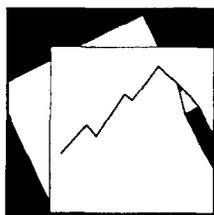


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## Assessing Fiscal Sustainability: A Cross-Country Comparison

*Enzo Croce and V. Hugo Juan-Ramón*

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**Assessing Fiscal Sustainability: A Cross-Country Comparison**

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**Abstract**

The views expressed in this Working Paper are those of the author(s) and do not necessarily represent those of the IMF or IMF policy. Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate.

To monitor fiscal sustainability, this paper proposes a recursive algorithm derived from the law of motion of the debt-to-GDP ratio, subject to a government reaction function that links convergence to the targeted debt ratio with primary fiscal surpluses. Based on quarterly estimates of this algorithm in the 1990s, 12 developed and developing countries are ranked according to their degree of sustainability. For a number of countries, the paper finds evidence of causality between the fiscal policy stance and growth-adjusted real interest rates.

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## I. INTRODUCTION

Determining whether the current fiscal policy stance is sustainable is important for policymakers because the answer may indicate the need for policy correction. There is consensus in the literature on the definition of fiscal sustainability. The concept refers to the future implications of current fiscal policies and, more precisely, to the question of whether the government can continue to pursue its set of budgetary policies without endangering its solvency. Yet, fiscal sustainability has proven difficult to measure. This paper proposes an operational criterion for assessing fiscal sustainability.

A fiscal policy stance can be thought of as unsustainable if over time it leads the government away from solvency. Thus, a good starting point for assessing fiscal sustainability is to check whether the conditions for government solvency are met. Following the theoretical literature, checking for solvency implies adopting a forward-looking approach that involves projecting future tax and spending measures—as well as a forecast of GDP growth and real interest rates—to determine whether the intertemporal budget constraint is satisfied.<sup>2</sup> However, solvency is only a necessary condition for fiscal sustainability: it requires that debt be fully repaid at some point in the future, even though *present* policies may not satisfy the government's intertemporal budget constraint.<sup>3</sup> By contrast, sustainability requires that solvency is achieved under *unchanged* fiscal policy stance.

The empirical literature has proposed two main approaches to assess sustainability in practice: sustainability tests and sustainability indicators. Sustainability tests aim at verifying whether the solvency condition has held for past budgetary policies and, based on these results, trying to infer lessons for the future.<sup>4</sup> The results have been mixed because the tests are very sensitive to the quality and quantity of data used and to the statistical procedures applied to the data: hence their findings have not been consistent, even when applied to the same countries and periods. But the main limitation of these tests as a guide to policymaking is that solvency within a sample period does not guarantee solvency in the future.

Following Buitert (1985) and Blanchard (1990), numerous proposals suggest using synthetic indicators to gauge fiscal sustainability in a way that allows for a simple interpretation of the results. The criteria followed for most of these indicators is whether current fiscal policies can stabilize either the ratio of public sector net worth to GDP (based on Buitert) or the debt-to-GDP ratio (based on Blanchard). But given the difficulty in obtaining reliable information

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<sup>2</sup> See Buitert (1985), Spaventa (1987), and Blanchard (1990).

<sup>3</sup> Even in the case of a country running large fiscal deficits and expected to continue to do so for many years, solvency can be formally reinstated by assuming that very large budgetary corrections will take place some time in the distant future, without specifying the technical and political economy considerations that would make those adjustments feasible.

<sup>4</sup> The classical reference is Hamilton and Flavin (1986), who tested whether the U.S. data supported the hypothesis that the transversality condition was met during the postwar years.

on government net worth, many studies favor the debt-to-GDP criterion. In this connection, the indicators are calculated by projecting government revenue and expenditure based on current policies. The estimated primary deficits and tax ratios are then compared with the permanent primary deficit (primary gap indicator) or the permanent tax ratio (tax gap indicator) required to keep the debt ratio constant. The resulting gaps will provide a measure of the sustainability of the current fiscal policy stance.

To monitor fiscal sustainability, this paper proposes an operationally simple recursive algorithm that is derived from the law of motion of the debt-to-GDP ratio, subject to the government's reaction function: when the actual debt-to-GDP ratio exceeds the target ratio, the government reacts by generating a primary surplus that is consistent with the convergence of the debt ratio to that target. Once this is achieved, the algorithm is expected to anchor policies to ensure that the target ratio is maintained over time. The sustainability indicator generated by this framework is similar to (albeit more general than) that proposed by Blanchard.<sup>5</sup> One advantage of our indicator over previous ones, including that of Blanchard, is that no estimations of future GDP and interest rates are required: the indicator generates its results based on current, past, and target values of relevant variables.

A sustainability indicator that it is easy to calculate and can be updated frequently (say on a quarterly basis) can help increase fiscal transparency, as long as the evolution of the algorithm and convergence to the debt-to-GDP target ratio can be monitored by the public. By announcing its commitment to keeping, on average, the value of the algorithm within the convergence region and making adherence to this objective easy to check, the government will not need to explain either its reaction function or any discretionary measures it deems appropriate. This fiscal policy strategy would be akin to the monetary policy strategy under inflation targeting. The adoption of an explicit target debt ratio requires government commitment to policy consistency. Consequently, it would follow a rules-based strategy. At the same time, it would leave to government discretion the decision on how to respond to unforeseen shocks.

The importance of identifying criteria to assess fiscal sustainability is discussed in a recent IMF paper (<http://www.imf.org/external/np/pdr/sus/2002/eng/052802.pdf>) which undertakes both baseline medium-term projections (ensuring consistency and clarity about the assumptions) and stress tests for deviations from the baseline in the assessment of sustainability. This paper also emphasizes the importance of monitoring the evolution of key indicators on a permanent basis. In this connection, our recursive fiscal sustainability indicator complements the projection scenarios, thus adding crucial inputs to the process of assessing sustainability. For example, it might happen that an improving baseline projection coincides with deteriorating current indicators, which should force the authorities to either revise the baseline projection or to make the case that the deterioration of current indicators

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<sup>5</sup> The difference between the fiscal sustainability indicator proposed in this paper and Blanchard's will be discussed in detail in Appendix V.

is temporary and does not affect sustainability in the long run. In this connection, the IMF paper also underscores, as we do in this paper, that appropriate coverage, good quality, and timeliness in the provision of data are essential for assessing sustainability under any framework.

To test how the algorithm will work in practice, we calculate the quarterly values of the indicator throughout the 1990s for 12 countries: 4 in the Western Hemisphere (Argentina, Brazil, Mexico, and the United States), 5 in Europe (Belgium, Ireland, Italy, Sweden, and Turkey), and 3 in Asia (Indonesia, Korea, and Thailand). This sample represents a very diverse group of countries in terms of their macroeconomic stability, public indebtedness, and composition and variability of public expenditures. All these countries, with the exception of the United States, have experienced episodes of currency crisis during the 1990s, which were followed by fiscal consolidation efforts. Therefore, the countries and period selected provide a good sample to test our indicator under a variety of conditions. Specifically, based on the behavior of its indicator, each country is classified according to its degree of sustainability. We also explore how different types of public expenditures affect fiscal sustainability. We found that unsustainable countries experienced larger increases in certain categories of public spending as a percent of GDP—wages, subsidies, and other current transfers—than countries classified as sustainable.

## II. CONCEPTUAL AND OPERATIONAL FRAMEWORK

### A. Fiscal Sustainability: Conceptual Aspects

Fiscal sustainability (and solvency) must be assessed in the context of the intertemporal budget constraint of the public sector. To keep the algebra simple, we will assume that net privatization proceeds, seigniorage, and revaluations of assets and liabilities are equal to zero. The financing needs of the public sector are defined as:

$$PSBR_t = (D_t - D_{t-1}) = PD_t + i_t D_{t-1} \quad (1)$$

That is,  $PSBR_t$ , the public sector borrowing requirement at time  $t$ , induces a change in the stock of total (domestic and foreign) debt,  $(D_t - D_{t-1})$ , to finance the primary deficit,  $PD_t$ , and interest payments on public debt,  $i_t D_{t-1}$ . Multiplying equation (1) by  $-1$ , we obtain:

$$PS_t = i_t D_{t-1} - (D_t - D_{t-1}) \quad (2)$$

where  $PS_t$  is the primary surplus of the public sector. By dividing equation (2) by GDP and rearranging terms, we obtain the law of motion of the debt-to-GDP ratio,  $d_t$  (the “debt ratio” henceforth):

$$d_t = \beta_t d_{t-1} - ps_t \quad (3)$$

in which  $ps_t$  represents the ratio of the primary surplus to GDP (the “primary surplus ratio” henceforth), and  $\beta_t = (1 + r_t) / (1 + g_t)$ ,  $r_t$  being the real interest rate and  $g_t$  the real GDP growth. This equation states that, in the absence of shocks and corrective policies,  $d_t$  will

increase over time in the presence of persistent primary fiscal deficits coupled with a real interest rate higher than the growth rate. Assuming for simplicity that  $\beta_{t+i} = \beta$ , that is, the discount factor stays constant from time  $t$  to time  $t+N$ , and solving equation (3) forward recursively for  $N$  periods, we obtain:<sup>6</sup>

$$d_t = \beta^{-1} ps_{t+1} + \beta^{-2} ps_{t+2} + \dots + \beta^{-N} ps_{t+N} + \beta^{-N} d_{t+N} \quad (4)$$

From equation (4) we can establish the formal condition for solvency: the public sector is solvent when the present discounted value of future primary surpluses is equal to the value of its outstanding stock of debt. This implies that  $d_{t+N} = 0$ , so that the last term of equation (4) is equal to zero. In other words, the public sector cannot be a net debtor in present value terms. This represents the strict condition for solvency, which requires that at some point the primary balance has to become positive.

