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African Department

The Determinants of Inflation in Selected African Countries: A Cross-Section Study

Prepared by Cyril Enweze 1/

Approved by Joseph G. Keyes

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I. Introduction

Many studies of inflation in developing countries, such as those by Bhatia [4], Dorrance [10], and Tun Wai [42] have tended to be concerned with determining, statistically, the effects of inflation on economic growth, rather than the determinants of inflation in these economies. However, some understanding of the determinants of inflation is a prerequisite to the formulation of meaningful fiscal and monetary policies in these countries, in order to avert the well-known consequences of inflation and to foster relative price stability. It is the purpose of this paper to formulate and test a simple model of the determinants of inflation in selected African economies.

Owing to the lack of research on inflation in African economies, the work reported here is the first, though preliminary, attempt to construct a model of inflation in African economies, using cross-section data. In so doing, the paper adapts the analytical framework provided by the quantity theory of money and the demand for money theory. In essence, a simple transformation of an appropriate demand for money function permits us to express the price level as a function of the quantity of money and of per capita output; first differencing further permits the expression of the variables in terms of rates of growth; two dummy variables are also included to test for the effects of selected economic policies on the rate of inflation. In contrast to several cross-section studies particularly of developing economies, which neither acknowledge an error-structure problem nor attempt to correct for this in the estimating process, this paper attempts to identify and correct such deficiencies in order to render the parameters and the use of the various test statistics meaningful.

Initially it was intended to cover a large sample of African countries in this study. However, only 15 countries could be covered,

1/ The author wishes to thank Rattan J. Bhatia (AFR), Edward A. Arowolo (AFR), H. Robert Heller (RES), and J. Artus (RES) for helpful comments, and to take sole responsibility for any errors which may remain.

largely because of data unavailability. These countries are Cameroon, Gabon, The Gambia, Ghana, Ivory Coast, Libya, Malawi, Malagasy Republic, Mauritius, Morocco, Senegal, Tanzania, Tunisia, Zaire, and Zambia.

The outline of the rest of the paper is as follows: Section II outlines the analytical basis for the derivation of the model of inflation in the countries under study, and discusses the coverage and definitions used in the paper. Section III discusses the empirical results, while Section IV provides some concluding remarks. Appendix I contains a brief outline of the basic statistical model, while Appendix II lists the sources and definitions of the data.

II. Analytical Framework

The Fisherian Equation of Exchange, commonly known as the Quantity Theory of Money, first enabled economists to write a simplified expression for the absolute level of prices in the economy, viz

$$P = \frac{M.V}{Q} \qquad (1)$$

where P = an index of the price of all goods sold in the economy, during a period.

M = the quantity of money in the economy during the period.

V = average velocity of circulation of the monetary stock; this is defined as total monetary expenditures (P.Q) divided by the money stock, M.

Q = a measure of the quantity of goods sold during the period.

In equilibrium, velocity is independent of money supply and is determined by such institutional factors as the public's payments habits, the extent of use of credit, etc. While it is not a constant, it is assumed to change only in the long run. Similarly, the quantity of goods sold in the economy is also independent of the money supply. As a result, from equation (1), we derive the familiar quantity theorist's postulation that a change in money supply will cause a proportionate change in the equilibrium level of prices.

However, equation (1) fails to take account of the fact that money is a commodity which yields services to the holders, who may desire to hold money as a medium of exchange to facilitate transactions. In order to be able to formulate a demand for money function which will recognize the role of money for transaction purposes, we may write the familiar Cambridge equation

$$Md = kY \quad \ldots\ldots \quad (2)$$

where Md = the demand for nominal money balances.

Y = money income (equivalent to $(P.Q)$ in the quantity theory formulation).

k = the factor of proportionality commonly known as the "Cambridge k," which is the inverse of the velocity of circulation.1/

While the economic properties of equation (2) are largely similar to those of equation (1), it focuses on the volatile and volitional determinants of k, such as the public's preferences and expectations, all of which could nullify the simple quantity theory result. It is nevertheless subject to some limitations. First, in its strict form, interest rates do not play any role in the demand for money. Similarly, unless it can be empirically demonstrated that, as implied by equation (2), variations in income are accompanied by equal proportional changes in the demand for nominal balances, the assumption of an income elasticity of demand for money equal to unity is unnecessarily restrictive.2/ However, a reformulation of equation (2) will permit us to apply the general theory of demand to the specific problem of the demand for money, and to expand it by incorporating other factors which affect the public's demand for money.

We can write the demand for nominal balances as

$$Md = \alpha (1+i)^{\beta_1} \cdot Y_p^{\beta_2} \quad \ldots\ldots \quad (3)$$

where Md = demand for nominal money balances.

i = some rate of interest representing the opportunity cost of holding money.

1/ In equilibrium, the quantity of money demanded must equal the quantity supplied. Substituting for Y, obtain $M = kP.Q$. Dividing through by k, obtain $M(1/k) = P.Q$,
where V is replaced by the reciprocal of k.

2/ From equation (2) and the definition of elasticity $(\eta) = \frac{M/M}{\Delta PQ/PQ}$, and substituting, obtain

$$\eta = \frac{k\Delta PQ}{kPQ} \cdot \frac{PQ}{\Delta PQ} = 1.$$

Y_p = real income.

α = constant.

β_1, β_2 = coefficients of interest rate and real income respectively.

