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Real Interest Rates, Real Exchange Rates, and Net Foreign Assets in the Adjustment Process

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Abstract

This paper analyzes the recent behavior of real exchange rates, the trade balance and the net foreign asset position of the United States in an intertemporal optimizing model of the world economy that incorporates heterogeneity across countries and imperfect international capital and good markets. While the model successfully tracks the dynamics of trade balances and net foreign assets it generates too much consumption smoothing and excessively volatile relative prices. Resolving these inadequacies simultaneously is difficult as the elasticity of substitution between tradables and nontradables affects in opposite ways the degree of consumption smoothing and the volatility of relative prices.

JEL Classification Numbers:

F31, F41, F47

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Contents

Summary

Consumption smoothing is a fundamental determinant of current account balances, as international trade allows consumers to smooth the effects of country-specific shocks to their incomes. In addition, international trade allows countries to change their external assets and liabilities in anticipation of future income shocks. Hence, the stock of net foreign assets is a reflection of both past and future consumption smoothing. In this paper, the time-series dynamics of net foreign assets in a dynamic, stochastic two-country consumption-smoothing model is studied. International risk sharing is constrained in the model by limits on the net indebtedness of a country, rigidities in international trade, the presence of nontraded goods, and the inability of consumers to issue any assets other than short-term bonds. The role of real exchange rates and real interest rates in the adjustment process is also studied.

Simulations are presented based upon observed output shocks for the United States and a composite rest-of-the-world aggregate during the post-Bretton Woods period. The model generates very persistent net foreign asset positions, a feature also found in the data. Similarly, the trade balance dynamics are captured well. The lack of frequent mean reversals in the simulated net foreign asset positions, even over long horizons, indicates that current output innovations dominate anticipated future endowment differences. This effect is further magnified by the positive correlation of output shocks across countries.

The simulation results for the post-Bretton Woods period obtained in the paper also indicate that, for standard preference parameters, the degree of exchange rate volatility associated with consumption-smoothing behavior is larger than the observed volatility of the multilateral U.S. dollar exchange rate. Perturbations to the framework indicate the robustness of the results, except in the case of an increase in the substitutability between tradable and nontradable goods, which naturally reduces exchange rate volatility and increases consumption smoothing.

I. Introduction

Persistent current account imbalances during the last 15 to 20 years have puzzled many economists since movements in the current account do not always appear to be related to corresponding movements in real exchange rates. Some observers of the international monetary system have wondered whether these imbalances are sustainable and raised the question of whether market-based adjustment mechanisms such as wealth and real exchange rate effects are sufficiently strong to correct seemingly large current account imbalances or whether supplementary policy measures are needed. Concerns about excessive current account imbalances have been expressed particularly in cases in which consumption was perceived to have been the main contributor to imbalances. This is somewhat surprising since the literature on the intertemporal approach of the current account recognizes consumption smoothing as a main determinant of current account balances. 1/ Judgments about the size of current account imbalances and the effectiveness of adjustment mechanisms are often based on a benchmark model. As in many domains of economics, however, there is no uniquely acknowledged benchmark model to explain and quantify a "normal" degree of consumption smoothing beyond solvency considerations. Similarly, the size of wealth and real exchange rate effects implied by consumption smoothing remain quite uncertain.

In this paper, we study the quantitative dimension of consumption smoothing in a stochastic, dynamic two-country general equilibrium model. We set up the model in such a way that both the real exchange rate and wealth as measured by the net foreign asset position play a role in the adjustment mechanism. A disaggregation of consumption into traded and nontraded goods, and incomplete financial markets, are the main devices to ensure a nontrivial role for both variables. The ability of our model to explain the time series behavior of net foreign asset positions, real interest rates, and real exchange rates for the United States during 1970-92 is analyzed. This strategy provides a benchmark model to address the question of the wealth and real exchange rate adjustment mechanisms in a market-clearing setting.

Wealth effects as well as the evolution of current account balances have received little attention so far in the quantitative modeling of international business cycles, despite their importance in policy discussions. The extraordinary amount of symmetry and of market completeness assumed in most papers, incorporated for analytical convenience, explains part of this lacuna. In a world populated by a large number of uniform representative agents and identical countries, the current account and the net foreign asset position are either irrelevant (in the

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1/ Obstfeld (1980, 1982) has analyzed consumption smoothing in deterministic dynamic models with agents optimizing over their entire life span. For more recent work in this area see, among others, Bayoumi and MacDonald (1994), Bayoumi and Klein (1995), and Ostry (1988).

case of a social planner solution) or zero (in the case of complete financial markets with perfect risk pooling). A model with a nontrivial net foreign asset position requires some form of heterogeneity across countries and/or some degree of market incompleteness. Both heterogeneity and incomplete asset markets are difficult to incorporate into dynamic stochastic general equilibrium models. Heterogeneity often implies that there is no well defined ergodic distribution for the variables of interest. Hence, incomplete financial markets are difficult to deal with, both on a conceptual and on a technical level. Unless one restricts the analysis to a small open economy, the distribution of wealth becomes an endogenous state variable.

Although the theoretical implications of market incompleteness on the existence of equilibria in dynamic general equilibrium models have been established in Clarida (1987, 1990), very few attempts have been made to test its empirical relevance. 1/ Baxter and Crucini (1993), Backus, Kehoe, and Kydland (1992), and Devereux, Gregory and Smith (1992), study savings-investment correlations in a one-good model without any role for exchange rates in a complete markets setup. Kollmann (1995) studies the role of productivity shocks and fiscal policy for the determination of the U.S. trade balance in an incomplete markets, one-good model. Asea and Mendoza (1995), Tesar (1993), and Backus and Smith (1993), incorporate nontraded goods in the analysis but do so in a complete markets setup, as does Bekaert (1994, Forthcoming) in a monetary model. Canova and Dellas (1993) and Correia, Rebelo and Neves (1995) present a small open-economy real business cycle model in which the net foreign asset position is allowed to vary to analyze trade balances dynamics.

The purpose of this paper is to integrate market incompleteness, tradables and nontradables in a model with transportation costs in a world with two large countries. We incorporate market incompleteness by limiting the ability of consumers to smooth consumption through the sale and purchase of a one-period real bond. Doing this in a model with traded and nontraded goods leads to a natural role for the real exchange rate in the international adjustment mechanism. Real rigidities are introduced with proportional shipping cost in international trade. Furthermore, we solve the model with techniques that have not yet been applied in an international context. Finally, we use the methodology developed in den Haan and Marcet (1994) to test for the accuracy of our approximation method.

We restrict our attention to a stochastic, stationary world economy in which agents are identical across countries in terms of their preferences. They differ by the exogenous, imperfectly correlated endowment streams of a tradable and two nontradable goods that they receive in every period. The focus is therefore entirely on consumption smoothing. Since markets are assumed to be incomplete, the exchange of intertemporal claims is necessary to insure against idiosyncratic income shocks. Real, one-period discount

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1/ In a closed-economy context, Telmer (1993), Heaton and Lucas (1993), and Lucas (1995) have studied the importance of market incompleteness to explain asset market puzzles such as the equity premium.

bonds are the only assets at the disposal of optimizing households in the two countries. If shocks in the endowment of tradable goods are not perfectly correlated across countries, agents can smooth their consumption of tradable goods by buying and selling one-period bonds. The balance of payments identity implies that these trades are the mirror image of the trades occurring in goods markets. Hence, consumers can only save by building up net foreign assets.

On average, there is no consumption smoothing as agents exchange only deviations from their constant mean endowments. The importance of consumption smoothing is reflected in the mean absolute value of the trade balance. The size of the trade balance depends mainly on the correlation structure and the persistence in endowment shocks, on the substitutability between traded and nontraded goods, and on precautionary behavior of consumers. We show that, using realistic time series inputs for the stationary components of output processes of tradable and nontradable goods, our model generates highly persistent net foreign asset positions. Persistent net foreign asset positions are, of course, a well-known characteristic of real economies. Paradoxically, this persistence is the consequence of limited consumption smoothing across the two countries. The large, positive correlation of outputs processes for tradable goods that are observed in industrial countries leaves agents with only few incentives to accumulate and decumulate assets over short time periods for precautionary motives. Hence, consumption smoothing occurs mostly in response to current income shocks and is largely independent of the current net foreign asset position. Given the high correlation of output shocks across countries, the current net foreign asset position is almost always nearly optimal. For this reason, accumulated past shocks are the most important factor for the determination of the net foreign asset position. In our numerical simulations, we solve the model based upon the observed output correlations for the post-Bretton Woods period. Unlike many other studies that impose certainty equivalence to solve the model, the solution technique used in this paper does not require such an assumption and can easily capture precautionary savings.

