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Flexible Estimation of Demand Schedules and Revenue under Different Auction Formats

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Abstract

From 1976 to 1980, the International Monetary Fund sold by sealed-bid auctions one-fifth of its gold stock and systematically experimented with auction format. Based on data from these auctions, this paper uses nonlinear estimation techniques to estimate demand curves under the alternative formats. Demand schedules at the uniform-price auctions were steeper and to the right of those at discriminatory-price auctions, upholding the predictions of bidding theory. Moreover, it is estimated that discriminating-price auctions yielded lower revenue than uniform-price auctions; Monte Carlo simulations suggest that this latter result is both robust and statistically significant.

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1/ International Monetary Fund and Board of Governors of the Federal Reserve System, respectively. The views expressed are our own and do not necessarily reflect the views of our institutions. Some of this material was presented at meetings of the American Economics Association (January 1995) and the Sixth Summer Institute on Game Theory (July 1995). In addition to participants at those meetings, we wish to thank Greg Belzer for excellent research assistance.

Contents

Summary

From 1976 to 1980, the International Monetary Fund (IMF) sold by sealed-bid auction one fifth of its gold stock to create a fund to assist developing countries. The IMF systematically experimented with auction technique, selling gold in 10 uniform-price and 35 discriminatory-price auctions. This paper uses nonlinear estimation techniques--local-fitting regressions approximating an unknown functional form--to estimate demand curves under alternative auction formats. It turns out that simple specifications using a small number of parameters can explain much of the variation in quantities. Demand schedules at the uniform-price auctions were steeper and to the right of those at discriminatory-price auctions, upholding the predictions of bidding theory. Based on these regressions, it is estimated that the IMF received an average payout at its discriminating-price gold auctions that was 0.06 percent lower than the payout at its uniform-price auctions. Monte Carlo simulations suggest that this result is both robust and statistically significant.

I. Introduction

There is a large academic literature on auctions, beginning with the important contributions of William Vickrey in 1961. Since then, a host of researchers have applied game theory and models that deal with imperfect information to the bidding problem. 1/ As a result, we know that there are a number of strategic similarities among auctions, as well as equivalence relationships concerning a seller's revenue. Moreover, we know that those results may be sensitive to assumptions about information, the cost of entering an auction, and trading opportunities before or after the auction. 2/ This sophisticated theorizing is now being put to the data, in large part thanks to the progress made in solving nonlinear optimization problems numerically that has made techniques such as simulated method of moments possible. 3/ However, for all this deepening of our understanding of bidder behavior, the basic logic still traces back to the shifting and flattening of demand schedules that dominated the discussion of this subject in the 1960s.

In this paper, we use nonlinear estimation techniques to estimate the twists and turns to demand curves under alternative auction formats. We undertake this estimation and perform various statistical tests on an important dataset, apparently neglected in the literature, covering an experiment in auction technique in a financial market setting. From 1976 to 1980, the International Monetary Fund (IMF) sold by sealed-bid auction one-fifth of its gold stock to create a fund to assist developing countries. There were forty-five auctions in total, and the award technique in the first twenty was varied in an announced and predictable manner to provide a reasonably controlled experiment. More specifically, ten of the first twenty auctions and the last twenty-five auctions awarded the gold at the prices that were bid (discriminatory-price format); the other ten awarded it at a single, market-clearing price at each auction (uniform-price format). The IMF press releases at the time provided a wealth of information about the specifics of delivery and, after the fact, the names of the participants and a detailed histogram of bids--the quantities bid sorted by price.

1/ The early references include Vickrey (1961) and Friedman (1964). Recent work is summarized and reviewed in McAfee and McMillan (1987) and Milgrom (1989).

2/ The effects of assumptions about information can be seen in Milgrom and Weber (1982); work on endogenous entry includes McAfee and McMillan (1987), Harstad (1990), and Levin and Smith (1994); and, multiple trading forums are considered in Bikhchandani and Huang (1989).

3/ This work includes Paarsch (1992) and Laffont and Vuoung (1993).

There are only a few empirical studies in the literature that have dealt with comparative revenue performance under discriminatory-price versus uniform-price auctions in a financial-market setting. 1/ Tenorio (1993) looks specifically at the impact of discriminatory-price and uniform-price formats, as well as various other auction characteristics, on bidder behavior and revenue equivalence in the context of foreign exchange auctions in Zambia. Umlauf (1993) studies revenues from the one-month Mexican Treasury bill auctions that were run using a discriminatory-price format from 1986 to mid-1990 and a uniform-price format afterward into 1991. In a companion paper also using data from the IMF gold auctions, Feldman and Reinhart (1995) consider whether actual bidding matched the predictions from optimal bidding rules. All three papers looking at price and quantity outcomes find that bid shading in discriminatory-price auctions was significant enough to pull seller revenue below that received in uniform-price auctions. Some other empirical studies of bidding behavior have looked at market prices in search of underpricing. 2/ For example, Cammack (1991) found that secondary market prices for three-month U.S. Treasury bills were a statistically significant 4 basis points above mean auction prices during the period from 1973 to 1984, results supported by the work of Spindt and Stolz (1991). 3/

1/ Several papers have studied revenue performance from the perspective of comparing sealed-bid and open auctions; see, for example, Mead (1966), Johnson (1979), Hansen (1985, 1986). Of course, in our case, discriminatory- and uniform-price formats are both examples of sealed-bid auctions. Reinhart (1992) and Feldman and Mehra (1993) discuss the terminology applied to auction formats, both in the academic literature and in the popular financial press.

2/ Such underpricing is sometimes taken as an indication of the so-called winner's curse (discussed in section 2 below), a factor present in operating discriminatory-price auctions that helps to explain why revenue might be relatively lower under this format than under the uniform-price format. The winner's curse is not, however, the only factor that can contribute to this result. Bikhchandani and Huang (1989) argue that the existence of resale coupled with the public announcement of winning bids generates additional revenues in uniform-price auctions above and beyond the gains that may result from the elimination of the winner's curse. Other factors can also affect expected revenue in multi-unit auctions, including bidders' attitudes toward risk, their access to information, and the potential for entry of other bidders.

3/ Of course, there is a wide body of empirical work examining the effects of auction format in other settings, including the sale of timber and oil rights, not to mention in controlled experiments. Hansen (1985) reviews the material on real-world sales. As to laboratory evidence, Smith (1982), for example, obtained inconclusive results in his analysis of revenue equivalence, based on controlled experiments for multi-unit auctions with bidders submitting single unit bids.

This present study adds to the limited body of empirical work on the revenue effects of auction format, both by using a dataset and market setting that are relatively unexplored in the literature and by estimating demand schedules using a flexible estimation technique that allows for considerable nonlinearities. This technique is the local-fitting methodology pioneered by Cleveland (1979), which is essentially weighted-least-squares estimation of a polynomial, usually linear or quadratic, through a moving window of the sample.

On organization, section 2 of this paper reviews the relevant literature on auction technique and bidding behavior and develops empirically testable implications. Section 3 discusses the dataset that is used. In section 4 we undertake the empirical work, which involves the estimation of demand schedules at individual auctions and for pooled samples. Given demand schedules, estimating revenue is a simple, although nonlinear, task, which we discuss in Section 5. We bootstrap standard errors for those estimates and find, at a high statistical confidence level, empirical evidence of better revenue performance under uniform-price auctions. Section 6 contains concluding remarks.

