



WP/04/230

# IMF Working Paper

---

## Can The Private Annuity Market Provide Secure Retirement Income?

*G.A. Mackenzie and Allison Schrager*

**IMF Working Paper**

Research Department

**Can The Private Annuity Market Provide Secure Retirement Income?**

Prepared by G.A. Mackenzie and Allison Schrager<sup>1</sup>

December 2004

**Abstract**

**This Working Paper should not be reported as representing the views of the IMF.**

The views expressed in this Working Paper are those of the author(s) and do not necessarily represent those of the IMF or IMF policy. Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate.

Annuity premiums are often assumed to be constant, although they can be expected to vary with the yield curve. Variations in premiums will become an important public policy issue as defined-contribution (DC) pension plans play an increasingly prominent role in providing retirement income. As DC plan holders retire, many will annuitize at least a part of their account balances. In the absence of current data on annuity prices, the paper relies on U.S. Treasury interest rate data to simulate the impact of interest rate variation on annuity premiums. For a spectrum of feasible interest rates, the variation in retirement income is not negligible.

JEL Classification Numbers: G22; H55

Keywords: Public pension systems; individual accounts; annuities

Author(s) E-Mail Address: gmackenzie@imf.org; acs171@columbia.edu

---

<sup>1</sup> IMF and Columbia University, respectively. We would like to thank Eytan Sheshinski, Mark De Broeck, Tony Webb, and FAD seminar participants for their useful comments and suggestions. We are solely responsible for any errors the paper may have.

Contents	Page
I. Introduction .....	3
II. Basic Features of Annuities and the Market for Annuities .....	4
A. The Life Annuity.....	4
B. Adverse Selection and Other Influences on the Demand for Annuities .....	5
C. Risks Confronting Annuity Suppliers .....	7
D. Previous Studies.....	8
III. Simulations .....	8
A. Sensitivity Analysis for Annuity Premiums.....	8
B. Methodology and Data.....	9
C. Results .....	10
D. Replacement Rates .....	12
E. Methodology and Data .....	12
F. Results.....	13
G. Decreasing Risk .....	15
IV. Conclusions and Future Research.....	16
Tables	
1. Estimated Premium for a One Dollar Annuity.....	10
2. Estimated Replacement Rates and their Variation.....	13
3. Expected Premiums of Portfolios of One Dollar Annuities for the General Male Population.....	15
Figures	
1. Premium for \$1.00 Annuity .....	11
2. United States Yield Curves (1989–2003) .....	14
References .....	18

## I. INTRODUCTION

Employer-provided defined-contribution (DC) pension plans are superseding the traditional defined-benefit plan in the United States. A parallel development on the international scene is the introduction of a DC individual accounts pillar in the public pension systems in a growing number of countries, which will enhance the role of DC plans in the provision of retirement income world wide.

The increasingly dominant role of DC plans has raised concerns over the variability of the income that these plans will provide. In the United States, nearly 58 million people participated in a DC plan in 1998, and they paid \$167 billion in contributions. The value of the assets of these funds totaled \$2,085 billion, or about 20 percent of GDP.<sup>2</sup> The addition of individual accounts to social security, as the President's Commission on Social Security has proposed, would increase the relative importance of DC plans further. Under such a plan, retirement income is at least partly financed by an account whose balance depends on the contribution rate and the rate of return that the account earns, not simply on the contributor's wage and salary history.

Most of the policy debate and research on the variability of DC retirement income has been focused on the accumulation phase, in particular, on the risks attendant on investing contributions in the stock and bond markets. Comparatively little attention has been given to the distribution phase. In studies of the investment risk associated with individual accounts, a standard assumption is that the retirement income per dollar of funds that accumulate in an individual account is a given (see, for example, Feldstein and Rangelova (2001) and Samwick and Skinner (2003)). This approach assumes away a potentially important source of risk to retirement income security.

A contributor to a DC plan approaching retirement confronts the important decision of what to do with the money that has accumulated in his or her account. With a defined benefit plan, that decision is made for him. He will typically receive a life annuity: that is, a regular stream of income, usually fixed in nominal terms and paid monthly for the rest of his life. Provided the plan sponsor is financially sound, a defined benefit pension has the great virtue of providing longevity insurance—insurance against the risk of outliving the wealth accumulated during the pensioner's working life. DC plan participants can also purchase a life annuity, although the life annuity is only one of a large number of annuity or annuity-like products available in U.S. financial markets.

Purchasing an annuity can be welfare enhancing for many DC plan participants, particularly if they will not receive a pension under an employer-provided defined benefit plan and their social security benefit is low relative to their income while working. However, annuity premiums—the cost of a given stream of income—are in principle a function of bond yields and can, as a result vary considerably over time. This variability creates additional risk for the DC plan participant over and above the investment risk he confronts during the accumulation phase. Even if his contributions were predetermined and earned a fixed rate of

---

<sup>2</sup> United States Department of Labor (2001–02).

return during the accumulation phase, he would still not receive a guaranteed retirement income. The DC plan participant cannot be certain before he annuitizes what premium he would have to pay for a given stream of annuity income.

The straightforward way of gauging the variability of annuity premiums over time would be to measure it directly. Since premiums vary by sex and age of the annuitant, a consistent time series would require the estimation of a representative premium for potential annuitants of a given age and sex. Data on annuity premiums offered by up to 140 insurance companies for both sexes and various ages for the period 1915 to 1993 are available from various issues of A.M. Best's Flitcraft Compend. Subsequently, similar information on a smaller number of companies for 1995–96 was published in Best's Review. Comparative Annuity Reports publishes data on the yield of various annuity products based on information provided by upward of 40 insurance companies. These sources have provided much useful information for studies of annuity premiums. It is, however, difficult to extract from them a consistent time series of a representative premium for an immediate life annuity for the more recent past.