The paper is organized as follows: in Section II, the stylized facts surrounding real exchange rates, real interest rates, and net foreign asset position are examined for a group of industrial countries. The model is laid out in Section III; the procedure and the calibration of parameters and of stochastic processes needed for the numerical solutions are then discussed in Section IV. The simulation results are presented in Section V. Section VI concludes the paper.

II. Stylized Facts

This section provides evidence on the magnitude, volatility, persistence, and correlations of an array of variables in international capital markets. In particular, we look at the means (if appropriate), standard deviations, maximum and minimum values, and persistence (as measured by the first autocorrelation coefficient and Cochrane's (1988), nonparametric V_k measure) of output and consumption, real interest rates, real exchange rates, trade balances, and net foreign asset positions. 1/

For our purposes, the United States is treated as the home country and seven other industrialized countries are treated as the "rest-of-the-world". Because of data limitations, we only include Belgium, Canada, Germany, Italy, Japan, Sweden and the United Kingdom. The variables for those countries are converted to a common currency using OECD PPP exchange rates and aggregated with U.S. export weights.

Imperfectly correlated stochastic processes in the production of tradable as well as nontradable goods across countries are a prerequisite for consumption smoothing across countries. 2/ An ergodic, steady state distribution for the variables of interest exists only if the stochastic processes for the output of tradables in both countries exhibit the same mean growth rate. In our relatively small sample, the mean growth rates clearly do not have this property. We abstract from this problem and assume that there is a common steady state growth rate among the major industrialized countries. In order to do so, we apply a Hodrick-Prescott filter to the output of tradable and nontradable goods in the United States and the rest of the world in order to isolate the stationary components of the output series. 3/

1/ Cochrane's V_k-statistics is a non-parametric measure for the persistence in a time series process. It is defined as $\hat{V}_k = 1 + 2\sum_{j=1}^{k}(1-\frac{j}{k+1})\hat{\rho}_j$, where $\hat{\rho}_j$ is the j-th autocorrelation coefficient. For a white noise process, which is not persistent by definition, the V_k-statistics is equal to one.

2/ Following Stockman and Tesar (1995), tradables output is defined as the sum of output in the following sectors: (1) agriculture, hunting, forestry and fishing; (2) mining; (3) manufacturing; (4) wholesale and retail trade, restaurants, and hotels; and (5) transport, storage, and communication. Output of nontradables comprises (1) electricity, gas and water; (2) construction; (3) finance, insurance, real estate and business services; (4) community, social, and personal services; and (5) government services. The sectoral data are taken from the OECD Annual National Accounts Database, the net foreign asset position data taken from Masson, Kremers and Horne (1993). See the appendix in Turtelboom (1995) for a detailed description of the data and sources.

3/ We use a weight for the relative variance of the growth component to the cyclical component of 100 in the calculation of the trend component, as it is standard in the literature. Baxter and King (1995), show that a weight of 10 comes close to replicating the properties of an ideal band-pass filter for the extraction of business cycle frequencies. We test the robustness of our result with respect to the weight chosen below.

Chart 1 presents the detrended output series for tradables and nontradables output for the United States and the rest of the world during 1970-92. The oil price shock of 1974 is reflected in the recession during the mid-1970s. The steep recession of the early 1980s was followed by a 7-year expansion and the recession of the early 1990s. It is worth noting that the U.S. series exhibit higher volatility than the output series for the rest of the world, a fact most likely driven by the aggregation of the other countries.

Since we want to analyze the role of exchange rates, trade balances, and net foreign asset positions in the consumption smoothing process, the stylized facts for those variables and their relation are summarized in the following six propositions:

1. *Total output is more correlated across countries than consumption.*

The fact that output series are strongly correlated across countries while the co-movements in consumptions are relatively weak is probably the best known stylized fact in the field and has been explored by, among others, Baxter and Crucini (1993) and Tesar (1993). Panel A in Table 1 shows that this proposition also holds in a multilateral context. The correlation between U.S.-GDP and ROW-GDP is 42.9 percent whereas the cross-country consumption correlation is only 18.4 percent.

2. *Consumption in each country is more correlated with output in the same country than with consumption in the other country.*

For the United States, panel A in Table 1 shows that the GDP-consumption correlation is 90.7 percent, slightly below the 92.7 percent in the ROW.

3. *The correlation between a country's tradables and nontradables output is higher than the cross-country correlation of tradables output. The latter is in turn larger than the correlation of nontradable outputs across countries. Output of tradables is also more volatile than nontradables output.*

Panel B of Table 1 shows that the correlation between the U.S. (ROW) tradables and nontradables output is 73.9 (90.1) percent. The cross-country correlation between tradables output is, however, only 49.8 percent. For nontradables, that correlation is only 36.1 percent. 1/ According to the standard deviations in Table 2, total output is also more volatile than consumption and the output of tradable goods is relatively more volatile

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1/ Stockman and Tesar (1995), who have a larger set of countries, do not obtain such a clearcut picture in their data: the correlation in the output of traded goods in one country with the output of nontraded goods in another is sometimes larger than the correlation between the outputs of traded goods in the same two countries.

Table 1. Contemporaneous Correlation Between Key Variables: 1970-92

Panel A. GDP and Consumption

United States	GDP	1.000	0.907	0.429	0.361
	Consumption		1.000	0.316	0.184
ROW	GDP			1.000	0.927
	Consumption				1.000

Panel B. Output

United States	Tradables	1.000	0.739	0.498	0.395
	Nontradables		1.000	0.288	0.361
ROW	Tradables			1.000	0.901
	Nontradables				1.000

Panel C. Real Interest Rates and Real Exchange Rates

United States	Real interest rate	1.000	0.933	-0.133
ROW	Real interest rate		1.000	-0.027
	Real exchange rate			1.000

Panel D. Relative Prices of Nontradables and Real Exchange Rate

United States	Relative price	1.000	-0.223	-0.935
ROW	Relative price		1.000	0.539
	Real exchange rate			1.000

Panel E. Trade Balance, Net Foreign Asset Position and Real Exchange Rate

United States	Trade balance 1/	1.000	0.324	0.537
United States	Net Foreign Asset Position 1/		1.000	0.259
	Real exchange rate			1.000

1/ As percentage of GDP.

Chart 1

Output of Tradables and Nontradables in the US
and the Rest of the World
(Average output normalized to 1)