II. Auction Theory

The gold auctions we study fall into a special class within the broad literature on bidding, as the items sold had close substitutes trading in an active secondary market and on a forward-delivery basis on organized futures markets. In effect, bidders at each auction were attempting to guess what others were guessing and what other markets were pricing at the same time to be the common value of the gold sold. Thus, in technical terms, these were common-values auctions, in which demand importantly depends on information that helps bidders outguess each other. As a result, to the extent that auction format reveals information about bidders' intentions, it also influences market expectations and, through that channel, total demand (in the manner described by Milgrom and Weber (1982)).

Discriminatory-price auctions. In this setup, the items auctioned are awarded to the highest priced bids, at the respective prices bid, until the auction stock is exhausted. Because participants pay differing amounts reflecting the strength of their tenders (with the most aggressive paying more than anyone else), there is an incentive for participants to shade bids toward the market consensus, as no one wants to pay more than necessary. Note that the expected gain to a participant from bidding equals:

$$\Pr\{b > a\}\bullet(v - b), \qquad (1)$$

which is the product of the probability that the bid, b, will win (i.e., that it is above a, the stop-out price or the lowest-priced winning bid) and the excess of the participant's assessment of the value of the gold, v, over the award price. As a consequence, a bidder's choice of b affects both the probability of wining and the profit attached to winning. A rational bidder trades between the two terms, lowering b toward the market consensus so as to increase the value in winning while accepting that this action lowers the probability of winning. In graphical terms, bid shading flattens the demand schedule, rotating it about the market consensus.

The effect of auction format on expectations compounds this shading. As all participants are guessing about the same price (where the gold will trade after the auction), a high bid signals a heightened probability of subsequent loss for that bidder. Planning on winning requires that the expected valuation of the items on auction be trimmed below the initial reading: that is, v is lower after the bidder finishes the thought exercise, "What do I learn from winning the auction that I did not know before?" This is the "winner's curse" and it causes aggressive bidders to rein in their enthusiasm by moving bids toward the perceived market consensus. Efforts to avoid the winner's curse may lead to the pooling of bids, as a group of investors are more likely to have a clearer view of the market consensus.

Uniform-price auctions. In this setup, auctioned items are awarded at the highest single price that just places the auction stock. In terms of incentives, aggressive bidders receive sure awards but pay a price closer to the market consensus. The expected value of winning is:

$$\Pr\{b > a\} \bullet (v - a), \qquad (2)$$

where the stop-out price, a, now also equals the award price. Single-price awards separate the probability of winning from the value of having won. There is no ability, then, to trade between the two terms in the product, so that participants bid as much as they truly value the gold. As a result, there should be none of the bid shading that marks the response to the winner's curse. Thus, the demand schedule associated with uniform-price awards should be steeper and to the right of the one associated with the discriminatory-price format. If bidders share a common assessment of the value of the gold, the fact that bids are now more informative as to the gold's underlying worth tends to raise valuations on average.

Among the contributions that Vickrey made in his seminal 1961 article, he established that the major auction formats provide equal proceeds to the seller when individual valuations are independent--when the subjective worth of the single item on the block is unrelated across bidders. Obviously, a gold auction violates this assumption, as the value agents place on the gold reflects an imperfect estimate of the price in subsequent market trading. Against this common-values backdrop, knowledge of the way that others would bid could importantly influence a bidder's valuation of the gold, with important consequences for revenue to the seller. Those analysts working with explicit models of bidder behavior in a common-values setting typically find that a uniform-price scheme produces higher revenue for the seller. 1/

These revenue implications can be spelled out in terms of the respective demand schedules under different auction formats, as detailed in Goldstein (1962) and illustrated in Figure 1. This figure shows that part of total revenue in a discriminating-price auction owes to charging winners the price that they bid, which for the current practice is measured by the area under the demand schedule labeled "discriminatory-price." Fear of the winner's curse leads participants to shade their bids and therefore discourages demand. By comparison, a single-price system turns part of that surplus back to the bidders, seen as the shifting out of the demand schedule. Revenue under that award technique is the area of the rectangle bounded by the auction stock and the single, market-clearing price.

The change in revenue by switching from a discriminatory- to a uniform-price format depends on the difference between the areas under the first demand schedule and that rectangle. Simple geometry reduces that problem to weighing the areas of two triangles, labeled in the figure as the loss from the inability to price discriminate and the gain from added demand. Formally, these two areas depend on the extent of the shift in demand and on the elasticity of that demand. 2/

Bidding theory suggests that these demand relationships are likely to be nonlinear, as are underlying valuations. 3/ But theory's most important prediction is that the slope and the position of these demand schedules are influenced by auction format. The bid shading that is the optimal response to discriminatory-price awards should flatten and shift the demand schedule inward relative to that observed under uniform-price awards. Moreover, if bidders share common valuations of the items sold, the seller loses more in revenue terms from that inward shift than gains from charging

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1/ The classic reference is Milgrom and Weber (1982). Exact conditions under which revenue increases when resale is possible, as was the case for the gold auctions, are given in Bikhchandani and Huang (1989). Also see Theorem 4 in Weber (1983). However, this result requires that auction participants be risk neutral. Some element of risk aversion in a common-values model of bidding renders the revenue rankings ambiguous. See Milgrom and Weber (1982), p. 1114.

2/ In general, an inelastic demand schedule with a wide price range would have to shift more to compensate for the failure to extract consumer surplus.

3/ Recall that the optimal bidding strategy under the uniform-price format is for bidders to bid their valuation. For example, with valuations that are independent, identical, and normally distributed across bidders, the shape of the resulting demand schedule would be akin to the shape of the cumulative density function corresponding to this normal distribution, rotated 90 degrees counterclockwise and situated in the usual price-quantity space for demand schedules.

bidders the price that was bid. Additionally, the winner's curse is reinforced as the number of bidder's increases. This is because it is prudent to bid more conservatively since, other things being equal, the range of the distribution of bids and thus the highest bid is likely to expand with the number of bidders. The extent of bid shading and the winner's curse increases the further away bids are from the market consensus.

III. The Data

Our dataset is based on the 45 gold auctions that the IMF ran from June 1976 to May 1980. Following these auctions, the IMF issued press releases that provided data on prices bid (expressed in intervals of US$1), as well as the number of bids and the total amount bid (expressed in thousands of ounces) in each price interval. The core of our dataset is from this source. The press releases also provided data on the total number of successful bidders, the total number of bids accepted, and the total number of valid bids. Preceding these auctions, the Fund issued invitations to bid, which detailed the terms and conditions for each of the various auctions. These are summarized in Table 1.

IV. Estimating Demand Schedules

1. Empirical strategy

Because theory suggests that the demand schedules should be nonlinear and that the form of their nonlinearity might change across auction format, a flexible estimation strategy is clearly needed. At the same time, we have only small samples for individual auctions, which dictates a particularly parsimonious estimation technique. Cleveland's (1979) local fitting (loess) meets these requirements. Loess regression, explained in Cleveland (1979) and Cleveland, Devlin, and Grosse (1988), as well as more informally in Cleveland (1993), involves a succession of weighted-least-squares regressions of a polynomial relationship, where the estimation moves incrementally through the sample.

