$

where X_i^q is the quantity of good i demanded by the qth consumer, α_i^q are share parameters, and σ_q is the substitution elasticity in consumer class q's CES utility function.

If consumers maximize these utility functions subject to the constraint that expenditures do not exceed income derived from the sale of endowments, the resulting demand functions are

$$Xi^q = \frac{\alpha_i^q \; I^q}{P_i^{\sigma q} \left(\alpha_1^q P_1^{1-\sigma_q} + \alpha_2^q P_2^{1-\sigma_q}\right)} \qquad \begin{matrix} i = 1,2 \\ q = 1,2 \end{matrix}$$

where Iq is individual q's income level.

With this structure, a "toy" model can be specified, with the following values of the parameters.

Production

	ϕ	δ	σ
Manufacturing (1)	1.5	0.6	2.0
Nonmanufacturing (2)	2.0	0.7	0.5

Consumption

	Endowments		Preference Parameters		
	K	L	α_1	α_2	σ
Rich households	25	0	0.5	0.5	1.5
Poor households	0	60	0.3	0.7	0.75

This model has been solved using Merrill's algorithm, which is an advanced variant of Scarf's method. The results are shown in Table 1. At the prices computed, total demand for each output exactly matches the amount produced. It follows that producer revenues equal consumer expenditures. It also is true, to a high degree of approximation, that the labor and capital endowments are fully employed and that consumer factor incomes equal producer factor costs. The cost per unit output in each sector matches the price, which means that economic profits are zero. The expenditure of each household exhausts its income. Thus, the solution closely approximates all of the properties of an equilibrium for this economy. The closeness of the approximation can be enhanced by increasing the amount of computation time allowed for the algorithm used in the solution.

Table 1. Equilibrium Solution: General Equilibrium for Illustrative Simple Model

Equilibrium prices

Manufacturing output	1.399
Nonmanufacturing output	1.093
Capital	1.373
Labor	1.000

Production

	Quantity	Revenue	Capital	Capital Cost
Manufacturing	24.992	34.894	6.212	8.529
Nonmanufacturing	54.379	59.436	18.789	25.797
Total		94.330	25.001	34.326

	Labor	Labor Cost	Total Cost	Cost Per Unit Output
Manufacturing	26.364	26.364	34.893	1.399
Nonmanufacturing	33.634	33.634	59.431	1.093
Total	59.998	59.998	94.324	

Demands

	Manufacturing	Nonmanufacturing	Expenditure
Rich households	11.514	16.674	34.333
Poor households	13.428	37.705	59.997
Total	24.942	54.379	94.330

	Labor Income	Capital Income	Total Income
Rich households	0	34.325	34.325
Poor households	60	0	60.000
Total	60	34.325	94.325

This illustrative example shows the kind of models that can be solved with the relatively new computer-based algorithms. However, it does not indicate how data are collected and incorporated and how taxes and other policy variables are introduced. Also, it is necessary to develop welfare economics techniques to compare the equilibria that result from alternative policies. Let us now turn to these issues.

3. Specification of policy models

a. The inclusion of taxes

The first modification that is desirable in the simple model outlined above is the inclusion of a system of taxes and government expenditures. Taxes may be imposed on the purchase of goods and services by consumers, the use of factors and intermediate inputs by producers, the receipt of income by consumers, and the final output of the various production sectors. The tax rates may differ for each good, consumer, and producer. The government uses the tax proceeds to finance transfer payments to consumers and to purchase final goods and services. Most of the models developed to date assume a balanced government budget, but recent work by Feltenstein (1983) incorporates a bond and money market into models of this type.

The method of including taxes and governments into the general equilibrium framework was first shown in Shoven and Whalley (1973) and Shoven (1974). Conditions for equilibrium become demand equals supply for each commodity, firms in operation break even after taxes, and government receipts (including bond sales and money issuance in Feltenstein's formulation) equal government expenditure. Walras's law now states that the gross-of-purchase-tax value of demands equals the value of endowments less personal taxes plus transfer payments. It continues to be the sum of the individual household after-tax budget constraints.

b. Equal yield tax comparisons

Often, in consideration of replacement of one system of taxes with an alternative system, the relevant policy constraint is that the replacement set of taxes should generate the same real government revenue as the original set. When economic behavior is itself a function of tax rates, the rates required for matching the yields cannot be easily determined. In fact, a full general equilibrium analysis is required to determine such rates correctly. The computational algorithms used can easily be extended to calculate not only an equilibrium for a new tax system but also a scaler that determines the level of tax rates. The user has some choice as to whether this scaler is additive or multiplicative to the rates in the tax system under examination and whether the equal yield rate adjustments (determined by the scaler) applies to all agents and taxed activities or just to a subset of them. This technique is described in Shoven and Whalley (1977).

