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# Skill Heterogeneity and Aggregation Bias over the Business Cycle

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## Abstract

This paper extends the equilibrium business cycle framework to incorporate ex ante skill heterogeneity among workers. Consistent with the empirical evidence, skilled and unskilled workers in the model face the same degree of cyclical variation in real wages although unskilled workers are subject to substantially higher procyclical variation in employment. Systematic cyclical changes in the average skill level of employed workers are shown to induce bias in aggregate measures of cyclical variation in the labor input, productivity, and the real wage. The introduction of skill heterogeneity improves the model's ability to match the empirical correlation between total hours and the real wage but the correlation between total hours and labor productivity remains higher than in the data.

JEL Classification Numbers:

E32, J20, J31

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## Summary

Equilibrium business cycle theorists have attempted to construct models that can match important patterns in aggregate macroeconomic data in terms of the relative cyclical variability and comovements with output of a set of key economic variables. These models have been successful at capturing many important features of postwar U.S. business cycles. However, one major deficiency of existing models is that they require a highly procyclical real wage, operating through the intertemporal substitution mechanism, to match the high degree of procyclical variation in aggregate hours in the U.S. economy. Furthermore, labor productivity is implied to be strongly procyclical and highly positively correlated with total hours and output. However, empirical studies using aggregate data have found that the average real wage is essentially acyclical. Economywide measures of average labor productivity have also been found to be weakly procyclical and not highly correlated with either total hours or output.

This paper constructs a model that explicitly incorporates skill heterogeneity of labor in order to examine if this feature improves the ability of a standard equilibrium business cycle model to describe the stylized facts of the business cycle. The specific empirical observation that motivates the model in this paper is that skilled workers face considerably less fluctuation in employment than unskilled workers. This implies that the average skill level of the employed workforce is countercyclical.

The cyclical variation in the quality of the labor force also implies that the average observed wage is likely to be a biased measure of cyclical movements in the mean of the underlying offer wage distribution (correcting for observed and unobserved worker heterogeneity). Similarly, measuring productivity simply as aggregate output divided by aggregate hours worked could potentially obscure the effect of cyclical variation in average worker quality. Moreover, even with a fairly standard production function, the introduction of heterogeneity in the labor force breaks the perfect correlation between average productivity and real wages that exists in homogeneous-agent business cycle models.

The results in this paper show that, for plausible parameter values, the aggregation effects discussed above can be quite large. However, although the introduction of heterogeneity improves the model's ability to match the empirical correlation between total hours and the real wage, the correlation between total hours and labor productivity remains higher than in the data. The model also does not perform well in improving the match between the predicted and actual ratio of the standard deviation of total hours relative to the standard deviation of productivity.

## I. Introduction

Equilibrium business cycle theorists have attempted to construct equilibrium models that can match important patterns in aggregate macroeconomic data in terms of the relative cyclical variability and the comovements with output of a set of key economic variables. Closed economy real business cycle (RBC) models have been successful at capturing many important features of the U.S. economy. However, one major drawback with standard RBC models, as pointed out by McCallum (1988), Christiano and Eichenbaum (1992), and others, is that RBC models require a strongly procyclical real wage, operating through the intertemporal substitution mechanism, to match the highly procyclical variation in aggregate hours in the U.S. economy. Further, since the standard Cobb-Douglas production function used in these models implies that the average product and marginal product of labor are proportional to the real wage, labor productivity is implied to be strongly procyclical and highly correlated with total hours and output. However, empirical studies using aggregate data have found that the average real wage is essentially acyclical. Economy-wide measures of productivity have also been found to be weakly procyclical and not highly correlated with total hours or output. These stylized facts appear to pose a problem for RBC proponents.

One aspect that is not captured by standard homogeneous-agent models of the business cycle is the possibility of cyclical variation in the average skill level of the employed workforce. Labor force heterogeneity is an important issue for business cycle theorists since it could potentially lead to bias in aggregate measures of macroeconomic quantities such as labor input, productivity, and the real wage. For instance, labor market studies using micro survey data have revealed the importance of accounting for skill heterogeneity in obtaining accurate measurements of wage cyclicality (see Stockman (1983), Bils (1985), Keane, Moffitt, and Runkle (1988), and Barsky, Solon, and Parker (1994)). Much of this work indicates that a composition-corrected measure of the real wage is more procyclical than an average measure of the real wage. In a similar context, labor economists such as Heckman (1984) have also argued that homogeneous-agent models of the business cycle do not capture many important aspects of labor market fluctuations.

The objective of this paper is to construct a model that explicitly incorporates skill heterogeneity of labor in order to examine if this feature improves the ability of a standard RBC model to describe the stylized facts of the business cycle. The model builds upon the earlier work of Kydland (1984) and Hansen and Sargent (1988) and is similar in spirit to the papers by Cho and Rogerson (1988) and Cho (1995). Cho and Rogerson (1988) examine how labor supply elasticities for different types of labor are influenced by fixed costs at the household level. Unlike in the Cho and Rogerson paper, the quasi-fixity of skilled labor in the present model arises from the effects of adjustment costs at the firm level on the demand for different types of labor. Cho (1995) examines the effect of ex-post skill heterogeneity in a model where workers face temporary idiosyncratic productivity shocks. The model in this paper differs from Cho's model in that workers are assumed to be ex-ante heterogeneous, although neither of these models captures the endogenous element of skill formation over the business cycle.

The specific empirical observation that motivates the model in this paper relates to the differences between skilled and unskilled workers in terms of employment variability. It has long been recognized that skilled and unskilled workers exhibit markedly different responses to the business cycle in terms of employment variation. Various studies, using different proxies for skills, have documented that skilled workers face considerably less fluctuation in employment than unskilled workers (see Rosen (1968), Clark and Summers (1981), and Kydland (1984)). Since skilled workers have more stable employment over the cycle than unskilled workers, it follows that the average skill level of the employed workforce is countercyclical. In a downturn, as less-skilled workers are laid off, the average skill level of the workforce starts rising. Similarly, as a recession ends and firms start hiring more workers, the average skill level of the employed workforce tends to decline (see, e.g., Hansen (1993)).

The differences in employment cyclicality for workers of different skill levels have implications for measurements of wage cyclicality. The cyclical variation in the quality of the labor force implies that the average observed wage is likely to be a biased measure of cyclical movements in the mean of the underlying offer wage distribution (correcting for observed and unobserved worker heterogeneity). In a recession, unskilled workers, who have lower mean wages than skilled workers, are more likely to be laid off. This implies that the average observed wage could have a countercyclical bias. The empirical studies cited earlier use micro panel data to control for variation in average worker quality over the cycle and generally find at least a weak countercyclical bias in aggregate wage measures.

Cyclical changes in the composition of the workforce also have important implications for the measurement of productivity over the business cycle. Measuring productivity simply as aggregate output divided by aggregate hours worked could potentially obscure the effect of cyclical variation in average worker quality. The effect of heterogeneity in the labor force needs to be taken into account in measuring the cyclicality of true underlying productivity. Further, even with a fairly standard production function, the introduction of heterogeneity in the labor force breaks the perfect correlation between average productivity and real wages that exists in homogeneous-agent RBC models.