We estimated demand curves for each auction using this technique. Recognizing that there are only a relatively small number of observations for each of the individual auctions and that statistical inference on this basis can be hazardous, we also estimated demand schedules under the two auction formats by normalizing the data and pooling it across individual auctions. These demand schedules are not derived from an explicit structural model. However, our reduced-form approach is the only practical way we have of incorporating (implicitly) the array of theoretical considerations about the market and its participants--for example, the degree of risk aversion, the structure of information, the underlying distribution of valuations, and the extent of correlation between the values different bidders place on the auctioned items. This approach is also sufficient to allow us to test statistically for differences in revenue performance between the two auction formats, even if these differences cannot be identified as having derived specifically from particular underlying theoretical considerations.

The quantity sold at the various auctions varied from 444,000 ounces to 780,000 ounces; to control for this substantial variation, we divided the quantity bid by the auction stock. The data were also expressed in logarithmic form before estimating demand curves. Thus, the normalization implies that there would be no effect on bid prices from changing the amount to be auctioned--a result that appeals intuitively to the common-value nature of the gold auctions and to the great extent of trading opportunities that existed to buy and sell gold before and after the auctions--as the quantity demanded adjusts proportionately with auction size. 1/ To control for the substantial variation in gold prices over the entire period, the price side of the various bids at the individual auctions (denominated in U.S. dollars) was divided by the secondary market price of gold from the close of the previous day. This variation was particularly apparent over the period of the last twenty-five discriminatory-price auctions, while the price of gold varied in a narrower range over the period of the first twenty auctions (Figure 2).

Two additional comments on the empirical strategy are relevant before turning to the estimation results themselves. First, the data for the first ten and last twenty-five discriminatory-price auctions were pooled separately. This is because the first ten discriminatory-price auctions were conducted within an overall plan to experiment with auction technique, and the price of gold varied in a relatively narrow range. After the twentieth auction, the experiment was dropped so that only discriminatory-price auctions followed; meanwhile, the price of gold sky-rocketed on world markets. Thus, the dividing line for splitting the sample for purposes of pooling data on the experience with the discriminatory-price format appears quite natural. Second, the dataset should be seen as representing a "*reasonably*" controlled experiment. As illustrated by Table 1, there was some variation in the details of the auctions other than the shifts in format alone, and these changes should be taken into account in estimating the demand schedules. In practice, however, there is no a priori reason to believe that the changing details would introduce a particular bias to the estimation results in favor of any one format. More importantly, institutional details other than auction format appeared to have been statistically unimportant. In a pooled regression of (normalized) auction bids on the various auction characteristics, only the variables for auction format are statistically significant at the usual significance levels.

Thus, we have forty-eight different samples, corresponding to the data on the forty-five individual auctions and the pooled datasets of the ten uniform-price auctions and the first ten and remaining twenty-five discriminatory-price auctions. We regressed the normalized quantities against the normalized prices, in part because it is easier to think of the

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1/ Indeed, there is a positive correlation between auction size and the volume of total bids at each auction.

nonlinearities that bidding theory expects in terms of influencing quantities tendered as a function of relative price.

In the local-fitting methodology of loess, three decisions have to be made: the degree of the polynomial, the width of the estimation window, and the weights attached to each observation in the rolling regressions. Cleveland's preferred local weighting function, which we adopt, is a tricube function, which has the effect of keeping to a minimum the importance of observations distant from the current one. We experimented with the other two choices, using both linear and quadratic polynomials and varying the width of the window relative to the total sample, and judged the result by some of Cleveland's preferred criteria--overall fit, whiteness of the residuals, and smoothness of the estimated function. The message from various loess estimations of these demand relationships using the different samples, not all of which we report for brevity's sake, is to keep the specification simple. In what follows, we describe the loess estimates of linear relationships in which the span of the estimation (the width of the estimation window relative to the total sample) equalled two thirds.

2. Estimation results

Figure 3 shows the results of estimating demand curves using normalized data for each of the first ten discriminatory-price auctions; Figure 4 shows the results for each of the first ten uniform-price auctions. Consistent with theoretical priors, the demand curves tended to be flatter at the discriminatory-price auctions in the relevant range of winning bids (by our normalization, this is shown by the portion of the figure to the left of zero on the horizontal axis). This is all the more apparent in Figure 5, which shows the estimated demand schedules for pooled samples from the first 20 auctions--results for the uniform-price format are in the upper panel and for the discriminatory-price format in the middle panel. The obvious and important point to note is that the demand schedule at the uniform-price auctions was steeper than at the discriminatory-price auctions--that is, bidders did shade to a significant degree, upholding the implications of theory. The results from pooled data for the last 25 discriminatory-price auctions, shown in the bottom panel, appear to leave the story unchanged. Summary statistics for each of the 45 individual auctions are reported in Table 2; results from the pooled samples are given in Table 3. As is quite evident from the two tables, simple specifications using a small number of parameters can explain much of the variation in quantities.

Beyond appearances, we must determine if the visible differences are in fact statistically significant. In Figure 6, we overlay the ninety-five percent confidence bands for the uniform-price and first ten discriminatory-price auctions. What matters to the seller is the behavior of demand at quantities up to the amount that is to be sold. As is evident, the demand for gold at the first ten uniform-price auctions was everywhere above that at the first ten discriminatory-price auctions. A similar exercise, not shown, would put the confidence bound about the last twenty-five discriminatory-price auctions even further below the uniform-price auction demand schedule.

V. Revenue from Alternative Auction Techniques

The consequences for revenue are more difficult to ascertain. In the uniform-price setup, revenue is simply the market-clearing price times the quantity sold. In the discriminatory-price setup, the seller seizes all consumer surplus by charging the price that the buyer bid so that total revenue equals the area under the demand curve. In either case, these are nonlinear calculations that make it difficult to determine standard errors about an estimate of revenue.

To get some sense of the confidence about an estimate of revenue from these two auction formats, we ran a Monte Carlo exercise, estimating these demand curves for randomly chosen subsets of the data and calculating the implied revenue to the auctioneer. To be specific, in the Monte Carlo exercise, we randomly drew samples of 100 observations from the aggregate normalized dataset for the first ten discriminatory-price auctions, estimated the demand schedule each time using our preferred loess specification and then calculated the implied revenue per unit of gold. This process was repeated 200 times. A similar procedure was followed for the uniform-price auctions.

Figure 7 plots the resulting frequency distribution of revenue under the two auction formats and reports various descriptive statistics on these distributions. According to these estimates, on average, the IMF received 0.48 percent below the recent secondary market price for each ounce of gold it sold in a discriminatory-price auction. In its ten uniform-price auctions, the average price was 0.42 percent below the secondary market price. The difference, 0.06 percent of the secondary market price, is statistically significant, at least judging by the t-test of equality of means. Put another way, for the same secondary-market price, the average revenue from uniform-price auctions was 0.06 percent greater than from discriminatory-price auctions (which is calculated as the percent difference between the averages, 99.58 and 99.52, reported in the table). More generally, the two distributions, based on various non-parametric tests reported in the table inset, differ significantly.

