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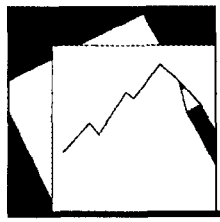

IMF Working Paper

Optimal Monetary Policy in a Small Open Economy with Habit Formation and Nominal Rigidities

Woon Gyu Choi and Yongseung Jung

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Optimal Monetary Policy in a Small Open Economy with Habit Formation and Nominal Rigidities

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Abstract

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Introducing habit formation into an open economy macroeconomic model with price stickiness, we examine the characteristics of an optimal monetary policy. We find that, first, the optimal policy rule entails interest rate smoothing and responds to the lagged values of the foreign interest rate and domestic technology shocks as well as their current values. Second, habit formation enriches the dynamics of the economy with a persistent, hump-shaped response of consumption to shocks. Finally, when habit formation does matter, the optimal policy rule achieves a greater welfare improvement over alternative policy rules by achieving lower macroeconomic variability.

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I. INTRODUCTION

The recent development in international finance that has become known as the new open economy macroeconomics (NOEM) incorporates imperfect competition and nominal rigidities into a dynamic general-equilibrium structure for an open economy to explore empirical issues such as excessive movements of exchange rates and liquidity effects (see, for example, Clarida and Galí, 1993; Eichenbaum and Evans, 1995). More recently, many authors have gone a step further, using NOEM, to study the properties of alternative monetary policies. Galí and Monacelli (2002) set up a small open economy version of the sticky-price model and analyze the properties of alternative monetary policy rules. They show that optimal monetary policy in such an economy responds to foreign interest rates and domestic technology shocks.

The monetary authority's typical policy action involves gradual changes in the interest rate target (Rudebusch, 1995; Choi, 1999). Such interest rate smoothing has often been considered as reflecting the monetary authority's desire to avoid distrupting or exerting undue stress on financial markets (Rudebusch, 1995; Clarida, Galí, and Gertler, 1999). Rudebusch (2002) suggests that interest rate smoothing evident in the estimated policy rules may reflect the persistent shocks that central banks face. Existing theory, however, largely fails to account for why central banks adjust interest rates in such a sluggish fashion (Clarida, Galí, and Gertler, 1999).[2]

One of the key features of aggregate data is the hump-shaped gradual response of spending to various shocks: for example, the hump-shaped response in nondurables and services consumption to a monetary policy shock. This observation raises a challenging question: what is the source of the gradual response to shocks? Fuhrer (2000) suggests that the inclusion of habit formation in the consumer's utility function significantly improves the short-run dynamic behavior of a model, which incorporates explicit expectations formation and frictions that allow monetary policy to have real effects. Some studies have attempted to improve the performance of sticky-price models in explaining business cycles by incorporating habit formation. Boldrin, Christiano, and Fisher (2001) show, in a two-sector real business cycle model, that habit formation helps explain the equity premium puzzle. Abel (1990, 1999) and Campbell and Cochrane (1999) also show that habit formation helps explain the time-varying equity premium. Jung (2002) shows that the introduction of habit formation into a sticky-price model improves model predictions for some selected variables, including output growth, at high frequencies.

The habit-formation specification incorporates the idea that a consumer's current utility is determined by current consumption relative to a reference level of consumption. Suppose that the reference level of consumption is the lagged level of aggregate consumption. If aggregate consumption increases today owing to a shock, the consumer will experience a higher utility from an additional unit of consumption tomorrow. Intuitively, under habit formation, the consumer gets "used" to a higher level of consumption, and the marginal utility of consumption gets "renormalized" at the higher reference level. As a result, the shock propagates consumption persistence, and past movements matter to policymakers who are concerned about macroeconomic variability. Hence a policy reaction to habit formation may entail smoothness in the policy instrument setting. As emphasized in Galí and Monacelli (2002), under high capital mobility, foreign interest rate changes and exogenous shocks have implications for a small economy's optimal

[2] Clarida, Galí, and Gertler (1999) suggest that model uncertainty and conservatism can also motivate a smoother path of interest rates than the certainty-equivalent policy implics.

monetary policy. This paper shows that in the NOEM framework, the existence of habit formation requires the optimal monetary policy to smooth interest rates in response to such shocks.

The paper introduces an Abel-type habit formation in consumption (Abel, 1990, 1999) into the NOEM framework with a Calvo-style price setting (Calvo, 1983) and producer currency pricing, based on a small open economy version of Obstfeld and Rogoff's (1995) redux model.[3] We derive an analytic form of the optimal monetary policy rule under domestic inflation targeting. To see the properties of the optimal monetary policy and its implications for the economy, we examine the dynamic responses of the model economy with the optimal monetary policy to domestic and foreign technology shocks. We also draw implications of openness of the economy for the optimal monetary policy. Further, we perform a welfare evaluation of the performance of the optimal monetary policy and alternative policy rules, using a second-order approximation of the consumer's utility.

The main findings are as follows. First, the optimal monetary policy in the small open economy with nominal rigidities and habit formation takes the form of interest rate smoothing. In addition to responses to the current foreign interest rate and domestic technology shocks that were emphasized in Galí and Monacelli (2002), we show that the optimal policy entails responses to the lagged domestic interest rate, the lagged domestic technology shocks, and the lagged foreign interest rate.

Second, habit formation enriches the dynamics of economic repercussions in response to shocks. We show that technology shocks have persistent effects on the small open economy with the optimal policy: in particular, consumption responses to shocks take on a pronounced hump shape. The openness of an economy affects the optimal policy coefficients and the variability of policy variables: for example, as the openness of an economy increases, the optimal policy in response to foreign shocks entails more responsive interest rates but less responsive exchange rates. Further, model calibrations under alternative policy rules suggest that, when habit formation does matter, the optimal policy rule achieves a greater welfare improvement over alternative policy rules by achieving lower macroeconomic variability.

The remainder of the paper is organized as follows. Section II presents a model of a small open economy with habit formation. Section III derives equilibrium conditions, and Section IV derives model dynamics and the optimal monetary policy rules. Section V analyzes the responses of the open economy to exogenous shocks and draws the implications of the degree of openness of the economy for the optimal monetary policy. The dynamic effects and welfare costs of alternative policy rules are also examined. Section VI concludes.

II. A Small Open Economy with External Habit Formation

A. Households with External Habit Formation in Consumption

Abel (1990, 1999) specified a simple recursive preference, in which a representative household derives utility from the level of consumption relative to a time-varying habit level. We assume that the utility function of the household takes the following form:[4]

[3] The choice of currency becomes an important issue when prices are sticky. Obstfeld and Rogoff (2000) suggest that nominal prices are preset in the currency of the producer (producer currency pricing). On the other hand, Betts and Devereux (1996, 2000) argue that prices are reset in consumers's currency (pricing to market or local currency pricing).

[4] With complete financial markets, each household's behavior can be rewritten in the same way as in Woodford (1996). For notational simplicity we suppress the index $j \in [0,1]$, which represents the variety as well as the identity of the household.

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{(C_t^d)^{1-\sigma}}{1-\sigma}-\frac{N_t^{1+\nu}}{1+\nu}\right)\right],\ 0<\beta<1, \tag{1}$$

where E_0 denotes the conditional expectations operator on the available information set in period 0, β is the household's discount factor, and $C_t^d = C_t/X_t$. C_t, X_t, and N_t represent the domestic household's consumption of composite goods, the time-varying habit level of consumption, and work hours in period t, respectively.

To make the discussion more concrete, a specific CES (constant elasticity of substitution) consumption index is assumed as follows:

$$C_t = \left[\theta^{\frac{1}{\psi}}C_{Ht}^{\,1-\frac{1}{\psi}} + (1-\theta)^{\frac{1}{\psi}}C_{Ft}^{\,1-\frac{1}{\psi}}\right]^{\frac{\psi}{\psi-1}},\ 0<\theta<1,\quad \psi>0, \tag{2}$$

where θ and $1-\theta$ are the shares of consumption allocated to domestic goods and imported goods, respectively. Here C_{Ht} and C_{Ft} are indices of consumption of domestic and foreign goods, which are given by the following CES aggregators of the consumed amounts of each type of good:

$$C_{Ht} = \left[\int_0^1 C_{Ht}(j)^{\frac{\phi-1}{\phi}}dj\right]^{\frac{\phi}{\phi-1}},\ C_{Ft} = \left[\int_0^1 C_{Ft}(j)^{\frac{\phi-1}{\phi}}dj\right]^{\frac{\phi}{\phi-1}}.$$

As is well known, ψ measures the intratemporal elasticity of substitution between domestic and foreign goods, and ϕ measures the elasticity of substitution among goods within each category.

Next, X_t summarizes the influence of past consumption levels on today's utility. The utility of a representative household depends on the ratio between consumption and habit. The stochastic sequence of habits $\{X_t\}_{t=0}^{\infty}$ is regarded as exogenous by the household and tied to the stochastic sequence of aggregate consumption $\{C_t\}_{t=0}^{\infty}$ as follows. For simplicity, X_t is specified as an external habit depending on only aggregate consumption as in Abel (1990, 1999). That is,

$$X_t = \widetilde{C}_{t-1}^{\kappa},$$

where $\widetilde{C}_{t-1}$ is aggregate past consumption. The parameter κ indexes the importance of habit formation: for example, if $\kappa = 0$, only absolute level of consumption matters (the standard model); and if $\kappa = 1$, only consumption relative to previous consumption matters. In this specification of habit formation, habit depends on one lag of consumption. Since there is a representative agent, aggregate consumption equals the household's consumption in equilibrium:

$$X_t = C_{t-1}^{\kappa}. \tag{3}$$

The household faces a time constraint such that

$$L_t + N_t \le \overline{N}, \tag{4}$$

where N_t and $\overline{N}$ denote the hours worked and the time endowment of the household, respectively.

The state of the economy, z_t, evolves according to a Markov process described by a density function $f(z_{t+1}, z_t)$, and there exists a complete asset market in the economy. In particular, we assume that there is a market for contingent one-period, home-currency-denominated bonds as in Woodford (1996) and Chari, Kehoe, and McGrattan (2000). That is, we assume that the representative household chooses a one-period, nominal contingent home-currency bond, $B(z_{t+1})$, in the asset market. $B(z_{t+1})$ pays one dollar if state z_{t+1} occurs next period and 0 otherwise, and $P_B(z_{t+1}, z_t)$ denotes the price of such a bond in period t when the state in period t is z_t. Then the one-period, nominal risk-free interest rate in period t is given by $R_t \equiv [\int P_B(z_{t+1}, z_t)dz_{t+1}]^{-1}$.

The household's budget constraint in period t is given by

$$\int_0^1 [P_{Ht}(j)C_{Ht}(j) + P_{Ft}(j)C_{Ft}(j)]dj + \int B(z_{t+1})P_B(z_{t+1}, z_t)dz_{t+1} \leq B_t + W_t N_t + \Gamma_t, \quad (5)$$

where $P_{H,t}(j)$ and $P_{F,t}(j)$ are the domestic-currency prices of home goods and of foreign goods, respectively, in period t. W_t and Γ_t denote the domestic nominal wage and lump-sum taxes (or transfers) in the domestic currency, respectively, in period t.

The optimal allocation of any given expenditure within each category of goods is given by

$$C_{Ht}(j) = \left[\frac{P_{Ht}(j)}{P_{Ht}}\right]^{-\phi} C_{Ht}, \quad C_{Ft}(j) = \left[\frac{P_{Ft}(j)}{P_{Ft}}\right]^{-\phi} C_{Ft}, \quad (6)$$

for all $j \in [0, 1]$. Here the price indices for domestic and foreign goods, which are expressed in domestic currency, are given by

$$P_{Ht} = \left[\int_0^1 P_{Ht}(j)^{1-\phi}\right]^{\frac{1}{1-\phi}}, \quad P_{Ft} = \left[\int_0^1 P_{Ft}(j)^{1-\phi}\right]^{\frac{1}{1-\phi}}.$$

Similarly, the optimal allocation of expenditure between domestic and foreign goods is given by

$$C_{Ht} = \theta\left[\frac{P_{Ht}}{P_t}\right]^{-\psi} C_t, \quad C_{Ft} = (1-\theta)\left[\frac{P_{Ft}}{P_t}\right]^{-\psi} C_t, \quad (7)$$

where $P_t = \left[\theta P_{Ht}^{1-\psi} + (1-\theta)P_{Ft}^{1-\psi}\right]^{\frac{1}{1-\psi}}$ is the consumer price index (CPI). As we assume that the Law of One Price holds, the price of foreign good j in domestic currency, $P_{Ft}(j)$, equals its price in foreign currency, $P_{Ft}^*(j)$, multiplied by the nominal exchange rate, S_t:

$$P_{Ft}(j) = S_t P_{Ft}^*(j).$$

In the rest of the world, a representative household faces a problem identical to the one outlined above. The only difference is that a negligible weight is assigned to consumption goods produced in the small economy ($\theta^* = 1$). Therefore, $P_t^* = P_{Ft}^*$ and $C_t^* = C_{Ft}^*$ for all t.

