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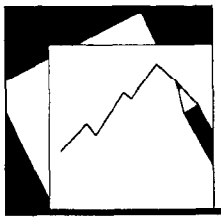

IMF Working Paper

An Analysis of the Underground Economy and Its Macroeconomic Consequences

Era Dabla-Norris and
Andrew Feltenstein

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An Analysis of the Underground Economy and Its Macroeconomic Consequences

Prepared by Era Dabla-Norris and Andrew Feltenstein[1]

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Abstract

This paper develops a dynamic computable general equilibrium model in which optimizing agents evade taxes by operating in the underground economy. The cost to firms of evading taxes is that they find themselves subject to credit rationing from banks. Our model simulations show that in the absence of budgetary flexibility to adjust expenditures, raising tax rates too high drives firms into the underground economy, thereby reducing the tax base. Aggregate investment in the economy is lowered because of credit rationing. Taxes that are too low eliminate the underground economy, but result in unsustainable budget and trade deficits. Thus, the optimal rate of taxation, from a macroeconomic point of view, may lead to some underground activity.

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Authors' E-Mail Addresses: edabla@imf.org; afeltenstein@imf.org

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I. INTRODUCTION

There is widespread belief that in many developing and transition countries, heavy tax burdens and excessive regulation imposed by governments that lack the capability to enforce compliance drives firms into the underground economy.[2] The presence of a large underground economy is a source of considerable revenue losses for the government and can significantly affect public finances and the quality of public administration (Loayza (1996) and Johnson et al. (1997)). The illegal nature of underground activity can also constrain private investment and growth. For instance, firms operating underground are often unable to make use of market-supporting institutions like the judicial system and courts and, as a result, may underinvest (De Soto (1989)). One important cost imposed by the inability to enforce legal contracts is the limited access to formal credit markets.

In this paper, we develop a simple intertemporal general equilibrium model that explores the link between tax rates, access to credit, and the size of the underground economy and examines the consequences of the underground economy for public finances and aggregate economic performance. In particular, we derive entry and exit into the underground economy as part of optimizing behavior that depends on taxes and interest rates. The cost to evading taxes is credit rationing by banks which reduce loans in relation to the firm's nonpayment of taxes. This assumption is consistent with the observation that it may be more difficult for tax evaders to borrow from a bank because to do so would require official documentation, especially if the bank requires collateral and if the process of hiding economic activity involves concealing the true ownership of assets. Since the size of the underground economy depends upon both endogenous and exogenous variables, our framework has scope for policy changes.

We develop numerical applications of our model using data for Pakistan. This should be viewed as an illustrative example of a developing country which faces problems from tax evasion and parallel markets for both goods and financial assets. Economic reform will depend upon policies that reduce the various forms of tax evasion, especially given the typical developing country's difficulties with controlling its budget deficit. There have been a number of empirical studies of the scope of the underground economy in developing countries (Ahmad and Masood (1995), Iqbal (1998)). Most of these studies use proxies, such as the amount of cash in circulation, or electricity consumption to estimate the size of the underground economy. In addition, they are not derived from optimizing models, but are based upon ad hoc empiricism. Accordingly, they can at best be used for partial equilibrium analysis and hence may lead to inaccurate conclusions.

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[2] As in Braun and Loayza (1993), the underground economy is defined as "a set of economic units which do not comply with one or more government imposed taxes and regulations, but whose product is considered legal."

Section II provides a brief overview of our modeling of the underground economy. Section III presents our dynamic general equilibrium model. Section IV discusses the parameterization of the model and presents the simulation results. Section V concludes.

II. Macroeconomic Background

Our paper models the benefits from operating in terms of the firm's desire to evade corporate taxes.[3] This assumption is consistent with the observation that in many developing countries, taxes on formal firms constitute a major source of government revenues, and narrow tax bases for formal firms have often resulted in governments imposing very high marginal tax rates.[4] The cost of operating in the underground economy is modeled in terms of the limited ability to borrow from the official banking system. Banks in the model are assumed not to have perfect information about the firm's true ownership of assets and its associated true tax obligation. We assume that due to collateral requirements credit is provided only in relation to the firm's implied ownership of assets, which is determined from its actual tax payment. The idea here is that in the face of default, banks can only seize those assets that have been officially declared by the firm. Hence, the higher the extent of tax evasion, the lower the implied value of firm assets, and the lower the amount of credit provided by the banking system.

We assume that firms can operate partially in the formal and partially in the underground economy. That part of their operation that takes place in the legal economy pays taxes and can borrow from the banking system. That part that is underground does not pay taxes and cannot borrow.[5] Admittedly this distinction is artificial, but captures some of the benefits and costs of operating in the underground economy discussed in the literature.

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[3] Johnson et al. (1998) find that a high corporate tax burden combined with ineffective and discretionary application of the tax system and other regulations influences the size of the underground economy.

[4] Burgess and Stern (1993) note that in developing countries, corporate income taxes represent 17.8 percent of total tax revenues as opposed to 7.6 percent in industrialized countries.