c. Production

The specification of production is somewhat more complex in the current computational general equilibrium models than in the illustrative model above. The key difference is that intermediate inputs are incorporated, often with a fixed coefficient technology. Substitution occurs only between primary factors in the production of value added, and then according to a CES or Cobb-Douglas function. The models in use vary in their level of disaggregation, with the number of production sectors varying between 4 and 33. In the case of fixed coefficients for intermediate inputs, the production function for each sector can be written as

$$Q = \text{Min}\left[\frac{1}{a_o}\ VA(K,L),\ \frac{X_N}{a_1} \cdots,\ \frac{X_N}{a_N}\right]$$

Most empirical models distinguish between industrial outputs and consumer goods for the simple reason that the data are classified differently. Industrial sectors involve such categories as forestry and fisheries, metal mining, and publishing and printing, while consumers purchase furniture, automobiles, and books. This fact is recognized in the models by incorporating a second stage of production, which converts industrial outputs into consumer goods. This technology is usually modeled as a fixed coefficient.

With some exceptions (for example, Fullerton (1982) and Derviş, de Melo, and Robinson (1982)), capital is modeled as fully mobile between production sectors and thus earns the same after-tax rate of return from each sector. Fullerton's model allows full mobility to new investment, which earns the same rate of return in all sectors engaged in new investment, but it fixes the industrial locale of capital, once it has been acquired. Derviş, de Melo, and Robinson have a similar "putty-clay" model of capital, although the allocation of investment may be set by arbitrary policy rules rather than by competitive rent seeking.

d. Consumption and saving

Computational or applied general equilibrium models were initially almost always static in nature, possibly including rather artificial saving and investment behavior. In recent years several of the models have been made dynamic, although this remains an area of active model development. The U.S. model with which the author is associated (along with his co-investigators John Whalley, Don Fullerton, Charles Ballard, and Larry Goulder) now computes a sequence of essentially static equilibria connected by saving and capital formation.

The 12 consumer classes in the U.S. model act as if they were maximizing the nested utility function

$$U = U[H(\prod_{i=1}^{N} X_i^{\lambda i}, \ell), C_F]$$

or some monotonic transformation of it, subject to their income constraint. The Xs are consumer goods (15 in number in the U.S. model), ℓ is leisure, and C_F is a composite commodity of future consumption.

Both H and U are CES functions. The parameters of those functions determine the shares of income devoted to each commodity, to saving (the provision for C_F), and to the "purchase" of leisure. They also

determine two key elasticities in the models--the elasticity of labor supply with respect to the real after-tax wage rate and the elasticity of saving with the real after-tax rate of return to capital.

In the U.S. model, consumers have myopic expectations regarding future prices and, in particular, regarding the future rate of return to capital. Future consumption is "acquired" by buying a fixed composition portfolio of real investments that offer an infinite annuity of returns. There has been some work on incorporating both perfect foresight and limited foresight into this model (Ballard and Goulder (1982)). Work is also being done to incorporate life-cycle behavior where a utility function such as

$$U = \int_0^T H(X, \ell)\, e^{-\delta t}\, dt$$

is maximized, subject to a lifetime wealth constraint.

e. Foreign trade

Applied general equilibrium modeling is used in the evaluation of customs unions, tariffs, and trade restrictions, and several models focusing on those issues have been developed (see, for example, Miller and Spencer (1977), Feltenstein (1980) and (1982), and Whalley (1982b)). Here, let us concentrate on the foreign trade specification of models basically designed for evaluating domestic tax policies.

International trade is usually modeled extremely simply. In the U.S. model, the standard specification is one which has a constant elasticity of export demand and has import supply equations. Trade balance is imposed, and there is no international mobility of capital. A richer specification of the foreign sector, including international capital markets, was investigated by Goulder, Shoven, and Whalley (1982). The impact of domestic tax policies was shown to be quite sensitive to international capital mobility and to the credit granted in the United States for foreign taxes paid. Other tax models (Keller (1980), Ballentine and Thirsk (1979)) include capital flows, while some (Slemrod (1981) and Auerbach, Kotlikoff, and Skinner (1981)) have no foreign sector at all.

f. Financial sectors

Current general equilibrium tax models clearly owe a great deal to the pathfinding work of Arnold Harberger (1959, 1962, 1966). He introduced the two-sector general equilibrium framework to public finance and was one of the first to investigate the issue of tax incidence as it is known today. In many ways, the proper approach to thinking of the current models is as super-Harberger models.