B. Firms with Calvo-Type Staggered Price Setting

Differentiated goods and monopolistic competition are introduced as in Dixit and Stiglitz (1977), which provides a convenient framework for aggregation of differentiated goods. Suppose that there are a continuum of firms producing differentiated goods and that each firm indexed by j, $0 \leq j \leq 1$, produces its product with a constant-returns-to-scale, concave production technology. Each domestic firm j takes P_{Ht} and aggregate demand as given and chooses its own product price $P_{Ht,t}(j)$.

In this economy, distortion occurs because of monopolistic competition in the goods market. The firm sets, on average, its price above marginal cost. In equilibrium this makes the marginal rate of substitution between consumption and labor different from their corresponding marginal rate of transformation. Since we assign no explicit value to money holdings, we can eliminate the distortion associated with the Friedman rule. Therefore, if prices were fully flexible, the equilibrium allocations would be efficient. For this reason we need to fully neutralize the effects of nominal rigidities and restore the allocation associated with the flexible-price equilibrium.

Under flexible prices, equilibrium implies a constant markup of size $\frac{\phi-1}{\phi}$, the same markup that prevails in the zero-inflation steady state of the model with nominal rigidities. Thus we

assume that the government fully offsets the distortion by subsidizing employment at a constant rate $\frac{1}{\phi}$ under the optimal monetary policy.[5]

Since the input markets are perfectly competitive, firm j's demand for labor is determined by its cost minimization as follows:

$$\begin{aligned} C\{W_t, Y_t(j)\} &\equiv \min_{N_t(j)}\{W_t(1-\tfrac{1}{\phi})N_t(j)\} \\ \text{subject to} \quad & Y_t(j) \leq A_t N_t(j), \end{aligned} \tag{8}$$

where A_t is a domestic technology shock in period t, and $Y_t(j)$ and $N_t(j)$ are the output and labor input of the jth firm in the home country, respectively. We assume that A_t follows an AR(1) process:

$$\log A_t = \rho_A \log A_{t-1} + \xi_{At}, \tag{9}$$

where $-1 < \rho_A < 1$, and ξ_{At} is an independently and identically distributed (i.i.d.) error term.

Next, to reflect the firms' price setting under monopolistic competition, we employ a discrete time version of the Calvo-style staggered price setting rule (Calvo, 1983) with producer currency pricing. Suppose that firms set prices in the currency of the producer in both domestic and foreign markets. Firms under monopolistic competition in the product markets set their own price in advance by maximizing the present discounted value of profits. Suppose that only a fraction, $1-\alpha$, of (randomly selected) firms set the new price, $P_{Ht,t}$, to home consumers optimally, and the other fraction, α, of firms, sets the current price at its previous level. Let $Y_{Ht,t+k}$ and $Y_{Ft,t+k}$ denote domestic and foreign demand in period $t+k$ facing firms that set their prices in period t, respectively. Let $P_{Ht,t+k}$ denote the prices in period $t+k$ that are predetermined in period t. Assuming that the probability of setting a new price level for each firm is independent of the time elapsed since the last price change, the firm's maximization problem can be written as follows:

$$\max \; E_t\left\{\sum_{k=0}^{\infty}(\alpha\beta)^k \frac{\Lambda_{t+k}P_t}{\Lambda_t P_{t+k}}[P_{Ht,t+k}(Y_{Ht,t+k}+Y_{Ft,t+k}) - MC_{t+k}(Y_{Ht,t+k}+Y_{Ft,t+k})]\right\}, \tag{10}$$

where $P_{Ht,t+k} = P_{Ht,t}$, Λ_t and Λ_{t+k} are the marginal utility of wealth in periods t and $t+k$ with $k = 0, 1, 2, \ldots, \infty$, and MC denotes the marginal cost of production.

III. Equilibrium for the Small Open Economy

A. Households' First-Order Conditions

The first-order conditions for the domestic household can be summarized as follows:

$$\frac{(C_t^d)^{-\sigma}}{X_t} = \Lambda_t \tag{11a}$$

$$N_t^{\nu} = \Lambda_t \frac{W_t}{P_t} \tag{11b}$$

$$\Lambda_t P_B(z_{t+1}, z_t) = \beta \frac{P_t}{P_{t+1}} \Lambda_{t+1} f(z_{t+1}, z_t), \tag{11c}$$

[5] Galí and Monacelli (2002) assume that the subsidy is financed through a lump-sum tax on households. Alternatively, the subsidy can be financed through an inflation tax. We consider the case of zero inflation for simplicity, and a constant inflation rate does not affect our results since related variables are expressed in log-linearized terms around the steady state.

and the time and budget constraints are in equations (4) and (5). Equation (11a) says that the marginal utility of the consumption good equals the marginal utility of wealth. Equation (11b) relates the marginal utility of leisure to the marginal utility of the real wage. Equation (11c) refers to the intertemporal decision of the household, that is, the decision on bond holdings.

Plugging equation (11a) into equation (11c), and taking conditional expectations on both sides, we have the following stochastic Euler equation:

$$\beta R_t E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{X_{t+1}}{X_t} \right)^{\sigma-1} \left(\frac{P_t}{P_{t+1}} \right) \right\} = 1. \tag{12}$$

B. Optimal Risk Sharing

To develop a relationship between the real exchange rate and marginal utilities of consumption in each region, we derive the first-order condition with respect to bond holdings of the rest-of-the-world consumer given by

$$\Lambda_t^* P_B(z_{t+1}, z_t) = \beta [\frac{P_t^*}{P_{t+1}^*} \frac{S_t}{S_{t+1}} \Lambda_{t+1}^*] f(z_{t+1}, z_t), \tag{13}$$

where asterisks denote foreign values of the corresponding domestic variables. Under the assumption of complete asset markets, arbitrage implies that the marginal utility of consumption of foreign residents is proportional to that of home residents multiplied by the real exchange rate, defined as $Q_t \equiv \frac{S_t P_t^*}{P_t}$. Combining equations (12) and (13), after iterating, leads to

$$\left(\frac{C_t^d}{C_t^{*d}} \right)^{\sigma} \frac{X_t}{X_t^*} = \mathbf{k}^{\sigma} Q_t, \tag{14}$$

where **k** is a constant that depends on initial conditions. The relative time-varying habit levels of the small economy and the rest of the world also affect the real exchange rate, which in turn affects the trade balance.

C. Firms' First-Order Conditions

The firm's cost minimization condition with a lump-sum subsidy can be written as

$$W_t(1 - \frac{1}{\phi}) = MC_t A_t, \tag{15}$$

where MC_t is the marginal cost of the firm in period t.

From the firm's discounted expected profit maximization, the first-order conditions of the newly determined prices in period t are given by

$$P_{Ht,t} = \frac{\phi - 1}{\phi} \frac{E_t[\sum_{k=0}^{\infty} (\alpha\beta)^k \frac{\Lambda_{t+k}}{P_{Ht+k}} Y_{t,t+k} MC_{t+k}]}{E_t[\sum_{k=0}^{\infty} (\alpha\beta)^k \frac{\Lambda_{t+k}}{P_{Ht+k}} Y_{t,t+k}]}, \tag{16}$$

where $Y_{t,t+k} = Y_{Ht,t+k} + Y_{Ft,t+k}$.

Since $P_{it}^{1-\phi} = \sum_{s=0}^{\infty} u_s P_{it-s,t}^{1-\phi}$ for $i = H, F$, $u_s = \alpha^s u_0$, and $u_0 = 1 - \alpha$, the price level satisfies the recursive form such that

$$P_{Ht}^{1-\phi} = (1 - \alpha) P_{Ht,t}^{1-\phi} + \alpha P_{Ht-1}^{1-\phi} \tag{17a}$$

$$(P_{Ft})^{1-\phi} = (1 - \alpha)(P_{Ft,t})^{1-\phi} + \alpha (P_{Ft-1})^{1-\phi}. \tag{17b}$$

If prices are flexible ($\alpha = 0$), the markup (the ratio of price to marginal cost) is constant ($\frac{\phi-1}{\phi}$).

D. Market-Clearing Conditions, Uncovered Interest Parity, and Some Identities

The equilibrium conditions for domestic and foreign goods markets are given by

$$Y_t = C_{Ht} + C^*_{Ht} \tag{18a}$$

$$Y^*_t = C^*_t, \tag{18b}$$

where C^*_{Ht} is the demand for domestic consumption goods from the rest of the world, and $C^*_t = C^*_{Ft}$.

With the assumption that $\theta^* = 1$, the consumer price index (CPI) definition for domestic and foreign countries can be rewritten as

$$P_t = [\theta P_{Ht}^{1-\psi} + (1-\theta)P_{Ft}{}^{1-\psi}]^{\frac{1}{1-\psi}} \tag{19a}$$

$$P^*_t = P^*_{Ft}. \tag{19b}$$

Next, the terms of trade, defined as the price of foreign goods in terms of domestic goods, is given by $\mathrm{T}_t \equiv \frac{P_{Ft}}{P_{Ht}}$. The real exchange rate can be written as

$$Q_t = \frac{\mathrm{T}_t P_{Ht}}{P_t}. \tag{20}$$

Since we assume complete international markets, the equilibrium prices of risk-free bonds denominated in foreign currency and in domestic currency are given by $S_t R^{*-1}_t = \int P_B(z_{t+1}, z_t) S_{t+1} dz_{t+1}$, and $R^{-1}_t = \int P_B(z_{t+1}, z_t) dz_{t+1}$, respectively. These can be combined with optimal sharing condition (12) to deduce an uncovered interest parity (UIP):

$$E_t\left[\frac{(C^d_{t+1})^{-\sigma}}{X_{t+1}P_{t+1}}\left(R_t - R^*_t \frac{S_{t+1}}{S_t}\right)\right] = 0. \tag{21}$$

That is, on average, the domestic interest rate equals an "adjusted" foreign interest rate.

The log-linearization of the CPI formula and the definition of the terms of trade yield

$$p_t = \theta p_{Ht} + (1-\theta)p_{Ft} = p_{Ht} + (1-\theta)\tau_t,$$

where lowercase letters denote the log of the respective variable. The definition of the real exchange rate and the above equation imply that

$$q_t = \tau_t + p_{Ht} - p_t = \theta\tau_t. \tag{22}$$

The (log) real exchange rate is proportional to the (log) terms of trade, with the proportionality coefficient being negatively correlated with the degree of openness.

Taking logs on both sides of equation (14), assuming that $\kappa = 1$ as in Abel (1990), and abstracting from a constant term,

$$c_t = \frac{1-\sigma}{\sigma} c_{t-1} + c^*_t + \frac{1-\sigma}{\sigma} c^*_{t-1} + \frac{1}{\sigma} q_t. \tag{23}$$

When $\sigma = 1$ and thus habit formation does not matter, optimal risk sharing says that $c_t = c^*_t + q_t$, implying that domestic consumption and the real exchange rate move one for one.

IV. Dynamics and Optimal Monetary Policy

A. Dynamics Around the Steady State for the Rest of the World

Aggregate Output and Demand Dynamics

It is convenient to start by discussing the simultaneous determination of consumption and output in the rest of the world. The preferences of the representative agent in the rest of the world are identical to those in the small open economy, but with a negligible weight on goods imported from the small open economy; that is, $\theta^* = 1$. The Euler equation for the rest-of-the-world consumer can be log-linearized as follows:

$$-\sigma E_t(c^*_{t+1}) + \sigma c^*_t + (\sigma - 1)[c^{*\kappa}_t - c^{*\kappa}_{t-1}] = -[r^*_t - E_t(\pi^*_{t+1})],$$

where lowercase letters denote the log of the respective variable.

Assuming that $\kappa = 1$ as in Abel (1990), the above equation combined with the market clearing condition $y^*_t = c^*_t$, implies

$$E_t(\Delta y^*_{t+1}) - \mathbf{a}\Delta y^*_t = \sigma^{-1}[r^*_t - E_t(\pi^*_{t+1})], \tag{24}$$

where $\Delta y^*_t \equiv y^*_t - y^*_{t-1}$ and $\mathbf{a} = \frac{\sigma-1}{\sigma}$. Without habit formation, the relationship collapses to that in Galí and Monacelli (2002).