More pragmatically, a broader (less stringent) condition for solvency can be derived by imposing weaker conditions on equation (4), for example, by requiring  $d_{t+N} = d^*$ , where  $0 < d^* < d_t$ . Under this definition, the present value of expected primary surplus ratios will reduce the debt ratio below the current level. The operational recursive algorithm that we propose in the next section is akin to this concept.

The above criterion for fiscal solvency will ensure that the intertemporal budget constraint is satisfied. It thus represents a necessary condition for fiscal sustainability. However, clear policy prescriptions cannot be derived from these expressions. The key difficulty is that all relevant variables are endogenous, so that the effect of fiscal measures on growth may in turn affect government revenues and expenditures, and interest rates, along with private savings and investment behavior. By contrast, solvency indicators implicitly assume that the projected paths of the primary balance, interest rates, and economic growth and inflation are independent. Thus, we need specific assumptions on the behavior of each of these variables to determine whether the current fiscal policy stance is sufficient to ensure its sustainability. At the same time, shocks that affect income growth and interest rates may affect the ability or willingness of the government to initiate fiscal consolidation to satisfy its intertemporal budget constraint. The indicator proposed in the next section tries to overcome these limitations.<sup>7</sup>

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<sup>6</sup> The constant discount factor can be thought of as the weighted average of future real interest rates adjusted for growth. As will be shown in Section II.B, when  $\beta_{t+i}$  varies over time the expression for the intertemporal budget constraint becomes somewhat more complex than in equation (4). The formal derivation of the latter equation, with full account given to the domestic and external component of debt, is presented in Appendix III.

<sup>7</sup> Obviously, the only way to fully address the issue of endogeneity is to specify sustainability within the framework of a general equilibrium model.

## B. Fiscal Sustainability: An Operational Recursive Algorithm

The framework used to derive our fiscal sustainability indicator includes equation (3), the law of motion of the debt ratio, and two additional equations needed to define our target variables and the government reaction function:

$$d_t = \beta_t d_{t-1} - ps_t \quad (3)$$

$$ps^* = (\beta^* - 1)d^* \quad (5)$$

$$ps_t = ps^* + \lambda_t (d_{t-1} - d^*) \quad (6)$$

In equation (5),  $ps^*$  and  $\beta^*$  are, respectively, the primary surplus ratio and the discount factor that would prevail once convergence to  $d^*$ , the target debt ratio, is attained. Equation (6) decomposes the primary surplus ratio into two components: (i) the primary surplus ratio ( $ps^*$ ) associated with the target debt ratio and (ii) the policy response to the gap between the observed debt ratio and the target debt ratio. The parameter  $\lambda_t$  indicates the intensity of the policy response at time  $t$ , given the debt-ratio gap in the previous period. Equation (6) is, therefore, a linear adjustment equation, that characterizes a fiscal rule or a policy reaction function.

From equations (3), (5), and (6), we derive the law of motion of the debt ratio, which includes the policy reaction parameter  $\lambda_t$ :

$$d_t = (\beta_t - \lambda_t) d_{t-1} - (\beta^* - \lambda_t - 1) d^* \quad (7)$$

Assuming that the debt ratio at time  $t-1$  is higher than the long-term objective for that ratio—that is, assuming  $d_{t-1} > d^*$ —equation (7) states that  $d_t$  would converge to  $d^*$ , if and only if  $|\beta_t - \lambda_t| < 1$ . Therefore, we propose to use  $(\beta_t - \lambda_t)$  as an indicator of fiscal sustainability:

$$IFS_t = (\beta_t - \lambda_t) = \left( \frac{1 + r_t}{1 + g_t} - \frac{ps_t - ps^*}{d_{t-1} - d^*} \right) \quad (8)$$

Values of  $IFS_t$  below 1 indicate a sustainable fiscal position, while values consistently above or equal to 1 signal unsustainability.<sup>8</sup>

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<sup>8</sup> The IFS indicator aims at gauging sustainability without explicit reference to the time frame set by the authorities to achieve convergence to the target debt ratio. In Appendix IV, we present an alternative, but closely related indicator which takes explicitly into account the time dimension of fiscal sustainability. Also, for the sake of simplicity, we do not show

The first component of the IFS algorithm,  $\beta_t$ , measures the spread between the observed real interest rate and the observed rate of growth at time  $t$ . This component is a lead indicator: a persistently higher spread would, other things being equal, lead to higher public indebtedness. The expected value of  $\beta$  should be about 1 for mature stable economies, higher than 1 for economies with relatively scarce capital and high financial intermediation costs, and much higher than 1 and more volatile in a context of economic and political uncertainty that generate expectations of rising inflation and default. We rule out the case of  $\beta^* < 1$ , which implies  $r < g$ , because this would lead, in a steady state, to “inefficient capital overaccumulation,” as indicated by Barro (1976).<sup>9</sup>

The second component of the algorithm,  $\lambda_t$ , measures the ratio between: (i) the deviation of the observed primary surplus ratio with respect to the primary ratio that would maintain the debt ratio at its target value; and (ii) the deviation of the observed public-debt ratio with respect to its target value. This component includes both *observed* and *target* values of the debt and primary surplus ratios.<sup>10</sup> The value of the target primary surplus ratio is obtained from equation (5), after assigning values to  $\beta^*$  and selecting the target debt ratio. The latter is a policy parameter that must be set by the authorities to enhance credibility and reduce vulnerability.<sup>11</sup> In the case of the 12 countries in our sample, we set  $d^*$  equal to the lowest value reached by the debt ratio during the period under review. As for  $\beta^*$ , its value for the United States was set at 1.006 (sample mean of the distribution of the observed values of  $\beta$ ):

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explicitly the stochastic shocks associated with each of the variables affecting IFS. Indeed, the observed values of interest and growth rates, primary fiscal balance, and the debt ratio in the previous period can be thought of as comprising both a benchmark value and a shock specific to each variable. Thus, both the debt ratio resulting from equation (7) and the IFS indicator defined in equation (8) fluctuate over time due to changes in policy as well as shocks. IFS fluctuations due to small and temporary shocks do not matter from the sustainability viewpoint; it only matters the region (above or below 1) where IFS fluctuates.

<sup>9</sup> In the same paper, Barro demonstrated that a competitive equilibrium would have to be in the efficient region where  $r > g$  (or  $\beta^* > 1$ ) in a steady state.

<sup>10</sup> A number of studies—Talvi and Vegh (2000), among others—propose algorithms based on structural rather than observed primary surplus ratios. But this procedure is difficult to apply because of the erratic nature of economic cycles in developing countries which makes it more difficult to predict them and to obtain reliable estimates of potential GDP. But aside from this technical difficulty, most would agree that a persistent deterioration of the primary surplus would lead to fiscal unsustainability, regardless of the sources behind the deterioration.

<sup>11</sup> Detragiache and Spilimbergo (2001) found that the likelihood of a debt crisis or a debt correction rises when the debt ratio is above 40 percent. Pattillo, Poirson, and Ricci (2002) have found that, on average, debt ratios above 35–40 have a negative impact on growth. Our choices of  $d^*$  for the countries in our sample are, in most cases, below the “danger” threshold level.

that is, we expect the real interest rate to be, in steady state, 0.6 percentage points higher than the real growth rate; and the same parameter was set at 1.02 and 1.03 for developed and developing countries, respectively (these values represent the median of the distribution of the observed values of  $\beta$  for each group of countries): thus, if a country's target debt ratio were, for example, 30 percent and  $\beta^*$  were 1.02, from equation (5) we can easily calculate the value of  $(ps^*)$ , which in this case, would be 0.6 percent of GDP.<sup>12</sup> Clearly, the estimated  $\lambda_t$  would not only reflect the intensity of the policy response to deviations of the primary surplus and debt ratios from their targets at time  $t$  (equation 6), but also the various shocks that affect these variables, as well as changes in tax and expenditure policies.

The observed algorithm encapsulates in one number (which should be compared basically with the threshold of 1) all the relevant variables for assessing fiscal sustainability, that is, real interest rate, growth rate, primary fiscal surplus, and current and target levels of public debt. If, because of negative shocks, seasonality factors, or bad policies, the observed primary surplus for a given quarter decreases,  $\lambda$  also falls (which in turn might trigger an increase in  $\beta$ ). As a result, the observed algorithm, IFS, increases in that given quarter. When IFS crosses the threshold of 1 and persists at that level, it would signal future fiscal sustainability problems. In subsequent quarters the value of the algorithm might improve because bad policies have been reversed, adverse shocks have faded or have been offset by new positive shocks.

Some shocks, such as a protracted real exchange rate (RER) misalignment, might persist for some time. A real appreciation of the local currency with respect to the long-term RER equilibrium would reduce the observed debt ratio. This is captured by the algorithm as a positive shock which helps the debt ratio to converge to the target ratio. But, in time, a fiscal sustainability problem might become manifest once the sudden correction of the RER takes place.<sup>13</sup> Of course, due to its partial equilibrium nature, our algorithm does not incorporate macroeconomic disequilibria. However, one could deal with protracted real appreciations by adjusting the target debt ratio downward by the magnitude of the misalignment.<sup>14</sup>

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<sup>12</sup> The results do not seem to be very sensitive to changes in the values of the parameter  $\beta^*$ . In the case of the United States, for instance, if the value of  $\beta^*$  were to increase from 1.006 to 1.01, the average value of IFS would increase from 0.962 to 0.970. Similarly, in the cases of Indonesia and Thailand, increases of  $\beta^*$  from 1.03 to 1.04 would increase the average values of IFS from 1.612 to 1.625 and from 1.022 to 1.033, respectively.

