

WP/02/237

# IMF Working Paper

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## Firm Investment, Corporate Finance, and Taxation

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**IMF Working Paper**

Fiscal Affairs Department

**Firm Investment, Corporate Finance, and Taxation**

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December 2002

**Abstract**

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This paper examines the intertemporal effect of corporate income taxation on the investment behavior of a firm that faces imperfect capital markets. It shows that when capital markets are imperfect, the optimizing firm goes through different phases of growth. In this dynamic setting, the effect of a corporate tax on profits varies over time. An increase in the corporate profit tax rate initially reduces investment, but the effect is reversed over time as the firm adjusts its financing policy to the new tax rate.

JEL Classification Numbers: H22, H25, H32

Keywords: Firm Investment, Imperfect Capital Markets, Corporate Income Taxation

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\* The author wishes to thank Tony Atkinson, James Mirrlees, Peter Sørensen, and Michael Keen for advice and comments.

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## I. INTRODUCTION

Reductions in corporate taxes are envisaged in many countries as a means of fostering capital accumulation and investment decisions by firms. In the United States, for example, there is considerable debate on the need to cut corporate taxation to support firm investment (Orszag, 2001; Gale, 2001). In the European Union, many countries have already reduced corporate taxes (Chennells and Griffith, 1997). However, despite the apparent link between corporate taxation and capital accumulation over time, very little of the theoretical debate has been interested in the time path of tax effects. The theoretical analysis of corporate taxation usually relies on cost-of-capital formulae that tend to assume steady state and to ignore the dynamics of tax effects (Sinn, 1991b); in fact, little of the current literature is based on any satisfactory analysis of the factors underlying the investment and financing behavior of firms over time. However, the effect of a tax change on investment varies over time as firms adjust to the tax change, and the timescale of this effect is critical for governments.

The aim of this paper is to provide a theoretical model of firm investment to examine the effect of corporate taxes over time. To this purpose, we need a dynamic theory of firm investment. We also need to envisage the mechanisms through which imperfections on capital markets affect the financing decisions of firms. In fact, the literature on taxation has demonstrated that the impact of taxes may be very sensitive to the firm's financing decisions (Atkinson and Stiglitz, 1980, Auerbach, 2002). To address these issues, this paper sets up a dynamic model of firm investment where capital markets are imperfect, and financial and real investment policies are simultaneously derived from the firm's decision problem. While this model involves a number of strong assumptions, it does capture some important features of tax effects that would otherwise be lost.

The general framework underlying the model of the firm is set up in Section II. This model is of the "adjustment costs" type that has been discussed a great deal in the literature, but incorporates in addition the firm's financial decisions. The role of financial decisions in the investment process has in fact been examined, among others, by Sinn (1991a); however, he did not consider debt financing and the possibility that along the accumulation path firms choose different combinations of alternative sources of financing. Since this is clearly central to the problem discussed in this paper, I have extended the analysis in this direction. This extended model of investment is used in Section III to determine the financial and real investment decisions of the firm over time. It is shown that the firm goes through different phases of growth and each phase corresponds to different financing decisions. Given the starting stock of capital, the firm initially enters a phase of "intensive" growth during which it finances new investment both by retaining earnings and by issuing new debt. While accumulating, it moves to a "consolidation" phase during which it fully finances investment by debt. Along this path, the firm approaches a steady-state stock of capital. This model differs from Sinn's (1991a) contribution in that along the accumulation path, the firm uses alternative financial instruments and resorts to different marginal sources of financing.

This model of investment is used in Section IV to examine the intertemporal effects of a corporate tax on profits. In discussing these effects, the standard method of comparative dynamics is

initially presented and criticized for it does not explicitly consider time. A different method for the dynamic analysis of taxation is then developed. This consists in directly examining the variations in the temporal path of investment as they derive from changes in the tax policy. A similar method has in fact been used by Atkinson (1971), to whose work I owe a considerable debt; however, the method is extended to a several variables problem and to include no endpoint conditions, which is the central framework of the problem discussed in this paper. The dynamic analysis shows that the effect of a corporate tax on profits varies over time. An increase in the corporate profit tax rate initially reduces investment, but this effect is reversed over time as the firm adjusts its financing policy to the new tax rate. In this setting, the level of investment is influenced by the marginal cost of capital but also by the availability of finance, therefore by the average cost of capital (as for example in Fazzari *et al.*, 1988). As a result, the corporate profits tax may affect investment even if it is neutral with respect to the marginal source of finance. The intertemporal perspective makes a great deal of difference to the way we think about the effect of corporate taxes and poses a number of problems. Section V deals with some of the problems of examining taxes in a dynamic model, problems that have received very little attention in the literature.

The theoretical results of this paper are useful for tax policy purposes. They suggest that reductions in corporate profit taxes increase firm investment, but this is only a temporary effect. Thus, lower tax rates, following for example fierce tax competition, should have only limited effect on capital accumulation of domestic firms. To confine the attention to the immediate impact of taxes, as in the standard literature, may be, therefore, misleading. Indeed, the intertemporal aspects can make a great deal of difference to the way we think about corporate tax policy.

## II. THE MODEL OF FIRM INVESTMENT WITH IMPERFECT CAPITAL MARKETS

To examine the effect of corporate taxation, we need a theory of firm investment and a model of how imperfections in capital markets affect the firm's financing decisions. In fact, the impact of taxes may be very sensitive to the firm's financing choices (Atkinson and Stiglitz, 1980) This section sets forth a dynamic model of investment where capital markets are imperfect and financial and real investment policies are simultaneously derived from the firm maximization problem.

### A. Firm Investment and Financing Constraints

We consider the case of a single firm that plans its investment over time and is concerned with how to finance the accumulation process. For simplicity, the firm is assumed to face market conditions that do not change over time. As a result of these assumptions, it takes real and financial decisions simultaneously, and has static expectations.

In the spirit of most of the theory of investment, we assume that when the firm invests in real assets it encounters some costs of installing new capital. In particular, the firm that at time  $t$  undertakes gross investment  $I_t$  must set aside  $I_t$  units as capital, together with  $c(I_t)$  units which are used during installation. The amount  $c(I_t)$  is used up in transforming goods into capital, and represents the cost of buying an increment in the stock of capital, namely the cost of adjusting capital upwards. For convenience, these costs are assumed to be nonnegative (with  $c(0) = 0$ ),

increasing (with  $c'(0) = 0$ ), and strictly convex, so that at each time the firm finds it convenient to accumulate only a finite amount of new capital. Models with strictly convex adjustment costs are often used in the literature of investment behavior to describe the accumulation decisions of firms, but they usually ignore financial decisions (see for example Eisner and Strotz, 1963; Lucas, 1967; Uzawa, 1969; Nickell, 1978; Abel and Blanchard, 1983). However, the basic premise of the present model is that financing decisions are important in examining the effect of taxes on firm investment behavior, and they have to be explicitly considered into the analysis.

Financial decisions are introduced into the model by assuming that the firm faces specific imperfections in capital markets. The first imperfection is a limitation on the firm's ability to issue new shares. This constraint is not new to economists, and the justification has been well developed in the microeconomic literatures on adverse selection and moral hazard (for example in Greenwald *et al.*, 1984). For simplicity, let us assume this constraint in the extreme form of no equity selling on the part of the firm, although results can in general be extended to include equity financing at the foundation of the firm (as in Sinn, 1991a). As a result of this assumption, the firm cannot issue new shares and has to finance new investment exclusively by means of retained profits and by borrowing in credit markets.<sup>1</sup>

The second imperfection is that the firm can borrow external funds, but has access only to imperfect credit markets. These are seen as decentralized markets characterized by search on the part of firms and lenders. On the one hand, firms look for lenders who can match their demand for funds. On the other hand, lenders try to select better borrowers or less risky debtors in a context where information is imperfect. In this setting, imperfections arise as a problem of coordination, one that requires an explicit and costly searching process for new funds. More specifically, to expand external finance the firm has to carry out costly search activities, and these searching costs are rising with the amount of new debt requirements.

To make this more concrete, let us denote by  $b_t$  the stock of debt at time  $t$ , and by  $\dot{b}_t \equiv db_t/dt$  its derivative with respect to time, that is the issues of new debt. At this point, we can define a searching or matching cost function  $\sigma(\dot{b}_t)$ . This is positive for any positive issue of new debt  $\dot{b}_t$ , and zero otherwise. Moreover, it is increasing in the new debt issues so that its first derivative  $\sigma'(\dot{b}_t)$  is positive as well. As a result, at each time, larger issues of new debt imply greater search costs to the firm.

To complete the building blocks of our model of accumulation, let us assume that the searching cost function  $\sigma(\dot{b}_t)$  is convex; thus, each further increase in new debt requires a successively larger increase in search expenses. This assumption captures the idea of a quality sequential search

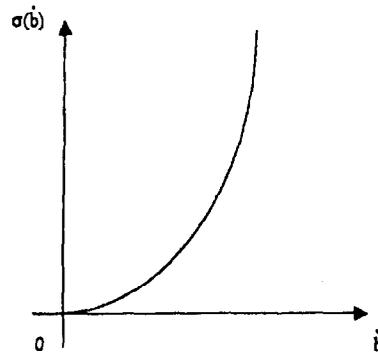
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<sup>1</sup>Indeed, empirical evidence suggests that firms tend not to issue shares and to primarily rely on internal flows and external debt. For instance, in the period from 1980 to 1985 in the United States, only 1,2 percent of gross investment by non-financial corporations was financed by share issues, while 67,8 percent was internally financed, and 31 percent was financed by debt (see Sinn, 1991a).

approach adopted by the firm, the first effort of which is supposed to be the most productive. For example, let us consider the firm that promotes new bonds issues by a sequence of advertisements. We would expect that the number of answers to successive announcements decreases, so that decreasing returns characterize these expenses. Put the other way around, the firm is facing uprisng costs of issuing new bonds.<sup>2</sup>

Armed with this information, we can draw the matching cost function shown in the following figure:

Figure 1: The Searching Cost Function



The matching or searching costs are nonnegative and rise at an increasing rate as  $b_t$  gets greater. We also assume, for convenience, that  $\sigma'(0) = A = 0$ , where  $A$  is a constant.

There is an analogy, in this model, between the treatment of search expenditures for new debt and the installing costs for investment. Search expenditures may be seen as buying an increment in the stock of “trust” or “informational capital” of the firm, for example, by providing information about the company. In that sense, Figure 1 can be regarded as a deterministic description of an uncertain world where the increase in the cost for external funds results from lack of information and fear of bankruptcy.

Ideally, the search process for external funds should be derived from first principles rather than being simply postulated. The matching cost function  $\sigma(b_t)$  ought to be achieved from a specification of the search technology used by firms and lenders; and, uncertainty should be treated more explicitly. However, to define the search process is beyond the scope of this analysis. In this sense, our representation is admittedly extreme in some respects.

The form and the position of function  $\sigma(\cdot)$  may well reflect a number of factors different from the issues of new debt, including the nature of the firm’s business, its outstanding debt liabilities

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<sup>2</sup>In practice, when debt issues exceed a “safety” level, lenders’ requirements (in terms of information, collateral, and other covenants) do increase. In the extreme case, they can prevent some borrowers from accessing credit arrangements. Thereby, credit costs may well be assumed to rise at an increasing rate with the amount of new funds.

and equity stock. Indeed, in large part of the theory of firm investment, the yields demanded by lenders are assumed to increase with the debt-equity ratio of the firm (Modigliani and Miller, 1958; Nickell, 1978; Steigum, 1983). However, this is equivalent to pre-fix the existence of an interior solution for the optimal debt to retained earnings ratio of the firm, and this is not a case of much interest for our purposes. We want a theory of firm investment that explains how—in the absence of taxation—the firm finances its investment by shifting across different combinations of financial instruments, and possibly choosing corner solutions. For this reason, I have preferred to confine the attention to the effects of current financing decisions because these avoid to pre-fixing the debt-retained earnings ratio. Indeed, this an important aspect which has implications for tax effects that are not always fully appreciated.

### B. The Firm's Problem

Let us assume that the firm acts optimally so as to maximize an objective function. Specifically, let us suppose that the firm chooses its real investment and financial strategies so as to maximize the present value of current and future net-of-tax cash flows discounted at an appropriate rate. Different objective functions could be, of course, proposed, but what the firm exactly maximizes is not important for our purposes. What is important for the analysis of tax effects is how the firm responds to changes in data like tax rates. In this respect, it is possible that the qualitative response to this practical question is more or less insensitive, within limits, to the precise nature of the firm's objective (see for example Solow, 1971). For simplicity, we then look at the firm's cash flow.<sup>3</sup>

To define the objective function of the firm, we need to determine its net-of-tax cash flow at each time. This is the difference between the firm's current receipts and disbursements net of any tax. At each time  $t$ , the firm receives revenue from new debt issues  $b_t$  and from its productive activity. For convenience, the firm is assumed to produce its output only with capital  $k_t$ , so that its revenue, profit and output can all be described by a function  $R(k_t)$ . This is increasing and strictly concave, thus  $R' > 0$  and  $R'' < 0$ , and for technical reasons, I assume that  $R''' > 0$  (which is a sufficient condition for the diagrammatic analysis).<sup>4</sup> On the expenditure side, the firm faces expenses for investment  $I_t$ , and for installing costs  $c(I_t)$ , as well as search costs for external financing  $\sigma(b_t)$ . Moreover, the firm borrows at an interest rate  $\rho$  that is supposed, for simplicity, to be constant; thus, at each time it has to pay interest  $\rho$  on debt outstanding at the beginning of the period.

As a result of these assumptions, and taking the capital good as numeraire, the before-tax cash

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<sup>3</sup>For alternative formulations of the firm's objective, and references to the literature, see Solow (1971). A recent discussion of the cash flow objective with an explicit treatment of uncertainty is in Dutta and Radner (1999).

<sup>4</sup>It is also assumed that the boundary Inada conditions hold, so that  $R(0) = 0$ ,  $R'(0) = \infty$ , and  $R'(\infty) = 0$ .

flow of the firm at any time  $t$  is:

$$\underbrace{R(k_t) + \dot{b}_t}_{\text{Revenue}} - \underbrace{I_t - c(I_t) - \sigma(\dot{b}_t) - \rho b_t}_{\text{Disbursements}}$$

For simplicity, let us consider only one kind of taxation:

- Corporate income tax - a corporate tax on the firm's profits.

This tax is assumed to be proportional and constant with tax rate  $\tau_C$ ; but the results can in general be extended to cover different tax schedules. For convenience, let us also assume that the proceeds from the tax are used outside the corporate sector, so as to ignore the effects of public spending.