Markup and Inflation Dynamics

Under the Calvo-type price-setting scheme, the optimal price $P^*_{t,t}$ can be approximated in the neighborhood of the zero-inflation steady state as follows:

$$p^*_{t,t} = \sum_{j=1}^{\infty}(\alpha\beta)^j E_t(\pi^*_{t+j}) - (1-\alpha\beta)\sum_{j=0}^{\infty}(\alpha\beta)^j E_t(\mu^*_{t+j}), \tag{25}$$

where $\mu^*_t = \log(\frac{P^*_t}{MC^*_t})$ is the (log) markup of a firm in the rest of the world in period t. This price-setting rule and the price of the rest of the world imply the following dynamics of inflation:

$$\pi^*_t = \beta E_t(\pi^*_{t+1}) - \gamma\hat{\mu}^*_t, \tag{26}$$

where $\hat{\mu}^*_t$ is the (log) markup, expressed as a deviation from its steady state (μ^*_t), and $\gamma \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$ (for more details, see Yun, 1996).

The log-linear approximation of the markup leads to

$$\mu^*_t = -[\sigma c^*_t + \nu n^*_t + (1-\sigma)c^{*\kappa}_{t-1} - a^*_t]. \tag{27}$$

Again assume that $\kappa = 1$ as in Abel (1990). Because the markup in the rest of the world is constant under flexible prices, the natural level of world output, y^*_{nt}, is given by

$$y^*_{nt} = \frac{\sigma-1}{\sigma+\nu}y^*_{nt-1} + \frac{1+\nu}{\sigma+\nu}a^*_t. \tag{28}$$

Therefore, letting $\hat{x}_t$ denote the value of variable x, expressed as a deviation from its steady state, the relationship between the markup gap and the output gap is given by

$$\hat{\mu}^*_t = -(\sigma+\nu)\hat{y}^*_t - (\sigma-1)\hat{y}^*_{t-1}, \tag{29}$$

where $\hat{\mu}^*_t \equiv \mu^*_t - \mu^*$ and $\hat{y}^*_t \equiv y^*_t - y^*_{nt}$.

Substituting equation (29) into equation (25), we have the so-called new Keynesian Phillips curve (NKPC):

$$\pi_t^* = \beta E_t(\pi_{t+1}^*) + \gamma(\sigma + \nu)\hat{y}_t^* + \gamma(\sigma - 1)\hat{y}_{t-1}^*. \tag{30}$$

With habit formation, the NKPC includes the lagged output gap $\hat{y}_{t-1}^*$. This term appears because habit formation affects the households' decision on work hours, and hence the lagged consumption (which equals the lagged output) affects the markup through the labor market clearing condition. Without habit formation, the NKPC has a standard form, which includes expected inflation and the current output gap but not the lagged output gap.

B. Dynamics Around the Steady State for the Small Open Economy

Aggregate Output and Demand Dynamics

For our small open economy, it can be shown that there exists a unique equilibrium for output, consumption, the terms of trade, and the nominal exchange rate.[6]

Market-clearing condition (for the domestic good j) in the small open economy suggests

$$\begin{aligned} Y_t(j) &= C_{Ht}(j) + C_{Ht}^*(j) \\ &= \left[\frac{P_{Ht}(j)}{P_{Ht}}\right]^{-\phi}\left[\left(\frac{P_{Ht}}{P_t}\right)^{-\psi}\theta C_t + \left(\frac{P_{Ht}}{S_t P_t^*}\right)^{-\psi}(1-\theta^*)Y_t^*\right] \\ &= \left[\frac{P_{Ht}(j)}{P_{Ht}}\right]^{-\phi} \mathrm{T}_t^{\psi} Y_t^*\left[Q_t^{\frac{1}{\sigma}-\psi}\mathbf{k}\theta\left(\frac{X_t}{X_t^*}\right)^{\kappa(1-\frac{1}{\sigma})} + (1-\theta^*)\right], \end{aligned} \tag{31}$$

for all $j \in [0,1]$, and the last equality makes use of optimal sharing rule (14).

Again assume that $\kappa = 1$. Substituting equation (31) into the definition of aggregate output $Y_t = \left[\int_0^1 Y_t(j)^{\frac{\phi-1}{\phi}} dj\right]^{\frac{\phi}{\phi-1}}$ and approximating domestic output with equation (14), we have

$$Y_t = (Y_t^*)^{1-\frac{\omega_\theta}{\theta}} C_t^{\frac{\omega_\theta}{\theta}} \left(\frac{X_t}{X_t^*}\right)^{\frac{(1-\sigma)}{\sigma\omega_\theta}(\omega_\theta-\theta^2)}, \tag{32}$$

where $\omega_\theta = 1 + (1-\theta)(1+\theta)(\sigma\psi - 1) > 0$. Alternatively, we can express the domestic consumer's demand for consumption goods as a weighted average of domestic output and foreign output with external habit adjustment terms as follows:

$$C_t = Y_t^{\frac{\theta}{\omega_\theta}} (Y_t^*)^{1-\frac{\theta}{\omega_\theta}} \left(\frac{X_t}{X_t^*}\right)^{\frac{(\sigma-1)}{\sigma\omega_\theta}(\omega_\theta-\theta^2)}. \tag{33}$$

Whitout habit formation, this can be written as in Galí and Monacelli (2002): $C_t = Y_t^{\frac{\theta}{\omega_\theta}} (Y_t^*)^{1-\frac{\theta}{\omega_\theta}}$.

Next, the log-linearization of the Euler equation yields

$$-\sigma E_t(\Delta c_{t+1}) + (\sigma - 1)\Delta c_t = -[r_t - E_t(\pi_{t+1})]. \tag{34}$$

Plugging the log-linearization of the UIP into the log-linearization of the terms of trade yields

$$\tau_t = [r_t^* - E_t(\pi_{t+1}^*)] - [r_t - E_t(\pi_{Ht+1})] + E_t(\tau_{t+1}).$$

[6] Assuming that T = 1 in the initial equilibrium, it can be shown that $Y = C = \mathbf{k}^{\sigma} Y^*$, implying zero net exports in the steady state (see Appendix I.A).

Using this equation and the log-linearization of the CPI formula, the Euler equation implies

$$E_t(\Delta c_{t+1}) = \frac{1}{\sigma(1-\mathbf{aL})}\{\theta[r_t - E_t(\pi_{Ht+1})] + (1-\theta)[r_t^* - E_t(\pi_{t+1}^*)]\}. \tag{35}$$

Markup and Inflation Dynamics

Equations (16) and (17a) lead to the Phillips curve as that in the rest of the world:

$$\pi_{Ht} = \beta E_t(\pi_{Ht+1}) - \gamma\hat{\mu}_t. \tag{36}$$

Plugging equations (11a) and (11b) into the markup determination equation, we have the following approximation of the (log) markup of the firm in the small open economy:

$$\begin{aligned}\mu_t &= -[\sigma c_t + \nu n_t + (1-\sigma)c_{t-1}^{\kappa} + (1-\theta)\tau_t - a_t] \\ &= -[\sigma y_t^* + (1-\sigma)y_{t-1}^* + \nu y_t + \tau_t - (1+\nu)a_t + \log\mathbf{k}],\end{aligned} \tag{37}$$

where the second line is obtained setting $\kappa = 1$ and using optimal risk sharing rule (14) and an aggregate relationship, $y_t = n_t + a_t$. The (log) markup, expressed as a deviation from its steady-state value (μ), is given by

$$\hat{\mu}_t = -[\sigma\hat{y}_t^* + (\sigma-1)\hat{y}_{t-1}^* + \nu\hat{y}_t + \hat{\tau}_t]. \tag{38}$$

Equations (36) and (38) imply a small open economy version of the NKPC as follows:

$$\pi_{Ht} = \beta E_t(\pi_{Ht+1}) + \gamma\sigma\hat{y}_t^* + \gamma(\sigma-1)\hat{y}_{t-1}^* + \gamma\nu\hat{y}_t + \gamma\hat{\tau}_t. \tag{39}$$

The open economy version of the NKPC says that domestic inflation depends positively on the terms-of-trade gap and the rest of the world's lagged output gap, in addition to the current domestic and foreign output gaps and expected domestic inflation. As an important feature arising from the introduction of external habit formation in consumption, the lagged domestic output gap also affects domestic inflation through the terms-of-trade gap, which depends on lagged domestic and foreign consumption as well as current domestic and foreign consumption. Without habit formation, the open economy NKPC becomes a standard one, with no lagged term.

C. Optimal Monetary Policy

Since we do not assign any explicit value to the holding of money balances, we need not consider the monetary distortion that is relevant to the discussion of the Friedman rule. Therefore resource allocation would be efficient if prices were flexible. Nominal rigidities are the only distortion in the economy that prevents it from optimality. As in King and Wolman (1996), Woodford (2002), and Galí and Monacelli (2002), the government can eliminate the distortion with a subsidy to employment. Under this circumstance the assumed subsidy $\frac{1}{\phi}$ to employment can fully neutralize monopolistic firms' market power distortion, making the markup constant at $1 - \frac{1}{\phi}$. The NKPC implies that the domestic inflation rate, π_{Ht}, should be zero at all times when the markup is constant for all firms. It also applies to the rest of the world.

Under the assumption of domestic inflation targeting (DIT) in both regions, we can set $P_t^* = P_{Ht} = 1$, without loss of generality. Then the definition of the terms of trade implies

$$\tau_t = s_t,$$

where s_t is the log of the nominal exchange rate. This relation and real exchange rate definition (20) imply that $\tau_t = s_t = (1/\theta)q_t$; that is, the terms of trade moves one for one with the nominal exchange rate, which moves proportionately to the real exchange rate.

Optimal Monetary Policy in the Rest of the World

Taking the difference of the Euler equation for the rest of the world yields

$$-\Delta E_t(\mu^*_{t+1}) = (\sigma+\nu)E_t(\Delta c^*_{t+1}) + (1+\nu)(1-\rho_{A^*})a^*_t + (\sigma-1)\Delta c^{*\kappa}_t.$$

Then, using equation (24) and setting $\kappa = 1$, the markup difference for the rest-of-the-world consumer can be written as

$$-\Delta E_t(\mu^*_{t+1}) = \frac{(\sigma+\nu)}{\sigma}\frac{r^*_t - E_t(\pi^*_{t+1})}{1-\mathbf{aL}} + (1+\nu)(1-\rho_{A^*})a^*_t + \frac{(1-\sigma)}{\sigma}\frac{r^*_{t-1}-\pi^*_t}{1-\mathbf{aL}}, \tag{40}$$

where $\mathbf{L}$ is a lag operator.

Suppose that the monetary authority stabilizes the firm's markup at the flexible price level. That is, $\mu^*_t = 0$ and $\pi^*_t = 0$ for all time t because the authority seeks to restore the flexible-price equilibrium. From the markup of the rest of the world's firm, the following should be satisfied:

$$\frac{(\sigma+\nu)}{\sigma}r^*_t + (1+\nu)(1-\rho_{A^*})a^*_t(1-\mathbf{aL}) + \frac{(1-\sigma)}{\sigma}r^*_{t-1} = 0.$$

Therefore the optimal monetary policy of the rest of the world can be written as

$$r^*_t = \rho^*_{r^*}r^*_{t-1} + \varphi_{A^*}a^*_t + \varphi_{A^*_{-1}}a^*_{t-1}, \tag{41}$$

where $\rho^*_{r^*} = \frac{\sigma-1}{\sigma+\nu} > 0$ for $\sigma > 1$ and $\varphi_{A^*} = -\frac{\sigma(1+\nu)(1-\rho_{A^*})}{\sigma+\nu} < 0$, $\varphi_{A^*_{-1}} = -\rho_{A^*}\varphi_{A^*}\frac{\sigma-1}{\sigma} > 0$ for $\sigma > 1$. The optimal monetary policy in the rest of the world follows an interest rate smoothing rule when there is external habit formation in consumption.[7] Note that, when $\sigma = 1$, habit formation plays no role. Without habit formation, the optimal monetary policy is given by a simple interest rate rule without any smoothing as in Galí and Monacelli (2002): $r^*_t = \varphi_{A^*}a^*_t$.