<sup>13</sup> Calvo, Izquierdo, and Talvi (2002) make the connection between fiscal sustainability and swings in the RER: “. . . unexpected stops in capital flows of a permanent nature can generate substantial swings in the RER, which may in turn lead to fiscal sustainability problems, particularly in relatively closed, highly indebted and dollarized emerging markets.” And, as the authors point out, the fact that often public sector debt is largely denominated in terms of tradables and government revenues comes mainly from nontradable activities leads to a larger increase in the observed debt ratio following a RER depreciation.

<sup>14</sup> Most often, currency crises are preceded by a period of RER appreciation. In these circumstances, the IFS algorithm falls, signaling an apparent improvement in the country's

Our empirical work consists of the following: first, we calculate the observed algorithm and its components, throughout the 1990s, for each of the countries in our sample and draw some conclusions from their behavior. We examine whether an improved fiscal stance (higher  $\lambda$ ) reduces the spread between the real interest rate and growth (lower  $\beta$ ). In addition, following Alesina and Perotti (1995, 1996), we investigate whether the composition and variability of public expenditures affect the behavior of the algorithm. In this regard, we examine whether sustainable countries exhibit some particular characteristics regarding the trend and variability of different kinds of public expenditures as percentage of GDP compared with unsustainable countries.

Finally, our sample countries are grouped according to the behavior of the IFS indicator during the 1990s. Specifically, countries are grouped into clusters, which help us match the varying degree of sustainability shown by countries in each cluster with the composition and variability of public expenditures.

### III. EMPIRICAL RESULTS

Figures with public debt ratios and IFS algorithms (with the two components  $\beta$  and  $\lambda$ ), are shown in Appendix II for each of the countries in our sample for the 1990s. Also, to facilitate the analysis, the 12 countries are grouped into three clusters based on the behavior of the IFS algorithm as follows:

**Cluster 1** groups countries with enduring problems of fiscal *unsustainability*. This cluster comprises countries for which the IFS was above the threshold of 1 at least 75 percent of the time during the 1990s.

**Cluster 2** groups countries with enduring fiscal *sustainability*. This cluster comprises countries for which the IFS was below the threshold of 1 at least 75 percent of the time during the 1990s.

**Cluster 3** groups countries not included in the rest of the clusters. This cluster comprises countries for which the IFS was above (or below) the threshold of 1 more than 25 percent and less than 75 percent of the time during the 1990s.

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sustainability position. However, when the RER correction takes place, the algorithm will increase abruptly reflecting fiscal unsustainability. This seems to have been the pattern for a number of countries in our sample (Indonesia and Korea in 1998, Mexico in 1994, and Thailand in 1997) as shown in Figure 2, Appendix II.

Table 1 shows the distribution of countries by clusters sorted according to the above definitions:

Table 1. Groupings of Countries According to their Degree of Sustainability

Cluster 1	Cluster 2	Cluster 3
Argentina	Belgium	Italy
Brazil	Indonesia	Korea
Turkey	Ireland	Sweden
	Mexico	Thailand
		United States

The grouping in the above table reflects the performance of the countries in the sample during the 1990s. Obviously, it should not be inferred in any way that this classification still applies today.

### The relationship between $\lambda$ and $\beta$

For each country, we analyze the issue of causality between the two components of the algorithm; in particular, we explore whether an improved fiscal stance (higher  $\lambda$ ) reduces the spread between the real interest rate and growth (lower  $\beta$ ). The Granger causality tests and the estimated coefficient of the vector autoregressions indicate that for about half of the countries in our sample (Belgium, Italy, Korea, Mexico, and Thailand) the Granger causality runs one-way from  $\lambda$  to  $\beta$ ; that is, an improved fiscal stance lowers the real interest-rate-growth gap (see Table 2).<sup>15</sup> As discussed in Appendix I, the quality of available quarterly fiscal data is luckluster. We expect the Granger causality from  $\lambda$  to  $\beta$  to become apparent for the rest of the countries in the sample as the quality of data improves.

### The behavior of the algorithm and its components

The first row in each cluster included in Table 3 shows the number of quarters as a percent of the total quarters during the 1990s (frequency) in which the IFS algorithm had values within the unsustainable region. The second and third rows show the frequency of  $\beta$  values being higher than  $\beta^*$ , and the frequency of  $\lambda$  assuming a negative value (implying primary deficits), respectively.

Table 3 reveals that the frequency of negative values of  $\lambda$  is higher than the frequency of high values of  $\beta$  for all unsustainable countries (Cluster 1). However, the frequency of

<sup>15</sup> In the case of the United States, the Granger causality runs both ways: from  $\lambda$  to  $\beta$  and vice versa.

negative values of  $\lambda$  is lower than the frequency of high values of  $\beta$  for all the other countries. This indicates that the fiscal stance, as measured by  $\lambda$ , matters the most because: (i) high interest rates might not affect the stock of debt immediately (and often high values of  $\beta$  are driven by high interest rates rather than by low growth), and (ii) improvements in the fiscal stance (increases in  $\lambda$ ) might lead to future improvements in the interest-rate-growth gap (decreases in  $\beta$ ).

### **The level and variability of specific types of adjustments in public expenditures**

There is a consensus on the fact that the composition of public expenditures plays an important role in the success of fiscal adjustments. In particular, Type-1 adjustments (cuts in social security and welfare, wages and salaries, subsidies and other current transfers, and public employment) work through credibility, wealth, and unit labor costs effects, to induce a more lasting consolidation of the budget and enhancing growth than do Type-2 adjustments (cuts in capital expenditures and labor tax increases).

This framework provides an additional criterion to examine the effects on sustainability of expenditure policies implemented by the countries in our sample. Specifically, we use a “hierarchical statistical method” to group countries based on combinations of the following four active variables:<sup>16</sup> (i) the percentage point increase in Type-1 expenditures as percentage of GDP during the 1990s, (ii) the percentage point increase in Type-2 expenditures as percentage of GDP during the 1990s, (iii) the variability (as measured by the coefficient of variation) of Type-1 expenditures as percentage of GDP during the 1990s, and (iv) the variability (as measured by the coefficient of variation) of Type-2 expenditures as percentage of GDP during the 1990s. Table 4 shows these and other descriptive variables.

As shown in Table 5, the clustering obtained by using the change in the level and variability of Type-1 expenditures better explain the grouping of countries according to their degree of sustainability as shown in Table 1. This is also revealed from a cursory review of Table 4, which shows unsustainable countries as having a larger increase in Type-1 expenditure as percentage of GDP than sustainable ones. An increase in this type of expenditures is associated with higher and more irreversible deficits, thus fueling expectations of unsustainability.

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<sup>16</sup> The hierarchical method groups countries into clusters and smaller clusters into larger clusters, thus forming a “tree” (or dendrogram). This is obtained by a mathematical algorithm that minimizes a distance (or dissimilarity) function, given by:  $d(a, b) = [N_a N_b / (N_a + N_b)] (X_a - X_b)'(X_a - X_b)$ , where  $X_a$  is the mean for cluster  $a$  with  $N_a$  objects and  $X_b$  is the mean for cluster  $b$  with  $N_b$  objects (objects are countries or clusters). As the tree shows the combination of clusters (among all possible combinations) that have minimum distance (more homogeneous), it is a good guide to decide the level of clustering that one would like to choose (see Karson, 1982).

Table 2. Pairwise Granger Causality Tests

Country	Null Hypothesis	Lags	Obs.	F-Statistic	Probability
Belgium	$\lambda$ does not Granger Cause $\beta$	4	36	2.733	0.053
	$\beta$ does not Granger Cause $\lambda$			1.303	0.298
Italy	$\lambda$ does not Granger Cause $\beta$	8	44	3.245	0.016
	$\beta$ does not Granger Cause $\lambda$			1.076	0.420
Korea	$\lambda$ does not Granger Cause $\beta$	4	21	15.885	0.000
	$\beta$ does not Granger Cause $\lambda$			0.205	0.928
Mexico	$\lambda$ does not Granger Cause $\beta$	3	28	7.009	0.003
	$\beta$ does not Granger Cause $\lambda$			1.544	0.237
Thailand	$\lambda$ does not Granger Cause $\beta$	4	23	4.381	0.026
	$\beta$ does not Granger Cause $\lambda$			0.304	0.868
United States	$\lambda$ does not Granger Cause $\beta$	4	44	5.515	0.002
	$\beta$ does not Granger Cause $\lambda$			4.209	0.007

Source: Authors' own calculations.