The results in this paper show that, for plausible parameter values, the aggregation effects discussed above can be quite large. However, although the introduction of heterogeneity improves the model's ability to match the empirical correlation between total hours and the real wage, the correlation between total hours and labor productivity remains higher than in the data. The model also does not perform well in improving the match between the predicted and actual ratio of the standard deviation of total hours relative to the standard deviation of productivity.

The paper is organized as follows. Section II motivates the model by reviewing some of the measurement and aggregation issues. Section III describes the model. Section IV presents results from numerical simulations of the model. Section V concludes.

## II. Measurement

The effect of aggregation bias on measurements of fluctuations in labor market aggregates has received considerable attention recently. Much of this work has focused on addressing the conventional wisdom that average real wages are acyclical, i.e., uncorrelated with the business cycle. The very low correlation between the average real wage and total hours in the postwar U.S. (Table 1, first panel) appears to confirm this notion. This stylized fact has long been used as a benchmark for evaluating representative agent business cycle models. However, these models typically yield predictions for variations in the mean of the distribution of wage offers for a homogeneous worker. The observed average real wage in the data could be biased as a measure of this mean because the average quality of workers is not constant over the cycle. As noted in the introduction, an extensive body of literature has documented that skilled workers have more stable employment patterns over the business cycle than unskilled workers. This implies that the average skill of the employed workforce is negatively correlated with the cycle, suggesting that movements in the average wage tend to understate the cyclical variation in the mean of the offer wage distribution.

An additional source of bias comes from the fact that unobserved (by the econometrician) characteristics of workers could simultaneously affect their real wage and their cyclical employment probabilities. Such selectivity bias could affect measures of wage cyclicality obtained from micro panel data that control only for observed worker characteristics. Controlling for aggregation as well as selectivity bias, the general conclusion appears to be that there is significant procyclical variation in the offer wage distribution (see, e.g., Keane, Moffitt, and Runkle (1989) and Barsky, Solon, and Parker (1994)).

In related work, using data from the Panel Study of Income Dynamics, Kydland and Prescott (1988) show that a human capital weighted measure of labor input is much less cyclically variable than aggregate manhours. Since unskilled workers have greater cyclical variation in employment, they account for a greater fraction of the variance in aggregate manhours. However, because unskilled workers have relatively smaller human capital weights, the use of these weights dampens the effect of fluctuations in unskilled employment on labor input variation. Kydland and Prescott calculate the average hourly wage over the sample period for each individual in the sample and use this as a measure of that individual's human capital weight. Over their sample period (1969-82), the percentage standard deviation of the conventional measure of aggregate hours is 1.42 (similar to the U.S. aggregate) while it is only 0.98 for a quality-adjusted measure of labor input. Both series have a correlation of about 0.75 with detrended output. The empirical elasticity with respect to output is also much higher for aggregate hours (0.45) than it is for the quality-adjusted labor input measure (0.29). Compensation per manhour has a percentage standard deviation of only 0.54, compared to 0.87 for the wage per efficiency-hour. The latter is also much more highly correlated with output deviations from trend (0.53) than is the former (0.14). In other words, when corrected for cyclical variation in average worker quality, the labor input has lower procyclical variation while the real wage and labor productivity are both more procyclical than is suggested by aggregate average measures of these variables.

The correspondence between model concepts and the measurement of stylized facts is of considerable importance for business cycle theorists (Prescott (1986)). The micro evidence discussed above strongly suggests that aggregate measures of employment, wage, and productivity fluctuations are contaminated by substantial aggregation bias induced by systematic changes in the composition of the workforce over the cycle.[1] Since it may not be possible to account for all sources of aggregation bias in empirical measures, it appears worthwhile to explicitly model heterogeneity in theoretical models of the business cycle and to assess the impact of heterogeneity on improving the match between theory and stylized facts. Before proceeding to analyze labor market heterogeneity, it is useful to impose some discipline on such an exercise by relating the theoretical work to empirical results on disaggregated measures of cyclical labor market fluctuations.

The relative cyclical variation of employment and real wages for skilled and unskilled labor is of crucial importance for the issues addressed in this paper. While it is well established that skilled workers have much less employment variability than unskilled workers, the evidence on relative wage variability is much less conclusive. Reder (1955, 1962) concludes that unskilled workers have more procyclical variation in wages than do skilled workers. Raisian (1983), on the other hand, finds evidence that skilled workers may face more procyclical variation in wages than unskilled workers.

More recently, using data from the National Longitudinal Survey of Young Men and correcting for various sources of aggregation and selectivity bias, Keane and Prasad (1993) conclude that, at the aggregate level, skilled and unskilled workers face essentially the same degree of cyclical variation in their offer wages.[2] That is, the relative offer wage differential between skilled and unskilled workers is acyclical. This result also mitigates the additional concern that differential patterns of wage variation for skilled and unskilled workers may exacerbate the bias inherent in average measures of wage cyclicality. The fact that skilled and unskilled workers have the same degree of cyclical wage variation is useful in calibrating the model developed in the next section.

## III. The Model

Consider a closed economy with a continuum of identical households that are uniformly distributed along the unit interval [0,1]. Each household consists of two infinitely-lived agents, one skilled (type 1) and the other unskilled (type 2). All agents are born at time zero and acquire their respective skill (or human capital) endowments at birth. Households in this economy have utility functions that are defined over consumption and leisure. While the household cares only about aggregate consumption, it attaches different weights to the utility derived from the leisure of skilled and unskilled workers. The household utility function is given by the specification

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[1] In the remainder of the paper, the term aggregation bias is to be interpreted broadly to include selectivity bias and other sources of bias in aggregate relative to disaggregate data.

[2] Barksy, Solon, and Parker (1994) report a similar finding using a different dataset.

$$U(c_t,\ell_{1t},\ell_{2t}) = \log c_t + \psi A \log \ell_{1t} + (1-\psi)A \log \ell_{2t} \qquad 0<\psi<1 \tag{1}$$

where $c_t$ represents household consumption and $\ell_{it}$ represents the leisure of a type i agent at time t. Each agent has a unit time endowment in every period, which can either be consumed as leisure or supplied as labor in return for a competitively determined real wage. Agents in this economy are restricted to work exactly $0<h<1$ hours and consume the remaining time endowment as leisure. This assumption of an exogenous fixed shift length is motivated by the observation that a substantial portion of the high frequency variation in aggregate man-hours in postwar U.S. data is attributable to fluctuations in employment rather than fluctuations in hours per worker. I assume that skilled and unskilled workers work the same shift lengths when employed.[3]

Since agents are restricted to work either zero hours or a fixed number of hours, household consumption sets are non-convex. Following Rogerson (1988) and Hansen (1985), the commodity space is convexified by expanding it to include employment lotteries. Households now choose a probability of their agents working in a given period, rather than a particular number of man-hours that the agents will supply. Hansen (1985) shows that, in an economy with homogeneous risk-averse agents and separable preferences, the employment lottery implies complete unemployment insurance. In the economy described here, households have the same level of consumption whether both their members are employed or unemployed, or only one of the two members is employed.