This paper uses an indirect method to gauge the potential variability of annuity premiums. Its aim is to shed light on the potential significance of this additional source of risk to DC plan contributors. The basic idea, which is explained more fully below, is that the variability of annuity premiums should reflect the variability of the slope and average height of the yield curve; life insurance companies, at least in the United States, fund their products mainly with investments in the bond market.

The first section provides a brief discussion of the key features of the annuity market. The second section describes the technique used to estimate interest rate variability. The paper then explains how estimates of annuity premiums can be derived from the distribution of bond yields. The same distribution is then used to estimate a spectrum of corresponding replacement rates. Next, a distribution of premiums are estimated for a portfolio of annuities purchased at different times. This strategy decreases variance without changing the expected value of the stream of annuity payments. The last section summarizes the paper's results, and proposes some directions that future research might take.

## **II. BASIC FEATURES OF ANNUITIES AND THE MARKET FOR ANNUITIES**

### **A. The Life Annuity**

Annuities can take many different forms. Specifically, annuities can be classified according to whether the income they pay is fixed or variable; whether the period of payment is for the remaining life of the annuitant (or until both the annuitant and a named beneficiary die), for a fixed term, or for the greater of the remaining life of the annuitant or a specified number of years; whether the purchase is made in a lump sum or a series of installments; whether regular payments begin immediately upon purchase, or are deferred for a specified period and so forth. The annuity considered here, and in much of the literature, is the single premium immediate life annuity—life annuity, for short—which an annuitant purchases with a lump sum and which immediately starts making regular payments fixed in nominal terms that continue until the annuitant dies.

The private annuity market charges women a higher premium for a life annuity than it charges men of the same age, because on average women live longer than men. A 64-year-old male purchasing a life annuity that will pay \$10,000 a year might have paid about \$130,000 in 2004, while a 64 year-old woman would pay an additional \$7,000.<sup>3 4</sup> In contrast, the U.S. social security system and the public pension systems of virtually every other country do not discriminate between men and women. This means that men, by dying earlier on average, subsidize women.

## **B. Adverse Selection and Other Influences on the Demand for Annuities**

In principle, a life annuity can confer a substantial benefit on its purchaser. If annuity premiums reflected the life expectancy of the general population, and administrative costs and profits were not excessive, the conditional rate of return on annuities should exceed the rates on bonds of comparable maturities.<sup>5</sup> Perhaps more importantly, annuities provide longevity insurance. They reduce the risk of both over and under consuming in retirement, and thus raise welfare. Very cautious individuals might accumulate more wealth than they will reasonably need to maintain their accustomed level of consumption in old age; the reckless might accumulate too little or run down their savings too quickly. A life annuity can increase the consumption and welfare of both classes of individuals. Brown (1999) estimates that annuities can increase lifetime utility by as much as 30 percent. The magnitude of welfare enhancement may vary depending on marital status, initial wealth, and degrees of risk aversion (Dushi and Webb (2004)). For many people, annuities should be welfare enhancing. According to the U.S. Government Accounting Office (2003), however, only 7.5 percent of the contributors to DC plans in the United States who retired between 1992 and 2000 received their plan distributions in the form of an annuity.

Many economists, including Friedman and Warshawsky (1990) have attributed the small size of the annuity market to adverse selection, which makes annuities undesirable for the bulk of the population. Adverse selection in the annuity market results from the fact that annuitants have significantly higher survival probabilities than the general population. A retiree with a terminal or life-threatening illness is unlikely to buy an annuity. As a result, insurance companies selling annuities will set premiums that reflect the greater longevity of annuitants. This makes annuities less attractive to the general population.

---

<sup>3</sup> These figures are based on quotations from WebAnnuities.com at <http://www.immediateannuity.com>, accessed on October 4, 2004.

<sup>4</sup> Conceivably, annuity premiums in the United States could use race as a pricing index, since the life expectancy of African Americans is less than that of other Americans of the same age and sex. Such discrimination is illegal, although it would reduce the cost of a private annuity to members of a group whose incomes are significantly below the national average.

<sup>5</sup> The price of an asset in the form of a stream of income that is paid only if the original purchaser is alive will be less than the price of the same asset if the stream of income is paid without conditions.

Some idea of the relative importance of adverse selection can be gleaned from measuring the expected return to annuitants from a life annuity and comparing it with the expected return for the general population. This measure, which Mitchell, Poterba, and Warshawsky (1999) have dubbed the money's worth ratio, or MWR, is calculated by estimating the expected discounted value of a regular payment of one dollar and then dividing the result by the premium per dollar of income (PPD) charged by a group of life insurance companies. The expected value is calculated for a stream of income payments by multiplying the payment in each year by the probability the annuitant will still be alive in that year, discounting to the present, and summing. The formula for the MWR below assumes that annuitants live for a maximum of  $T$  years, and that the probability of surviving to year  $i$  equals  $P_i$ . The discount factor for payments in year  $j$  after annuitization is  $1/(1+r_j)^j$ , where  $r_j$  is the rate of interest at the time the annuity is contracted for bonds of maturity  $j$  years. An insurance company that has a MWR of one would in practice be making a loss, given that it must incur administrative costs.

$$MWR = \frac{\sum_{i=1}^T \frac{P_i}{(1+r_i)^i}}{PPD} \quad (1)$$

Mitchell, Poterba, and Warshawsky estimate the MWR using mortality rates for the general population and the annuity-buying population in the United States. The difference between these ratios, which is an indication of the degree of adverse selection in the annuity market, is sizable. The MWR for a 65-year-old male, using the interest rate on treasury bonds to discount future payments, is estimated to be 0.816 for the general population and 0.916 for the annuity purchasing population. This gap may explain in part why the market for life annuities is so small in the United States and most other countries.