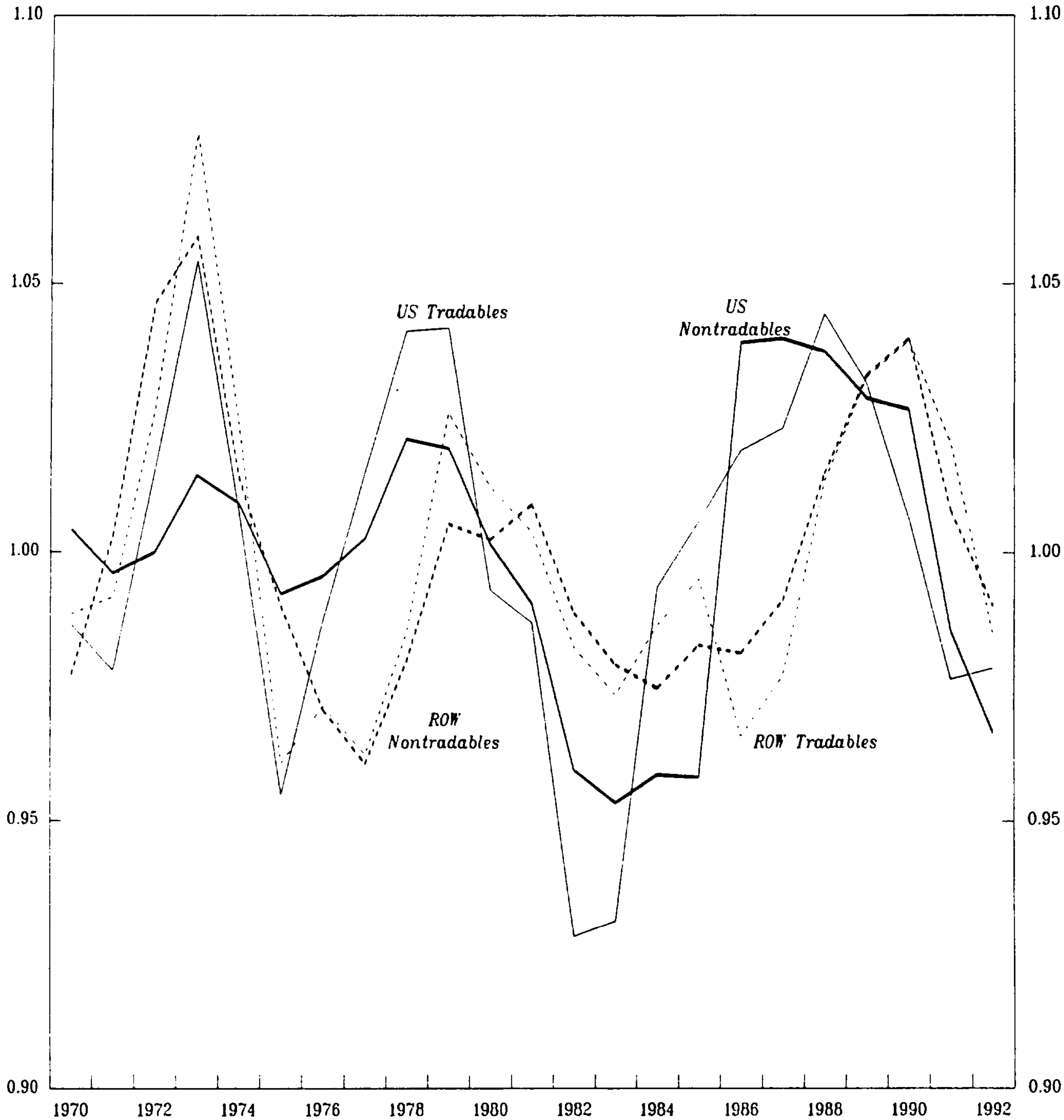

Table 2. Univariate Moments for Actual Series: 1970-92

Variable	Mean	STD	Maximum	Minimum	rho	Cochrane
United States real interest rate	3.00	4.8600	9.460	-6.610	0.866	2.74
ROW real interest rate	2.52	3.3400	6.520	-4.890	0.866	2.76
United States price of nontradables	1.00	0.0185	1.050	0.963	0.431	1.89
ROW price of nontradables	1.00	0.0068	1.010	0.985	0.486	1.98
United States net foreign assets (percent of GDP)	3.29	8.8600	13.400	-18.700	0.969	2.94
United States trade balance (percent of GDP)	-1.06	1.0900	0.851	-3.150	0.828	2.65
United States real exchange rate	1.00	0.0124	1.030	0.971	0.570	2.14
United States GDP	2.00	0.0281	2.040	1.940	0.640	2.27
ROW GDP	2.00	0.0268	2.070	1.960	0.594	2.20
United States consumption	2.00	0.0206	2.030	1.960	0.646	2.27
ROW consumption	2.00	0.0252	2.070	1.960	0.624	2.26
United States output of tradables	1.00	0.0336	1.050	0.928	0.538	2.06
United States output of nontradables	1.00	0.0270	1.040	0.953	0.640	2.28
ROW output of tradables	1.00	0.0293	1.080	0.961	0.500	2.01
ROW output of nontradables	1.00	0.0256	1.060	0.961	0.652	2.33

than the output of nontradable goods (a standard deviation of 0.0336 versus 0.0293 in the United States and 0.027 versus 0.0256 in the ROW). The fact that the output series for the rest of the world are less volatile than the same series is probably a consequence of the aggregation of eight countries.

4. *Real CPI-based interest rates are highly volatile, very persistent, and strongly correlated across countries.* 1/

Table 2 reports that the standard deviation of the real interest rate exceeds its mean in both countries. The first-order autocorrelation coefficient is 86.6 percent in both countries with a Cochrane-measure also indicating very strong persistence. Panel C in Table 1 indicates a cross-country correlation of 93.3 percent.

5. *The relative prices of nontraded goods to traded goods are much less volatile and persistent processes than those for real interest rates. Similarly, the real exchange rate is also much less volatile and persistent than real interest rates.*

The first fact, reported in Table 2, indicates that the volatility of the CPI-based real interest rates is largely driven by the tradables component. Since our definition of the real exchange rate is closely linked to nontradables prices, it is not surprising that the main time series characteristics of nontradables prices--i.e., low volatility, moderate persistence--carry over to the real exchange rate. The relative prices of nontradable goods in terms of tradable goods are not strongly correlated (Table 1, Panel D).

6. *Net foreign asset positions are highly volatile, only weakly correlated with the real exchange rate, and highly persistent over time.*

The most striking characteristic of the net foreign asset position of the United States is its high degree of persistence. 2/ Panel E in Table 1 indicates that the net foreign asset position as a percentage of GDP is only weakly correlated with both the trade balance as a percentage of GDP (32.4 percent) and the real exchange rate (25.9 percent). Movements in the trade balance are much more correlated contemporaneously with the real exchange rate (53.7 percent). In addition, the net foreign asset position exhibits strong autocorrelation, with a first-order autocorrelation of 96.9 percent and a Cochrane measure of 2.94, indicating very strong persistence in the net foreign asset position. 3/ It is this picture of a

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1/ Real consumption based interest rates are ex post real interest rates based on a CPI inflation measure.

2/ A lack of data prevented us from calculating an aggregate net foreign asset position for the rest of the world.

3/ For comparison, for an AR(1) process with an autoregressive coefficient of 0.5 (0.9) with two lags, the Cochrane measure equals 1.83 (2.74).

world with limited consumption smoothing, small volatility of nontradables prices, highly volatile real interest rates, and highly persistent net foreign asset positions that we try to capture with the model in the next section.

III. Incomplete Markets and Asset Trading in a Two-Country General Equilibrium Model

1. Overview of the model

We now develop a two-country, stochastic dynamic general equilibrium model that incorporates both a disaggregation of consumption into traded and nontraded goods and incomplete as well as imperfect asset markets. In addition, we capture real rigidities in goods markets by explicitly incorporating transportation costs in international trade. The relevance of nontraded goods to explain the obvious lack of complete consumption risk sharing has been well established in the literature. 1/ Many economists claim that incomplete financial markets as well as financial market imperfections account for some of the many asset pricing anomalies found in the empirical literature. 2/ In the context of a two-country model, incomplete financial markets also provide a vehicle for the net foreign asset position to have a meaningful role in the determination of real exchange rates and real interest rates and to ensure nontrivial current account dynamics. Almost all quantitative studies of international business cycle linkages so far rely on the assumptions of complete financial markets and/or perfect risk pooling. 3/ They imply a zero current account balance at any point in time and current account dynamics that are entirely driven by extrinsic factors. 4/

In our model, each country i (i=1,2) is inhabited by a large number of households with identical preferences who maximize the present value of consuming tradable goods $T_{i,t}$ and nontradable goods $NT_{i,t}$ according to the time separable utility function:

1/ See, for example, Cole and Obstfeld (1991), Tesar (1993), Lewis (1994), and Stockman and Tesar (1995).

2/ See Aiyagari and Gertler (1991) and Heaton and Lucas (1993).

3/ Lucas (1982) emphasized the analytical convenience of the assumption of perfect risk pooling in this class of models. Baxter and Crucini (1991) deviate from the standard settings in the literature by allowing incomplete financial markets to play a role in their two-country, one good model.

4/ Note, that in a typical solution with perfect risk pooling, wealth effects do not play any role as the capital account dynamics is reduced to valuation effects.

$$U_t = E_t\left[\sum_{s=0}^{s=\infty} \beta^s \frac{1}{1-\gamma}\left\{\left[T_{i,t+s}^{1-\alpha} + NT_{i,t+s}^{1-\alpha}\right]^{\frac{1}{1-\alpha}}\right\}^{1-\gamma}\right] \tag{1}$$

β is the subjective discount factor ($0 < \beta < 1$), $(1/\gamma)$ the elasticity of intertemporal substitution of total consumption, and $(1/\alpha)$ the elasticity of intratemporal substitution between traded and nontraded goods. E_t denotes the expectations operator conditional on information available at time t. It is assumed that all information concerning shocks in any period t is known before agents make their decisions.