Table 3. Analysis of the Algorithm and Its Components

	<i>Cluster 1</i>	Argentina <sup>1</sup>	Brazil	Turkey		
<b>Frequency</b>						
$\beta - \lambda > 1$	83%	97%	78%	77%		
$\beta > \beta^*$	68%	38%	83%	61%		
$\lambda < 0$	68%	100%	53%	70%		
	<i>Cluster 2</i>	Belgium	Indonesia	Ireland	Mexico	
<b>Frequency</b>						
$\beta - \lambda > 1$	21%	23%	22%	9%	21%	
$\beta > \beta^*$	40%	43%	24%	30%	46%	
$\lambda < 0$	1%	0%	8%	0%	0%	
	<i>Cluster 3</i>	Italy	Korea	Sweden	Thailand	United States
<b>Frequency</b>						
$\beta - \lambda > 1$	49%	64%	48%	59%	57%	46%
$\beta > \beta^*$	43%	61%	48%	56%	43%	40%
$\lambda < 0$	40%	39%	43%	46%	52%	40%

Sources: Authors' calculations based on sources detailed in Appendix I.

<sup>1</sup> Estimations based on unofficially adjusted series for the primary fiscal balance as calculated by Teijeiro (2001). Using the official quarterly primary fiscal balance for the period 1994:4–2000:4, the results for that column will be 72 percent, 60 percent, and 28 percent, respectively.

Table 4. Active and Descriptive Variables by Countries and Clusters

	<i>Cluster 1</i>	<b>Argentina</b> 1989-00	<b>Brazil</b> 1987-98	<b>Turkey</b> 1989-00		
<b>Active Variables</b>						
Δ Type-1 Exp. / GDP	<i>10.4</i>	8.1	11.3	11.0		
Δ Type-2 Exp. / GDP	<i>2.0</i>	0.6	2.7	1.8		
cv (Type-1 Exp. / GDP) 1/	<i>0.19</i>	0.15	0.20	0.23		
cv (Type-2 Exp. / GDP) 1/	<i>0.19</i>	0.13	0.22	0.15		
<b>Descriptive Variables</b>						
Δ Public Debt / GDP	<i>14.0</i>	17.7	11.5	16.8		
Per Capita Growth	<i>1.9</i>	2.5	1.5	2.2		
Inflation	<i>127.7</i>	9.7	192.9	77.3		
	<i>Cluster 2</i>	<b>Belgium</b> 1987-98	<b>Indonesia</b> 1988-99	<b>Ireland</b> 1986-97	<b>Mexico</b> 1988-99	
<b>Active Variables</b>						
Δ Type-1 Exp. / GDP	<i>0.4</i>	-3.4	2.5	-13.3	4.4	
Δ Type-2 Exp. / GDP	<i>-1.4</i>	-1.8	-0.4	-3.6	-1.2	
cv (Type-1 Exp. / GDP) 1/	<i>0.12</i>	0.02	0.24	0.10	0.14	
cv (Type-2 Exp. / GDP) 1/	<i>0.11</i>	0.02	0.18	0.07	0.14	
<b>Descriptive Variables</b>						
Δ Public Debt / GDP	<i>-2.2</i>	2.0	9.6	-51.1	-1.0	
Per Capita Growth	<i>2.1</i>	1.5	3.4	5.7	1.4	
Inflation	<i>12.0</i>	1.9	13.5	2.3	19.3	
	<i>Cluster 3</i>	<b>Italy</b> 1988-99	<b>Korea</b> 1986-97	<b>Sweden</b> 1988-99	<b>Thailand</b> 1989-00	<b>United States</b> 1989-00
<b>Active Variables</b>						
Δ Type-1 Exp. / GDP	<i>0.3</i>	2.2	3.3	-2.9	2.5	-0.1
Δ Type-2 Exp. / GDP	<i>1.2</i>	0.3	4.0	-3.0	6.9	1.3
cv (Type-1 Exp. / GDP) 1/	<i>0.05</i>	0.06	0.11	0.09	0.13	0.05
cv (Type-2 Exp. / GDP) 1/	<i>0.07</i>	0.05	0.19	0.14	0.41	0.06
<b>Descriptive Variables</b>						
Δ Public Debt / GDP	<i>6.2</i>	18.3	23.8	22.2	39.3	2.5
Per Capita Growth	<i>2.1</i>	1.5	2.9	0.6	0.0	2.2
Inflation	<i>2.9</i>	3.7	5.0	1.9	4.8	2.7

Sources: Authors' calculations based on sources detailed in Appendix I.

1/ cv stands for coefficient of variation.

Table 5. Distribution of Countries by Clusters

Based on the Statistical Method of Hierarchical Clustering (active variables: increase in the level and variability of Type-1 public expenditures)

Cluster 1	Cluster 2	Cluster 3
Argentina	Belgium	Korea
Brazil	Ireland	Indonesia
Turkey	Sweden	Italy
	United States	Mexico
		Thailand

### Policy lessons

- The proposed recursive IFS algorithm seems suitable to monitor the fiscal stance of a country. Also, it warns of the need to rectify policies when the algorithm hovers long enough in the unsustainable region ( $IFS > 1$ ). However, for this indicator to be an effective monitoring device, good quality quarterly fiscal series have to be made available.
- When used to formulate and communicate fiscal policy objectives and results, the IFS algorithm may help enhance fiscal transparency. To achieve this, the fiscal policy strategy may include the following steps that the government could take: (i) announcing the target for the debt ratio; (ii) announcing its commitment to policies that would keep the IFS algorithm within the sustainable region, on average, without having necessarily to spell out its fiscal policy reaction function or any discretionary measure; (iii) disclosing the IFS algorithm and its components to the public on a quarterly basis; and (iv) explaining ex post movements in the IFS algorithm.
- Countries defined as fiscally sustainable exhibited during the 1990s low increases and low variability in Type-1 public expenditures compared with the other countries.
- Prudent fiscal policies improve the algorithm, first, by improving  $\lambda$  and, subsequently, by improving  $\beta$ , other things being the same. Following a successful (permanent) fiscal consolidation (higher  $\lambda$ ), improved expectations of sustainability seem to have a positive effect on  $\beta$ .

#### IV. CONCLUSIONS

This paper proposes a simple algorithm that can be used to monitor the fiscal stance recursively and to develop a fiscal policy strategy. As a monitoring device, the IFS algorithm seems to work well, judging from its performance and that of its components during the 1990s for each of the 12 countries in the sample. Obviously, the analysis in this paper should not be interpreted as a projection for these countries' sustainability in the future.

Three countries (Argentina, Brazil, and Turkey) are classified as having been fiscally unsustainable in the 1990s, because their IFS algorithms are greater than 1 (unsustainable region) more than 75 percent of the time. Countries for which the algorithm was greater than 1 less than 25 percent of the time during the 1990s were classified as sustainable (Belgium, Indonesia, Ireland, and Mexico). And the rest of the countries in our sample (Italy, Korea, Sweden, Thailand, and the United States) represented intermediate cases, as their IFS indicator moved from the unsustainable to the sustainable region or vice versa during part of the 1990s. As expected, the behavior of public debt ratios mirrors that of the algorithm.

During the 1990s, unsustainable countries had a larger increase in Type-1 public expenditures, and a larger coefficient of variation for this variable, than the other countries. The obvious policy conclusion is that large increases in Type-1 expenditures (wages, current transfers, and welfare), which are generally irreversible and fuel expectations of permanent deficits down the road, are to be avoided. This result is consistent with the findings reported by Alesina and Perotti for some episodes of fiscal adjustment in Europe during the 1980s. Unsustainable countries also showed greater variability in both types of expenditures than the rest of the countries; this might reflect stop-and-go policies, weak institutions, and more vulnerability to shocks.

For Belgium, Italy, Korea, Mexico, and Thailand, Granger causality tests and estimated vector autoregressions support the hypothesis that improvements in the fiscal stance—as measured by component  $\lambda$ —reduce the future values of the spread between real interest rates and growth rates—as measured by component  $\beta$ . More empirical research is needed on this issue as new, better, and higher frequency fiscal data become available.

### DATA DESCRIPTION AND PREPARATION

The data are from various sources. For most of the countries, the primary fiscal balance for the central government was obtained from the IMF's annual *Government Finance Statistics (GFS)*. Type-1 public expenditures (social security and welfare, wages and salaries, and subsidies and other current transfers) and Type-2 public expenditures (capital expenditures and taxes on payroll and workforce) were also from the *GFS*. Population, the consumer price index, and the nominal exchange rate were from the IMF's *International Financial Statistics*. Nominal interest rates and nominal and real GDP were from the *International Financial Statistics*, the World Economic Outlook (*WEO*) database of the IMF, and central bank websites of the selected countries. The public debt of the central government was from the *GFS*, *WEO*, and respective government websites. For the United States, all quarterly variables were provided by the IMF's Western Hemisphere Department. For Mexico, the quarterly primary fiscal balance was provided by the Mexican authorities. And for Argentina, the annual "corrected" primary fiscal balance was provided by the Center for Public Studies, Buenos Aires, Argentina.<sup>17</sup>

To compute the algorithm and perform statistical tests, we prepared the data as follows. When the quarterly series was not available (as in the case of Argentina, Belgium, Indonesia, Ireland, Korea, Sweden, Thailand, and Turkey), the annual primary fiscal balance was divided by four to obtain the quarterly series. This series was then annualized by accumulating the previous four quarters. Domestic and external real interest rates, as well as the weighted average of the two rates, were calculated using the formula shown in Appendix III. The annual growth rate in each quarter was obtained by comparing the real GDP for that quarter with that of the same quarter in the previous year. In all cases, the real interest rate was calculated on a monthly basis and then smoothed using a three-month moving average.

There are a number of caveats. Data limitations forced us to use for each country the central government primary fiscal balance (thus excluding state and local governments). Also, each of the countries under study had different budgetary treatment for recording costs related to the financial crises of the 1990s.