Optimal Monetary Policy in the Small Open Economy

To introduce the optimal monetary policy, as we show in Appendix I.B, we can rewrite the markup in difference form as follows:

$$\begin{aligned}\frac{\Delta E_t(\mu_{t+1})(1-\mathbf{aL})\sigma}{\Omega} &= -\{[r_t - E_t(\pi_{Ht+1})] - \rho_r(r_{t-1}-\pi_{Ht}) - \rho_{r^*}[r^*_t - E_t(\pi^*_{t+1})] \\ &= -\rho_{r^*_{-1}}(r^*_{t-1}-\pi^*_t) + \Theta_A(1-\mathbf{aL})a_t\},\end{aligned} \tag{42}$$

where $\Omega = \sigma + \nu + (1-\theta^2)\nu(\sigma\psi - 1)$, $\rho_r = -\frac{(1-\sigma)\theta - \mathbf{a}\sigma(1-\theta)[1+\nu\psi(1+\theta)]}{\Omega}$, $\rho_{r^*} = -\frac{(\sigma+\nu\theta)(1-\theta)+(1-\theta)\nu-\sigma(1-\theta)[1+\nu\psi(1+\theta)]}{\Omega}$, $\rho_{r^*_{-1}} = -\frac{\{(1-\sigma)(1-\theta)-\mathbf{a}\sigma(1-\theta)[1+\nu\psi(1+\theta)]\}}{\Omega}$, and $\Theta_A = \frac{\sigma(1+\nu)(1-\rho_A)}{\Omega}$.

The optimal domestic monetary policy is also implied by $\mu_t = 0$ and $\pi_{Ht} = 0$ for all time t.

$$\begin{aligned}0 = {} & [r_t - E_t(\pi_{Ht+1})] - \rho_r(r_{t-1}-\pi_{Ht}) - \rho_{r^*}[r^*_t - E_t(\pi^*_{t+1})] \\ & - \rho_{r^*_{-1}}(r^*_{t-1}-\pi^*_t) + \Theta_A(1-\mathbf{aL})a_t.\end{aligned}$$

[7] To eliminate the indeterminacy that would otherwise be associated with the interest rate rule that depends on exogenous variables only, we add an extra term of inflation with a coefficient greater than unity to equations (41) and (43) in the simulation of the model as in Galí and Monacelli (2002) (see also, for a detailed discussion, Bernanke and Woodford, 1997; Clarida, Galí, and Gertler, 1999).

Rewriting the domestic optimal monetary policy,

$$r_t = \rho_r r_{t-1} + \rho_{r^*} r_t^* + \rho_{r^*_{-1}} r_{t-1}^* - \Theta_A(1 - \mathbf{aL})a_t. \tag{43}$$

The optimal monetary policy is a function of its own lagged interest rate as well as the world's current and lagged interest rates. It also depends on current and lagged domestic productivity shocks. Regarding the signs on the policy rule coefficients, several points are noteworthy. First, the sign of the interest rate smoothing parameter, ρ_r, is ambiguous. A sufficient condition for $\rho_r > 0$ is given by $\sigma > 1$ and $\sigma\psi > 1$, which is likely to be satisfied for empirically reasonable values. Also, ρ_r is less than 1 for empirically plausible values of the parameters. In this case the optimal policy involves interest rate smoothing as in a generalized-Taylor rule (Clarida, Galí, and Gertler, 1999). Since ρ_r does not depend on ρ_A, interest rate smoothing stems from habit formation rather than from persistence of shocks.

Second, ρ_r^* is positive and less than 1 for empirically reasonable values of σ and ν. $\rho_{r^*} = \frac{\nu(1-\theta^2)(\sigma\psi-1)}{\sigma+\nu+(1-\theta^2)\nu(\sigma\psi-1)}$ is also positive and less than 1 for $\sigma > 1$ and $\sigma\psi > 1$. $\rho_{r^*_{-1}} = \frac{(1-\theta)(\sigma-1)[1+\nu\psi(1+\theta)]}{\sigma+\nu+(1-\theta^2)\nu(\sigma\psi-1)}$ is also positive and less than 1 for $\sigma > 1$ and $\sigma\psi > 1$.Therefore the domestic optimal monetary policy involves a positive comovement of the domestic interest rate with that of the rest of the world, and thus some degree of exchange rate smoothing. Also, notice that the coefficient on foreign interest rates increases with the degree of openness, implying that the optimal policy requires greater sensitivity of the domestic interest rate to the foreign interest rate when openness increases.

Third, $\Theta_A = \frac{\sigma(1+\nu)(1-\rho_A)}{\Omega} > 0$. Thus the interest rate responds negatively to the current productivity shock. This is quite intuitive, since stabilization of inflation requires to absorb downward pressures on domestic prices exerted by a productivity increase. With a lag, however, the interest rate responds positively to the domestic productivity shock ($0 < \mathbf{a} < 1$).

Finally, all slope coefficients in the optimal policy rule depend on the degree of openness $(1 - \theta)$ as well as the degree of risk aversion (σ) and of substitutability between home and foreign goods (ψ). When $\sigma = 1$ and thus habit formation plays no role, $\rho_r = \rho_{r^*_{-1}} = \mathbf{a} = 0$, implying that the optimal policy responds only to the current foreign interest rate and domestic technology shocks.

D. Alternative Monetary Policy Rules

CPI Inflation Targeting

Suppose that the monetary authority of the small economy employs CPI inflation targeting (CIT), which requires that

$$\pi_t = 0$$

for all t. Suppose further that the rest of the world pursues an optimal monetary policy with $\pi_t^* = 0$. Under this regime we can set $p_t = p_t^* = 0$ without loss of generality. From the definition of the real exchange rate, we can see that the real exchange rate should have the same value as the nominal exchange rate. Since $p_{Ht} = -(1 - \theta)\tau_t$ and $q_t = \theta\tau_t$, we have $q_t = s_t = -\frac{\theta}{1-\theta} p_{Ht}$.

Then the markup can be rewritten as

$$\mu_t = \frac{1}{(1-\theta)}\left\{1 + \nu\left[\frac{\theta^2}{\sigma}\mathbf{b}(\mathbf{L}) + \psi(1-\theta^2)\right]\right\} p_{Ht} + (1+\nu)a_t \tag{44}$$
$$- \left[(\sigma+\nu)\frac{1+\nu}{\sigma+\nu}\mathbf{c}(\mathbf{L})a_t^* + (1-\sigma)\frac{1+\nu}{\sigma+\nu}\mathbf{c}(\mathbf{L})a_{t-1}^*\right],$$

where $\mathbf{b}(\mathbf{L}) = (\mathbf{1} - \mathbf{aL})^{-1}$ and $\mathbf{c}(\mathbf{L})=(1 - \frac{\sigma-1}{\sigma+\nu}\mathbf{L}^{-1})$. Plugging this into the NKPC leads to a second-order stochastic difference equation of the equilibrium domestic inflation rate. Moreover, the long effects of productivity shocks on the domestic price level are exemplified by the lagged productivity shocks as well as the current productivity shocks. When there is no habit formation in consumption, the markup is given by

$$\mu_t = \frac{1}{(1-\theta)}\{1 + \nu[\theta^2 + \psi(1-\theta^2)]\}p_{Ht} + (1+\nu)(a_t - a_t^*).$$

Exchange Rate Peg

The small open economy can pursue an exchange rate peg. Assuming that the rest of the world pursues the optimal monetary policy, the exchange rate peg implies $\tau_t = -p_{Ht}$ and $q_t = -p_t$. Then the markup can be rewritten as

$$\mu_t = \left\{1 + \nu\left[\frac{\theta^2}{\sigma}\mathbf{b}(\mathbf{L}) + \psi(1-\theta^2)\right]\right\}p_{Ht} + (1+\nu)a_t \qquad (45)$$
$$- \left[(\sigma+\nu)\frac{1+\nu}{\sigma+\nu}\mathbf{c}(\mathbf{L})a_t^* + (1-\sigma)\frac{1+\nu}{\sigma+\nu}\mathbf{c}(\mathbf{L})a_{t-1}^*\right].$$

Substituting this into the NKPC leads to a second-order stochastic difference equation for the domestic inflation rate. There is a high persistence in the domestic price level, which leads to persistence in the real and nominal exchange rates as in CIT.

The sign and qualitative pattern of the markup response under a exchange rate peg are the same as those derived for the CIT regime. The difference between an exchange rate peg and CIT stems from the coefficient on domestic prices. If the economy is fully open ($\theta = 0$), there is no difference between the two regimes because domestic prices are entirely determined by foreign prices. In the general case ($0 < \theta < 1$), an exchange rate peg leads to a greater response of the markup, and hence domestic productivity shocks have a greater effect on domestic prices compared to the CIT regime. This is because under an exchange rate peg, equilibrating the trade balance falls solely on price adjustments, whereas under CIT both domestic prices and the exchange rate can adjust.

V. Monetary Policy Rules and Exogenous Shocks

We now examine the dynamic effects of domestic and foreign technology shocks, given a certain degree of openness. We consider the effects of shocks on the key variables: consumption, output, (nominal) interest rates, the nominal exchange rate (NER), the real exchange rate (RER), the terms of trade, domestic inflation, CPI inflation, the (domestic) markup, and the trade balance. Then we look at how the openness of the economy affects the effects of the optimal monetary policy. Finally, we compare the dynamic effects and welfare costs of alternative policy rules.

The baseline model of this paper sets the intertemporal elasticity of consumption, σ, equal to 3. Note that $1/\sigma$ measures the elasticity of the expected consumption growth to the real interest rate. Although many real business cycle models assume $\sigma = 1$, many empirical studies on consumption tell us to be more cautious and conservative in choosing the value : σ is about 2 or more (see Hall, 1988). We assume that $\nu = 1$, implying a unit labor supply elasticity, that is, $N_w = 1$. We choose a conservative value for ν. Empirical micro labor studies suggest that the intertemporal elasticity of labor supply is no greater than 1, whereas real business cycle models take the value of 4 (for example, Yun, 1996). We assume that a steady-state markup value, μ, equals 1.1, which implies that ϕ, the elasticity of substitution between differentiated goods, is 11. The nominal rigidity parameter, α, equals 0.75, a value consistent with an average period of one year between price adjustments. We assume $\beta = 0.99$, implying a riskless return of about 4 percent in the steady state. The parameter for

the technology shock process, ρ_A, is assumed to be 0.9. The intratemporal elasticity of domestic and foreign goods, ψ, equals 1.5 as in Chari, Kehoe, and McGrattan (2000).

A. Dynamic Effects of a Domestic Technology Shock

Figure 1 shows the dynamic effects on the key variables of a positive unit shock to domestic technology (A_t) when the degree of openness is 0.5. By design, the optimal monetary policy implies no response of domestic inflation or the markup. The response of consumption to the shock is hump-shaped, because, with habit formation, households gradually adjust their consumption profile to the shock. The hump-shaped response of consumption, however, disappears when the degree of openness is 1, because the rest of the world economy completely dominates the small open economy. The degree of openness determines whether the response of output to the shock is hump-shaped. This is because an expenditure-switching factor is proportional to the terms of trade under the optimal monetary policy, and the effect of foreign consumption can dominate that of domestic consumption in determining output, depending on the degree of openness.

Also, a positive domestic technology shock leads to a persistent reduction of the domestic interest rate from its steady-state level under the optimal monetary policy, as implied by a negative coefficient on a_t and interest smoothing in equation (43). Moreover, with habit formation, the persistence of domestic interest rates is increased owing to policy responses to the lagged variables. Since the interest rate of the rest of the world does not respond to the domestic shock, UIP implies an initial depreciation of the nominal exchange rate, calling for a subsequent appreciation to its steady-state level. With price stickiness, nominal exchange rate overshooting results in a real appreciation. The optimal policy requires the same responses of the terms of trade and the nominal exchange rate, to which the response of the real exchange rate is proportional by the factor θ. The initial depreciation accompanies an initial increase in CPI inflation through the tradable sector. Finally, the trade balance response (the response of net exports divided by the steady-state output) reflects an increase in net exports with the real depreciation.

B. Dynamic Effects of a Foreign Technology Shock

Figure 2 shows the dynamic effects of a positive technology shock in the rest of the world when the degree of openness is 0.5. In the face of such a shock, the overseas monetary authority, following the optimal monetary policy rule (41), lowers the interest rate to stabilize inflation and the marginal cost of production in the rest of the world. The optimal monetary policy for the small open economy implies that the domestic monetary authority also lowers its interest rate in response to the decrease in the world interest rate.

However, the degree of the domestic interest rate response to the world interest rate is less than unity ($0 < \rho_{r^*} < 1$). Therefore UIP implies an initial appreciation in terms of the nominal exchange rate, followed by subsequent depreciations to its steady-state level. As the domestic interest rate decreases, domestic consumption increases with a hump shape. Unlike consumption, output can decrease because the initial nominal appreciation accompanies a real appreciation with price stickiness and causes the trade balance to deteriorate. The initial nominal appreciation accompanies a decrease in CPI inflation.