VI. Concluding Remarks

Auction theory provides a rich set of testable hypotheses concerning bidding behavior and seller revenue across auctions formats. Two problems, however, have kept the development of empirical testing from matching the rapid expansion of theory. First, auction theory can be quite complicated, because the strategic bidder must make a decision in light of the likely course of the competition's behavior. A modeler has to capture what everyone knows, when they know it, and how they show that knowledge to each other. As a result, explicit, closed-form solutions of interesting auction problems--which is what the empiricist needs to start from--are few and far between. The second difficulty is that data on controlled auction experiments are even rarer. Sellers are interested in outcomes, so that they structure their auction formats in the manner that they anticipate will generate the most revenue based on assessments of the likely number of bidders and the strength of demand. Further complicating the picture, many sellers are risk averse, which makes them loathe to experiment with auction technique.

We studied an exception to that general rule, the IMF's sale of gold from 1976 to 1980, in a manner that captured the logic, if not the exact parameter restrictions, of modern work in auction theory. Our results suggest that economic agents do restrict their stated demands when the auctioneer attempts to extract bidders' consumer surplus by charging the price bid, exactly in the manner William Vickrey and Milton Friedman predicted thirty years ago. The evidence, flexible estimation of demand schedules, also suggests that the net effect of that attempt by the seller to discriminate by price lowers total revenue, in line with the conclusion from stylized models of bidder behavior offered by Paul Milgrom and Robert Weber in 1982. We showed these results by estimating demand schedules using William Cleveland's loss technique, which uses local fitting to approximate an unknown functional form. This apparatus, not much used in micro-economics, extracts a considerable amount of information from the data with only a few parameters. In this regard, it is particularly useful when only aggregated information about bidding behavior is available in the public domain.

The key result--that the IMF received an average payout at its discriminating-price gold auctions that was 0.06 percent lower than the payout at its uniform-price auctions--appears robust and statistically significant. 1/ A modest difference can cumulate to a considerable sum, given that the IMF sold 23-1/2 million ounces of gold. If the 0.06 percent difference across auctions owed only to the variations in selling technique, then the IMF would have raised US$2-3/4 million more had it conducted forty-five uniform-price auctions rather than the ten uniform-price and thirty-five discriminating-price auctions that it actually sponsored.

1/ By way of perspective, as was noted in the Introduction, Elizabeth Cammack (1991) found in her study of the Treasury market that new three-month bills sold at a price that was 0.04 percent below comparable seasoned issues.

Table 1

Summary of IMF Gold Auctions

Auction Number	Date	Type	Press Release #	Size (troy ounces)	Total Bids (troy ounces)	Minimum Bid (troy ounces)	Deposit	Payment (Days) 1/	Delivery Place	Delivery Time (Days) 2/
1	6/02/76	U	76/34	780,000	2,320,000	2,000	$50,000	28	FRBNY	30
2	7/14/76	U	76/46	780,000	2,114,000	1,200	$50,000	28	FRBNY	42
3	9/15/76	D	76/70	780,000	3,662,400	1,200	$50,000	28	FRBNY	42
4	10/27/76	D	76/80	779,200 3/	4,214,400	1,200	$50,000	28	FRBNY	42
5	12/08/76	U	76/94	780,000	4,307,200	1,200	$50,000	21	BOE	30
6	1/26/77	U	77/2	780,000	2,003,200	1,200	$50,000	14	FRBNY	28
7	3/02/77	D	77/12	524,400	1,632,800	1,200	$50,000	14	FRBNY	28
8	4/06/77	D	77/16	524,800	1,278,000	1,200	$50,000	14	FRBNY	28
9	5/04/77	D	77/24	524,800	1,316,400	1,200	$50,000	14	FRBNY	28
10	6/01/77	U	77/40	524,800	1,014,000	1,200	A	9	FRBNY	23
11	7/06/77	U	77/47	524,800	1,358,400	1,200	A	9	BOF	28
12	8/03/77	U	77/53	524,800	1,439,200	1,200	A	9	BOE	23
13	9/07/77	D	77/62	524,800	1,084,400	1,200	A	9	FRBNY	23
14	10/05/77	D	77/74	524,800	971,200	1,200	A	9	FRBNY	23
15	11/02/77	D	77/83	524,800	1,356,400	1,200	A	9	BOE	23
16	12/07/77	U	77/88	524,800	1,133,600	1,200	A	9	FRBNY	23

Table 1
(Continued)

Summary of IMF Gold Auctions

Auction Number	Date	Type	Press Release #	Size (troy ounces)	Total Bids (troy ounces)	Minimum Bid (troy ounces)	Deposit	Payment (Days) 1/	Delivery Place	Delivery Time (Days) 2/
17	1/04/78	U	77/94	524,800	984,800	1,200	A	9	FRBNY	23
18	2/01/78	U	78/3	524,800	598,400	1,200	A	9	BOF	27
19	3/01/78	D	78/13	524,800	1,418,000	1,200	A	9	FRBNY	23
20	4/05/78	D	78/17	524,800	1,367,600	1,200	A	9	FRBNY	23
21	5/03/78	D	78/24	524,800	3,104,000	1,200	A	9	BOE	23
22	6/07/78	D	78/31	470,000	1,072,400	1,200	A	9	FRBNY	23
23	7/05/78	D	78/42	470,000	797,200	1,200	A	9	FRBNY	23
24	8/02/78	D	78/53	470,000	1,467,600	1,200	A	9	FRBNY	23
25	9/06/78	D	78/62	470,000	773,200	1,200	A	9	FRBNY	23
26	10/04/78	D	78/66	470,000	805,600	1,200	A	9	BOE	23
27	11/01/78	D	78/74	470,000	689,600	1,200	A	9	FRBNY	23
28	12/06/78	D	78/83	470,000	1,965,200	1,200	A	9	BOF	23
29	1/03/79	D	78/92	470,000	1,479,600	1,200	A	9	FRBNY	23
30	2/07/79	D	79/6	470,000	1,489,600	1,200	A	9	FRBNY	23
31	3/07/79	D	79/12	470,000	1,534,400	1,200	A	9	BOE	23
32	4/04/79	D	79/18	470,000	1,186,800	1,200	A	9	FRBNY	23
33	5/02/79	D	79/28	470,000	1,514,800	1,200	A	9	FRBNY	23

Table 1
(Concluded)

Summary of IMF Gold Auctions

Auction Number	Date	Type	Press Release #	Size (troy ounces)	Total Bids (troy ounces)	Minimum Bid (troy ounces)	Deposit	Payment (Days) 1/	Delivery Place	Delivery Time (Days) 2/
34	6/06/79	D	79/36	444,000	1,452,400	1,200	A	9	FRBNY	23
35	7/03/79	D	79/44	444,000	1,518,800	1,200	A	10	FRBNY	24
36	8/01/79	D	79/52	444,000	1,138,800	1,200	A	9	FRBNY	23
37	9/05/79	D	79/62	444,000	1,646,000	1,200	A	9	BOF	23
38	10/10/79	D	79/72	444,000	665,600	1,200	A	9	FRBNY	23
39	11/07/79	D	79/81	444,000	1,798,400	1,200	A	9	FRBNY	23
40	12/05/79	D	79/95	444,000	1,746,000	1,200	A	9	BOE	23
41	1/02/80	D	79/101	444,000	1,342,400	1,200	A	9	FRBNY	23
42	2/06/80	D	80/8	444,000	1,939,600	1,200	A	9	FRBNY	23
43	3/05/80	D	80/15	444,000	1,412,400	1,200	B	9	FRBNY	23
44	4/02/80	D	80/24	444,000	802,800	1,200	B	9	FRBNY	23
45	5/07/80	D	80/31	443,200 3/	1,822,000	1,200	B	9	FRBNY	23