In every period, household i receives random endowments $X_{i,t}$ of the tradable good and $Y_{i,t}$ of the nontradable good. Tradables and nontradables outputs are assumed to be exogenous. 1/ The output series can be thought of as being driven by mixture of shocks, such as technology and government spending shocks. It is at this level that heterogeneity arises in our model: endowments are different across countries but not within countries. Households in the two countries are constrained to trading a one-period discount bond $b_{i,t+1}$ which can be acquired for q_t units of the tradable good in period t and which pays out one unit of the tradable good in period t+1. Goods markets are assumed to be imperfectly integrated because it is costly to ship goods between countries. These costs take the form of a proportional loss κ per unit of tradable good shipped and are borne by the importer. The wealth constraint of a representative household in country i is then given by:

$$T_{i,t}+p_{i,t}NT_{i,t}+\max[0,\kappa(T_{i,t}-X_{i,t}]+q_t b_{i,t+1} \leq X_{i,t}+p_{i,t}Y_{i,t}+b_{i,t}, \tag{2}$$

where $p_{i,t}$ is the price of the nontradable good in country i in terms of the world price of the tradable good in period t. The left-hand side of equation (2) sums the amount of resources spent on tradables and nontradables consumption, the shipping costs incurred if the country runs a trade deficit, and the purchases of bonds. This spending must be less than the endowments received during period t and the stock of net foreign assets carried over from the previous period.

The structure of heterogeneity in the model implies that trade in goods as well as in bonds occurs only across countries. The amount of bonds purchased in period t, $b_{i,t+1}$, is therefore equivalent to the net foreign

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1/ The assumption of an exogenous production structure is primarily made for convenience given our focus on consumption smoothing. Endogenous production decisions are clearly a desirable extension of our model as changes in productive capacity across countries do not need to be solely linked to consumption decisions in a world with capital mobility. They also allow for a much richer dynamics in real exchange rates and wealth, which in that case is not only given by the net foreign asset position, but also by the domestic capital stock.

asset position in period t+1 since the stock of "outside" bonds is assumed to be zero. Both countries have a zero net foreign asset position in the first period.

Our assumption that a real bond denominated in units of the tradable goods is the only available financial instrument is admittedly arbitrary. It is, however, an important benchmark case since bonds and bank credit/deposits still are by far the most important vehicles of international financial intermediation. 1/ This asset market set-up also has the advantage that self-insurance implies trade in assets. If stocks for all four sectors were available, implying that the space of the exogenous stochastic process can be spanned completely (as in Tesar (1993) for example), there would be no trade in equities provided that the ideal portfolio is already held in the initial period. If this were not the case, all dynamics would be transitional and would not be relevant for the computation of a stationary equilibrium. 2/ Continuous asset trading requires some imperfect spanning; given this condition, the restriction of financial instruments to bonds is a natural starting point. 3/

Financial markets are also assumed to be imperfect because there are limits to the process of asset accumulation and decumulation. Households can only borrow up to a maximum amount $\bar{b}$:

$$b_{i,t+1} \geq \bar{b} < 0 \ . \tag{3}$$

The borrowing constraint can be motivated in several ways. First, it is a crude representation of sovereign default risks or, in a less extreme form, of increasing risk premia related to rapid increases in external indebtedness. Second, borrowing constraints are a way to ensure the existence of an equilibrium. Although it has been shown that infinite horizon, incomplete asset market economies in which debt is restricted by a borrowing constraint coincide in equilibrium with the same type of economies in which debt is only constrained by a transversality condition (Magill and Quinzii (1994)), the transversality conditions are not helpful in the numerical computation of equilibria. Although transversality conditions only limit the growth rate of debt, they do not prevent debt from becoming

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1/ Arbitrary restrictions on the structure of financial markets is a general problem in general equilibrium models with incomplete financial markets (Geanakoplos (1990)). Allen and Gale (1994) show how the market structure can be endogenized by incorporating the production decision of financial intermediaries. An extension along this line is outside the scope of this study, however.

2/ This does not imply that a stationary equilibrium will be reached from any initial condition.

3/ For a theoretical exposition of these and related issues, see Giavazzi and Wyplosz (1984) and Krugman (1994).

very large from a practical point of view. For that reason, a borrowing constraint seems to be more appropriate for our application.

There are also several resource and finance constraints that hold in equilibrium. In particular, the consumption of tradable goods in the two countries equals the aggregate endowment of tradable goods:

$$\sum_{i=1}^{i=2} T_{i,t} = \sum_{i=1}^{i=2} X_{i,t}, \forall t. \tag{4}$$

In both countries, the consumption of nontradable goods equals the endowment of nontradable goods:

$$NT_{i,t} = Y_{i,t}, \ \forall t. \tag{5}$$

The supply of bonds equals its demand implying zero excess demand and initial wealth is assumed to be zero for both countries:

$$\sum_{i=1}^{i=2} b_{i,t+1} = 0, \ \forall t. \tag{6}$$

$$b_{1,0} = b_{2,0} = 0 \tag{7}$$

Equations (1) through (7) present a complete description of the model. Agents optimize consumption intertemporally in equation (1) subject to the budget constraint in equation (2), a liquidity constraint in equation (3) and the standard resource constraints in equations (4) through (7). We now turn to the equilibrium conditions.

2. Rational expectations equilibrium

In equilibrium, each household maximizes equation (1) subject to constraints (2) and (3), taking the expected equilibrium law of motion for real interest rates and the real exchange rates as well as endowments as given.

The model consists of two sets of state variables: the exogenous output series for tradables and nontradables $Z_t = (X_{i,t}, Y_{i,t})$ (i=1,2) and the endogenous state variable $b_{1,t}$ (= - $b_{2,t}$). The two-country assumption facilitates the computation of the equilibrium as the distribution of wealth is completely described by $b_{1,t}$. We assume that the exogenous stochastic variables follow a Markov process such that their current values (and past values up to the order of the process) contain all the necessary information needed for the projection of future values. We stack all these variables into the vector ψ_t. This vector provides a complete description of the state of the world in period t and permits the definition of a recursive, competitive equilibrium in the spirit of Mehra and Prescott (1980). Hence, a stationary, rational expectations equilibrium can be characterized as follows:

Equilibrium is defined as a set of recursive sequences for prices $p_{1,t}(\psi_t)$, $p_{2,t}(\psi_t)$, and $q_t(\psi_t)$ and for quantities $T_{1,t}(\psi_t)$, $T_{2,t}(\psi_t)$, $b_{1,t}(\psi_t)$, and $b_{2,t}(\psi_t)$ for t=1,...,T such that:

1. *Households optimize, that is, the following set of equation holds for t=1,....,∞ and 1=1, 2:*

$$p_{i,t}(\psi_t) = \left(\frac{T_{i,t}(\psi_t)}{Y_{i,t}}\right)^{\alpha}, \tag{8}$$

$$\begin{aligned}
&[(T_{i,t}(\psi_t))^{1-\alpha} + Y_{i,t}^{1-\alpha}]^{\frac{\alpha-\gamma}{1-\alpha}}[T_{i,t}(\psi_t)]^{-\alpha} = \\
&\max[[(T_{i,t}(\psi_t,\bar{b})^{1-\alpha}+Y_{i,t}^{1-\alpha}]^{\frac{\alpha-\gamma}{1-\alpha}}[T_{i,t}(\psi_t,\bar{b})]^{-\alpha}, \\
&q_t^{-1}(\psi_t)\beta E_t[(T_{i,t+1}(\psi_{t+1}))^{1-\alpha}+Y_{i,t+1}^{1-\alpha}]^{\frac{\alpha-\gamma}{1-\alpha}}[T_{i,t+1}(\psi_{t+1}]^{-\alpha}]]
\end{aligned} \tag{9}$$

$$\begin{aligned}
&T_{i,t}(\psi_t)+\max[0,\kappa(T_{i,t}(\psi_t)-X_{i,t})]+q_t(\psi_t)b_{i,t+1}(\psi_t) \\
&\quad = X_{1,t}+b_{1,t}(\psi_{t-1}), \quad i=1,2 \;;
\end{aligned} \tag{10}$$

2. *Markets are in equilibrium, that is the following set of equations holds for t=1,...,∞:*

$$T_{1,t}(\psi_t)+T_{2,t}(\psi_t)=X_{1,t}+X_{2,t} \tag{11}$$

$$b_{1,t+1}(\psi_t)+b_{2,t+1}(\psi_t)=0 \tag{12}$$

$$b_{1,0} = b_{2,0} = 0 \tag{13}$$

Equation (8) provides the equilibrium price of nontradables in terms of tradables which is, not surprisingly, an increasing function of the ratio of tradables and nontradables consumption. The Euler condition in equation (9) has the standard asset pricing interpretation: the real rate of return on the discount bond equals the ratio of the discounted marginal utility of consuming tradable goods in period t+1 over the marginal utility of consuming tradable goods in period t unless the household faces a binding borrowing constraint. In that case, the marginal utility of consuming the tradable goods today is entirely determined by the limit net foreign asset position and the budget constraint. Equations (10), (11), (12), and (13) report the budget and resource constraints and the initial condition for the net foreign asset position. The real exchange rate is defined as the ratio of the two price levels:

$$REER=\left(\frac{1+(P_{1,t})^{\frac{1-\alpha}{\alpha}}}{1+(P_{2,t})^{\frac{1-\alpha}{\alpha}}}\right)^{\frac{\alpha}{\alpha-1}} \tag{14}$$

The model, as it stands in the last subsection, has no closed form solution. For this reason, we will rely on numerical solution procedures to characterize the equilibrium. It is useful, however, to explore some of the qualitative aspects of the model before numerically solving the model in a simple linearized version.