Equation (3) states that the demand for nominal balances is a function of the rate of interest and of real income. However, in empirical studies for the advanced economies, controversy still surrounds the appropriate interest rate argument in the demand for money equation. Thus, although several studies have documented the significance of the rate of interest, opinion is sharply divided as to the appropriate interest rate variable to be included in the money demand function.1/ More importantly, however, it has been found (Cagan [8]) that even in the advanced economies, the significance of the rate of interest which represents the opportunity cost of holding money rather than bonds is usually swamped by that of the rate of inflation, i.e. the opportunity cost of holding money rather than goods during periods of rapidly rising prices. In such cases, the true cost of holding money during periods of rapidly rising prices may be better approximated by the rate of change of the price level. Similarly, in a study of the demand for money in less developed economies, Adekunle [1] found that in these economies changes in prices satisfactorily approximated the yield on real assets.2/ Consequently, we may substitute the rate of change in the price level in the demand for money function for the African economies as a more appropriate measure of the opportunity cost of holding money, and rewrite equation (2) in the following form:

$$Md = \alpha P^{\beta_1} Y_p^{\beta_2} \quad \ldots\ldots \quad (4)$$

where P is the rate of change of the general price level.

From equation (4) it is clear that changes in money growth may increase nominal expenditures, by producing initially an excess of supply

1/ Strictly speaking, this debate is peripheral to our main objective, and the reader is referred to Brunner and Meltzer [7] and Bronfenbrenner and Mayer [6] for discussion of these issues. In a nutshell, proponents of the short-term rate of interest include Laidler [31] and Heller [24], while Hamburger [22] and Lee [33] propose the long-term rate. In part, some of the contradiction arises from the fact that in several cases the various proponents were studying different time periods and/or were using various definitions of the money supply.

2/ See also Dorrance [10, p. 6] and Friedman [14].

over demand for money. At less than full employment and given a sufficient output elasticity, the increase in aggregate expenditures which the increase in money generates will tend to increase real output. However, if for some reason[1] supply is inelastic to the growth of aggregate expenditure, the increase in nominal expenditures will tend to be dissipated in higher prices.

Following Harberger's proposal [23] we can rearrange equation (4) in terms of prices,

$$P = \left(\frac{1}{\alpha}\right)^{1/\beta_1} Md^{1/\beta_1} Y_p^{\beta_2/\beta_1} \quad \ldots\ldots\ldots\ldots (5)$$

In order to express equation (5) in terms of rates of percentage changes we take first differences and obtain:

$$\Delta P = \alpha + \beta_1 \Delta M + \beta_2 \Delta Y_p + v \quad \ldots\ldots\ldots\ldots (6)$$

As written, equation (6) expresses the rate of inflation as a function of the rate of increase in money supply and the rate of growth of real output. The nature and properties of v are discussed in Appendix I.

Several empirical studies have documented for other economies the appropriateness of money supply changes as a determinant of inflation.[2] In a recent study, Heller [25] notes that, at a global level, there also exists a fairly strong relationship between changes in money supply (via increases in international reserves) and the rate of inflation.[3] Fleming [13], in somewhat more muted terms, echoed the same views, but in addition to assailing the monetarists' proposition of a two-year lag

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1/ Such as the existence of full employment, technological constraints, low productivity, unfavorable weather conditions, etc.

2/ See Mikesell [37] and Harberger [23], for example.

3/ Heller observes [pp. 3-4] that, at a national level, "The increases in national money supplies prompted by international reserve increases will in turn have an important impact on national inflation rates. The channels via which monetary changes are translated into price changes are many and varied, as economic units rearrange their financial and real asset portfolios in response to monetary changes. The end result...being [through an increase in aggregate demand]...an increase in the nation's nominal GNP. Supply conditions play an important role in the division of the nominal GNP increase into real product and price increases. If velocity is secularly stable and monetary changes do not have a significant impact on real output in the long run, we can expect that monetary changes will be reflected after a certain adjustment period in changes in national price levels."

between monetary expansion and variations in price inflation, he also casts serious doubts on the "familiar [European] proposition that the United States [pursued] unduly expansionary policies, thus creating enormous balance of payments deficits which, by expanding the reserves of other countries and forcing an expansion of money, exported and almost imposed inflation on the outside world." He argues that, most European countries "had considerable ability to offset the effects of the influx of central bank reseves on commercial banks' reserves and hence on money supply," through open market sales, government borrowing, and increases in reserve requirements, and that, in this as in other areas responsible for inflation,1/ the existence of "...a regime of national states and open economies...makes it easier for peoples to do what they want--either to control inflation or to give way to it" [p. 15.]. Finally, single-country simultaneous-equation models of inflation also indicate that money supply changes (operating via reserve changes or changes in domestic credit) are an important determinant of the rate of inflation. Thus, Otani's imported inflation model [38] delineates money supply changes in the case of inflation arising from an increase in the world market prices of exports, and, in the case of inflation arising through higher import prices, its direct transmission through the components of the price index itself, while Aghevli [2] focuses on monetary and inflationary implications of financing higher levels of government expenditures through an expansion of bank borrowing.

In this study, which does not purport to measure imported inflation, no attempt is made to distinguish between changes in money supply, arising from changes in international reserves and/or changes in domestic credit by the banking system.2/ Indeed, essentially, the paper merely sets out to test, for the group of African economies, the basic and fundamental proposition that, as in other economies, an increase in the money

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1/ These include: the reluctance of host countries to regulate xenocurrency banks whose activities add to the expansion of private liquidity and the growth of world reserves; the failure of countries to avail themselves of the options open to them (within limits) through the present system of exchange rate variability to follow policies designed to attune their economies to officially desired target levels of demand; the paucity of official intervention in exchange rates; the inadequacy of official action to limit swings in effective exchange rates; the resistance in some cases to a downward trend in the value of a currency in inflationary situations; the role played by national states in fostering cost-inflationary cartel action such as the OPEC; and (by implication) the failure of several large countries to avail themselves of wage and price controls as an anti-inflationary weapon. For details see Fleming [13].