Adverse selection is by no means the only inhibiting influence on the demand for life annuities. The demand for them is reduced by social security's provision of an indexed life annuity (Benitez-Silva (2003)), by the need to provision for unexpected large or lumpy expenditures (e.g. health or nursing home costs), by the attractiveness of competing investments, like a house, and by the desire to leave a bequest. The irrevocable character of a life annuity, whose payments stop with the death of the annuitant, may also reduce its attractiveness. By using other instruments, the annuitant may be able to mimic the desirable features of an annuity without forsaking as much financial flexibility (Milvesky (1998)).

Nonetheless, a life annuity can be a good investment, particularly for an individual without an employer-provided pension to supplement the basic safety net that social security provides. Declines in the relative importance of the social security pension benefits and an increased role for DC plans can be expected to increase the demand for these instruments.

### **C. Risks Confronting Annuity Suppliers**

Life insurance companies face various kinds of risk when they underwrite a life insurance policy or an annuity. Two of particular importance are mortality and investment risk. Mortality risk takes two forms. The first is the risk that with a given life expectancy for the population as a whole, the experience of a particular life insurer is adverse. The second is uncertainty about the life expectancy of the population as a whole. The first type of risk should in principle be inversely related to the number of annuitants, and can be minimized by holding a sufficiently large portfolio of individual policies. The second is a more basic risk of the insurance business. It may be imperfectly hedged by issuing life insurance policies. Governments could create a hedge against this particular risk by issuing “survivor” bonds whose value would vary positively with the actual life expectancy of the population (Blake (1999)). Governments would thus assume this second kind of mortality risk.

Investment risk comes from the fact that the insurance company funds its annuities by buying fixed interest securities. Assuming, for simplicity, that an insurance company’s payments to its annuitants can be accurately estimated, the company could in principle avoid risk entirely by purchasing a portfolio of bonds that is structured so that the amount maturing in a given year plus interest payments on the stock outstanding at that time would equal the estimated annuity payments that would be made in that same year. This would require that the maturity spectrum of bonds encompass the maximum lifespan of annuitants.

In practice, some of the members of a cohort of annuitants aged 65 can live for 35 or more additional years. However, the longest maturity now available for US government bonds off the shelf is 20 years. The maximum maturity for outstanding bonds, auctioned in the secondary market, is about 27 years, which is the maximum period over which the insurance company can accurately predict the rate of return on its assets.<sup>6</sup> If bond returns plunge during the life of the annuity, the insurer may not earn the necessary rate of return to match the expected stream of payments to its annuitants.

In practice, the financing of an annuity is more complicated, since insurance companies invest in equities, which tend to vary negatively with bond prices, and in other assets. In addition, hedging strategies can partially insulate the impact on portfolios of unexpected changes in interest payments. That said, in the case of the United States, investments in credit instruments dominate the portfolios of life insurers. In Europe, equities are a much more important investment (International Monetary Fund (2004)). The bursting of the equity bubble in 2000 has, however, sparked a debate over the wisdom of a heavy equity exposure.

In order to keep its analysis tractable, the paper will make the assumption that life annuities are funded 100 percent by fixed interest securities. This assumption implies that the premium for a life annuity in a competitive market should be a function of the level and shape of the yield curve as well as the life expectancies of annuitants.

---

<sup>6</sup> The U.S. Treasury last issued 30-year bonds at the auction of October 2001.



## D. Previous Studies

Studies in this area are few. The compulsory annuity market in the United Kingdom provides a time series of annuity premiums, which has been used by Cannon and Tonks (2003) and Finkelstein and Poterba (2000).

Cannon and Tonks estimate the relationship between interest rates, annuity premiums and replacement rates for the United Kingdom. They construct a time series of annuity premiums for 1972–2002. They find that over time annuity and U.K. consul (a perpetual bond) rates are highly correlated and that each has a unit root. They reject the hypothesis that the two series are cointegrated, however. For the purpose of their simulations and on the basis of this high correlation they assume the annuity premium can be proxied by the sum of the consul rate and a constant. Cannon and Tonks then estimate replacement rates for a British worker who invests his pension in a variety of assets. Their replacement rates are highly variable, as they range from 0.4 to 1.0. However, this variability reflects both the variability of returns to contributions, and wage variation. It is not clear how much of the variation derives from the annuity premium and how much from the accumulation phase.