One severe shortcoming of these models is the total absence of financial markets. They are "real" models solving for relative prices, but there are no debt instruments, money, financial intermediation, or deficits. Integrating financial and real markets in these models is perhaps the current area of greatest research activity. Feltenstein (1983) has added to this general model money and government bonds as well as foreign exchange markets. Slemrod (1981) has attempted to incorporate modern portfolio behavior on the part of consumers, while Fullerton and Gordon (1981) have begun to deal with issues of corporate financial policy and behavior toward risk.

Given that countries experience large government deficits, current account trade imbalances, and sizable accumulated foreign debts, the inclusion of these features in policy models is clearly important. The general issue of the "crowding out" of private sector investment through government borrowing can also be addressed in this framework.

g. Data requirements and parameter specification

In applying general equilibrium models, a complete equilibrium data set must be assembled. This includes factor usage by industry, an input-output table, consumer expenditures by commodity and incomes by source, government expenditures and tax collections, and information on foreign trade and investment. The normal practice is to gather this data from available sources for a particular year. In general, such data are inconsistent. For example, total labor payments by employers do not match total labor income. To be useful, the data must be adjusted for consistency. This requires some judgment as to which data are most reliable and which should be changed so as to be consistent.

The consistent data represents what is often referred to as the "benchmark" equilibrium. The strong assumption is made that the data represent an equilibrium of the economy. The construction of data sets of this type is described in St. Hilaire and Whalley (1980), Piggott and Whalley (forthcoming), and Ballard, Fullerton, Shoven and Whalley (forthcoming). Since the benchmark data are usually presented in value terms, units must be chosen for goods and factors in order to obtain separate price and quantity observations. A commonly used type of unit conversion, originally adopted by Harberger, is to choose units for both goods and factors that have a price of unity in the benchmark equilibrium.

With the benchmark observation at hand, parameters are then chosen so that the solution to the model explicitly replicate the benchmark data. This procedure is termed "model calibration." Values of the parameters thus generated can then be used to solve for a different equilibrium under alternative policy regimes. This is usually termed a "counterfactual" or "policy replacement" equilibrium.

The typical calibration procedure involves only one year's data or one single observation, which may be an average over a number of years. Depending on the complexity of functional forms used, the data may not uniquely identify the parameters. With Cobb-Douglas functions, a single benchmark observation serves to uniquely identify the values of the parameters, since expenditure and factor shares by sector are known. With other functions, it is typically the case that an infinite number of combinations of parameters can replicate the data in the required manner. In such cases, extraneously specified elasticities usually serve as identifying restrictions. Once specified, these allow the other parameters to be determined uniquely from the equilibrium observation.

The extraneous specification of elasticities can be thought of as determining the curvature and position of isoquants and indifferent surfaces. If Cobb-Douglas preference functions are chosen, a single observation of a point and slope at that point of an indifference curve is sufficient to uniquely determine the parameters of the function. If CES functions are used, extraneous values of substitution elasticities are required, since the curvature of indifference curves, described by the single elasticity parameter, is not given by benchmark data. In the case of linear expenditure system demand functions, income elasticities are determined, once the origin coordinates for utility measurement are known. The current procedure in setting the additional parameters is to scan empirical literature to select appropriate values of substitution elasticities for the underlying utility and production functions. The primary role of calibration is thus to determine the shares and unit parameters in these functions, once elasticities are known. No statistical test of the chosen model specification is involved, since a deterministic procedure is employed for calculating the values of the parameters from the equilibrium observation. This entire procedure is clearly dependent on both the accuracy of the assembled data and the assumption that it represents an equilibrium. Also, the key role played by elasticities used in these models becomes immediately apparent.

Once the calibration procedure is completed, a full model is available and can be used for policy analysis. As indicated in Figure 1, any policy change can be specified and a counterfactual equilibrium for a new policy regime can be computed. Policy appraisal then proceeds on the basis of pair-wise comparisons of counterfactual and benchmark equilibria. If further policy changes are to be evaluated, the specification of policy changes is repeated.