In calculating taxable profits, we assume that the firm is not necessarily allowed to deduct all costs. It can deduct expenses for installing new capital, and interest payments to bondholders, but it cannot always deduct all search costs for external funds. Indeed, in many countries, debt expenses are not fully deductible. More specifically, the search process may involve costs that do not correspond to actual expenses or costs that are not recognized as such under tax provisions, for example, the cost of providing collateral and guarantee funds. Formally, let us assume that only a fraction  $a$  of the total searching costs  $\sigma(\dot{b}_t)$  is permitted as a deduction, with  $0 \leq a \leq 1$ ; where if  $a < 1$ , then some costs upon search are not tax deductible.

At this point, we can define the net-of-tax cash flow of the firm at each time  $t$  as:

$$\left( R(k_t) - \rho b_t - c(I_t) - a\sigma(\dot{b}_t) \right) \Upsilon_C - I_t + \dot{b}_t - (1-a)\sigma(\dot{b}_t)$$

This is the after-tax profits net of earnings reinvested in the firm  $(I_t - \dot{b}_t)$  and net of nondeductible search costs  $(1-a)\sigma(\dot{b}_t)$ .

The firm's objective function can be now defined as the discounted sum of each single period net-of-tax cash flow:

$$v(0) = \int_0^{\infty} \left\{ \left( R(k_t) - \rho b_t - c(I_t) - a\sigma(\dot{b}_t) \right) \Upsilon_C - I_t + \dot{b}_t - (1-a)\sigma(\dot{b}_t) \right\} e^{-\theta t} dt \quad (1)$$

where  $\Upsilon_C \equiv 1 - \tau_C$ , and future net-of-tax cash flows are discounted at whatever rate  $\theta$  is appropriate, given the rate of interest  $\rho$ , the riskiness of the business, and whatever else may matter. In particular, we would expect the discount rate  $\theta$  to be larger than the market interest rate, so as to justify the existence of productive activities even when capital markets are imperfect (see for example Kalecki, 1937; Hahn and Matthews, 1964; Uzawa, 1969; Nickell, 1978).

In choosing its policies, the firm has to satisfy a number of technological and legal constraints. The most obvious constraint is that investment increases the stock of capital. If there is no physical

depreciation, then net investment  $\dot{k}_t \equiv dk_t/dt$  equals gross investment  $I_t$ , and

$$\dot{k}_t = I_t \quad (2)$$

hence, whatever is invested augments the existing stock of capital. In addition, the firm finances new investment  $I_t$  by retaining profits and by borrowing. If  $s_t$  denotes the profits retained in the firm at time  $t$ , that is its saving, then we have the financing constraint

$$I_t = s_t + \dot{b}_t \quad (3)$$

In what follows, we assume for simplicity that the firm can neither reduce its outstanding debt (or lend money) nor decreases the stock of own capital, so that  $\dot{b}_t \geq 0$  and  $s_t \geq 0$ . These assumptions are admittedly restrictive, but they are motivated by the attempt to treat one difficulty at a time and simplify the analysis. They imply that the firm cannot run down capital ( $I_t \geq 0$ ) and allow us to exclude financial capital issues; indeed, real capital accumulation is what we are interested in.<sup>5</sup>

As a result of these constraints, the firm may be seen as designing its investment and financial policy so as to maximize the objective function (1) subject to the capital accumulation constraint (2), the financing condition (3), and the nonnegativity constraints on  $\dot{b}_t$  and  $s_t$ . In formalizing this decision process, we need to choose the appropriate decision or control variables of the problem. In particular, the financing constraint (3) allows us to reduce the number of controls, and to only use investment  $I_t$  and the new issues of debt  $\dot{b}_t$ . Retained earnings are, in fact, determined as a residual, the difference between total investment and new debt issues ( $s_t = I_t - \dot{b}_t$ ). Hence, the firm has only two margins along which to maximize its objective.

Armed with this information, the firm's maximization problem can be stated as follows:

$$\max_{I_t, \dot{b}_t} v(0) = \max_{I_t, \dot{b}_t} \int_0^{\infty} \left\{ \left( R(k_t) - \rho b_t - c(I_t) - a\sigma(\dot{b}_t) \right) \Upsilon_C - I_t + \dot{b}_t - (1-a)\sigma(\dot{b}_t) \right\} e^{-\theta t} dt$$

subject to

$$\dot{k}_t = I_t$$

$$I_t = s_t + \dot{b}_t$$

and to the nonnegativity constraints

$$\dot{b}_t \geq 0 \quad s_t \geq 0$$

where the initial values of the state variables at time  $t = 0$  are given, with  $b(0) = b_0 \geq 0$ , and  $k(0) = k_0 > 0$ , so that the firm can get production off the ground.

It may be noticed that this problem differs from the standard cost of adjustment model in the search cost term  $\sigma(\dot{b}_t)$ , and in the presence of financial imperfections. Indeed, in the absence of

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<sup>5</sup>For an extended version of this model, including reductions in the stock of real capital and accumulation of financial capital (lending money), see Palomba (2001).

an equity finance constraint and with no search cost for debt, investment could be indifferently financed by all means; the firm's problem would then reduce to the usual cost of adjustment formulation (as for example in Abel and Blanchard, 1983).

### C. Optimality Conditions

We can now apply the standard techniques of the optimal control theory, and solve the firm's maximization problem for the path of the controls  $I_t$  and  $\dot{b}_t$ , given the tax rate. To this end, we can restrict the attention to the case where  $\theta > \rho$ , that is the discounting rate exceeds the market interest rate; indeed, this is the only possible case. If we assumed, for example, that  $\theta \leq \rho$ , then the optimal plan would involve continuous accumulation of financial assets (with a negative  $\dot{b}_t$  representing financial accumulation); but, this possibility has been ruled out by imposing the constraint  $b_t \geq 0$ .

Introducing the multipliers  $q_{1t}$  and  $q_{2t}$  the constraints (2) and (3), the firm's maximization problem can be reformulated in terms of the Lagrangian function:

$$L = \left( R(k_t) - \rho b_t - c(I_t) - a\sigma(\dot{b}_t) \right) \Upsilon_C - I_t + \dot{b}_t - (1-a)\sigma(\dot{b}_t) + \\ q_{1t}I_t - q_{2t}\dot{b}_t + \mu_{bt}\dot{b}_t + \mu_{st}(I_t - \dot{b}_t)$$

where  $\mu_{bt}$  and  $\mu_{st}$  are the Lagrange multipliers associated with, respectively, the nonnegativity constraints on  $b_t$  and  $s_t$ , and the  $q_{1t}$  and  $q_{2t}$  functions are the co-state variables of the problem. These variables can be interpreted as giving the tax-adjusted marginal value of the associated state variables at time  $t$  in terms of values at  $t$ . The optimal investment and financing policy of the firm can be now calculated as a maximum of  $L$ .

The necessary first-order conditions for optimality and the multiplier equations of the problem can be set out by differentiating the Lagrangian  $L$  with respect to, respectively, the control variables  $I_t$  and  $\dot{b}_t$ , and the states  $k_t$  and  $b_t$  to give

$$\tilde{q}_{1t} = \frac{1}{\Upsilon_C} + c'(I_t) - \tilde{\mu}_{st} \\ \tilde{q}_{2t} = \frac{1}{\Upsilon_C} - \frac{(1-a\tau_C)}{(1-\tau_C)}\sigma'(\dot{b}_t) - \tilde{\mu}_{st} + \tilde{\mu}_{bt} \\ \dot{\tilde{q}}_{1t} = \theta\tilde{q}_{1t} - R'(k_t) \\ \dot{\tilde{q}}_{2t} = \theta\tilde{q}_{2t} - \rho$$

where  $c'$  and  $\sigma'$  denote the derivatives of functions  $c(\cdot)$  and  $\sigma(\cdot)$  with respect to their arguments,  $\dot{\tilde{q}}_{xt} \equiv \frac{d\tilde{q}_{xt}}{dt}$  and where, for convenience, I have rescaled up the multipliers of the problem by the

term  $\Upsilon_C$ , so that

$$\tilde{q}_{1t} = \frac{q_{1t}}{\Upsilon_C} \quad \tilde{q}_{2t} = \frac{q_{2t}}{\Upsilon_C} \quad \tilde{\mu}_{bt} = \frac{\mu_{bt}}{\Upsilon_C} \quad \tilde{\mu}_{st} = \frac{\mu_{st}}{\Upsilon_C}$$

Moreover, at the optimum the following complementary slackness conditions must hold:

$$\begin{aligned} \mu_{bt} &\geq 0 & \dot{b}_t &\geq 0 & \mu_{bt} \dot{b}_t &= 0 \\ \mu_{st} &\geq 0 & s_t &\geq 0 & \mu_{st} s_t &= 0 \end{aligned}$$

Finally, on the boundary, optimality also requires the transversality conditions

$$\lim_{t \rightarrow \infty} q_{1t} e^{-\theta t} k_t = 0$$

$$\lim_{t \rightarrow \infty} q_{2t} e^{-\theta t} b_t = 0$$

These conditions may be simplified. We can solve the differential equations in  $\tilde{q}_{1t}$  and  $\tilde{q}_{2t}$  subject to, respectively, the transversality condition for capital and debt to give

$$\begin{aligned} \tilde{q}_{1t} &= e^{\theta t} \int_t^{\infty} R'(k_v) e^{-\theta v} dv \\ \tilde{q}_{2t} &= \frac{\rho}{\theta} \end{aligned}$$

Thus, at the optimum, the tax-adjusted shadow price of capital (the marginal  $q$  in a  $q$ -model of investment) is equal to the present discounted value of the future marginal products. As in the standard cost of adjustment model, it does not directly depend on financial means (Abel and Blanchard, 1983).

The second equation tells us that along the optimal path, the tax-adjusted shadow cost or marginal valuation of debt is constant, and it depends both on the market interest rate,  $\rho$ , and on the expected rate of return  $\theta$ . In particular, since  $\theta > \rho$ , the marginal cost of debt is lower than one. This suggests that, at the margin, debt costs less than what it can buy and is likely to be a “bargain” for the firm. In choosing its optimal borrowing policy, the firm shall trade off this advantage against the search costs for obtaining new funds, and the choice of borrowing will depend on the level of search costs, an issue we discuss later.

Looking for the optimal growth path of the firm means finding a temporal paths of the controls  $I_t$  and  $b_t$  that solve the optimality conditions given above, and that at the optimum satisfies the dynamics and the constraints of the system.

Along the optimal path the firm may adopt different financial policies and be in different regimes or activity phases. In particular, because of condition (3), if the firm invests in new capital

( $I_t > 0$ ), then it has to either issue new debt ( $\dot{b}_t > 0$ ) or retain earnings ( $s_t > 0$ ) or both ( $\dot{b}_t > 0$ ,  $s_t > 0$ ); but, if it does not invest, then it neither retains profit nor collects external funds ( $I_t = \dot{b}_t = s_t = 0$ ). Because of the nonnegativity constraints, investment cannot be negative. This leaves us with four feasible regimes or activity phases:

- Regime 1.      $I_t = \dot{b}_t = s_t = 0$             *The stationary state*
- Regime 2.      $I_t > 0$ ,  $\dot{b}_t > 0$ ,  $s_t > 0$     *The singular solution*
- Regime 3.      $I_t > 0$ ,  $\dot{b}_t > 0$ ,  $s_t = 0$     *The pure debt-financing policy*
- Regime 4.      $I_t > 0$ ,  $\dot{b}_t = 0$ ,  $s_t > 0$     *The pure internal growth policy*

The names of these regimes reflect different strategies of investment financing.

### III. THE OPTIMAL ACCUMULATION PATH OF THE FIRM

To examine the effects of taxes on firm behavior, we need to determine the optimal accumulation path of the firm. This is a combination of different financial regimes that satisfies the Maximum Principle's optimality requirements. In turn, each regime is characterized by its own specific first-order conditions and dynamics, and to find the overall path, we need to investigate the different regimes separately. In this section, I begin to examine the stationary states in regime 1 and the dynamics of the model. I then consider the investment and financial decision rules of the firm in the other regimes and the channels through which the tax affects the behavior of the firm. Finally, these results are combined together to determine the firm's policy along the optimal accumulation path and the dependence of all variables with respect to the tax.

#### A. Stationary States and Dynamics

Let us start by examining the stationary states in regime 1, and by describing the general dynamics of the model. While these do not tell us about the optimal policies of the firm, they are useful to illustrate the behavior of the model.

##### **Regime 1 and stationary states**

In financial regime 1, the firm is in a stationary state; it does not invest in new capital and does not take any financing decision either, so that  $I_t = \dot{b}_t = s_t = 0$ . As a result of this behavior, both the stock of capital and the level of outstanding debt are constant, and we can denote these steady-state values by  $k_s$  and  $b_s$ , respectively.

In this regime, the optimal behavior of the firm is described by the first-order conditions and dynamics discussed in the previous section, but these greatly simplify. Since the stock of capital is constant, the integral for  $\tilde{q}_{1t}$  can be explicitly solved so as to have

$$\tilde{q}_{1t} = \frac{R'(k_s)}{\theta} \tag{4}$$

From this equation and the condition that  $\tilde{q}_{2t} = \frac{\rho}{\theta}$ , it can be deduced that in the steady state both the co-state variables  $\tilde{q}_{1t}$  and  $\tilde{q}_{2t}$  are constant, so that

$$\dot{\tilde{q}}_{1t} = \dot{\tilde{q}}_{2t} = 0 \quad (5)$$

From these results, it follows that in financial regime 1 there are no dynamics, and once the firm is in this activity phase it does not find it desirable to move away.

At this point, we can explicitly characterize the possible stationary states of the model. Let us consider the first two optimality conditions. Provided that  $c'(0) = 0$  and  $\sigma'(0) = A = 0$ , and using the nonnegativity property of Kuhn-Tucker multipliers, we obtain that at the optimum

$$\tilde{q}_{1t} \leq \tilde{q}_{2t} = \frac{\rho}{\theta} \quad (6)$$

In the steady state, the shadow cost of capital  $\tilde{q}_{1t}$  is, therefore, not greater than its replacement cost in terms of external funds  $\tilde{q}_{2t} = \frac{\rho}{\theta}$ . That is, its value does not compensate for the cost that a marginal addition of capital would generate when financed by borrowing, even if the cost of borrowing,  $\frac{\rho}{\theta}$ , is smaller than one.

