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## Internal Models-Based Capital Regulation and Bank Risk-Taking Incentives

*Paul Kupiec*

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## **IMF Working Paper**

Monetary and Exchange Affairs Department

### **Internal Models-Based Capital Regulation and Bank Risk-Taking Incentives**

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#### **Abstract**

<p>The views expressed in this Working Paper are those of the author(s) and do not necessarily represent those of the IMF or IMF policy. Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate.</p>
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Advocates for internal model-based capital regulation argue that this approach will reduce costs and remove distortions that are created by rules-based capital regulations. These claims are examined using a Merton-style model of deposit insurance. Analysis shows that internal model-based capital estimates are biased by safety-net-generated funding subsidies that convey to bank shareholders when market and credit risk regulatory capital requirements are set using bank internal model estimates. These subsidies are not uniform across the risk spectrum, and, as a consequence, internal model regulatory capital requirements will cause distortions in bank lending behavior.

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## I. INTRODUCTION

In the early 1990's, banking interest groups successfully convinced banking regulators to allow the use of bank internal risk measurement models as a basis for setting market risk capital requirements.<sup>2</sup> More recently, banking associations and risk management consultancies have argued that banks should be allowed to use their internal credit risk model estimates as a basis for setting credit risk capital requirements.<sup>3</sup> Those that advocate an internal models-based approach to regulation reason that the use of internal models will result in regulatory capital requirements that are more closely aligned with the so-called "economic capital" allocations set by bank managers for operational purposes, and thereby lower regulatory compliance costs and create fewer distortions in credit and securities markets. "The New Basel Accord Consultation Document," (2001) (NBA) shows that these arguments have resonated with regulators. In "The Overview of the New Basel Accord," (2001), the Basel Committee on Banking Supervision (BCBS) states that an objective for the New Accord is to place "greater emphasis on banks' own assessment of the risks to which they are exposed in the calculation of regulatory capital charges (paragraph 5)."

Notwithstanding stated objectives, the NBA does not propose that capital regulations be based on the full use of bank internal credit risk models. Rather, it proposes a link between credit risk capital requirements and bank internal loan classification schemes. In the internal rating-based (IRB) approaches, credit risk weights are set according to a credit's anticipated probability of default. While the IRB approaches do not use bank internal models directly, the regulatory capital requirements generated under the IRB approaches are calibrated using buffer stock capital allocations estimated from industry standard credit value-at-risk (VaR) models that are applied to a stylized bank loan portfolio.

The decision to base regulatory capital on measurements that are designed to be consistent with banks' internal risk measurement processes is a deliberate attempt to harmonize regulatory capital requirements with the best practices of internationally active banks.<sup>4</sup> This objective reflects the Committee's view that the "ultimate responsibility for managing risks and ensuring that capital is held at a level consistent with a bank's risk profile remains with that bank's management."<sup>5</sup>

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<sup>2</sup> Basel Committee on Banking Supervision (1995).

<sup>3</sup> See, *inter alia*, ISDA (2001), Institute of International Finance (2001), The Financial Services Roundtable (2001), or KPMG (2001).

<sup>4</sup> Basel Committee on Banking Supervision (2001), "Overview of The New Basel Capital Accord," paragraphs 36 and 99–102.

<sup>5</sup> *Ibid.*, paragraph 30.

The existing Basel Accord Market Risk Amendment and banking industry calls for the full recognition of internal model estimates in regulatory capital calculations for credit risk raise an important unresolved policy issue as to whether or not it is possible to design effective regulatory capital requirements that mirror the capital measures that banks themselves use in their own internal management processes. While it is clear that recent BCBS regulations and consultative documents have embraced the goal of harmonizing capital regulation with bank internal processes, it is troubling that the BCBS has yet to discuss the nature of the externalities that are being addressed by the newly proposed capital regulations or provide analysis that supports the claim that bank's internal capital allocation model estimates can be harnessed to control the underlying market failure(s) that mandate regulation.

If the need for bank regulation is based on the existence of externalities, it is important to understand if, and how, these externalities can be measured and controlled using the internal processes that banks have designed for their own profit-maximization objectives. While the goal of harmonizing regulatory capital guidelines with those used by banks in their internal risk management processes is appealing, by virtue of the implicit promise of reduced regulatory burden, it is far from clear that an internal models approach for regulatory capital will control the externalities that mandate bank regulation.

This paper will analyze the internal model capital allocation process within a bank that sets its capital structure to satisfy a buffer stock capital constraint using VaR measures of risk exposure in both the market and credit risk settings. This approach, in which debt finance is maximized subject to a limit imposed by a maximum acceptable probability of default on the bank's funding debt, has become the commonly recognized standard for setting bank internal model "economic capital" allocations.

The discussion will revisit internal models capital allocation measures and show that buffer stock capital requirements can be estimated using an appropriately constructed VaR measure augmented by an estimate of the equilibrium interest payments required by bank debt holders. The procedures required to set accurate buffer stock capital allocations differ from those discussed in the VaR literature and BCBS consultative documents.<sup>6</sup> The capital allocation methodology discussion not only highlights the shortcomings in existing discussions of capital allocation, but by identifying the importance of funding debt interest payments in the capital calculation, it provides the key for understanding how bank internal capital estimates are affected by the externalities that are engendered by under-priced bank safety net guarantees.

The analysis demonstrates that the funding cost subsidies enjoyed by banks—subsidies that are generated by implicit guarantees or under-priced explicit deposit insurance—will reduce the internal buffer stock capital allocations selected by banks. As a consequence, if banks are allowed to use their internal models to set an accurate buffer stock capital allocation consistent

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<sup>6</sup> See Basel Committee on Banking Supervision, (1999) for a description of the internal models process for setting credit risk capital allocations.

with a probability of default selected by supervisors, the required amount of capital will be smaller than capital that would be required in the absence of deposit insurance. In setting their internal capital allocations, banks recognize the benefits of implicit or explicit deposit insurance and use a subsidized interest rate to calculate their internal model-based estimate of “economic capital.” From the bank’s point of view, the approach is appropriate and satisfies regulatory requirements. The bank’s own internal model estimates will however internalize the safety net funding cost subsidy, and the bank’s estimates of buffer stock capital requirements are downward biased compared to an institution that does not benefit from a safety net funding subsidy. Not only will this bias ensure that the bank’s insurance guarantee is valuable, it will create distortions in a bank’s lending behavior.

The interest rate subsidy that will be captured by a bank under an internal models approach to capital is not uniform across investments. For the shareholders of a capital-constrained bank, an internal models approach to capital will make some investments significantly more profitable than others. As a consequence, the use of internal model “economic capital” requirements for regulatory capital purposes will not remove regulation-induced distortions in bank lending behavior. Rather, the use of internal models capital regulation will create a new set of distortions in financial markets that have yet to be recognized by regulators or those promoting internal models approaches for bank regulatory capital.

The upshot of the analysis is that, if banks enjoy a financing cost advantage because of implicit or explicit safety net guarantees on their liabilities, bankers and supervisors should never agree on the “economic capital” that is required under a buffer stock capital objective function. If capital regulations are required to control the externalities created by under-priced deposit insurance, they cannot be set equal to a bank’s internal model capital estimate. Internal models based capital allocations must be modified to remove the distortions created by under-priced safety net guarantees, or the safety net subsidies will create distortions in bank lending activities.

An outline of the paper follows. Section II reviews the economics associated with the provision of underpriced safety net guarantees to banking institutions. Section III discusses buffer stock capital allocation techniques that are based on VaR model estimates. Section IV considers the internal model buffer stock capital estimates that would be produced by a bank that enjoys safety net related financing cost advantages. Section V calculates, in both the market and credit risk settings, the *ex ante* value of the safety net related profits that would be earned by bank shareholders who are allowed to operate under an internal models approach to regulatory capital requirements. Section VI considers the potential distortions in bank lending activities that are engendered by internal models-based capital regulations. Section VII concludes the paper.