3. Net foreign assets and international adjustment in equilibrium

In this subsection we use a linearized version of the model to investigate the qualitative nature of the time series properties of net foreign assets and the factors determining the degree of their persistence. We also show that this certainty equivalence version of our model implies that the net foreign asset position is a random walk with no drift unless a borrowing constraint is imposed.

In order to illustrate the mechanism driving the evolution of net foreign assets in the model, we initially abstract from borrowing constraints or shipping costs and linearize the model around a deterministic steady state. 1/ Since consumers in both countries have similar taste parameters and the output of tradables and nontradables is stationary, the net foreign asset position is naturally zero in a deterministic steady state. Hence, on average, consumers in each country consume their endowments and the trade balance is zero. The law of motion for net foreign assets in this certainty-equivalence formulation is given by:

$$\hat{b}_{1,t+1}=\left(\frac{1+q}{q}\right)\hat{b}_{1,t}-\left(\frac{1}{q}\right)\hat{b}_{1,t-1}+\left(\frac{1}{2q}\right)[\Delta\hat{x}_{1,t}-\Delta\hat{x}_{2,t}+\phi\,(\Delta\hat{y}_{1,t}-\Delta\hat{y}_{2,t})] \tag{15}$$

where $\phi = (1-\alpha)(1-\gamma)/(\alpha(1-\gamma)-1)$, a hat over a variable denotes the percentage deviation from the steady state value (the net foreign asset position is denoted as a percent of the endowment in tradable goods of

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1/ King, Plosser, and Rebelo (1988), introduced this technique into the real business cycle literature.

country 1, and a bar refers to a steady state value. 1/ Equation (15) reflects the essence of the consumption smoothing process which is driven by a two-country permanent income mechanism. The solution to this difference equation for the home country's net foreign assets is given by the sum of expected future, as well as past, excess growth rates of tradables and nontradables:

$$\hat{b}_{1,t+1}=\frac{1}{2(1-\bar{q})}\{E_t[\frac{1}{\bar{q}}\sum_{s=1}^{\infty}\bar{q}^{\,s}\{(\Delta\hat{x}_{2,t+s}-\Delta\hat{x}_{1,t+s})-\phi\ (\Delta\hat{y}_{2,t+s}-\Delta\hat{y}_{1,t+s})\}]+ [\sum_{s=0}^{\infty}(\Delta\hat{x}_{2,t-s}-\Delta\hat{x}_{1,t-s})-\phi\ (\Delta\hat{y}_{2,t-s}-\Delta\hat{y}_{1,t-s})]\} \quad (16)$$

It is useful to simplify equation (16) to:

$$\hat{b}_{1,t+1}=\hat{b}_{1,t}+\frac{1}{2\bar{q}}\{E_t[\frac{1}{\bar{q}}\sum_{s=1}^{\infty}\bar{q}^{\,s}\{(\Delta\hat{x}_{2,t+s}-\Delta\hat{x}_{1,t+s})-\phi\ (\Delta\hat{y}_{2,t+s}-\Delta\hat{y}_{1,t+s})\}]. \quad (17)$$

Equation (15) and solutions (16) and (17) reveal several fundamental aspects of cross-country consumption smoothing under a certainty equivalence regime. First, there are no wealth effects because the current wealth position is recursively determined by the expected future and past path of tradable goods endowments as shown in solution (16). In other words, current wealth has no mean reverting effects . Only the forward looking element in equation (17) changes the inherited net foreign asset position $b_{1,t}$, which is thus a random walk. 2/ It is important to note that solution (17) satisfies the transversality condition which restricts the growth rate of net foreign assets to be smaller than $1/\beta$. Second, endowment shocks in nontradable goods can either encourage or discourage consumption smoothing in tradable goods. If tradables and nontradables are substitutes, shocks to the endowment of nontradable goods discourage consumption smoothing. 3/ Third, changes in the real interest rate only depend on the expected relative deviations of all endowment shocks between t and t+1, but not on the current wealth position.

1/ See Sargent (1979) for the solution of this difference equation.

2/ In fact, it can be shown that the deviations of tradable consumption from the steady state value do not depend on the current net foreign asset position. It is clear that this result depends crucially on the assumption that we have abstracted from borrowing constraints or other imperfections in the derivation.

3/ This is the case if the elasticity of the marginal utility from tradable goods consumption with respect to the marginal consumption from nontradables consumption, $(1-\alpha)(1-\gamma)/(\alpha(1-\gamma)\ -1)$, is positive.

Wealth effects in the traditional sense do not play in the certainty equivalence version of the model. It is clear that a linearization without borrowing constraints yields a very poor approximation of an equilibrium solution. In addition, the random walk nature of net foreign asset positions is clearly unappealing from an economic perspective. Even if borrowing constraints are in force to limit net foreign asset positions, the issue of whether linearization-based solution techniques have desirable properties remains open. The most important effect of linearization techniques is the elimination of precautionary behavior. Precautionary behavior in itself is not sufficient to circumvent the random walk problem because the possibility of borrowing renders a zero consumption event impossible: For reasonably behaved endowment processes, the expected value of discounted future endowment streams is always positive. Hence, strong precautionary behavior of agents is unlikely to occur and it would hardly limit foreign indebtedness. The question then becomes whether the elimination of precautionary behavior with borrowing constraints has only minor or serious drawbacks. The limited knowledge on precautionary behavior in the context of dynamic stochastic general equilibrium models makes it impossible to give a definitive answer to this question. Nevertheless, it is worthwhile to point out that precautionary behavior is particularly important in situations when agents in one country are close to being constrained in their foreign borrowing: the larger probability of being subjected to a sequence of low consumption if hit by a sequence of bad endowment shocks when having a large stock of foreign debt makes agents more risk averse. Hence, linearization techniques provide unreliable equilibrium prices and quantities near the borrowing constraint. For this reason, we have chosen a solution technique that allows for the approximation of a truly stochastic equilibrium.

IV. Solution Methodology

In this section, the time-series behavior of the exogenous variables is characterized and a brief overview of our numerical solution method, the method of parametrized expectations, is presented. 1/ We refer the reader to Danthine and Donaldson (1995), den Haan (1994), and den Haan and Marcet (1990) for a more technical exposition.

1. Vector-autoregression model for the exogenous variables

The model needs to be calibrated for the numerical solution procedure to be implemented. We adopt a mixed calibration strategy. The law of

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1/ See Taylor and Uhlig (1992) for an overview of alternative solution techniques for multiperiod rational expectations models in the context of a neoclassical, closed-economy growth model. The parametrized expectations algorithm was chosen for its simplicity. Given the experience of den Haan (1994), and given that there is only one endogenous state variable, this algorithm can be expected to work quite well in this application.

motion for the exogenous state variables is estimated using the data described in Section II. The parameters for the utility function (1) are taken from previous studies. We set α, the inverse of the coefficient of intratemporal substitution between tradables and nontradables, equal to 2. The discount factor β is set to 0.973 and gamma, the inverse of the coefficient of intertemporal substitution, is set to 3. Few studies are available on the proper size of shipping costs and we put them at 5 percent of the trade deficit. The borrowing limit to net foreign assets is set to 20 percent of tradables output.

The output of tradable goods and nontradable goods in the United States and the rest-of-the-world is used to calibrate the exogenous shock processes. The raw data indicate that the output for tradable goods does not have the same trend in the United States and the rest-of-the-world. Since consumption is an inherently stationary phenomenon, we extract the stationary component from the output series by applying a Hodrick-Prescott filter with a weight of 100. We assume that the law of motion for this series can be approximated by a finite order VAR model. The Sims (1980) Likelihood Ratio for the determination of the optimal lag length indicates that, starting from a predetermined initial lag order of three, a second order VAR is appropriate. The parameter estimates and the variance-covariance matrix for this VAR are reported in Table 3.