[5] In reality, the underground firm may not be able to borrow from official banks but may still be able to invest by borrowing from secondary lenders who charge higher than market interest rates and are willing to incur high risks. Huq and Sultan (1991) note that in Bangladesh, while borrowing rates from commercial banks were around 12 percent, firms dependent on noninstitutional sources to meet their financing needs paid rates between 48 to 100 percent.

We ignore the effects of corruption in generating an underground economy.[6] Presumably, the size of the underground economy may itself serve as a proxy for the extent of corruption in the economy. This might be appropriate if the country in question has relatively low statutory tax rates, so the incentives to evade taxes would not exist unless doing so were not easy. Alternatively, a large underground economy can be a result of firms seeking to escape a predatory bureaucracy in the formal economy (Shleifer (1997), Johnson et al. (1998)). Of course, the presence of a large underground economy could indicate high costs of ensuring compliance rather than corrupt officials.

Our approach also assumes that firms can evade taxes without any real risk of detection or punishment. From a modeling perspective, it is difficult to determine just what the penalty for tax evasion should be. That is, should it be a criminal penalty or a fine? Should it be proportional to the size of the tax evasion, or should it be a flat rate? In addition, what is the probability of apprehension faced by a tax evader? Presumably this probability should itself be some function of the enforcement technology, the amount of evasion, as well as the amount of money spend on enforcement. A problem with that modeling strategy is that, as there is no clear mapping from the enforcement technology and how spending affects the probability of being caught to real data, it does not offer a reasonable framework for quantitative evaluations. Moreover, Shleifer and Vishny (1993) point out that where public pressure on corruption or the enforcement ability of the government is relatively weak - as we believe to be the case in many developing countries - this is in fact a fitting assumption.

In our model, the decision to operate in the underground economy depends on the firm's present value of the future stream of returns on marginal investment relative to the return on the corporate capital tax rate. If the marginal rate of return is higher than the corporate tax rate, the firm chooses to operate in the above ground economy, since it is profitable to borrow and pay taxes. If, on the other hand, the tax rate is greater than the marginal rate of return on investment, the firm chooses to operate in the underground economy. However, we assume that the firm does not make a bipolar choice. That is, it reduces its tax payments and borrowing for investment proportionally to the difference between the rate of return and the tax rate. Hence if the rate of return were 0, then the firm would pay no taxes and carry out no borrowing for investment. If tax rates and rates of return on investment are equal, then the firm pays the full tax rate and invests.

In this framework, one could measure the size of the underground economy by aggregating the value of all lost tax revenues and comparing it to the revenues that would accrue if rates were low enough so as to generate no underground activity. The ratio of the two would then provide a measure of the share of the underground economy in total economic activity. We would thus compare two simulated equilibria.

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[6] Sarte (2000) develops a model which examines the link between the size of the informal economy and corrupt bureaucracies. He shows that in the presence of a corrupt bureaucracy, it may be efficient to locate economic activity in the informal economy if the costs of informality are low. However, he ignores credit rationing by banks.

III. A General Equilibrium Specification

In this section we develop the formal structure of a dynamic general equilibrium model that endogenously generates an underground economy. Much of the structure of our model is designed to permit numerical implementation. Our model has n discrete time periods. All agents optimize in each period over a 2 period time horizon. That is, in period t they optimize given prices for periods t and $t+1$ and expectations for prices for the future after $t+1$. When period $t+2$ arrives, agents reoptimize for period $t+2$ and $t+3$, based on new information about period $t+2$.

Our model structure is related to a number of earlier papers, starting with Strotz (1956). Here preferences are inconsistent over time, primarily because the future does not turn out as anticipated. Thus it may be optimal for agents to commit themselves for a few periods into the future. They may be better off, however, if they reoptimize at some later date, based on their own changed preferences or changes in economic variables. This is quite different from the notion of time inconsistency of Kydland and Prescott (1977), where rational behavior by economic agents itself leads to inconsistencies in what would otherwise be an optimal government plan.

1. Production

There are 8 factors of production and 3 types of financial assets:

1-5 Capital types
6. Urban labor
7. Rural labor
8. Land
9. Domestic currency
10. Bank deposits
11. Foreign currency

The five types of capital correspond to five aggregate nonagricultural productive sectors.[7] An input-output matrix, A_t, is used to determine intermediate and final production in period t. Corresponding to each sector in the input-output matrix, sector-specific value added is produced using capital and urban labor for the nonagricultural sectors, and land and rural labor in agriculture.

Assuming that there are more than five sectors in the economy, the different factors would be allocated across the economy so that agriculture uses land and rural labor, and all other sectors use one of the five capital types plus urban labor.

[7] We could have any number of capital types without affecting the structure of the model.

Accordingly, capital is perfectly mobile across a given subsector, but is immobile across other subsectors. Labor, on the other hand, may migrate from the rural to the urban sector.[8]

The specific formulation of the firm's problem is as follows. Let y_{Ki}^{j}, y_{Li}^{j} be the inputs of capital and urban labor to the jth nonagricultural sector in period i. Let Y_{Gi} be the outstanding stock of government infrastructure in period i. The production of value added in sector j in period i is then given by:

$$va_{ii} = va_{ii}\left(y_{Ki}^{j}, y_{Li}^{j}, Y_{Gi}\right) \tag{1}$$

where we suppose that public infrastructure may act as a productivity increment to private production.

