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A Model of the Bimetallic System

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Abstract

This paper formalizes Irving Fisher's century-old model of bimetallism and adds the important "disequilibrium" dynamics to deal with the long periods during which bimetallic countries were on effective monometallic standards. It resolves a long standing puzzle in the bimetallic literature regarding the remarkable stability of the gold/silver price ratio in the nineteenth century by modeling the bimetallic mint ratio as a regulating barrier to the gold/silver price ratio. It thus provides a clean-cut example of a target-zone model that--in contrast to other such models in the literature--exhibits the main predicted nonlinearities in the data.

JEL Classification Numbers:

E42, F33, N20

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### Summary

Irving Fisher first explicitly described the workings of the nineteenth century system of international bimetallism. He considered the dynamics of the system in "equilibrium" when both gold and silver coins circulate in the bimetallic money supply. In essence, a continuously adjusting relative circulation of gold and silver would ensure that the market ratio of gold and silver remained close to the bimetallic mint ratio. An increase in the relative market price of one of the metals would spark a transfer of that metal from the money supply to the bullion markets, bringing its price back down.

In practice, the bimetallic "buffer stock" was never large enough to stabilize effectively the market ratio. The desired concurrent circulation of gold and silver coins was rarely achieved. Bimetallic countries went through alternating periods of effective monometallism, so that bimetallic arbitrage could not stabilize the market ratio. However, the market ratio was remarkably stable, presenting the bimetallic literature with a long-standing puzzle. If not bimetallic arbitrage, what was it that stabilized the market ratio?

The answer lies in adding to Fisher's story "disequilibrium" dynamics that are relevant for the significant periods when bimetallic countries were on effective monometallic standards. This paper provides a model that formalizes Fisher's description of international bimetallism and adds the necessary forward-looking expectations. It recognizes that the alternating episodes of effective bimetallism and effective monometallism should be seen as regime switches between a fixed market ratio and a floating market ratio. In effect, therefore, the bimetallic mint ratio acted as a regulating barrier to the market ratio. Thus, the forward-looking expectations of the bullion market traders tended to have a stabilizing effect on the market ratio, even when Fisher's arbitrage mechanism was inoperative. This stabilizing effect is very similar to that caused by central bank intervention in an exchange rate target zone.

The empirical validity of the model is investigated in the paper by using standard tests from the literature designed to detect target zone effects. Data on the distribution and volatility of the market ratio and data on interest rate differentials between the gold standard and silver standard countries are largely consistent with the model. This result is in contrast to earlier studies on the empirical relevance of target zone effects in other exchange rate regimes, which have found no significant evidence of these effects.



## I. Introduction

In a bimetallic monetary system, the currency is defined in terms of two metals, usually gold and silver. The rationale behind this is to exploit the difference in value between the two metals to provide a set of full-bodied coins with a wide range of face values. 1/ Small-denomination coins are coined in silver, while large-denomination coins are coined in gold. Without the existence of token coins, monometallic systems would be inconvenient: under a gold standard, gold coins of a value appropriate for day-to-day transactions would be excessively small; should the country opt for a silver standard instead, large payments would require a great weight of coins.

It has long been recognized, however, that in practice the concurrent circulation of gold and silver coins was rarely achieved in bimetallic money supplies. This was a result of the law attributed to Sir Thomas Gresham, which tells us that since gold and silver also trade in bullion markets "good money drives out bad money:" the type of coin that was relatively more valuable in the bullion market--the good money--would not circulate at face value. 2/ It would either be melted down to disappear completely, or would command a premium over its face value in the other metal--the bad money. Fisher (1894) first formally modeled this mechanism, and I will refer to it as "Fisher's arbitrage mechanism."

Since its publication, Fisher's model has provided the main theoretical framework for the literature on bimetallism. It shows that under the right circumstances, the relative price of a unit of gold in units of silver in the bullion markets--the market ratio--is stabilized at the relative price in the monetary system--the mint ratio. This is a result of profit opportunities that arise when the two are not equal: when the market ratio rises above the mint ratio, the value of gold coins in terms of silver rises above their face value. It pays therefore to transfer gold coins from the monetary system to the bullion market. Similarly, when the market ratio is below the mint ratio it pays to sell silver coins in the bullion market. In both cases, the resulting increase in the supply of the expensive metal in the bullion market tends to return the market ratio to the mint ratio.

For over a century now, the literature on bimetallism has been struggling with a puzzle presented by the evidence on the nineteenth century bimetallic experience. On the one hand, as Chart 1 shows, the market ratio

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1/ Full-bodied coins are coins with an intrinsic metal value equal to their face value, whereas token coins are coins with a face value higher than their intrinsic value. Redish (1990, 1992) discusses the importance of token coinage for the choice between a bimetallic and monometallic system. She argues that only after the technology became available to manufacture token coins that were difficult to counterfeit did a monometallic gold standard become feasible.

2/ Kindleberger (1984) notes that Gresham's Law was actually discovered two centuries earlier by Nicolas Oresme in 1360.

was very stable around the mint ratio of France, the most important bimetallic country of the era. In the 50 years between 1823 and 1873, the market ratio remained in a 6 percent band around the French ratio of 15.5, despite the major fluctuations in the relative supplies of gold and silver evidenced by Chart 2. Fisher's model would suggest that a continuously adjusting relative circulation of gold and silver coins in France acted as an equilibrating mechanism for the bullion markets. Indeed, Friedman (1990a) uses the observed stability of the market ratio to argue that gold and silver coins must have circulated side by side in France during this time.

This conclusion is in conflict, however, with the direct quantitative and anecdotal evidence. It is generally accepted--Friedman calls this view "conventional"--that during most of this time, France alternated between different effective monometallic standards, with coins of only one metal in general circulation. Fisher's arbitrage mechanism was inoperative, since it comes into play only when both metals circulate side by side at face value. But if Fisher's mechanism cannot account for the observed stability of the market ratio, then what can?

The problem is that the literature has been arguing from within Fisher's framework, which tells only part of the story on bimetallism. It deals only with the dynamics of bimetallism "in equilibrium," when both gold and silver are in circulation in the bimetallic money supply. The model in this paper adds the important "disequilibrium" dynamics, relevant for the significant periods when bimetallic countries were on effective monometallic standards. It recognizes that the alternating episodes of effective bimetallism and effective monometallism should be seen as regime switches between a fixed market ratio and a floating market ratio. In effect, the bimetallic mint ratio acted as a regulating barrier to the market ratio, so that the forward-looking expectations of the bullion-market traders tended to have a stabilizing effect on the market ratio even when Fisher's arbitrage mechanism was inoperative.