The model in this paper has two independent employment lotteries, one for skilled workers and another for unskilled workers. The expected utility of each household is then defined over total household consumption and the probability of employment of each type of worker. The law of large numbers permits a one-to-one mapping between probabilities at the household level and aggregate fractions in this economy. Hence, the preferences of the representative household can now be written more concisely as

$$\log c_t - \psi a n_{1t} - (1-\psi) a n_{2t} \tag{2}$$

where $a \equiv -A(\log(1-h))/h$, and $n_{it} = \alpha_{it} \cdot h$ is the total number of hours worked by type i workers.

The economy has a single firm in which all households have equal shares of ownership. The firm hires labor and rents capital each period to produce the single

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[3] The average length of the workweek is not very sensitive to skill level. In the NLS data (see Keane and Prasad (1993) for a description of this micro panel dataset), the reported "usual weekly hours" for most workers tend to cluster around the figure of 40 hours a week. About 60 percent of the observations for weekly hours lie between 38 and 42 hours.

non-storeable consumption good using a production technology of the form

$$y_t = \theta_t K_t^{\alpha} N_t^{1-\alpha}. \tag{3}$$

Output at time t is denoted by $y_t$. The exogenous productivity or "technology" shock in the economy is given by $\theta$. Physical capital, denoted by K, is assumed to be fully utilized in all states of the world. The income share of capital is given by $\alpha$. Labor input, $N_t$, is given by the following CES function of skilled and unskilled labor:

$$N_t = \left\{ \omega \left[ n_{1t} - \frac{d}{2} (n_{1t} - n_{1,t-1})^2 \right]^{1-\nu} + n_{2t}^{\;1-\nu} \right\}^{\frac{1}{1-\nu}}. \tag{4}$$

The parameter $\omega > 1$ captures the higher productivity of skilled labor relative to that of unskilled labor. The real resource cost of adjusting the level of skilled labor is given by the quadratic adjustment cost term embedded in the production function.[4] This adjustment cost may be interpreted, for instance, as reflecting the training cost involved in hiring new skilled workers. The hiring and firing costs for skilled workers are assumed to be symmetric.[5] The parameter $d \geq 0$ determines the magnitude of this adjustment cost. With $d = 0$, the model would have the property that the ratio of skilled to unskilled employment would remain constant over the cycle.

The exogenous productivity shock $\theta$ is assumed to follow a first-order autoregressive process of the form

$$\theta_{t+1} = \rho\theta_t + \varepsilon_{t+1}. \qquad 0<\rho<1 \tag{5}$$

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[4] This specification provides a straightforward interpretation of output as a function of capital and the effective units of labor. For instance, if a new worker must be trained, then it is natural to assume that, of the hours supplied by that worker to the firm, some fraction is used to acquire training and only the remainder is available for the production process. Hansen and Sargent (1988) use a similar adjustment cost specification to examine the relatively higher cyclical variability of overtime relative to straight-time employment.

[5] An alternative justification for the adjustment cost on skilled labor could be in terms of firm-specific capital. For instance, the firm-specific capital embodied in skilled workers would be lost each time a separation occurred between a firm and a skilled worker, implying a real resource cost. However, since the model does not keep track of the identities of individual workers, the justification for these adjustment costs in terms of firm-specific capital would be tenuous. Also see Oi (1962) for one of the earliest models where skilled labor is postulated to be a quasi-fixed factor.

The productivity shock innovation $\varepsilon$ is identically and independently distributed across time and is drawn from a log-normal distribution with mean $1-\rho$ and standard deviation $\sigma_\varepsilon$. Following Hansen (1985), this distribution is assumed to have a positive support with a finite upper bound, in order to ensure that output is always positive. Since the mean of $\varepsilon$ is $1-\rho$, equation (5) implies that the unconditional mean of $\theta$ is unity. In each period, agents make decisions after the realization of the productivity shock is observed and is common knowledge.

The representative household in this economy takes the wage rates for the two types of labor, $w_{1t}$ and $w_{2t}$, and the real interest rate $r_t$ as given. The household's problem is to choose a sequence $\left\{c_t, n_{1t}, n_{2t}, x_t\right\}_{t=0}^{\infty}$ to maximize, given $K_0$ and $\theta_0$,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log c_t - \psi a n_{1t} - (1-\psi) a n_{2t} \right\} \qquad 0<\beta<1 \tag{6}$$

subject to

$$c_t + x_t \le h n_{1t} w_{1t} + h n_{2t} w_{2t} + r_t K_t + \chi_t, \text{ and} \tag{7}$$

$$K_{t+1} = (1-\delta)K_t + x_t. \tag{8}$$

Profits earned by the representative firm, if any, are distributed on a lumpsum basis among households and are denoted above by $\chi_t$. The evolution of the physical capital stock $K_t$ is given by equation (8), with $\delta$ being the constant depreciation rate and $x_t$ representing gross investment.[6] Combining the representative household's first order conditions for its labor supply choices yields the following relation between $w_{1t}$ and $w_{2t}$ that must be satisfied for any optimal interior solution for $n_{1t}$ and $n_{2t}$:

$$w_{1t} = \frac{\psi}{1-\psi} w_{2t}. \tag{9}$$

This condition results from the specification of the representative household utility function, which is linear in the leisure of both types of agents. The proportionality between the two wage rates will play an important role later in the analysis.

Since there are no distortions in this economy, an extension of standard welfare

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[6] Since all households in this economy are identical, no distinction is made between household and aggregate capital stocks in order to simplify the notation.

theorems, drawn from Rogerson (1988), allows us to exploit the fact that the competitive outcome in this economy corresponds to the solution of a social planning problem. The social planning problem is to choose a contingency plan for $(c_t, n_{1t}, n_{2t}, x_t)$ to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log c_t - \psi a n_{1t} - (1-\psi) a n_{2t} \right\} \tag{10}$$

subject to

$$c_t + x_t \le \theta_t K_t^{\alpha} N_t^{1-\alpha}, \text{ where } N_t = \left\{ \omega \left[ n_{1t} - \frac{d}{2} (n_{1t} - n_{1,t-1})^2 \right]^{1-\nu} + n_{2t}^{1-\nu} \right\}^{\frac{1}{1-\nu}} \tag{11}$$

$$K_{t+1} = (1-\delta) K_t + x_t, \text{ and} \tag{12}$$

$$\theta_{t+1} = \rho\theta_t + \varepsilon_{t+1}. \tag{13}$$

The state vector for this problem is $[K_t \ n_{1,t-1} \ \theta_t]'$. The control vector is $[c_t \ n_{1t} \ n_{2t} \ x_t]'$. Assuming that the aggregate resource constraint (11) is binding in each period, the analysis can be limited to the control vector $[n_{1t} \ n_{2t} \ x_t]'$. The solution to the recursive problem described above is a set of time-invariant decision rules (or policy functions) that express the optimal values of the choice variables as functions of the state vector. Since it is not possible to obtain closed-form expressions for the policy functions, the strategy adopted in this paper is to derive a linear-quadratic model that approximates the social planning problem. Before proceeding to solve numerically for the policy functions, however, it is useful to describe some equilibrium relationships that characterize the optimal policy functions.

As noted before, all agents of a particular skill level earn the same income in any given period, whether or not they happen to be employed in that period (depending on individual realizations from the lottery). Thus, the model is not designed to explain variations in consumption patterns across employed and unemployed workers or between skilled and unskilled workers.