The above impulse response analysis suggests that habit formation and nominal rigidities play a role in persistence of responses to shocks. Nominal rigidities affect macroeconomic variability since the aggregate demand responds to policy changes. Without habit formation, consumption and labor supply, in response to shocks, overshoot instantaneously and then monotonically decrease to their steady state values. With habit formation, however, households, caring about aggregate

consumption, gradually adjust their consumption and labor supply to shocks. As a result, there occurs a persistent, and hump-shaped response of consumption. Consistent with Fuhrer's (2000) empirical finding, our model displays a hump-shaped consumption response owing to habit formation. In the absence of habit formation, however, the response of consumption (and output regardless of the degree of openness) will show a monotone shape (see Galí and Monacelli, 2002).

C. Openness and the Optimal Monetary Policy

The degree of openness, $1 - \theta$, plays a key role in shaping the response of output to shocks. It also determines the size of the response of variables to real shocks. Again, by design, the optimal monetary policy implies no response of domestic inflation or the markup, regardless of the degree of openness. Figure 3A displays the impulse responses to a domestic technology shock when monetary authorities in both regions pursue the optimal monetary policy: the open-circle line depicts the response of a less open economy ($\theta = 0.9$); the solid line that of an open economy ($\theta = 0.5$); and the filled circles that of a fully-open economy ($\theta = 0$). First, output shows a hump-shaped response to the shock when the degree of openness is low enough. This response reflects the idea that, as the degree of openness decreases, the share of foreign consumption in domestic output and thus the expenditure-switching effect decrease, and that the effect of the domestic consumer's external habit on output becomes more pronounced. Second, when the economy is less open, stabilizing the domestic marginal cost and inflation requires a more active response of monetary policy, which leads to a more volatile response of consumption and exchange rates. Conversely, when the economy is fully open, domestic consumption does not respond at all, whereas output and the trade balance respond most to the domestic technology shock.

With reasonable parameter values —for example, $\sigma = 3$ and $\nu = 1$— the coefficient of the lagged interest rate is on the order of 0.6, so that the degree of interest rate smoothing is substantial, consistent with the parameter estimate using the actual data (see, for example, Clarida, Galí, and Gertler, 1999). This paper, therefore, helps us understand why the monetary authority chooses a smooth path of interest rates, contrary to what existing theory based on a general-equilibrium framework would predict.

Figure 3B shows impulse responses to a positive technology shock to the rest of the world. In a less open economy, the optimal monetary policy renders domestic interest rates less responsive and exchange rates more responsive to the foreign technology shock. For example, UIP implies a bigger interest rate difference between home and the rest of the world when $\theta = 0.9$ than when $\theta = 0.5$. When the economy is less open, with a smaller alignment of domestic interest rates and a greater adjustment of the real exchange rate, consumption, net exports, and thus output are less affected by such a shock. Conversely, when the economy is fully open ($\theta = 0$), with a high responsiveness of domestic interest rates and no responsiveness of the real exchange rate, consumption, net exports, and output are more responsive to the foreign shock than when the economy is not fully open.

D. Dynamic Effects of Alternative Monetary Policy Rules

Figure 4 displays the effects of domestic or foreign technology shocks under alternative policy rules: the optimal monetary policy, that is, a domestic price index inflation targeting rule (DIT, open-circle line), a CPI inflation targeting rule (CIT, asterisks), and an exchange rate peg (PEG, filled circles). We maintain the assumption of the adherence to the optimal monetary policy rule in the rest of the world, for comparison purposes. Under DIT, by design, the output gap, domestic inflation, and the markup show no response to the shock. Under alternative policy rules, however, these three variables respond to the shock.

Figure 4A shows the case of a positive domestic technology shock. Under CIT, consumption increases more sluggishly and domestic inflation falls after the shock. The real exchange rate response becomes more hump-shaped, as implied by the optimal risk sharing. The domestic nominal interest rate rises, and the exchange rate depreciates moderately, supporting upward pressures on CPI inflation, as implied by UIP. However, the rise in the interest rate and the smaller depreciation of the exchange rate, compared with what is seen under DIT, result in less of a boost to net exports and a fall in the output gap. Under PEG, the output gap, markup, and domestic inflation respond more to the shock by limiting exchange rate flexibility than under other policy rules.[8] Since the foreign interest rate does not change, UIP implies no domestic interest rate response, while declines in inflation and the real exchange rate accompany an increase in net exports.

Figure 4B shows the responses to a positive foreign technology shock. The overseas monetary authority lowers the interest rate to stabilize the markup and inflation. This leads the domestic monetary authority to lower its interest rate, which in turn leads to a real appreciation as implied by UIP. Again, the output gap, the domestic markup, and the domestic inflation rate do not respond to the shock under DIT. Under CIT and PEG, consumption responds more to the foreign productivity shock, with sharper decreases in the interest rate. With a smaller real appreciation, the positive response of consumption dominates the negative response of net exports, resulting in an initial rise in the output gap. The decline in the domestic interest rate induces increases in consumption and domestic inflation, which result in a decrease in net exports. Although these responses under CIT and PEG are quite similar, CIT puts more weight on controlling inflation and less on the nominal exchange rate than does PEG. As a result, under CIT, decreases in the nominal exchange rate accompany bigger declines in the real exchange rate, which imply bigger decreases in the trade balance and smaller increases in the output gap than under PEG.

Finally, in both figures the real exchange rate and the terms of trade are more stable under PEG than under the optimal monetary policy. Notice also that because of the loss of monetary policy autonomy, consumption, the output gap, and inflation gap are more volatile under PEG than under CIT that allows for a partial stabilization. In this regard, the next section examines the welfare implications of the choice of policy rule.

E. Welfare Cost of Alternative Policy Rules

We employ both a microfounded model and a welfare criterion, based directly on the representative consumer's utility as in Woodford (2002) and Galí and Monacelli (2002). In evaluating alternative policies, we rely on a quadratic approximation of the welfare criterion, which brings the framework closer to the linear-quadratic models extensively used in the Keynesian literature (for the quadratic approximation, see Appendix I.C). The welfare loss function evaluates the distortions existing in the economy. The expected welfare loss of any policy that deviates from strict domestic inflation targeting can be rewritten as

$$\widehat{\mathbf{W}} = -\frac{\mathbf{w}}{2}\Big[(\sigma - 1)\mathrm{V}(\Delta\hat{y}_t) + \frac{\psi}{\gamma}\mathrm{V}(\pi_{Ht}) + (1+\nu)\mathrm{V}(\hat{y}_t)\Big], \tag{46}$$

[8] Our model here does not account for financial market frictions. Choi and Cook (2002) consider the case of a small open economy with financial frictions: a bank balance sheet channel and a currency mismatch problem exist owing to a substantial debt dollarization. In this case an exchange rate peg may be preferable to inflation targeting, because a depreciation causes the bank balance sheet to deteriorate and reduces the credit supply, inducing an adverse effect on economic activity.

where the measure of variability for any variable z is defined by

$$V(z) \equiv (1-\beta)E\left[\sum_{t=0}^{\infty}\beta^t E_0 z_t^2\right].$$

External habit formation in consumption results in an increase in the output gap variability, which reduces welfare, as captured by the first term in equation (46). When there is no external habit —for example when $\sigma = 1$— the first term in the loss function disappears.

Utility gains from consumption and leisure are represented by inflation and the output gap in terms of welfare. The response of consumption and leisure to a shock varies with the source of the shock. Also, as shown in Figure 4, transmission variables such as interest rates and exchange rates respond differently to shocks from different sources. Nevertheless, the simulation of the model suggests that the optimal risk sharing renders the variability of output and inflation independent of the source of a shock. Since the welfare of the economy does not depend on the source of a shock, we report the case of a domestic technology shock below.

Figure 5 shows how our welfare measure under alternative policy rules varies with the degree of openness when there is a domestic technology shock. As in Galí and Monacelli (2002), when the economy is closed ($1-\theta = 0$), DIT, which has zero welfare loss regardless of the degree of openness, coincides with CIT, whereas PEG implies a larger welfare loss; and as the economy becomes more open, CIT and PEG converge to DIT as a limiting case. Unlike that of Galí and Monacelli, our model incorporates habit formation in consumption, which involves more persistent responses of the economy to shocks. With habit formation (curves with symbols), both CIT and PEG have larger welfare losses, except for the limiting cases of openness. This implies that the choice of a policy rule becomes more important when habit formation does matter.

VI. Conclusion

In this paper, we have introduced habit formation into a small open economy model with a Calvo-type price setting and producer currency pricing. We derive an optimal monetary policy rule in a closed form and compare its characteristics with those of alternative policy rules. As in Galí and Monacelli (2002), the influence of foreign monetary policy on the monetary policy of a small open economy is explicitly formulated in a general-equilibrium framework. Unlike Galí and Monacelli, we show that habit formation affects the dynamic characteristics of an optimal monetary policy.

Our optimal policy rule implies that interest rate smoothing is attributable to habit formation in the household's preferences, rather than to the monetary authority's concern about financial market stability. This result does not rely on Rudebusch's (2002) suggestion that interest rate smoothing stems from the persistent shocks that the monetary authority faces. However, it is consistent with his general interpretation that interest rate smoothing reflects endogenous policy responses to a variety of influences that cannot be captured easily: that is, we can interpret persistent shocks as (unobservable) habit persistence. In addition, habit formation renders the optimal monetary policy responsive to lagged domestic productivity shocks and the lagged foreign interest rate. Thus habit formation increases the persistence of optimal monetary policy responses to shocks.

Our findings from model calibrations suggest policy implications as follows. First, changes in consumption and the exchange rate are more persistent under our optimal monetary policy rule than those under the optimal policy rule that does not account for habit formation. The consumption response to shocks shows a hump shape because habit formation leads to a gradual adjustment of consumption to shocks. The output response, however, may not show a hump shape for a highly

open economy because of the effect on consumption of foreign goods, which becomes stronger with the degree of openness. Second, a foreign technology shock is transmitted to the domestic interest rate through an optimal monetary policy rule. As a result, reconciliation of the policy of a small open economy with that of the rest of the world can reinforce a comovement of domestic consumption with foreign consumption. Third, our optimal monetary policy rule, consistent with rational expectations and the optimization framework, suggests that the degree of openness of an economy affects the optimal policy coefficients and the variability of policy variables such as interest rates and exchange rates. With a higher degree of openness, the interest rate becomes less responsive to domestic technology shocks, but more responsive to foreign technology shocks. However, the exchange rate becomes less responsive to both domestic and foreign technology shocks when the degree of openness increases. Finally, domestic inflation targeting tends to outperform CPI inflation targeting and an exchange rate peg by a wider margin when the economy exhibits habit formation in consumption than otherwise, implying that the choice of a policy rule becomes more important in welfare consideration when habit formation does matter.

For simplicity, our model abstracts from the government budget constraint and fiscal policy. In future research, optimal monetary and fiscal policy (see, for the case of a closed economy with sticky prices, Schmitt-Grohé and Uribe, 2001) can also be introduced in the NOEM framework. In contrast to our producer currency pricing environment, under local currency pricing, inflation is not so responsive to changes in the exchange rate, and a wedge is driven between the domestic interest rate setting and the exchange rate change that arises from foreign monetary policy and foreign technology shocks. It will also be of interest to examine the case where a small open economy presets nominal prices in the consumers' currency. Local currency pricing may dampen the effect of openness on monetary policy, rendering the optimal monetary policy more responsive to domestic technology shocks than to foreign shocks.[9]

Future research should examine the implication of incomplete markets for monetary policy in countries with persistent, large current account deficits. Lane and Milessi-Ferreti (2001) provide an empirical link between net foreign asset positions and the real exchange rate. The results of this paper suggest that, under complete markets, the dynamics of the current account do not matter for monetary policy. However, under incomplete markets, the dynamics of the current account do matter because then, besides the distortions in the markup, market incompleteness should be dealt with. If the dynamics of the current account affect the real exchange rate, the optimal monetary policy would possibly entail a response to net foreign asset positions.

[9] The degree of openness can also be associated with the extent to which the economy combines producer currency pricing and local currency pricing. More weight can be placed on the former when the economy is more open. For example, the introduction of the euro may reduce the openness of the European countries towards the world market in the sense that the share of imports in output for the European countries is smaller than that for a pre-euro European country. Devereux, Engel, and Tille (1999) argue that the introduction of the euro insulates European consumer prices from U.S. exchange rate changes since lower transaction costs with the euro tend to tilt firms toward local currency pricing.