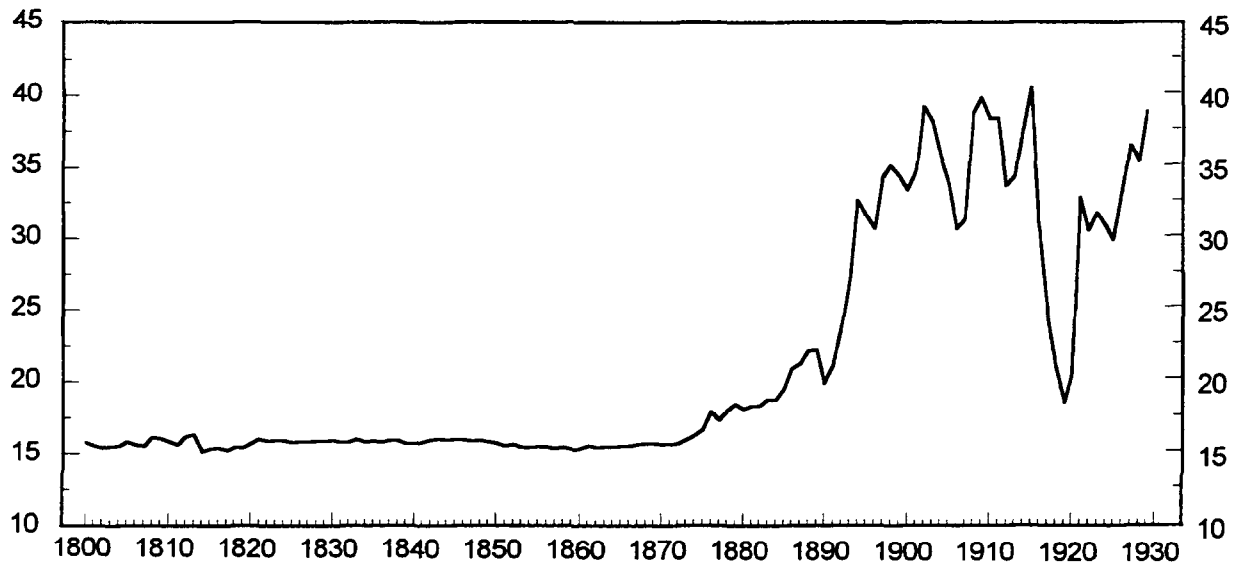
This stabilizing effect is very similar to that caused by central bank intervention in an exchange-rate target zone. There is one important difference, however. Traditional central-bank-enforced target zones usually suffer from credibility problems, resulting from the possibility that the central bank will fail to defend the boundaries of the target zone. Bimetallism modeled with the mint ratio as a regulating barrier to the market ratio does not suffer from these credibility problems. The regulating barrier is perfectly credible because its existence depends not on the whims of a central bank, but on individuals exploiting profit opportunities. <sup>1/</sup> Thus, seen in this light, the bimetallic system provides a very clean-cut example of a "target-zone" system.

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<sup>1/</sup> As I will show below, the risk of "realignment" is also absent, since the authorities have no incentive to change the mint ratio when Fisher's arbitrage mechanism is operational.

CHART 1

RATIO OF GOLD TO SILVER PRICE 1800-1930

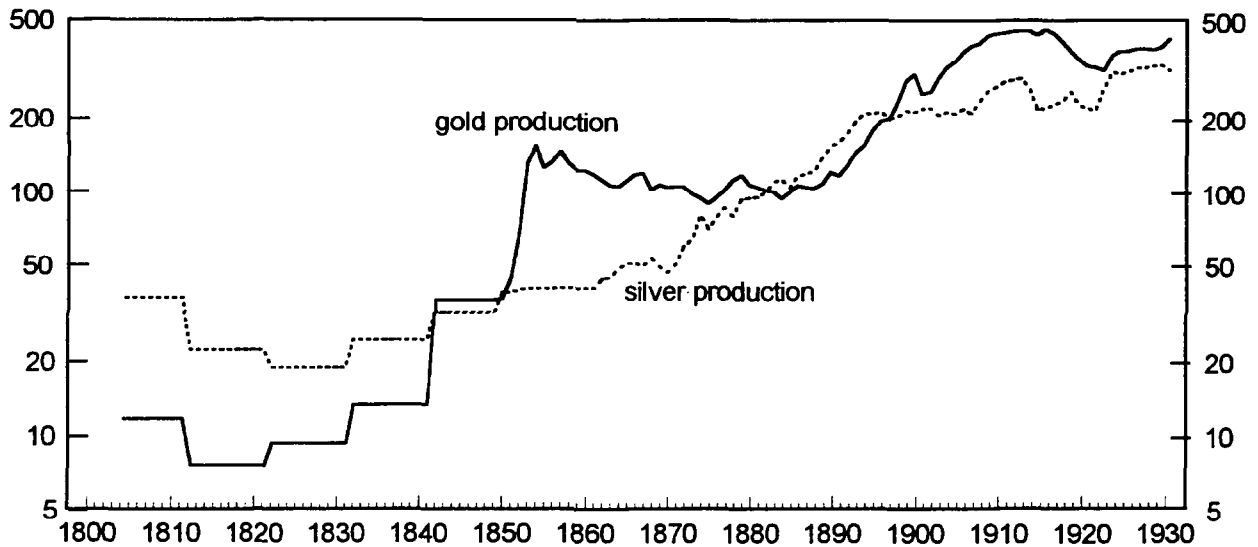


Sources: Laughlin (1968), Appendix II; U.S. Treasury Department (1944), p. 91.

CHART 2

WORLD GOLD AND SILVER PRODUCTION 1800-1930

U.S. coining value, in million dollars (log scale)



Sources: U.S. Congress (1894), p. 103; various Annual Reports of the Director of the Mint.



This is reflected in the empirical investigation in section 4 of the paper. Data on the distribution and volatility of the market ratio and data on interest-rate differentials between the gold-standard and silver-standard countries are largely consistent with the model. This is in contrast to earlier studies on the empirical relevance of target-zone effects in other exchange-rate regimes that have found no significant evidence of these effects.

## II. The Bimetallic System in the Nineteenth Century

To make the desired concurrent circulation of gold and silver coins possible, the monetary authorities of a bimetallic country stood ready to mint coins of a fixed weight and value in both metals. For example, between 1803 and 1873 the French franc was defined as 0.2903 grams of fine gold and 4.500 grams of fine silver. By exchanging French gold and silver coins at their face value 15.5 grams of silver could therefore be obtained for 1 gram of gold. It is important to note that the mint did not trade gold for silver itself, and did not attempt to fix the relative price of the metals directly. <sup>1/</sup> Trading coins already present in the money supply was therefore the only way to trade the metals at the fixed bimetallic ratio.

Even though the mint did not fix the market ratio directly, the system did promote stability of the market ratio in an indirect way. Fisher's arbitrage mechanism predicts that, if both gold and silver coins circulate side-by-side, a continuously adjusting relative circulation of gold and silver coins could stabilize the market ratio around the bimetallic mint ratio. An incipient rise in the market ratio would spark arbitrage transferring gold from the monetary system to the bullion market, whereas an

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<sup>1/</sup> This is the reason countries could maintain the widely different mint ratios depicted in Chart 4 without creating an opportunity for unlimited arbitrage. If the mints had taken it upon themselves to trade gold for silver at the fixed mint ratio, arbitrageurs could have in principle made unlimited profits. In the 1820s for example, anyone could then have bought gold at the U.S. mint and sold it at the Spanish mint for an immediate gross profit of 11 percent. Clearly, the U.S. mint would have run out of gold very quickly, while the Spanish mint would have run out of silver. This is why the mints were not in the business of fixing the market ratio.

incipient fall would trigger arbitrage removing silver from circulation. 1/ The bimetallic money supply acts as a buffer stock to the bullion market, where quantity adjustment has taken the place of price adjustment as the force that equates supply and demand. 2/

Like any scheme designed to fix a relative price, however, a bimetallic system cannot permanently fix the market ratio. Unless the bimetallic money supply is infinitely large, sustained upward pressure on the market ratio, for example, would eventually lead to the removal of all gold from monetary circulation. The "buffer stock" that was the supply of gold coins in the money supply has then been exhausted. Bimetallism has degenerated into effective monometallism, with only silver coins in circulation. When this happens, Fisher's mechanism is rendered temporarily inoperative and the market ratio is left to "float."

The bimetallic system in the nineteenth century centered around France, and its money supply has long been thought to have provided the bimetallic

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1/ Rolnick and Weber (1986) propose a much less universally valid version of Gresham's Law in which Fisher's mechanism might not operate: unless this would be prohibitively expensive--as in the case of small-denomination coins--undervalued coins would trade at a premium instead of being melted down. They offer anecdotal evidence that suggests the practice was widespread. Greenfield and Rockoff (1992) offer evidence to the contrary, however, and claim the traditional version of Gresham's Law was generally valid. In any case, I will assume here that if coins trade at a premium, they have in effect been transferred to the bullion market. Under this assumption, the act of bimetallic arbitrage takes place when coins that previously traded at their face value trade at a premium for the first time.