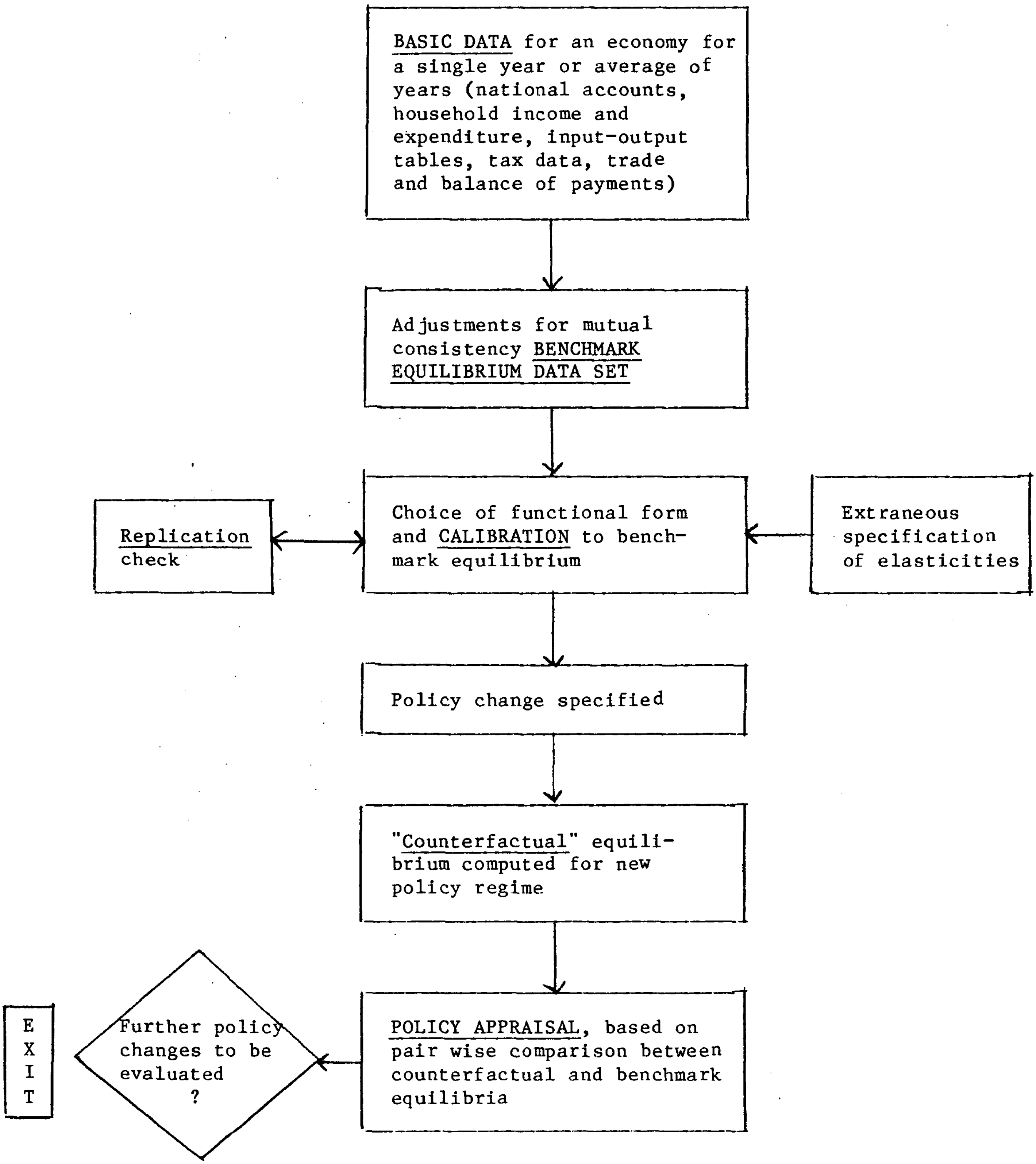
There are a number of reasons why this calibration approach is so widely used, rather than a more direct econometric approach, in setting the parameters for applied models. First, in some of the models, many thousands of parameters are involved; to estimate all of these parameters of the model simultaneously by using time series methods would require an unrealistically large number of observations. Second, the way in which benchmark data sets are used to generate the values of the parameters under calibration involves taking an observation in value terms and then decomposing it into separate price and quantity observations. Benchmark equilibrium prices, by construction, represent unity in each benchmark equilibrium. This makes it difficult to sequence equilibrium observations with consistent units through time, as would be required for time series estimation. These problems, combined with the difficulty of incorporating equilibrium restrictions into a satisfactory estimation procedure, have thus far largely excluded complete econometric estimation of general equilibrium systems, although some progress in this direction has been made in recent work by Mansur (1981). Mansur, for instance, notes the difficulties in simply writing down a likelihood function for a maximum likelihood procedure incorporating full-equilibrium restrictions. He suggests a partitioning approach, using segmented production and demand systems, with a third segment incorporating their equilibrium interdependence. Other attempts at econometric estimation of complete equilibrium systems occur in the work of Allingham (1973) and Jorgenson (1983).

h. Welfare evaluations

A counterfactual equilibrium is computed and is compared with the observed economy (which is assumed to represent an equilibrium). In the case of a dynamic model, a dynamic path of prices and endowments is computed (the capital endowments being endogenous). This is compared

Figure 1

Flow Chart for a Typical Applied General Equilibrium Model



with the path of the economy when there is no policy change. For the U.S. model, the investigators have assumed that the base year's data (1973) represent not only a static equilibrium, but one which lies on a steady-state growth path. Thus, without any policy change, relative prices remain constant and the economy simply gets larger in a completely balanced manner. When an unanticipated policy change is announced, the economy goes through a transition period but eventually resettles into a new steady-state growth path. The model thus computes both the transition path and the long-run comparative steady states.