A. The Steady State

We can characterize the perfect-foresight, zero-inflation steady state of the small open economy model, taking Y^* as given and setting $A_t \equiv 1$, for all t. In the steady state, $\frac{(1-\eta)W}{P_H} = (1 - \frac{1}{\phi})$, where η is the employment subsidy, by which the small open economy's policymaker guarantees the optimality of the flexible-price equilibrium allocation, as in Galí and Monacelli (2002). From the definition of the CPI, $\frac{P}{P_H} = [\theta + (1-\theta)\mathrm{T}^{1-\psi}]^{\frac{1}{1-\psi}} \equiv g(\mathrm{T})$, with $g'(\mathrm{T}) > 0$. Combining these two equations, we have

$$CY^{\nu} = \frac{1 - \frac{1}{\phi}}{g(\mathrm{T})(1-\eta)}, \tag{A.1}$$

since $C = X$ and $C^* = X^*$ in the steady state.

Next, notice that optimal risk sharing in the steady state implies

$$C = h(\mathrm{T})\mathbf{k}^{\sigma}C^*, \tag{A.2}$$

where $h(\mathrm{T}) \equiv \frac{\mathrm{T}}{g(\mathrm{T})}$ with $h'(\mathrm{T}) > 0$, which links the real exchange rate to the terms of trade in the steady state.

Combining equations (A.1) and (A.2), we have the following relationship between output and exchange rate:

$$Y = \left[\frac{1 - \frac{1}{\phi}}{\mathbf{k}^{\sigma}C^*\mathrm{T}(1-\eta)}\right]^{\frac{1}{\nu}}. \tag{A.3}$$

Given foreign output $Y^*(= C^*)$ and other parameter values, output is decreasing in the terms of trade, and $\lim_{\mathrm{T}\to 0} Y = \infty$ and $\lim_{\mathrm{T}\to\infty} Y = 0$.

On the other hand, market-clearing conditions require

$$\begin{aligned} Y &= C_H + C_H^* \\ &= \theta g(\mathrm{T})^{\psi} C + (1-\theta^*)\mathrm{T}^{\psi} Y^* \\ &= \theta g(\mathrm{T})^{\psi} h(\mathrm{T})\mathbf{k}^{\sigma} Y^* + (1-\theta^*)\mathrm{T}^{\psi} Y^*. \end{aligned} \tag{A.4}$$

Output is increasing in the terms of trade and $\lim_{\mathrm{T}\to 0} Y = 0$ and $\lim_{\mathrm{T}\to\infty} Y = \infty$. Therefore there is a unique steady-state value of output and the terms of trade.

In addition, assuming that $\mathrm{T} = 1$ in the initial equilibrium, we have

$$\begin{aligned} Y &= \theta\mathbf{k}^{\sigma} Y^* + (1-\theta^*)Y^* \\ &= \mathbf{k}^{\sigma} Y^* + [(\theta-1)\mathbf{k}^{\sigma} + (1-\theta^*)]Y^*. \end{aligned}$$

When $\mathbf{k}^{\sigma} = \frac{1-\theta^*}{1-\theta}$, we have $Y = \mathbf{k}^{\sigma} Y^*$ and $C = \mathbf{k}^{\sigma} C^* h(\mathrm{T}) = \mathbf{k}^{\sigma} C^* = \mathbf{k}^{\sigma} Y^*$, implying $Y = C$ and zero net exports in the steady state.

B. The Optimal Monetary Policy

Taking a first difference of the markup equation for the domestic country and assuming $\kappa = 1$, the domestic markup is given by

$$\Delta E_t(\mu_{t+1}) = -(\sigma+\nu\theta)\Delta E_t(c_{t+1}) - \nu(1-\theta)\Delta E_t(c^*_{t+1}) - (1-\sigma)\Delta c_t - (1-\theta)[1+\nu\psi(1+\theta)]E_t(\Delta\tau_{t+1}) - (1+\nu)(1-\rho_A)a_t. \quad \text{(B.1)}$$

Substituting equation (37) and the terms-of-trade equation into the above markup equation to eliminate the consumption terms,

$$\begin{aligned}\Delta E_t(\mu_{t+1}) = &-\frac{(\sigma+\nu\theta)}{\sigma(1-\mathbf{aL})}\{\theta[r_t - E_t(\pi_{Ht+1})] + (1-\theta)[r^*_t - E_t(\pi^*_{t+1})]\} \\ &- \frac{(1-\theta)\nu}{\sigma(1-\mathbf{aL})}[r^*_t - E_t(\pi^*_{t+1})] - (1+\nu)(1-\rho_A)a_t \\ &- \frac{(1-\sigma)}{\sigma(1-\mathbf{aL})}[\theta(r_{t-1} - \pi_{Ht}) + (1-\theta)(r^*_{t-1} - \pi^*_t)] \\ &- (1-\theta)[1+\nu\psi(1+\theta)]\{[r_t - E_t(\pi_{Ht+1})] - [r^*_t - E_t(\pi^*_{t+1})]\}.\end{aligned} \quad \text{(B.2)}$$

Rearranging the terms yields

$$\begin{aligned}\Delta E_t(\mu_{t+1})(1-\mathbf{aL})\sigma = &-\{(\sigma+\nu\theta)\theta + \sigma(1-\theta)[1+\nu\psi(1+\theta)]\}[r_t - E_t(\pi_{Ht+1})] \\ &- \{(1-\sigma)\theta - \mathbf{a}\sigma(1-\theta)[1+\nu\psi(1+\theta)]\}(r_{t-1} - \pi_{Ht}) \\ &- \{(\sigma+\nu\theta)(1-\theta) + (1-\theta)\nu - \sigma(1-\theta) \\ &\times [1+\nu\psi(1+\theta)]\}[r^*_t - E_t(\pi^*_{t+1})] - \{(1-\sigma)(1-\theta) \\ &- \mathbf{a}\sigma(1-\theta)[1+\nu\psi(1+\theta)]\}(r^*_{t-1} - \pi^*_t) \\ &- \sigma(1+\nu)(1-\rho_A)(1-\mathbf{aL})a_t,\end{aligned}$$

which can be rewritten as equation (43) in the main text.

C. Welfare Approximation

We derive the utility-based loss function. The average utility flow of the representative household is given by

$$\omega_t = U(C_t, X_t) - V(N_t) = \frac{(C_t/X_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\nu}}{1+\nu}. \quad \text{(C.1)}$$

Taking a Taylor expansion of the utility function with respect to consumption, we have

$$\begin{aligned}U(C_t, X_t) = &\ U(\overline{C}, \overline{X}) + U_C(C_t - \overline{C}) + U_X(X_t - \overline{X}) + U_{CX}(C_t - \overline{C}) \\ &\times (X_t - \overline{X}) + \frac{1}{2}U_{CC}(C_t - \overline{C})^2 + \frac{1}{2}U_{XX}(X_t - \overline{X})^2 + o(\|\xi\|^3),\end{aligned} \quad \text{(C.2)}$$

where $C_t = X_t = \overline{C}$, $U_C = \overline{C}^{-1}$, $U_{CC} = -\sigma\overline{C}^{-2}$, $U_X = -\overline{C}^{-1}$, $U_{XX} = -(\sigma-2)\overline{C}^{-2}$, $U_{XC} = -(1-\sigma)\overline{C}^{-2}$, and $o(\|\xi\|^n)$ denotes terms that are of order higher than n^{th}, in the bound $\|\xi\|$ on the amplitude of the relevant shocks. Applying log approximation with a second-order Taylor expansion, we have $(C_t - \overline{C})/\overline{C} = (c_t + \frac{1}{2}c_t^2) + o(\|\xi\|^3)$ and $(X_t - \overline{X})/\overline{X} = (x_t + \frac{1}{2}x_t^2) + o(\|\xi\|^3)$, where we approximate $\ln C_t$ around $\ln\overline{C}$, and $c_t \equiv \ln(C_t/\overline{C})$. Substituting these relations into equation (C.2),

$$U(C_t, X_t) = \mathbf{w}\Delta c_t + \frac{1}{2}\mathbf{w}(1-\sigma)\Delta c_t^2 + t.i.p. + o(\|\xi\|^3),$$

where $\mathbf{w} = \frac{\theta}{\omega_\theta}$, and *t.i.p.* indicates terms independent of monetary policy. Here, the consumption ratio rather than the consumption level enters into the household's utility function. Similarly, we have

$$V(N_t) = V(\overline{N}) + V_N \overline{N}\left[n_t + \frac{1}{2}(1+\nu) n_t^2 \right] + o(\|\xi\|^3),$$

where $N_t = \overline{N}$ in the steady state.

Therefore, welfare loss is given by

$$\omega_t = \mathbf{w}\Delta c_t + \frac{1}{2}(1-\sigma)\mathbf{w}\Delta c_t^2 - V_N \overline{N}\left[n_t + \frac{1}{2}(1+\nu) n_t^2 \right] + t.i.p. + o(\|\xi\|^3).$$

Assume $Ey_t = E(\ln Y_t - \ln Y_n) = 0$. Then, following Woodford (2002) and Galí and Monacelli (2002), the above can be rewritten as

$$\omega_t = \frac{1}{2}(1-\sigma)\mathbf{w}\Delta y_t^2 - \mathbf{w}\left[u_t + \frac{1}{2}(1+\nu) y_t^2 \right] + t.i.p. + o(\|\xi\|^3), \tag{C.3}$$

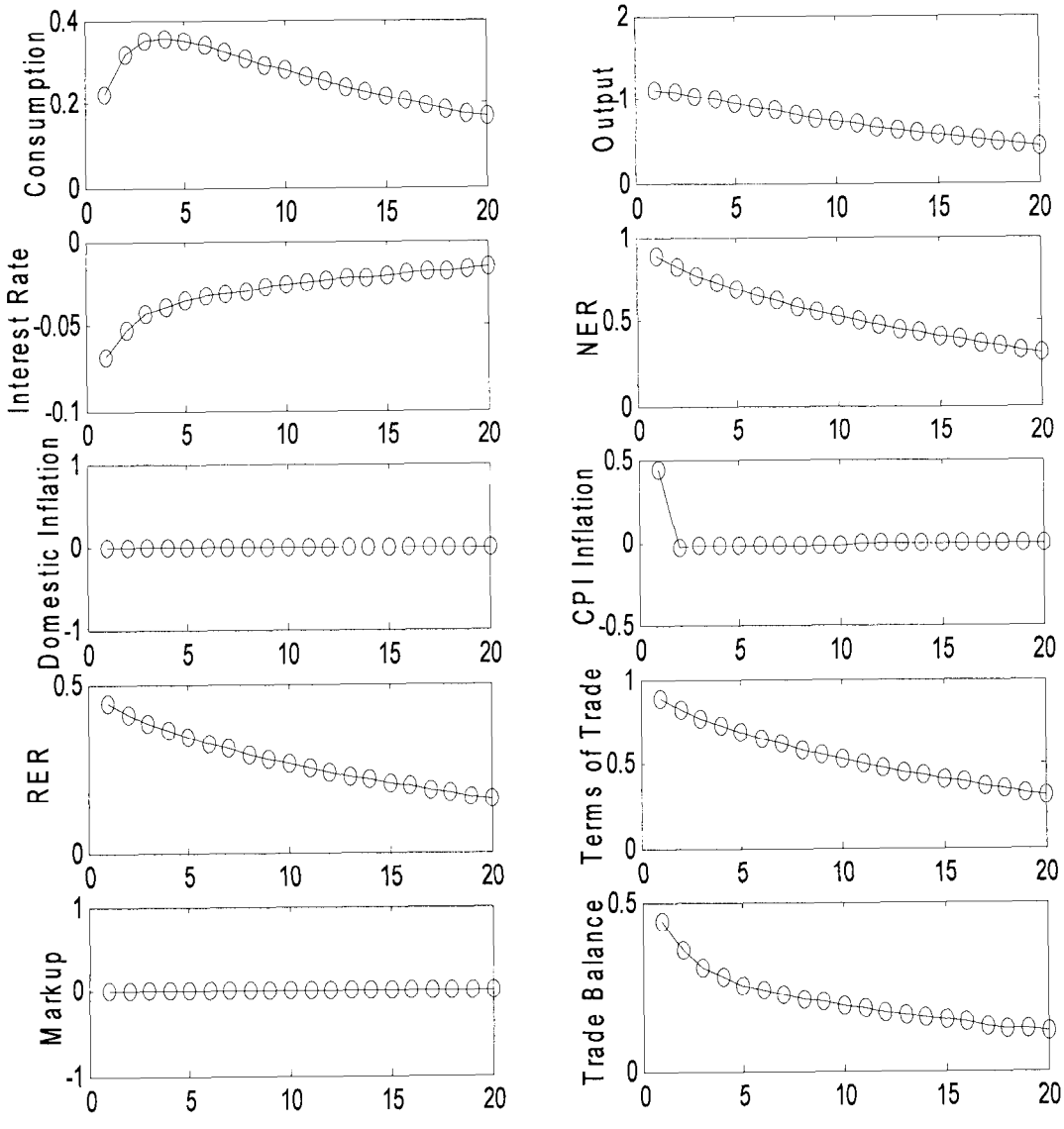
where $u_t = \log \int_0^1 \left[\frac{P_{Ht}(j)}{P_{Ht}} \right]^{-\phi} dj = \frac{\phi}{2} var_j[p_{Ht}(j)] + o(\|\xi\|^3)$.