2/ Bimetallic arbitrage will not completely fix the market ratio. Because of costs connected to arbitrage, such as melting and minting costs, the ratio will fluctuate in a band around the bimetallic ratio. Arbitrage will not be profitable until the market ratio has moved a certain distance away from the bimetallic ratio. In analogy with gold points under a gold standard, I will call the points at which bimetallic arbitrage becomes profitable "gold-silver points." It is not immediately clear how far these gold-silver points were from the mint ratio. The main part of the costs of arbitrage was seigniorage charged by the mint, since bimetallic arbitrage could be conducted entirely in domestic bullion markets, eliminating the need to ship the metals over long distances. The only other major costs were melting costs and the cost of trading gold for silver in the bullion market, which were most likely small. The seigniorage charged by the French mint amounted to 1.0 percent for silver and 0.19 percent for gold, so that the upper gold-silver point was probably close to 15.7 and the lower gold-silver point close to 15.4. This refers to the gold-silver points with respect to the market ratio prevailing in France, however. If we compare the French mint ratio with the market ratio in London, the gold-silver points would most likely be farther away from the mint ratio, since arbitrage between the French monetary system and the London bullion market does of course involve the shipping of bullion across the Channel.

buffer to keep the market ratio around the French mint ratio of 15.5. According to Friedman (1990a, p. 89):

France [was] a major participant in the market for silver and gold, important enough to be able to peg the price ratio despite major changes in the relative production of silver and gold.

This ability of France to peg the market ratio has traditionally been attributed to Fisher's arbitrage mechanism: bimetallic arbitrage between the French monetary system and the bullion markets kept the market ratio fixed. There is a problem with this argument, however. It is widely recognized that during the period 1820-70, France and the other bimetallic countries were mostly on effective monometallic standards. Bimetallic arbitrage tended to quickly remove the dearer metal from ordinary circulation at face value, and then cease. In fact, from at least the late 1830s until 1850, and again during the 1860s, Fisher's arbitrage mechanism was all but inoperative. <sup>1/</sup>

A model of the bimetallic standard must therefore explain the remarkable stability of the market ratio throughout the nineteenth century without relying on arbitrage as the sole stabilizing mechanism. In the next section I will present a model that adds the dynamics of bimetallicism in "disequilibrium," when only the "bad" money remains in circulation. A key feature of this model is the insight from exchange-rate target-zone theory that the prospect of arbitrage at the bimetallic ratio affects the market ratio even when it is away from the mint ratio. This effect is similar to that of intervention barriers in a target zone, and provides stability to the market ratio when Fisher's mechanism cannot operate.

### III. A Dynamic Model of the Bimetallic System

To build a comprehensive model of an international monetary system that includes countries on bimetallicism, I consider the log-linear model of the exchange rate originally used by Flood and Garber (1983) and subsequently

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<sup>1/</sup> In Oppers (1993) I discuss the anecdotal evidence and show how we can use data on the exchange rate of a bimetallic currency to determine the relative circulation of gold and silver in the money supply. As an example, the paper shows that by the end of the 1850s silver coins could no longer be obtained at face value in France. Either they had disappeared through arbitrage, or they had started to trade at a premium over gold coins. In either case, further bimetallic arbitrage was impossible.

adopted by the target-zone literature. 1/ It gives a simple framework to analyze exchange-rate determination in a one-good, two-country world. Assume that there are three groups of countries: gold-standard countries, collectively indicated by GS; silver-standard countries, collectively indicated by SS; and a number of bimetallic countries, indicated by  $BIM_i$ ,  $i=1, \dots, n$ . Depending on the position of the market ratio compared to the mint ratio, the money supply of  $BIM_i$  will consist of only gold coins, only silver coins, or both gold and silver coins. I will assume that if both metals are in circulation, the market price of gold and silver is fixed at the  $i$ th country's mint ratio through Fisher's arbitrage mechanism. 2/ If the market ratio is below the mint ratio, only gold circulates, and if the market ratio is above the mint ratio, only silver circulates.

With this setup, we can consider two separate currency areas: a gold area and a silver area. The size of each area is determined by the composition of the currency circulation of each country  $BIM_i$ . The exchange market acts as the bullion market in our simplified world: gold and silver coins of the three countries are traded in the exchange market at the market ratio. Arbitrage takes place between bimetallic money supplies and the exchange markets in the way described in the previous section. Coinage is gratuitous and thus bullion and coins are perfect substitutes. 3/ Therefore, without loss of generality, we can assume that gold and silver held for investment or speculative purposes are held as currency only. The relative market price of gold and silver is equal simply to the exchange rate between the gold and silver areas.

Our world can be captured in a composite-good model of two areas, the gold area and the silver area, as follows:

$$m_G(t) - p_G(t) = \alpha_G + \beta_G y_G(t) - \gamma_G i_G(t) + v_G(t) \quad (1)$$

$$m_S(t) - p_S(t) = \alpha_S + \beta_S y_S(t) - \gamma_S i_S(t) + v_S(t) \quad (2)$$

$$p_G(t) + x(t) = p_S(t) \quad (3)$$

1/ The theory of stochastic process switching and target zones, developed by Flood and Garber (1983, 1991), Froot and Obstfeld (1991a, 1991b), Krugman (1991), and Smith (1991) has been used to model many phenomena. They include Britain's return to gold in 1925 and entry in the EMS (Miller and Sutherland 1992), exchange rate behavior after the Louvre accord (Lewis 1990), the EMS as a target zone (Delgado and Dumas 1991 and Svensson 1992a), and a collapsing gold standard (Krugman and Rotemberg 1990 and 1992).

2/ For simplicity, I am abstracting from the existence of gold points here. It is assumed that bimetallic arbitrage is costless.

3/ "Free" coinage refers to the right to have precious metals coined on demand. Coinage is "gratuitous" if there is no charge for it, i.e., the weight of the coins received is equal to the weight of the bullion supplied to the mint.

$$i_G(t) = i_S(t) - E_t(dx)/dt \quad (4)$$

All variables except the interest rate are in logarithms, where  $m$  is the money supply,  $p$  is the price level,  $i$  is the interest rate, and  $x$  is the exchange rate between the gold and silver areas, equal to the market ratio of the gold and silver prices.  $E_t(dx)/dt$  is the expected rate of change in  $x(t)$ , which depends on the information set available at time  $t$ .  $v(t)$  is a random shock to money demand. A subscript  $G$  indicates the gold area, and  $S$  indicates the silver area. Equations 1 and 2 describe how in each area real money demand must equal real money supply. As in Flood and Garber, I will assume for simplicity that  $\gamma_G = \gamma_S$ . Equation 3 implies PPP, and Equation 4 indicates that uncovered interest parity holds.

The variables  $y$  are defined as follows:

$$y_G = \ln(Y_{GS} + \sum_{i=1}^n (1 - \lambda_i) Y_{BIM_i}) \quad (5)$$

$$y_S = \ln(Y_{SS} + \sum_{i=1}^n \lambda_i Y_{BIM_i}) \quad (6)$$

where  $Y$  is the level of output.  $\lambda_i$  is 0 when the market ratio  $x$  is below the country's mint ratio ( $\bar{x}_i$ )--only gold is in circulation--and  $\lambda_i$  equals 1 when  $x > \bar{x}_i$ , so that the money supply of  $BIM_i$  consists of only silver coins. With  $x = \bar{x}_i$ ,  $\lambda_i$  is between 0 and 1, and both metals are in circulation. In effect,  $y_G$  is a measure of world output "covered" by the gold money supply, and  $y_S$  is a measure of world output covered by the silver money supply. In periods when coins of both metals circulate in  $BIM_i$ , that country belongs partly to the gold area and partly to the silver area. Its output and demand shocks will then be allocated to a currency area in proportion to the relative circulation of gold and silver. <sup>1/</sup>

We can combine Equations 1 through 4 to get

$$m_G(t) - m_S(t) + x(t) = \alpha_G - \alpha_S + \beta_G y_G(t) - \beta_S y_S(t) + \gamma E_t(dx)/dt + v_G(t) - v_S(t) \quad (7)$$

or

$$x(t) = w(t) + k(t) + \gamma E_t(dx)/dt \quad (8)$$

where

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<sup>1/</sup> This can be safely done since in this case the exchange rate between the two areas is fixed at the bimetallic mint ratio, so the price-level movements and the interest rate are equal in both areas by Equations 3 and 4.