## III. SIMULATIONS

### A. Sensitivity Analysis for Annuity Premiums

The estimation of the variation in annuity premiums requires two steps. The first is the estimation of the relationship between the premium, the interest rate term structure and the probability that annuitants survive a given number of years. Equation 2 expresses this relationship in the form of the premium per dollar of annual income (PPD), where  $P_i$  stands for the probability that the typical annuitant will survive to year  $i$ , and  $r_{0j}$  stands for the interest rate on a bond maturing in  $j$  years and issued in year 0. Assuming the insurance company is risk neutral, the expected present discounted value of the annuity payment stream equals the PPD:

$$PPD = \sum_{i=1}^T \frac{P_i}{(1 + r_{0,i})^i}. \quad (2)$$

Equation (2) simply reexpresses equation (1), assuming that the MWR equals one. This assumption would normally mean that the premium per dollar of income would be underestimated, given the existence of administrative costs, and perhaps, relaxing the assumption of risk neutrality, a risk premium to reflect the risks to the insurer just described. Nonetheless, because the paper is focusing on the variability of premiums, and because these latter components of cost should be relatively stable, the basic results should not be materially affected.

The formulation also assumes that the law of large numbers applies, so that insurance companies can predict perfectly the number of annuitants surviving to each year. The estimates of survival probabilities are derived from the U.S. Social Security Administration's cohort life table. Specifically, the data pertain to 64-year-old men and women in 2003. It is

assumed that the annuity is paid for 30 years or until the pensioner dies, whichever comes first. The omission of payments in years 31 and greater is not really consequential, because the probability of surviving to age 95 is small enough that excluding the final payments made to the annuitants who live past that age will have only a marginal impact on the results.

Although the U.S. Treasury has stopped issuing 30-year bonds, it still quotes a long-term average rate. This is the average yield of securities that mature in 25 or more years; it is based on quotations for bonds on the secondary market with an original maturity of 30 years. By October 2011, assuming no longer-term securities are issued, there will no longer be securities that mature in more than 20 years. Consequently, it will be that much harder for an annuity provider to match its expected liabilities with a portfolio of assets of equivalent maturities.

## B. Methodology and Data

Because the distribution of annuity premiums is determined by the variation in bond yields, the second step of the procedure requires the creation of a frequency distribution for interest rates across the maturity spectrum. The method used was to derive estimates of implicit forward one-year interest rates from the available term structure data. The one-year future rate for period  $t-1$ ,  $r_{t-1,t}$ , is calculated from equation (3) below, where  $r_{0,t}$  is the interest rate on a bond issued in period 0 and maturing in period  $t$ , and  $r_{0,t-1}$  is the interest rate on a bond issued in period 0 and maturing in period  $t-1$ .

$$r_{t-1,t} = \frac{(1+r_{0,t})^t}{(1+r_{0,t-1})^{t-1}} - 1 \quad (3)$$

As an example, an observation on the term structure of interest rates on a given day includes estimates of interest rates on securities with a remaining maturity of one and two years. The formula would then be used to calculate the implied or expected rate on a one-year bond maturing in year two (i.e.  $r_{1,2}$ ).

The term structure data set is not complete. The vector of interest rates observed on a given day includes the rates of bonds with maturities of one, two, three, five, seven, ten, twenty, and thirty years. If no maturities are available between year  $t$  and year  $t+n$ , the one-year forward rates for each of the  $n$  years in the period from  $t$  to  $t+n$  would be calculated using the following formula (equation (4)), where  $r_{t,t+1}$  stands for the one-period forward rate for each year between  $t$  and  $t+n$ .

$$(1+r_{t,t+1}) = \sqrt[n]{(1+r_{0,t+n})^{t+n} / (1+r_{0,t})^t} \quad (4)$$

Once the short rates for all available maturities are calculated, they can be used to construct a complete term structure that can be simulated. Calculating the short rates is expedient because their first and second moments can be easily derived and used in a simulation to construct a complete term structure.

The data used in the estimation of the short rates are daily U.S. treasury bond yields from the period January 1990 through January 2002. The mean of each short rate and the covariance matrix are estimated and used as parameters to simulate short rates following a multivariate normal distribution (equation (5)) where  $\mu_t$  is the mean of each one year forward rate and  $\Sigma_{t-1,t-1}$  is an element of the covariance matrix.

$$r_{t-1,t} \sim N(\mu_{t-1,t}, \Sigma_{t-1,t-1}) \quad (5)$$

The study assumes simply that the forward rates implied by a given term structure will have a distribution that can be inferred from a large sample. The basic assumption is that the variance–covariance matrix of the vector of future short-term rates is stable. Each vector of simulated one-year forward rates is used to construct one observation for the term structure. This method gives an accurate distribution of interest rates that can be used in equation (2) to estimate a distribution of annuity premiums.

The mortality statistics used here are for the general population. Mitchell, Poterba, and Warshawsky (1999) point out that the insurance company actually uses the mortality rates of the annuity buying population. To see how adverse selection affects the premium it is also estimated using cohort adjusted mortality rates for 65-year-old male annuity buyers.<sup>7</sup>

### C. Results

The distribution of forward one-year rates is simulated 100,000 times. The summary statistics for the corresponding premiums for men and women and male annuity buyers are given in Table 1.