If we now substitute equation (4) in previous condition (6), we obtain the optimal investment rule in this regime:

$$R'(k_s) \leq \rho \quad (7)$$

Equation (7) determines the firm's optimal steady-state level of capital as a function of the market rate of interest. It may be noticed that the steady-state stock of capital is determined like in a model with perfect capital markets, in which  $\rho$  is the market interest rate. However, condition (7) does not identify a unique steady-state stock of capital and unlike a standard model with adjustment costs, it defines an infinite set of possible stationary states. This is the consequence of our simplifying assumption that capital cannot be disinvested ( $I_t \geq 0$ ), and the result would not hold in a more general version of the model.<sup>6</sup>

It is worth noting that the possible stationary states are independent of the corporate income tax. However, an exception arises if we drop the assumption  $\sigma'(0) = A = 0$  and allow for  $A > 0$ . In this case, the steady-state condition becomes  $R'(k_s) \leq \rho + \frac{1-a\tau_c}{1-\tau_c} A\theta$ , and if  $a < 1$ , then the tax does affect the range of possible stationary states.<sup>7</sup>

We can illustrate the set of possible stationary states in the  $(k, \tilde{q}_1)$  phase diagram shown in Figure

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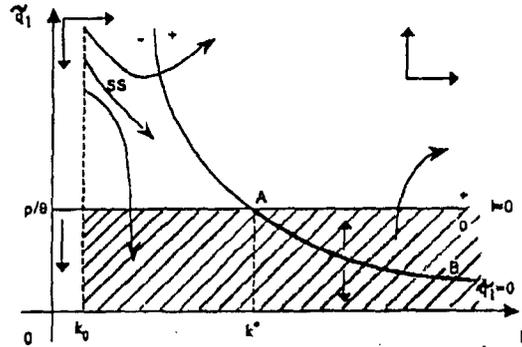
<sup>6</sup>For a general version of this model with no constraints on state variables, see Palomba (2001). For a discussion of standard cost of adjustment models, and references to the literature, see for example Nickell (1978), Abel and Blanchard (1983) and, for their use in models of growth, Atkinson (1999).

<sup>7</sup>For a detailed discussion of this case, see Palomba (2001).

2. All points in the positive orthant of this figure are feasible, except for points on the vertical axis above the origin; under the previous assumptions,  $k_0$  is in fact positive, and capital cannot be reduced (as  $I_t \geq 0$ ), so that the stock of capital can never be zero. The points along the horizontal axis also are excluded by the Inada conditions on  $R(k_t)$  which do not allow  $\tilde{q}_1$  to be zero.

The set of possible stationary states can be depicted by considering conditions (5) and (6). The locus  $\dot{\tilde{q}}_1 = 0$  in the figure corresponds to condition (5) (recall that, from the optimality conditions,  $\tilde{q}_{1t} = \theta \tilde{q}_{1t} - R'(k_t)$ , therefore  $\dot{\tilde{q}}_1 = 0$  implies that  $\tilde{q}_{1t} = \frac{R'(k_t)}{\theta}$ ); this is a downward sloping curve, and its concavity properties derive from the assumptions made on the marginal product function  $R'(k)$ . Equation (6) requires that  $\tilde{q}_1 \leq \frac{\rho}{\theta}$ ; this condition identifies an area below the  $\tilde{q}_{1t} = \frac{\rho}{\theta}$  locus in the figure where  $I = 0$ . The points on the  $\dot{\tilde{q}}_1 = 0$  locus and below the  $\frac{\rho}{\theta}$  line (the line  $AB$ ) satisfy both the optimality conditions (5) and (6), and identify the set of stationary-state levels of capital  $\{k_s\}$  compatible with the optimality condition (7). In the special case when  $\sigma'(0) = A > 0$  and  $a < 1$ , condition (6) becomes  $\tilde{q}_1 \leq \frac{\rho}{\theta} + \frac{1-a\tau_C}{1-\tau_C} A\theta$ , and the  $I = 0$  line moves upward. As noted above, there is a wide set of possible steady states. However, once the firm is at one of these points, it can stay there for ever, since there are no dynamics, and all transversality conditions are satisfied.

Figure 2: Steady States and Dynamics



### Dynamics

At this point, we can use the phase diagram of Figure 2 to sketch the directions of movement compatible with the optimality conditions, and to conjecture a possible solution to the firm's problem.

From Figure 2, it can be deduced that anywhere above the  $I_t = 0$  locus capital  $k_t$  is increasing; in fact, the marginal valuation of new capital,  $\tilde{q}_{1t}$ , exceeds the cost of borrowing money,  $\frac{\rho}{\theta}$ , and the necessary condition (6) for a stationary state is violated. In this case, the firm finds it desirable to invest against borrowing external funds, and  $I_t > 0$ . Similarly, the stock of capital  $k_t$  is constant at points below the  $I_t = 0$  locus where  $\tilde{q}_1 \leq \frac{\rho}{\theta}$ , and it is not optimal to invest in new capital. In the case of the  $\dot{\tilde{q}}_1 = 0$  locus, the marginal value of capital  $\tilde{q}_{1t}$  is increasing at points above the locus, where  $\tilde{q}_{1t} > \frac{R'(k_t)}{\theta}$  and, from dynamics,  $\dot{\tilde{q}}_1 > 0$ ; but, it is decreasing at points below the locus,

where  $\tilde{q}_1 < 0$ . The arrows in Figure 2 demonstrate these possible directions of motions, and the trajectories illustrate the dynamics of the model consistent with the optimality conditions.

As it is clear from the figure, there are three possible sets of equilibria: the set of stationary states along the  $\tilde{q}_1 = 0$  locus and below the  $\frac{\rho}{\theta}$  locus, namely the line  $AB$ ; the points along the horizontal axis; finally, the points at the top right-hand side of the phase diagram where both capital and the co-state variable  $q_1$  take infinite values. However, only the trajectory  $SS$  that converges to  $A$  and the set of stationary states on the  $AB$  line satisfy the optimality conditions of the problem. All other paths either violate the transversality conditions or do not satisfy some of the optimality requirements. The trajectory going to infinity violates the transversality condition for capital, and cannot, therefore, be optimal. The paths leading to a zero marginal value of capital (the horizontal axis) are not possible either, as they violate the Inada conditions and the fact that the equilibrium cannot be on the horizontal axis. It follows that the only possible optimal paths are those leading towards one of the stationary state levels of capital, and these are the only long-run equilibria of the model.

The nature of the optimal solution crucially depends on the initial conditions of the problem. For example, let us assume that the starting stock of capital  $k_0$  is greater than the smallest possible steady state level of capital  $k^*$  (see Figure 2), then there is no optimal path, and any trajectory would violate some of the conditions of the problem. In this case, the only long-run equilibrium has to be the initial stock of capital itself, if this is a stationary state of the model (that is,  $k_0$  has to be on the line  $AB$  in Figure 2). If  $k_0 < k^*$ , there may exist an optimal path such as the trajectory  $SS$  which approaches point  $A$  and leads to the steady state level of capital  $k^*$ . Along this path the firm accumulates capital, and has to finance the growth process adequately. This is the only optimal accumulation path of the model, and this is the case on which I will focus my attention.

## B. The Investment and Financial Decision Rules of the Firm

How does the firm choose its investment and financial policy along the accumulation path? How does corporate taxation of profits affect this policy? Along the accumulation path, the firm may grow through different financial regimes. To determine the firm's optimal policy, we then need to examine its decisions in each regime separately. The different regimes can be then joined together to characterize the firm's optimal policy along the accumulation path and the dependence of all variable with respect to the tax rate.

### Regime 2 and the singular solution

Let us start considering financial regime 2. In this regime the firm invests in new capital, and finances investment both by issuing new debt and by retaining earnings, so that the decision variables  $I_t$ ,  $\dot{b}_t$ , and  $s_t$  are positive.

In this regime, the complementary slackness conditions require the Kuhn-Tucker multipliers  $\mu_{bt}$  and  $\mu_{st}$  to be zero, and the first of the optimality conditions becomes:

$$\tilde{q}_{1t} = \frac{1}{\Upsilon_C} + c'(I_t) \quad (8)$$

This condition contains a straightforward result; namely, that given the profits tax, the rate of investment (relative to the stock of capital) is a function of the marginal valuation of installed capital  $\tilde{q}_{1t}$  (like in a standard  $q$ -model). More specifically, since the adjustment cost function is strictly convex ( $c''(\cdot) > 0$ ), we can define an inverse function  $\varphi(\cdot)$  to give the investment rule of the firm,

$$I_t = \varphi\left(\tilde{q}_{1t} - \frac{1}{\Upsilon_C}\right) \quad (9)$$

where from the properties of  $c(\cdot)$ , it follows that  $\varphi'(\cdot) > 0$  and  $\varphi(0) = 0$ , so that at  $\tilde{q}_{1t} = \frac{1}{\Upsilon_C}$  investment is zero.

Equation (9) describes the firm's optimal investment rule as a decreasing function in the tax factor  $\frac{1}{\Upsilon_C}$ , and increasing in the rescaled marginal valuation of capital  $\tilde{q}_{1t}$ . It may be noticed that, in this regime investment is always positive, therefore  $\tilde{q}_{1t}$  has to be greater than  $\frac{1}{\Upsilon_C} > 1$ ; that is, this regime is characterized by  $\tilde{q}_{1t} > 1$ . It is also interesting to observe that for each given stream of capital,  $\tilde{q}_{1t}$  is independent of the tax rate as  $\tilde{q}_{1t} = e^{\theta t} \int_t^\infty R'(k_v) e^{-\theta v} dv$ . It then follows that the profit tax reduces the investment of the firm for any given  $\tilde{q}_{1t}$ . This caveat notwithstanding, it should be noticed that equation (9) is not a closed form solution for investment because  $I_t$  affects  $k_t$  and thereby  $\tilde{q}_{1t}$ . As a result, we cannot conclude from (9) that the tax has a negative effect on the "temporal" path of investment, a question I discuss further below. Finally, equation (9) also brings out that investment is independent of the firm's financial decisions and how responsive these decisions are to the tax.

In this regime, finance does not explicitly affect investment decisions, but the different sources of funds are not equivalent. The second of the optimality conditions in Section II shows, in particular, the optimal borrowing rule of the firm:

$$\dot{b}_t = \omega\left(\frac{1 - \tau_C}{1 - a\tau_C} \left(\frac{1}{\Upsilon_C} - \frac{\rho}{\theta}\right)\right) \equiv \bar{\omega} \quad (10)$$

where  $\omega(\cdot)$  is the inverse of the search cost function  $\sigma'(\cdot)$ , with  $\omega'(\cdot) > 0$ , and we have substituted the shadow cost of debt  $\tilde{q}_{2t} = \frac{\rho}{\theta}$ , so that  $\bar{\omega}$  is a positive constant as  $\frac{\rho}{\theta} < 1$ .

From this, we deduce that in financial regime 2 the firm issues a constant and positive amount of new debt  $\bar{\omega}$ , and this is independent of its investment decisions, but it does depend on the tax rate  $\tau_C$ . A higher tax rate increases the issues of debt, as  $\frac{d\bar{\omega}}{d\tau_C} > 0$ , and the effect is larger the higher is the tax deductibility parameter  $a$ . In fact, with profit taxation, interest and debt costs deductibility reduce the tax burden, thereby encouraging the firm to choose debt financing.

If we now substitute equations (9) and (10) in the financing constraint  $I_t = \dot{b}_t + s_t$ , and rearrange,

we can determine the firm's retaining profit rule,

$$s_t = I_t - \dot{b}_t = \varphi \left( \tilde{q}_{1t} - \frac{1}{\Upsilon_C} \right) - \bar{w} > 0 \quad (11)$$

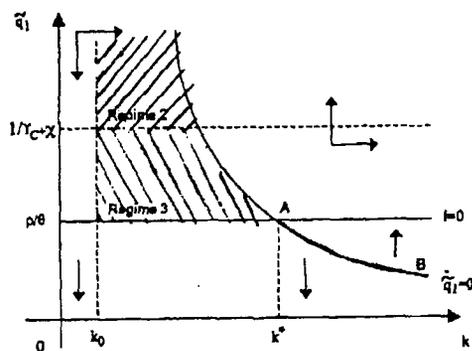
where both investment and debt issues depend on the profit tax rate  $\tau_C$ . From previous relationships, it follows that for any given  $\tilde{q}_{1t}$ , the firm's saving  $s_t$  is a decreasing function of the tax rate  $\tau_C$ . Equations (9), (10) and (11) describe the investment and financing policy of the firm and the dependence with respect to the tax rate in regime 2.

At this point, we need to investigate how the level of investment and the combination of the different sources of funds vary in this regime. We have already seen that  $\tilde{q}_{2t}$  does not change over time, and that the new debt issues are constant. Therefore, it only remains to examine the dynamics of  $\tilde{q}_{1t}$  and the path of investment. In this regime, the marginal valuation of capital is steadily decreasing so that  $\dot{\tilde{q}}_{1t} < 0$  (see Appendix I). From this result and equation (9), it follows that investment is monotonically decreasing over time together with the marginal valuation  $\tilde{q}_{1t}$ .

We conclude that, in financial regime 2, the firm invests in new capital at a decreasing rate, while issuing a constant amount of new debt. Given this optimal use of debt (equation (10)), there are, however, sufficiently attractive investment opportunities remaining, and the firm finances this extra investment by retaining earnings (equation (11)). As a result, investment tends to be financed by retained earnings at a decreasing rate (and by debt at an increasing rate), and these earnings are the marginal source of financing.

It is convenient to illustrate these conclusions in the standard phase diagram shown in Figure 3. The top shadowed area in this figure denotes the possible values for the co-state variable  $\tilde{q}_{1t}$  and the stock of capital  $k_t$  during financial regime 2. This area lies below the  $\dot{\tilde{q}}_1 = 0$  locus since during this regime  $\dot{\tilde{q}}_{1t} < 0$  (see Appendix I). The marginal value of capital  $\tilde{q}_{1t}$  has to also be greater than  $\frac{1}{\Upsilon_C}$ ; thus, there exists a regime 2-lower border at some  $\tilde{q}_{1t} = \frac{1}{\Upsilon_C} + \chi$ , where  $\chi > 0$ . At  $\tilde{q}_{1t} = \frac{1}{\Upsilon_C} + \chi$ , the firm leaves regime 2, and this constitutes the lower limit for this activity phase as shown in the figure. The arrows indicate the optimal directions of motion described above, with capital be increasing as  $\tilde{q}_{1t}$  decreases.

Figure 3: Steady States, Dynamics, and Financial Regimes



### Regime 3 and pure debt-financed growth

In the case of financial regime 3, the firm entirely finances investment by borrowing funds and is, therefore, in a pure debt-financing regime where  $I_t = \dot{b}_t$ . In this regime, the complementary slackness conditions imply that the Kuhn-Tucker multiplier  $\mu_{bt}$  is zero.

If we substitute the first of the optimality conditions into the second, we obtain the firm's optimal investment borrowing rule:

$$c'(I_t) + \sigma'(\dot{b}_t) \frac{(1 - a\tau_C)}{(1 - \tau_C)} = \tilde{q}_{1t} - \frac{\rho}{\theta} \quad (12)$$

where the shadow cost of debt  $\tilde{q}_{2t} = \frac{\rho}{\theta}$  has been substituted in the optimality conditions.