2. Method of parametrized expectations

The key problem for any solution algorithm is the calculation of the conditional expectations of the discounted, future marginal utilities from consuming the tradable and nontradable goods in equation (9). If households in the two countries are not liquidity constrained, the first-order conditions (9) can be stacked and written as follows:

$$q_t(\psi_t)[(T_t(\psi_t))^{1-\alpha}+Y_t^{1-\alpha}]^{\frac{\alpha-\gamma}{1-\alpha}}[T_t(\psi_t)]^{-\alpha}= \beta E_t[(T_{t+1}(\psi_{t+1}))^{1-\alpha}+Y_{t+1}^{1-\alpha}]^{\frac{\alpha-\gamma}{1-\alpha}}[T_{t+1}(\psi_{t+1})]^{-\alpha} \tag{18}$$

The functional form of the conditional expectation $E_t[\ U_T\ (.,.)\ ]$ on the right-hand side of this equation is unknown and has to be approximated numerically. The method of parametrized expectations, developed by den Haan and Marcet (1990), and extended to a heterogenous agent setting by den Haan (1994), expresses this conditional expectation as a polynomial function $f(\psi_t,\lambda,\zeta,\delta)$ of the state variables ψ_t:

$$f(\psi_t,\lambda,\zeta,\delta)=[(\frac{\psi_t^{\lambda}-1}{\lambda})\zeta]'\delta \tag{19}$$

Table 3. VAR for Exogenous Stochastic Processes

	$x_{1,t}$	$y_{1,t}$	$x_{2,t}$	$y_{2,y}$
$x_{1,t-1}$	1.0105 (0.0457)	0.818 (0.334)	0.107 (0.222)	0.092 (0.199)
$x_{1,t-2}$	-0.292 (0.507)	0.850 (0.371)	-0.344 (0.247)	-0.224 (0.221)
$y_{1,t-1}$	-0.266 (0.514)	-0.408 (0.376)	0.048 (0.250)	0.0373 (0.224)
$y_{1,t-2}$	0.136 (0.378)	-0.402 (0.276)	0.336 (0.184)	0.268 (0.165)
$x_{2,t-1}$	-1.412 (0.880)	-1.800 (0.644)	-0.124 (0.429)	-0.230 (0.384)
$x_{2,t-2}$	-0.365 (0.916)	-1.090 (0.671)	0.268 (0.446)	0.546 (0.400)
$y_{2,t-1}$	1.231 (0.868)	1.497 (0.636)	1.484 (0.423)	1.284 (0.378)
$y_{2,t-2}$	0.056 (0.978)	0.916 (0.716)	-1.179 (0.476)	-1.206 (0.426)
Constant	6895.600 (2696.800)	6749.100 (1974.400)	2082.400 (1313.100)	1709.200 (1175.900)
σ	184.769	135.270	89.962	80.565
AR(1)	1.514 [0.244]	1.866 [0.200]	3.324 [0.096]	4.453 [0.059]
Normality	11.786 [0.003]	0.250 [0.882]	3.556 [0.170]	3.122 [0.210]
ARCH(1)	0.158 [0.700]	0.316 [0.587]	0.033 [0.589]	0.024 [0.880]
Vector Portmateau (3)	42.729			
Vector AR(1)	F(16, 15)	1.428 [0.248]	chi ^{2} (16)	33.507 [0.067]
Vector Normality	Chi^2 (8)	9.314 [0.317]		

Notes: $X_{1,t}$ is the output of tradable goods in the United States, $Y_{1,t}$ the output of non-tradable goods in the United States. Series with subscript 2 stand for the save variables in the Rest of the World. The standard errors of the coefficients are reported in paranthesis. σ is the standard error of the estimate. AR(1) is the $\chi^2(1)$-statistics of a Lagrange multiplier test for first-order serial correlation in the residuals of an equation. The marginal significance level is shown in brackets below. Normality denotes the $\chi^2(2)$-statistics of a joint test that the skewness of the residuals of an equation is equal to zero and, similarly, that the kurtosis is equal to three. ARCH(1) denotes the $\chi^2(1)$-statistics of a LM multiplier test for first-order autoregressive conditional heteroskedasticity in the residuals of an equation. Vector Portmanteau (3) is the multivariate version (variance-covariance matrix) of a single time series Portmanteau test with 3 lags. It has a $\chi^2(16)$-statistics under the null hypothesis that the residuals are not serially correlated. Vector AR(1) stands for $\chi^2(16)$-statistics of the multivariate extension of an LM test for first-order serial correlation in the residuals. Finally, Vector Normality denotes the $\chi^2(8)$-statistics of a multivariate test that the skewness of the residuals is zero and that their kurtosis is equal to three. Doornik and Hendry (1994) discuss these tests in more detail.

where λ is the Box-Cox parameter, ς is an integer which denotes the order of the polynomial, and δ is a vector of parameters. den Haan and Marcet (1990) and den Haan (1994) demonstrate that a λ set to either 1 or 0 works quite well in practice. The parametrized expectation (PE) algorithm is therefore a mapping in the vector of coefficients δ for a given ς and λ.

The challenge is to find a vector δ that approximates the right-hand side of equation (19) as well as possible. This can be done by the following four-step iterative procedure. First, the time series of the exogenous variables need to be characterized. The law of motion for the endowments of tradables and nontradables for both countries is approximated by the vector autoregressive model described above. A sample of size T (e.g. T=200) for the exogenous state variables is generated by bootstrapping the residuals from the VAR.

Second, an initial estimate for the vector δ must be found. The choice of this vector is critical to assure a smooth convergence of the approximation procedure. In the context of a consumption smoothing model, a natural candidate for the choice of the initial vector is the solution of the model under autarchy. This can be done easily by imposing the assumption that there is no trade in asset and goods markets between countries for all periods t=1,...,T. In that case, the trade balance is zero in every period, as is the net foreign asset position $b_{i,t+1}$, and the other endogenous variables, q_t, $T_{i,t}$, $NT_{i,t}$, and $p_{i,t}$, can be easily calculated using a nonlinear equation solver.

Third, this initial estimate for δ can be used to generate time series for the endogenous variables in the model. Since both countries start off with a zero net foreign asset position in the first period, we can use the fitted value for $f(\psi,\lambda,\varsigma,\delta)$ to calculate the first period's consumption and interest rate and, imposing the budget constraint, next period's net foreign asset position. If the borrowing constraint is binding for any of the two countries, the equilibrium values need to be recomputed with the first-order condition for the other country only since the budget constraint of the constrained country determines the equilibrium level of tradables consumption given the world bond price q_t. The equilibrium values for the endogenous variables from this simulation can then be plugged into the right-hand side of equation (19), and a new estimate for the vector δ, call it δ_p, can be obtained by performing the multivariate regression:

$$arg\ \min_{\delta}\ (\frac{1}{T})\sum_{t=1}^{T}\ [U_T(T(W_t^0),Y)-f(\psi_t,\lambda,\varsigma,\delta_0)]'\cdot[U_T(T(W_t^0),Y)-f(\psi_t,\lambda,\varsigma,\delta_0)] \quad (20)$$

Fourth, we repeat this exercise until convergence is achieved. In iteration p+1, the guess for δ_p is used to compute $f(\psi,\lambda,\varsigma,\delta)$ and to solve for the endogenous variables, taking into account all relevant constraints and equilibrium conditions. This process is repeated until the difference

$|\delta_p - \delta_{p-1}|$ is small relative to a convergence criterion ν. Since the existence of a solution in the form of a unique coefficient vector δ_p such that $|\delta_p - \delta_{p-1}| < \nu$, is by no means certain for all combinations of parameter values and stochastic processes for Z_t, Marcet and den Haan (1994), propose to modify the iteration scheme for δ as follows:

$$\delta_p = \kappa\hat{\delta} + (1-\kappa)\delta_{p-1} \tag{21}$$

where δ denotes the current estimate according to regression (14). We now address the accuracy of this approximation method.