$$k(t) = \alpha_G - \alpha_S + m_S(t) - m_G(t) + v_G(t) - v_S(t) \quad (9)$$

and

$$w(t) = \beta_G y_G(t) - \beta_S y_S(t) \quad (10)$$

The dynamics of the system are driven by  $k(t)$ , a combination of the difference between the money supplies and money demand shocks in the gold and silver area and money demand shocks in the gold and silver area. It is assumed to follow a continuous-time random walk without drift, as in Krugman (1991):

$$dk = \sigma dz \quad (11)$$

where  $\sigma$  is a constant and  $dz = u(t)(dt)^{1/2}$ , and  $u(t)$  is iid normal, so that  $z$  follows a standard Brownian motion (Wiener) process. <sup>1/</sup>

To solve for the market ratio  $x$ , we need to find a general solution  $x = g(w + k)$  that satisfies Equation 8 as long as the fundamentals follow a random walk. As a second step, we will impose the boundary conditions, leading to a unique solution for  $x$  as a function of the fundamentals. The general solution  $x = g(w + k)$  can be found by applying Itô's lemma to the function  $g(\cdot)$ , to get:

$$E_t(dx)/dt = (\sigma^2/2) g''(w+k) \quad (12)$$

Substituting into (8), we obtain:

$$g(w+k) = (w+k) + (\gamma\sigma^2/2) g''(w+k) \quad (13)$$

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<sup>1/</sup> This assumes that demand for precious metals for nonmonetary uses is constant, and the supply of precious metals is dependent exclusively on technological and environmental factors, such as mining technology and new discoveries of metal deposits. Thus, after allowing for (constant) nonmonetary demand, the impact of the supply of precious metals on the monetary system comes as a series of random shocks to the supply of monetary gold and silver. These assumptions are somewhat restrictive and do not allow for things like an extended relative excess of gold production over silver production, such as gold rushes. More complex processes for  $k$ --notably autoregressive ones--can be imagined, but these do not allow for the explicit analytical solutions arrived at by Krugman. See Miller and Weller (1989) for a discussion.

This is a second-order differential equation, with a general solution:

$$g(w+k) = (w+k) + Ae^{\theta(w+k)} + Be^{-\theta(w+k)} \quad (14)$$

where

$$\theta = (2/\gamma\sigma^2)^{1/2} \quad (15)$$

The values of the integration constants A and B depend on the boundary conditions, the relationship between x and the fundamentals at the mint ratios. As shown in Flood and Garber (1991), in simple target-zone models involving a reflecting barrier there is an infinite number of solutions for A and B. They can only be uniquely determined by specifying the exact size and timing of the intervention undertaken by the central bank. They show that the smooth-pasting outcome familiar from Krugman (1991) is the limiting case of a value-matching condition that requires that there is no change in the exchange rate when it hits the boundary. Smooth pasting results when the authorities commit to intervene infinitesimally at the boundary.

A similar value-matching condition determines A and B in the model of the bimetallic system. The main difference arises from the different nature of the boundary itself. In the mainstream target zone model, increases in fundamentals are offset by intervention in such a way that the exchange rate does not exceed the upper limit. Decreases in fundamentals, however, are not offset at the upper boundary, and would move the exchange rate immediately back into the band. Thus, since fundamentals are a random walk, the expected change in the exchange rate is negative. By (12), it is equal to:

$$\begin{aligned} E(dx)/dt &= (\sigma^2\theta^2/2) [Ae^{\theta(w+k)} + Be^{-\theta(w+k)}] \\ &= (1/\gamma) [Ae^{\theta(w+k)} + Be^{-\theta(w+k)}] \end{aligned} \quad (16)$$

It is constant at this value as long as the exchange rate is at the upper boundary of the target zone.

The mint ratio as an arbitrage barrier is different in the following way. Assume there are two bimetallic countries, BIM<sub>1</sub> and BIM<sub>2</sub>, with BIM<sub>1</sub>'s mint ratio ( $\bar{x}$ ) higher than that of BIM<sub>2</sub> ( $\underline{x}$ ), and the market ratio in between the two initially. When the market ratio hits BIM<sub>1</sub>'s mint ratio from below, a further increase in the market ratio is prevented by arbitrage that replaces gold coins with silver coins in BIM<sub>1</sub>. But with silver coins now in circulation, a subsequent decrease in fundamentals will not immediately move the market ratio down and away from the mint ratio, as is the case in the target-zone model. Instead, the drop in fundamentals will be offset by arbitrage replacing the newly present silver coins with gold coins. It is still true that the expected change in the market ratio is given by (16) as long as the market ratio is below the mint ratio. As soon as x hits  $\bar{x}$ , however, it jumps to zero, since the market ratio is fixed by arbitrage offsetting both increases and decreases in fundamentals.

This jump in  $E(dx)/dt$  leads to the following value matching condition:

$$\begin{aligned}\bar{x} &= w_1 + k + (1/\gamma) [Ae^{\theta(w_1+k)} + Be^{-\theta(w_1+k)}] \quad \text{as long as } x < \bar{x} \\ \bar{x} &= w_0 + k \quad \text{as soon as } x = \bar{x} \\ \Rightarrow w_0 - w_1 &= (1/\gamma) [Ae^{\theta(w_1+k)} + Be^{-\theta(w_1+k)}]\end{aligned} \quad (17)$$

The increase in  $E(dx)/dt$  from a negative value to zero is offset by a decrease in the value of  $w$ . As long as the market ratio is below the mint ratio,  $BIM_1$  is using gold as its only medium of exchange and  $\lambda_1 = 0$ , so that by (10)

$$w_1 = \beta_G \ln(Y_{GS} + Y_{BIM_1}) - \beta_S \ln(Y_{SS} + Y_{BIM_2}) \quad (18)$$

When the market ratio reaches the mint ratio,  $BIM_1$ 's currency becomes part gold and part silver, so that  $Y_{BIM_1}$  is divided between the gold and silver currencies, and

$$w_0 = \beta_G \ln(Y_{GS} + (1-\lambda_1)Y_{BIM_1}) - \beta_S \ln(Y_{SS} + \lambda_1 Y_{BIM_1} + Y_{BIM_2}) \quad (19)$$

for  $\lambda_1 \in (0,1)$ . So when  $E_t(dx)/dt$  increases discontinuously,  $\lambda_1$  jumps from zero to a positive value. Therefore,  $y_S = \ln(Y_{SS} + (1-\lambda_1)Y_{BIM_1})$  jumps up and  $y_G = \ln(Y_{GS} + (1-\lambda_1)Y_{BIM_1})$  jumps down, instantaneously decreasing  $w$  to leave the market ratio  $x$  unchanged. Note that  $y_S$  and  $y_G$  jump even though the actual levels of output in the gold and silver areas,  $Y_{GS}$  and  $Y_{SS}$  remain unchanged. The jump in  $w$  is a result of the jump in  $\lambda_1$ , the share of silver coins in the bimetallic money supply. We can think of this as a rush to mint silver at the bimetallic mint that instantaneously converts a share  $\lambda_1$  of the money supply from gold to silver.