Sector j pays income taxes on inputs of capital and labor, given by t_{Kij}, t_{Lij}, respectively, in period i. The interpretation of these taxes are that the capital tax is a tax on firm profits, while the labor tax is a personal income tax that is withheld at source.

We suppose that each type of sectoral capital is produced via a sector-specific investment technology that uses inputs of capital and labor to produce new capital. Investment is carried out by the private sector and is entirely financed by domestic borrowing.[9]

Let us define the following notation:

C_{Hi} = The cost of producing the quantity H of capital.
r_i = The interest rate in period i.
P_{Ki} = The return to capital in period i.
P_{Mi} = The price of money in period i.
δ_i = The rate of depreciation of capital.

Suppose, then, that the rental price of capital in period 1 is P_1. If C_{HI} is the cost-minimizing cost of producing the quantity of capital, H_1, then the cost of borrowing must equal the present value of the return on new capital. Hence:

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[8] We assume that the labor market is not segmented and there is no wage differential between workers in the underground and the formal economy.

[9] We assume that all foreign borrowing for investment is carried out by the government, so that, implicitly, the government is borrowing for the private investor but the debt incurred is publicly guaranteed.

$$C_{H1} = \sum_{i=2}^{N} \left[\frac{P_{Ki}(1-\delta)^{i-2} H_1}{\prod_{j=1}^{i-1}(1+r_j)} \right] \tag{2}$$

where r_j is the interest rate in period j, given by:

$$r_j = 1/P_{Bj}$$

where P_{Bj} is the price of a bond in period j. The tax on capital is implicitly included in the investment problem, as capital taxes are paid on capital as an input to production.

The decision to invest depends not only the variables in the above equation, but also upon the decision the firm makes as to whether it should pay taxes.[10] This decision determines the firm's entry into the underground economy. We assume that the firm's decision is based upon a comparison of the tax rate on capital with the rate of return on new capital. If the tax rate on capital is less than the corresponding rate of return, the firm pays the full tax. If the tax rate is greater than the return to new capital, then the firm pays less than the full capital tax. That is, it withdraws, at least partially, into the underground economy.[11] Formally, suppose that we were in a two period world. Suppose that:

$$\frac{P_{K2}}{1+r_1} \geq t_{K1}$$

that is, the present value of the return on one unit of new capital is greater than the current tax rate on capital. In this case we assume the investor pays the full tax rate on capital inputs. Suppose, on the other hand, that:

$$\frac{P_{K2}}{1+r_1} \leq t_{K1}$$

Here the discounted rate of return is less than the tax rate, and the firm will attempt to reduce its tax payments by moving into the underground economy. The extent to which the firm goes into the underground economy is determined by the gap between

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[10] In reality, we can regard the tax rate on capital as the generalized tax rate, including taxation, regulation, and corruption (bribes).

[11] Clearly this is an ad hoc assumption. We wish to capture the notion that the decision whether or not to pay taxes is based on the relationship between the return to investment and the tax rate on capital.

the tax rate and the rate of return to investment. That is, the firm pays a tax rate of $\bar{t}_{K1}$ where:

$$\bar{t}_{K1} = t_{K1}\left[1 - \left(\frac{t_{K1} - \frac{P_{K2}}{1+r_2}}{t_{K1}}\right)\right]^{\alpha} \qquad (3)$$

Here $0 \leq \alpha$ and higher values of α lead to lower values of taxes actually paid. That is, the ratio $\frac{\bar{t}_{K1}}{t_{K1}}$ reflects the share of the sector that operates in the above ground economy. Hence α represents a firm-specific behavioral variable. An "honest" firm would set $\alpha = 0$, while a firm that is prone to evasion would have a high value for α.[12]

If a sector can avoid paying taxes, as above, by going into the underground economy, why does it pay taxes at all? That is, why does it simply not set $\bar{t}_{K1} = 0$? In the next section we develop a simple approach that supposes that a firm's refusal to pay taxes reduces its ability to borrow from the commercial banking system. Thus a firm's desire to invest will constrain its evasion of tax payments.

2. **Banking**

The banking sector in our model is quite simple and is meant to capture some of the key features and problems in many developing countries. We will suppose that there is one bank for each nonagricultural sector of the economy. There are 5 such sectors, and hence 5 banks. Such sectoral specialization of the banking system reflects the reality of many developing countries.

We contend that the underground economy affects different sectors in a non-uniform way. Indeed, tax evasion in one sector may benefit the sector at a micro level but may be harmful to the macro economy. Tax evasion varies across sectors not only because of the behavior of firms in that sector, but also because different banks could have varying attitudes towards lending to clients who have evaded taxes.

Each bank lends primarily to the sector with which it is associated. The banks are, however, not fully specialized in the sector they correspond to. We make the simplifying assumption that each bank holds 50 percent of the outstanding debt of its particular sector. It then holds 12.5 percent of the debt of each of the remaining 4 sectors. Hence bank 3, for example, holds 50 percent of the debt of sector 3, and 12.5 percent of sectors

[12] Here α could be a function of the enforcement technology. In our model, however, we assume that there is no enforcement technology or means to enforce tax compliance.