## II. THE VALUE OF GOVERNMENT SAFETY NETS FOR BANK SHAREHOLDERS

### A. Background

This section adopts a Merton (1977) modeling framework to establish the value of an implicit or explicit fixed rate deposit insurance guarantee to the shareholders of a participating bank. We assume the existence of a government agency that implicitly or explicitly insures the value of banks' liabilities, and consider the present value of the claims of three bank stakeholders: equity, insured debt, and the deposit insurance authority. For simplicity, we assume that insurance is provided at a fixed *ex ante* rate normalized to 0 and so the insurance is costless to the bank.<sup>7</sup> Following Merton, the analysis does not consider information asymmetries that may arise in the context of the valuation of bank shares, and assumes that the value of bank assets are transparent to equity market investors.

For purposes of the internal models analysis that follows, it is useful to consider the market value of stakeholder claims under alternative bank investment opportunity sets. The first bank investment opportunity set considered allows a bank to invest only in equity-type instruments and is identical to the model analyzed in Merton (1977). Another type of investment opportunity set considered allows banks to invest only in short-term risky discount bonds where the phrase "short-term" means "of the same term to maturity as bank liabilities." The final type of investment opportunity set considered allows banks to invest only in long-term risky discount bonds, where long-term means a term to maturity that exceeds the maturity of bank's insured liabilities.

### B. Asset Value Dynamics

Before considering specific expressions for the market values of stakeholder positions in these alternative settings, it is appropriate to consider the characteristics of the asset price dynamics that will underlie all stakeholder valuations. In the BSM model, the firm's underlying assets evolve in value according to geometric Brownian motion and have future values that exhibit so-called "market risk" in the vernacular of risk managers. In this setting, the process of selecting the bank's debt-equity funding mix under an objective of maximizing the use of debt finance, subject to a maximum default rate on the bank's funding debt, is a market risk capital allocation problem. In the market risk setting, the VaR calculation is applied to the physical probability distribution for the firm's asset value, at a horizon equal to the desired maturity of the firm's funding debt.

Under the assumptions of the BSM model, the value of the firm's assets evolve following,

$$dA = \mu A dt + \sigma A dz \quad (1)$$

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<sup>7</sup> As a point of comparison, it should be noted that the U.S. deposit insurance premium rate is currently 0 for well-capitalized banks.

where  $dz$  is a standard Weiner process. If  $A_0$  represents the initial value of the firm's assets, and  $A_T$  the value of the firm's assets at time  $T$ , Ito's lemma implies,

$$\ln A_T - \ln A_0 \sim \phi \left[ \left( \mu - \frac{\sigma^2}{2} \right) T, \sigma \sqrt{T} \right], \quad (2)$$

where  $\phi[a,b]$  represents the normal density function with a mean of "a" and a standard deviation of "b". Equation (2) defines the physical probability distribution for the end-of-period value of the firm's assets,

$$\tilde{A}_T \sim A_0 e^{\left( \mu - \frac{\sigma^2}{2} \right) T + \sigma \sqrt{T} \tilde{\varepsilon}} \quad (3)$$

where  $\tilde{\varepsilon} \sim \phi[0,1]$ .

When the underlying assets or claims on these assets are traded, equilibrium absence of arbitrage conditions impose restrictions on the underlying asset's Brownian motion's drift term,  $\mu = r_f + \lambda\sigma$ , where  $\lambda$  is the market price of risk associated with the firm's assets. It will be useful subsequently to use this equilibrium relationship. Define  $dA^\eta = (\mu - \lambda\sigma)A^\eta dt + A^\eta \sigma dz$ .  $dA^\eta$  is the "risk neutralized" geometric Brownian motion process that is used to value derivative claims after an equivalent martingale change of measure. The probability distribution of the underlying end-of-period asset values after the equivalent martingale change of measure,  $\tilde{A}_M^\eta$ , is,

$$\tilde{A}_T^\eta \sim A_0 e^{\left( r_f - \frac{\sigma^2}{2} \right) T + \sigma \sqrt{T} \tilde{\varepsilon}} \quad (4)$$

### C. Deposit Insurance Value Under an Equity Investment Opportunity Set

In the absence of deposit insurance, if there are no taxes, transactions are costless, short sales are possible, trading takes place continuously, if investors in asset markets act as perfect competitors, if the risk-free term structure is flat, and a firm issues only pure discount debt, and the firm's assets evolve in value following geometric Brownian motion, Black and Scholes (1973) and Merton (1974) demonstrated that: (1) the value of a firm's equity is equivalent to the value of a European (Black-Scholes) call option written on the firm's underlying assets; the call option has a maturity equal to the maturity of the firm's debt and a strike price equal to the par value of the firm's debt; and (2) the market value the firm's debt issue is equal to the market value the issue would have if it were default risk free, less the market value of a Black-Scholes put option written on the value of the firm's assets; the put option has a maturity equal to the maturity of the debt issue and strike price equal to the par value of the discount debt.

If  $B_0$  represents the discount bond's initial equilibrium market value, and  $Par$  represents its promised payment at maturity date  $M$ , the BSM model requires,

$$B_0 = Par e^{-r_f M} - Put(A_0, Par, M, \sigma), \quad (5)$$

where  $r_f$  represents the risk free rate and  $Put(A_0, Par, M, \sigma)$  represents the value of a Black-Scholes (put) option on an asset with an initial value of  $A_0$ , a strike price of  $Par$ , a maturity of  $M$ , and an instantaneous return volatility of  $\sigma$ . The default (put) option is a measure of the credit risk of the bond. The larger the bond's credit risk, the greater is the discount in its market value relative to a default riskless discount bond with identical par value and maturity.

Now assume that the bank can issue discount debt claims that are insured by the government. If the bank's debt is insured, its initial equilibrium market value is  $Par e^{-r_f M}$  as investors require only the risk free rate of return on the debt issue. If the deposit insurer does not charge for insurance, the initial market value of the bank's equity is given by  $Call(A_0, Par, M, \sigma) + Put(A_0, Par, M, \sigma)$ , where  $Call(A_0, Par, M, \sigma)$  represents the value of a Black-Scholes call option on an asset with an initial value of  $A_0$ , a strike price of  $Par$ , a maturity of  $M$ , and an instantaneous return volatility of  $\sigma$ .

The provision of costless deposit insurance provides the bank's shareholders with an interest subsidy on the bank's debt. This interest subsidy has an initial market value equal to  $Put(A_0, Par, M, \sigma)$ .<sup>8</sup> Absent any regulatory constraints, it is well known that the bank shareholders maximize the *ex ante* value of their wealth by maximizing the present market value of the interest subsidy on their debt, or equivalently by maximizing the value of  $Put(A_0, Par, M, \sigma)$ . By selecting the bank's investment assets and capital structure, the bank's shareholders maximize the value of their insurance guarantee by maximizing the credit risk of the insured debt claims issued by the bank.

#### D. Deposit Insurance Value Under Alternative Bank Investment Opportunity Sets

The *ex ante* value of a deposit insurance guarantee was derived by Merton (1977) in the context of a bank that purchased assets that evolve in value according to geometric Brownian motion. Assets such as these have return characteristics similar to equity type investments. As a consequence, the Merton (1977) results do not characterize the deposit insurance value enjoyed by the shareholders of a bank that invests in risky fixed income investments where the payoffs in favorable return states are limited by the terms of its loan contracts.

Using the intuition of the Merton results, it is straight-forward to derive deposit insurance values under alternative bank investment opportunity sets, provided it is possible to establish the equilibrium value of the bank's debt in the absence of any deposit insurance guarantee. In the

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<sup>8</sup> If the insurer were to charge an *ex ante* fee for insurance coverage, the market value of the insurance subsidy would be given by  $Put(A_0, Par, M, \sigma)$  less the *ex ante* fee.

absence of an insurance premium, the deposit insurance value is equal to the value of the default option on the bank's insured debt. This default option value is, in turn, equal to the difference between the market value the debt would have if it were a risk free claim, and its equilibrium market value inclusive of credit (default) risk. The key to estimating the insurance value is the ability to establish the initial equilibrium market value that the bank's insured debt claims would command in the absence of the insurance guarantee. In the following two sub-sections, this approach is used to derive the insurance value for alternative investment opportunity sets in which the bank can purchase only Black-Scholes-Merton risky discount bond investments.