By taking a quadratic approximation around natural output and a zero inflation rate and abstracting from those terms that are independent of monetary policy, we obtain the following welfare loss function:

$$\mathbf{W} \simeq -\frac{\mathbf{w}}{2} \sum_{t=0}^{\infty} \beta^t \left[(\sigma - 1)\Delta \hat{y}_t^2 + \frac{\psi}{\gamma} \pi_{Ht}^2 + (1+\nu)\hat{y}_t^2 \right], \tag{C.4}$$

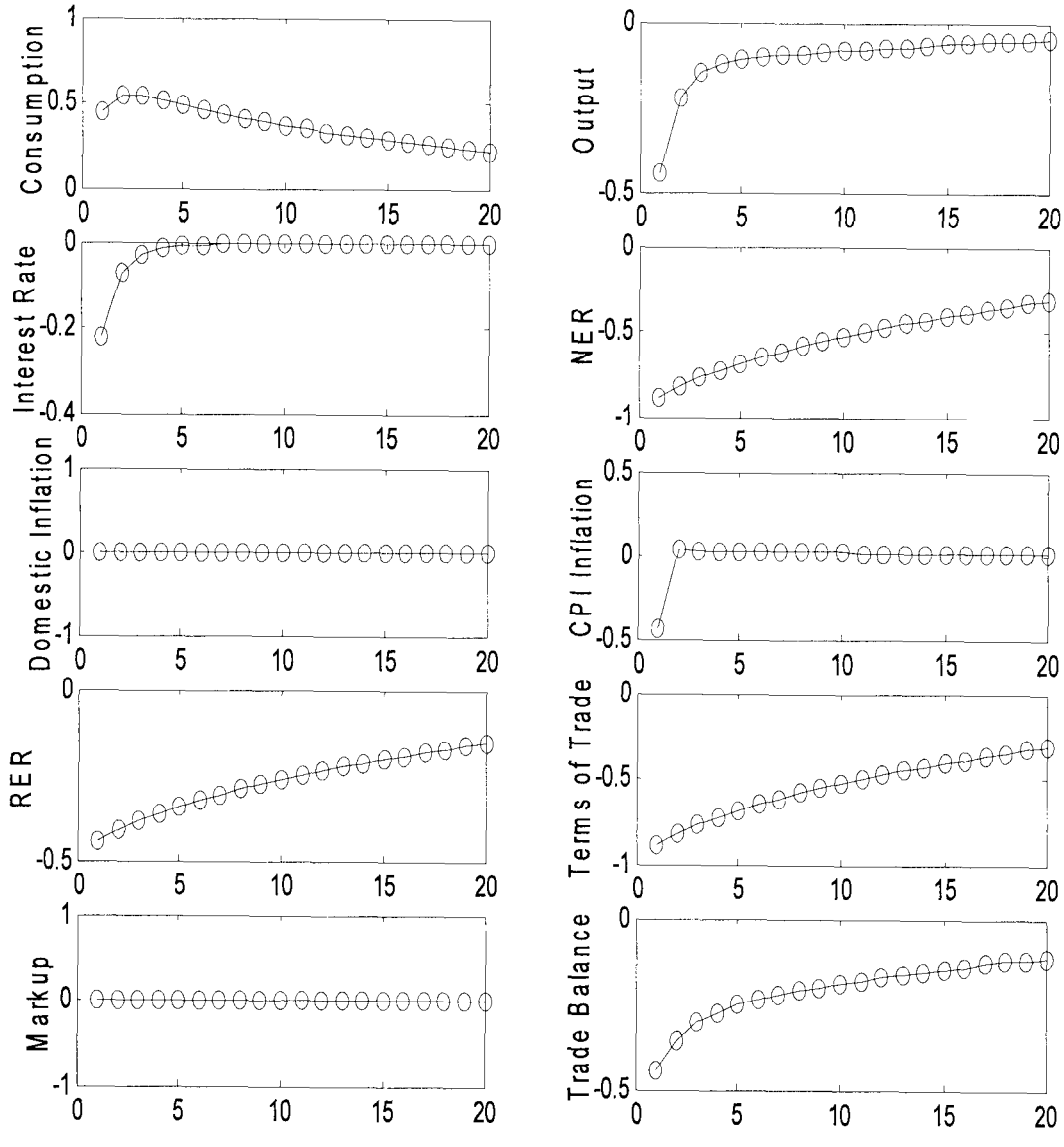
where $\gamma = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$. Then taking unconditional expectations, we can obtain the expected welfare loss, corresponding to equation (46) in the main text.

Figure 1. Dynamic Effects of a Positive Domestic Technology Shock



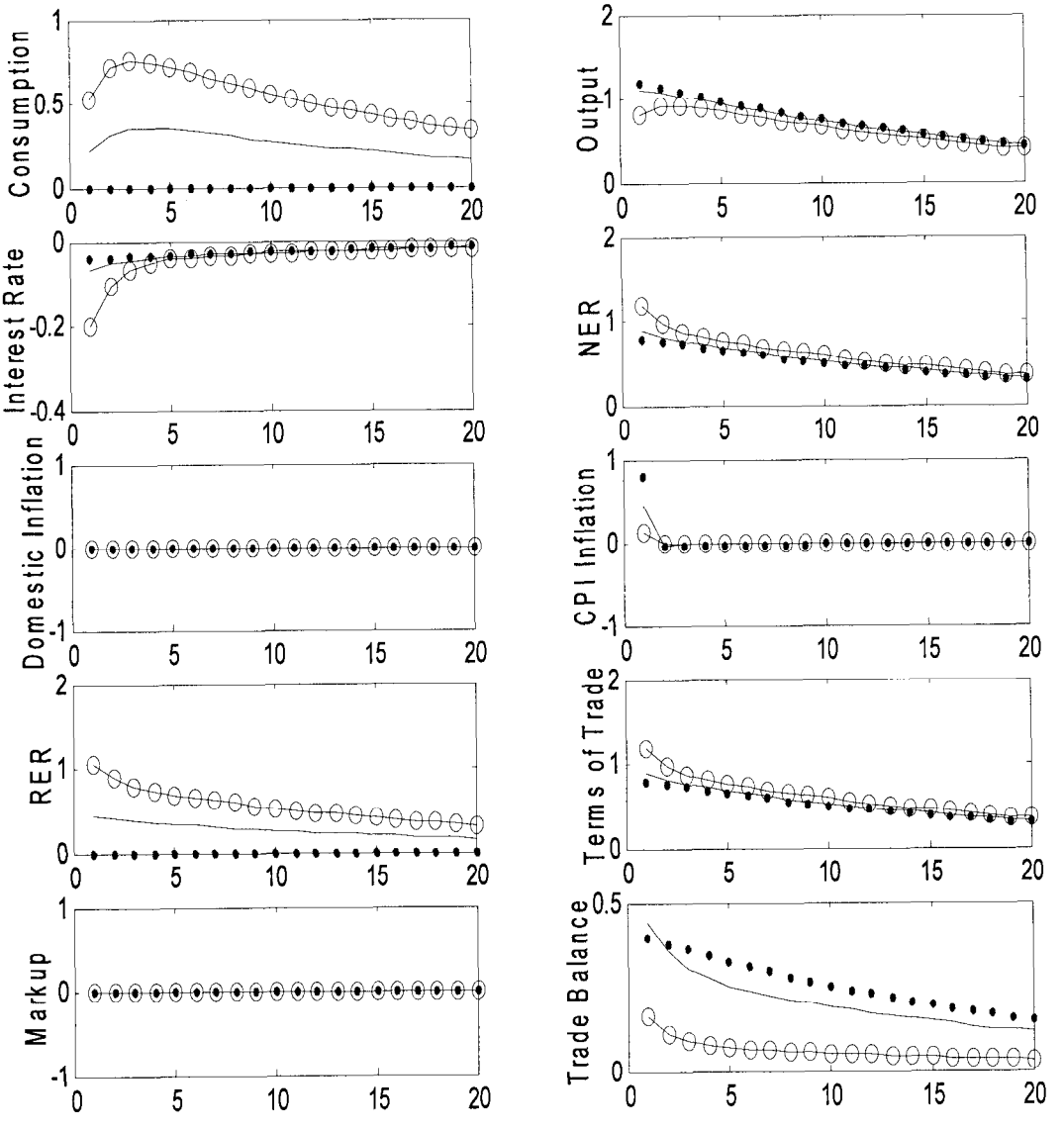
Notes: The impulse responses to a unit innovation in a_t, measured as percentage deviations from the steady state except for the trade balance response, are depicted by open-circle lines. The trade balance response are measured as the response of net exports divided by the steady-state output. NER and RER denote the nominal exchange rate and the real exchange rate, respectively. A positive response by RER means a depreciation relative to the steady-state level.

Figure 2. Dynamic Effects of a Positive Foreign Technology Shock



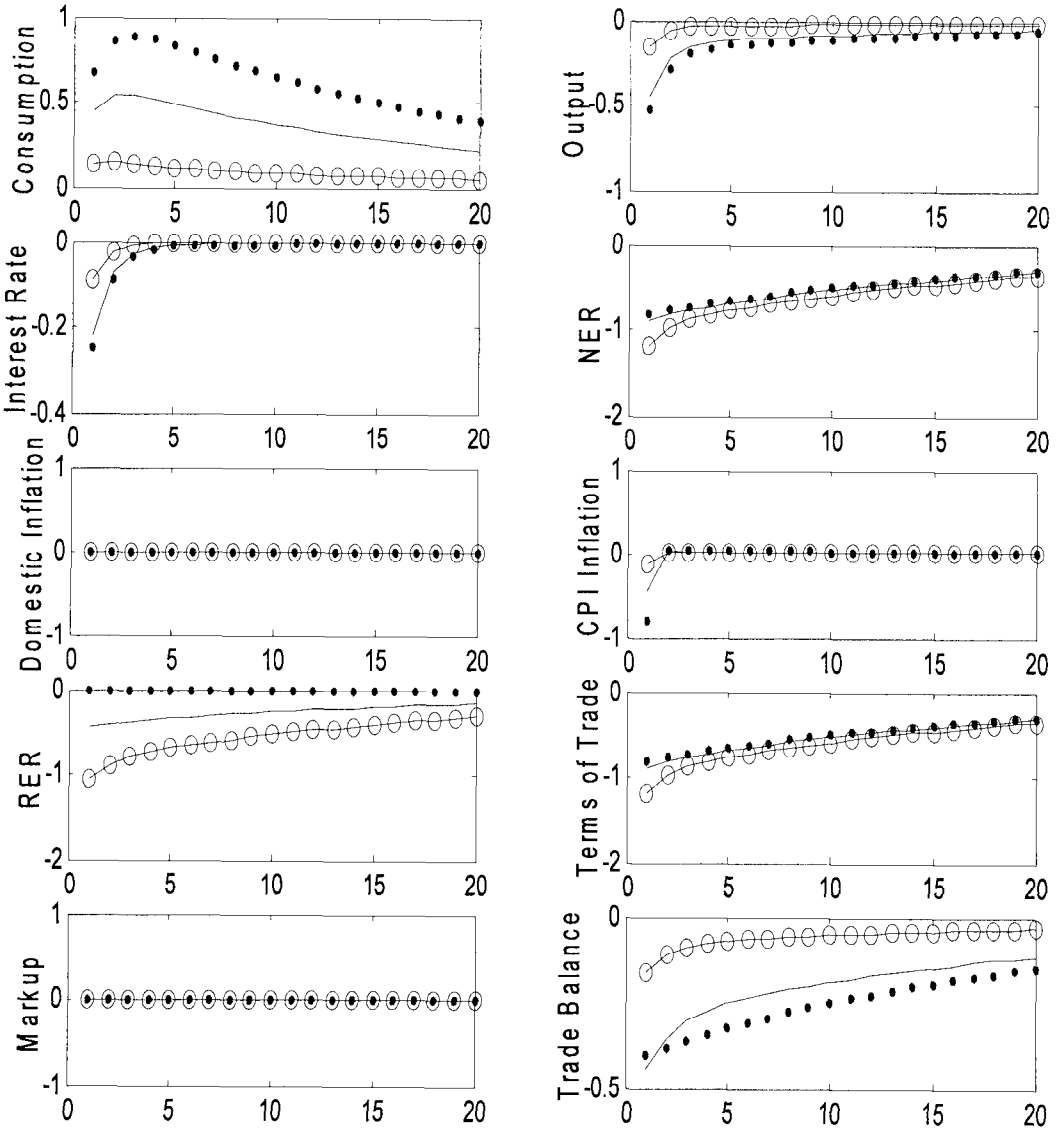
Notes: The impulse responses to a unit innovation in a_t^*, measured as percentage deviations from the steady state except for the trade balance response, are depicted by open-circle lines. The trade balance response are measured as the response of net exports divided by the steady-state output. NER and RER denote the nominal exchange rate and the real exchange rate, respectively. A positive response by RER means a depreciation relative to the steady-state level.