The term on the left-hand side of this equation gives the marginal cost of investing by issuing new debt adjusted by the tax, while the term on the right-hand side represents the net marginal advantage that results from financing investment by debt (this is the difference between the rescaled marginal valuation of capital,  $\tilde{q}_{1t}$ , and the shadow cost of debt,  $\frac{\rho}{\theta}$ ). At the optimum, the firm trades off these advantages against the marginal costs of investing, and will invest until the advantages have been fully exploited.

It may be noticed that in this regime if  $a = 1$ , then the profit tax has no effect on the firm's investment function. That is, full debt-cost deductibility and debt financing imply tax neutrality. But, if only a fraction of search costs is permitted as a deduction so that  $a < 1$ , then the tax does change the investment rule of the firm and, for each value of the multiplier  $\tilde{q}_{1t}$ , it results in lower investment.

At times, it may be more straightforward to reformulate firm's optimal investment-borrowing policy as a function of the rescaled shadow price  $\tilde{q}_{1t}$  and of the profit tax rate  $\tau_C$ :

$$I_t = \Psi \left( \tilde{q}_{1t} - \frac{\rho}{\theta}, \tau_C \right) = \dot{b}_t \quad (13)$$

where from the properties of  $c'(\cdot)$  and  $\sigma'(\cdot)$ , it follows that  $\Psi'_{\tilde{q}_1}(\cdot) > 0$ , and  $\Psi(0, \tau_C) = 0$ , so

that at  $q_{1t} = \frac{\rho}{\theta}$  investment is zero; in addition,  $\Psi'_{\tau_C}(\cdot) < 0$ , but  $\Psi'_{\tau_C}(\cdot) = 0$  when  $a = 1$ .

Equation (13) describes the investment policy of the firm in regime 3. From this, it turns out that investment takes place until the marginal value of installed capital,  $\tilde{q}_{1t}$ , equals the marginal cost of debt. Since in this regime investment is positive, it follows that  $\tilde{q}_{1t}$  must always be greater than  $\frac{\rho}{\theta}$ ; but, it can be greater or lower than one, without violating the necessary conditions for an optimum. The shadow value of capital  $\tilde{q}_{1t}$  can be smaller than one because in this regime, at the margin, the firm finances new investment by borrowing external funds; in turn, the cost of external funds can be as low as  $\frac{\rho}{\theta}$  and, under previous assumptions, this is smaller than one.

At this point, we need to investigate the dynamics characterizing the firm's optimal policy in this regime. As shown in Appendix I, the marginal valuation of capital  $\tilde{q}_{1t}$  in regime 3 is steadily decreasing over time, so that  $\dot{\tilde{q}}_{1t} < 0$ . From equation (13), it then follows that both investment and new debt issues are decreasing.

The above conclusions are illustrated in the phase diagram of Figure 3. The lowest shadowed area in this figure denotes the possible values for the co-state variable  $\tilde{q}_{1t}$  and the stock of capital during financial regime 3. This area lies below the  $\dot{\tilde{q}}_1 = 0$  locus since during regime 3 the marginal valuation of capital is decreasing, and  $\dot{\tilde{q}}_{1t} < 0$  (see Appendix I). In this regime, the shadow value of capital  $\tilde{q}_{1t}$  also has to be greater than  $\frac{\rho}{\theta}$ , as investment is positive (recall (13)); thus,  $\tilde{q}_{1t} = \frac{\rho}{\theta}$  will be lower limit of regime 3. Finally, an upper constraint will also exist; indeed, above the  $\tilde{q}_{1t} = \frac{1}{\tau_C} + \chi$  locus the level of investment differs from the new debt issues and the firm can no longer be in regime 3. The arrows demonstrate the optimal directions of motion described above, with capital steadily increasing as  $q_{1t}$  decreases.

#### **Regime 4 and pure internal growth**

Finally, it only remains to consider the alternative strategy offered by financial regime 4. In this regime, the firm invests in new capital and entirely finances investment by retaining earnings. As a result, the firm is in a pure internal growth phase where  $I_t = s_t$ , and external funds play no role. In this regime, the complementary slackness conditions require the Kuhn-Tucker multiplier  $\tilde{\mu}_{st}$  to be zero.

Provided that  $\sigma'(0) = A = 0$ , we can reformulate the first-order conditions and dynamics for debt as:

$$\tilde{q}_{2t} = \frac{1}{\tau_C} + \tilde{\mu}_{bt}$$

$$\tilde{q}_{2t} = \frac{\rho}{\theta}$$

These optimality conditions identify a path that is not, in general, optimal. On the one hand, the second of the optimality conditions implies that the rescaled shadow cost of debt  $\tilde{q}_{2t}$  is not smaller than  $\frac{1}{\tau_C}$  because from the Kuhn-Tucker conditions  $\tilde{\mu}_{bt} \geq 0$ , so that  $\tilde{q}_{2t} \geq 1$ . On the other hand, the marginal valuation of debt  $\tilde{q}_{2t}$  is such that  $\tilde{q}_{2t} = \frac{\rho}{\theta}$  and, under our assumptions, this is smaller than one, that is  $\tilde{q}_{2t} < 1$ . This is clearly a contradiction; thus, financial regime 4 cannot be part of

the firm's optimal growth path. This follows because, in this model, borrowing is an attractive option for the firm ( $\rho < \theta$ ) as the marginal cost for searching external funds,  $\sigma'(\cdot)$ , becomes close to zero for sufficiently small amounts.<sup>8</sup>

SUMMING UP. - Armed with these results, we conclude that given the set of stationary states in regime 1, financial regimes 2 and 3 are the only feasible activity phases. Therefore, the optimal growth path of the firm can only go through these two regimes. In each regime, there are different channels through which the corporate profits tax influences the firm's behavior. In the case of  $a < 1$ , the profit tax affects the investment and debt functions of the firm both during regime 2 (recall equations (9) and (10)) and during regime 3 (equation (13)). However, this result depends on the search costs deductibility parameter  $a$ , and if  $a = 1$ , then the tax has no effect on the investment-borrowing policy of the firm in regime 3. Nonetheless, the tax is, in general, neutral in the steady state.

### C. The Optimal Growth Path of the Firm

The optimal accumulation path of the firm may be seen as a combination of the different financial regimes described above. The combination is not, of course, arbitrary. It has to satisfy the transversality conditions and the Maximum Principle's general requirement that there be no jumps in the state and co-state variables. We have already seen, however, that the discounting and the market interest rates are constant, so that the co-state variable  $\tilde{q}_{2t} = \frac{\rho}{\theta}$  is necessarily continuous. The growth path can then be uniquely determined in a  $(k, \tilde{q}_1)$  phase diagram used above, where  $k_t$  and  $\tilde{q}_{1t}$  are the only state and co-state variables that we require to be continuous.

Crucial to the characterization of the optimal growth path is the choice of the starting point or initial stock of capital. The earlier analysis has demonstrated how different starting stocks of capital may lead to different optimal patterns and to alternative steady states. For convenience, let us confine the attention to the case of a small level of initial capital; in particular, let us assume that the starting capital,  $k_0$ , is smaller than the smallest steady state level  $k^*$ , as illustrated in Figure 4. This assumption is clearly restrictive, but the alternative condition that  $k_0 \geq k^*$  would imply no accumulation at all, and this is not a case of much interest to the present purposes. As a result of this assumption, there is only one direction of motion compatible with optimality, and this leads the firm to approach the steady-state stock of capital  $k^*$ . This is shown in Figure 4 by the continuous line  $SS$ .

The question at issue here is then to find, if it exists, a succession of financial regimes such that, given any tax rate, the firm starts off from the initial condition  $k_0 \leq k^*$  and, following a continuous path, approaches the steady-state and long-run equilibrium level of capital  $k^*$  in financial regime 1. To determine such an accumulation path, it is convenient to adopt a backward solution strategy. Assume then to be at the long-run equilibrium  $k^*$ , and according to the optimality conditions let us trace out a complete reverse accumulation path, till to reach the initial stock  $k_0$ .

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<sup>8</sup>For a discussion of the conditions under which debt may not be an optimal source of funds, see Palomba (2001).

Following the backward strategy, the firm is initially in the stationary state  $k^*$  where it acts according to financial regime 1 (see Figure 4). In this regime, the firm follows the *laissez faire* investment rule (7), and the level of capital  $k^*$  is such that  $R'(k^*) = \rho$ . In this steady state, optimality also requires that

$$\tilde{q}_{1t} = \tilde{q}_{2t} = \frac{\rho}{\theta}$$

thus, the shadow price of capital  $\tilde{q}_{1t}$  is equal to its replacement cost in terms of external funds  $\tilde{q}_{2t}$ . That is, the value of capital just compensates for the cost that an additional (infinitesimal) unit of capital would generate when financed by borrowing.<sup>9</sup>

In this steady state, all transversality conditions are satisfied, and  $\dot{\tilde{q}}_{1t} = \dot{\tilde{q}}_{2t} = 0$ ; it follows that the firm can stay at this stage for ever as there is no incentive to invest and move away.

As a result of this behavior, financial regime 1 cannot be a moving phase. It cannot be a starting phase either, as  $k_0 < k^*$  and the firm has to raise enough capital to get there. Thus, regime 1 can only be a final phase, and  $k^*$  is the long-run equilibrium of the model.

With financial regime 1 as the final phase, it remains to explain how we can approach the long-run equilibrium  $k^*$ , and the potential candidates are the financial regimes 2 and 3.

Financial regime 2 cannot lead to the long-run equilibrium  $k^*$ , as this would imply a jump in the optimal path of the co-state variable  $\tilde{q}_{1t}$  (see figure 4). In this regime, the marginal valuation of capital is, in fact, greater than one, that is  $\tilde{q}_{1t} > 1$ . But, in the stationary state,  $\tilde{q}_{1t} = \frac{\rho}{\theta}$  so that, under our assumptions,  $\tilde{q}_{1t} < 1$ . It follows that any move from financial regime 2 to the steady state  $k^*$  requires a forbidden jump in the co-state variable  $\tilde{q}_{1t}$ , and the Maximum Principle's requirement of continuity would be violated.

It should be clear, at this point, that the only regime potentially leading to the stationary state is financial regime 3. Indeed, in this regime the first-order optimality conditions define a set of paths for the co-state variable  $\tilde{q}_{1t}$  in the  $(k, \tilde{q}_1)$  space, one of which can eventually approach the stationary state  $k^*$ . In particular, the slope of these paths is given by

$$\frac{d\tilde{q}_t}{dk_t} = \frac{\tilde{q}_{1t}}{k_t} = \frac{\theta\tilde{q}_{1t} - R'(k_t)}{k_t}$$

Since in regime 3 the marginal valuation of capital is decreasing over time, that is  $\dot{\tilde{q}}_{1t} < 0$  (see Appendix I), and capital steadily rises, so that  $\dot{k}_t > 0$ , it follows that the slope of these paths is negative. As a result, the co-state variable  $\tilde{q}_{1t}$  is monotonically decreasing as capital rises, and the arrows in Figure 4 illustrate these directions of motion. From equation (13), it can be then deduced that in regime 3 the rate of investment is monotonically decreasing, and the firm will eventually approach a point where  $I_t = 0$ . This is the case when  $\tilde{q}_{1t} = \frac{\rho}{\theta} = \frac{1}{\Upsilon_C} - \tilde{\mu}_{st}$  (recall (13)),

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<sup>9</sup>As far as the steady-state Kuhn-Tucker multipliers are concerned, from the optimality conditions, we have that at  $k_t = k^*$ ,  $\tilde{\mu}_{bt} = 0$  and  $\tilde{\mu}_{st} > 0$  with  $\tilde{\mu}_{st} = \frac{1}{\Upsilon_C} - \frac{\rho}{\theta}$ .

and the convergence to such a point is ensured by the fact that in this regime  $\tilde{q}_{1t} < 0$ , while at the same time  $\tilde{\mu}_{st} > 0$  (see Appendix I). Once at this point,  $\tilde{q}_{1t} = \tilde{q}_{2t} = \frac{\rho}{\theta}$ , and capital is such that  $R'(k_t) = \rho$ , thereby the firm is at the long-run equilibrium in regime 1 where  $k_t = k^*$ , and there is no incentive to move away.

We then conclude that from financial regime 3, an optimal growth path exists such that the firm converges towards the long-run equilibrium level of capital  $k^*$  in financial regime 1. This conclusion is most obvious from the phase diagram of Figure 4, where the continuous optimal path is drawn such that the firm in financial regime 3 approaches the level of capital  $k^*$  in regime 1 with no jumps. With regime 3 as the only phase approaching regime 1, and regime 1 as the necessary final phase, there only remains the question of the role of financial regime 2; is this regime needed at all?

It can be shown that optimality requires the firm to go from financial regime 2 to regime 3, so that regime 2 is the necessary link between the initial condition  $k_0$  and the optimal growth path. To illustrate this point, let us examine the firm's optimal policy in regime 2. In this regime, the firm invests in new capital and accumulates at a decreasing rate, as  $\tilde{q}_{1t}$  decreases over time (recall equation (9)). At the same time, it issues a constant amount of new debt, as described by the optimal debt rule (10); thus, the firm increasingly finances investment by debt. As a result of this behavior, there will be a level of  $\tilde{q}_{1t}$ , e.g.  $\tilde{q}_{1t} = \frac{1}{\Upsilon_C} + \chi > 1$ , at which investment eventually equals the new debt issues  $\dot{b}_t$ , and  $I_t = \dot{b}_t$ . At this point, the firm would finance new investment entirely by debt, thereby shifting to the financial rule of regime 3. We conclude that regime 2 is the necessary link between the sufficiently small initial condition  $k_0$  and the activity phase 3.