### E. Maturity-Matched Risky Discount Bonds

Assume that the bank can only invest in BSM risky discount bonds and that it funds these investments with equity and its own discount debt issue. Moreover, assume that the bank's investment opportunity set is restricted to discount bonds that are matched in maturity to the discount debt that the bank issues. The initial market value of the bank's bond investment is given by,

$$B_0 = Par_p e^{-r_f M} - Put( A_0, Par_p, M, \sigma ) \quad (6)$$

where  $Par_p$  represents the par value of the purchased discount bond.

Define  $Par_F$  to be the par value of the discount bond that the bank issues to fund the bond purchase. In the absence of an insurance guarantee, if the maturity of the bank's funding debt matches the maturity of the firm's asset (both equal to  $M$ ), then the end-of-period cash flows that accrue to debt holders are given by,

$$Min[Min(\tilde{A}_M, Par_p), Par_F] \quad (7)$$

Because  $\tilde{A}_M$  is the only source of uncertainty determining bank bond holder payoffs, the initial market value of the funding debt is given by discounting (at the risk free rate) the expected value of (7) taken with respect to the equivalent martingale probability distribution of the end-of-period asset's value,  $\tilde{A}_M^\eta$ ,

$$E^\eta [Min[Min(\tilde{A}_M, Par_p), Par_F]] e^{-r_f M}, \quad (8)$$

where  $E^\eta[\cdot]$  represents the expectations operator with respect to the probability density of  $\tilde{A}_M^\eta$ .

Applying the intuition of Merton (1977), if the bank's funding debt is costlessly guaranteed by the government, the value of the insurance guarantee that accrues to bank shareholder' is given by,

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<sup>9</sup> Alternatively, Geske (1979) provides a closed form expression for the value of the compound option.

$$Par_F e^{-r_f M} - E^\eta \left[ \text{Min} \left[ \text{Min} \left( \tilde{A}_M, Par_p \right), Par_F \right] \right] e^{-r_f M} \quad (9)$$

### F. Long-Term Risky Discount Bonds

Assume that the bank's investment opportunity set is composed of BSM risky discount bonds that have maturities that exceed the maturity of the bank's funding discount liabilities. When the bank's funding debt is of a shorter maturity ( $T$ ), than the discount bond purchased by the bank (maturity  $M$ ), then the end-of-period cash flows that accrue to the bank's debt holders are given by,

$$\text{Min} \left[ \left( Par_p e^{-r_f(M-T)} - \text{Put} \left( \tilde{A}_T, Par_p, M - T, \sigma \right) \right), Par_F \right]. \quad (10)$$

In the absence of any insurance guarantee, the initial equilibrium value of the funding debt can be calculated as the discounted value (at the risk free rate) of the expected value of expression (10) taken with respect to the equivalent martingale probability density  $\tilde{A}_T^\eta$ ,

$$E^\eta \left[ \text{Min} \left[ \left( Par_p e^{-r_f(M-T)} - \text{Put} \left( \tilde{A}_T, Par_p, M - T, \sigma \right) \right), Par_F \right] \right] e^{-r_f T}. \quad (11)$$

Using the intuition of Merton's original results, in the absence of an insurance premium, the *ex ante* value of the deposit insurance guarantee under this bank investment opportunity set restriction is given by,

$$Par_F e^{-r_f T} - E^\eta \left[ \text{Min} \left[ \left( Par_p e^{-r_f(M-T)} - \text{Put} \left( \tilde{A}_T, Par_p, M - T, \sigma \right) \right), Par_F \right] \right] e^{-r_f T}. \quad (12)$$

### III. BUFFER STOCK CAPITAL ALLOCATION USING VALUE-AT-RISK

It is useful to review the conventional intuition that underlies the use of VaR approaches for setting buffer stock capital allocations. In the case of an equity or traditional loan or bond investment, a buffer stock capital allocation is the equity portion of a funding mix that can be used to finance an asset (portfolio) in a way that maximizes the use of debt finance subject to a maximum acceptable probability of default on the funding debt.<sup>10</sup>

VaR is commonly defined to be the loss amount that could be exceeded by at most a maximum percentage of all potential future value realizations at the end of a given time

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<sup>10</sup> We make no claim that this objective function formally defines a firm's optimal capital structure—indeed it almost certainly does not. It is, however, the objective function that is consistent with VaR-based capital allocation schemes and an approach commonly taken by banks according to the Basel Committee on Banking Supervision's (1999) survey results.

horizon.<sup>11</sup> By this definition, VaR is determined by a specific left-hand critical value of a potential profit and loss distribution, and by convention, losses are reported as positive values. The other determinant of VaR is the right boundary against which the loss is measured. In capital allocation applications, the right hand boundary of the VaR measure is critically important and yet its importance is not widely recognized in the literature.

In both the market and credit risk setting, it is common to find the right hand boundary of the VaR measure set equal to the expected value of the end-of-period value distribution. While this approach is common, and the VaR measure is said to produce estimates of so-called “unexpected losses,” it will not produce accurate estimates of buffer stock capital requirements. In a buffer stock capital application, in both the market and the credit risk setting, it is important that VaR be measured relative to the initial market value of the asset or portfolio that is being funded. It should be noted that this VaR measure is not the measure that is specified in the Basel Internal Models Approach for Market Risk, or the VaR measure typically used in credit VaR capital allocation measures. Regardless, the accuracy of VaR-based buffer stock capital is compromised if VaR is measured from a different right hand boundary the end-of-period value distribution.<sup>12</sup>

Consider the use of a 1 percent, one-year VaR measure to determine the necessary amount of equity funding for an investment under a buffer stock approach for capital. If VaR is measured relative to the asset or portfolio’s initial value, by definition, there is less than a 1 percent probability that the asset’s value will ever post a loss that exceeds its 1 percent VaR risk exposure measure. That is, if the firm chooses an amount of equity finance equal to its 1 percent VaR, the implication is that there is less than a 1 percent chance that any loss in its underlying assets’ values will ever exceed the value of the firm’s equity. A common but flawed interpretation is that this equity financing share will ensure that there is at most a 1 percent chance that the firm will default on its debt.

Assume that VaR will be measured from the asset’s initial market value and that VaR measures are completely accurate in the sense that there is no statistical error in measuring the asset’s end-of-period market value distribution. In the case of discount debt or an equity asset, VaR can never exceed  $V_0$ , the initial market value of the investment. If the firm were to set the share of equity funding equal to the asset’s 1 percent VaR measure,  $VaR(.01)$ , the amount of debt finance required to fund the asset would be  $V_0 - VaR(.01)$ . The flaw in the aforementioned VaR capital allocation logic is that if the firm borrows  $V_0 - VaR(.01)$ , *it must pay back more than  $V_0 - VaR(.01)$  if it is to avoid default*. The simple intuition that underlies the VaR approach for

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<sup>11</sup> This definition can be found *inter alia* in Duffie and Pan (1997), Hull and White (1998), Jorion (1996 and 1997), Beder (1995), and Marshall and Siegel (1997).

<sup>12</sup> See Kupiec (1999, 2001, or 2002a), for further discussion.

capital allocation ignores the interest payment that must be made on funding debt. An unbiased buffer stock capital allocation rule is to set equity capital equal to 1 percent VaR (calculated appropriately) *plus the interest that accrues on the funding debt over the VaR horizon*. Using a correct VaR measure—one in which the VaR’s right-side boundary is set by the asset’s initial market value—and augmenting the VaR estimate by the interest payments that will be required by investors who purchase the funding debt, the VaR methodology can, in theory, provide perfectly accurate measures of buffer stock capital for bond or equity type investments. This is true in both the market risk and the credit risk setting. The required VaR calculation, while modified compared to many discussions of VaR measures, does not present any new technical issues. The complication is introduced by the necessity of obtaining estimates of the required interest payments on funding debt—a calculation that requires the use of an asset pricing model.

#### IV. USING VaR TO CALCULATE CAPITAL IN THE ABSENCE OF BANK SAFETY-NET PROTECTIONS

When calculating credit VaR for buffer stock capital purposes, the credit VaR horizon must be equal to the maturity of the funding debt issue. Any other credit VaR horizon will produce capital allocations with technical insolvency rates that differ from the intended target.<sup>13</sup> Technical insolvency occurs when the value of the promised maturity payment on the bank’s liabilities exceeds the equilibrium market value of the bank’s assets.