Table 1. Estimated Premium for a One Dollar Annuity

	Mean	Standard Deviation	Maximum	90th percentile	Median	10th percentile	Minimum
Male general population	\$8.99	\$0.68	\$12.37	\$9.89	\$8.96	\$8.15	\$6.93
Female general population	\$9.97	\$0.82	\$14.54	\$11.04	\$9.91	\$8.96	\$7.34
Male annuitant population	\$9.74	\$0.78	\$14.02	\$10.76	\$9.69	\$8.78	\$7.15

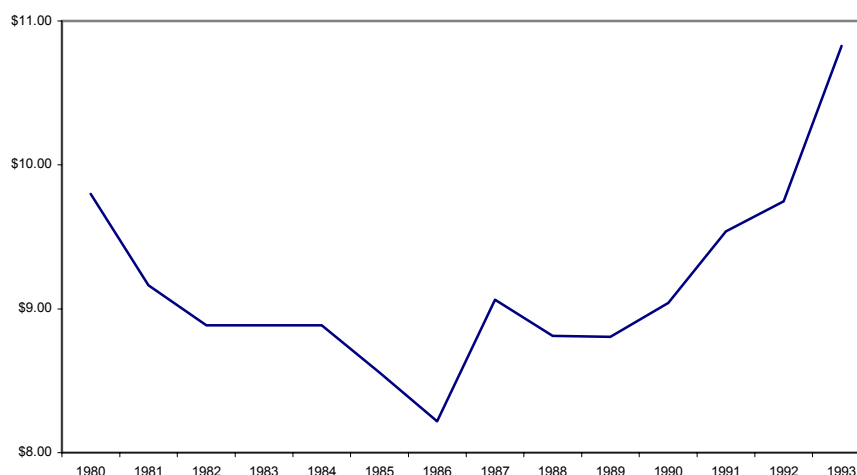
For both sexes, the premium per dollar is substantially lower than the premium per dollar that annuity providers are now quoting. For a 65-year old man, the estimated mean premium for an annuity that pays \$10,000 a year is \$89,983 with a standard deviation of \$6,824. The population that falls into the tail of the distribution pays a significantly higher premium. Specifically, an annuitant can expect to be charged a premium that is about \$15,000 more than the mean with a probability of about 5 percent.

<sup>7</sup> Available from the Society of Actuaries.

Premiums for women are significantly higher than premiums for men. A woman can expect to pay about 8.3 percent more for the same annuity payment. Because the same interest rate distribution is used, the variance as a percent of the premium is not greatly changed. However, the standard deviation, about \$8,200, is a larger fraction of the annual annuity payment. Discriminating by gender thus entails two disadvantages for women. They can expect to pay more than men on average, and the standard deviation of their premium is a larger fraction of the income that the annuity pays.

Because A.M. Best's data are not readily available for 1990–2002, the average premium offered by five large firms was calculated for an earlier period, namely, 1980–1993 (see figure 1).<sup>8</sup> The average premium during this earlier period for an annuity that paid \$12,000 a year to a 65-year-old male was \$104,933 with a standard deviation of \$10,841. A women's average premium was \$113,760 with a standard deviation of \$11,277. The 1980–1993 period and the 1990–2002 period are not directly comparable; the large standard deviation in the data from the earlier period is most likely the result of the large fluctuations in bond yields during the 1980s and their generally higher level.

**Figure 1. Premium for \$1.00 Annuity  
(5 large firms)**



Source: A.M. Best.

The general decline in nominal interest rates since the 1980s probably accounts for the fact that the average premium for the earlier period is lower than the premium simulated with data from 1990–2002, even though the simulations assumed no administrative costs. This inequality holds for both annuity buyers and the general population. In the annuity-

---

<sup>8</sup> Because data are unavailable on the volume of all individual firm sales, it is impossible to estimate a sample premium. Following Warshawsky (1988) and Mitchell et al. (1999), the mean premium offered by large firms is selected to represent the market.

buying estimation the premium per dollar is \$9.74 while the historical data imply a premium per dollar of \$8.74. Nonetheless, the variance of the A.M. Best data does not differ hugely from the simulated variance.

#### D. Replacement Rates

The replacement rate—the ratio of retirement income to average disposable income at the end of working life—is an indicator of how a person fares in retirement relative to his working life. To determine how the variation in annuity premiums will actually affect the standard of living of a participant in a defined contribution plan, the replacement rate is estimated using illustrative assumptions about salary growth, the contribution rate, and the rate of return to contributions.

#### E. Methodology and Data

It is assumed that the contributor works and saves for  $N$  years. Each year his income grows at the rate  $g$  and he saves a fraction,  $s$ , of it. His savings go into a pension fund, from which no withdrawals are permitted, that earns a certain annual rate of return  $r$ . When the contributor retires, he uses all of the funds accumulated in his account to purchase an annuity. Given these assumptions, the precise dollar amount of his starting income has no bearing on what his replacement rate will be. An increase in that amount will simply raise the value of his accumulated savings proportionately.

With these assumptions, the value of the premium is determined by equation (6), where  $W_0$  is the starting salary:

$$Premium = \sum_{i=1}^N s \cdot W_0 \cdot (1+g)^{i-1} \cdot (1+r)^{N-i+1} \quad (6)$$

As before, it is assumed that the contributor/pensioner receives a fixed nominal annuity. He is assumed to live for a maximum of  $M$  years in retirement. The insurance company is risk neutral, faces no transactions costs, and offers an actuarially fair premium, so that the annuity premium is equal to the expected present discounted value of the stream of annuity payments. Consequently,  $A$  is determined by equation (7):

$$A = \frac{Premium}{\sum_{i=1}^M \frac{P_i}{(1+r_{0,i})^i}} \quad (7)$$

The replacement rate ( $RR$ ) is defined to be the ratio of the annuity payment to average income, net of saving, during the last  $n$  years of work. Income taxes are not taken into account, so that the replacement rate can be expressed as equation (8):

$$RR = \frac{\sum_{i=1}^N s \cdot (1+g)^{i-1} \cdot (1+r)^{N-i+1}}{\frac{(1-s)}{n} \left[ \sum_{i=N-n+1}^N (1+g)^i \right] \sum_{i=1}^M \frac{P_i}{(1+r_i)^i}} \quad (8)$$

For the purposes of the simulations, the parameters of this simple model assume the following values:  $g$ , the rate of growth—3 percent;  $r$ , the rate of return—3 percent; and  $s$ , the rate of saving—10 percent;  $N$ , working years—40;  $M$ , potential retirement years—30;  $n$ , number of years at end-career used for replacement ratio calculation—5.