1,2,4, and 5. Similarly, it makes 50 percent of the loans to sector 3 and 12.5 percent of the loans to the other 4 sectors.[13] We make this assumption of diversification of assets in order to allow for a situation in which a the firm that evades taxes, and, thereby, enters the underground economy, should receive varying degrees of credit rationing from the different banks to which it applies for loans.

We choose a simple approach to determine the degree of credit rationing that firms face. Our premise is that banks have no direct way of knowing whether specific firms operate in the underground economy. We assume that banks only care about the amount of capital that they estimate the firm may have. If the firm defaults on its loan, then this represents the best estimate of the amount that the bank could seize. The bank would, presumably, be willing to lend an amount equal to at least the estimated firm capital. If the firm requests a loan larger than its estimated capital, the bank may choose to grant the full loan, or it may choose to restrict the loan amount. This restriction would depend, in turn, upon the bank's degree of risk aversion.

How can the bank estimate the value of the firm's capital, if this information is not directly revealed by the firm? We assume the borrower is required to show the bank his tax returns in order to obtain a loan. There is a single, flat corporate tax rate that the borrowing firm faces. Hence, suppose that T_{K1} represents taxes actually paid by the borrower in period 1. This is known to the bank, as the potential borrower is required to present his tax returns. Thus if the borrower fully complied with his tax obligation, and hence carried out no underground activity, the value of his capital, $\hat{K}_1$, would be given by:

$$\hat{K}_1 = \frac{T_{K1}}{t_{K1}}$$

Accordingly, the bank would be willing to lend at least $\hat{K}_1$ to the borrower, as this would represent a minimum estimate of the value of the firm's capital, which could be seized in the event of a default.[14] Our approach has some similarity to Kiyotaki and Moore (1997) in which there are credit limits on loans. These limits are determined by estimates of collateral which, in turn, are determined by estimates of durable asset holdings by borrowers. Here tax payments are used to estimate the value of the durable asset of the borrower, as the asset cannot be directly observed.

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[13] Clearly these percentages are arbitrary and should serve only for illustrative purposes. We could have any initial pattern of distribution of bank assets across the different sectors.

[14] We have not explicitly incorporated bankruptcies and defaults in this model, for the sake of simplicity. However bankruptcies and corresponding bank contractions can be introduced as in Ball and Feltenstein (2001) and Blejer, Feldman, and Feltenstein (2002).

Suppose, however, that the amount the firm wishes to borrow, C_{HI}, as in equation (2), such that:

$$C_{HI} > \hat{K}_1$$

In this case the bank lends an amount L_1, where $L_1 < C_{HI}$, as the bank would not be able to seize the full value of the loan in the case of a default. The situation we have described would, in the case of perfect certainty, have credit rationing when the estimated value of the firm's capital is less than its loan request. If the firm's capital is greater than its loan request, there would be no credit rationing.

In a more realistic case of uncertainty about both the true value of the firm, as well as about the bank's own ability to seize the firm, one might expect the lending process to be somewhat different. Accordingly, we will suppose that a simple functional form determines bank lending as a function of the amount requested as well as the estimated value of the firm's capital. We define the amount the bank lends, L_1, as:

$$L_1 = C_{HI}\left[\frac{\frac{\hat{K}_1}{C_{HI}}}{1+\frac{\hat{K}_1}{C_{HI}}}\right]^{\gamma} = C_{HI}\left[\frac{\hat{K}_1}{C_{HI}+\hat{K}_1}\right] \qquad (4)$$

Here γ represents a measure of risk aversion by the bank. If $\gamma = 0$, there are no credit restrictions, and the bank ignores estimates of the borrower's estimated net worth. As γ rises, the bank increasingly restricts lending if the term in brackets is less than 1. If the firm pays no taxes, hence operating entirely in the underground economy, $\hat{K}_1 = 0$ and hence $L_1 = 0$, that is there is, no lending. If $\frac{\hat{K}_1}{C_{HI}}$ increases, as would be the case if the value of the firm increases relative to its borrowing request, then $L_1 \Rightarrow C_{HI}$, that is, the bank lends the full value of the request.

Thus if a firm operates entirely in the underground economy it will not be able to borrow to finance investment. If banks are highly risk averse, they will never lend more than a firm's estimated net worth, which is based on its tax return. This tax return therefore represents all the information the bank needs in order to determine its response to a request for a loan.

3. Consumption

There are two types of consumers, representing rural and urban labor. We suppose that the two consumer classes have differing Cobb-Douglas demands. The consumers also differ in their initial allocations of factors and financial assets.

The consumers maximize intertemporal utility functions, which have as arguments the levels of consumption and leisure in each of the two periods. We permit rural-urban migration which depends upon the relative rural and urban wage rate. The consumers maximize these utility functions subject to intertemporal budget constraints. The consumer saves by holding money, domestic bank deposits, and foreign currency. He requires money for transactions purposes, but his demand for money is sensitive to changes in the inflation rate. The consumer pays taxes on his consumption, and does not have any direct contact with the underground economy. That is, he pays the full nominal rates under all circumstances.