The credit VaR profit and loss distribution differs according to whether the horizon corresponds to the maturity of the credit risky asset or a shorter period of time. Similar to a market risk capital calculation when the maturity of the assets is undefined, in the case of credit risk capital calculations when the bank’s funding debt matures before its fixed income assets, the buffer stock capital is set to maintain technical solvency with a given target rate at the maturity date of the debt funding issue. The following sub-sections describe the buffer stock capital allocations that are required to limit the technical insolvency rate to  $\alpha$  in the absence of deposit insurance.

##### A. Market Risk Capital Allocation

Let  $\Phi(x)$  represent the cumulative density function for a standard normal random variable evaluated at  $x$ , and  $\Phi^{-1}(\alpha)$ , the inverse of this function evaluated at  $0 \leq \alpha \leq 1$ . In the context of the BSM model, the market risk VaR measure that is appropriate for calculating an equity capital allocation consistent with a target default rate of  $\alpha$  for a funding debt maturity of  $T$  is given by,

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<sup>13</sup> For further discussion, see Kupiec (2002a).

$$VaR(\alpha) = A_0 - A_0 e^{\left(\frac{\mu - \sigma^2}{2}\right)T + \sigma\sqrt{T} \Phi^{-1}(\alpha)} = A_0 \left[ 1 - e^{\left(\frac{\mu - \sigma^2}{2}\right)T + \sigma\sqrt{T} \Phi^{-1}(\alpha)} \right] \quad (13)$$

$A_0 - VaR(\alpha) = A_0 e^{\left(\frac{\mu - \sigma^2}{2}\right)T + \sigma\sqrt{T} \Phi^{-1}(\alpha)}$  is the maximum par value of discount debt that can be issued without violating the firm's target default rate. The BSM debt pricing condition (expression (6)) can then be used to determine the initial market value of this debt issue. The difference between the initial market value of the debt and its par value is the equilibrium interest compensation that must be offered to the firm's debt holders. In the BSM model setting, the interest payments are,

$$A_0 \left[ e^{\left(\frac{\mu - \sigma^2}{2}\right)T + \sigma\sqrt{T} \Phi^{-1}(\alpha)} - e^{\left(\frac{\mu - \sigma^2}{2}\right)T + \sigma\sqrt{T} \Phi^{-1}(\alpha) - r_f T} \right] + Put \left( A_0, A_0 e^{\left(\frac{\mu - \sigma^2}{2}\right)T + \sigma\sqrt{T} \Phi^{-1}(\alpha)}, T, \sigma \right). \quad (14)$$

This interest amount must be added to  $VaR(\alpha)$  to calculate the true equity capital allocation needed to achieve the target default rate on funding debt. The true amount of equity required to achieve a target default rate of  $\alpha$  on funding debt of maturity  $T$  is given by,

$$A_0 \left[ 1 - e^{\left(\frac{\mu - \sigma^2}{2}\right)T + \sigma\sqrt{T} \Phi^{-1}(\alpha) - r_f T} \right] + Put \left( A_0, A_0 e^{\left(\frac{\mu - \sigma^2}{2}\right)T + \sigma\sqrt{T} \Phi^{-1}(\alpha)}, T, \sigma \right). \quad (15)$$

## B. Credit Risk Capital

### Held-to-Maturity (HTM) Credit VaR Capital Allocation

At maturity, the payoff of the firm's purchased bond is given by,  $Min[Par_p, \tilde{A}_M]$ . The credit risk VaR measure appropriate for credit risk capital allocation is given by,

$$VaR_{Credit}(\alpha) = B_0 - Min \left[ Par_p, A_0 e^{\left(\frac{\mu - \sigma^2}{2}\right)M + \sigma\sqrt{M} \Phi^{-1}(\alpha)} \right] \quad (16)$$

where  $B_0$  is the initial market value of the purchased discount debt given by expression (6), and  $\alpha$  is the target default rate on the funding debt. If  $\alpha$  is sufficiently small (which will be

assumed), the expression  $Min \left[ Par_p, A_0 e^{\left(\frac{\mu - \sigma^2}{2}\right)M + \sigma\sqrt{M} \Phi^{-1}(\alpha)} \right]$  simplifies to  $A_0 e^{\left(\frac{\mu - \sigma^2}{2}\right)M + \sigma\sqrt{M} \Phi^{-1}(\alpha)}$ ,

and consequently, the expression for credit VaR is,

$$VaR_{Credit}(\alpha, M, M) = B_0 - A_0 e^{\left[ \mu - \frac{\sigma^2}{2} \right] M + \sigma \sqrt{M} \Phi^{-1}(\alpha)} \quad (17)$$

In order to maintain notational consistency with the mark-to-market capital allocation discussion that follows, the notation for credit VaR is defined to include three arguments: the target default rate  $\alpha$ , the maturity of the funding debt issue,  $M$  (the second argument), and the

maturity of the credit risky asset,  $M$ .  $B_0 - VaR_{Credit}(\alpha, M, M) = A_0 e^{\left[ \mu - \frac{\sigma^2}{2} \right] M + \sigma \sqrt{M} \Phi^{-1}(\alpha)}$  determines the maximum par value of the funding debt that is consistent with the target default rate. The initial market value of this funding debt issue is given by,

$$E^\eta \left[ \text{Min} \left[ \text{Min}(\tilde{A}_M, Par_p), A_0 e^{\left[ \mu - \frac{\sigma^2}{2} \right] M + \sigma \sqrt{M} \Phi^{-1}(\alpha)} \right] \right] e^{-r_f M} \quad (18)$$

These relationships define the equilibrium required interest payment on the funding debt,

$$A_0 e^{\left[ \mu - \frac{\sigma^2}{2} \right] M + \sigma \sqrt{M} \Phi^{-1}(\alpha)} - E^\eta \left[ \text{Min} \left[ \text{Min}(\tilde{A}_M, Par_p), A_0 e^{\left[ \mu - \frac{\sigma^2}{2} \right] M + \sigma \sqrt{M} \Phi^{-1}(\alpha)} \right] \right] e^{-r_f M} \quad (19)$$

Expressions (17) and (19) imply that the initial equity allocation consistent with the target default rate  $\alpha$  is given by,

$$B_0 - E^\eta \left[ \text{Min} \left[ \text{Min}(\tilde{A}_M, Par_p), A_0 e^{\left[ \mu - \frac{\sigma^2}{2} \right] M + \sigma \sqrt{M} \Phi^{-1}(\alpha)} \right] \right] e^{-r_f M} \quad (20)$$

### Mark-to-Market (MTM) Credit VaR

When the bank's funding debt matures at date  $T$  before the bank's risky discount bond's maturity,  $M$ ,  $T < M$ , the  $T$ -period,  $\alpha$  level credit VaR is given by,

$$VaR_{Credit}(\alpha, T, M) = B_0 - \left( Par_p e^{-r_f(M-T)} - \text{Put} \left( A_0 e^{\left[ \mu - \frac{\sigma^2}{2} \right] T + \sigma \sqrt{T} \Phi^{-1}(\alpha)}, Par_p, M - T, \sigma \right) \right) \quad (21)$$

$B_0 - VaR_{Credit}(\alpha, T, M)$  determines the maximum par value of the funding debt that satisfies the target default rate constraint; this value is given by,

$$Par_F(\alpha, T, M) = Par_p e^{-r_f(M-T)} - \text{Put} \left( A_0 e^{\left[ \mu - \frac{\sigma^2}{2} \right] T + \sigma \sqrt{T} \Phi^{-1}(\alpha)}, Par_p, M - T, \sigma \right) \quad (22)$$

The arguments in the notation for  $Par_F(\alpha, T, M)$  conform with those in  $Var_{Credit}(\alpha, T, M)$ .