Equations 17 through 19 and symmetrical conditions for the lower mint ratio yield a relationship between  $A$ ,  $B$  and the initial rushes of arbitrage at the upper and lower mint ratios. The size of the initial rush determines the value of  $E(dx)/dt$  at the boundary, and therefore the strength of the stabilizing effect on the market ratio. The full set of solutions is characterized by the range  $g'(w+k) \in [0,1]$  at the boundaries. If there are no initial rushes, then  $E(dx)/dt = 0$ ,  $A$  and  $B$  are zero and the function  $g$  is linear, with  $g'(w+k) = 1$ . On the other extreme, the largest initial rush occurs when  $g'(w+k) = 0$ . This is the smooth-pasting outcome familiar

from the target-zone literature, and leads to the largest "target-zone" effects. 1/

In what follows, I will consider only the smooth pasting solution, assuming that there is a large number of individuals all expected to try to take advantage of a profit opportunity as soon as it arises. This basically means that as soon as the market ratio hits the mint ratio from above many holders of gold coins will attempt to transfer them to the bullion market to make a profit. This plausible assumption would lead to the largest possible bout of initial arbitrage, and the largest target zone effects. 2/

The lower panel of Chart 3 shows the market ratio as a function (g) of the fundamentals under the smooth-pasting outcome. We see the familiar "S" shape of the function, where the market ratio is bent away from its free floating path through the target-zone effects. When the market ratio approaches the upper mint ratio from below--through an increase, say, in the monetary supply of silver--speculators are induced to hold more of the silver area's currency because silver is expected to appreciate: the market ratio has "more room to go down than to go up." The effect on the economy of the silver area is clear from the upper panel of the diagram: the increase in its money supply leads to a decrease in the interest rate and a positive interest-rate differential ( $i_G - i_S$ ). This differential sustains the expected appreciation of silver through Equation 4. Both curves are drawn for a value of the short-run interest semielasticity of  $\gamma = 7.295$ , the midpoint of the low and high long-run (1867-1975) estimates reported for the United States and the United Kingdom by Friedman and Schwartz (1982, table

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1/ Note the difference with the traditional target zone model, where infinitesimal intervention is required to obtain the target zone effects. Because of the different nature of the boundary--a hybrid between an absorbing and reflecting barrier--the largest target-zone effects occur when a large initial rush of arbitrage takes place.

2/ A quick back-of-the-envelope calculation shows that there is evidence that this might have been the relevant case for nineteenth century bimetallism. For the values of  $\gamma$  and  $\sigma$  used in Figure 2 and for plausible relative sizes of the gold area (GS, consisting of the United Kingdom and the United States), the silver area (SS), and France (BIM);  $\lambda$  for the smooth-pasting solution is between 0.9964 (GS=0.4, SS=0.5, BIM=0.1) and 0.9988 (GS=0.2, SS=0.6, BIM=0.2). So between 0.12 percent and 0.36 percent of the French money stock would have had to have been converted "instantaneously" when the market ratio reached the mint ratio. To see that this is plausible, realize that the total stock of coins in 1849 was at most 3 billion francs (total cumulative silver coinage since 1820). A 0.12 percent and 0.36 percent share of this would be 3.6 million and 10.8 million francs, respectively. Total gold coinage in France in 1850 amounted to 85 million francs, most of it associated with six weeks' worth of arbitrage in response to the drop in the market ratio below the French mint ratio on November 22, 1850. That is around 14 million francs per week, or 3 million francs per working day, close to what is needed for the smooth-pasting outcome to prevail.

6.11). As an estimate for the value of  $\sigma$  I used the standard deviation of the first differences of monthly observations of the market ratio over the period 1880 to 1885,  $\sigma = 0.0058$ . 1/ The maximum difference between the market ratio and the free-floating ratio at the upper mint ratio is -0.177, while the maximum instantaneous interest-rate differential is 0.15 percent. 2/

At  $s_1$ , the market ratio hits the mint ratio. A rush of arbitrage occurs in  $BIM_1$ , with silver coins replacing undervalued gold coins. Silver flows from the silver area to  $BIM_1$ , and gold flows from  $BIM_1$  to the gold area through the exchange market. These bullion flows adjust the money supplies to equate interest rates in each area, and cause the fundamentals ( $w + k$ ) jump to point  $s_1'$ . As long as both metals remain in circulation in  $BIM_1$ , the fundamentals will remain at this point, increases in  $k$  being offset by decreases in  $w$ .

The diagram also shows how the area above the upper mint ratio displays symmetrical target-zone effects. In essence, the bimetallic mint ratio is the upper boundary of a lower band and the lower boundary of an upper band. 3/ There is a range of fundamentals for which the bimetallic system is stable--that is, both metals are in circulation in the bimetallic country and the market ratio is fixed through Fisher's arbitrage mechanism. If the monetary supply of silver increases by so much that bimetallic arbitrage threatens to render the bimetallic money supply effectively monometallic in silver, an arbitrage attack occurs--the counterpart to the initial rush of arbitrage--that removes the last gold from circulation in  $BIM_1$ , decreasing ( $w + k$ ) from  $s_1'$  to  $s_1''$  in the diagram. At the same time, the interest-rate differential ( $i_G - i_S$ ) jumps to a negative value. Beyond  $s_1''$ ,  $x$  increases according to the function  $h$ .

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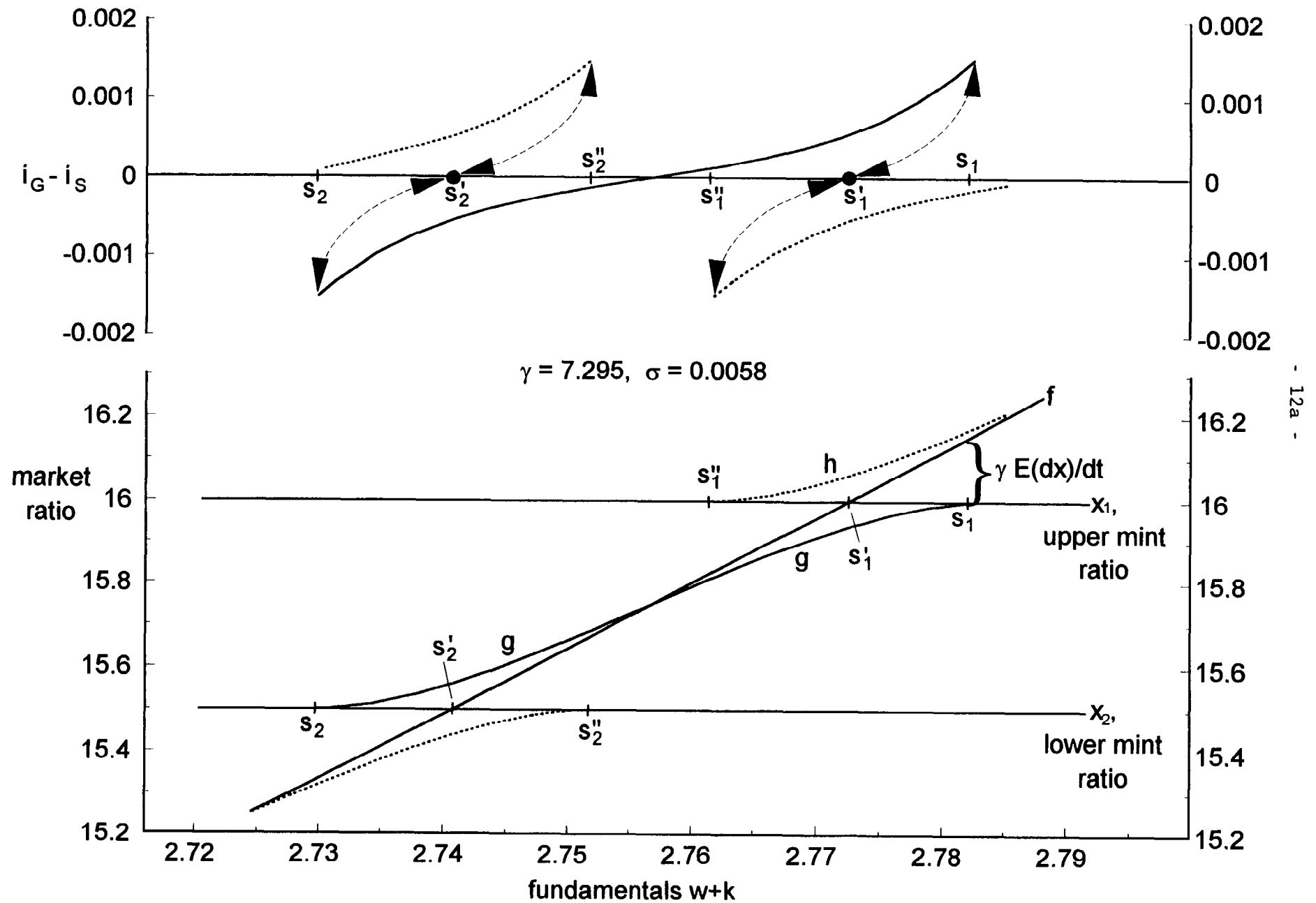
1/ Notice that  $\sigma$  is equal to the standard deviation of the first differences of the fundamentals. During the period 1880 to 1885 the bimetallic system had been abandoned, so that the market ratio was equal to the fundamentals:  $x = w + k$ .  $\sigma$  can therefore directly be measured by looking at the variability of the market ratio.