The switch from regime 2 to regime 3 is shown in the phase diagram of Figure 4, and can be explicitly calculated. Let  $t_1$  be the point in time at which the firm changes regime, and financial regime 3 starts. At this point, investment  $I_{t_1}$  has to equate the new issues of debt  $\dot{b}_{t_1}$ , and given equations (9) and (10), the function  $\varphi(\cdot)$  can be inverted to give  $\tilde{q}_{1t_1}$  as

$$\tilde{q}_{1t_1} = \frac{1}{\Upsilon_C} + \varphi^{-1} \left[ \omega \left( \frac{1 - \tau_C}{1 - a\tau_C} \left( \frac{1}{\Upsilon_C} - \frac{\rho}{\theta} \right) \right) \right] = \frac{1}{\Upsilon_C} + c'(\bar{\omega}(\tau_C)) = \frac{1}{\Upsilon_C} + \chi(\tau_C) \quad (14)$$

where from the properties of  $\varphi(\cdot)$ , it turns out that  $\varphi^{-1}(\cdot) = c'(\cdot)$ ; in addition, let us assume that  $c'(\bar{\omega}) = \chi > 0$ , where  $\chi'_{\tau_C} > 0$ . Unfortunately, we cannot find the value of the constant  $\chi$  (and the switching point  $\tilde{q}_{1t_1}$ ), without specification of the  $\varphi$  and  $\omega$  functions.<sup>10</sup>

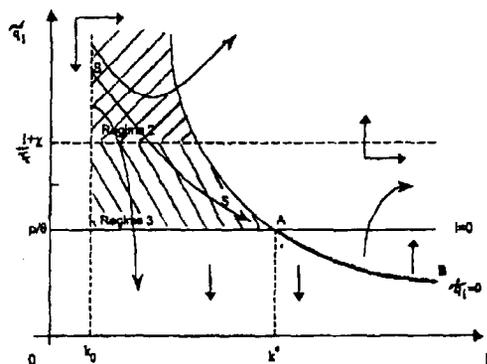
The switching value  $\tilde{q}_{1t_1}$  crucially depends on the shadow cost of debt  $\frac{\rho}{\theta}$  and the tax rate  $\tau_C$ . More specifically, equation (14) brings out that the lower the ratio  $\frac{\rho}{\theta}$ , and the higher the tax rate, the greater is the  $\chi$  value and the tax term  $\frac{1}{\Upsilon_C}$ , and therefore the higher is the switching point for the  $\tilde{q}_{1t}$  variable. This conclusion can be seen in Figure 4 where the  $\tilde{q}_1 = \frac{1}{\Upsilon_C} + \chi$  locus would move

<sup>10</sup>It may be interesting to observe that along the optimal path,  $\tilde{q}_{1t} < 0$  and,  $\tilde{q}_{1t}$  is converging to a stationary level, i.e.  $\lim_{t \rightarrow \infty} \tilde{q}_{1t} = \frac{\rho}{\theta} < 1$ . It follows that a finite time  $t_1$  necessarily exists such that  $\tilde{q}_{1t_1} = \frac{1}{\Upsilon_C} + \chi > 1$ , where  $\chi > 0$ ; hence, the financial regime 3 is always reached.

up as the  $\frac{e}{\theta}$  ratio decreases, thereby leading the financial regime areas to change.

At this point, we have demonstrated that regime 2 is the necessary link between the initial condition  $k_0$  and regime 3; in turn, regime 3 is the only phase approaching regime 1 that represents the necessary final state. Armed with this information, we can summarize our conclusions by drawing the overall optimal growth path in the phase diagram of the following figure:

Figure 4: The Optimal Growth Path of the Firm



Note: The AB line denotes the steady state levels of capital in regime 1.

This figure shows that there is a unique accumulation path converging to a stationary state, the downward-sloping path  $SS$ . This starts from the initial condition  $k_0 < k^*$  and approaches the stationary state and long-run equilibrium level of capital  $k^*$ . The transversality conditions require the system to be on this unique stable path. The stationary state  $k^*$  is approached monotonically with  $k_t$  growing and  $\tilde{q}_{1t}$  falling.

Along this optimal growth path, the firm goes through different financial regimes. It starts off from regime 2 where the marginal valuation of capital  $\tilde{q}_{1t}$  is greater than one and investment is positive. While accumulating, the firm finds it optimal to shift to the financial regime 3. In this regime,  $\tilde{q}_{1t}$  is initially greater than one but, as capital accumulates, it decreases to a value lower than unity till to approach the steady state in which there is no investment.

Each of these regimes corresponds to different policy choices on the part of the firm; it may then be useful to summarize our findings in terms of the firm's financial and investment decisions.

**Proposition 1** *Given an initial condition  $k_0$ , a phase of growth initially arises during which the firm finances new investment both by retaining earnings and by issuing new debt, the regime 2. During this regime, retained earnings are the marginal source of funds. While accumulating capital, the firm moves to a phase of pure debt-financed growth, regime 3, during which capital is accumulated at a decreasing rate and debt is the marginal source of funds. Along this path, the firm approaches a level of capital that equates its marginal product to the market interest rate (regime 1).*

From Figure 4 it may be noticed that, with a different initial stock of capital, the firm might start off from financial regime 3. Alternatively, when the initial condition is such that  $k_0 \geq k^*$ ,

then there may be no dynamics at all; in this case the only optimal policy is to stay at the initial situation.

This model differs from Sinn's (1991a) model of investment in the way the firm finances its accumulation path. In Sinn's model there is a foundation moment and a single internally financed phase of growth. The existence of this accumulation phase depends on the presence of dividend taxation. In the absence of taxation, there would be no accumulation path, and capital would freely adjust to its steady state level. In the present model, we obtain a multiphase accumulation path whose existence is independent of taxation.

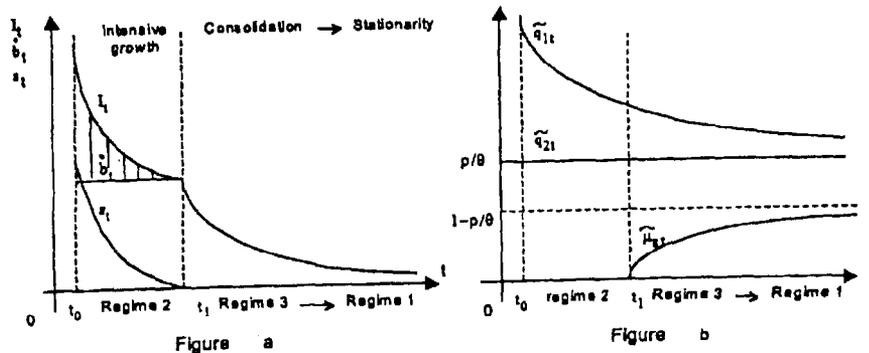
In each regime, the profits tax has different effects on investment and borrowing. In financial regime 2, for each value of the multiplier  $\tilde{q}_{1t}$ , the profit tax reduces investment and rises new debt issues (recall equations (9) and (10)). During financial regime 3, the effect of the tax depends on the search costs deductibility parameter  $a$ . For any  $\tilde{q}_{1t}$ , if  $a = 1$ , then the tax has no effect on the investment-borrowing policy of the firm; but, if only a fraction of search costs is permitted as a deduction so that  $a < 1$ , then for each value of the multiplier  $\tilde{q}_{1t}$ , the tax result in lower investment and lower borrowing.

As the reader will have already detected, previous conclusions refer to the pattern of capital accumulation, and they are independent of time. The investigation of the financial and investment policy over time may, however, provide a useful insight into the decision-making process of the firm and the effect of corporate taxation.

#### D. The Life Cycle of the Firm: Investment and Financial Policies Over Time

One of the merits of the model described here is to provide a life-cycle story for the firm, something which is usually missing, even from dynamic models of the firm (which tend to assume steady state). In particular, armed with previous information, we can draw the optimal temporal paths of the variables  $I_t$ ,  $b_t$ , and  $s_t$  shown in Figure 5. This figure also illustrates the patterns for the co-state variables  $\tilde{q}_{1t}$  and  $\tilde{q}_{2t}$ , and the multiplier  $\tilde{\mu}_{st}$  (Figure 5b).<sup>11</sup>

Figure 5: The Life Cycle of the Firm: Investment and Financial Policy over Time



<sup>11</sup>The concavity properties shown in Figure 5 follow from specific assumptions on the third grade derivatives of functions  $c(\cdot)$  and  $\sigma(\cdot)$ .

The optimal path for investment  $I_t$  is obtained by combining equations (9) and (13) with the temporal path of the co-state variable  $\tilde{q}_{1t}$  (see Figure 5b). In particular, given the tax rate  $\tau_C$ , in each regime investment  $I_t$  is an increasing function of  $\tilde{q}_{1t}$ , and this is steadily decreasing over time (see Appendix I). As a result, the investment rate is also decreasing monotonically. This is shown in Figure 5a where the path of investment starts at  $t_0$ , and approaches a steady-state value of nil investment. At the early stages of development the firm is in an “intensive growth” phase where investment expenses  $I_t$  are particularly large. This initial policy of high investment is followed by a reduced growth, or “consolidation” phase where investment expenses are lower and capital accumulates at a lower and decreasing rate. The firm then approaches a phase of “stationarity” where no further investment is desirable.

Along this accumulation path, the firm adopts different financing strategies and acts according to a clear financial hierarchy. In the “intensive growth” phase, the firm follows the financial rule of regime 2 (see Figure 5a). It prefers to finance investment by borrowing up to its maximum debt capacity. But, as shown in the figure, given this optimal debt policy, the firm has sufficiently attractive investment opportunities remaining (the shadowed area), and it finances the higher level of investment by retaining earnings. These are considered a residual or marginal source of funds. As the firm grows, its investment opportunities reduce, and it is eventually able to finance investment expenses by borrowing only. At this point, the firm shifts to financial regime 3, the pure debt-financing regime. From this, we deduce that the firm acts according to a financial hierarchy: it prefers debt to retained earnings along the entire accumulation path. This preference depends on the cost of debt and is independent of taxation.

From Figure 5a, it may be noted that the optimal path of investment has different slopes at the different phases, so that the speed of accumulation changes with the financial regime. We conclude that, in this model, financial constraints do affect the firm’s real investment decisions and the optimal accumulation path.<sup>12</sup>

<sup>12</sup>The different speed in capital accumulation in regimes 2 and 3 derives from the optimal investment rules in each regime. In regime 2, optimality requires that  $c'(I_t) = \tilde{q}_{1t} - \frac{1}{\Upsilon_C}$ , and the slope of the function  $I_t$  with respect to time is:

$$\dot{I}_t = \dot{\tilde{q}}_{1t} \frac{1}{c''(\cdot)} < 0$$

where dots denote the derivatives with respect to time. In financial regime 3, instead, the optimality conditions imply that:

$$\dot{I}_t = \dot{\tilde{q}}_{1t} \frac{1}{c''(\cdot) + \sigma''(\cdot) \frac{(1-\alpha\tau_C)}{(1-\tau_C)}} < 0$$

From these equations, it turns out that the slope of the investment curve in the two regimes is different, and at the switching point  $t_1$ , given  $\tilde{q}_{1t}$ , there must be a kink as illustrated in Figure 5a.

### E. An Example with Quadratic Cost Functions

To make previous results more concrete, it may be useful to work out a specific example. In particular, let us consider the more restrictive class of quadratic cost functions and constant returns to scale

$$c(I_t) = \alpha_0 + \frac{\alpha}{2} I_t^2 \quad \sigma(\dot{b}_t) = \beta_0 + \frac{\beta}{2} \dot{b}_t^2 \quad R(k_t) = \left( \delta_0 - \frac{\delta}{2} k_t \right) k_t$$

where the constants  $\alpha_0$  and  $\beta_0$  are nonnegative,  $\alpha, \beta, \delta_0, \delta$  are positive, and  $\delta_0 > \theta > \rho$  (which is a sufficient condition for the steady-state level of capital to be positive).<sup>13</sup> This example is difficult to solve for the entire accumulation path, but is specially informative if we confine the attention to the firm which starts off from financial regime 3.

Let us solve our problem for this special case. In particular, let us suppose to initially be in the steady state and solve the firm's problem by working backward in time. From previous optimality conditions, it follows that the long-run equilibrium level of capital  $k^*$  is:

$$k^* = \frac{\delta_0 - \rho}{\delta}$$

where  $\delta_0 > \rho$ , so that  $k^* > 0$ . As noted above, this is independent of the profit tax.

Approaching this steady state, the firm is in financial regime 3 (Figure 4). From the optimality condition (12), and the dynamic equations for  $\tilde{q}_{1t}$ , it follows that at the optimum,

$$\begin{aligned} \left( \alpha + \beta \frac{1 - a\tau_C}{1 - \tau_C} \right) \dot{k}_t &= \tilde{q}_{1t} - \frac{\rho}{\theta} \\ \dot{\tilde{q}}_{1t} &= \theta \tilde{q}_{1t} + \delta k_t - \delta_0 \end{aligned}$$

This is a nonhomogeneous and autonomous system of two linear first-order differential equations in  $k_t$  and  $\tilde{q}_{1t}$ , and its general converging solution can be written as:

$$\begin{aligned} k_t &= \tilde{c}_2 e^{\tilde{r}_2 t} + \frac{1}{\delta} (\delta_0 - \rho) \\ \tilde{q}_{1t} &= \left( \alpha + \beta \frac{1 - a\tau_C}{1 - \tau_C} \right) \tilde{c}_2 \tilde{r}_2 e^{\tilde{r}_2 t} + \frac{\rho}{\theta} \end{aligned} \tag{15}$$

where  $\tilde{r}_2$  is the negative root of the characteristic equation associated with the system and it depends on the tax rate  $\tau_C$  (as  $\tilde{r}_2 = \frac{1}{2} \left( \theta - \sqrt{\theta^2 + \frac{4\delta}{\alpha + \beta \frac{1 - a\tau_C}{1 - \tau_C}}} \right)$ ), and  $\tilde{c}_2$  is an arbitrary constant. To determine this constant, we can use the value of the function  $k_t$  at the initial time  $t = 0$ ; this

<sup>13</sup>It may be noticed that in this example  $R'''(\cdot) = 0$  and this differs from the assumptions used in previous sections where  $R'''(\cdot) > 0$ . However, the assumption of  $R'''(\cdot) > 0$ , it should be recalled, was introduced only to simplify the diagrammatic discussion; thus, it may be omitted, without jeopardizing the analysis.

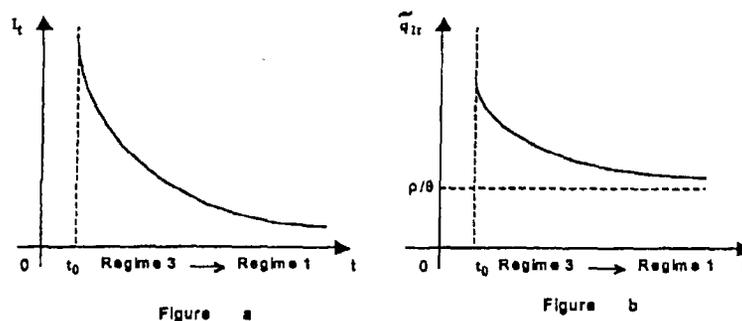
gives us

$$\tilde{c}_2 = k_0 - \frac{1}{\delta}(\delta_0 - \rho) < 0$$

which is negative and independent of the tax rate.

These equations provide an explicit solution to the model of accumulation described above. From this, it can be noticed that the long-run equilibrium is approached monotonically with capital  $k_t$  growing steadily ( $\dot{k}_t > 0$ ) and  $\tilde{q}_{1t}$  falling ( $\dot{\tilde{q}}_{1t} < 0$ ), and both paths depend on the tax rate. This information may be also used to determine the optimal pattern of investment shown in Figure 6.