3. Accuracy test

It is clear from the previous discussion that the accuracy of the numerical procedure critically depends upon the approximation of the conditional expectation in equation (12). In the parametrized expectations method, this implies that the choice of the order in the polynomial used is of paramount importance. In order to check the accuracy of the polynomial, den Haan and Marcet (1994) propose a GMM-type test utilizing the fact that under rational expectations, the error term from the approximation should be serially uncorrelated since the expectations operator is conditioned upon all information available at time t. Hence, the error terms should be white noise and the row vector of residuals ξ_t from equation (12), defined as

$$\xi_t = \beta E_t[U_T(T_{t+1}(\psi_{t+1}), Y_{t+1})] - q_t(\psi_t)U_T(T_t(\psi_t), Y_t) \tag{22}$$

should not be predictable with information at time t. The test statistic DHM(i,j) is defined as follows:

$$\begin{aligned} DHM(i,j) &= T * B_T A_T^{-1} B_T \\ B_T &= (1/T)\sum_{t=1}^{T} [\xi_t \otimes \psi_{t-i}^j] \\ A_T &= (1/T)\sum_{t=1}^{T} [\xi_t \otimes \psi_{t-i}^j]' \cdot [\xi_t \otimes \psi_{t-i}^j] \end{aligned} \tag{23}$$

and is distributed with χ^2 with ($\dim(\xi) * \dim(\psi) * i$) degrees of freedom under the null hypothesis that ξ_t is a martingale difference vector series. It is important to perform this test for both agents jointly since this captures not only the predictability of individual consumer's endogenous variables but also of market movements. In addition to the simulation results, Section V also reports the results of this accuracy test.

V. Simulation Results

For our base simulation, we have used a second order polynomial ($\varsigma=2$) in which the state variables enter linearly ($\lambda=1$) to approximate the conditional expectations in equation (19). The model is simulated for 200 years. The discount factor β is set to .973 and α is set to 2. In this case, the elasticity of substitution between tradables and nontradables is 0.5. The coefficient of risk aversion, $1/\gamma$, is set to 1/3. A Hodrick-Prescott filter, with smoothing parameter equal to 100, is used for detrending the series. The output series for the rest of the world are trade-weighted sums of the national outputs of tradables and nontradables respectively. The statistics reported in Tables 4 and 5 are averages over 100 simulations.

Table 4 reports the univariate statistics implied by the model. It is clear that the model generates too much volatility in the price of nontradables in the model and hence in the exchange rate. The model underpredicts the autocorrelation for the United States nontradables price (32.2 versus 43.1 percent) but slightly overpredicts the autocorrelation of the same variable in the ROW (52.8 versus 48.6 percent). This divergence is driven by the relatively higher volatility of the United States output series. This excess volatility in the price of nontradables does, however, not carry over to the trade balance which is less volatile than the trade balance observed during 1970-92. Since consumption smoothing is driving the trade balance dynamics, it is not surprising that the average trade balance is very close to zero in the simulations and exhibits a degree of autocorrelation consistent with the output patterns.

The model is very successful in replicating the persistence in the net foreign asset position. The first order autocorrelation coefficient of the home country's net foreign asset position is 0.985 in the model and 0.969 in the data. Although the model underestimates the average United States net foreign asset position it does generate about 65 percent of the observed volatility. These discrepancies partly reflect the difference in the sample period. There is more tendency towards mean reversion in our simulation sample of 200 years than there is in our data sample of 22 years. The net foreign asset position of the home country is, on average negative whereas the home country trade balance is, on average, positive. This finding is remarkable since these moments are averages over 100 simulations runs. The fact that the home country's tradables and nontradables endowment is skewed towards negative shocks whereas the opposite holds for the ROW drives this result. In addition, the large positive correlation between endowment shocks in traded goods across countries leads to a synchronization of incentives across countries. Agents in the two countries therefore wish to buy and to sell real bonds at the same time. It follows that net foreign asset positions can only gradually be offset over time. There is no scope for frequent mean reversals under these circumstances.

This last finding is likely to explain one of the most noticeable anomalies of the model, the volatility of the consumption-based real

Table 4. Univariate Moments for Simulation with HP-100, α=2, β=.973, and γ=3

Variable	Mean	STD	Maximum	Minimum	rho	Cochrane
United States real interest rate	2.79	8.4200	31.60	-17.900	0.194	1.37
ROW real interest rate	2.86	8.0300	28.90	-16.900	0.222	1.43
United States price of nontradables	0.99	0.0491	1.14	0.862	0.322	1.64
ROW price of nontradables	1.01	0.0419	1.13	0.902	0.528	2.05
United States net foreign assets (percent of GDP)	-11.00	5.8000	2.23	-22.000	0.985	2.97
ROW net foreign assets (percent of GDP)	11.50	5.6600	21.10	-2.270	0.986	2.97
United States trade balance (percent of GDP)	0.25	0.9090	2.66	-2.150	0.681	2.36
ROW trade balance (percent of GDP)	-0.20	0.8910	2.26	-2.470	0.682	2.36
United States real exchange rate	1.01	0.0343	1.10	0.920	0.603	2.21
United States GDP	1.99	0.0690	2.17	1.760	0.381	1.76
ROW GDP	2.01	0.0592	2.16	1.830	0.407	1.82
United States consumption	1.99	0.0699	2.18	1.760	0.399	1.80
ROW consumption	2.01	0.0655	2.18	1.810	0.465	1.93
United States output of tradables	1.00	0.0330	1.08	0.894	0.530	2.06
United States output of nontradables	1.00	0.0271	1.07	0.925	0.612	2.22
ROW output of tradables	1.00	0.0276	1.07	0.921	0.518	2.04
ROW output of nontradables	1.00	0.0241	1.06	0.933	0.679	2.36
United States consumption of tradables	1.00	0.0276	1.07	0.904	0.498	2.00
ROW consumption of tradables	1.01	0.0263	1.07	0.921	0.505	2.01
United States consumption of nontradables	1.00	0.0271	1.07	0.925	0.612	2.22
ROW of consumption nontradables	1.00	0.0241	1.06	0.933	0.679	2.36

interest rates. The level of the real interest rates in this no-growth simulation set-up is determined by the discount factor β. The standard deviation of the simulated real interest rate series exceeds the actual value found in the data by a factor of 2 to 2.5 and the model fails to underestimates the autoregressive pattern of the interest rates by a factor of 4. The limited scope for mean reversion in the net foreign asset position and this excess volatility of interest rates are related to the two agent assumption and the assumption of a small elasticity of intertemporal substitution, an issue which is explored further in this section.

Table 5 presents the cross-sectional moments implied by the model. Panel A indicates that the model still predicts an excess degree of consumption smoothing, 70.3 percent vis-à-vis 18.4 percent in the data, despite the presence of liquidity constraints and transportation costs. This excess consumption smoothing is linked to the excess volatility in the price of nontradables and the real exchange rate. The exchange rate clearly works too well in breaking the domestic savings-investment correlation. This failure of the model is also linked to the excess correlation of outputs across countries. Although the GDP of the United States and the rest-of-the-world exhibits a correlation of 42.9 percent during our sample period, the model implies a correlation of 90 percent. This fact might seem puzzling because these two countries are treated as endowment economies and hence output of tradables and nontradables is exogenous. Its explanation lies again in that the price of nontradable goods adjusts not only consumption but significantly smooths the share of nontradables in GDP, measured in tradables. This high cross-country correlation for GDP explains the very high degree of correlation between domestic output and consumption despite the presence of consumption smoothing. Panel B reports the output correlations from bootstrapping the VAR. As can be readily seen, the cross-moments between output of tradables and nontradables closely matches the actual moments. The real interest rate and exchange rate correlations reported in Panel C have mixed results. The cross-country correlation between real interest rates closely matches the actual correlation but the correlation between interest rates and exchange rates has the opposite sign. The simulated correlation between the relative price and the real exchange rate reported in Panel (6) closely matches the actual correlations. In contrast to the actual data, the trade balance and net foreign asset position of the United States are negatively correlated. A similar finding, also reported in Panel E, holds for the correlation of the net foreign asset position and the real exchange rate. The implied correlation between the trade balance and the real exchange rate, however, has the correct sign but is much higher than during 1970-92 for the United States.