Here, and in what follows, we will use x to denote a demand variable and y to denote a supply variable. In order to avoid unreadable subscripts, let 1 refer to period i and 2 refer to period $i+1$. The consumer's maximization problem is thus:

$$\max\ U(x),\quad x\ =\ (x_1,x_{Lu1},x_{Lr1},x_2,x_{Lu2},x_{Lr2}) \tag{5}$$

such that:

$$(1+t_i)P_i x_i+P_{Lui}x_{Lui}+P_{Lri}x_{Lri}+P_{Mi}x_{mi}+P_{Bi}x_{Bi}+e_i P_{Bfi}x_{BFi}\ =\ C_i \tag{5a}$$

$$P_{K1}K_0+P_{A1}A_0+P_{Lu1}L_{u1}+P_{Lr1}L_{r1}+P_{M1}M_0+r_0B_0+P_{B1}B_0+e_1P_{BF1}B_{F0}+TR_1\ =\ N_1$$

$$P_{K2}(1-\delta)K_0+P_{A2}A_0+P_{Lu2}L_{u2}+P_{Lr2}L_{r2}+P_{M2}x_{M1}+r_1x_{B1}+P_{B2}x_{B1}+e_2P_{BF2}x_{BF1}+TR_2\ =\ N_2$$

$$C_i\ =\ N_i$$

$$\log\ P_{Bi}x_{Bi}-\log\ e_iP_{BFi}x_{BFi}\ =\ \alpha+\beta(\log\ r_i\ -\ \log\frac{e_{i+1}}{e_i}\ r_{Fi}) \tag{5b}$$

$$\log(L_{ui}/L_{ri})\ =\ a_1+a_2\log\frac{P_{Lui}-P_{Lri}}{P_{Lui}+P_{Lri}} \tag{5c}$$

$$\log P_{Mi}x_{Mi}=a+b\log(1+t_i)P_ix_i \tag{5d}$$

$$P_{B2}x_{B2}\ =\ d_0+d_1(1+t_2)P_2x_2+d_2\left[\frac{r_2-\pi_2}{1+\pi_2}\right] \tag{5e}$$

where:

P_i = price vector of consumption goods in period *i*.

x_i = vector of consumption in period *i*.

C_i = value of aggregate consumption in period *i* (including purchases of financial assets).

N_i = aggregate income in period *i* (including potential income from the sale of real and financial assets).

t_i = vector of sales tax rates in period *i*.

P_{Lui} = price of urban labor in period *i*.

L_{ui} = allocation of total labor to urban labor in period *i*.

x_{Lui} = demand for urban leisure in period *i*.

P_{Lri} = price of rural labor in period *i*.

L_{ri} = allocation of total labor to rural labor in period *i*.

x_{Lri} = demand for rural leisure in period *i*.

a_2 = elasticity of rural/urban migration.

P_{Ki} = price of capital in period *i*.

K_0 = initial holding of capital.

P_{Ai} = price of land in period *i*.

A_0 = initial holding of land.

δ = rate of depreciation of capital.

P_{Mi} = price of money in period *i*. Money in period 1 is the numeraire and hence has a price of 1.

x_{Mi} = holdings of money in period *i*.

P_{Bi} = discount price of a certificate of deposit in period *i*.

π_i = domestic rate of inflation in period *i*.

r_i, r_{Fi} = the domestic and foreign interest rates in period *i*.

x_{Bi} = quantity of bank deposits, that is, CD's in period *i*.

e_i = the exchange rate in terms of units of domestic currency per unit of foreign currency in period *i*.

x_{BFi} = quantity of foreign currency held in period *i*.

TR_i = transfer payments from the government in period *i*.

a, b, α, ß = estimated constants.

d_i = constants estimated from model simulations.

The left-hand side of equation (5a) represents the value of consumption of goods and leisure, as well as of financial assets. The next two equations contain the value of the consumer's holdings of capital and labor, as well as the principal and interest that he receives from the domestic and foreign financial assets that he held at the end of the previous period. The equation $C_i = N_i$ then imposes a budget constraint in each period.

Equation (5b) says that the proportion of savings made up of domestic and foreign interest bearing assets depends upon relative domestic and foreign interest rates, deflated by the change in the exchange rate. Finally, equation (5c) is a migration equation that says that the change in the consumer's relative holdings of urban and rural labor depends on the relative wage rates. Equation (5d) is a standard money demand equation in which the demand for cash balances depends upon the domestic rate of inflation and the value of intended consumption.

In period 2 we impose a savings rate based on adoptive expectations, as in equation (5e). The constants (d_i) are estimated by a simple regression analysis, based on the previous periods. Thus if we are in period *t*, where *t* the end of a two-period segment, then the closure saving rate for period *t* is determined by nominal income and the real interest rate. The constants are updated after each two period segment by running a regression on the previous $t-2$ periods. Thus savings rates are endogenously determined by intertemporal maximization in period *t*, but are determined by adoptive expectations in period $t+1$.[15]

[15] Since the only information the consumer has about the future is the real interest rate, adoptive expectations is, in this case, equivalent to rational expectations.