Without a social welfare function, it is impossible to unambiguously state that one equilibrium or a path of equilibria is better than an alternative, unless the improvement follows Pareto's law--that is, everyone is better off. This is, unfortunately, rarely the case. What the investigators do in this model in measuring the change in economic efficiency or the welfare of a policy change is analogous to the measurement of costs and benefits in cost-benefit analysis--that is, apply the Kaldor criterion: a situation is superior if the winners could compensate the losers, even if this compensation does not take place. The criterion has well-known theoretical shortcomings (Skitovsky showed, for instance, that it need not be transitive), but it is widely used for policy evaluation. In the U.S. model, the investigators calculate the dynamic or static compensating variation for each household and sum these for an overall welfare measure. The government's expenditures do not enter into this calculation--an omission that is less serious owing to the equal revenue-equal expenditure constraint described above; that is, the government has the same real resources available to it under both the old and new policy regimes.

It should be noted that the general equilibrium approach offers a very complete description of the economy for alternative policy scenarios. Substantial information is lost in the endeavor to compare the equilibria with a single number. The changes in welfare for each consumer can be computed, and the changes in factor usage, expenditure patterns, and industrial output levels can be examined. It should also be said that, for large policy changes, such as the institution of a new tax regime or the construction of a project such as the Aswan Dam, only a general equilibrium analysis can capture the interactive effects.

4. Applications

In this section, let us review some of the applications that have been completed using the U.S. model. It should be stated at the outset that there are a large number of other models that have been used for policy evaluation in other countries. These include Miller and Spencer's (1977) assessment of the United Kingdom's entry into the European Community, Whalley's (1975) evaluation of the major 1973 U.K. tax reform package, Feltenstein's (1980) analysis of trade restrictions in Argentina, Serra-Puché's (1981) policy model for Mexico, Whalley's (1982b) examination of the effects of the Tokyo Round trade agreement, and John Piggott's (1979) evaluation of Australian tax policy. Similar models are being used for development policy (see Derviş, de Melo, and Robinson (1982)), energy economics (Hudson and Jorgenson (1978) and Borges and Goulder (1982)), and even economic history (James (1981)).

The U.S. model consists of a production sector of 19 products, 16 consumer goods, and 12 consumer groups. It is a dynamic model incorporating the complete tax system (federal, state, and local personal income taxes, corporate taxes, sales and excise taxes, social security taxes, etc.). The benchmark data set represents the 1973 economy. The model's development was financed by the U.S. Treasury Department, and it is currently in use there, most recently in evaluating flat-tax proposals.

The policy that has received the most attention in the United States from general equilibrium modelers is the integration of the U.S. corporate and personal income tax probably because Harberger originally examined the incidence and efficiency consequences of the corporate income tax with his two-sector model. Corporate equity capital is taxed twice in the United States in that the earnings of firms are subject to the 46 per cent corporate income tax. After-tax earnings are either distributed as dividends and taxed at the personal level or retained. If retained, the earnings may lead to partially taxable capital gains. Capital income from other sectors, particularly real estate and to some extent agriculture, is lightly taxed. The result is an inefficient allocation of capital across sectors and, quite possibly, a distortion of the consumption/saving decision.

Another policy that has been evaluated with the U.S. general equilibrium model is the possibility of taxing consumption rather than income at the personal level. This could be accomplished by first establishing the household's income and then allowing a deduction for all saving. As the tax would be direct, it could have special tax exemptions for the blind, the elderly, those with large families, etc., and could have increasing marginal rates. The advocates of a consumption tax argue that it does not distort the consumption/saving decision, as does an income tax, and that it is better to base taxes on a household's withdrawals from the social product (consumption) than on a rough approximation of their contribution to it (income).

Before evaluating the consumption tax, it is important to recognize that the United States already has a partial consumption tax since roughly half of saving is not subject to tax. Thirty per cent is saved through retirement plans and life insurance, where the tax is deferred until withdrawal (as with a consumption tax). Another 20 per cent of saving is in the form of new housing construction. Housing must be purchased with after-tax dollars (i.e., the saving/investment is not deductible), but the return on it, imputed or otherwise, is very lightly taxed. Thus, it is not taxed twice, as with an income tax; its treatment is more nearly analogous to a consumption tax.