The den Haan-Marcet (1994) test for the accuracy of simulations indicate that the residuals from equation (12) in the baseline simulation do have the properties of martingale difference series. The test statistics defined in equation (22) is distributed as a χ^2 with 30 degrees of freedom. The average value of this test over 100 simulations is 36.2, which rejects the null hypothesis of serial correlation in the error terms at 1 percent level. This indicates that our second-order polynomial expansion including

Table 5. Correlations in Simulation with HP=100, α=2, β=.973, and γ=3

Panel A. GDP and Consumption					
United States	GDP	1.000	0.966	0.900	0.835
	Consumption		1.000	0.832	0.703
ROW	GDP			1.000	0.964
	Consumption				1.000
Panel B. Outputs					
United States	Tradables	1.000	0.756	0.506	0.408
	Nontradables		1.000	0.356	0.434
ROW	Tradables			1.000	0.898
	Nontradables				1.000
Panel C. Real Interest Rates and Real Exchange Rates					
United States	Real interest rate		1.000	0.934	-0.319
ROW	Real interest rate			1.000	-0.170
	Real exchange rate				1.000
Panel D. Relative Prices of Nontradables and Real Exchange Rate					
United States	Relative price		1.000	-0.102	-0.795
ROW	Relative price			1.000	0.679
	Real exchange rate				1.000
Panel E. Trade Balance, Net Foreign Asset Position and Real Exchange Rate					
United States	Trade balance 1/		1.000	-0.345	0.870
United States	Net Foreign Asset Position 1/			1.000	0.329
	Real exchange rate				1.000

1/ As percentage of GDP.

lagged variables of the output series is a good approximation of the expected discounted marginal utility of consuming tradables and nontradables in the next period.

The results might still be colored by our specification of the preferences and the output series. Hence, we check the robustness of our results to different parameters in the utility function. Tesar (1993), among others, has identified the substitutability between tradables and nontradables as a key parameter for the degree of consumption smoothing. In this section, we revisit this issue empirically and look at the effects of a higher degree of intratemporal substitution on consumption smoothing and the real exchange rate, trade balance, and net foreign assets. The results in Tables 6 and 7 are obtained from simulating the model for $1/\alpha$ equal to 2 rather than 0.5. Not surprisingly, this higher degree of intratemporal substitution between tradables and nontradables reduces the price volatility in the model. This is intuitive because consumers are now more willing to substitute tradables and nontradables in the face of the same endowment shocks. Hence the real interest rates, the prices of nontradables, and the real exchange rate all exhibit much less volatility bringing them much closer to the observed volatility during 1970-92. This increased desire to substitute tradables and nontradables leads, naturally, to a higher degree of consumption smoothing, which is reflected in the higher volatility of the trade balance without affecting its mean. The net foreign asset positions are only marginally affected, both in their mean and volatility, which once again illustrates the disconnection between consumers' wealth and their consumption decisions. Predictably, overall consumption is now more stable than in the previous simulation despite the fact that tradables consumption is now more volatile. Table 7 bears out the implications of this change for the cross-moments in the model. Panel A indicates a higher degree of consumption smoothing but a much lower correlation between GDP across countries. In addition, domestic consumption-GDP correlations are also reduced significantly. Real interest rate and exchange rate correlations, reported in Panel D changed very little although the correlations between relative prices and the real exchange rate are much more pronounced. The behavior of the trade balance, net foreign assets, and real exchange rate correlations also change very little (Panel E). 1/

1/ Canova and Dellas (1993) showed that results in their study of the trade balance in the international real business cycle model were very sensitive to the detrending method. In order to check the sensitivity of our results to the use of the HP filter with parameter 100, we reran the simulations with an HP filter with parameter 10. Baxter and King (1995) showed that this detrending method is very close to their proposed band-pass filter which we could not use due to the short sample period. The results change only very little using this filter. In addition, we aggregated the ROW tradables and nontradables output series using actual exchange rates rather than PPP values. Again, the results do not change qualitatively. Results from these two alterations are available from the authors.

Table 6. Univariate Moments for Simulation with HP=100, α=0.5, β=.973, and γ=3

Variable	Mean	Standard Deviation	Maximum	Minimum	rho	Cochrane
United States real interest rate	2.78	6.1300	22.50	-12.200	0.293	1.57
ROW real interest rate	2.82	6.1100	22.20	-12.200	0.291	1.57
United States price of nontradables	1.00	0.0152	1.04	0.955	0.380	1.76
ROW price of nontradables	1.00	0.0130	1.04	0.968	0.588	2.18
United States net foreign assets (percent of GDP)	-14.00	5.4000	0.43	-23.300	0.975	2.95
ROW net foreign assets (percent of GDP)	13.90	5.3800	23.20	-0.448	0.972	2.94
United States trade balance (percent of GDP)	0.30	1.2200	3.52	-2.910	0.676	2.35
ROW trade balance (percent of GDP)	-0.27	1.2100	2.92	-3.480	0.676	2.35
United States real exchange rate	1.00	0.0120	1.04	0.971	0.592	2.18
United States GDP	2.00	0.0551	2.13	1.820	0.561	2.12
ROW GDP	2.00	0.0483	2.12	1.860	0.552	2.11
United States consumption	1.00	0.0518	2.13	1.830	0.556	2.11
ROW consumption	2.01	0.0484	2.13	1.860	0.554	2.11
United States output of tradables	1.00	0.0330	1.08	0.894	0.530	2.06
United States output of nontradables	1.00	0.0271	1.07	0.925	0.612	2.22
ROW output of tradables	1.00	0.0276	1.07	0.921	0.518	2.04
ROW output of nontradables	1.00	0.0241	1.06	0.933	0.679	2.36
United States consumption of tradables	1.00	0.0296	1.07	0.899	0.494	1.99
ROW consumption of tradables	1.01	0.0275	1.08	0.922	0.531	2.06
United States consumption of nontradables	1.00	0.0271	1.07	0.925	0.612	2.22
ROW of consumption nontradables	1.00	0.0241	1.06	0.933	0.679	2.36

Table 7. Correlations in Simulation with HP=100, α=0.5, β=.973, and γ=3

Panel A. GDP and Consumption

United States	GDP	1.000	0.897	0.783	0.957
	Consumption		1.000	0.949	0.898
ROW	GDP			1.000	0.874
	Consumption				1.000

Panel B. Outputs

United States	Tradables	1.000	0.756	0.506	0.408
	Nontradables		1.000	0.356	0.434
ROW	Tradables			1.000	0.898
	Nontradables				1.000

Panel C. Real Interest Rates and Real Exchange Rates

United States	Real interest rate	1.000	0.983	-0.318
ROW	Real interest rate		1.000	-0.239
	Real exchange rate			1.000

Panel D. Relative Prices of Nontradables and Real Exchange Rate

United States	Relative price	1.000	-0.417	-0.870
ROW	Relative price		1.000	0.809
	Real exchange rate			1.000

Panel E. Trade Balance, Net Foreign Asset Position and Real Exchange Rate

United States	Trade balance 1/	1.000	0.295	0.931
United States	Net Foreign Asset Position 1/		1.000	0.267
	Real exchange rate			1.000

1/ As percentage of GDP.

VI. Conclusion

This paper develops a heterogenous agent, international business cycle model with incomplete markets that incorporates tradables and nontradables, real rigidities in international trade, and liquidity constraints in asset markets. We provide the first implementation of the parametrized expectations technique in the international business cycle literature and simulate the model for the United States vis-à-vis an aggregate of most G-10 countries for 1970-92 to study the behavior of the real exchange rate, trade balance, and net foreign asset position during this period. We also test the accuracy of our approximation method.

The model is highly successful in generating a very persistent net foreign asset position and a reasonably plausible trade balance for the United States. In accordance with other studies, the results show that introducing nontradable goods improves the ability of the model to obtain more reasonable cross-country correlations for total consumption. However, the resolution of this puzzle implies a degree of real exchange rate volatility that is much higher than the volatility of the actual series and cross-country output correlations that are similarly out of line. The results show that increasing the degree of intratemporal substitution between tradables and nontradables consumption significantly improves the ability of the model to match exchange rate and GDP moments, but at the cost of worsening cross-country consumption correlations. Hence, the introduction of nontradables seems to be a mixed blessing at best.

The framework in this paper can be extended in several directions to loosen the link between consumption smoothing and net foreign assets. First, the effect of production decisions on net foreign assets can be analyzed by endogenizing capital formation in the production process. Second, the model maps all heterogeneity across countries in different endowment patterns but treats consumers within a country as homogenous. Clearly, different preference structures, population dynamics and tax systems can affect net foreign assets and exchange rates and it is certainly worthwhile to incorporate these differences by relaxing the homogeneity assumption within a country. An overlapping generation framework as developed in Buiter (1981) might provide a convenient setup to address these issues. Finally, real rigidities can be more carefully modelled beyond the transportation costs in international trade in this model.

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