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<sup>17</sup> We also computed the IFS algorithm and its components for Argentina using the official quarterly data for the primary fiscal balance for the period 1994:4–2000:4. This series was kindly provided by the Argentine authorities (see Appendix II).

## BRIEF ANALYSIS BY COUNTRY AND THE BEHAVIOR OF THE ALGORITHM AND DEBT RATIOS IN THE 1990S

Figures 1 and 2 show, respectively, debt ratios and IFS algorithms for each of the twelve countries.

### **Argentina**

The IFS algorithm shows an unsustainable fiscal stance during the 1990s, explained mostly by inadequate primary fiscal balances rather than high spreads between the real interest rates and the growth rates. In the same period, Argentina's public sector debt to GDP ratio rose by 17 percentage points. Specifically, the IFS algorithm captures the shocks that affected Argentina during the 1990s, including the "tequila effect" in 1995, and the Russian and the Brazilian crises in August 1998 and in January 1999, respectively.<sup>18</sup> Fiscal sustainability problems become even more apparent when an adjusted primary fiscal balance series is used.<sup>19</sup>

### **Belgium**

The debt-to-GDP ratio grew from about 115 percent in 1990 to about 130 percent in 1993, to decrease thereafter reaching about 115 percent in 1998. Consistent with this, the IFS algorithm hovered around 1 from 1990 to 1993, and improved thereafter due to a better fiscal stance and a lower real interest-rate-growth gap.

### **Brazil**

The IFS algorithm has consistently shown an unsustainable fiscal stance throughout the period. Both components of the algorithm contribute to the overall unsustainability: inadequate primary fiscal balances and relatively high spreads between the real interest rate

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<sup>18</sup> Although those shocks were clearly beyond the government's control, fiscal policy continued to be expansionary in this period. As Mussa (2002), states " . . . the deficit never came in well below the target; generally it was just below or even slightly above the permitted limit. Indeed, during the five-year period from 1995 through 1998, the deficit of the Argentine government was within the quarterly limits prescribed at the beginning of each year under the IMF-supported program less than half of the time."

<sup>19</sup> This adjusted primary fiscal balance series was taken from Teijeiro (2001), who states that the official measure of the fiscal deficit during the 1990s was rigged to give an appearance of fiscal responsibility and inspire investors confidence. Some of the problems of the official data include: (a) failure to record expenditures paid with government bonds; (b) failure to register interest on the public debt that were capitalized; (c) recording income from privatizations as recurring revenues; and (d) treating many expenditures financed by the World Bank and the Inter-American Development Bank as off-budgetary items.

and the growth rate. Brazil's public debt-to-GDP ratio slightly decreased in the first half of the 1990s (from about 38 percent in 1991 to about 30 percent in 1996) to increase steadily thereafter reaching about 50 percent in 2000 (and about 56 percent by the end of 2001).

### **Indonesia**

The algorithm consistently hovered within the sustainable region during the period 1990–97. However, fiscal policy following the October 1997 crisis has not been successful in stabilizing the algorithm back to the sustainable region. The debt-to-GDP ratio hovered around 40 percent of GDP through mid-1997, when it increased steadily reaching 102 percent by the end of 1999.

### **Ireland**

The algorithm fluctuated slightly below the threshold of 1 from 1990 to 1994, and it improved substantially thereafter as a result of better fiscal stance and lower real interest-rate-growth gap. The public debt-to-GDP ratio has been relatively constant at about 90 percent between 1990 and 1994, and it decreased steadily thereafter, reaching 40 percent in 2000.

### **Italy**

The algorithm hovered above 1 from 1990 to 1994, around 1 from 1995 to 1996, and below 1 from 1997 to 2000. The improvement in the algorithm over time reflects improvements of both of its components. The public debt-to-GDP ratio closely mirrors the algorithm: during 1990–94, it increased from about 85 percent to about 110 percent; during 1995–96, it remained about constant at 110 percent; and during 1997–2000, it decreased to 100 percent.

### **Korea**

The algorithm hovered in the sustainable region from 1995 until the crisis of December 1997, when it first jumped temporarily to unsustainable values and it improved gradually and steadily thereafter. The debt-to-GDP ratio mirrors the algorithm from 1994 to the third quarter of 1997, the public debt-to-GDP remained relatively constant at about 13 percent, and it increased steadily thereafter reaching about 40 percent in the first quarter of 2000.<sup>20</sup>

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<sup>20</sup> In the case of Korea, we should acknowledge that there was consensus among analysts on the fact that fiscal unsustainability was not an issue after 1997 despite the jump in the public debt ratio. The idea is that given the low initial level of Korea's debt ratio, this ratio was still "outside the danger zone" even after the jump. However, in our opinion, since the "danger zone" would be known only imprecisely, it might not be a good practice to ignore debt growth until it reaches such a danger zone.

### **Mexico**

The fiscal sustainability algorithm was in the sustainable region before the end-1994 crisis. Following the crisis period, the algorithm moved back to the sustainable region and remained there throughout the decade. Except for brief periods (crises bouts) the spread of real interest rates and growth rates was moderate in Mexico; thus, the behavior of the algorithm is fully explained by the primary fiscal balance developments. Mexico's public debt-to-GDP ratio hovered about 25 percent until end-1994; it jumped to about 40 percent as a result of the 1995 crisis, and it fluctuated between 35 percent and 40 percent until 1999.

### **Sweden**

From 1990 to 1992, the IFS algorithm jumped to the unsustainable region, but decreased steadily thereafter crossing into the sustainable region in 1998. The steady decline of the algorithm after 1993 reflects the continued improvement of the fiscal stance (as reflected by the behavior of  $\lambda$ ). The debt-to-GDP ratio increased during the period 1990–97, but decreased somewhat thereafter.

### **Thailand**

The IFS algorithm has consistently hovered in the sustainable region until the crisis of July 1997, when it indicated unsustainability, showing only a partial recovery in more recent years. From 1994 to the fourth quarter of 1997, the debt-to-GDP ratio remained relatively constant at about 15 percent, but decreased steadily thereafter to about 55 percent in the first quarter of 2000.

### **Turkey**

The IFS algorithm showed an unsustainable fiscal position from 1990 until 2000, and the debt-to-GDP ratio increased from about 30 percent to about 50 percent in the same period.

### **United States**

The algorithm fluctuated above the threshold of 1 from 1990 to 1993; around 1 during 1994–95; and below 1 from 1996 to 2000. The public debt-to-GDP ratio closely mirrored these three phases: it increased, remained fairly constant, and then decreased.

Figure 1. Debt/GDP  
(In percent)

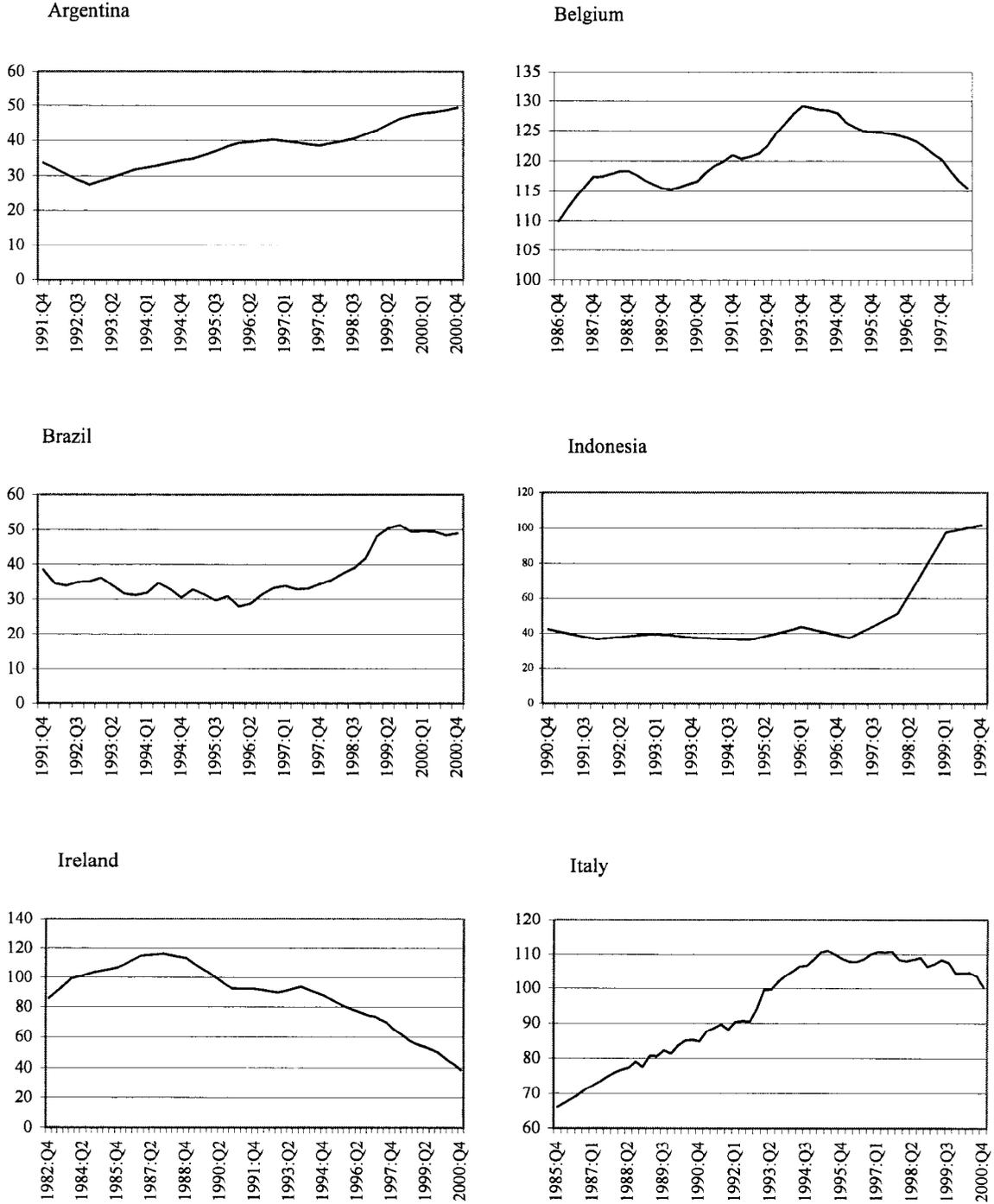


Figure 1 (continue). Debt/GDP  
(In percent)