Notes: U = Uniform price auction.
D = Discriminatory price auction.
A = Greater of $25,000 or $10 per ounce of final bid.
B = $40 per ounce of final bid.
FRBNY = Federal Reserve Bank of New York.
BOE = Bank of England.
BOF = Bank of France.
1/ = Time from auction date to payment due date.
2/ = Time from auction date to delivery date.
3/ = 800 ounces could not be awarded because they did not reach the minimum award of 1,200 ounces under the terms and conditions of the auction.

Table 2

Summary Statistics for Estimated Demand Schedules

Auction -Number	Auction Date	Auction Type	Number of Observations	Effective # of Parameters	Residual Standard Error	Multiple R-Squared
1	Jun. 02, 76	U	18	3.37	0.53	0.95
2	Jul 14, 76	U	13	3.13	0.28	0.97
3	Sep. 15, 76	D	30	3.51	0.93	0.82
4	Oct. 27, 76	D	21	3.48	0.36	0.81
5	Dec. 08, 76	U	23	3.65	0.53	0.95
6	Jan. 26, 77	U	19	3.37	0.42	0.98
7	Mar. 02, 77	D	16	3.82	1.09	0.85
8	Apr. 06, 77	D	12	4.09	0.75	0.94
9	May 04, 77	D	14	3.72	0.87	0.91
10	Jun. 01, 77	U	11	3.78	0.51	0.96
11	Jul. 06, 77	U	9	3.78	0.55	0.97
12	Aug. 03, 77	U	10	3.18	0.46	0.97
13	Sep. 07, 77	D	8	3.44	0.46	0.98
14	Oct. 05, 77	D	10	3.18	0.80	0.93
15	Nov. 02, 77	D	10	3.77	0.78	0.95
16	Dec. 07, 77	U	10	3.34	0.47	0.98
17	Jan. 04, 78	U	10	4.34	0.31	0.99
18	Feb. 01, 78	U	10	3.33	0.34	0.99
19	Mar. 01, 78	D	13	3.12	0.40	0.98
20	Apr. 05, 78	D	11	3.28	0.54	0.97
21	May 03, 78	D	8	3.32	1.24	0.87
22	Jun. 07, 78	D	6	2.93	0.16	0.96
23	Jul. 05, 78	D	7	3.76	1.17	0.93
24	Aug. 02, 78	D	11	3.45	0.74	0.94
25	Sep. 06, 78	D	9	4.64	1.28	0.84
26	Oct. 04, 78	D	7	3.29	0.77	0.91
27	Nov. 01, 78	D	17	3.16	0.44	0.97
28	Dec. 06, 78	D	8	3.32	0.12	0.99
29	Jan. 03, 78	D	12	3.42	0.54	0.95
30	Feb. 07, 79	D	13	3.75	0.06	0.95
31	Mar. 07, 79	D	8	3.32	0.55	0.97
32	Apr. 04, 79	D	8	3.32	0.85	0.91
33	May 02, 79	D	9	4.21	1.17	0.91
34	Jun. 06, 79	D	12	3.92	1.20	0.75
35	Jul. 03, 79	D	7	3.77	0.19	0.92
36	Aug. 01, 79	D	9	3.68	0.21	0.96
37	Sep. 05, 79	D	14	3.75	0.13	0.95
38	Oct. 10, 79	D	28	3.94	0.77	0.87
39	Nov. 07, 79	D	30	3.11	0.66	0.91
40	Dec. 05, 79	D	16	4.19	0.66	0.92
41	Jan. 02, 80	D	23	4.32	0.22	0.93
42	Feb. 06, 80	D	31	3.62	0.49	0.92
43	Mar. 05, 80	D	27	3.63	0.35	0.97
44	Apr. 02, 80	D	33	3.34	0.59	0.86
45	May 07, 80	D	54	3.68	0.60	0.80

Note: All demand schedules use Loess estimation of a linear relationship with span= 2/3, and the errors assumed to be gaussian.

Table 3

Summary Statistics for Estimated Demand Schedules for Pooled Samples

Sample	Number of Observations	Effective # of Parameters	Residual Standard Error	Multiple R-Squared
10 Uniform Price	133	4.02	0.91	0.84
First 10 Discriminating Price	145	4.08	0.98	0.80
Remaining 25 Discriminating Price	407	3.98	1.13	0.56

Note: All demand schedules use Loess estimation of a linear relationship with span= 2/3, and the errors assumed to be gaussian.