Using the expression for  $Par_F(\alpha, T, M)$ , the initial market value of the funding debt issue is,

$$E^n \left[ \text{Min} \left[ \left( Par_p e^{-r_f(M-T)} - \text{Put}(\tilde{A}_T, Par_p, M-T, \sigma) \right), Par_F(\alpha, T, M) \right] \right] e^{-r_f T} \quad (23)$$

and the equilibrium required interest payment on the funding debt is,

$$Par_F(\alpha, T, M) - E^n \left[ \text{Min} \left[ \left( Par_p e^{-r_f(M-T)} - \text{Put}(\tilde{A}_T, Par_p, M-T, \sigma) \right), Par_F(\alpha, T, M) \right] \right] e^{-r_f T} \quad (24)$$

Expressions (21) and (24) imply that the equity allocation consistent with a target default rate of  $\alpha$  is given by,

$$B_0 - E^n \left[ \text{Min} \left[ \left( Par_p e^{-r_f(M-T)} - \text{Put}(\tilde{A}_T, Par_p, M-T, \sigma) \right), Par_F(\alpha, T, M) \right] \right] e^{-r_f T} \quad (25)$$

## V. SAFETY NET INSURANCE VALUE UNDER AN INTERNAL-MODELS APPROACH TO CAPITAL

If the bank's liabilities are insured by an implicit or explicit government guarantee, it may be reasonable for investors to view these claims as risk free, or at least nearly so. For purposes of this analysis, we assume that insured bank liabilities are priced by investors as if they are risk free claims even though there is positive probability that the bank will default on these claims and create losses that will be borne by the deposit insurer.

If investors view insured bank liabilities as if they are riskless, the initial market value of bank claims will increase relative to identical claims issued by a non-insured entity. That is, for any given par value of a discount liability offered to investors by a bank, the initial market value of this discount issue will be greater if the bank is insured. The reduction in interest expense enjoyed by the insured bank allows its shareholders to invest less equity (compared to a non-insured business) in order to establish a given target rate for technical insolvency. In other words, given two banks that are identical in all respects except that one is (costlessly) insured and the other is not, the insured bank's shareholder will be required to invest less in order to ensure that the bank's technical insolvency rate is  $\alpha$ .

The interest rate subsidy enjoyed by insured banks has an important implication for buffer stock capital allocations. Suppose a regulatory authority mandates that an insured bank has sufficient capital to ensure that, at the end of some specific horizon, the bank remains technically solvent in at least  $100 * (1 - \alpha)$  percent of all outcomes. If the insured bank takes into account the safety net related interest subsidy in its internal capital allocations, it can meet the regulatory mandated target solvency rate with less equity capital than would be required by an otherwise identical non-insured institution. The reduction in the buffer stock equity capital requirement is equal to the reduction in the interest cost on the bank's debt.

In the market risk setting, over a buffer stock capital allocation horizon of  $T$ , for a bank investing in an asset with an initial value of  $A_0$ , and price dynamics consistent with expression

(3), it is straight-forward to show that the insurance related reduction in the interest cost is equal to,

$$Put \left( A_0, A_0 e^{\left[ \mu - \frac{\sigma^2}{2} \right] T + \sigma \sqrt{T} \Phi^{-1}(\alpha)}, T, \sigma \right). \quad (26)$$

Interpreting the results in the context of the Merton model and the discussion in Section II, it is clear that the reduction in interest costs enjoyed by an insured bank are equal to the *ex ante* value of the insurance guarantee enjoyed by bank shareholders, when the par value of the

bank's discount debt is  $Par_F = A_0 e^{\left[ \mu - \frac{\sigma^2}{2} \right] T + \sigma \sqrt{T} \Phi^{-1}(\alpha)}$ . Alternatively, expression (26) is Merton's expression for the value of a costless insurance guarantee that will be enjoyed by the bank's shareholders under this internal models approach to market risk capital requirements.

In the case of credit risk and HTM buffer stock capital requirements, the interest subsidy on the bank's debt is given by,

$$A_0 e^{\left[ \mu - \frac{\sigma^2}{2} \right] M + \sigma \sqrt{M} \Phi^{-1}(\alpha) - r_f M} - E^\eta \left[ \text{Min} \left[ \text{Min}(\tilde{A}_M, Par_p), A_0 e^{\left[ \mu - \frac{\sigma^2}{2} \right] M + \sigma \sqrt{M} \Phi^{-1}(\alpha)} \right] \right] e^{-r_f M} \quad (27)$$

where  $M$  is the maturity of the fixed income asset, the funding debt, and the VaR horizon. Again, it is apparent that the interest subsidy is identical to the deposit insurance guarantee value (expression(9)) evaluated at the par value of funding debt set by the regulatory determined

minimum solvency rate,  $Par_F = A_0 e^{\left[ \mu - \frac{\sigma^2}{2} \right] M + \sigma \sqrt{M} \Phi^{-1}(\alpha)}$ .

In the MTM credit risk setting, where the bank's funding debt has a maturity of  $T$ , and the fixed income asset that is purchased by the bank has a par value of  $Par_p$  and a maturity of  $M$ ,  $M > T$ , the VaR horizon is  $T$ , and the interest subsidy is given by,

$$Par_F(\alpha, T, M) e^{-r_f T} - E^\eta \left[ \text{Min} \left[ \left( Par_p e^{-r_f(M-T)} - Put(\tilde{A}_T, Par_p, M-T, \sigma) \right), Par_F(\alpha, T, M) \right] \right] e^{-r_f T} \quad (28)$$

where  $Par_F(\alpha, T, M)$  is the par value of the bank's funding debt set by the buffer stock VaR calculations,

$$Par_F(\alpha, T, M) = Par_p e^{-r_f(M-T)} - Put \left( A_0 e^{\left[ \mu - \frac{\sigma^2}{2} \right] T + \sigma \sqrt{T} \Phi^{-1}(\alpha)}, Par_p, M-T, \sigma \right). \quad (29)$$

Expression (28) is identical to expression (12) evaluated at the par value for the bank's funding debt that satisfies the regulatory internal models solvency requirement, and as such, it represents

the *ex ante* value of the insurance guarantee to the bank's shareholders under an internal models approach for regulatory capital.

## **VI. BANK RISK-TAKING INCENTIVES UNDER AN INTERNAL-MODELS APPROACH TO CAPITAL**

Expressions 26-28 represent the deposit insurance values that a bank generates under internal model approaches for setting regulatory capital requirements for market and credit risks. In each case, the insurance value that is generated depends on the risk characteristics of the investment that the bank is funding. The insurance subsidy is not uniform; its value can be altered by altering the risk characteristics of the equity or the discount bond in which the bank invests. Consequently, an internal models approach to setting regulatory capital will stimulate banks' demand for investments that offer the most attractive insurance subsidy benefits.

It is assumed that bank shareholders attempt to maximize their wealth. If equity markets are competitive and there are no asymmetric information costs associated with new equity issuance, existing bank shareholders will be able to capture the safety net subsidies associated with all new investments, and the existing shareholders will maximize their wealth by raising new equity capital and investing in all fair-valued investments that generate a positive safety net funding subsidy.

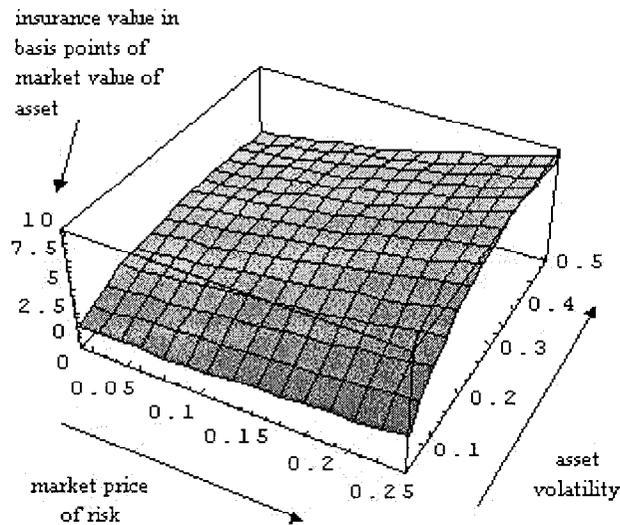
If, however, there are costs associated with issuing new equity shares, the existing shareholders will not capture the full value of the safety net funding subsidy and shareholders may not find it optimal to exploit all fair-valued investments with positive safety net subsidies. Instead, when raising outside equity capital is costly, the shareholder maximization problem must recognize the tradeoffs between the transactions and asymmetric information costs required to raise outside equity and the corresponding benefits that can be attained from exploiting available safety net guarantees. In the extreme case in which outside equity issuance costs are prohibitive, existing shareholders will allocate their equity capital across investments in order to maximize the value of the insurance subsidy per dollar of equity invested.