## F. Results

The short rates are again simulated 100,000 times, and a replacement rate is calculated for each simulation. The summary statistics of these figures for men and women in the general population and the male annuity-buying population are given in Table 2.

Table 2. Estimated Replacement Rates and their Variation

	Mean	Standard Deviation	Maximum	90th percentile	Median	10th percentile	Minimum
Male General Population	0.54	0.041	0.74	0.59	0.54	0.49	0.35
Female General Population	0.49	0.040	0.65	0.54	0.48	0.44	0.33
Male Annuitant Population	0.50	0.039	0.72	0.55	0.50	0.45	0.35

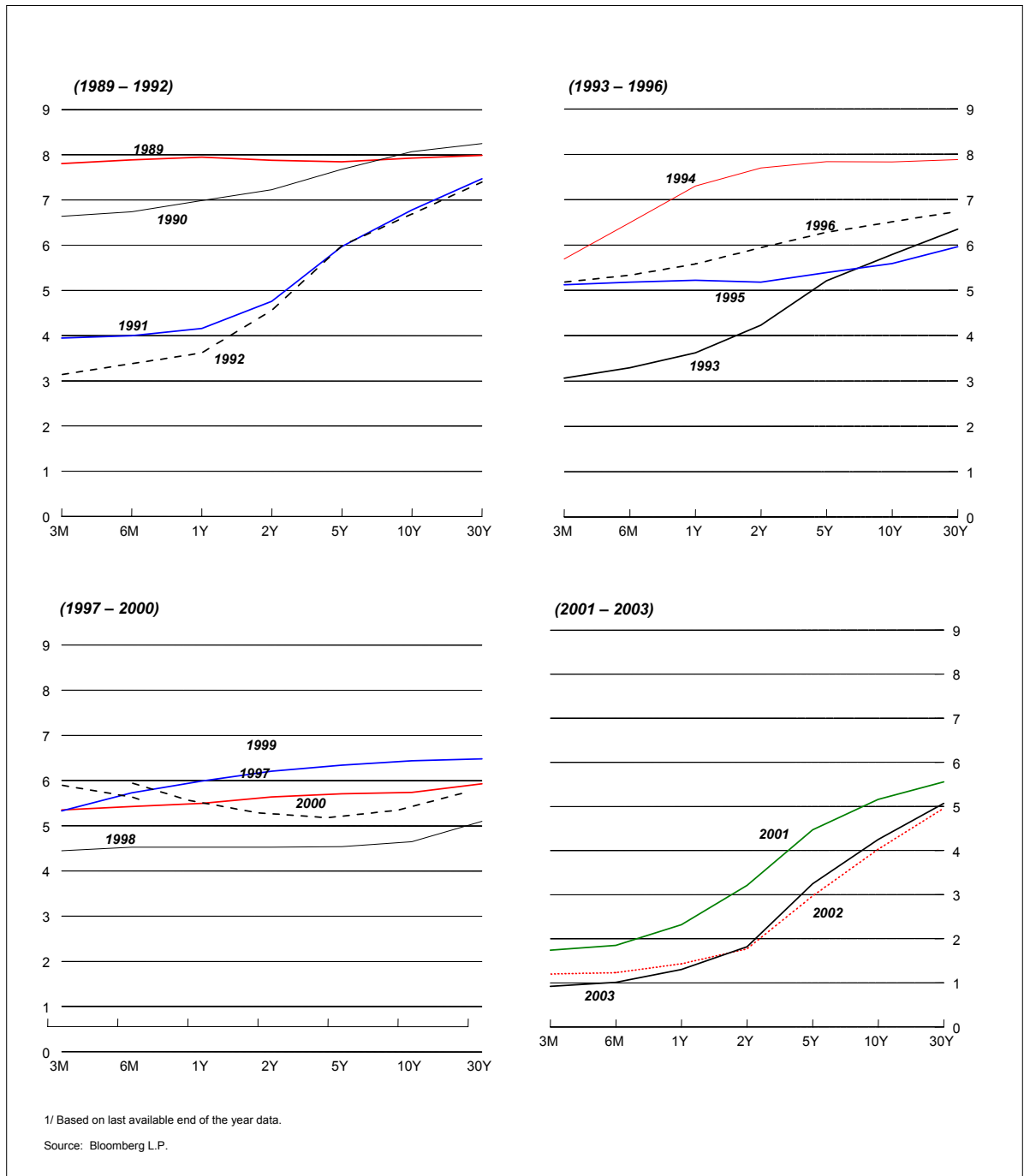
The mean replacement rate for the general male population is about 54 percent, and the standard deviation is about 4 percent. While the standard deviation of the distribution of replacement rates is not large, there is a small probability that a male could have a replacement rate as low as 35 percent and as high as 74 percent of his working income.