2/ The results are similar for different estimates of  $\gamma$ . The low estimate of Friedman and Schwartz,  $\gamma = -2.85$ , leads to  $x - f = -0.108$  and  $(i_G - i_S) = 0.24$  percent, while the high estimate,  $\gamma = -11.84$ , leads to  $x - f = -0.226$  and  $(i_G - i_S) = 0.12$  percent.

3/ The combination of two one-sided target zones is a concept introduced by Krugman and Rotemberg (1990, 1992) to model a collapsing gold standard. The application to bimetallicism is especially interesting since, in contrast to the hypothetical case of a collapsing gold standard, it allows for empirical analysis of the model.

CHART 3

THE MARKET RATIO AND INTEREST RATES AS A FUNCTION OF FUNDAMENTALS





#### IV. An Empirical Investigation

Empirical studies have not been successful in finding significant target-zone effects in a number of different exchange-rate regimes. Svensson (1992b) provides a survey of the empirical work done in the literature. An extensive study was done by Flood, Rose, and Mathieson (FRM, 1991), who tested the most important empirical implications of the traditional target-zone model with data on the EMS, Bretton Woods, and the gold standard, and concluded there is little empirical support for the model in most of these regimes. In this section I will provide evidence on three of the four kinds of tests reported by Svensson and FRM, concerning the distribution and volatility of the market ratio inside the arbitrage bands, and the empirical relationships between the market ratio and interest-rate differentials. 1/

The data are taken from the London and Amsterdam stock exchanges as reported in the *Course of the Exchange* and the *Prijscourant der Effecten*, respectively. The data cover the years 1840-49, a decade particularly appropriate in the present context because it covers a period of bimetallism in "disequilibrium:" none of the relevant bimetallic countries experienced simultaneous circulation of gold and silver coins. In Chart 4 we note that the market ratio fluctuated between the mint ratios of the United States and France in the 1840s. Both of these countries were on effective monometallic standards at the time. The mint ratio of the United States had been slightly above the market ratio since 1834, indicating the country was on an effective gold standard. France's ratio had been below the market ratio since the Napoleonic era, so that silver had become the basis of the currency by the late 1830s. With the market ratio in between the mint ratios of two effectively monometallic countries, the 1840s was a period in which we expect the "target-zone effects" of bimetallism to be most evident.

The nonlinear relationship between fundamentals and the market ratio has two implications for its behavior in the stability band formed by the French and U.S. mint ratios. First, the distribution of the ratio in this band should be U-shaped, with the market ratio spending more time near the edges than in the middle of the band. Second, the volatility of the ratio near the edges should be reduced compared to that in the middle of the band. Both implications have been clearly rejected by the data for the EMS, Bretton Woods, and the gold standard (FRM, 1991), with the distribution of the exchange rate mostly hump-shaped, and no clear patterns in the volatility of the exchange rate.

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1/ A fourth test that looks at the relationship between the fundamentals and the market ratio is omitted here, since data on fundamentals--basically the monetary supplies of gold and silver--could only be obtained on a yearly basis for the period 1851-73. During this period, target zone effects might be observed only over the years 1859-67, when Fisher's mechanism was most likely not operative. This leaves us with too few datapoints to be informative.

Charts 5 and 6 show that the data for the bimetallic system are more consistent with the nonlinear model than the regimes studied by FRM. Chart 5 gives the distribution of the market ratio in the stability band created by the French and U.S. mint ratios. Although not U-shaped, the distribution of the data in Chart 5 does display a peak near the upper mint ratio; the market ratio spent most of its time near one of the mint ratios, instead of in the middle of the band. This is in contrast to the distributions found by FRM, which were mostly hump shaped with most of the density in the center of the target-zone bands.

Chart 6 gives the average absolute change in the market ratio as a function of its position in the band. As expected, the market ratio is most volatile in the middle, and less so close to the upper mint ratio. The somewhat larger volatility near 15.5 might be caused by the small sample of observations in that range.

Data on interest-rate differentials turn out also to be supportive of the model. As explained above, in the presence of two differing mint ratios the model predicts an interest differential favoring the gold currency when the market ratio is near the upper mint ratio and favoring the silver currency when it is near the lower mint ratio.

It is difficult to get detailed direct data on short-term interest rates for the mid-nineteenth century. We can obtain indirect data on interest rates, however, from the exchange-market quotes on bills of exchange. These were bills drawn on a foreign payer, payable in a foreign currency. For the major currencies, both spot and forward bills were traded. The two-month forward bills on London traded in Amsterdam, for example, were payable in pounds two months from the date of transaction. They were quoted in guilders and had to be paid for at the time of purchase. Delivery of the currency did not take place until two months later, so that, in effect, purchase of such a bill amounted to a combination of a spot purchase of pounds and a two-month time deposit in pounds. We can therefore use the spread between the spot and forward bills to calculate a short-term interest rate: annualizing the percent spread between the spot and forward pound bill in Amsterdam yields the two-month interest rate on pounds.

Using Amsterdam quotes on spot and forward bills of exchange, I calculated monthly averages of two-month interest rates for the Pound--a gold currency--and the Franc--a silver currency--for the years 1840 to 1849. <sup>1/</sup> Chart 7 shows a scatterplot of monthly averages of the market ratio in

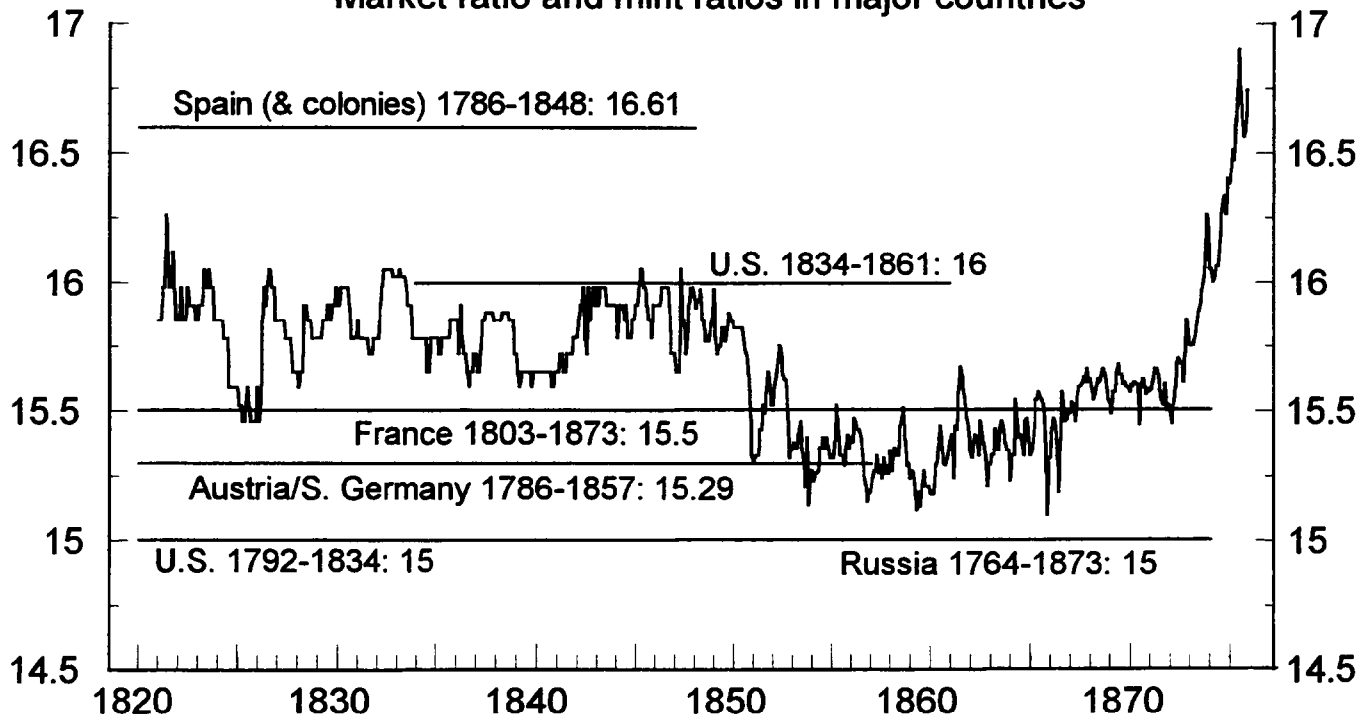
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<sup>1/</sup> For an indication of the accuracy of this procedure, I obtained a directly observable interest rate from the *Prijscoorant* of 1845, the guilder "Belening" rate, and compared it with the interest rate implicit in the spread between the spot and forward guilder bills of exchange on Amsterdam that were traded in London and quoted in the *Course of the Exchange*. For the 102 usable observations, the correlation coefficient was 0.91. A regression of the implicit rate on the belening rate had an  $R^2$  of 83 percent, with a coefficient of 1.11 and a standard error of 0.10.