Table 2 presents the dynamic efficiency gains for a consumption tax and corporate tax integration. The figures are in 1973 dollars. The key parameters of the model are set at 0.4 for the saving elasticity and 0.15 for the labor supply elasticity. The elasticity for factor substitution in value added varies by industry, but is generally slightly less than unity. The gain in efficiency depends on how the lost revenue is compensated for. For example, if a consumption tax is instituted by making 80 per cent of saving deductible (over and above the 20 per cent currently saved through new housing acquisition), the first row of Table 2 shows that the gain would be $686 billion if the revenue shortfall was made up with lump-sum tax increases. However, if marginal tax rates are increased in a multiplicate manner (everyone's ratio is multiplied by a common $X > 1.0$), the gain is $621 billion, while if they increased in an additive manner ($t' = t + X$), the welfare measure increases by $636 billion. These numbers are about 1.25 per cent of the present value of future national income, expanded to include the value of leisure. The discount rate used is each consumer's after-tax rate of return to capital before the tax change, which averaged a real rate of return of 4 per cent.

The second row of Table 1 shows the welfare gains of integrating the two income tax systems. The results are more sensitive to the replacement tax used for maintaining government revenues, both because integration involves the loss of more tax receipts and because it does not stimulate saving, capital formation, and growth as much as the consumption tax. The third row combines the policies of the first two systems and shows that the efficiency improvement is approximately additive.

Since 80 per cent of total savings are deductible under the plans of rows 1 and 3 and 20 per cent of total savings flow into tax-favored housing, these plans capture the intertemporal effects of a full consumption tax. However, since any savings can be used for housing, these plans leave an intersectoral distortion in favor of owner occupancy. The plan of row 4 allows full deductibility of savings and eliminates the preference for housing. Gains are larger, as expected. The efficiency gain of the plan in row 4 relative to the current tax system is roughly $1.5 trillion with lump-sum revenue replacement, $1,350 billion with multiplicative marginal rate surcharges, and $1,390 billion with additive marginal rate surcharges. Row 5 examines a partial move toward a consumption tax (halfway between the current 30 per cent sheltering of retirement plans and the 80 per cent of row 1), while row 6 exempts all

Table 2. Dynamic Welfare Effects in Present Value of Compensating Variations Over Time

(In billions of 1973 dollars) 1/

Tax Replacement	Types of Scaling to Preserve Tax Yield		
	Lump sum	Multiplicative	Additive
1. Consumption tax (80 per cent savings deduction)	686.167 (1.376)	620.652 (1.245)	636.002 (1.275)
2. Corporate tax integration with indexation of capital gains	731.550 (1.467)	338.858 (0.680)	448.541 (0.889)
3. Consumption tax with integration	1429.503 (2.867)	999.813 (2.005)	1135.083 (2.276)
4. Pure consumption tax with integration	1500.881 (3.010)	1344.423 (2.696)	1388.410 (2.784)
5. Partial consumption tax (55 per cent savings deduction)	328.268 (0.658)	289.999 (0.582)	298.180 (0.598)
6. Full savings deduction with housing preference	991.704 (1.989)	962.633 (1.931)	964.370 (1.934)
7. Pure income tax without integration	-579.177 (-1.162)	-471.653 (-0.946)	-496.861 (-0.996)
8. Pure income tax with integration	128.298 (0.257)	-22.596 (-0.045)	21.422 (0.043)

1/ The numbers in parentheses represent the gain as a percentage of the present discounted value of consumption plus leisure in the base sequence. This number is $49.863 trillion for all comparisons and accounts for only the initial population.

saving from taxation, leaves the housing preference unchanged, and results in a personal income tax subsidy to saving. However, since this subsidy offsets the corporate income tax, which is left in place, total efficiency is enhanced relative to the plan shown in row 1.

The results shown in rows 7 and 8 indicate that the United States could move to a pure income tax and integrate the corporate tax with no loss in efficiency, but that a pure income tax alone would lose efficiency. For row 7, the tax base is increased, since imputed income from housing is included and existing savings deductions are eliminated. Thus, the tax rate can be lowered, rather than raised, in order to maintain government revenues. Results in row 7 show that moving to a pure income tax alone involves an efficiency loss of $579 billion if marginal tax rates are not lowered--primarily because the intertemporal distortions of the current system are worsened. However, if the marginal rates are reduced, the efficiency loss to the economy is lowered to roughly $470 billion. The improvement in the interindustry allocation of capital (resulting primarily from the taxation of the return to owner-occupied housing) tends to offset the deterioration in the intertemporal efficiency (now reduced by the marginal rate adjustments). Row 8 shows the results from a comprehensive single level income tax plan involving corporate tax integration as well. Such a tax system lowers revenues and thus necessitates a rate increase to maintain the yield. When the rates are adjusted either multiplicatively or additively, the net efficiency impact of the package is negligible.