A woman's replacement rate is about 5 percentage points, or 10 percent lower than that of the average man. The standard deviation is almost equal. If an individual accounts reform of social security were to completely or largely eliminate the retirement benefit that the Old-age, Survivors and Disability Insurance program now pays, price discrimination by sex would raise some important public policy issues. The complications entailed by price discrimination are mitigated to some extent by the risk pooling that accompanies marriage or cohabitation, since couples can each annuitize their income or purchase a joint annuity. Their total annuity income will be a weighted average of the rate at which each annuitizes. However, the impermanence of contemporary marriage makes this a less than an ideal solution.

The simulated replacement rate of a male annuitant is about 4 percentage points lower than that of the general population, although its standard deviation is nearly identical.

To judge from the considerable fluctuations to which short-term interest rates are subject, the estimated variance in annuity prices could seem modest. However, medium-and long-term interest rates, which would in principle account for most of the variation in annuity premiums, vary less than short-term rates. This is shown in the fluctuations of the yield curve's slope throughout the 1990s in the figure below.

Figure 2. United States Yield Curves (1989–2003) 1/  
(in percent)



### G. Decreasing Risk

To alleviate the variability of annuity prices, Alier and Vitas (2001) recommend that retirees make staggered purchases over a period of several years. This strategy builds a portfolio of annuities over a period of time before retirement. It will hedge some of the interest rate risk to the extent that it allows the annuitant to smooth the effect of fluctuations of bond prices on his retirement income.

To see how purchasing multiple annuities decreases premium variance, the simulated bond yields are used to estimate the average premium a retiree would pay if he bought a portfolio of annuities with the purchases staggered over time. It is assumed that the regular payment of each annuity that the retiree buys is the same. If he wanted \$1 of annuity income and his portfolio consisted of two annuities, he would receive \$0.50 from each. Two additional simplifying assumptions are made: that the annuities begin to make payments at the same time, and that bond yields are not correlated over time. Consequently, the premium per dollar of the portfolio is the mean of the premiums for each annuity. The mean and standard deviation of the expected premium for portfolios of different sizes are given in Table 3.

Table 3. Expected Premiums of Portfolios of One Dollar  
Annuities for the General Male Population

Number of annuities in portfolio	2	5	10
Mean	\$9.00	\$8.99	\$9.00
Standard Deviation	\$0.48	\$0.30	\$0.22

Even if the annuitant buys just two annuities, the standard deviation of the premium drops by nearly 30 percent, while the expected value does not change. The more annuities the retiree buys, the smaller the variance, which converges to zero as the portfolio grows.<sup>9</sup>

These findings do not imply that a strategy of “annuity diversification” is optimal. The model neglects the fixed cost element in the purchase of an annuity, which can make multiple annuity purchases quite expensive. The marginal benefit to diversifying diminishes

---

<sup>9</sup> Assuming costless financial intermediation and full knowledge of survival probabilities, the premium for a deferred life annuity should be less than the premium for an immediate annuity. Specifically, the premium for a deferred annuity purchased in year  $t-1$  should equal the premium for an immediate annuity times a discount factor equal to  $(1/(1+r_t)) * P_t | t-1$ , where  $r_t$  is the one-year interest rate and  $P_t | t-1$  is the probability of surviving to year  $t$  conditional on being alive in the previous year. If this term, which would be close to one for annuitants aged about 65, is ignored, then the forward value of the premium for the deferred annuity in the year  $t$  should equal the premium of the immediate annuity.



as more annuities are purchased. Modeling the optimal number of annuities in a portfolio would require some sort of assumption about transactions costs. In addition, as Blake (1999) points out, a market for deferred annuities requires even longer-range mortality projections than the market for immediate annuities. Blake suggests that this property explains the thinness of the market (speaking of the United Kingdom).

#### **IV. CONCLUSIONS AND FUTURE RESEARCH**

Volatility in the asset accumulation phase of a DC pension is not the only source of retirement income risk. It may be optimal for a retiree with a DC scheme to annuitize at least a part of his accumulated contributions, but when he does so he exposes himself to another source of risk. The paper's working hypothesis is that annuity prices are determined by the interest rates on government debt. As a result, treasury bond yield fluctuations cause uncertain annuity premiums. In the simulations the paper describes, the standard deviation of the replacement rates for a pensioner who faces no other source of risk is about 4 percentage points. This is not a trivial variation.

Given the critical assumption of no transactions costs (or at least no lump-sum element in the cost), a strategy of phased purchases of a portfolio of annuities reduces the variance while maintaining the same expected value. Given the fixed cost element, perhaps a one-time contract of staggered annuity purchases might be a way of reducing variance without excessive administrative cost. However, for a prospective retiree to commit himself in this manner might require considerable foresight, and might seem to many like an unattractive limitation of the freedom to choose. Deferred annuitization also aggravates the funding problem for the provider.

The annuity literature has emphasized repeatedly that adverse selection distorts the annuity market. The simulations here show that the costs of this distortion to the consumer are substantial. In the U.S. market, an annuitant can pay about 8 percent more for an annuity than he would in the absence of adverse selection. The replacement rate is on average about 4 percent lower than it would be if annuity premiums reflected the longevity of the general population. Adverse selection is not the focus of this paper, but it is an important part of the annuity market and certainly worth considering further in future research.

Price discrimination on the basis of gender does not alter the variance of retirement income, but does mean that women can expect a replacement rate that is less than that for men. As a larger percentage of the population purchases annuities it is conceivable that premiums may discriminate by race and socioeconomic status, as well as by gender. If private annuity income were to replace the Social Security benefit, discrimination would become a key issue in pension reform.