Subsequent sections consider the attractiveness of alternative bank investments under internal model regulatory capital requirements. The analysis considers the profitability of investment alternatives when equity can be issued costlessly at equilibrium market prices as well as the relative attractiveness of alternative investments when banks are capital constrained.

### A. Market Risk

The safety net subsidy under a market risk internal model regulatory capital requirement is given by expression (26). The value of the subsidy depends *inter alia* on the volatility of the equity investment and the market price of risk, two asset characteristics that can be selected by the bank.<sup>14</sup> Figure 1 plots the insurance value surface under a 1-year, 1 percent internal models market risk capital requirement for alternative choices of volatility and the market price of risk under the maintained assumptions that the asset has an initial value of 100, and the risk free rate of interest is 5 percent. Figure 1 shows that the value of the insurance subsidy is an increasing function of the market price of risk, and a concave function of asset volatility. Conditional on the market price of risk, there is an interior value of asset volatility that maximizes the value of the safety net subsidy generated by the investment. The relationship between the market price of risk and asset volatility that maximizes the insurance value (conditional on a risk free rate of 5 percent) is illustrated in Table 1.

**Figure 1. Deposit Insurance Value Under an Internal Model Market Risk Capital Requirement**



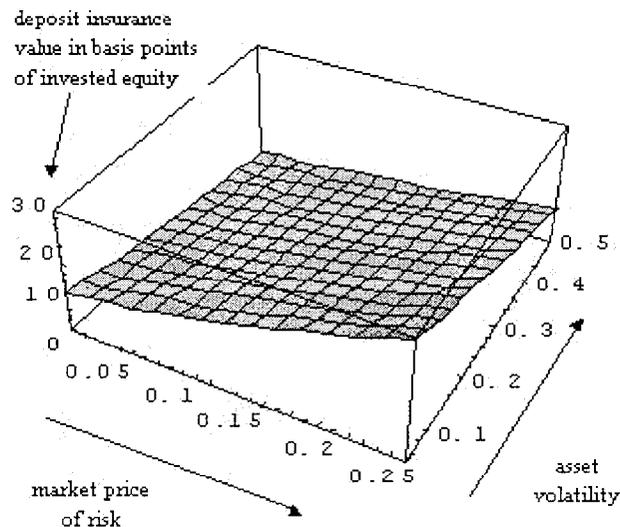
<sup>14</sup> In a more general model in which multiple risk factors are priced in equilibrium, the bank can also select which risk factor or factors it wishes to be exposed to.

Table 1. Asset Volatility Levels that Maximize the Insurance Value for Alternative Market Prices of Risk

Market Price of Risk	Volatility that Maximizes Subsidy	Maximum Insurance Value
0.0	0.339	0.045
0.5	0.334	0.052
0.1	0.349	0.061
0.15	0.354	0.072
0.2	0.359	0.084
0.25	0.365	0.098

Figure 2 plots the insurance value surface measured in basis points per dollar of invested shareholder equity for alternative choices of volatility and the market price of volatility risk under the maintained assumptions of Figure 1 (an initial value of 100 and risk free rate of 5 percent). Figure 2 shows that the value of the insurance subsidy per dollar of equity invested under a one-year, 1 percent market risk VaR capital requirement is an increasing function of the market price of risk and a decreasing function of asset volatility. The most profitable market risk bank investments—measured on a per dollar of required equity capital basis—are assets with low volatility and a high risk premium. Under a market risk internal models capital requirement, these assets have low capital requirements, and consequently a bank can earn the asset’s associated risk premium on a large portion of the asset’s value.

**Figure 2. Deposit Insurance Value per Dollar Invested Equity Under an Internal Model Approach to Market Risk Capital**



## B. Credit Risk

### Held-to-Maturity (HTM) Internal Models Capital Requirement

Consider initially the insurance value generated under a credit risk internal models approach to regulatory capital, when the maturity of the assets are identical to the maturity of the insured bank liabilities and both have a maturity of one-year. While this special case is not consistent with the short-term (on demand) nature of many bank liabilities, the choice of a one-year maturity horizon is consistent with modern banking practices regarding credit risk capital allocation. According to a recent survey conducted by the Basel Committee on Banking Supervision, many banks use a one-year horizon when calculating their internal economic capital allocations for credit risk using credit VaR models.<sup>15</sup>

In the HTM case, the value of the safety net subsidy generated under an internal models approach to credit risk capital is given by expression (27). The value of the insurance subsidy depends on specific characteristics of the purchased bonds, including the bond's par value, the market value of the bond's supporting assets, their return volatility, and the market price of risk. The par value, initial asset value, and asset return volatility also determines the bond's credit risk as measured by the discount bond's default option value. Figure 3 plots the default option value surface of the underlying risky discount debt for alternative values for the bond's par value and the supporting assets' volatility, when the supporting assets have an initial market value of 100 and the risk free rate is 5 percent. Figure 3 shows that the bond's credit risk is an increasing function of the bond's par value and the underlying assets' return volatility.

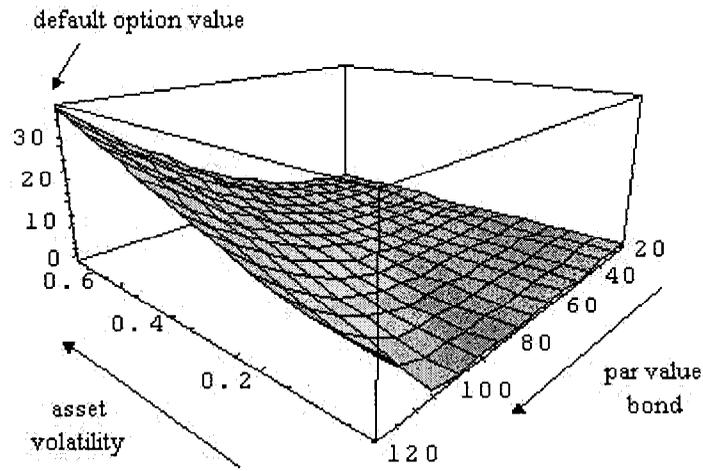
The insurance value surface generated under a one-year, 1 percent internal models capital requirement for the one-year discount bonds is plotted in Figure 4 under the assumption that the market price of risk is 10 percent. A comparison of Figures 3 and 4 shows that the peak of the insurance value surface corresponds to a set of discount bonds that have only modest credit risk. Under a 1 percent internal model capital constraint, these bonds allow the bank to use considerable funding leverage.

Figure 5 illustrates the tradeoff between insurance value, asset volatility, and the market price of risk for a bond with a par value of 85, supported by assets with an initial market value of 100. Figure 5 shows, that while the market price of risk has an influence on the insurance value in the credit risk setting, the effect is second order relative to the importance of asset volatility given the par value of debt. Similar tradeoffs are implicit for alternative par value choices.

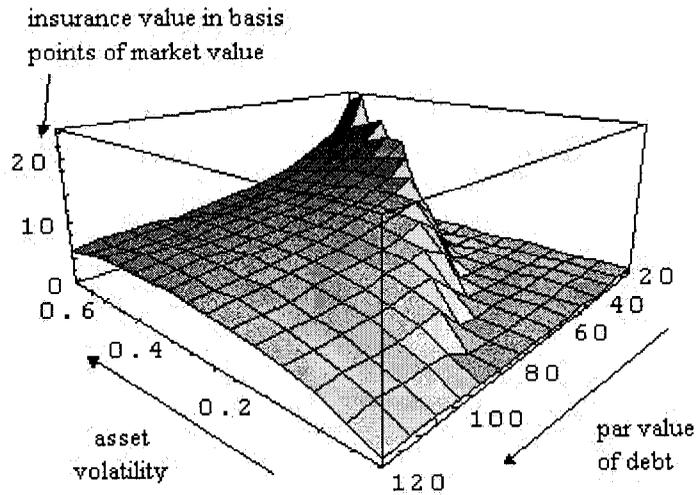
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<sup>15</sup> See Basel Committee on Banking Supervision (1999) for further discussion.