Figure 6: The Optimal Investment Path with Quadratic Cost Functions



This path is obtained by combining the solution for the co-state variable  $\tilde{q}_{1t}$  (shown in Figure 6b), with the optimality condition for investment (13). Since  $I_t$  is an increasing function of  $\tilde{q}_{1t}$ , and this is decreasing, it follows that the investment rate is also decreasing monotonically, and the path depends on the tax rate  $\tau_C$ .<sup>14</sup>

#### IV. DYNAMIC EFFECTS OF CORPORATE INCOME TAXATION

What can we deduce from this about the effect of the profit tax? In considering this question, I am interested in the immediate impact of the tax and in its effects over time. While the standard analysis of tax incidence has mainly concentrated on the impact of taxes on the optimal stock of capital (Atkinson and Stiglitz, 1980; Auerbach, 2002), taxes have important intertemporal implications which are not always fully appreciated.

In order to examine these intertemporal effects, we need in principle to characterize the full dynamic path and the dependence of all variables with respect to the tax. Unfortunately, we do not know the optimal path explicitly, and we cannot calculate the impact of the tax over time. Nevertheless, we can attempt to qualitatively characterize these effects by means of the well-known comparative-dynamics analysis (see for example Kamien and Schwartz, 1991). As I shall argue below, this approach is far from fully satisfactory, but it provides a convenient starting

<sup>14</sup>As an alternative, the reader may derive Figure 6a by differentiating the first of equations (15) with respect to time.

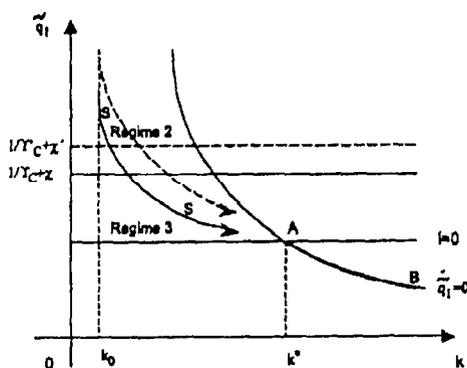
point for our discussion.<sup>15</sup>

### A. The Method of Comparative Dynamics

The method of comparative dynamics consists in comparing the pattern of accumulation described in a conventional phase diagram (as in Figure 4), with that deriving from a parametric change in tax provisions. To investigate the effects of profit taxation, let us then examine how the optimal accumulation path of the firm changes with variations in the corporate profits tax rate. In particular, let us assume that at the initial time  $t = 0$  an unexpected marginal increase in the tax rate  $\tau_C$  occurs. Let us also assume that the change persists for the entire future beyond period 0, so as to consider the effect of a single change on the entire accumulation path. Moreover, suppose for simplicity that the firm is in regime 2 when the tax rate rises, and confine the attention to the general case of  $a < 1$ , that is only a fraction of search costs is permitted as a deduction.

At this point, we can use the diagrammatic analysis, and illustrate the effects of the higher profit tax rate in the  $(k, \tilde{q}_1)$  phase diagram of Figure 7.

Figure 7: The Comparative Dynamics Analysis of Profit Taxation



As shown in this figure, the increase in the tax rate shifts the  $\tilde{q}_{1t} = \frac{1}{r_C} + \chi(\tau_C)$  line upwards (the dotted line), while leaving the loci  $I = 0$  and  $\dot{\tilde{q}}_{1t} = 0$  unaffected. As noted above, the tax has no effect on the steady-state or long-run equilibrium level of capital.<sup>16</sup>

This result does not tell us anything about the accumulation decisions, and we need to examine the equilibrium path. Along this path, the higher profit tax discriminates in favor of debt and against retained earnings. Since in the initial regime 2 retained earnings are the marginal source of finance, it follows that investment has to initially decrease as a result of the tax change. At the

<sup>15</sup>For an application of the comparative-dynamics analysis to analyzing the effect of tax changes in models with adjustment costs (with no financial imperfections), see Abel and Blanchard (1983).

<sup>16</sup>These comparative statics results can be found by differentiating the switching value  $\tilde{q}_{1t} = \frac{1}{r_C} + \chi(\tau_C)$  with respect to  $\tau_C$  (see previous section where  $\chi'_{\tau_C} > 0$ ), and by noting that the equations of the loci  $I_t = 0$  and  $\dot{\tilde{q}}_{1t}$  are independent of the tax rate.

same time, a greater debt reduces the burden of taxation, and thus increases the marginal value or shadow price of capital  $\tilde{q}_{1t}$ ; hence,  $\tilde{q}_{1t}$  is initially higher. If we go back to Figure 7, the higher  $\tilde{q}_{1t}$  requires the optimal path to initially jump upwards as shown by the dashed line; in addition, we know that investment is initially reduced. However, the steady-state stock of capital remains unaffected, thus the investment corresponding to each level of capital has to increase above its original path at some point. Given the investment equations (9) and (13), this necessarily requires a higher value of  $\tilde{q}_{1t}$ . As shown in Figure 7, the new path then steadily lies above the original one; along this path, investment is initially lower than on the original pattern, but there must be some level of capital for which it is higher. A formal proof of these results is given in Appendix II.<sup>17</sup>

As the reader will have already detected, time does not explicitly enter in the analysis of comparative-dynamics, and this gives rise to a number of problems. The fundamental problem with this method is that it rests on the assumption that tax effects are examined with respect to a given level of capital. This “equal capital repercussion” assumption simplifies the analysis, but it prevents the method from providing information on the effect of the tax on the “temporal” path of investment. For example, we cannot know the speed of capital accumulation as this depends on the stock of capital at each instant which, in turn, is affected by the tax. As a result, at any time investment may rise above and fall below, its original path, depending on the effect of the tax on the stock of capital. But, this is like to redefine the question at issue. We want to know whether investment will be higher after a tax change; we cannot say that it will be higher if  $\tilde{q}_{1t}$  is higher given a stock of capital, since that simply shifts the question about whether  $\tilde{q}_{1t}$  is higher. Therefore, the problem with comparative-dynamics analysis is that, while it may tell us about the relation between the investment  $I_t$  and the co-state variable  $\tilde{q}_{1t}$  for each given level of capital, it does not tell us about the impact on  $I_t$  at all instants  $t$ , since  $\tilde{q}_{1t}$  depends on capital at each time, a variable that is, in turn, affected by the policy change.

These theoretical considerations may be thought to be of little practical relevance, but they have implications of great importance for policy purposes. Governments are likely to be more concerned with the impact of taxes over time than with the effect for a hypothetical stock of capital. In many countries, the discussions regarding tax reforms have centered round the timing of tax effects. Governments want to know, for example, whether investment in three years time will be higher as a result of a proposed reform. However, if we rely on the method of comparative-dynamics, then the intertemporal effect of taxation is a question to which no definite answer can be given. In this respect, the method is not a particularly appealing tool of analysis, and we should run explicit simulations. As an alternative, we need a different approach to the dynamic analysis of taxation, an issue which has received very little attention in the literature.

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<sup>17</sup>It is important to realize that we examine infinitesimal changes in the tax rate under the assumption that the initial rate is positive. Because of continuity properties of the model, it is therefore legitimate to assume that the tax change does not affect the initial standing regime of the firm. However, when the initial tax rate is zero, it may well be the case that the introduction of the tax encourages the firm to transfer to a different regime. In this special case, previous results hold under the condition that the firm does not change regime as a result of the tax.

## B. The Method of Direct Variations

To consider the effect of taxes over time, we develop an alternative method of dynamic analysis, what we can call “the method of direct variations.” This consists in directly examining the variations in the temporal path of investment as they derive from a change in the tax rate. While this approach involves the examination of systems of differential equations which is not always straightforward, it results in a very simple diagrammatic analysis, and does permit a rigorous discussion of the intertemporal effects of taxes.

In order to simplify the analysis, let us examine first the firm which is in the initial regime 2 when the tax rate rises, and then the firm which is in regime 3 when the tax increase occurs under the common assumption that search costs are fully deductible; finally, let us consider the implications of introducing partial search costs deductibility.

### The mixed-financed investment regime

Let us start assuming that the firm is in the mixed financial regime 2 when the statutory tax rate  $\tau_C$  rises. Let us also consider for convenience the restrictive case of  $a = 1$ ; that is, costs upon search are fully tax deductible. This assumption is clearly strong, and it will be relaxed later; nevertheless, it provides a useful benchmark for the examination of more complicated cases.

The effect of the profit tax on investment may be seen by differentiating the optimality conditions for  $I_t$ , and the equation of motion for  $\tilde{q}_{1t}$  along the accumulation path with respect to  $\tau_C$ . In particular, in the starting regime 2, we can differentiate the optimality condition (8) for  $I_t$ , and the dynamic equations for  $\tilde{q}_{1t}$  so as to have:<sup>18</sup>

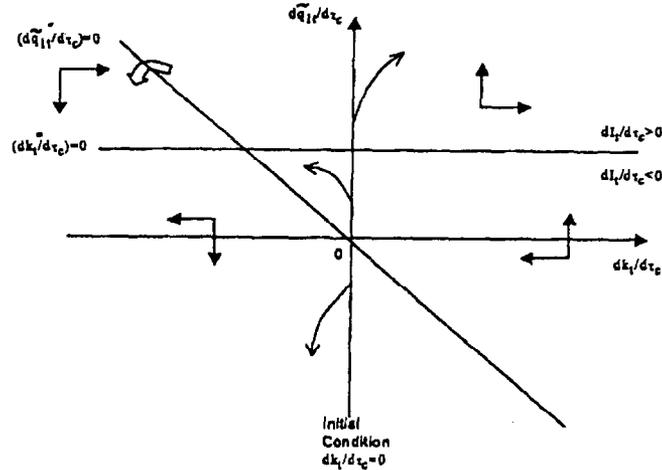
$$\begin{aligned} c''(I_t) \frac{d}{d\tau_C} \left( \frac{dk_t}{dt} \right) &= c''(I_t) \frac{d}{dt} \left( \frac{dk_t}{d\tau_C} \right) = \frac{d\tilde{q}_{1t}}{d\tau_C} - \frac{1}{\Upsilon_C^2} \\ \frac{d}{d\tau_C} \left( \frac{d\tilde{q}_{1t}}{dt} \right) &= \frac{d}{dt} \left( \frac{d\tilde{q}_{1t}}{d\tau_C} \right) = \theta \frac{d\tilde{q}_{1t}}{d\tau_C} - R''(k_t) \frac{dk_t}{d\tau_C} \end{aligned} \quad (16)$$

These equations give us a nonhomogeneous system of two linear differential equations in  $\frac{dk_t}{d\tau_C}$  and  $\frac{d\tilde{q}_{1t}}{d\tau_C}$ , from which we can draw the phase diagram shown in Figure 8. This diagram illustrates the loci  $\frac{d}{dt} \left( \frac{d\tilde{q}_{1t}}{d\tau_C} \right) = 0$  and  $\frac{d}{dt} \left( \frac{dk_t}{d\tau_C} \right) = 0$  derived from previous equations, and the directions of motion associated with the system. In drawing this figure, it must be realized that  $R''(k_t)$  varies over time, and the  $\frac{d}{dt} \left( \frac{d\tilde{q}_{1t}}{d\tau_C} \right) = 0$  line has, therefore, a changing slope around the origin. Nonetheless, under previous assumptions  $R''(k_t) < 0$  and  $R'''(k_t) > 0$  for all  $k_t$ , so that the slope of this line is negative, and increasing with the stock of capital. In particular, as capital accumulates over time, the slope of the  $\frac{d}{dt} \left( \frac{d\tilde{q}_{1t}}{d\tau_C} \right) = 0$  line steadily rises, with the locus being

<sup>18</sup>The reversal of the order of differentiation is legitimate since the variables  $\tilde{q}_{1t}$  and  $k_t$  are continuous in  $\tau_C$ .

flatter. This is illustrated in the following figure by the double arrow demonstrating the direction of this change.<sup>19</sup>

Figure 8: The Basic Diagram



As an initial condition we have  $\frac{dk_0}{d\tau_C} = 0$  because the stock of installed capital is predetermined in the short-run and invariant to taxation; therefore, we have to start on the vertical initial condition line of the diagram. According to the directions of motion, this initial condition defines the three possible directions of adjustment illustrated in the figure. To determine which of these is optimal, we clearly need a terminal condition. However, this is *a priori* undetermined because along the optimal path the firm moves from the financial regime 2 to regime 3. The terminal condition for regime 2 has to be then obtained as initial condition of the adjustment process in regime 3. At this point, the adjustment in this regime has to be clearly discussed.

During regime 3, the effect of a higher profit tax can be seen from the equation of motion for  $\tilde{q}_{1t}$ , and the optimality condition (12) characterizing  $I_t$  in this regime. In particular, differentiating these equations with respect to  $\tau_C$ , we have:

$$\frac{d}{d\tau_C} \left( \frac{d\tilde{q}_{1t}}{dt} \right) = \frac{d}{dt} \left( \frac{d\tilde{q}_{1t}}{d\tau_C} \right) = \theta \frac{d\tilde{q}_{1t}}{d\tau_C} - R''(k_t) \frac{dk_t}{d\tau_C} \quad (17)$$

$$\left( c''(I_t) + \sigma''(\dot{b}_t) \frac{1 - a\tau_C}{1 - \tau_C} \right) \frac{d}{dt} \left( \frac{dk_t}{d\tau_C} \right) = \frac{d\tilde{q}_{1t}}{d\tau_C} - \sigma'(\dot{b}_t) \frac{1 - a}{(1 - \tau_C)^2}$$

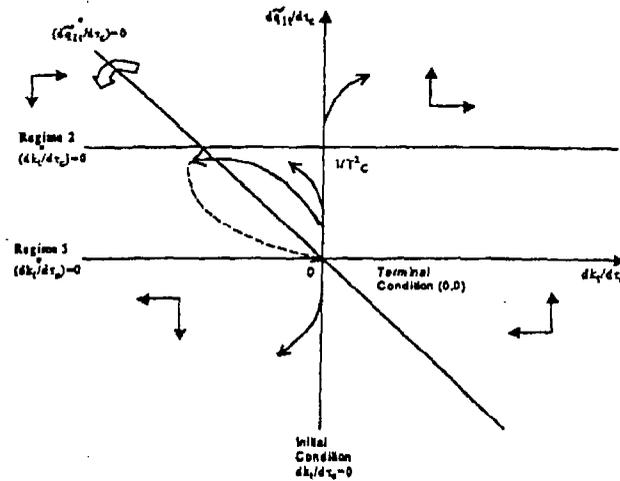
where since  $a = 1$ , the term multiplying  $\sigma''(\dot{b}_t)$  on the left-hand side of the second equation is equal to one, while the last term on the right-hand side disappears.