While analytical expressions for the decision rules for employment cannot be obtained, it is possible to analyze the main determinants of the relative variability of skilled and unskilled employment. The adjustment cost on skilled labor leads to a relatively higher degree of serial correlation in the employment of skilled labor and makes it less sensitive to variations in output than unskilled labor. The variation in skilled employment is, inter alia, a function of the adjustment cost parameter d and the parameter $\rho$, which governs the persistence of the technology shock. The ratio of the cyclical variation in skilled employment relative to unskilled employment will tend to be positively related to the degree of persistence in the technology shock process and negatively related to the size of the adjustment cost parameter. The steady-state relationship between the levels of employment of the two types of labor is given by

$$n_1 = \vartheta \cdot n_2 \qquad \text{where} \quad \vartheta = \left[\frac{\omega(1-\psi)}{\psi}\right]^{\frac{1}{\nu}}. \tag{14}$$

Loosely speaking, the parameters $\omega$, $\psi$, and $\nu$ determine the relative levels of employment of skilled and unskilled labor while the parameters $\rho$ and d determine the relative cyclical variability of these two types of labor.

With positive adjustment costs, it is intuitive that skilled workers would face much less cyclical variation in employment than unskilled workers. In a recession, skilled workers would be less likely to be laid off, thereby raising the average skill level of the employed workforce. In an expansion, since it is more costly to increase the level of skilled employment in the short run in response to transitory productivity shocks, relatively higher numbers of unskilled workers are likely to be hired. This would reduce the ratio of skilled to unskilled employment and thereby lower the average skill level of the employed workforce. This discussion suggests that the average skill level of the employed workforce is countercyclical in this model. A more adequate measure of labor input than aggregate manhours can be calculated here using the definition of labor input in equation (4). This measure controls for the systematic movement of low or high productivity workers into or out of employment over the business cycle and, hence, provides an unbiased measure of the cyclical variation in labor input. If skilled workers have less variable employment than unskilled workers, then labor input is likely to have much less cyclical variation than an unweighted measure of aggregate hours.

Turning next to labor compensation, spot wages in this model are determined in a competitive labor market. From the production function, the marginal products for the two labor inputs are given by

$$MP_{1t} = \theta_t K_t^{\alpha} N_t^{\frac{\nu-\alpha}{1-\nu}} \left(1-\alpha\right) \left[\omega\left(n_{1t} - \frac{d}{2}(n_{1t}-n_{1,t-1})^2\right)^{-\nu}\right]\left[1 - d(n_{1t}-n_{1,t-1})\right], \text{ and} \tag{15}$$

$$MP_{2t} = \theta_t K_t^{\alpha} N_t^{\frac{\nu-\alpha}{1-\nu}} \left(1-\alpha\right) \left[n_{2t}^{-\nu}\right]. \tag{16}$$

The marginal product of skilled labor, shown in (15), is evaluated only in terms of current output. Skilled workers have an asset value to the firm that is not incorporated in this expression. This is, however, subsumed in the social planning problem since the maximization over $n_{1t}$ would incorporate a term that can be interpreted as the future value of the current stock of skilled labor. The household's maximization problem indicates that, in equilibrium, $n_{1t}$ and $n_{2t}$ will adjust to keep the ratio of the two wage rates constant in every period.[7] Hence, equilibrium wage rates in this economy are calculated in the following manner:

$$w_{2t} = MP_{2t} \quad \text{and} \quad w_{1t} = \frac{\psi}{1-\psi}\, w_{2t}. \tag{17}$$

Unskilled workers are paid their marginal product in each period and equation (9) is then used to pin down $w_{1t}$.[8] Note that, using equations (15) and (16), the steady state ratio of the skilled wage to the unskilled wage is given by

$$\frac{w_1}{w_2} = \omega \cdot \left[ \frac{n_1}{n_2} \right]^{-\nu} \tag{18}$$

Substituting equation (14) into equation (18) yields a ratio that is identical to the equilibrium ratio of the two wage rates obtained from the household's optimization problem, confirming that $w_1$ and $w_2$ are market-clearing wage rates.

The average wage in this economy, obtained by dividing total earnings by aggregate hours, is likely to be a biased measure of cyclical variation in the mean of the offer wage distribution. As noted earlier, if unskilled (low wage) workers have more procyclical employment than skilled (high wage) workers, then the average observed wage would have a countercyclical bias. A measure of wage cyclicality that controls for this source of aggregation bias can be obtained by dividing total labor earnings by the labor input in this model, yielding a measure of compensation per effective unit of labor input. Similarly, efficiency-weighted productivity can be constructed as a measure of output per unit of effective labor input. If labor input fluctuates less than aggregate hours, this definition suggests that, for given variation in output, efficiency-weighted productivity may have greater cyclical variation than an unweighted measure of productivity.

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[7] The hourly wage rates are given by $h^{-1}w_{1t}$ and $h^{-1}w_{2t}$. Since the shift length h is fixed and identical for skilled and unskilled workers, the distinction between the shift wage and the hourly wage is not important here.

[8] This suggests that profits may not necessarily be zero in every period. While the discounted expected profit stream of the firm will indeed equal zero, profits may be nonzero away from the steady state. This is not inconsistent with a competitive equilibrium since the adjustment cost in the model arises at the level of the firm. Profits, if any, are transferred in a lumpsum fashion to households (which, as stated earlier, own equal shares in the representative firm).

## IV. Results

In this section, I present results from numerical simulations of the model developed above, using the conventional strategy of calibrating the model by using parameter values that have been independently estimated in various studies. To solve the model numerically, I implement a solution procedure proposed by Hansen and Sargent (1988). First, a linear-quadratic approximation to the social planning problem described in the previous section is derived. The approximating model is then solved and the implied linear systems representation for the observables in the model is obtained.

I begin by discussing the model calibration procedure. The first set of results comes from simulations of a homogeneous-agent model, which is a special case of the model in this paper. This provides a benchmark for a numerical assessment of aggregation effects using the heterogeneous-agent model. Finally, the sensitivity of the results to the calibrated parameter values is examined.

### 1. Model Calibration

I adopt the values for the parameters $\{\beta, h, a, \alpha, \delta, \rho,\}$ directly from standard papers in the real business cycle literature, such as Hansen (1985) and Prescott (1986). The standard deviation of the innovation to the productivity shock ($\sigma_\varepsilon$) is chosen such that the mean standard deviation of output for the benchmark homogeneous-agent model matches the standard deviation around trend of quarterly U.S. GDP over the period 1954:1-1994:4. That leaves four parameters to be determined: d, $\omega$, $\nu$, and $\psi$.

The parameter $\psi$ determines the steady-state ratio of the (hourly/weekly) wage of skilled workers to the (hourly/weekly) wage of unskilled workers. In the NLS data, the average wage premium for workers with a college degree is about 40 percent of the wage paid to workers without a degree. However, college-educated workers represent only about 23 percent of the person-year observations in the sample, while the model in this paper divides the labor force equally into skilled and unskilled categories. Using labor market experience as an additional proxy for skill, I add enough workers (sorted by experience level) to the category of skilled workers so that this category comprises half of the sample. The average wage for the skilled half (college-educated or more experienced) of the sample chosen in this manner is roughly 43 percent more than the average wage of the unskilled half (non-college educated and relatively less experienced) of the sample. This implies a value of about 0.59 for $\psi$.