CHART 4

RATIO OF GOLD TO SILVER PRICE 1815-1873

Market ratio and mint ratios in major countries

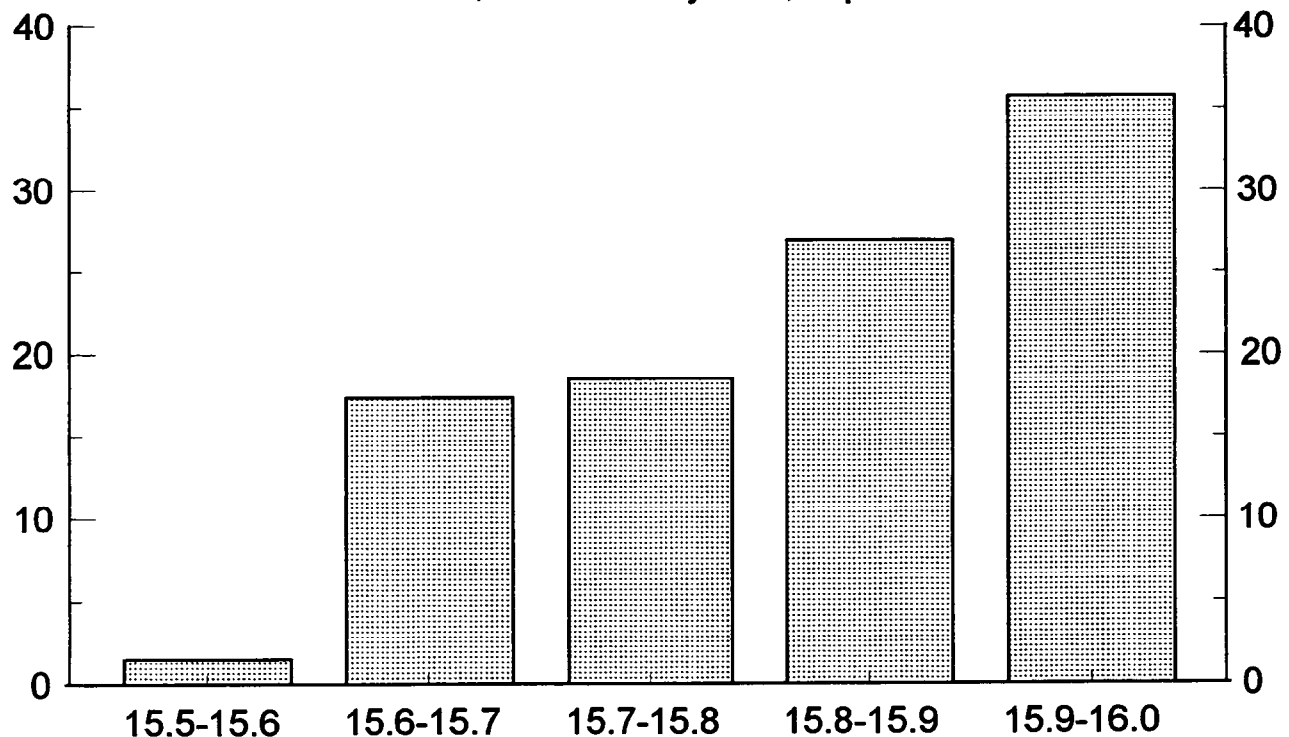


Source: The Course of the Exchange.

CHART 5

FREQUENCY DISTRIBUTION OF MARKET RATIO IN THE BAND

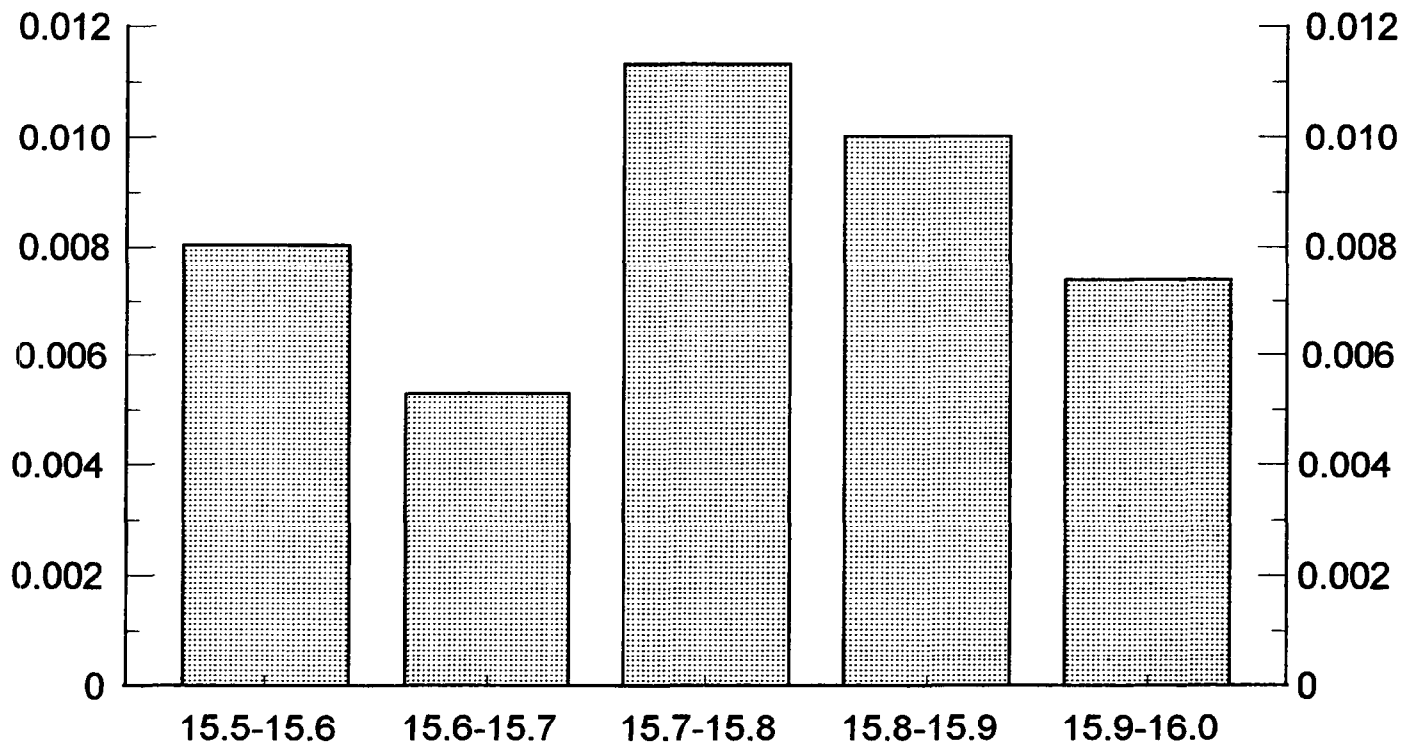
1840-1849, twice-weekly data, in percent



Source: The Course of the Exchange, author's calculations.