The results in Table 2 are sensitive to the elasticities incorporated in the model. For example, the $621 billion from row 1, with a multiplicative scaling of the marginal tax rates, becomes $411 billion if the uncompensated saving elasticity is zero and $1,279 billion if this elasticity is 2.0. A more thorough evaluation of these results appears in Fullerton, Shoven, and Whalley (1982).

Table 3 provides information on how long the economy takes to resettle into a steady-state growth path after a tax change occurs. Once the economy has completely adjusted to the new policy regime, all relative prices will again remain constant. In the case of consumption tax proposals, the new steady state is characterized by a higher capital intensity and a lower relative return to capital. The results of Table 3 indicate that, for the cases with a 0.4 savings elasticity, roughly 40 per cent of the adjustment is completed after 10 years and 80 per cent is completed after 30 years. The economy then asymptotically approaches the new steady-state growth path. The transition is accomplished much more rapidly with a savings elasticity of 2.0, despite the fact that the total adjustment is larger. Adjustments in capital/labor ratios proceed in patterns similar to the adjustments of the price ratios in Table 3.

Table 3. Time Path for the Ratio of the Rental Price of Capital to the Wage Rate

Plan Number 1/	Savings Elasticity	Revenue Replacement	Prechange	Factor Price Ratios Time period (years)					
				0	10	20	30	40	50
1	0.4	Lump sum	1.00	1.00	0.93	0.89	0.86	0.84	0.83
1	0.4	Additive	1.00	0.99	0.92	0.88	0.85	0.84	0.83
3	0.4	Additive	1.00	1.19	1.04	0.96	0.92	0.89	0.88
1	0.0	Additive	1.00	0.99	0.94	0.91	0.89	0.87	0.86
1	2.0	Additive	1.00	0.99	0.80	0.79	0.79	0.79	0.79
7	0.4	Additive	1.00	0.97	1.00	1.02	1.04	1.06	1.07

1/ Refers to the row number in Table 2.

Interestingly, Ballard and Goulder (1982) find that the adjustment to a new steady state is slightly slower with perfect foresight and the institution of a consumption tax, since consumers are deterred from additional saving by the recognition that future capital deepening will depress the rate of return to capital.

In previous literature, estimates of the length of the long run vary widely. Sato (1963) finds the adjustment to be extremely long (more than 100 years), while Summers (1981) and Hall (1968) find it to be surprisingly short (about 5 years). It is difficult to completely reconcile these various findings, but it is clear that a prime determinant is the strength of substitution effects in the model used for the analysis.

Charles Ballard, John Whalley, and the author (1982) made another set of computer runs, asking a question of more theoretical interest: What are the efficiency costs of the entire U.S. tax system? This question is of interest because efficiency issues are often treated as minor ones relative to those of economic stability. Our aim was also to estimate the marginal cost of a government dollar raised by increasing taxes. In the past efficiency costs have frequently been quoted as fractions of gross national product or as the deadweight loss relative to the revenue raised. The former measure is ridiculous if the question is whether a tax on automobile tires or restrictions on steel imports are inefficient. The latter measure--the average distortion per dollar raised--does not often give the right answer either (average figures seldom do in economics). What has been computed, therefore, is the marginal distortionary cost per marginal dollar raised for each of the major tax systems in the United States.

Our estimate for the hypothetical experiment of removing the entire tax system and replacing it with a set of lump-sum levies (proportional to income and with sales taxes actually paid so as to minimize income and wealth transfers) is that the present value of welfare would increase by $3.3 trillion, which is roughly 6.7 per cent of national income plus leisure or 10 per cent of national income. The primary result of removing all marginal taxes is tremendous capital deepening. The net of tax rental-wage ratio immediately climbs by 113 per cent and gradually sinks from there to become 30 per cent higher than its present value in the new steady-state growth path. The capital-labor ratio is 50 per cent higher after 50 years. The labor supply also grows, being 19 per cent higher in the first period, because leisure is no longer the ultimate tax shelter that it is under the current system.

These results are sensitive to the values of the key elasticity parameters, as shown in Table 4, although the general picture is preserved. The standard case is shown in the second row. The total loss represents 3.55 per cent of expanded national income, even when both the uncompensated labor supply and saving elasticities are zero.

Table 4. Sensitivity Analysis with Respect to Key Parameters of Deadweight Loss of the Total Tax System

Labor Supply Elasticity	Saving Elasticity	Welfare Gain (trillions of 1973 dollars)
0.15	0.0	2.231 (4.48) 1/
0.15	0.4	3.338 (6.69)
0.15	2.0	8.236 (16.52)
0.0	0.0	1.772 (3.55)
0.0	0.4	2.709 (5.43)
0.0	2.0	7.017 (14.07)

1/ The figures in parentheses express the welfare gains as a percentage of the total present value of welfare from consumption and leisure, which is $49.863 trillion.