Figure 1

Demand Schedules and Their Revenue Implications
under Different Auction Formats

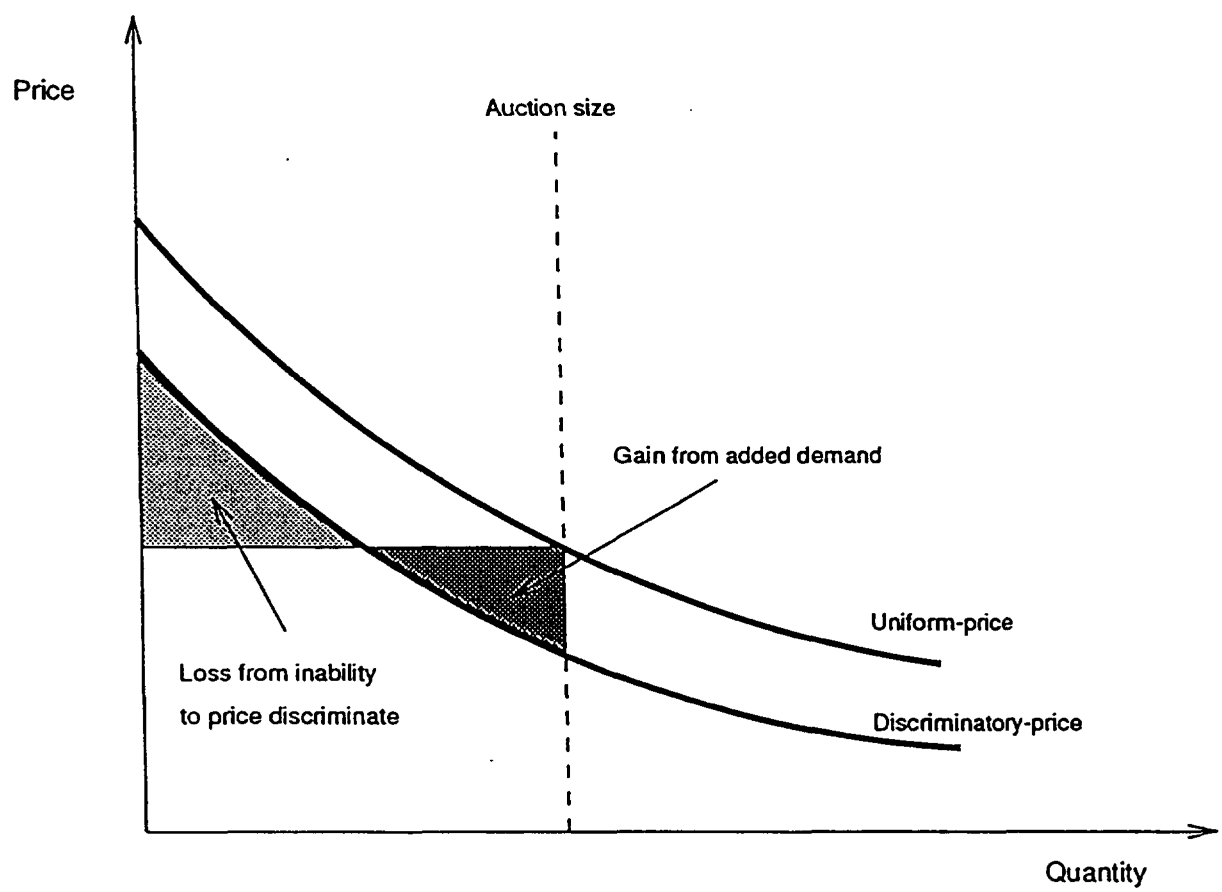


Figure 2

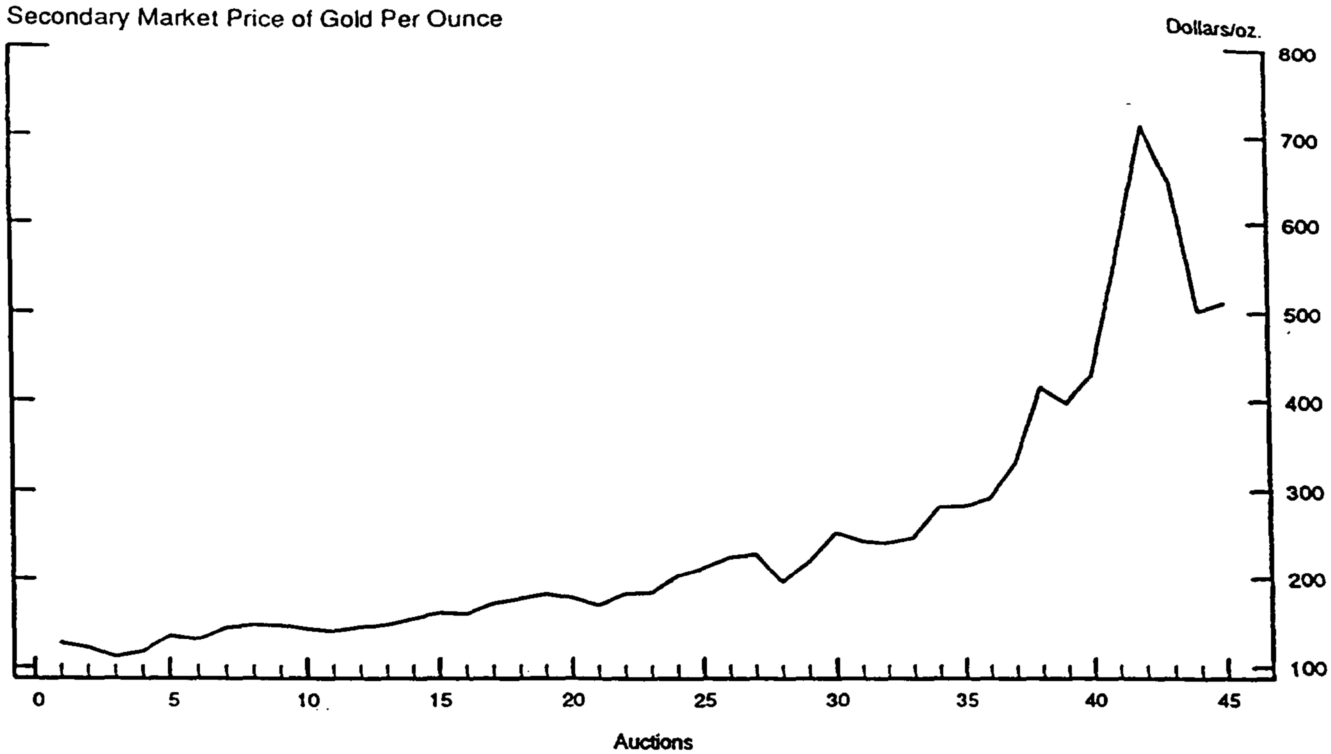
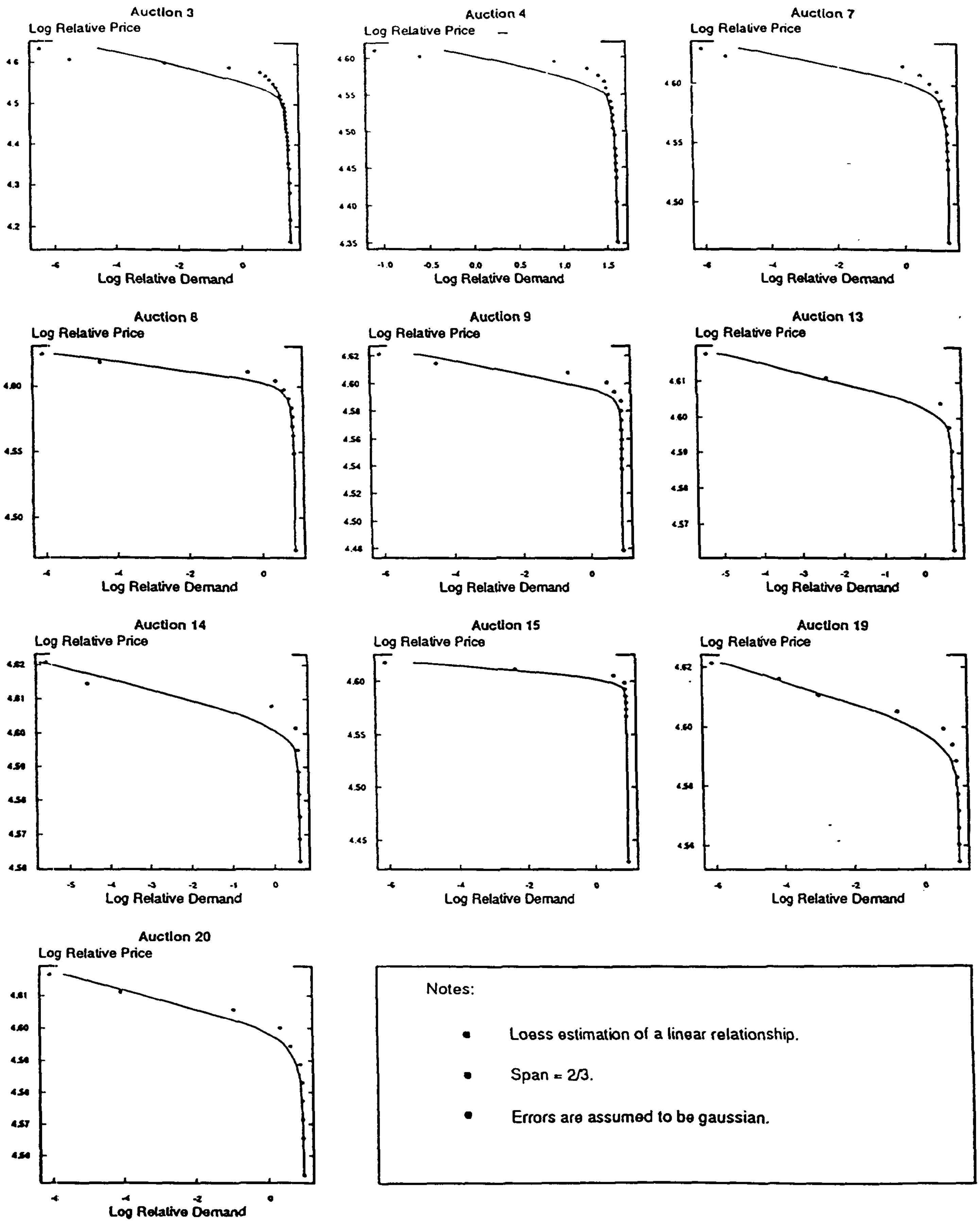