4. The Government

The government collects personal income, corporate profit, and value-added taxes, as well as import duties. It pays for the production of public goods, as well as for subsidies. In addition, the government must cover both domestic and foreign interest obligations on public debt. The deficit of the central government in period 1, D_1, is then given by:[16]

$$D_1 = G_1+S_1+r_1B_0+r_{F1}e_1B_{F0}-T_1 \tag{6}$$

where S_1 represents subsidies given in period 1, G_1 is spending on goods and services, while the next two terms reflect domestic and foreign interest obligations of the government, based on its initial stocks of debt. T_1 represents tax revenues, which is partially determined by firms' entry into the underground economy.

The resulting deficit is financed by a combination of monetary expansion, as well as domestic and foreign borrowing. If Δy_{BG1} represents the face value of domestic bonds sold by the government in period 1, and C_{F1} represents the dollar value of its foreign borrowing, then its budget deficit in period 2 is given by:

$$D_2 = G_2+S_2+r_2(\Delta y_{BG1}+B_0)+e_2r_{F2}(C_{F1}+B_{F0})-T_2 \tag{7}$$

where $r_2(\Delta y_{BG1}+B_0)$ represents the interest obligations on its initial domestic debt plus borrowing from period 1, and $e_2r_{F2}(C_{F1}+B_0)$ is the interest payment on the initial stock of foreign debt plus period 1 foreign borrowing.

The government finances its budget deficit by a combination of monetization, domestic borrowing, and foreign borrowing. We assume that foreign borrowing in period i, C_{Fi}, is exogenously determined by the lender. The government then determines the face value of its bond sales in period i, Δy_{BGi}, and finances the remainder of the budget deficit by monetization.[16] Hence:

$$D_i = P_{Bi}\Delta y_{BGi} + P_{Mi}\Delta y_{Mi} + e_iC_{Fi}$$

5. The Foreign Sector

The foreign sector is represented by a simple export equation in which aggregate demand for exports is determined by domestic and foreign price indices, as well as world income. The specific form of the export equation is:

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[16]As before, 1 denotes period i and 2 denotes period i+1.

$$\Delta X_{no} = \sigma_1 \left[\frac{\pi_1}{\Delta e_i + \pi_{Fi}} \right] + \sigma_2 \Delta y_{wi}$$

where the left-hand side of the equation represents the change in the dollar value of exports in period i, π_i is inflation in the domestic price index, Δe_i is the percentage change in the exchange rate, and π_{Fi} is the foreign rate of inflation. Also, Δy_{wi} represents the percentage change in world income, denominated in dollars. Finally, σ_1 and σ_2 are corresponding elasticities.

The combination of the export equation and domestic supply responses determines aggregate exports. Demand for imports is endogenous and is derived from the domestic consumers' maximization problems. Foreign lending is assumed to be exogenous. Thus gross capital inflows are exogenous, but the overall change in reserves is endogenous. Finally, we will suppose that the exchange rate is fixed.

IV. Simulations

In this section we carry out simulations designed to give some qualitative notion of the implications for the economy of tax evasion and entry into the underground economy. We use data from Pakistan, but this should be viewed as having only a tenuous relationship to the economy of that country.[17] We first consider a base line scenario and then carry out certain counterfactual exercises designed to analyze the effects of alternative tax policies in reducing the size of the underground economy.

In order to use our model for counterfactual simulations, we first generate an equilibrium using benchmark policy parameters. We run the macroeconomic model forward for eight years,[18] giving tax rates and public expenditures their estimated values. In particular, we assume an effective corporate tax rate of 13 percent. We also suppose that the central bank maintains a fixed exchange rate, with the rate being fixed at the level of the first year.

Table 1 shows the results of the benchmark simulation. It may be worth making a few remarks concerning the simulated values. We do not wish to make comparisons with actual historical data, given the illustrative nature of this example. First notice that our

[17] We have used various parameters derived from Iqbal (1994, 1998) as well as Feltenstein and Shah (1993) in order to implement the functional forms of our model.

[18] In practice, we take 1993 as the base year. By this we mean that initial allocations of factors and financial assets are given by stocks at the end of 1992. We have data for fiscal and other policy parameters for the next 8 years, that is, through 2000.

model generates moderate rates of growth in real GDP for the first seven periods, after which real growth stagnates. This is primarily the result of the fixed nominal exchange rate, which becomes progressively overvalued. The budget deficit improves, and then fluctuates as activity in the underground economy declines. At the same time, however, credit rationing fluctuates.. Similarly, interest rates decline and then begin to rise.

It is useful to observe the change in participation of the different sectors in the underground economy. We see that sectors 2 and 3 both have a share of their activity in the underground economy during the initial periods. Over time, their underground activity falls as a share of their total output. The reason for this decline is that the rate of return to capital slowly rises over time, as real GDP rises more rapidly than does investment. However the rate of change in investment is not uniform across sectors, so underground activity in sector 2 falls more rapidly than in sector 3. We thus see that underground activity may be cyclical.