2/ For the same reasons, it was felt unnecessary to disaggregate and weight the consumer price index by domestic and external components.

stock is a key element or at least a significant permissive factor in inflationary situations.1/ Therefore, this study is limited to the period prior to the 1970s, when import prices were fairly stable. Even in cases of externally induced inflation from the export side, the potential increase in money stock can be ameliorated by pursuing appropriate stabilization policies, as pointed out by Fleming [13]. Suffice it to say that, in cases where there has been reluctance on the part of monetary authorities to mitigate the monetary impact of an increase in external reserves, and where output is sluggish, the increase in money supply will tend to increase aggregate demand and the rate of inflation. Similarly, although import price increases tend to be reflected directly in the consumer price index, depending on the weight of imported items in the index, this is so largely insofar as, in the final analysis, the monetary authorities are willing to permit an increase in money supply (via domestic credit expansion) to finance higher levels of imports, rather than pursue probable deflationary policies.2/ As a result, changes in money supply may still be viewed as a significant inflationary factor, in cases of inflation induced by higher levels of imports, although, as Otani [38] shows, the mechanism by which this is achieved may be less direct.

The responsiveness of output is usually expected to act as a dampener on the rate of inflation by absorbing some of the increase in aggregate demand in an economy. However, it is further posited, in this paper, that the extent to which two countries which have the same growth in output will weather inflationary tendencies will also depend on their respective rates of growth of population, and that the rate of growth of per capita real output may be the more appropriate argument in situations in which population is rising rapidly, or if countries exhibit significantly divergent rates of population growth. If per capita output is rising rapidly, this may be expected to reduce demand pressures and the rate of inflation. In contrast, a low or stagnant rate of increase in per capita real output will result in aggregate demand pressures working themselves through price increases.

Several administrative measures may also have some bearing on the actual rates of inflation experienced in various countries or in the same country over time, although there is as yet no consensus. As Fleming [13, pp. 14-15] points out, opinion is divided among economists about the feasibility of wage and price controls as an anti-inflationary

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1/ See, for example, Mikesell [37] and Harberger [23].

2/ The choice of the phrase "probable deflationary policies" is deliberate. If high priced imports are effectively curtailed through credit policy but domestic output fails to expand, inflation may still result, unless, of course, the curtailment of money demand via credit is also accompanied by (fiscal) measures designed to siphon off purchasing power from the public.

instrument. Nevertheless, he documents the successful use of this instrument by several relatively advanced but small economies. While it may not be strictly accurate to imply that any of the countries under study in this paper have at any time implemented a full-fledged "incomes policy" as such, it is still empirically recognized that some of these countries have at some periods instituted some form of price controls and/or refrained from awarding substantial wage boosts.

Accordingly, it was felt that a model of the determinants of inflation should attempt to take account of these specific administrative actions which countries sometimes take and which may be expected to exert independent effects over and above those generated by changes in money supply and in the rate of growth of per capita output. In several of the countries under study, minimum wages prevail, and these tend to be stable for several years at a time. In certain cases, however, these wages were renegotiated during the sample period, and substantial wage hikes were awarded. In other cases general across-the-board wage boosts, including salary increases for public servants, were made. Even in the absence of an increase in money supply, or, pari passu, with such an increase, it is expected that such autonomous wage boosts will increase aggregate demand and, depending on the responsiveness of real output, will cause inflation in varying degrees.1/ To take account of the effect of such economic policy actions which have a direct implication for the rate of inflation, one "dummy" variable was incorporated into the model to ascertain the impact of wage awards; similarly, another dummy variable was introduced to test the effectiveness of price control in reducing the rate of inflation in the selected African economies. On the basis of these modifications, equation (6) may be expanded, to read:

$$\Delta P = \alpha + \beta_1 \Delta M + \beta_2 \Delta Y\eta + \beta_3 D_g + \beta_4 Dp + v \quad \text{.................} \quad (7)$$

where D_g and D_p are the dummy variables for wage boosts and price controls, respectively, and v has the same properties as in equation (6) above [Appendix I].

III. Empirical Results

For estimation purposes, several problems arose with regard to the empirical definition of some of the arguments in equation (7). In different economies, and even in the same economy, different indices may be used to measure the rate of inflation. However, the usual arguments surrounding the reliability and/or choice of one or the other of these

1/ Harberger [23].

price indices are not discussed here,1/ neither do we discuss the well-known limitations of such indices as measures of price changes. For the purpose of this study, changes in the cost-of-living index have been used as a measure of the rate of inflation in the African countries under study.

Similarly, as in all studies of this nature, the relevant empirical concept of the money stock has to be defined. Traditionally, economists have defined money in a "narrow" sense, that is, as the sum of currency and demand deposits. While there is general agreement that, according to Meltzer [36], the relevant definition must satisfy the criterion of substitutability,2/ recent controversy, however, has centered on whether a broader definition may not be more appropriate. Although Feige [12] and Brunner and Meltzer [7] favor a "narrow" definition of money, Friedman [14] and Laidler [30] maintain that the appropriate definition of money must include time and savings deposits. Indeed, using Meltzer's criterion, some writers, notably Lee [33], Chetty [9], and Lydall [34], have argued that an even broader definition of money may be more appropriate. Lee proposes the inclusion of time and savings deposits, as well as the liabilities of financial intermediaries, e.g., savings and loan shares, in the definition of money. In addition to these assets, Chetty would also include mutual bank deposits in money supply. For the African countries under study, however, there is no proliferation of financial assets, and their money markets, where they exist, are essentially rudimentary.3/ Furthermore, given the relative fixity of interest rates during the period covered on time and savings deposits (that is, iTD) in these countries, and the fact that substitutability stemming from financial assets is probably very small, this study opts for the traditional and narrow definition of money supply.