Secondary Market Price of Gold Per Ounce
Dollars/oz.
800
700
600
500
400
300
200
100
0
5
10
15
20
25
30
35
40
45
Auctions

Figure 3

Estimated Demand Schedules at the First Ten Discriminating-Price Auctions

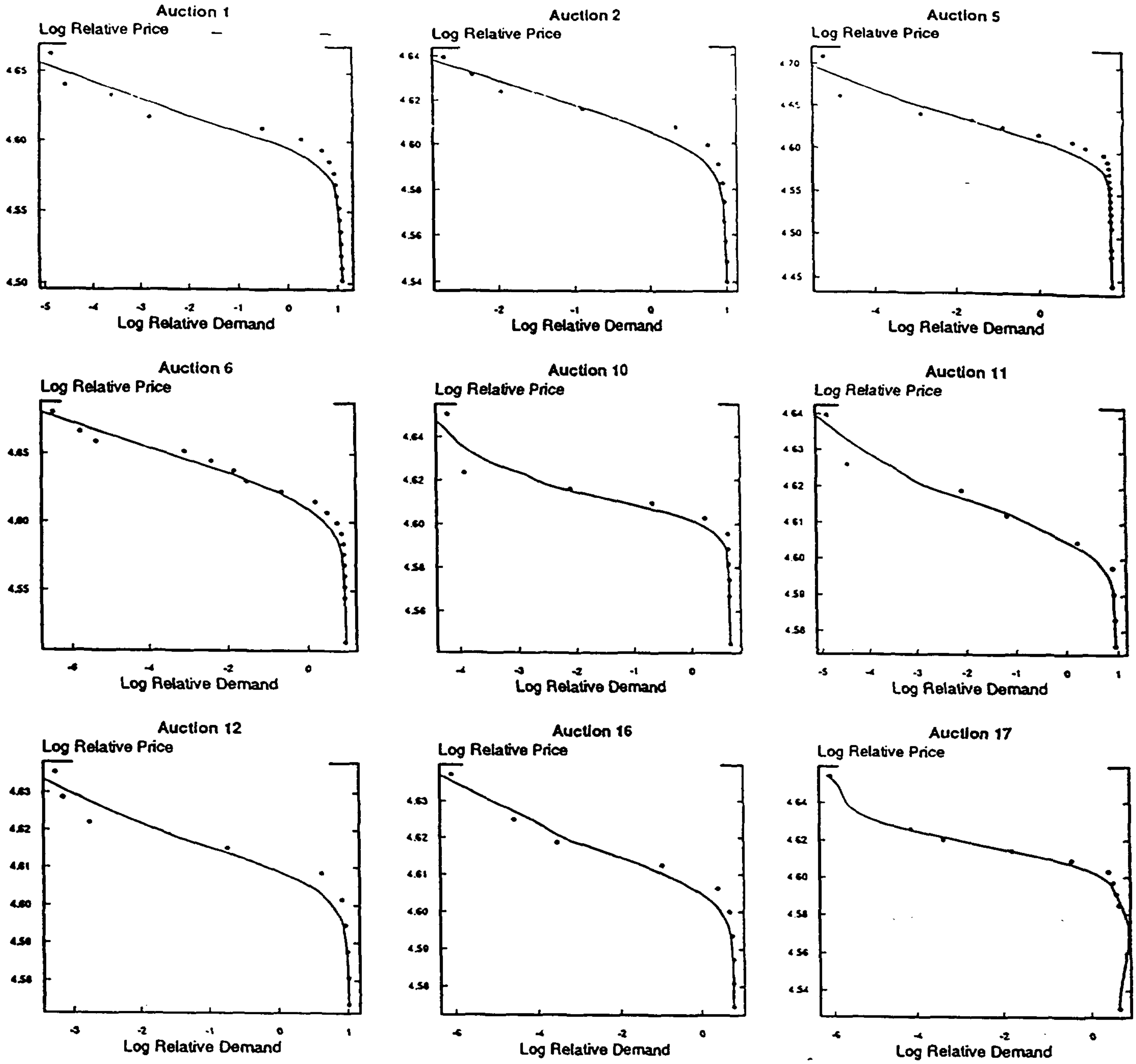


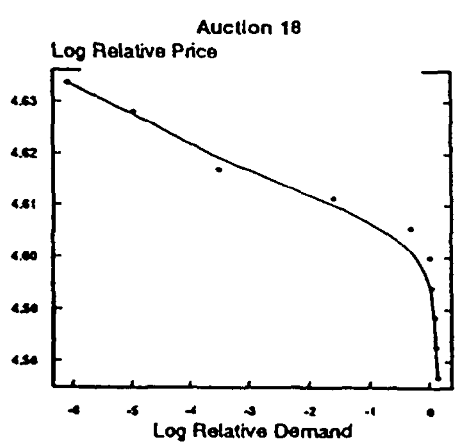
Notes:

- Loess estimation of a linear relationship.
- Span = 2/3.
- Errors are assumed to be gaussian.