Table 1. Base Case

Period	1	2	3	4	5	6	7	8
Nominal GDP 1/	100.0	116.9	106.1	125.1	142.9	159.1	197.5	234.5
Real GDP 1/	100.0	108.4	103.9	105.7	112.9	112.1	119.9	119.4
Price level	100.0	107.8	102.1	118.4	126.6	141.4	164.8	196.7
Interest rate	11.0	0.5	10.9	2.7	11.7	11.5	14.4	16.5
Budget deficit 2/	-6.0	-6.3	0.2	-4.8	1.7	-3.5	0.6	-4.6
Trade balance 2/	-7.6	-7.4	-8.9	-8.3	-9.7	-8.6	-10.9	-10.7

	Net capital stock at end of period 8 1/	Percent of sector in underground economy (Period)				K/C 3/ (Period)				Credit granted (Percent) 4/ (Period)			
		2	4	6	8	2	4	6	8	2	4	6	8
Sector 1	100.0	0.0	0.0	0.0	0.0	0.5	5.9	0.9	3.2	100.0	35.0	85.7	47.3
Sector 2	100.0	12.6	0.6	0.0	0.0	11.8	18.0	21.7	30.0	100.0	92.2	94.7	95.6
Sector 3	100.0	20.9	13.3	17.7	10.9	2.1	3.9	4.1	5.7	100.0	67.3	79.7	80.3
Sector 4	100.0	0.0	0.0	0.0	0.0	7.4	12.0	14.1	14.2	100.0	88.3	92.3	93.3
Sector 5	100.0	0.0	0.0	0.0	0.0	1.6	5.5	4.8	5.8	100.0	62.3	84.7	82.7

1/ Normalized to period 1 of the base case.
2/ As a percent of GDP.
3/ Ratio of estimated value of firm's capital to the value of its loan request (as in eq. 4).
4/ Percentage of requested financing that is actually granted to the firm.

Suppose now that the government moves to a high tax regime. That is, the government increases the capital tax rate from its current 13 percent to 23 percent. Obviously this is an arbitrary change, but could be viewed as a typical instrument for reducing the budget deficit. Table 2 shows the outcomes of this exercise. As might be expected, the increase in the corporate tax rate has a deflationary impact upon the economy. In addition, there is a decline in real GDP, primarily due to the decline in investment in all sectors, as can be seen from a comparison of the final capital stock in all sectors in Tables 1 and 2.

There are, however, certain unexpected outcomes in this simulation. We see that, with the exception of sector 5, all sectors move partially into the underground economy. They gradually move back into the legal economy as the corresponding investment stagnation, caused partially by credit rationing, results in a higher return to new capital.[19] Since sectors 1 - 4 are evading taxes, they are also having their credit restricted, as compared to Table 1.

Hence, raising the corporate income tax rate has negative consequences beyond those that one might normally expect. The entry of firms into the underground economy leads to a decline in the tax base so that there is only a modest improvement in the budgetary situation. At the same time, credit has been rationed to the non-tax paying firms, leading to reductions in investment. Thus the tax increase appears to lead to few benefits.

[19] If we compare credit granted in Tables 1 and 2, we see that the percentage of requested loans that is actually granted generally declines in Table 2.

Table 2. Capital Tax Rate Increase

Period	1	2	3	4	5	6	7	8
Nominal GDP 1/	100.5	117.0	105.5	123.8	128.2	144.2	172.2	200.9
Real GDP 1/	100.5	108.7	103.9	105.1	110.7	110.8	117.4	116.8
Price level	99.9	107.6	101.5	117.8	115.8	130.2	146.7	172.1
Interest rate	11.0	0.4	8.8	-0.4	13.5	12.2	13.4	14.1
Budget deficit 2/	-5.4	-6.2	0.9	-4.4	2.3	-3.6	2.0	-3.8
Trade balance 2/	-7.8	-7.4	-9.0	-8.4	-8.9	-7.9	-10.0	-9.7

	Net capital stock at end of period 8 1/	Percent of sector in underground economy (Period)				K/C 3/ (Period)				Credit granted (Percent) 4/ (Period)			
		2	4	6	8	2	4	6	8	2	4	6	8
Sector 1	98.2	64.4	58.9	56.4	23.3	0.2	12.8	0.4	9.3	100.0	18.3	92.7	26.3
Sector 2	98.6	79.1	77.8	75.4	71.2	5.6	10.2	10.7	13.6	100.0	85.4	91.1	91.4
Sector 3	91.2	85.3	85.7	87.4	80.0	0.8	3.4	1.6	3.1	100.0	39.6	77.0	62.0
Sector 4	96.6	61.4	63.1	63.0	48.0	3.2	6.4	6.3	8.4	100.0	76.3	86.7	86.3
Sector 5	97.9	0.0	0.0	0.0	0.0	1.4	5.0	4.3	5.2	100.0	58.7	83.3	81.3

1/ Normalized to period 1 of the base case.
2/ As a percent of GDP.
3/ Ratio of estimated value of firm's capital to the value of its loan request (as in eq. 4).
4/ Percentage of requested financing that is actually granted to the firm.