With regard to output measures, constant-price national income data are not available for many of the African countries in the sample. Furthermore, although current-price national income data are available for

1/ For a good discussion of these issues, see Bronfenbrenner and Holzman [5].

2/ Usually measured by the cross-elasticity between two money-type assets, say, DD (demand deposits) and TD (time deposits). Suppose the earnings on these two assets are designated as iDD and iTD, respectively; the cross-elasticity of DD with respect to TD (i.e., $\eta DD.TD$) is defined as the percentage change in the quantity of DD demanded divided by the percentage change in iTD. In symbols,

$$\eta DD.TD = \frac{\Delta DD/DD}{\Delta iTD/iTD}$$

Substitution is said to exist if ΔiTD is positive and ΔDD is negative.

3/ Arowolo [3].

the sample countries in the period covered, appropriate price deflators for the main economic sectors could not be assembled. Consequently, as in an earlier effort,1/ since national income deflators are not generally available, the cost-of-living index was used to deflate the current-price national income data.

On the basis of the statistical model outlined in Appendix I, regression estimates of the parameters of equation (7) were estimated, while at the same time ensuring that the usual tests of significance will be appropriate and meaningful. The following specific alternative hypotheses, which require the execution of one-tailed tests on the significance of the parameters, will be verified:

$\Delta M \quad \beta_o > 0$

$\Delta Y_n \quad \beta_1 < 0$

$D_g \quad \beta_2 > 0$

$D_p \quad \beta_3 < 0$

D_g and D_p are formulated such that

D_g = 1 if wage awards

0 otherwise . . . . (8)

and D_p = 1 if price control

0 otherwise . . . . (9)

Except for the dummy variables, the data relate to annual average rates of growth over the period 1968-70. As a springboard for most of the empirical work in this section, equation (7) was estimated by ordinary least squares, with the following results:

$$\Delta P = \underset{(0.80841)}{-1.03912} + \underset{(3.3136)}{0.3992\Delta M} = \underset{(0.65589)}{0.130855\Delta Yn}$$

$$= \underset{(0.72729)}{0.60865\, D_g} \quad \underset{(-1.08565)}{-0.86263\, D_p}$$

$$R^2 = 0.4844 \qquad \ldots\ldots\ldots\ldots \qquad (10)$$

1/ Enweze [11].

(figures in parentheses are t-ratios). Owing to the statistical limitations of equation (10) (Appendix I), the significance of the parameters of this equation are not amenable to reliable interpretation. However, it is interesting to point out, in passing, that the coefficient for per capita output comes out with the opposite sign from what was expected (implying that increases in per capita real output lead to inflation), and that the equation explains about 48 per cent of the rate of inflation in the sample countries.1/

In spite of the practical difficulty of distinguishing between "pure" and "mixed" heteroskedasticity, 2/ the application of Glejser's test permitted the identification of the approximate form of heteroskedasticity. Following Glejser [16] as a first step, ordinary least squares was fitted; then trial runs on $|V|$, which was retrieved from the regression runs, suggested an approximately mixed pattern of heteroskedasticity (i.e., $M_o = 0$ and $m_1 \neq 0$), at a 5 per cent level of significance,3/ implying:

1/ Similarly, estimates to determine whether heteroskedasticity in the error term takes the form of proportionality between the residual variance of the error term and the rate of growth of money supply were unsatisfactory, as the following equation shows:

$$\frac{\Delta P}{(\Delta M)} = \underset{(1.17576)}{3.23948}\ \frac{\alpha}{\Delta M} \underset{(-0.03278)}{-0.09830} + \underset{(2.54706)}{0.35557}\ \frac{\Delta Yn}{\Delta M} - \underset{(-10597)}{0.91048}\ \frac{Dg}{\Delta M} - \underset{(-0.98345)}{0.09719}\ \frac{Dp}{\Delta M}$$

$$R^2 = 0.4111 \qquad \ldots\ldots\ldots\ldots (11)$$

Notice that, in this equation, some of the independent variables have "wrong" signs (for example, $\frac{\Delta Yn}{\Delta M}$, and $\frac{Dg}{\Delta M}$) and that the explanatory power of the equation is only 41 per cent, compared to 48 per cent when ordinary least squares was used. These results underscore the need for a reasonable approximation regarding the probable form of heteroskedasticity (Appendix I).

2/ See Glejser [16].

3/ The estimated equation yielded

$$|\hat{V}| = \underset{(2.36173)}{1.53356} - \underset{(-0.946601)}{5.98019}\Delta M^{-1}$$

Apparently, Glejser considers a significance level of 11 per cent also reasonable, although he would prefer a 5 per cent level. See Glejser [16], p. 319.

$$\hat{V} = \sigma^2 \, (MO + m_1 \Delta M^{-1})^2 \quad \text{. . . . .} \quad (12)$$

On estimating the final equation (16) in the Appendix, we obtain:

$$\frac{(\Delta P)}{(m_o + m_1 \Delta M^{-1})} = \underset{(-1.29817)}{\frac{-2.83082}{(m_o + m_1 \Delta M^{-1}}} + \underset{(2.73008)}{0.61105} \frac{(\Delta M)}{(m_o + m_1 \Delta M^{-1})}$$

$$\underset{(-0.38528)}{-0.07640} \frac{\Delta Yn}{(m_o + m_1 \Delta M^{-1})} + \underset{(0.60137)}{0.56374} \frac{Dg}{(m_o + m_1 \Delta M^{-1})}$$

$$\underset{(-2.25587)}{-1.31969} \frac{Dp}{(m_o + m_1 \Delta M^{-1})}$$

$$R = 0.9255 \quad \text{. . . . .} \quad (13)$$

The figures in parentheses under the regression coefficients are the t-ratios, and the $\bar{R}^2$ statistic in this case serves as an indicator of the fit of the equation, rather than as its explanatory power. In equation (13) all the coefficients have the expected signs, and the coefficients of that equation are subject to the usual interpretations.1/ In general, as expected, the rate of inflation is affected positively by the growth of money supply but negatively by the growth of per capita real output. Indeed, the impact of the rate of increase in money supply on the rate of inflation is highly significant at the 1 per cent level; but the coefficient of per capita real output falls short of significance, although, as in Harberger [23, p. 237], it has the expected negative sign. Similarly, the dummy variable for price control is significant at the 5 per cent level, suggesting that in the sample countries price control has tended to dampen the potential increase in the rate of inflation. No attempt, however, is made to assess the distortionary effects of this. Finally, although the coefficient of wage awards is positive as expected, as in the case of per capita output this variable also falls short of significance. In general, these results conform to