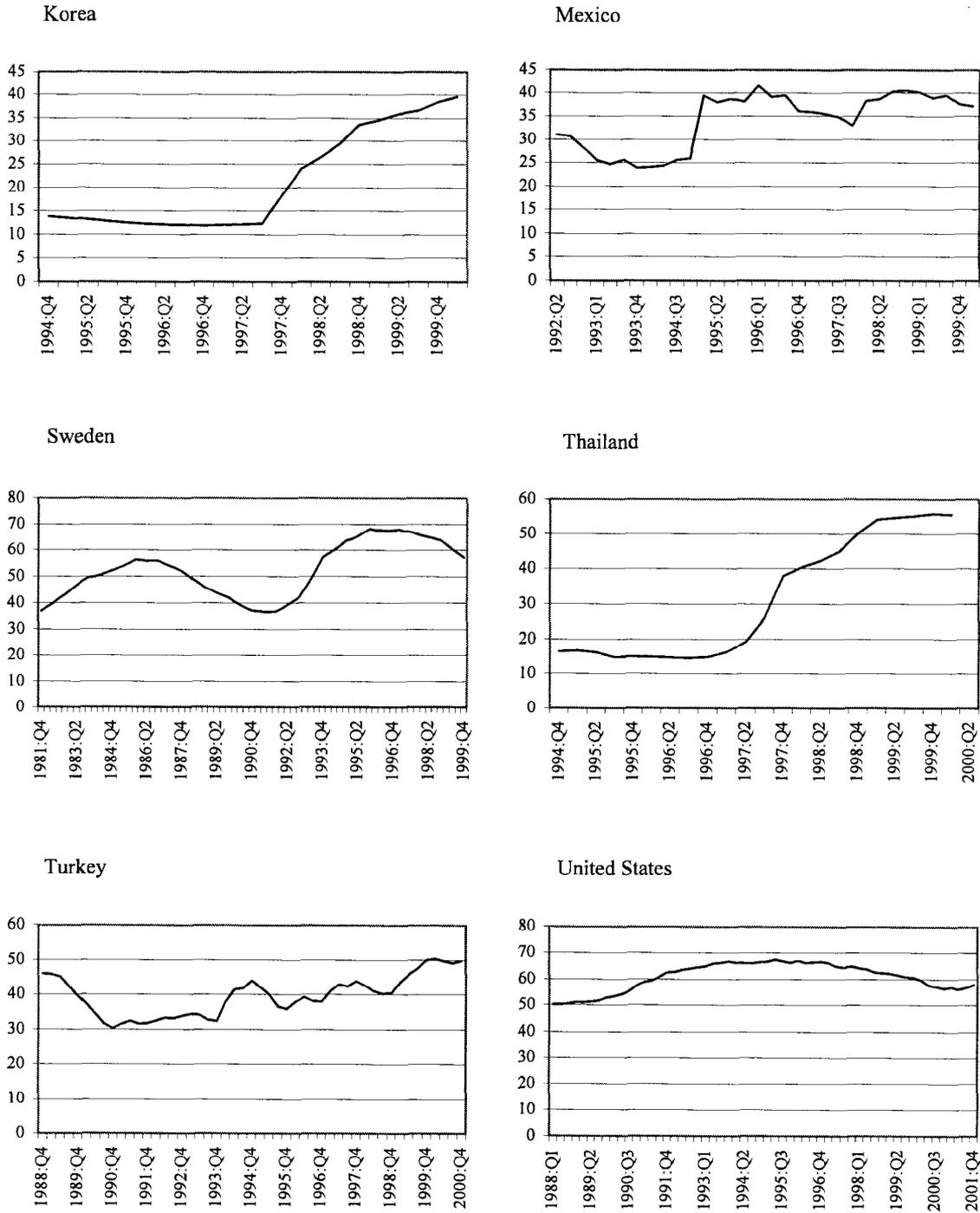


Figure 2. Fiscal Sustainability Indicators

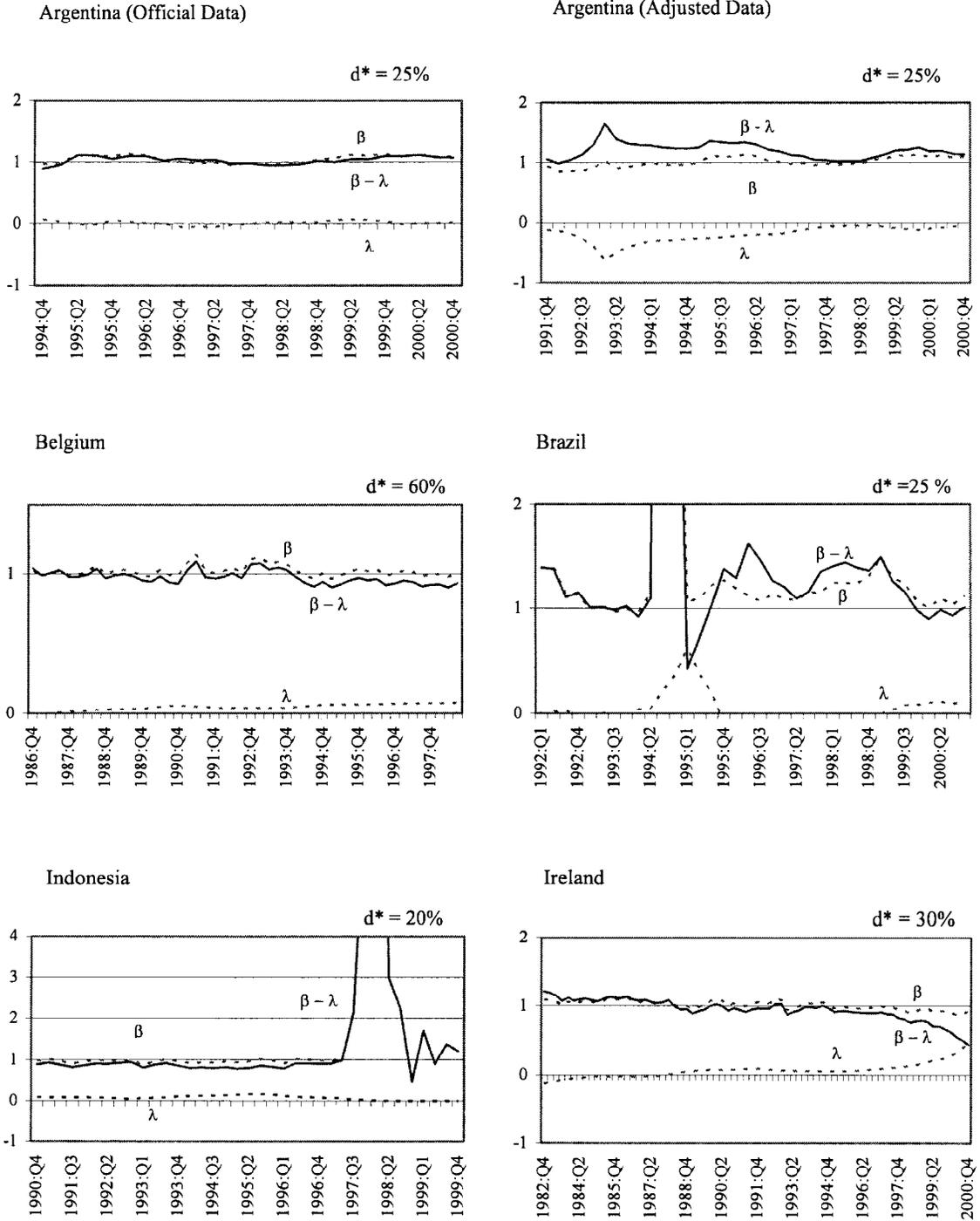
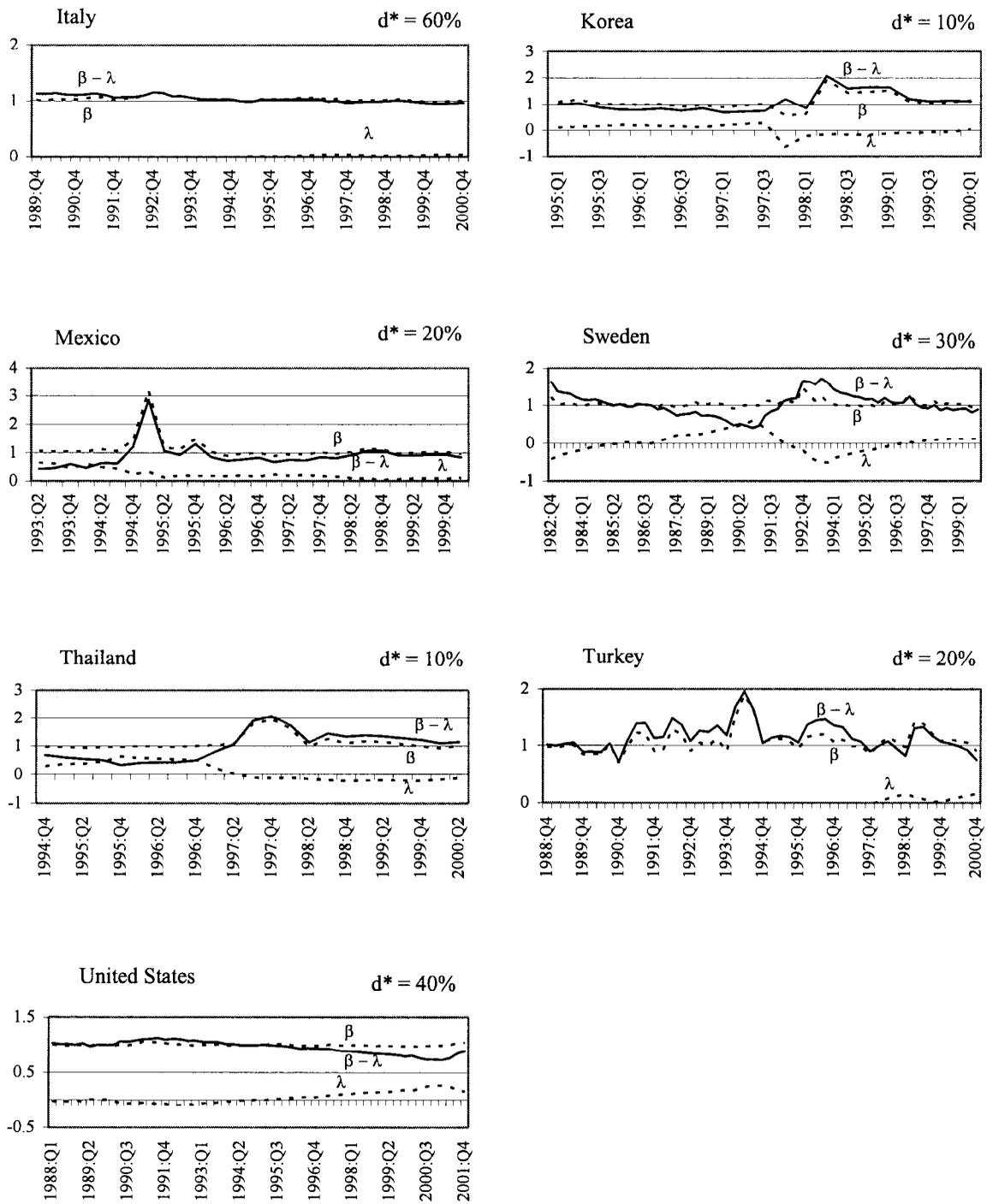