Suppose now that the government decides to move in the opposite direction. That is, it lowers taxes. Such a policy might be carried out as an attempt to create something like a Laffer effect that increases tax revenues by increasing economic activity in response to lower taxes, while reducing the attractiveness of entry into the underground economy. As an extreme example, we will reduce the corporate income tax rate to 3 percent, from the 13 percent in the base case. Clearly the intent of such a policy would be to stimulate growth by increasing both investment and consumption. At the same time, lower tax rates would presumably discourage underground activity and therefore enhance the tax base. Table 3 gives the outcomes of the low tax case

Table 3. Capital Tax Rate Decrease

Period	1	2	3	4	5	6	7	8
Nominal GDP 1/	103.7	117.2	120.5	130.4	184.1	219.5	248.2	306.9
Real GDP 1/	101.4	108.0	107.1	106.7	117.7	118.2	123.4	122.5
Price level	102.2	108.6	112.5	122.3	156.4	185.7	201.1	250.5
Interest rate	11.2	2.8	11.7	11.1	15.2	17.6	26.5	38.0
Budget deficit 2/	-7.4	-8.2	-2.2	-7.6	-1.7	-5.9	-5.2	-11.3
Trade balance 2/	-8.0	-7.3	-9.5	-7.9	-10.8	-10.8	-12.0	-12.4

	Net capital stock at end of period 8 1/	Percent of section in underground economy (Period)				K/C 3/ (Period)				Credit granted (Percent) 4/ (Period)			
		2	4	6	8	2	4	6	8	2	4	6	8
Sector 1	104.3	0.0	0.0	0.0	0.0	1,1	5.4	1.6	3.7	100.0	51.7	86.3	61.7
Sector 2	100.0	0.0	0.0	0.0	0.0	38.7	51.3	78.6	90.9	100.0	97.5	98.1	98.7
Sector 3	108.5	0.0	0.0	0.0	0.0	6.1	11.2	10.7	14.3	100.0	85.7	91.7	91.3
Sector 4	102.7	0.0	0.0	0.0	0.0	14.0	22.7	22.7	30.2	100.0	93.3	95.7	95.8
Sector 5	104.8	0.0	0.0	0.0	0.0	3.0	8.3	7.2	10.9	100.0	75.3	89.3	87.3

1/ Normalized to period 1 of the base case.
2/ As a percent of GDP.
3/ Ratio of estimated value of firm's capital to the value of its loan request (as in eq. 4).
4/ Percentage of requested financing that is actually granted to the firm.

Again, there are some unexpected changes, as compared to the base case. We see that, although there is no underground activity, the rate of capital formation has increased significantly in only one sector. This is largely due to the fact that the budget deficit has more than doubled, leading to crowding out of private investment by public borrowing. There is a corresponding rapid rise in the interest rate, which tends to outweigh the impact on investment of the tax decrease. Indeed, the rate of investment has slowed significantly by the final period. At the same time, the average annual inflation rate rises from 10.1 percent to 13.7 percent. Also, the trade balance deteriorates as increases in the monetary base combine with the assumed fixed exchange rate regime. As might be expected, the share of credit requested that is actually granted has risen, as compared to Table 1. As firm's fully pay their taxes, the corresponding bank estimate of their net worth rises, leading to lower credit restrictions.

Hence, we may conclude that the low tax regime is not sustainable over time, due to increases in the budget and trade deficits, even though it eliminates underground economic activity and reduces credit rationing. Accordingly, we may conclude that it might well be possible to have tax rates that induce some underground behavior, yet are nonetheless optimal for the overall economy. At the same time, moderate tax increases can lead to entry into the underground economy and credit rationing that have a significant recessionary impact on the economy.

V. Summary and Conclusion

We use a dynamic general equilibrium structure in which optimizing firms compare the rate of return on investment with the corporate tax rate to analyze the determinants and effects of an underground economy for public finances and the macroeconomy. If the tax rate is higher than the return to investment, the firm moves into the underground economy, that is, it engages in tax evasion. At the same time, a firm that evades taxes is subject to credit rationing by banks.

We carry out a series of simulations of the model, based on stylized data from Pakistan. Since we have not estimated any parameters, our results should be viewed as having only a tenuous relationship to reality. A benchmark simulation, using actual tax rates, shows that entry into the underground economy can have a cyclical nature, as the rate of return on investment changes. A second simulation raises the corporate tax rate, as a possible anti-budget deficit policy. This turn out to be counterproductive with a large amount of production fleeing to the underground economy, thereby lowering the tax base and actually increasing the deficit. Aggregate investment in the economy and, hence, growth, is lowered due to greater credit rationing by the banking system. A third simulation reduces the corporate tax rate, with the intent of creating Laffer curve effects. This policy does, indeed, eliminate underground activity, but at the cost of high rates of inflation, increased budget deficit, and a loss of foreign reserves. Hence this scenario is not sustainable in the long run.

We may thus conclude that it is possible that, in the absence of any flexibility to adjust expenditures, an economy may have to accept some underground activity, that is, tax evasion as part of an otherwise acceptable tax program. We have not considered the possibility of a government enforcement technology that might reduce the incidence of tax evasion. Also, we have not looked at the impact of productive government spending on infrastructure and property rights protection in reducing the underground economy. These may represent directions for future research

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