1/ The reader should be wary of interpreting these coefficients as inflation elasticities, since a logarithmic form of equation (13) was not specified.

Harberger's [23, p. 248], who finds that in the case of the Chilean inflation there are "typically higher partial correlations between monetary variables and the rate of inflation than between wage changes and the rate of inflation."

IV. Summary and Conclusions

This paper is essentially a preliminary and exploratory attempt to formulate and test a model of the determinants of inflation in a selected number of African countries. Deriving from the analytical construct of the demand for money function, the paper attempted to determine specifically the extent to which the rate of inflation is determined by the rate of increase in money supply and in per capita output, and also by specific economic policies which were implemented at various times in the different countries. The overall fit of the estimated equation, coupled with the significance of the rate of growth of money supply, seems to provide an indicative verification of the central and/or permissive role of money in inflationary situations. Similarly, in an indicative sense, the study has generated further supporting empirical evidence about the probable effectiveness of price controls in dampening inflation in a selected group of African economies. While the adverse effects of price controls have not been examined in this study, it is hoped that the implications of the results pertaining to this variable will generate further interest in this area, particularly in the light of the tendency of African countries to resort to such controls. In view of the total information provided by the estimated equation, however, it would appear that the relative significance of the money supply variable underscores the need for more prudent monetary policies at the same time that administrative action may be needed to restrain the rate of inflation. In any case, the results indicate that efforts to control prices, on the one hand, may be aborted if, on the other hand, such measures exist side by side with rapid monetary expansion, and/or lack of real growth.

However, none of the above may be construed as indicating that the model presented and tested in this paper has performed in a totally satisfactory fashion. As indicated above, some of the findings are tentative. Furthermore, as in all single-equation methods, a relatively simple approach was followed for analytical and empirical reasons, as no attempt could be made in a cross-section study to incorporate the simultaneous interaction of various sectors of the economy. Nevertheless, these limitations are not such as to nullify the indicative relevance of the results. It is also hoped that the work reported here may be viewed as representing a modest contribution to the study of important economic phenomena in African economics, as a prelude to the formulation of comprehensive macroeconomic models for these economies.

Estimation of the Cross-Sectionally Heteroskedastic Model

This Appendix presents a short summary of the technique used in estimating the cross-section model in this paper. The presentation relies heavily on Glejser [16], Goldberger [17], Goldfeld and Quandt [18] and [19], and Johnston [27].

Throughout this Appendix, the following notation has been used:

ΔP = rate of inflation measured by change in the cost-of-living index.

ΔM = rate of increase in money supply (narrow definition).

ΔY = per capita rate of growth of real output.

Dg = dummy variable for wage control.

Dp = dummy variable for price control.

e = vector of residuals.

v = vector of disturbances.

E = mathematical expectation.

I = identity matrix.

Σ = covariance matrix.

n = number of observations.

k = number of parameters estimated in the regression equation.

S = sum of squared residuals in ordinary least square regression.

c = number of central observations in the Goldfeld-Quandt test [18].

Suppose that, as in the classical linear model,

$$Y = X\beta + v \qquad (1)$$

where X is a nonstochastic n x k matrix of full rank, β is a k x 1 vector of parameters, and v is the n x 1 vector of disturbances with

$$E(v) = 0 \qquad (2)$$

and $E(vv') = \sigma 2\phi$ . . . . (3)

where ϕ is some positive definite matrix of disturbances.

In the simple classical case, ϕ is the identity matrix, and the BLUE estimate of β is given by

$$\beta = (X'X)^{-1}X'X \quad \ldots\ldots\ldots\ldots \quad (4)$$

with covariance matrix

$$\Sigma = \sigma\ (X'X)^{-1} \quad \ldots\ldots\ldots\ldots \quad (5)$$

An unbiased estimate of σ^2 is provided by

$$\hat{\sigma}2 = \frac{e'e}{n-k} \quad \ldots\ldots\ldots\ldots \quad (6)$$

so that the equivalent unbiased estimate of (5) is given by $\hat{\sigma}^2\ (X'X)^{-1}$

However, in the heteroskedastic model used in this paper, the BLUE of β is provided by the Aitken's estimator:

$$\beta = (X'\ \phi^{-1}X)^{-1}\ (X\ \phi^{-1}Y) \quad \ldots\ldots\ldots\ldots \quad (7)$$

with covariance matrix

$$\tilde{\Sigma} = \sigma^2(X\ \phi^{-1}X)^{-1} \quad \ldots\ldots\ldots\ldots \quad (8)$$

An unbiased estimate of equation (8) will therefore be given by

$$\tilde{\sigma}2 = \frac{e\ \phi^{-1}e}{n-k} \quad \ldots\ldots\ldots\ldots \quad (9)$$

where $\phi \neq I$ . . . . (9a)

As a result of equations (7) to (9), if β is estimated by equation (4), as in many cross-section studies, instead of by the appropriate estimator (7), β will no longer have minimum variance,1/ and, because of the existence of an unknown positive definite matrix ϕ, in equation (8), the usual formula for covariance is no longer appropriate, and the standard test for statistical significance is invalidated.