Figure 3A. Openness and Responses to a Domestic Technology Shock



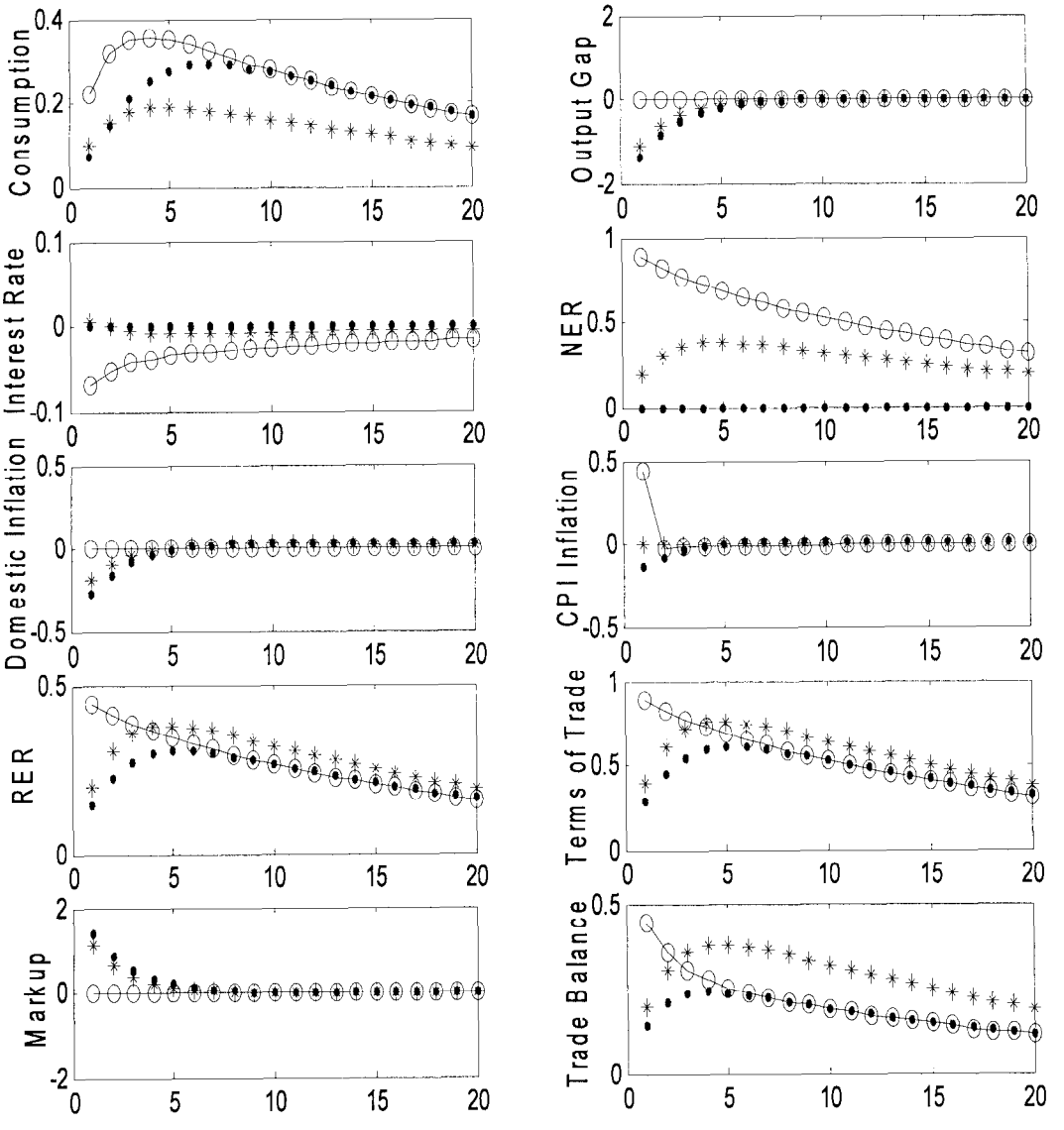
Notes: See notes to Figure 1. Impulse responses are for a less-open economy (θ=0.9; open-circle line, -o), an open economy (θ=0.5, solid line, –), and a fully-open economy (θ=0, filled circles, ••).

Figure 3B. Openness and Responses to a Foreign Technology Shock



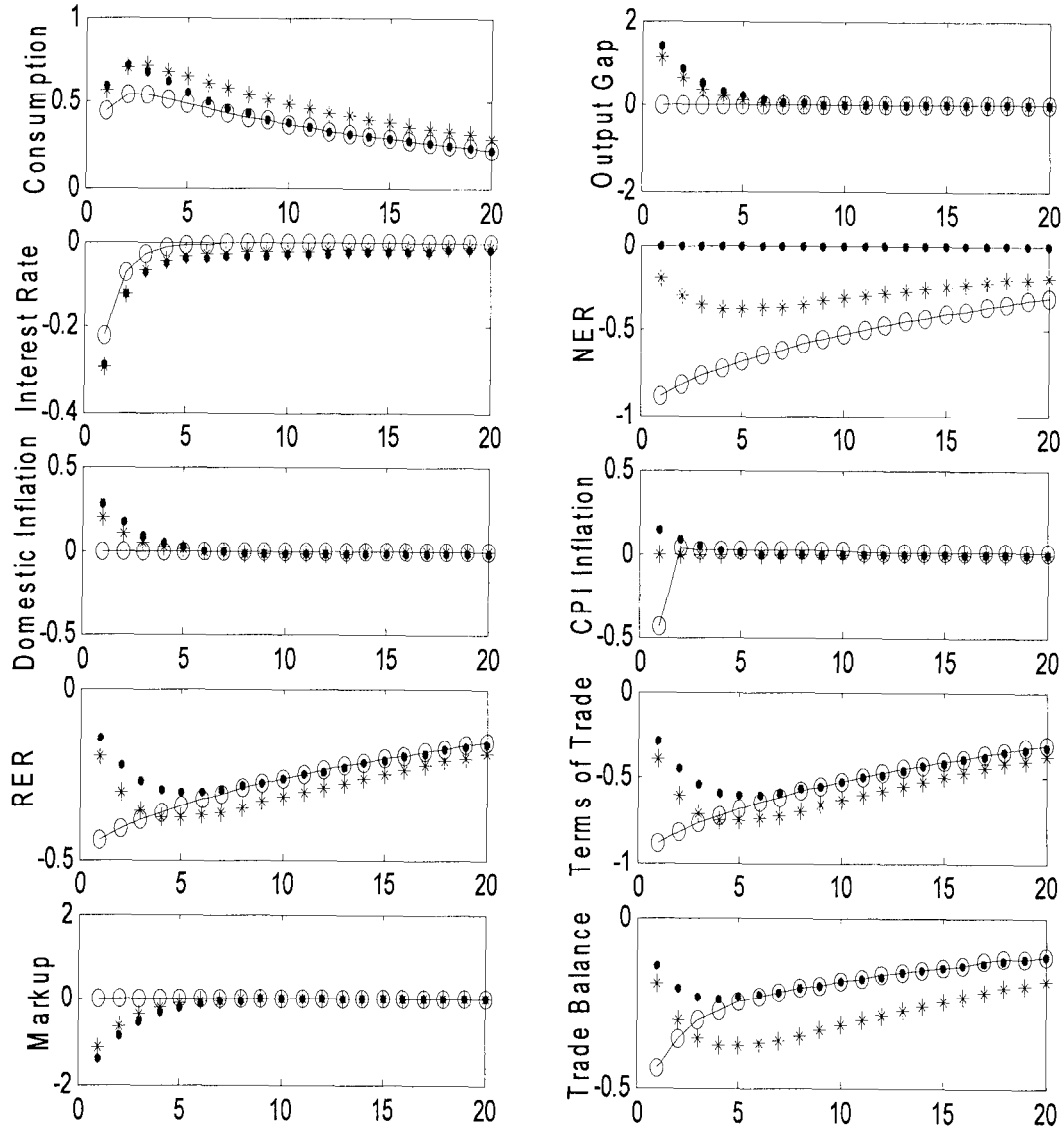
Notes: See notes to Figure 2. Impulse responses are for a less-open economy (θ=0.9; open-circle line, -o), an open economy (θ=0.5, solid line, –), and a fully-open economy (θ=0, filled circles, ••).

Figure 4A. Responses to a Domestic Technology Shock Under Alternative Policy Rules



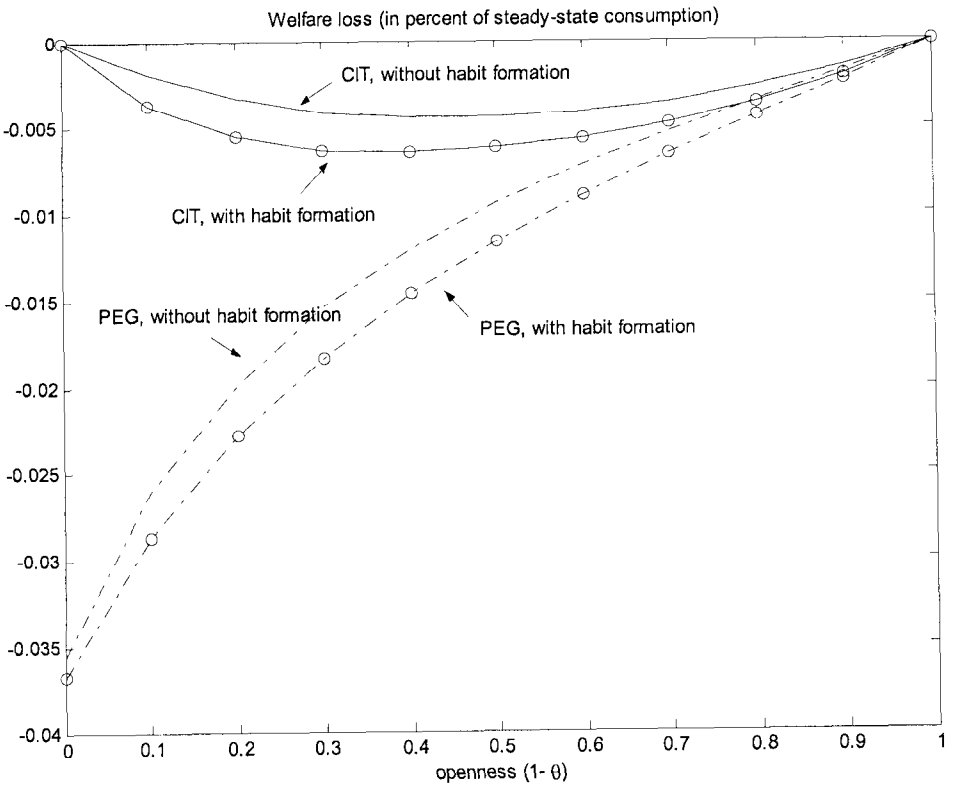
Notes: See notes to Figure 1. Impulse responses are under a domestic price index inflation targeting rule (DIT, open-circle line, -o), a CPI inflation targeting rule (CIT, asterisks, **), and an exchange rate peg (PEG, filled circles, ••).

Figure 4B. Responses to a Foreign Technology Shock Under Alternative Policy Rules



Notes: See notes to Figure 2. Impulse responses are under a domestic price index inflation targeting rule (DIT, open-circle line, -o), a CPI inflation targeting rule (CIT, asterisks, **), and an exchange rate peg (PEG, filled circles, ••).

Figure 5. Welfare Loss and Openness:
Alternative Policy Rules With and Without Habit Formation



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