CHART 6

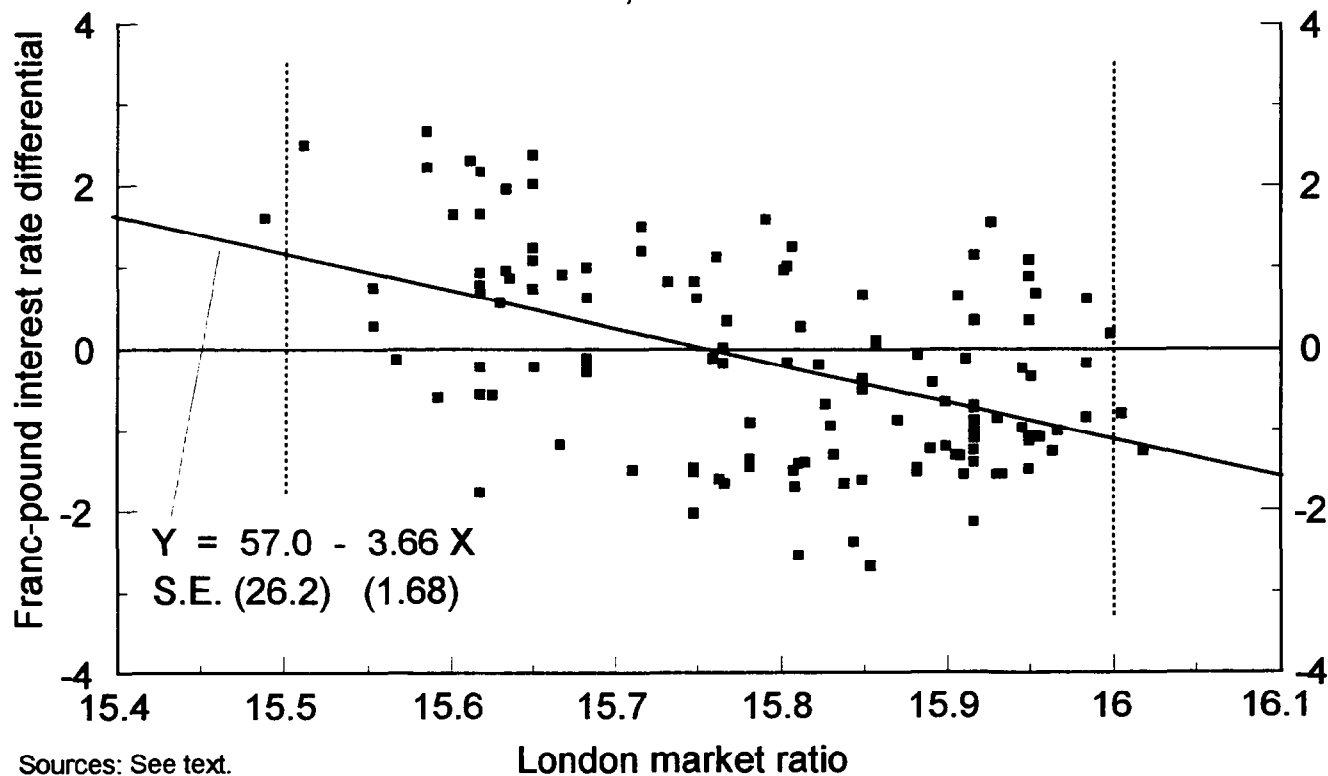
**AVERAGE ABSOLUTE CHANGE IN MARKET RATIO  
BY POSITION IN THE BAND, 1840-1849**



Source: The Course of the Exchange, author's calculations.

CHART 7

**LONDON RATIO VS. FRANC-POUND INTEREST RATE DIFFERENTIAL,  
1840-1849, MONTHLY DATA**



Sources: See text.

London against the calculated interest differential between the Pound and the Franc. The data is a bit noisy, but the included regression line shows a clear positive relationship: when the market ratio is near the lower mint ratio of 15.5, an interest-rate differential favoring silver tends to develop. This differential sustains the expected rise in the market ratio, that is, the expected depreciation of silver. Analogously, when the market ratio moves towards 16, a differential favoring gold tends to develop. This sustains the expected depreciation of gold.

A confrontation of the dynamic nonlinear model of bimetallism with the data does not lead to the clear rejection other target-zone models have suffered. The distribution of the market ratio in the band, the volatility of the market ratio as a function of its location in the band, and--most importantly--the relationship between the market ratio and interest-rate differentials are consistent with the model.

One question remains: why do we find target-zone effects in the bimetallic data when they have not been found for other regimes? The major reason perhaps is that the bimetallic system does not suffer from the credibility problems that plague other target-zone regimes. Under a traditional target-zone system, the threat of the central bank failing to defend the edges of the band leads to a weakening of the target-zone effects, and might even lead to the perverse results FRM have observed. The case of bimetallism is different because the arbitrage at the edges of the band formed by two different mint ratios--the system's equivalent of central bank intervention--does not actually depend on the actions of a central bank. It depends only on the profit motive of individuals, and therefore will always take place; there is no threat of the edge of the band not being "defended."

The second difficulty with traditional target zones, the threat of realignment, is also much less important under bimetallism. The reason is that the monetary authorities have no incentive to "realign"--change the mint ratio--when the market ratio is at the edge of the arbitrage band. To see this, realize that the main objective of bimetallism is to have concurrent circulation of gold and silver coins. This is possible only when the market ratio is at the mint ratio, i.e., at the edge of the arbitrage band. Therefore, with the market ratio close to the mint ratio the monetary authorities actually have no incentive to realign. At the edge of the band is exactly where they want the market ratio to be.

## V. Conclusion

This paper has developed a model of the bimetallic system combining Fisher's arbitrage mechanism with elements of stochastic switching theory. It shows that the alternating episodes of true bimetallic circulation and effective monometallism should be seen as regime shifts between a fixed and floating market ratio. This leads to target-zone effects when the market ratio is near the bimetallic mint ratio: the anticipation of arbitrage occurring at the mint ratio stabilizes the market ratio even when it is away

from the mint ratio and no arbitrage is actually taking place. This resolves the long-standing puzzle in the bimetallic literature concerning the long periods of stability in the market ratio in the absence of arbitrage possibilities in the major bimetallic countries.

During the first three-quarters of the nineteenth century, the combination of several different mint ratios among major countries created a series of adjacent two-sided bimetallic stability bands. This can be seen in Chart 4. During 1834-1861, the bimetallic mint ratio of the United States formed the upper barrier of a stability band with France's mint ratio as the lower barrier. Similarly, until 1857, the Austrian mint ratio acted as the lower barrier of a stability band with France's mint ratio as the upper barrier. Thereafter, the Russian mint ratio of 15 fulfilled this role. During periods of relatively minor shocks to the supply of gold and silver, the market ratio remained stable within one such band. Large shocks to the relative supply of the precious metals were absorbed by a relatively quick change of currency that took place in one of the currency areas, moving the market ratio into an adjacent stability band.

France was the pivot of this system, experiencing such a change of currency twice during the nineteenth century--in the early 1850s and again in the late 1860s. Contrary to what is often assumed in the literature, however, she was not the only major stabilizing player in the bimetallic system. Other countries officially on the bimetallic standard--such as the German states and the United States--played an essential stabilizing role by forming the counterpart to France's one-sided stabilization potential, thereby creating full-fledged bands in which the gold-silver price ratio remained remarkably stable.

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