Figure 2 (continue). Fiscal Sustainability Indicators



### DERIVATION OF KEY EQUATIONS IN THE TEXT

The government deficit for period  $t$ , which is assumed to be financed by net domestic and external indebtedness, is decomposed into the primary deficit ( $PD_t$ ) and interest payment ( $IP_t$ ). The total outstanding debt comprises domestic debt expressed in domestic currency ( $D_t^D$ ) and external debt expressed in *foreign* currency ( $D_t^*$ ). Denoting  $E_t$  as the *average* exchange rate, defined as domestic currency per unit of foreign currency between the end of period  $t-1$  and the end of period  $t$ , the external debt can be expressed in *domestic* currency as  $D_t^E \equiv E_t D_t^*$ .

The government budget constraint for period  $t$  in term of domestic currency is

$$PD_t + IP_t \equiv (D_t - D_{t-1}) + E_t(D_t^* - D_{t-1}^*) \quad (9)$$

Denoting  $i_t^D$  and  $i_t^*$  as the *average* representative interest rates on the domestic and external debt at time  $t$ , respectively, interest payments can be expressed as:

$$IP_t \equiv i_t^D D_{t-1}^D + E_t i_t^* D_{t-1}^* \quad (10)$$

Equation (10) implicitly assumes that there is no interest incurred for period  $t$  on the debt acquired between the end of time  $t-1$  and the end of time  $t$ .

Defining primary surplus as  $PS_t \equiv -PD_t$  and using equation (10), we can rearrange equation (9) as:

$$D_t^D + E_t D_t^* = (1 + i_t^D) D_{t-1}^D + E_t (1 + i_t^*) D_{t-1}^* - PS_t \quad (11)$$

Dividing both sides of the above equation by  $Y_t$  (nominal GDP at time  $t$ ) and defining

$d_t^D \equiv \frac{D_t^D}{Y_t}$ ,  $d_t^E \equiv \frac{D_t^E}{Y_t} \equiv \frac{E_t D_t^*}{Y_t}$  and  $ps_t \equiv \frac{PS_t}{Y_t}$ , equation (11) becomes

$$d_t^D + d_t^E = (1 + i_t^D) d_{t-1}^D \left( \frac{Y_{t-1}}{Y_t} \right) + (1 + i_t^*) d_{t-1}^E \left( \frac{E_t}{E_{t-1}} \right) \left( \frac{Y_{t-1}}{Y_t} \right) - ps_t \quad (12)$$

Expressing the change of the nominal gross domestic product in terms of the growth rate of the real gross domestic product ( $g$ ) and the inflation rate ( $\pi$ ) as  $Y_t / Y_{t-1} \equiv (1 + \pi_t)(1 + g_t)$ , and

defining the rate of change in the average nominal exchange rate as  $e_t \equiv (E_t / E_{t-1}) - 1$ , we can then rewrite equation (12) as:

$$d_t^D + d_t^E = \frac{(1 + i_t^D)}{(1 + \pi_t)(1 + g_t)} d_{t-1}^D + \frac{(1 + i_t^*)(1 + e_t)}{(1 + \pi_t)(1 + g_t)} d_{t-1}^E - ps_t \quad (13)$$

By defining the real interest rates on the domestic and external debt expressed in *domestic* currency as  $r_t^D \equiv \frac{(1 + i_t^D)}{(1 + \pi_t)} - 1$  and  $r_t^E \equiv \frac{(1 + i_t^*)(1 + e_t)}{(1 + \pi_t)} - 1$ , respectively, we can write equation (13) more concisely as:

$$d_t^D + d_t^E = \frac{(1 + r_t^D)}{(1 + g_t)} d_{t-1}^D + \frac{(1 + r_t^E)}{(1 + g_t)} d_{t-1}^E - ps_t \quad (14)$$

Defining the ratio of total debt expressed in domestic currency to the nominal GDP as  $d_t \equiv d_t^D + d_t^E$  and multiplying the right-hand side of equation (14) by  $\frac{d_{t-1}}{d_{t-1}}$ , we obtain:

$$d_t = \frac{(1 + r_t^D)}{(1 + g_t)} \left( \frac{d_{t-1}^D}{d_{t-1}} \right) d_{t-1} + \frac{(1 + r_t^E)}{(1 + g_t)} \left( \frac{d_{t-1}^E}{d_{t-1}} \right) d_{t-1} - ps_t, \quad (15)$$

or

$$d_t = \frac{(1 + r_t)}{(1 + g_t)} d_{t-1} - ps_t \quad (15')$$

where the weighted real interest rate relevant for the total debt ( $r_t$ ) is defined as:

$$r_t \equiv r_t^D \frac{d_{t-1}^D}{d_{t-1}} + r_t^E \frac{d_{t-1}^E}{d_{t-1}}$$

In other words,  $1 + r_t \equiv (1 + r_t^D) \left( \frac{d_{t-1}^D}{d_{t-1}} \right) + (1 + r_t^E) \left( \frac{d_{t-1}^E}{d_{t-1}} \right)$ .