The ratio of the level of employment of skilled to unskilled labor restricts either $\omega$ or $\nu$. Choosing the sample of skilled workers as described above, skilled workers are found to have about 45 percent more average hours worked in a year than unskilled workers.[9] Unfortunately, there is little direct evidence available to restrict

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[9] It should be noted that this could potentially overstate the ratio of average hours worked for skilled relative to unskilled workers. Workers are considered unemployed for the whole year if they report a zero wage for the survey period. This could cause more unskilled workers to be assigned zero hours in any year since they have, on average,

$\nu$, which determines the elasticity of substitution between skilled and unskilled labor. Estimates by Halvorsen and Ford (1979) indicate a range from about zero to 0.65 for industries in the manufacturing sector. Setting $\nu$ equal to zero and to the same value as the parameter alpha represent interesting special cases of the model. The effects of using alternative values of $\nu$ will be discussed below. The choice of $\nu$ pins down $\omega$ using equation (14).

The adjustment cost parameter d cannot be determined by a comparison of steady state values. In the NLS data, college-educated workers have much less cyclical variability in employment than uneducated workers. However, a worker's experience level does not appear to have much effect on the cyclical variability of employment probabilities. I pick d to yield a ratio of the standard deviation of skilled to unskilled employment of between one-third and one-fourth. This is marginally higher than the variability of employment of college-educated workers relative to workers without a college degree. Experiments with alternative values of d will be discussed later.

To summarize, the parameter values used in simulations of the basic heterogeneous agent model are as follows: $\beta$ = 0.99; h = 0.53; a = 2.85; $\psi$ = 0.59; $\alpha$ = 0.36; $\delta$ = 0.025; $\rho$ = 0.95; $\sigma_\varepsilon$ = 0.0067; and d = 100. The choice of $\nu$ (and $\omega$) will be discussed below. The statistics reported in this section are for logged and Hodrick-Prescott filtered versions of the simulated time series. The residuals from this procedure are interpretable as percentage deviations of the series from their respective trends.

**2. The Homogeneous-Agent Model**

The model studied here can be specialized to Hansen's (1985) homogeneous-agent model with indivisible labor by setting $\psi$ = 0.5, $\omega$ = 1, $\nu$ = 0, and d = 0. Numerical simulations of this model provide a benchmark for examining the effects of incorporating skill-heterogeneity in an RBC framework. Results from stochastic simulations of the homogeneous-agent model are presented in the second panel of Table 1. Aggregate hours are strongly procyclical and the ratio of the standard deviation of total hours to that of productivity is 2.30, somewhat more than the value of this ratio in the data (1.81). An important shortcoming of this model is that it predicts a correlation of 0.62 between productivity and total hours, while the empirical correlation is close to zero. Since the real wage and productivity are proportional in the homogeneous-agent model, all relevant statistics are identical for productivity and the real wage. This model also counterfactually predicts a high degree of correlation between labor productivity and cyclical output.

**3. The Heterogeneous-Agent Model**

For the baseline heterogeneous-agent model, I pick $\nu$ = 0.4, which is roughly in the middle of the range of estimates reported by Halvorsen and Ford (1979). Using the

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lower employment probabilities than skilled workers.

Table 1

Comparison of the stochastic properties of the models relative to postwar U.S data.

| | U.S. data 1954:1–1994:4 | Homogeneous–agent model | | Heterogeneous–agent model | | | |
|---|---|---|---|---|---|---|---|
| | | | | Average measures | | Efficiency units | |
| sd(y) | 1.60 | 1.60 | (0.22) | 1.24 | (0.15) | | |
| sd(c) / sd(y) | 0.75 | 0.34 | (0.03) | 0.38 | (0.05) | | |
| sd(x) / sd(y) | 3.46 | 3.09 | (0.10) | 3.83 | (0.17) | | |
| sd(h) / sd(y) | 1.15 | 0.76 | (0.02) | 0.57 | (0.03) | 0.47 | (0.03) |
| sd(p) / sd(y) | 0.64 | 0.33 | (0.03) | 0.52 | (0.03) | 0.60 | (0.03) |
| sd(w) / sd(y) | 0.57 | 0.33 | (0.03) | 0.41 | (0.04) | 0.47 | (0.04) |
| sd(h) / sd(p) | 1.81 | 2.30 | (0.22) | 1.10 | (0.12) | 0.79 | (0.08) |
| corr(c,y) | 0.88 | 0.81 | (0.04) | 0.62 | (0.08) | | |
| corr(x,y) | 0.90 | 0.98 | (0.01) | 0.96 | (0.01) | | |
| corr(h,y) | 0.88 | 0.96 | (0.01) | 0.93 | (0.03) | 0.91 | (0.03) |
| corr(p,y) | 0.49 | 0.80 | (0.04) | 0.91 | (0.03) | 0.95 | (0.02) |
| corr(w,y) | 0.37 | 0.80 | (0.04) | 0.73 | (0.06) | 0.86 | (0.04) |
| corr(h,p) | 0.08 | 0.61 | (0.08) | 0.69 | (0.09) | 0.74 | (0.08) |
| corr(h,w) | 0.18 | 0.61 | (0.08) | 0.44 | (0.11) | 0.60 | (0.11) |

Notes: The term "sd" stands for standard deviation and "corr" represents the correlation coefficient. The variable definitions are as follows: y–output; c–private consumption; x–gross fixed investment; h–total hours; p–labor productivity; and w–real wage. Seasonally adjusted quarterly data for the United States were taken from the U.S. Macroeconomic Database maintained by WEFA.
Output (real GDP) and its components are measured in 1987 constant dollars. Data on total hours, labor productivity, and the real wage were obtained for the nonfarm business sector. All statistics reported in this table are for data logged and detrended using the Hodrick–Prescott filter with a smoothness parameter of 1600. The model statistics represent the means of 100 stochastic simulations. Each simulation was 164 periods long, matching the sample size for the U.S. data. Standard deviations of the sample distributions are in parentheses.

ratio of steady state skilled to unskilled employment, this implies that $\omega = 1.67$. Results from simulations of this model are reported in the third panel of Table 1. Compared to the homogeneous-agent model, this model implies lower variability in total hours relative to output, although the variability of productivity and the real wage relative to output are both higher.[10] This is attributable to the adjustment cost for changing the employment level of skilled labor, leading to lower variation in total hours. The ratio of the standard deviation of total hours relative to that of productivity is lower in this economy than in the homogeneous-agent economy.

The correlation between productivity and hours is not very different in the two models. However, a key difference is that the correlation between total hours and the real wage drops by almost 30 percent, bringing this correlation closer to the empirical correlation. This is largely attributable to the composition effect. By construction, real wages for both types of labor move in proportion to each other and, hence, have the same percentage standard deviation and correlations with output and total hours. However, skilled employment is much less variable and is not as highly correlated with cyclical output as is unskilled employment.[11] Thus, although the skilled and unskilled wage are both highly positively correlated with total hours (correlation of 0.85, with a standard deviation of 0.04), the cyclical variation in the composition of the workforce reduces the correlation between the average wage and total hours. Note that the real wage and productivity are not perfectly correlated in the heterogeneous-agent economy. Although the predicted correlation is still very high (0.93, with a standard deviation of 0.02), it is a little closer to the empirical correlation of about 0.6 between these two variables.