Table 5. Relationship Between First Period Marginal Tax Revenue Collections and Marginal Change in Welfare (Annualized to First Period) for Various Parts of the Tax System

(1) Marginal Rates Increased by 1 Per Cent	(2) Increase in Total Tax Revenues ($ billion)	(3) Increase in Transfers ($ billion)	(4) Increase in Government Tax Payments on Labor Use ($ billion)	(5) Net Increase in Government Revenue (2)-(3)-(4)	(6) Decrease in Utility ($ billion)	(7) Marginal Welfare Loss per Dollar of Revenue (6)÷(5)-1	(8) Marginal Welfare Loss Holding Transfers Fixed
All marginal rates	3.331	1.101	0.233	1.997	3.520	0.763	0.52
Capital taxes by industry	0.615	0.205	0.021	0.389	0.694	0.784	0.51
Labor taxes by industry	0.767	0.242	0.156	0.369	0.475	0.287	0.19
Consumer purchase taxes	0.330	0.108	0.010	0.212	0.398	0.877	0.63
Output taxes	0.131	0.043	0.004	0.084	0.129	0.536	0.43
Motor vehicles taxes	0.013	0.003	0.000	0.010	0.013	0.300	0.23
Marginal income taxes	1.895	0.633	0.055	1.207	2.209	0.830	0.57

Table 5 contains the results of computing the marginal distortionary costs of the U.S. tax system. If all marginal tax rates were multiplied by 1.01, the effect would be to raise government receipts by $3.331 billion. Transfers to consumers amount to just under a third of government revenues and the question is whether this fraction of a marginal tax increase would also be returned to households as transfers. Table 5 computes the marginal cost of funds for exhaustive expenditures under the assumptions that transfers are adjusted when receipts increase and that they are held fixed.

If transfers increase, then $1.101 billion is returned to households. Further, of the $3.331 billion raised, the government itself pays $0.233 billion. Netting out the transfers and the government's own tax payments, the funds available for a public project are $1.997 billion, shown in column 5. The decrease in consumer welfare is $3.520 billion, or 1.76 times as much as the money available for the government project. Column 7 reflects this $0.76 distortionary cost per dollar transferred to the public sector. If transfer payments are not increased, the government ends up with more net revenue and households have a larger decrease in utility, with the net result that consumers lose 1.52 times as much as the government raises. This is reflected in column 8.

The implications of these numbers, if accepted, are great. The $1.76 or $1.52 private cost of a marginal government dollar means that cost-benefit studies that use unity as the critical benefit-cost ratio support projects that are socially inefficient. They do this by not taking into account the resource waste caused by the distortionary taxes used to raise the additional revenues. The correct critical ratio would be 1.76 or 1.52, depending on how transfer payments react to the enlarged government budget. At a more theoretical level, the results indicate that the Samuelson conditions for the optimal provision of public goods should include

$$\sum_{\text{cons}} MRS_{X,G} = \lambda \cdot MRT_{X,G}$$

where λ is 1.76 or 1.52. Public goods not only use up resources in their manufacture but are a cost to economic welfare in the distortionary taxes they necessitate.

Rows 2-7 of Table 5 compute the marginal cost of each major type of tax. If these tax types were used in a third-best optimal manner, they would each have the same marginal distortionary cost per dollar raised. This would minimize the total deadweight loss for a given revenue using this given set of instruments. The results of columns 7 and 8 show that the U.S. tax system is far from this optimality condition. An additional flat tax on labor by industry (payroll tax) could raise a dollar for as little as $1.19 in private welfare, whereas increasing the 1973 personal income tax rates would result in government dollars that cost 1.57 each at the margin, even if transfer payments are frozen.

The results of this table are elaborated on in Ballard, Shoven, and Whalley (1982). They illustrate a kind of analysis that cannot appropriately be carried out with partial equilibrium techniques.

5. Conclusion

General equilibrium analysis has developed from an abstract economic theory to a computational procedure and now to a tool that can be used for policy purposes. It is not always the appropriate technique; certainly, there are many issues that are best examined at a very fine level of detail by using partial equilibrium methods. The model also is inappropriate for very short-run forecasting of business and the business cycle and does not yet have the many rigidities (unions, rent controls, monopolies, transaction costs, etc.) that characterize real economies. Nonetheless, for analyzing the likely medium-run to long-run adjustments of an economy to a large policy change, the applied general equilibrium model seems to be appropriate and to be ready for application.

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