As for directions for future research, the replacement rates could be estimated for a person who faces both asset and wage risk. The simple model we have used could be enriched further by including taxes and different income classes. Taking account of asset risk would be particularly important if an asset portfolio is heavily invested in equities because the realized rate of return, and hence the funds available to buy an annuity, is greatly affected by the timing of retirement. According to Alier and Vittas (2001), the replacement rate of an

American fully invested in equities would have been 180 percent higher if he retired in 1995 instead of 1999. It is also important to know whether lower-than-average annuity premiums offset the impact on retirement income of below average stock market performance during working life, or vice versa. This correlation of premiums with stock market returns would tend to reduce the unpredictability of retirement income.

Another direction is to consider inflation risk. If consumers are sufficiently concerned about inflation risk they may be hesitant to purchase nominal annuities. The market for indexed annuities can be expected to develop as the stock of Treasury Inflation-Protected Securities (TIPS) increases. As the market for TIPS develops a history, it should be possible to simulate the impact of real interest rate fluctuations on the premiums of indexed annuities.

Annuities are generally regarded as a risk-free way to guarantee retirement income until death. The paper has shown that annuities are not immune from risk, inasmuch as the premium per dollar of regular income can vary substantially.

## REFERENCES

- Alier, Max, and Dimitri Vittas, 2001, "Personal Pension Plans and Stock Market Volatility," in *New Ideas About Old Age Security: Toward Sustainable Pension Systems in the 21st Century*, ed. by Robert Holzman and Joseph E. Stiglitz (Washington: World Bank).
- Benitez-Silva, Hugo, 2003, "The Annuity Puzzle Revisited," MRRC Working Paper WP2003-05 (Ann Arbor, MI: Michigan Retirement Research Center).
- Blake, David, 1999, "Annuity Markets: Problems and Solutions," *The Geneva Papers on Risk and Insurance—Issues and Practice*, Vol. 24, No. 3 (July).
- Brown, Jeffrey R., 1999, "Private Pensions, Mortality Risk, and the Decision to Annuitize," NBER Working Paper No. 7191 (Cambridge, MA: National Bureau of Economic Research).
- , Olivia S. Mitchell, James M. Poterba, and Mark J. Warshawsky, 2001, *The Role of Annuity Markets in Financing Retirement* (Cambridge, MA: MIT Press).
- Burtless, Gary, 2002, "How Would Financial Risk Affect Retirement Income Under Individual Accounts?" An Issue in Brief Center for Retirement Research at Boston College No. 5 (Chestnut Hill, MA: Center for Retirement Research Boston College).
- Cannon, Edmund, and Ian Tonks, 2003, "UK Annuity Rates and Pension Replacement Ratios 1957–2002," UBS Pensions Research Program Discussion Paper No.8 (London: London School of Economics).
- Dushi, Irena and Tony Webb, 2004, "Annuitization: Keeping Your Options Open," CRR Working Paper 2004-04 (Chestnut Hill, MA: Center for Retirement Research Boston College).
- Feldstein, Martin and Elena Rangelova, 2001, "Individual Risk in an Investment-Based Social Security System," NBER Working Paper No. 8074 (Cambridge, MA: National Bureau of Economic Research).
- Finkelstein, Amy and James Poterba, 2000, "Adverse Selection in Insurance Markets: Policyholder Evidence from the UK Annuity Market," NBER Working Paper No. 8045 (Cambridge, MA: National Bureau of Economic Research).
- Friedman, Benjamin M. and Mark Warshawsky, 1990, "The Cost of Annuities: Implications for Saving Behavior and Bequests," *Quarterly Journal of Economics*, Vol. 105, No. 1, pp. 135–154.
- International Monetary Fund, 2004, *Global Financial Stability Report*, World Economic and Financial Surveys (Washington, April).

- James, Estelle and Dimitri Vittas, 1999, "Annuity Markets in Comparative Perspective: Do Consumers Get Their Money's Worth?" paper prepared for World Bank Conference, "New Ideas on Old-Age Security" (Washington).
- Milevsky, Moshe, 1998, "Optimal Asset Allocation Towards the End of the Life Cycle: To Annuitize or Not to Annuitize?" *The Journal of Risk and Insurance*, Vol. 65, No. 3, pp. 401–426.
- Mitchell, Olivia, James Poterba, and Mark Warshawsky, 1999, "New Evidence on the Money's Worth of Individual Annuities," *American Economic Review*, Vol. 89, No. 5, pp. 1299–1318, reprinted as Chapter 4 in Brown et al., 2001.
- Poterba, James, 1997, "The History of Annuities in the United States," NBER Working Paper No. 6001 (Cambridge, MA: National Bureau of Economic Research).
- Samwick, Andrew, and Jonathan Skinner, 2003, "How Will 401(k) Pension Plans Affect Retirement Income?" *American Economic Review*, Vol. 94, No. 1, pp. 329–343.
- U.S. Government Accounting Office, 2003, "Private Pensions: Participants Need Information on Risks They Face in Managing Pension Assets at and During Retirement," Report to Congressional Requestors GAO-03-8 (Washington).
- U.S. Department of Labor, Pension, and Welfare Benefits Administration, 2001–2002, "Private Pension Plan Bulletin Abstract of 1998 Form 5500 Annual Reports," Number 11 (Winter).
- Warshawsky, Mark, 1988, "Private Annuity Markets in the United States," *Journal of Risk and Insurance*, Vol. 55, No. 3, pp. 518–528, reprinted as Chapter 3 in Brown et al., 2001.