A comparison of the raw aggregates and efficiency-weighted measures in this economy provides an indication of the magnitude of aggregation bias induced by skill heterogeneity. Relative to the standard deviation of output, the standard deviation of efficiency-weighted hours (i.e., the labor input) is less than that of total hours. On the other hand, both productivity and the real wage have higher cyclical variability relative to output when measured in efficiency units rather than as economy-wide averages. The ratio of the standard deviation of the labor input relative to productivity in efficiency units is substantially lower than the corresponding ratio for total hours and average productivity. Another striking effect of aggregation bias occurs in the case of the hours-real wage correlation, which is about 40 percent higher when measured in efficiency units.

This model produces exactly the type of aggregation effects that have been found by Kydland and Prescott (1988). Although the direction of bias is the same, the

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[10] Note that the standard deviation of output in the heterogeneous-agent economy is below that in the homogeneous-agent economy, largely because of the lower cyclical variation in one of the factor inputs--labor. To match the empirical standard deviation of output, the standard deviation of the productivity shock would have to be increased to about 0.0080.

[11] The percentage standard deviation of skilled (unskilled) employment is 0.35 (1.16) and its correlation with output is 0.57 (0.92).

magnitude of the aggregation bias in average measures of labor input, productivity, and the real wage is smaller than the magnitudes reported by these authors. This may partly be attributable to the very broad skill disaggregation of the labor force used in this model.

An additional dimension in which one might evaluate the performance of RBC models is their prediction for the elasticity of labor with respect to output.[12] In the U.S. economy, this elasticity is close to unity over the period 1954:1-1994:4. The homogeneous- agent model produces a value of 0.73. In this dimension, the heterogeneous-agent model appears to perform poorly as it yields an elasticity of only 0.53. However, the elasticity of quality-weighted hours with respect to output, approximately 0.43, is even lower than the elasticity for the aggregate hours measure, which is exactly the finding reported by Kydland and Prescott.

Figure 1 shows the impulse response functions of some of the observables in the baseline heterogeneous-agent model to a unit positive innovation in the technology shock. Panel 1 shows that the cost of adjusting the level of skilled employment causes it to respond sluggishly unlike unskilled employment which takes a large upward jump when the shock to productivity occurs and then falls off rapidly. Panel 2 reveals an interesting pattern. Total hours rise sharply when the shock hits and then decline rapidly, while the skill-weighted measure of hours has a much smoother response. Initially, total hours rise more than hours in efficiency units since unskilled workers are hired in a greater proportion. Skilled workers are hired gradually but are also laid off more gradually as the shock wears off. Hence, after a few periods, the impulse response for efficiency-weighted hours rises above that of total hours. Panel 3 shows that the impulse response for productivity in efficiency units rises faster and then falls off at a faster rate than that of average productivity. This follows from the nature of the impulse responses for total hours and skill-weighted hours. Finally, the fourth panel indicates that the average wage has a more subdued response to the shock then the efficiency-weighted wage.

Table 2 reports the results of varying a few key parameters in the model. The results reported above do not, in most cases, appear to hinge on the particular values chosen from the calibration exercise for any of the key parameters. In general, the direction of bias in average aggregate measures of the labor input, the real wage, and productivity is not sensitive to the choice of the parameters examined in this table, although the magnitude of bias is somewhat sensitive to certain parameters. The cyclical variability of skilled employment relative to unskilled employment is, as expected, affected by the choice of certain parameters such as the adjustment cost parameter d.

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[12] This elasticity is given by $\mathrm{cov}(h_t, y_t)/\mathrm{var}(y_t)$ or, alternatively, may be computed as $\mathrm{corr}(h_t, y_t)\cdot\sigma_h \,/\, \sigma_y$, where $\sigma_i$ represents the standard deviation of series i. Earlier discussion in the literature has focused on the labor elasticity of output, which is simply the reciprocal of the measure discussed above. I work with the former measure since it is the measure that Kydland and Prescott (1988) report in their paper.