1/ Using the Gauss-Markov theorem, recall, from equations (1) and (4), that the estimator of β in the dependent variable Y is given by

$$\hat{\beta} = \left[(X'X)^{-1}X' + A\right]Y \quad \text{(i)}$$

where A is a k x n matrix and unbiasedness for β implies

$$E(\hat{\beta}) = E\left[(X'X)^{-1}X' + A\right]\left[X\beta + e\right] \quad \text{(ii)}$$

$$= \beta + AX\beta \quad \text{(iii)}$$

$$+ \beta \quad \text{(iv)}$$

In general, A is restricted to the class of matrices for which AX = 0, that is, homogeneous linear equations. Recall also that the covariance matrix of β is given by

$$E\left[(\hat{\beta} - \beta)\ (\hat{\beta} - \beta)'\right] \quad \text{(v)}$$

$$= E\left[(X'X^{-1})\ X' + A\right] Y - \beta)\ (\left[(X'X)^{-1}X' + A\right] Y - \beta)\right] \quad \text{(vi)}$$

On substituting Xβ + v for Y and using AX = 0, this simplifies to

$$\sigma^2\left[(X'X)^{-1} + AA'\right] \quad \text{(vii)}$$

which, because A is a null matrix, reduces to

$$= \alpha^2\ (X'X)^{-1} \quad \text{(viii)}$$

where the diagonal elements of the equation are the minimized variances of β with respect to A.

Since, however, in the heteroskedastic case $\phi \neq I$, we obtain instead

$$\Sigma = \sigma^2\ (X'\ \phi^{-1}X)^{-1} + \sigma^2\ A\ \phi\ A' \quad \text{(ix)}$$

or, simply, equation (8) plus a positive semi-definite matrix.

Similarly, for the same reason, the usual estimate given by σ^2 in equation (6) will now be biased.1/

1/ The proof of this is as follows:

Substitute equation (1) into equation (4) and obtain

$$\hat{\beta} = \beta + (X'X)^{-1}X'v \quad \ldots\ldots\ldots\ldots \quad \text{(i)}$$

Recall that, by definition, e is given by

$$e = X\beta + v - X\hat{\beta} \quad \text{(ii)}$$

On further substitution [(ii) into (i)], obtain

$$e = X\beta + v - X\left[\beta + (X'X)^{-1} X'V\right] \quad \ldots\ldots\ldots \quad \text{(iii)}$$

which may be simplified into

$$e = \left[I - X(X'X)^{-1}X'\right] v \quad \text{(iv)}$$

where the matrix $\left[I - X(X'X)^{-1}X'\right|$ is symmetric and idempotent, so that

$$e'e = v'\left[I - X\,(X'X)^{-1}X'\right]v \quad \ldots\ldots\ldots \quad \text{(v)}$$

Taking expectations on equation (v), obtain

$$E(e'e) = E\ \mathrm{tr}\ \left(v'\left[I - X(X'X)^{-1}X'\right]v\right) \quad \ldots\ldots\ldots \quad \text{(vi)}$$

which, on rearrangement, gives

$$E(e'e) = \mathrm{tr}\ \left[(I - X(X'X)^{-1})X'\right)E(vv') \quad \ldots\ldots\ldots \quad \text{(vii)}$$

Recall also that, by definition,

$E(e'e) = \sigma^2\ \phi$, so that we may simplify equation (vii) to obtain

$$E(e'e) = \sigma^2\ \mathrm{tr}(\left[I - X(X'X)^{-1})X'\right]\ \phi) \quad \ldots\ldots\ldots \quad \text{(viii)}$$

$$= \sigma^2\ \mathrm{tr}\ \left[(\phi - X(X'X)^{-1}X'\phi)\right] \quad \ldots\ldots\ldots \quad \text{(ix)}$$

$$= \sigma^2\ \left[\mathrm{tr}\phi - \mathrm{tr}(X'X)^{-1}\ (X'\phi X)\right] \quad \ldots\ldots\ldots \quad \text{(x)}$$

Observe, from equation (ix) in the preceding footnote, that

$$X'\phi X = X'X - X'(I - \phi)X \quad \ldots\ldots\ldots \quad \text{(xi)}$$

These results show that the use of equation (4) instead of equation (7) to estimate β, will lead to inefficient estimates with larger sampling variances than necessary, and that the estimated covariance matrix will also be biased, so that standard tests of statistical significance are inappropriate.

(Footnote 1 continued)

Hence the last two terms of equation (x) are equivalent to

$$I - (X'X)^{-1}X' (1 - \phi)X \quad \ldots\ldots\ldots\ldots \quad \text{(xii)}$$

with trace given by

$$k - \text{tr}\,[X'X) \quad X'(I - \phi)X] \quad \ldots\ldots\ldots\ldots \quad \text{(xiii)}$$

Assuming ϕ is normalized, so that trϕ = n then, substituting (xiii) into (x), obtain

$$E(e'e) = \sigma^2 \,[n - k + \text{tr}(X'X)^{-1}X'(I - \phi)X] \quad \ldots\ldots \quad \text{(xiv)}$$

From (xiii), we have

$$E(\tilde{\sigma}^2) = \sigma^2 + \frac{\sigma^2 \text{tr}\,[(X'X)^{-1}X'(I - \phi)X]}{n - k} \quad \ldots\ldots \quad \text{(xv)}$$

from which it follows that

$$E(\hat{\sigma}^2(X'X)^{-1}) = \Sigma + \sigma^2 \,\frac{\text{tr}[(X'X)^{-1}X' \ (I - \phi)X]}{n - k}(X'X)^{-1} \quad . \quad \text{(xvi)}$$

$$= \Sigma\hat{\beta} + \sigma^2(X'X)^{-1}X' \ (I - \phi)X(X'X)^{-1}$$

$$+ \sigma^2 \,\frac{\text{tr}[X'X)^{-1}X'(I - \phi)X]}{} \,(X'X)^{-1} \quad \text{(xvii)}$$

with the last two terms giving some idea of the bias.