These conditions define a homogeneous system of two linear differential equations. As in the

<sup>19</sup>It should be noticed that if  $R''' = 0$ , as in the previous example, then the slope of the locus  $\frac{d}{dt} \left( \frac{d\tilde{q}_{1t}}{d\tau_C} \right) = 0$  would not change.

case of regime 2, we can draw a phase diagram in  $\left(\frac{dk_t}{dr_C}, \frac{d\tilde{q}_{1t}}{dr_C}\right)$  space where the line  $\frac{d}{dt}\left(\frac{dk_t}{dr_C}\right) = 0$  overlaps the horizontal axis, and the locus  $\frac{d}{dt}\left(\frac{d\tilde{q}_{1t}}{dr_C}\right) = 0$  shows a varying, and increasing slope. This is illustrated in figure 9 where I also draw the loci for regime 3, so that moving from regime 2 to regime 3, the  $\frac{d}{dt}\left(\frac{dk_t}{dr_C}\right) = 0$  line jumps from the initial value  $\frac{d\tilde{q}_{1t}}{dr_C} = \frac{1}{r_C^2}$  down to  $\frac{d\tilde{q}_{1t}}{dr_C} = 0$ .

Figure 9: The Effects of an Increase in the Corporate Profits Tax During Regime 2 (case  $a = 1$ , Full Search Costs Deductibility)



Since along the optimal path the firm approaches the stationary state in which the profit tax is neutral (recall the long-run equilibrium condition), we must approach the terminal point  $\frac{dk_t}{dr_C} = \frac{d\tilde{q}_{1t}}{dr_C} = 0$ . Armed with this information, we can deduce that the adjustment path in regime 3 can start off from (and regime 2 can therefore terminate at) neither a point in the bottom left-hand quadrant, nor in the top right-hand one. For if we were to begin in one of these areas, we would never approach the terminal condition. Using the analysis for regime 2, the adjustment path has to begin between the origin of the axes and the point  $\frac{1}{r_C^2}$  on the initial condition line. The only path converging to the terminal point is, therefore, that illustrated in Figure 9, where the continuous line denotes the adjustment process in regime 2, and the dashed part the effect during regime 3.

From the optimal adjustment path of Figure 9, it can be deduced that the immediate effect of the profit tax is to reduce the investment of the firm, but this effect reverses over time. During the initial regime 2,  $\frac{dI_t}{dr_C}$  is negative (the path is below the  $\frac{d}{dt}\left(\frac{dk_t}{dr_C}\right) = 0$  line), and the tax reduces investment. After this initial effect, the firm moves to regime 3; the  $\frac{d}{dt}\left(\frac{dk_t}{dr_C}\right) = 0$  locus jumps down, and the higher tax rate leads to a greater investment which eventually offsets the initial negative impact as in the steady state the tax is neutral. We then conclude that the effect of the tax on investment varies over time.

Indeed, there is an important distinction, in this model, between the short-run impact of the

tax, which is negative, and its intertemporal and long-run implications, which compensate the initial impact. As a result, the profit tax does not necessarily reduce investment along the entire accumulation path. However,  $\frac{dk_t}{dr_C}$  is everywhere negative, so that the profit tax does reduce the stock of capital at every point along the adjustment path.

The second important effect of the increase in the profit tax rate is on the firm's financing policy. From the equation (10), it may be seen that in the initial regime 2 the tax induces the firm to increase the issues of new debt. We can then conclude that an increase in the profit tax has initially two effects, it: (1) reduces investment spending, and (2) increases debt issues. As a result, the leverage ratio of the firm is at each time greater. In this respect, the profit tax can be seen as an instrument that temporarily reduces investment and permanently affects the firm's financial structure.

These conclusions may appear remote from reality, but they have a straightforward economic interpretation. Intuitively, the increase in the profit tax rate lowers the return to capital, and if at the margin the firm finances investment by retaining earnings (as in regime 2), then it finds desirable to reduce investment. But, the tax also discriminates against retained profits and in favor of debt finance. Since in regime 3 debt is the marginal source of finance, it follows that once in this regime the firm will take advantage of the lower cost of external funds and will invest more. These conclusions are not totally new to empirical economists either; in fact, a number of studies suggest that profit taxes do work as an incentive to borrow funds.<sup>20</sup> That profit taxation reduces investment also is a common result. In their recent study on 'tax policy and investment,' for example, Hassett and Hubbard observe that:

"Recent studies (*based on firm-level data*) appear to have reached a consensus that the elasticity of investment with respect to the user cost of capital (*including taxes*) is between  $-0.5$  and  $-1.0$ ." (Hassett and Hubbard, 1997, p. 365)

However, what our analysis suggests is that the negative impact of the tax on investment is likely to be only a temporary effect. This consideration is important for policy purposes, and clearly brings out the relevance of carrying out an explicit dynamic analysis of taxation.

### **Debt-financed investment**

The discussion above applies to the firm which is in regime 2 when the tax rate rises; but, the analysis may be extended to the firm which is in the debt-financed investment regime 3 at the time of the tax change. This discussion is particularly important because firms with different financing policies when the tax changes may react differently to taxation (Atkinson and Stiglitz, 1980).

In this case, the profit tax becomes a cash-flow tax and it affects neither the investment nor the financing policy of the firm. With debt-financed investment and full cost deductibility ( $a = 1$ ), the tax is, in fact, equivalent to multiplying the firm's objective function in section 2 by the constant

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<sup>20</sup>For discussion of the effects of taxation on firm's debt policy, and references to the literature, see for example Graham (1996) and Auerbach (2002).

$\Upsilon_C$ . As a result, there are no effects on the optimality conditions at the margin, and the profit tax does not influence the behavior of the firm. It thus follows that when costs for external funds (in this model, interest payments and search expenses) are fully deductible, and investment is financed at margin by debt, then the profit tax invariably burdens on the infra-marginal returns to investment, leaving the marginal conditions for optimality unaffected. In this sense, the tax may be said to be “neutral.”

Formally, the neutrality of the profit tax can be seen by differentiating the dynamic equations for  $\tilde{q}_{1t}$  and  $k_t$  characterizing regime 3 with respect to  $\tau_C$ . This gives us the previous system of differential equations (17) where the term  $\sigma' \left( \frac{b_t}{(1-\tau_D)^2} \right)$  is zero as  $a = 1$ . Similarly, we can draw a phase diagram in  $\left( \frac{dk_t}{d\tau_C}, \frac{d\tilde{q}_{1t}}{d\tau_C} \right)$  space as in illustrated in the previous Figure 9 where the  $\frac{d}{dt} \left( \frac{dk_t}{d\tau_C} \right) = 0$  locus overlaps the horizontal axis. We must begin along the initial condition line  $\frac{dk_0}{d\tau_C} = 0$ , and approach the point  $\frac{dk_t}{d\tau_C} = \frac{d\tilde{q}_{1t}}{d\tau_C} = 0$ . It follows that the only possible adjustment path is that which does not move from the origin of axes. For, if we were to begin above or below the origin, we should never approach the terminal point, and optimality conditions would be violated. As a result, the profit tax does not affect the optimal path of investment.

It may be noticed that in this special case the immediate impact of the tax coincides with the long-run effect on the optimal stock of capital, and we have a unique conclusion about the effect of taxation over time. Not surprisingly, this conclusion coincides with the predictions from the standard theory of corporate taxation which concentrates on the effects on the optimal stock of capital (see for example Atkinson and Stiglitz, 1980, Lecture 5).

### Implications of partial search costs deductibility

So far we have discussed the effect of the profit tax for the special case where  $a = 1$ ; it may be now interesting to relax this assumption, and look at the general case of  $a < 1$ . In this event, costs upon search are not entirely deductible; indeed, this is the most likely case in practice because debt costs are not, in general, fully deductible.

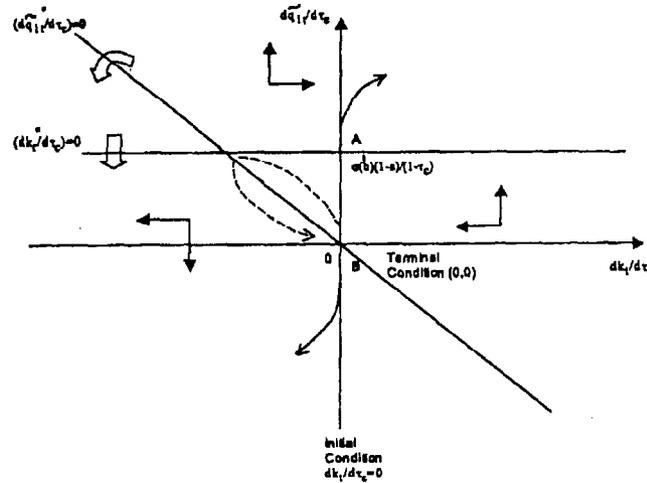
For simplicity, let us consider the case of the mature and debt-financed company which is in regime 3 when the profit tax rate  $\tau_C$  rises.<sup>21</sup> Making use of the method of direct variations, the effect of the tax on investment can be seen by differentiating the dynamic equation for  $\tilde{q}_{1t}$ , and the optimality condition (12) characterizing  $I_t$  in this regime with respect to  $\tau_C$ . This gives us a pair of differential equations like those in the previous system (17). Again, we can draw a phase diagram in  $\left( \frac{dk_t}{d\tau_C}, \frac{d\tilde{q}_{1t}}{d\tau_C} \right)$  space as illustrated in Figure 10, where we show the loci  $\frac{d}{dt} \left( \frac{d\tilde{q}_{1t}}{d\tau_C} \right) = 0$  and  $\frac{d}{dt} \left( \frac{dk_t}{d\tau_C} \right) = 0$ , and the optimal directions of motion.

As initial condition we have  $\frac{dk_0}{d\tau_C} = 0$  since the initial stock of capital is unchanged by the tax;

<sup>21</sup>For a discussion of the more complex case of the firm that is in regime 2 when the tax rate rises, see Palomba (2001)

moreover, in the steady state the tax is neutral, and we must therefore approach the terminal point  $\frac{dk_t}{d\tau_C} = \frac{d\tilde{q}_{1t}}{d\tau_C} = 0$ . Given the directions of motion, we can now determine the adjustment path.

Figure 10: The Effects of an Increase in the Profit Tax when Investment Is Financed by Debt (case  $a < 1$ , Partial Search Costs Deductibility)



If we were to begin above the line  $\frac{d}{dt} \left( \frac{dk_t}{d\tau_C} \right) = 0$ , or below the origin, we should obviously never approach the terminal condition. This means that we must follow a path beginning between the points A and B on the initial condition line. The only adjustment path is therefore that illustrated in the figure.

As shown in the figure, along the adjustment path, investment initially decreases as a result of the higher tax rate, then it rises as the firm approaches the stationary point, even if the marginal source of funds does not change. This result has a straightforward economic interpretation. In this model, the effect of the tax on investment depends on the marginal cost of capital, but it is also influenced by the availability of finance, namely by the average cost of capital (as for example in Fazzari *et al.*, 1988). The higher profit tax rate initially increases the marginal cost of capital and reduces investment. However, as the firm continues to invest by borrowing funds, its average cost of capital decreases, and it will eventually find desirable to invest more than it would have in the absence of the tax increase. Whether the initial impact of the tax has any enduring effects depends, of course, on the form of functions  $c(I_t)$ ,  $\sigma(b_t)$  and  $R(k_t)$ , and on the term  $a$ , thus the timescale of the model may change with the exact specification of these aspects.

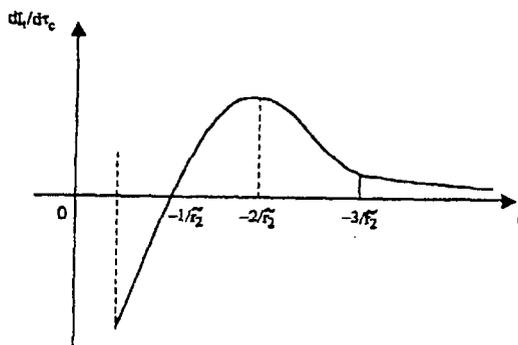
If we confine the attention to the case of quadratic cost functions, we can calculate the precise effect of the tax on the pattern of investment, namely the function  $\frac{dI_t}{d\tau_C}$ . In particular, differentiating the optimal path for capital  $k_t$  from equation (15) with respect to time, and then with respect to the tax rate  $\tau_C$ , we obtain:

$$\frac{dI_t}{d\tau_C} = \frac{d\tilde{r}_2}{d\tau_C} \tilde{c}_2 e^{\tilde{r}_2 t} (1 + \tilde{r}_2 t)$$

where  $\tilde{c}_2$  is negative and independent of the tax rate  $\tau_C$ , and  $\frac{d\tilde{r}_2}{d\tau_C}$  is positive.<sup>22</sup>

The function  $\frac{dI_t}{d\tau_C}$  can be illustrated in the following figure:

Figure 11: Effects of an Increase in the Profit Tax Rate with Quadratic Cost Functions and Debt-Financed Investment (case  $a < 1$ )



It may be noticed from this figure that the function  $\frac{dI_t}{d\tau_C}$  is the analogue of the adjustment process described for the general case, with the effect of the tax varying over time. Investment initially decreases as a result of the increase in the profit tax rate; then, it rises till to compensate the initial effect with the tax being neutral in the long-run.

This simple example also allows us to consider an important question underlying the dynamic analysis of taxation, that of the timescale of the tax effect. Indeed, the speed of the adjustment may make a great deal of difference to the way we think about the effect of taxes. In this example with quadratic cost functions, we can calculate explicitly the timescale of the tax effect. For instance, suppose for convenience that the parameters  $\alpha$ ,  $\beta$ , and  $\delta$  are similar and equal to 0.5, that the discounting rate  $\theta$  is 0.3, and that the government levies a profit tax rate of 40 percent per year. Then, it can be seen that the switching point  $-\frac{1}{\tilde{r}_2}$  in Figure 11 is 1.8. In this case, the initial negative impact of the tax is clearly limited over time as it takes only 1.8 years for the initial impact to be reversed. It may also be interesting to observe that the timescale of the tax effect depends on the initial tax rate. A lower tax rate, for example 10 percent, increases the switching period to 2.7 years. Put the other way around, the highest the tax rate, the fastest is the adjustment process to a tax change. In fact, the higher the tax rate, the stronger is the incentive for the firm to increase borrowing and, therefore, the faster is the reduction in the average cost of capital. As a result, the shorter will be the negative impact of the tax rate increase on investment. These results are important for policy purposes; they mean that governments that lower high tax rates

<sup>22</sup>This result follows by differentiating  $\tilde{r}_2$  (see the example in Section III above) with respect to  $\tau_C$ , so as to have:

$$\frac{d\tilde{r}_2}{d\tau_C} = \frac{\delta\beta(1-\tau_C)^2}{\left[ (\alpha + \beta(1-a\tau_C))^3 (\theta^2(\alpha + \beta(1-a\tau_C)) + 4\delta(1-\tau_C))^{\frac{1}{2}} \right]} > 0$$

can only hope to have temporary positive effects on the investment of domestic firms. This is of course only an example, and the parameterization is admittedly arbitrary, but this simple “back to an envelope” calculation clearly brings out the importance of considering the intertemporal effects in the analysis of tax policy.