Defining the spread between the weighted real interest rate relevant for the total debt and the rate of growth of the real domestic product as  $\beta_t \equiv \frac{(1 + r_t)}{(1 + g_t)}$ , we obtain equation (3):

$$d_t = \beta_t d_{t-1} - ps_t$$

Alternatively, the law of motion for  $d_t$  can be expressed as:

$$d_t = \frac{1}{\beta_{t+1}} d_{t+1} + \frac{1}{\beta_{t+1}} ps_{t+1} \quad (16)$$

Iterating equation (16) forward up to time  $N$ , we have

$$d_t = \sum_{s=1}^N \left( \frac{ps_{t+s}}{\prod_{k=1}^s \beta_{t+k}} \right) + \frac{d_{t+N}}{\prod_{k=1}^N \beta_{t+k}} \quad (17)$$

In a special case where  $\beta_s = \beta$  and  $ps_s = ps$  for all  $s \in [t+1, t+N]$ , equation (17) becomes

$$d_t = ps \sum_{s=1}^N \left( \frac{1}{\beta^s} \right) + \frac{d_{t+N}}{\beta^N} = ps \frac{(1-\beta^N)}{\beta^N(1-\beta)} + \frac{d_{t+N}}{\beta^N} \quad (18)$$

**AN ALTERNATIVE ALGORITHM FOR FISCAL SUSTAINABILITY**

Solving for  $ps$ , from equation (18) we derive:

$$ps = d_t \beta^N \frac{(1-\beta)}{(1-\beta^N)} - d_{t+N} \frac{(1-\beta)}{(1-\beta^N)} = d_t \left( \beta^N - \frac{d_{t+N}}{d_t} \right) \frac{(1-\beta)}{(1-\beta^N)} \quad (18')$$

Defining  $\gamma \equiv \frac{d_{t+N}}{d_t}$  and  $psd \equiv \frac{ps}{d_t}$ , we obtain:

$$psd = \frac{(\beta^N - \gamma)(1-\beta)}{(1-\beta^N)} = \frac{(\beta-1)(\gamma - \beta^N)}{(1-\beta^N)} \quad (19)$$

We can derive another indicator for fiscal sustainability (in addition to IFS) from equation (19), which is restated below:

$$psd^* = \frac{ps^*}{d_t} = \frac{(\beta-1)(\gamma^* - \beta^N)}{(1-\beta^N)} \quad (19')$$

where  $psd^*$  is the optimal constant primary surplus as a percentage of the current outstanding debt, and  $\gamma^*$  is the ratio between the target debt ratio for period  $t+N$  and the current debt ratio (that is,  $d^*/d_t$ ). A value of  $\gamma^*$  of 1 (less than 1) indicates a policy of maintaining constant (reducing) the debt ratio.

Equation (19') shows that the constant primary surplus as a percentage of the outstanding debt at end of period  $t$  needed to achieve an operational definition of sustainability (given by the value of  $\gamma^*$ ) depends on the relevant horizon,  $N$ , and the real interest rate adjusted for growth,  $\beta$ . Fiscal sustainability could be gauged by systematically comparing the *actual* primary surplus as a percentage of the outstanding debt,  $psd$ , with the optimal primary surplus,  $psd^*$ , given by equation (19'). If  $psd$  were lower than  $psd^*$  most of the time, the fiscal stance would be unsustainable and the public debt-to-GDP ratio would increase, and vice versa.

We can, therefore, derive an alternative recursive algorithm from equation (19') as follows: substituting in equation (19')  $psd^*$  for the actual primary surplus as a percentage of public debt,  $psd$ , and solving for  $\gamma$ , we would obtain the following recursive algorithm:

$$IFS' \equiv \gamma = \beta^N + psd \frac{(1-\beta^N)}{(\beta-1)} \quad (20)$$

In equation (19'),  $\gamma^*$  is a policy target and  $psd^*$  the value needed to achieve that target, given  $\beta$  and  $N$ ; however, in equation (20), IFS' is the ratio of the projected value of the public debt as percent of GDP for the period  $t+N$  to the current debt as a percentage of GDP—that is,  $d_{t+N} / d_t$ . Our proposed alternative algorithm, IFS', summarizes in one number the sustainability of fiscal policy under the assumption that current conditions remain unchanged. This means that the actual primary surplus as a percentage of the public outstanding debt,  $psd$ ; the relevant horizon,  $N$ ; and  $\beta$  should all remain constant. In reality, as those conditions are not constant, and, as a result, the algorithm has to be computed recursively.

The recursive values of IFS' should be plotted and compared with a threshold of 1. Values of IFS' that are about 1 indicate that future debt-to-GDP ratios will remain at about their current levels if there are no changes in the fundamentals ( $psd$ ,  $N$  and  $\beta$ ). By the same token, a value of IFS' above (below) 1 indicates that the future debt-to-GDP ratios will increase (decrease) if there are no changes in the fundamentals. Furthermore:

1. when  $psd$  increases (decreases), IFS decreases (increases);
2. when  $psd = \beta - 1$ , IFS = 1;
3. when  $\beta$  increases (decreases), IFS increases (decreases);
4. when  $\beta = 1$ , IFS =  $1 - psd * N$ ;
5. when  $\beta = 1$  and  $psd = 0$ , IFS = 1; and
6. when  $N$  increases, IFS increases for  $\beta > 1$  and  $psd \leq \beta - 1$  (and decreases for  $\beta > 1$  and  $psd > \beta - 1$ , or for  $\beta < 1$  and  $psd > 0$ ).

Point (6) indicates the trade-off between the government horizon,  $N$ , and the real interest rate adjusted for the growth rate,  $\beta$ . For example, for countries with  $\beta > 1$  (developing countries), an increase in the government horizon,  $N$ , would not decrease the debt-to-GDP ratio if the primary surplus as a percentage of the current stock of debt remains below a minimal threshold (that is,  $psd < \beta - 1$ ).

**SOME SUSTAINABILITY INDICATORS PROPOSED IN THE LITERATURE**

Rudin and Smith (1994) proposed an algorithm (called the  $U$  statistic) defined as  $D/(D+PS)$  to gauge sustainability, where  $D$  is the stock of net government liabilities and  $PS$  is the primary surplus over some relevant period. Values of  $U$  greater than 1 define a fiscal policy stance that, if continued, would lead to insolvency (and the opposite would signal values of  $U$  less than 1). Of course, as Rudin and Smith point out, a value of  $U$  greater than one in a single period is probably no cause for alarm. It is only when  $U$  exceeds 1 for prolonged periods that there is cause for concern.

We argue that the  $U$  statistic should be compared not with 1 (which, as we will see is a special case) but with a number that takes into account the value of the parameters  $\beta$ ,  $\gamma$ , and  $N$ , as defined earlier. It is straightforward to show that Rudin and Smith's algorithm,  $U$ , can be expressed as a function of  $psd$ , as follows:

$$U \equiv \frac{D}{D+PS} = \frac{1}{1+psd} = \frac{1-\beta^N}{(1-\gamma)+\beta(\gamma-\beta^N)}$$

Therefore, the  $U$  statistic should be compared with the right-hand expression, which could be larger, smaller, or equal to 1, depending on the relevant values for the parameters involved. The right-hand expression will be equal to 1 in the special case when both  $\gamma$  and  $\beta$  are equal to 1. This is a case when the level of the ratio of debt to GDP is considered to be at its optimal level and the real rate of interest is the same as the growth rate of real GDP. This case might be more relevant for developed countries in general.<sup>21</sup>

Blanchard (1990) suggested basically two indicators of fiscal sustainability—the primary gap and the medium- and long-term tax gap—derived from the government intertemporal budget constraint with a finite horizon. The primary gap indicator (PGI) measures the adjustment in the primary balance needed to stabilize the outstanding public debt ratio:

$$PGI = ps - ps^* = ps - (r-g)d_0$$

where  $ps$  is the current primary balance,  $ps^*$  is the constant primary balance that stabilizes the debt ratio at its current level, and  $r$  and  $g$  are the real rate of interest and the growth rate, respectively. Blanchard suggests using constant values for  $r$  and  $g$ , say the averages over the last 10 years or so. However, he finds that this primary gap indicator does not take into account changes in economic fundamentals and policies. To resolve this, Blanchard proposes the tax gap indicator (TGI), which measures the required adjustment in the tax ratio needed to stabilize the outstanding public debt ratio:

$$TGI = t^* - t = (\text{Avg. over the current and next } n \text{ years primary spending as percent of GDP}) + (r-g) d_0 - (\text{Avg. current tax revenue as percent of GDP})$$

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<sup>21</sup>Rudin and Smith (1994) study sustainability, using the  $U$  statistic, for Canada during 1937–84, and for the United States during 1890–1986.

where  $t$  is the average current tax revenue as percent of GDP, and  $t^*$  is the tax ratio consistent with a stabilized initial debt ratio, over an  $n$ -year horizon, approximated by the projected path of no interest expenditures and transfers as a percent of GDP, real interest rate, rate of growth, and the initial debt ratio. The horizon might be two years (medium-term gap) or longer (long-term gap).

For the sake of comparison, our proposed algorithm is similar to Blanchard's, once convergence is reached. While his algorithms capture the adjustment needed in the primary surplus (or in the tax ratio) to stabilize the outstanding debt ratio, ours captures convergence conditions towards a target debt ratio. Unlike Blanchard's tax gap indicator, our algorithm does not require explicit forecast of either primary surplus, real interest rate, or rate of growth; instead, it implicitly treats the current values of these variables as if they were to remain constant into the future. This naïve feature is counterbalanced by the recursiveness nature of the algorithm. Operationally, our algorithm should be recalculated quarterly; therefore, shocks to the current debt ratio, the primary surplus, the real interest rate, or to the growth rate are taken into account.

From the government intertemporal budget constraint with infinite horizon, Talvi and Vegh (2000) proposed the following algorithm to assess fiscal sustainability:

$$I_t^* = \frac{r - g}{1 + g} d_{t-1} - \overline{ps}$$

where  $r$  is the real interest rate,  $g$  is the rate of growth,  $d$  is the public debt-to-GDP ratio, and  $\overline{ps}$  is the constant primary surplus as percentage of GDP, whose present discounted value over an infinite horizon is equal to the projected trajectory of this variable. Thus, the projected fiscal stance (summed up in  $ps$ ) would be considered sustainable if  $I^*$  is less than or equal to zero, and unsustainable if  $I^*$  is larger than zero. In practice, it is not easy to calculate the sustainability indicator  $I^*$  due to the difficulty of projecting the primary fiscal balance from now to infinity. Furthermore, the authors suggest that a structural (macroadjusted) primary deficit should be calculated, which is the balance that would prevail at normal times. This, of course, adds to the difficulty of the exercise.

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