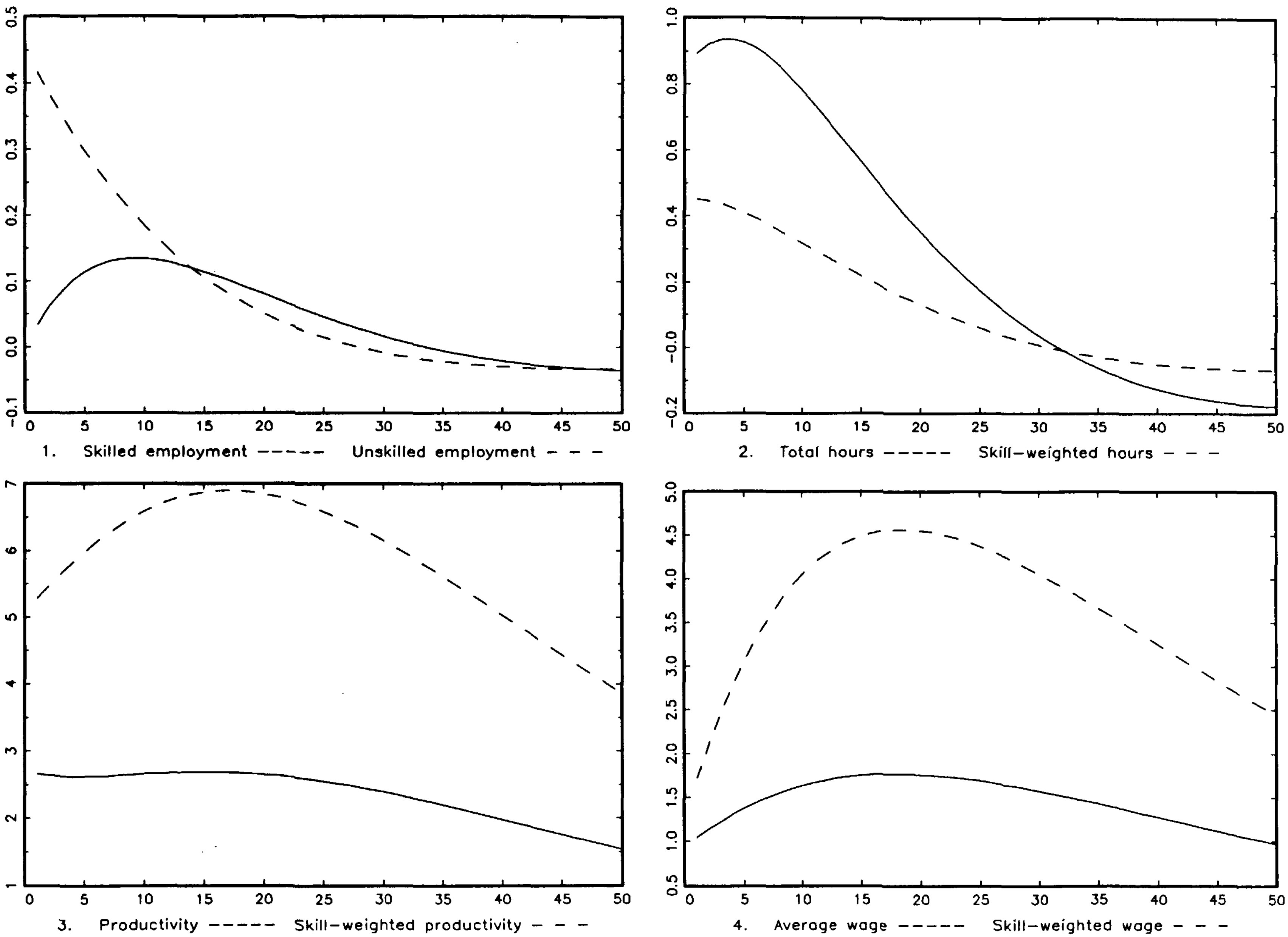
Table 2

Sensitivity of the heterogeneous–agent economy to calibrated parameters

| | nu = 0.1 | | nu = 0.55 | | phi = 0.66 | | d = 200 | |
|---|---|---|---|---|---|---|---|---|
| | Avg. measures | Efficiency units | Avg. measures | Efficiency units | Avg. measures | Efficiency units | Avg. measures | Efficiency units |
| sd(y) | 1.39 (0.21) | | 1.39 (0.20) | | 1.27 (0.17) | | 1.22 (0.15) | |
| sd(h) / sd(y) | 0.75 (0.03) | 0.61 (0.02) | 0.80 (0.16) | 0.64 (0.12) | 0.59 (0.04) | 0.46 (0.04) | 0.55 (0.03) | 0.43 (0.02) |
| sd(p) / sd(y) | 0.36 (0.03) | 0.45 (0.03) | 0.67 (0.09) | 0.71 (0.08) | 0.56 (0.06) | 0.66 (0.05) | 0.54 (0.04) | 0.63 (0.03) |
| sd(w) / sd(y) | 0.32 (0.04) | 0.39 (0.03) | 0.63 (0.10) | 0.64 (0.09) | 0.51 (0.08) | 0.55 (0.06) | 0.42 (0.05) | 0.48 (0.04) |
| sd(h) / sd(p) | 2.10 (0.23) | 1.36 (0.12) | 0.80 (0.16) | 0.64 (0.12) | 1.06 (0.15) | 0.70 (0.09) | 1.02 (0.10) | 0.69 (0.06) |
| sd(n1) / sd(n2) | 0.09 (0.01) | | 0.42 (0.04) | | 0.38 (0.04) | | 0.19 (0.02) | |
| corr(h,y) | 0.96 (0.01) | 0.95 (0.02) | 0.77 (0.12) | 0.76 (0.13) | 0.87 (0.06) | 0.85 (0.08) | 0.92 (0.03) | 0.91 (0.04) |
| corr(p,y) | 0.79 (0.05) | 0.92 (0.03) | 0.88 (0.04) | 0.92 (0.03) | 0.86 (0.04) | 0.93 (0.03) | 0.92 (0.03) | 0.96 (0.01) |
| corr(w,y) | 0.64 (0.07) | 0.86 (0.04) | 0.70 (0.08) | 0.80 (0.06) | 0.66 (0.07) | 0.84 (0.05) | 0.72 (0.06) | 0.86 (0.04) |
| corr(h,p) | 0.57 (0.09) | 0.76 (0.07) | 0.39 (0.18) | 0.46 (0.19) | 0.50 (0.15) | 0.60 (0.15) | 0.70 (0.10) | 0.77 (0.09) |
| corr(h,w) | 0.39 (0.11) | 0.67 (0.09) | 0.13 (0.17) | 0.27 (0.20) | 0.23 (0.15) | 0.46 (0.16) | 0.41 (0.13) | 0.60 (0.12) |

**Notes: The reported statistics are the means of 100 stochastic simulations, each 164 periods long. Standard deviations of the sample distributions are in parentheses. For the baseline heterogeneous–agent model, the parameter values are as follows: nu = 0.4; phi = 0.59; and d = 100. In the baseline model, sd(n1) / sd(n2) = 0.30 (std. dev.: 0.03). Also see notes to Table 1.**

Figure 1. Impulse Response Functions



## V. Conclusions

In this paper, I constructed a closed economy RBC model that allowed for ex-ante skill heterogeneity among workers. Numerical simulations of the model showed that the magnitude of aggregation bias induced by ignoring skill-heterogeneity could be quite large in measurements of cyclical variation in the labor input, productivity, and the real wage. While the model revealed magnitudes of aggregation bias consistent with recent empirical studies using micro data, it performed only moderately well in terms of improving the "fit" of RBC models to actual data. Relative to a benchmark homogeneous agent model, the model with skill heterogeneity did better in matching the empirical correlation between total hours and the real wage, but the implied correlation between total hours and labor productivity remained higher than in the data. The model also did not perform well in improving the match between the predicted and actual ratio of the standard deviation of total hours relative to the standard deviation of productivity.

A possible avenue for improving the model's performance would be to experiment with less restrictive production function specifications. For instance, there is some evidence to suggest that capital and skilled labor are complements, while capital and unskilled labor are substitutes (see Rosen (1968) and the references in Keane and Prasad (1995)). However, the available evidence is far from robust and is very sensitive to the econometric specification, sample period, and choice of industry.

Heterogeneity in the labor market is an important issue that needs to be recognized in the interpretation of stylized facts and the construction of theoretical models that attempt to replicate these stylized facts. This paper has contributed to the literature showing that it is possible to fruitfully study such aggregation issues within the framework of equilibrium business cycle models.

## Appendix

This appendix describes the procedure used to derive a linear-quadratic approximation to the social planning problem described in the paper, solve the approximating model, and derive the implied linear systems representation for the observables in the model. The solution technique described in this section draws on the work of Hansen and Sargent (1988).

### A.1 Linear-Quadratic Approximation for the Nonlinear Model

This section describes the steps involved in obtaining a linear-quadratic approximation to the planning problem described in equations (10) - (13) in the paper.

(i) Set the technology shock to its unconditional mean of unity and set all variables equal to their steady state values. For all t, $c_t = c$, $n_{1t} = n_1$, $n_{2t} = n_2$, $K_t = K$, $k_t = k$, $\theta_t = 1$. In the steady state, net investment is equal to zero and gross investment is just sufficient to replace depreciated physical capital. That is, $k = \delta K$. We now have a nonstochastic version of the original planning problem.

A set of four steady state conditions must be satisfied in equilibrium. The first three conditions are essentially the first order conditions for the planner's problem of choosing optimal consumption and labor supply when faced with appropriate marginal products that correspond to competitively determined real wages. The Lagrange multiplier on the aggregate resource constraint is denoted by $\lambda$. The fifth condition equates the marginal product of capital to its rental rate. This condition may be viewed as the first order condition for the firm's problem of choosing the optimum quantity of capital.