**Figure 3. Bond Characteristics and Credit Risk**



**Figure 4. Insurance Value Under an HTM Internal Model Capital Requirement**



**Figure 5. Insurance Value and the Market Price of Risk Under an HTM Internal Model Capital Requirement**

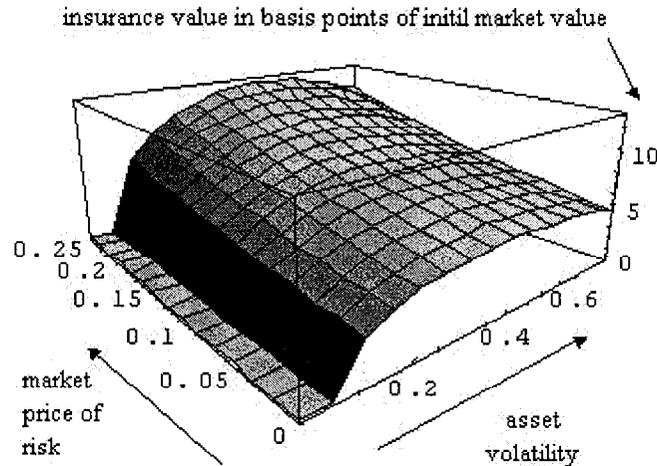


Figure 6 revisits the insurance value surface pictured in Figure 4, and plots the surface when insurance value is measured in basis points per dollar of required equity capital—the insurance value measure that is relevant for the shareholders of a capital constrained bank. The face of the cliff in Figure 6 corresponds with the peak of the mountain ridge in Figure 4. The high plateau at the top of the cliff in Figure 6 corresponds to the bonds in Figure 4 that populate the minimal credit risk “lowlands” in the northeast quadrant of Figure 4. Under the 1 percent internal models capital requirement, the bonds in this region—bonds with minimal credit risk—can be fully financed with insured deposits. Since bank shareholders make no investment but accrue fully the credit risk premium paid by these bonds (however small), the bonds on the plateau above the cliff face in Figure 6 represent a pure arbitrage from the perspective of a bank’s shareholders.

### **Mark-to-Market (MTM) Internal Models Capital Requirement**

In the MTM examples that follow, a long maturity bond is funded with one-year bank discount debt and equity, under the assumptions that the use of debt finance is maximized subject to a 1 percent maximum default rate on the bank’s funding debt. The value of the safety net subsidy generated under an internal models approach to credit risk capital, given by expression (28), depends on the purchased bond’s par value, its maturity, the market value of the bond’s supporting assets as well as their return volatility, and the market price of risk. The par value, maturity, initial asset value, and asset return volatility also determine the bond’s credit risk as measured by the discount bond’s default option value.

**Figure 6. Insurance Value (in basis points) of Required Equity Capital**

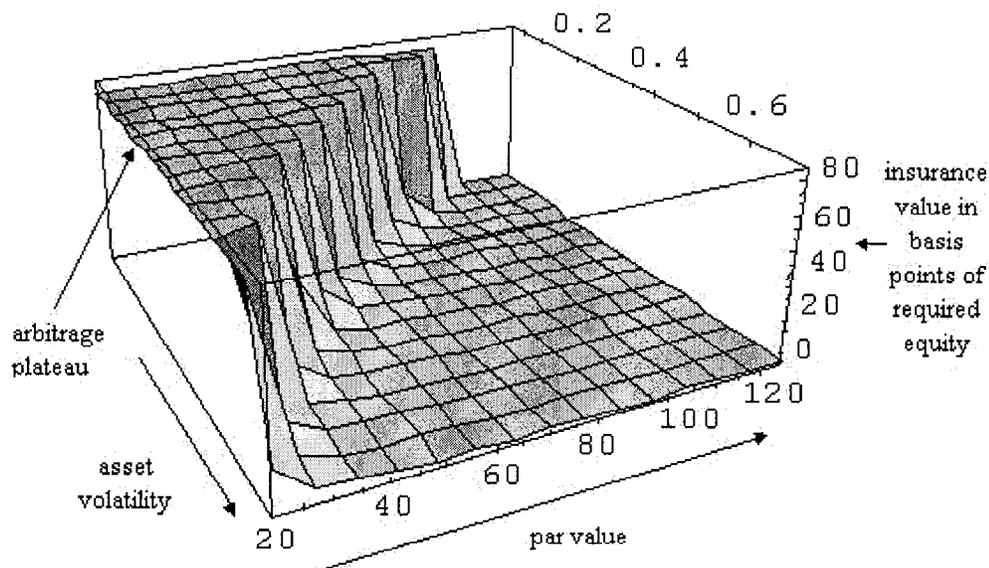
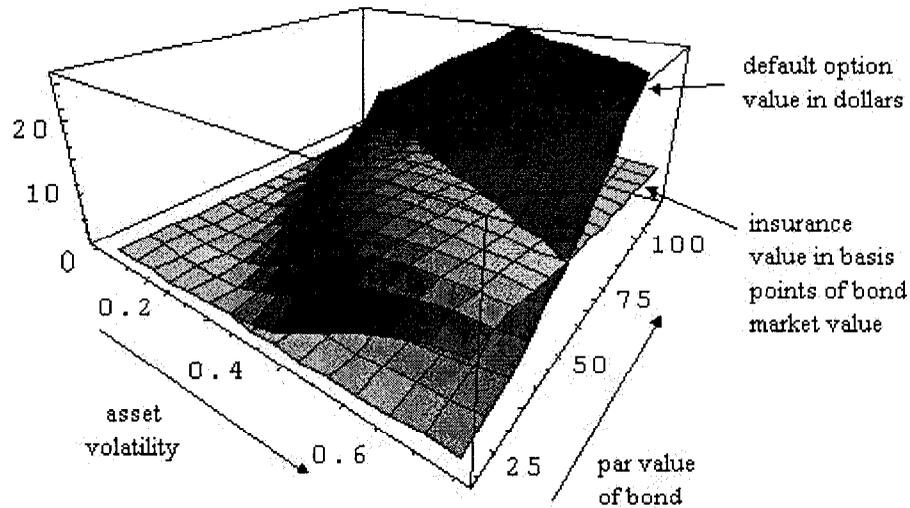


Figure 7 plots, for a two-year bond, the default option value surface of the underlying risky discount debt, and the insurance value measured in basis points of the two-year bond's market value for alternative values for the bond's par value, and the supporting assets' volatility under the maintained assumptions that the supporting assets have an initial market value of 100, and the risk free rate is 5 percent. Figure 7 shows that, while credit risk is an increasing function of the bond's par value and the underlying assets' return volatility, the insurance value per dollar of asset value generated by the safety net funding subsidy is not maximized by selecting the bonds with the greatest credit risk. The bonds in the southern-most quadrant of Figure 7 have the largest insurance values per dollar of bond market value under the internal models capital requirement, but have only modest credit risk.

Figure 8 plots the insurance value surface pictured in Figure 7 when the insurance value is measured in terms of basis points per dollar of required equity capital under the 1 percent, one-year internal models capital rule. Figure 8 suggests that the shareholders of a capital constrained bank would find it most profitable to invest in bonds with very little credit risk. The arbitrage plateau in Figure 8 corresponds with the credit-risk "lowlands" in the far western triangle area of Figure 7. Under a 1 percent, one-year internal models capital requirement, this set of bonds can be fully financed with insured deposits. While these bonds offer tiny credit risk spreads, the risk premia accrue entirely to the bank's shareholders who, under this internal models capital regulation, are required to invest nothing to receive them.