The extent of bias and inefficiency is, in general, not known, both being functions of the independent variables and of the term ϕ, which is unknown, and which in the heteroskedastic case is given by the diagonal matrix

$$\Phi = \sigma^2 X_i^{\lambda} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

and X_i is the independent variable associated with heteroskedasticity.

As stated above, nothing is known about ϕ, but it has the general form

$$\phi = \begin{bmatrix} \sigma_1^2 & 0 & \ldots & 0 \\ 0 & \sigma_2^2 & \ldots & 0 \\ 0 & 0 & \ldots & \sigma_n^2 \end{bmatrix} \quad \ldots\ldots\ldots\ldots \quad (11)$$

and we need to impose some structure on ϕ to make the problem tractable. Thus, in cases where there is proportionality between e and X_1 (i.e., ΔM),1/ an appropriate structure can be imposed, such that

$$\sigma^2 \begin{bmatrix} X_{11}^2 \; {}^{2/} & 0 & \ldots & 0 \\ 0 & X_{1,2}^2 & \ldots & 0 \\ 0 & 0 & & X_{1,15}^2 \end{bmatrix} \begin{bmatrix} \frac{1}{X_{1,1}^2} & 0 & \ldots & 0 \\ 0 & \frac{1}{X_{1,2}^2} & \ldots & 0 \\ 0 & 0 & \ldots & \frac{1}{X_{1,15}^2} \end{bmatrix} = \sigma_1^2 \quad (12)$$

1/ Empirical estimates failed to support this hypothesis (see text equation (11).

2/ Since the scalar σ^2 commutes.

Insofar as the Goldfeld-Quandt parametric test [18] could be used on small samples, we adopted this test for our sample countries. It was assumed that σ_1^2 was monotonically related to ΔM, with individual observations ΔM and c = 2, subject to the constraint that

$$n - k > k \quad \ldots \quad (13)$$

While the test statistic1/ $R = \frac{S_2}{S_1}$ could not be executed due to sample-size constraints, under the alternative hypothesis values of R will tend to be larger, since the values of ΔM are larger for the one set of residuals than for the other, and the corresponding set of squares of residuals will tend to be larger as well.

With $n_1 = 7$, $c = 2$, and $n_2 = 6$, we obtained markedly different estimates for the sum of squared residuals, such that

$$S_1 = 0.8277 \text{ and } S_2 = 0.0747$$

where S_1 and S_2 = sum of squared residuals of OLS estimates for $n_1 = 7$ and $N_2 = 6$, respectively.

Having established the existence of heteroskedasticity, Glejser's test [16] was used to determine the approximate pattern. Thus in equation (1) we posit

$$v = vP(z) \quad \ldots \quad (14)$$

Where P(z) is a nonnegative polynomial, z and v are an independent and a random variable respectively. Specifically, after testing for several functional forms, we finally obtain significant estimates for Mo and m_1 in the form:

$$|v| = Mo + m_1 M^{-1} \quad \ldots \quad (15)$$

1/ It is distributed as

$$F \approx \left[\frac{(n-c-2k)}{2}, \frac{(n-c-2k)}{2}\right]$$ degrees of freedom under the null hypothesis of homoskedasticity.

On the basis of this information, we proceeded to transform the estimating equation (see equation (7) in the text) to obtain

$$\frac{\Delta P}{(Mo+m1\Delta M^{-1})} = \frac{\alpha}{(Mo+m1\Delta M^{-1})} + \beta_o \frac{\Delta M}{(Mo+m1\Delta M^{-1})} + \beta_1 \frac{\Delta Yn}{(Mo+m1\Delta M^{-1})}$$

$$+ \frac{\beta_2 \; Dg}{(Mo+m1\Delta M^{-1})} + \frac{\beta_3 \; Dp}{(Mo+m1\Delta M^{-1})} + e \qquad (16)$$

which is of a form that lends itself to linear estimation (see text equation (13).

Data: Sources and Description

Prices	= cost-of-living index, 1963 = 100. IMP, *International Financial Statistics*.
Money supply	= currency plus demand deposits. IMF, *International Financial Statistics*.
Income	= Gross Domestic Product at current prices. IMF, *International Financial Statistics*.
Population	= annual level of population IMF, *International Financial Statistics*.
Wage "dummy"	= (See text) IMF, *Annual Consultation Reports*.
Price control "dummy"	= (See text) IMF, *Annual Consultation Reports*.

BIBLIOGRAPHY

1. Adekunle, Joseph O., "The Demand for Money: Evidence from Developed and Less Developed Economies", IMF Staff Papers, Vol. XV, July 1968, pp. 220-259.

2. Aghevli, Bijan, "Monetary Expansion and Inflation in Indonesia: 1954-72", DM/75/6.

3. Arowolo, Edward A., "Money Markets in African Countries", DM/72/65.

4. Bhatia, Rattan J., "Inflation, Deflation and Economic Development", IMF Staff Papers, Vol. VIII, November 1960, pp. 101-114.

5. Bronfenbrenner, M. and F.D. Holzman, "A Survey of Inflation Theory", in Surveys of Economic Theory, Vol. 1, American Economic Association and the Royal Economic Association, 1967, pp. 46-108.

6. Bronfenbrenner, M. and Thomas Mayer, "Liquidity Functions in the American Economy", Econometrica, 28 (October 1960), pp. 810-834.