### **Summing up**

The analysis of the effect of profit taxation over time has often been thought to be of little practical importance, but this discussion suggests that it does deserve attention. Previous results show that an important distinction exists between the immediate impact of the tax and its implications over time and on the optimal steady-state stock of capital. For example, an increase in the profit tax rate may reduce investment by firms, but the negative impact of the tax is only temporary and it is shown to be reversed and offset over time. This result depends on how the tax influences the marginal source of finance, the availability of finance and, therefore, the average cost of capital. Clearly, the exact time path of the tax effect varies with the accumulation model and the assumptions made about the functioning of capital markets. However, independently of the possible assumptions, changes in the effect of changes in the corporate profits tax should still vary over time.

These conclusions are important for policy purposes; if the benefits of a tax cut are only temporary, then lower taxes, following for example a tax reform, may have only limited effect on capital accumulation of domestic firms. The relevance of these intertemporal implications depends, of course, on the timescale of the effects. Unfortunately, whether the initial effect is enduring is influenced by the specific parameterization of the model of firm investment we use. Nonetheless, it should be clear that to confine the attention to the immediate impact of taxes may be misleading; indeed, the intertemporal aspects can make a great deal of difference to the way we think about corporate taxation of profits.

## **V. THE EFFECTS OF CORPORATE TAXATION—COMPARING WITH THE LITERATURE**

In previous sections, I have discussed the effect of a corporate profits tax; I now consider how the results given above can be used to shed light on some of the most debated issues in the literature on corporate taxation. In particular, I examine two questions: the concept of tax effect in a dynamic model, and the role of financial optimization in determining the effects of taxes. The discussion of these issues is of great importance for understanding the advantage of our dynamic approach with endogenous financial decisions compared to the conventional cost-of-capital formulae (Sinn, 1991b; Auerbach, 2002). However, it involves a number of problems of considerable complexity. As a result, the conclusions reached in this section are only very limited in scope. Indeed, the primary aim of the section is not to present results, but to bring out the advantages and the problems of examining taxes in a dynamic model with financial imperfections, aspects which have received very little attention in the literature.

### A. The Concept of Tax Effect in a Dynamic Model

The principal argument for reforming corporate taxation is that of affecting the investment behavior of firms, and this is clearly the first issue we should discuss. However, the concept of tax effect on investment is not as straightforward as may appear, and we need to exactly define what is meant by “affecting investment.”

The underlying concepts of tax effect have received very little attention in the literature on corporate taxation. The standard cost-of-capital analysis is usually carried out in highly stylized models of accumulation in which there are no stock adjustments, and no dynamics of growth. In this context, the initial impact of the tax coincides with its effect on the long-run level of capital, and there are no intertemporal implications; in fact, this is a single-period analysis.<sup>23</sup> However, in terms of the dynamic model described here, this approach means that if a tax does not affect the long-run level of capital, then it may be said to be neutral, even if it changed the level of investment at each single instant along the accumulation path. There would be neutrality because the effects at different instants offset each other, eventually leading to a unchanged long-run stock of capital. This brings out the difficulties implicit in using models of accumulation with no explicit dynamics for the analysis of the effects of taxes on investment; indeed, investment is typically a flow variable.

While we have learned a lot from single-period investigations, the study of corporate taxation ought to be based on explicit dynamic models. The effects of taxes should be examined in terms of the impact on investment at each instant, and not just on the long-run level of capital. In this sense, we should regard as nonneutral a tax which affects the investment decisions at each time, even though this implies a unchanged long-run level of capital. In fact, governments are more concerned with the timing of policy repercussions than with the long-run impact. On these grounds, I have considered the effect of a corporate profits tax in terms of its immediate, and dynamic impact on firm investment, as well as in terms of the long-run effect on the stock of capital.

It should be noticed that this intertemporal approach repeats some of the results of the conventional analysis as to the effect of taxes on the long-run level of capital, but, it gives rather different results if we consider the overall effect over time. We may have a situation in which a particular tax is neutral with respect to the long-run equilibrium level of capital, but by changing the intertemporal path of investment it leads to different levels of investment at each instant. Moreover, we should consider that in adopting this intertemporal approach the speed of the tax adjustment can make a great deal of difference to the way in which we think about the effect of tax policies. For example, even if tax changes were to bring about an initial temporary reduction in investment, this would not cause the same concern as if the effects were enduring. This is important for the analysis of tax reforms. If the path of tax effects varies over time, then the choice of a tax policy becomes

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<sup>23</sup>For discussion on taxation of the firm, and references to the literature, see for example Nickell (1978), Atkinson and Stiglitz (1980), Auerbach (1983, 1984), Sinn (1987, 1991b), and more recently Sorensen (1994), Mintz (1995), Head (1997), Auerbach (2002).

complicated. Suppose that a country has to decide whether to cut taxes or not. It will have to compare the different paths of investment corresponding to alternative levels of domestic tax rates. In this setting, the choice will depend on the temporal horizon of the government: a strategy with appealing initial effects may have undesired implications in the long-run. It should be clear that obtaining analytical results on these bases is going to be extremely difficult, even with strong simplifying assumptions. While for simplicity I have ignored the timescale of the tax effects and strategic considerations, these qualifications should be borne in mind in interpreting the results.

### **B. Effects of Corporate Taxation and Imperfections in Capital Markets**

The second objection to the conventional or cost-of-capital analysis of corporate taxation is that it does not explicitly incorporate imperfections in capital markets. The analysis derives predictions on how the effect of taxes depend on the firm's financing decisions (see Atkinson and Stiglitz, 1980), but no satisfactory analysis is provided of the financing decision-making process of the firm. Ideally, the financial behavior of the firm should be determined simultaneously with its investment decisions, and the cost of tax and financial distortions ought to be considered together. Unfortunately, this approach has rarely been taken in the literature. But, these considerations may have important implications for the analysis of taxation. In the model described here, for example, the effect of taxes varies even if the marginal source of finance remains unchanged. The effect varies with the stage of development of the firm and the average cost of capital. More interestingly, in any given country, the same tax policy change, for example lower corporate taxation, may have different effects depending on the "average" firm. Clearly, the dynamics of the tax effects could have been different with alternative assumptions about the firm's investment decision process, and if firms could rely, to some extent, on equity finance, but the tax effect would still vary over time.

It is not, of course, suggested that the standard cost-of-capital approach is unimportant; however, it is interesting that even when the marginal form of finance is given, we may still get different conclusions depending on the firm's financial constraints. In this respect, the conventional approach to corporate taxation is not fully satisfactory, and a more complex view of the problem should be taken. Models of accumulation with explicit dynamics and financial constraints have to be used. This may give rise to a *plethora* of different models, but it would eventually lead to a better understanding of the "black box" of the firm's decision process, and of the effects of corporate taxation.

## **VI. CONCLUSIONS**

The primary aim of this paper has been to set forth a simple dynamic model of investment to discuss the determinants of firm behavior and examine the effect of taxes over time. In particular, the paper brings into the analysis financial constraints as these are clearly necessary if we are to examine the impact of taxes on firm's investment decisions.

The model of investment described in this paper shows that financial and real investment decisions are closely related; indeed, the firm behavior is more complex than it is often supposed. Financing constraints influence the firm's investment strategy over time and its entire accumulation process.

The impact of financial constraints on the firm's financing decisions is also less straightforward than generally thought. It is true, for example, that the cost of borrowing funds discriminates debt in favor of internal saving. But, this does not necessarily imply a financial hierarchy with firms preferring internal funds to external finance; the result depends on each firm-specific characteristics. Indeed, firms may behave in a number of different ways.

This intertemporal model is then used to examine how taxes alter the firms' investment decisions over time. Setting the discussion out in this dynamic setting shows that the effect of taxation is more subtle than often supposed. In particular, we need to distinguish between the short-term impact of taxes and the effects along the temporal path of accumulation. For example, an increase in the profit tax rate may reduce investment, but the negative impact of the tax is only temporary. The dynamics of the tax effects depend on the stage of development of the firm and its financing constraints, and enterprises at different stages of development and with different financing constraints respond in a variety of ways to the same tax change.

The conclusions reached in this paper are limited in scope, and rely on a very special model of firm investment and financial constraints. They are sufficient, however, to cast light on a substantive issue: the effect of profit taxes on firm investment over time. The dynamics of the tax effect depends, of course, on the model of investment and the specific financing constraints we utilize, but the effect of a tax change will always vary over time. This conclusion is important for policy purposes. If the benefits of a tax cut are only temporary, then lower taxes, following for example a tax reform or fierce tax competition, may have only limited effect on capital accumulation of domestic firms. Clearly, the choice of a tax reform strategy depends on the optimizing time horizons of governments.

### OPTIMAL PATHS

#### A. Derivation of the optimal path of the co-state variable $\tilde{q}_{1t}$

In this appendix, we show that during financial regimes 2 and 3 optimality conditions require the co-state variable  $\tilde{q}_{1t}$  to steadily decrease over time, so that

$$\dot{\tilde{q}}_{1t} < 0$$

**Proof.** From the multiplier equation for  $\tilde{q}_{1t}$ , at the optimum,

$$\dot{\tilde{q}}_{1t} = \theta \tilde{q}_{1t} - R'(k_t)$$

and, using the transversality condition for capital, we have

$$\tilde{q}_{1t} = e^{\theta t} \int_t^{\infty} R'(k_v) e^{-\theta v} dv$$

Integrating by parts this integral, we can also write

$$\tilde{q}_{1t} = \frac{R'(k_t)}{\theta} + \frac{e^{\theta t}}{\theta} \int_t^{\infty} R''(k_v) \dot{k}_v e^{-\theta v} dv$$

At this point, from the previous multiplier equation  $\dot{\tilde{q}}_{1t} < 0$  implies that  $\tilde{q}_{1t} < \frac{R'(k_t)}{\theta}$ , therefore,

$$\dot{\tilde{q}}_{1t} \leq 0 \iff \frac{R'(k_t)}{\theta} + \frac{e^{\theta t}}{\theta} \int_t^{\infty} R''(k_v) \dot{k}_v e^{-\theta v} dv \leq \frac{R'(k_t)}{\theta}$$

that, in turn, requires

$$\dot{\tilde{q}}_{1t} < 0 \iff \frac{e^{\theta t}}{\theta} \int_t^{\infty} R''(k_v) \dot{k}_v e^{-\theta v} dv < 0$$

Since both in the financial regime 2 and in regime 3,  $R''(k_t)$  is negative, while  $\dot{k}_t > 0$ , it follows that  $\dot{\tilde{q}}_{1t}$  is always negative, as it was to be shown. ■

#### B. Derivation of the optimal path of the multiplier $\tilde{\mu}_{st}$

This appendix shows that during financial regime 3 the Kuhn-Tucker multiplier  $\tilde{\mu}_{st}$  has a positive slope, so that

$$\dot{\tilde{\mu}}_{st} > 0$$

**Proof.** From the first-order conditions characterizing regime 3

$$\tilde{\mu}_{st} = \frac{1}{\Upsilon_C} - \frac{\rho}{\theta} - \frac{(1 - a\tau_C)}{(1 - \tau_C)} \sigma'(b_t)$$

and at the optimum  $\dot{b}_t = I_t = \Psi(\tilde{q}_{1t} - \frac{\rho}{\theta}, \tau_C)$ . (see Section III)

Differentiating these expressions with respect to time, we have

$$\begin{aligned}\tilde{\mu}_{st} &= -\sigma''(\cdot) \ddot{b}_t \frac{(1 - a\tau_C)}{(1 - \tau_C)} \\ \ddot{b}_t &= \Psi'(\cdot) \tilde{q}_{1t}\end{aligned}$$

where  $\tilde{q}_{1t} < 0$  (see Appendix I.A), and  $\Psi'(\cdot) > 0$ , so that  $\ddot{b}_t < 0$ .

At this point, from the expression for  $\tilde{\mu}_{st}$ , and since  $\sigma''(\cdot) > 0$ , it follows that,

$$\tilde{\mu}_{st} > 0$$

as it was to be shown. ■

### THE COMPARATIVE-DYNAMICS ANALYSIS OF PROFIT TAXATION

In this appendix we use the method of comparative dynamics to examine the effects of an increase in the profit tax rate on the firm's accumulation path (Section IV). To determine these effects, let us denote the new tax rate  $\tau_{C_2}$ , and let  $\tau_{C_2} > \tau_{C_1}$  where  $\tau_{C_1}$  is the original rate of taxation. The implications of this tax change are illustrated in the phase diagram of Figure 7 in Section IV (using dashed lines). The neutrality of the corporate profit tax in the steady state has been already discussed in section IV. Furthermore, we have also showed that the higher profit tax leads to a higher  $\tilde{q}_{1t} = \frac{1}{\gamma_C} + \chi$  locus (the switching point between regimes 2 and 3). We now show that the optimal path corresponding to  $\tau_{C_2}$  lies initially above the original path as in Figure 7, by demonstrating that the new path can neither be uniformly lower than nor cross from below the old path. From Figure 7, we can deduce that a steadily lower accumulation path would imply a smaller  $\tilde{q}_{1t}$  corresponding to each level of capital; in turn, from the investment rules (9) and (13) this also means a lower level of investment. But, the steady-state stock of capital is unaffected by the tax, and the investment corresponding to each level of capital must, therefore, rises at some point. As a result, the new pattern of accumulation cannot steadily lies below the original path, and it has to necessarily cross the original pattern of accumulation. However, the new path cannot cross the original one from below. Using Figure 7, we can notice that the new path crosses the original pattern from below if at the intersection point its slope is relatively greater (the path is flatter). The slope of the optimal path is

$$\frac{d\tilde{q}_{1t}}{dk} = \frac{\dot{\tilde{q}}_{1t}}{\dot{k}_t} = \frac{\theta\tilde{q}_{1t} - R'(k_t)}{I_t}$$

Moving from the initial conditions toward the steady state, it is easily verified that at any possible intersection point the required inequality implies that  $\tau_{C_2} \leq \tau_{C_1}$ , which is a contradiction; therefore, the new path cannot cross the old one from below. As a result, a higher profit tax rate has to result in a new path initially lying above the original one. At this point, the new path may steadily lies above or cross the original path. However, if it crosses the original path, then at any point below this path, the new pattern should be flatter (it has a greater slope) and, given the level of capital, this requires that  $\tau_{C_2} \leq \tau_{C_1}$ , which is again a contradiction. Therefore, the new pattern of accumulation must steadily lie above the original path as illustrated by the dashed line in Figure 7, and discussed in Section IV.

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