**Figure 7. Default Option Value in Dollars and Insurance Value (in basis points) of Bond Market Value Under a 1-Year MTM Capital Requirement on a 2-Year Bond**



**Figure 8. Insurance Value (in basis points) of Required Equity Under a 1-Year MTM Capital Requirement on a 2-Year Bond**

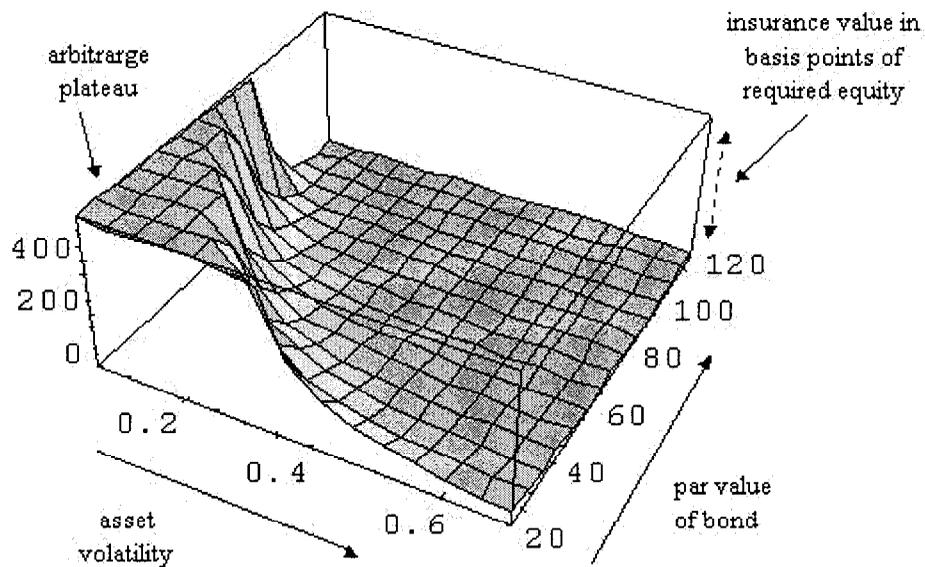


Figure 9 plots the default option value surface of the underlying risky discount debt, and the insurance value under a one-year MTM credit VaR capital requirement for a five-year bond under alternative values for the bond's par value and its supporting assets' volatility, under the maintained assumption that the supporting assets have an initial market value of 100, the risk free rate is 5 percent. The insurance value is measured in basis points of the bond's initial market value. Figure 9 is similar in appearance to Figure 7, only the five-year bond supports smaller insurance values. Credit risk is an increasing function of the bond's par value and the underlying assets' return volatility, but the insurance value per dollar of bond value is not maximized by selecting the bonds with the greatest credit risk. The five-year bonds in the southern-most triangle-shaped area of Figure 9, bonds with only modest credit risk, have the largest insurance values per dollar of bond market value under the one-year, 1 percent internal models capital requirement for credit risk.

**Figure 9. Default Option Value in Dollars and Insurance Value (in basis points) of Bond Market Value Under a 1-Year MTM Capital Requirement on a 5-Year Bond**

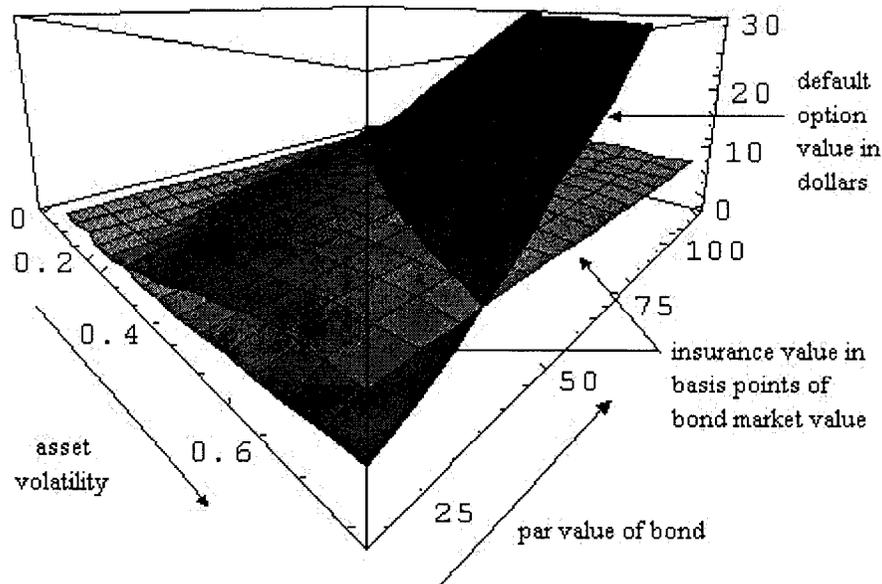
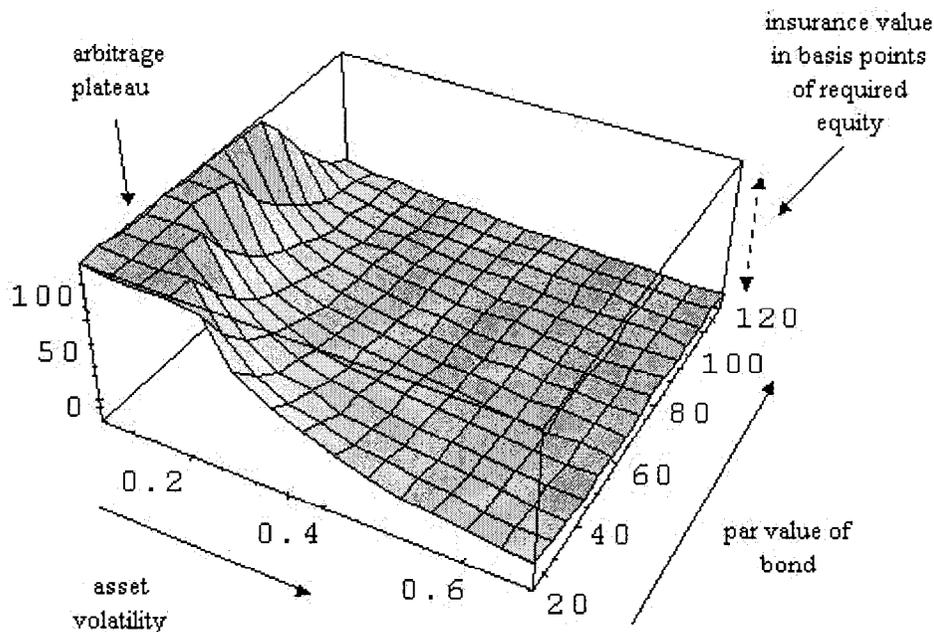


Figure 10 revisits the bonds analyzed in Figure 9 and plots insurance values measured in terms of basis points of the equity capital required under the internal models capital regulation. Figure 10 suggests, again, that the shareholders of a capital constrained bank will prefer to invest in five-year bonds that have very little credit risk. The credit risky bonds pictured in the western most quadrant of Figure 9 are the bonds that populate the arbitrage plateau in Figure 10, and represent a pure arbitrage to bank shareholders under this internal models approach to capital.

**Figure 10. Insurance Value (in basis points) of Required Equity Under a 1-Year MTM Capital Requirement on a 5-Year Bond**



## VII. CONCLUSIONS

Those advocating that banks use their internal capital allocation models to set regulatory capital requirements have failed to appreciate that the safety net-related funding-cost subsidies enjoyed by banks will alter their internal capital allocation decisions. If banks enjoy a funding cost subsidy, banks' internal models will produce downward-biased estimates of the "economic capital" required by a risky investment. These lower capital requirements can satisfy regulatory default rate constraints and yet still ensure that bank shareholders earn unpriced safety net engendered profits that are transfers which ultimately are a potential expense of the government insurer. The shareholder profits (transfers) generated under an internal models-based capital regulation are not uniform with respect to the risk profiles of alternative market and credit risk investments. As a consequence, bank investment decisions may be influenced by the relative

magnitudes of the funding-cost subsidies on alternative investment opportunities, and not by the true economic profits associated with these investment alternatives.

Supervisors have yet to fully embrace a bank internal models-based approach for credit risk regulatory capital requirements. The IRB approaches of the New Basel Accord proposal do, however, use a risk-weighting scheme that has been calibrated based upon the results of internal model simulations using stylized bank credit portfolios. While the IRB's lower bound on the admissible probability of default precludes a pure arbitrage opportunity, the results of Kupiec (2002b) are consistent with the internal models-based results reported herein and show that the IRB approaches produce the largest insurance value for low-risk bonds. Again, as would be predicted based upon the internal models analysis of this paper, Kupiec (2002b) documents that bonds with greater credit risks offer more modest safety-net-related profits under the proposed IRB calibrations.

The results of this analysis suggest that an internal models-based approach to bank capital regulation will not eliminate regulation-induced distortions in bank lending behavior. Instead, a pure internal models-based approach to regulatory capital promises to substitute a different set of economic distortions for those that exit under the current system of rule-based regulatory capital. This is true in part because bank internal model estimates are influenced by the externality that the capital regulations are attempting to control.

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