7. Brunner, Karl and Allan H. Meltzer, "Predicting Velocity: Implications for Theory and Policy", Journal of Finance, 18 (May 1963), pp. 319-354.

8. Cagan, Phillip, "The Monetary Dynamics of Hyperinflation", in Studies in the Quantity Theory of Money, ed. Milton Friedman (University of Chicago Press), pp. 25-117.

9. Chetty, V.K., "On Measuring the Nearness of Near-Moneys", American Economic Review, 59 (June 1969), pp. 270-281.

10. Dorrance, Graeme S., "The Effects of Inflation on Economic Development", IMF Staff Papers, Vol. X, March 1963, pp. 1-44.

11. Enweze, Cyril, "Structure of Public Expenditures in Selected Developing Countries: A Time Series Study", The Manchester School of Economic and Social Research, No. 4, December 1973, pp. 430-463.

12. Feige, Edgar, "The Demand for Liquid Assets: A Temporal Cross Section Analysis", Englewood Cliffs Company, 1964.

13. Fleming, J. Marcus, "International Aspects of Inflation", DM/75/66.

14. Friedman, Milton, "The Quantity Theory of Money: A Restatement" in Studies in the Quantity Theory of Money, Chicago, 1967.

15. -----------------, "The Demand for Money: Some Theory and Empirical Results", Journal of Political Economy, 67, (June 1959), pp. 327-351.

16. Glejser, H., "A New Test for Heteroskedasticity", Journal of American Statistical Association, 1969, Vol. 64, pp. 316-323.

17. Goldberger, Arthur S., Econometric Theory (John Wiley and Sons, Inc., New York, 1964).

18. Goldfeld, Stephen M. and Richard E. Quandt, "Some Tests for Homoskedasticity", Journal of American Statistical Association, 1965, Vol. 60, pp. 539-547.

19. ----------------------, "Nonlinear Methods in Econometrics", North Holland Publishing Company, 1972.

20. Haberler, Gottfried, "Inflation, its causes and cures: (With a New Look at Inflation in 1966)", American Enterprise Institute for Public Research, Washington, D.C., July 1966.

21. Hague, D.C., (ed.) "Inflation", International Economic Association, St. Martin's Press, Inc., New York, 1962.

22. Hamburger, Michael J., "The Demand for Money in Households, Money Substitutes and Monetary Policy", Journal of Political Economy, 74 (December 1966), pp. 600-623.

23. Harberger, Arnold C., "The Dynamics of Inflation in Chile", in Measurement in Economics: Studies in Mathematical Economics and Econometrics in Memory of Yehuda Grunfeld (Stanford University Press, Stanford, California, 1963).

24. Heller, H.R., "The Demand for Money: The Guideline from the Short Run Data", Quarterly Journal of Economics, 79 (June 1965), pp. 291-303.

25. ----------------------, "International Reserves and World-Wide Inflation", DM/75/63. (See also IMF Staff Papers, 61-87).

26. Johnson, Harry G., "Inflation and the Monetarist Controversy", North Holland Publishing Company, Amsterdam-London 1972.

27. Johnston, J., Econometric Methods (2nd edition), McGraw Hill, New York, 1972.

28. Kmenta, Jan, Elements of Econometrics (The Macmillan Company, New York, 1971).

29. Kuh, Edwin, "The Validity of Cross-Sectionally Estimated Behavior Equations in Time Series Applications", Econometrica, 27 (1959), pp. 197-214.

30. Laidler, David, "Some Evidence on the Demand for Money", Journal of Political Economy, 74 (February 1966), pp. 55-68.

31. ---------------, "The Rate of Interest and the Demand for Money: Some Empirical Evidence", Journal of Political Economy, 74 (December 1966), pp. 545-555.

32. Latane, Henry A., "Cash Balances and the Interest Rate: A Pragmatic Approach", Review of Economics and Statistics, 36 (November 1954), pp. 456-460.

33. Lee, T.H., "Alternative Interest Rates and the Demand for Money: The Empirical Evidence", American Economic Review, 57 (December 1967), pp. 1168-1181.

34. Lydall, Harold, "Income, Assets and the Demand for Money", Review of Economics and Statistics (February 1958), pp. 1-14.

35. Malinvaud, E., Statistical Methods of Econometrics, North Holland Publishing Company, Amsterdam-London, 1970.

36. Meltzer, Allan H., "The Demand for Money: The Evidence from the Time Series", Journal of Political Economy, 71 (June 1963), pp. 219-246.

37. Mikesell, Raymond F., Survey of the Alliance for Progress: Inflation in Latin America, U.S. Government Printing Office, Washington, 1967.

38. Otani, Ichirio, "Inflation in an Open Economy: A Case Study of the Philippines", DM/74/117.

39. Rousseaus, Stephen W. (ed.), "Inflation, its causes, consequences and control", Proceedings of a Symposium, Department of Economics, New York University, January 31, 1968. (Participants include Robert M. Solow, Albert G. Hart, Phillip Cagan, and James Tobin).

40. Solow, Robert M., Price Expectations and the Behavior of the Price Level, Manchester University Press, 1969.

41. Symposium, "Ten Economists on the Inflation", Reprint from Review of Economics and Statistics, Vol. XXX, No. 1, February 1948. (Participants included Seymour E. Harris, Sumner H. Slichter, Frederick C. Mills, Joseph S. Davis, Gottfried Haberler, K.E. Boulding, Fritz Machlup, M. Kalecki, Abba P. Lerner, and Richard Heflebower).

42. Wai, U Tun, "The Relation Between Inflation and Economic Development: A Statistical Inductive Study", IMF Staff Papers, Vol. VII, October 1959, pp. 302-317.