Using these steady state conditions and making some substitutions, it is possible to express the steady state value of $n_1$ as a function of the underlying parameters of the model $\{\alpha,\omega,\psi,a,\delta,\nu\}$. It is then straightforward to derive functions that linearly relate $n_1$ to the steady state values of other variables in the original return function. Thus, for a given set of parameters $\{\alpha,\omega,\psi,a,\delta,\nu\}$, it is possible to explicitly calculate the stationary values of skilled and unskilled employment, capital, output, consumption and investment.

(ii) Let the state vector for the problem be denoted by $X_t = [\ 1\ \theta_t\ K_t\ n_{1,t-1}\ ]'$ and let the control vector be denoted by $U_t = [\ n_{1t}\ n_{2t}\ k_t\ ]'$

Substitute the constraints of the original planning problem into the objective function and thereby rewrite the return function as a constrained optimization problem. Take a second order Taylor series expansion of the return function around the stationary values that were computed in the previous step. This yields the following approximation

$$\log c_t - \psi an_{1t} - (1-\psi)an_{2t} \approx [X_t' \ U_t'] \ \Theta \begin{bmatrix} X_t \\ U_t \end{bmatrix} \qquad \text{(A1)}$$

where $\Theta$ is a symmetric 7x7 matrix. The elements of this matrix are functions of the parameters $\{\alpha,\omega,\psi,a,\delta,d\}$. The matrix $\Theta$ may be partitioned as follows

$$\Theta = \begin{bmatrix} Q & W \\ W' & R \end{bmatrix}$$

where Q is a 4x4 symmetric negative definite matrix, R is a 3x3 symmetric negative semidefinite matrix, and W has dimension 4x3. Using this partition, the approximation may be written in a simpler form

$$\log c_t - \psi an_{1t} - (1-\psi)an_{2t} \approx X_t'QX_t + U_t'RU_t + 2U_t'W'X_t. \qquad \text{(A2)}$$

The approximation to the return function now has a quadratic representation which has to be maximized subject to a constraint that describes the evolution of the state vector. The law of motion for the state vector is

$$X_{t+1} = AX_t + BU_t + G\varepsilon_{t+1} \qquad \text{(A3)}$$

where $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1-\rho & \rho & 0 & 0 \\ 0 & 0 & 1-\delta & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, and $G = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$.

These steps yield a quadratic objective function that has to be maximized subject to a linear constraint. Thus, we have obtained a linear-quadratic approximation to the original social planning problem. With the matrices $\{X,U,Q,R,W,A,B,G\}$ as defined above, the planning problem is to choose a sequence of functions $\left\{U_t(X_t)\right\}_{t=0}^{\infty}$ to maximize

$$E \sum_{t=0}^{\infty} \beta^t \{ X_t'QX_t + U_t'RU_t + 2U_t'W'X_t \} \qquad \text{(A4)}$$

subject to an initial condition $X_0$ and the linear law of motion for the state vector as specified in (A3).

Since the state transitions in this economy follow a first-order Markov process, it should, in principle, be possible to solve for a time-invariant function $U(\cdot)$ that expresses the controls as functions of the current state vector. Next, I turn to the derivation of this time-invariant decision rule for the controls.

### A.2 Solving the Linear-Quadratic Model

The linear-quadratic problem described above is essentially a standard stochastic discounted linear optimal regulator problem. The solution to this problem is an invariant decision rule of the form

$$U_t = -FX_t \tag{A5}$$

The matrix F is given by the expression

$$F = (R + \beta B'PB)^{-1}(W' + \beta B'PA) \tag{A6}$$

where P is a symmetric positive definite matrix that is defined as the unique limit of iterations, as $j \rightarrow \infty$, on the following matrix Ricatti difference equation:

$$P_{j+1} = Q + \beta A'P_jA - (W + \beta A'P_jB)\ (R + \beta B'P_jB)^{-1}(W' + \beta B'P_jA). \tag{A7}$$

The starting value for iterations on the Ricatti equation is $P_0 = [0]$.

### A.3 A Linear Model for the Observables

Given the state vector $X_t$ and the controls $U_t = -FX_t$, the variables of interest can now be directly solved for as nonlinear functions of the states and controls. Apart from the controls themselves, there are a few variables whose time-series properties are of interest in this analysis. Using the various functional relationships described in the earlier sections, these observable quantities can be directly calculated as nonlinear functions of the states and controls. The observables and the functional relationships that link them to the states and controls are then stacked into a vector denoted by $Y^1_t$ and the functional relationships are summarized by $Y^1_t = H^*(X_t)$, where $H^*(\cdot)$ is nonlinear. One would, at this stage, like to use the well-developed theory that is available for linear dynamic systems. This requires that the observables in $Y^1$ be expressed as linear, rather than nonlinear, functions of the state vector. In other words, one would like to derive a relationship of the form

$$Y^1_t = H\cdot X_t \tag{A8}$$

where H is a matrix with column dimension 4. The row dimension of H is determined by the number of variables included in the system. To obtain the H matrix, I adopt the following procedure suggested by Hansen and Sargent (1988):

(i) Generate a series of 200 realizations of the technology shock process using a random number generator for the innovations to the shock.

(ii) Using these realizations, obtain time series on the state vector and the controls using (A3) and (A5).

(iii) Obtain time series on the observables using the nonlinear relationships summarized in $H^{*}(\cdot)$.

(iv) Linearly regress the realizations of the observables on the realizations of the state vector. The matrix of regression coefficients is the required H matrix.

This procedure involves approximating a set of nonlinear relationships by a corresponding set of linear relationships. A measure of the quality of the approximation is provided by the adjusted $R^2$ obtained from each of these regressions. In virtually all the experiments performed in this paper, the smallest adjusted $R^2$ from the linear regressions was more than 0.9998, indicating that the linear functions summarized in H are good approximations to the nonlinear functions that relate observables to the state vector.

The measured quantities discussed above and the controls can now be stacked into one vector that can be expressed as a function of the state vector.

$$Y_t = C \cdot X_t \tag{A9}$$

where $Y_t = \begin{bmatrix} Y^1_t \\ U_t \end{bmatrix}$ and $C = \begin{bmatrix} H \\ -F \end{bmatrix}$. The model for the observables can now be summarized as follows:

$$Y_t = C \cdot X_t \tag{A10}$$

$$X_{t+1} = (A-BF)\, X_t + G\varepsilon_{t+1}. \tag{A11}$$

It is then straightforward to derive the following moving average representation for the state vector:

$$X_t = [I - (A-BF)L]^{-1} G\varepsilon_t \tag{A12}$$

where L is the lag operator and I is a 4x4 identity matrix. Conditions on A, B, and F that were discussed earlier ensure that the matrix [I - (A-BF)L] is invertible, i.e., that a moving average representation exists for $X_t$. Combining (A.18) and (A.19), the model for the observables can be written as

$$Y_t = \upsilon(L)\varepsilon_t \tag{A13}$$

where $\upsilon(L) = C[I - (A-BF)L]^{-1}G$ is a polynomial in the lag operator and $\varepsilon$ is a scalar white noise. Thus, we have derived a model that represents the vector stochastic process $Y_t$ as a vector one-sided distributed lag of the innovations to the technology shock, i.e., the scalar white noise $\varepsilon_t$.

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