Figure 4

Estimated Demand Schedules at the Ten Uniform-Price Auctions



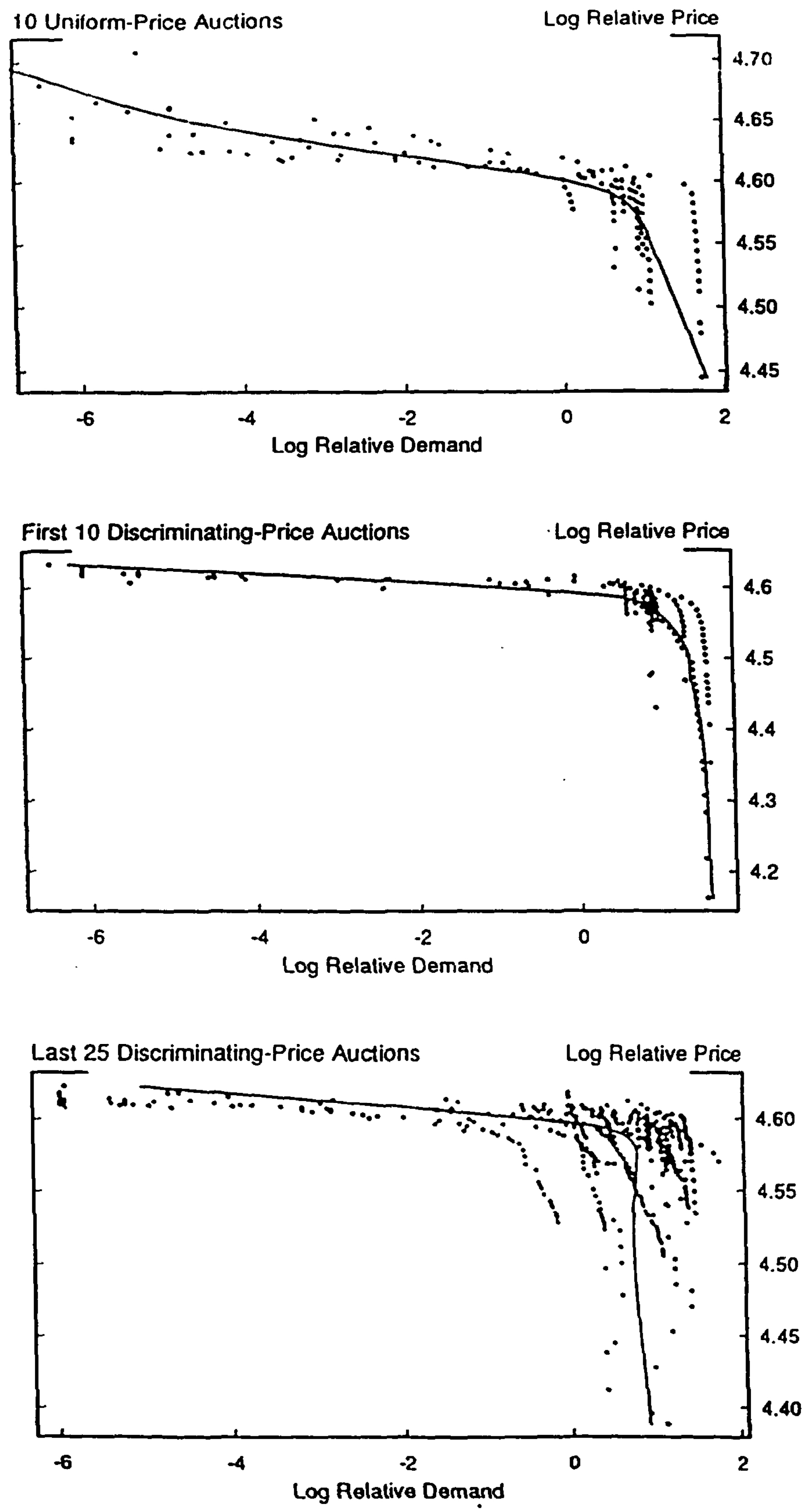


Notes:

- Loess estimation of a linear relationship.
- Span = 2/3.
- Errors are assumed to be gaussian.

Figure 5

Estimated Demand Schedules for Pooled Samples



Note: Loess estimate of linear relationship using span of 2/3.

Figure 6

Uniform-Price vs Discriminating-Price Auction Demand Curves

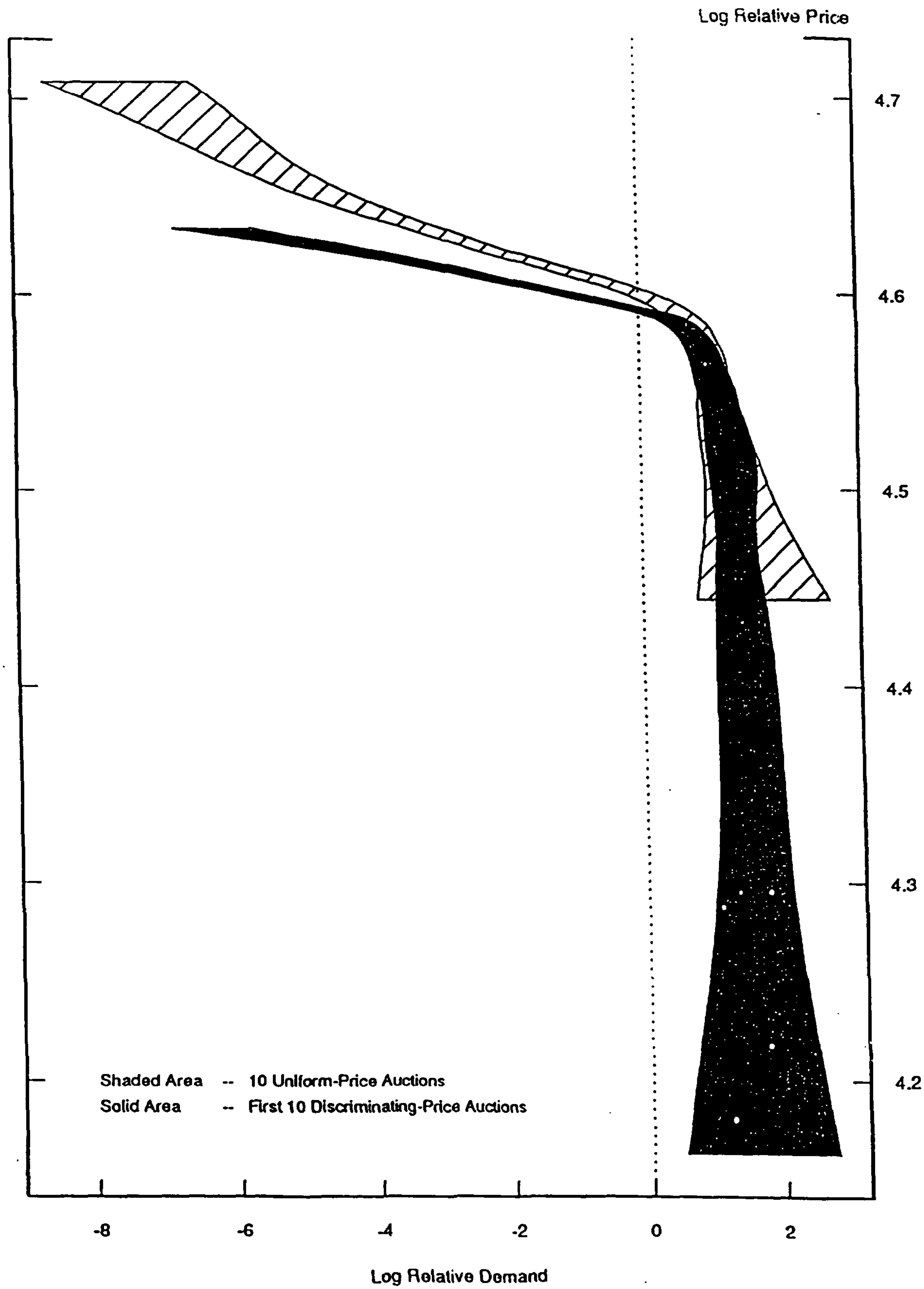
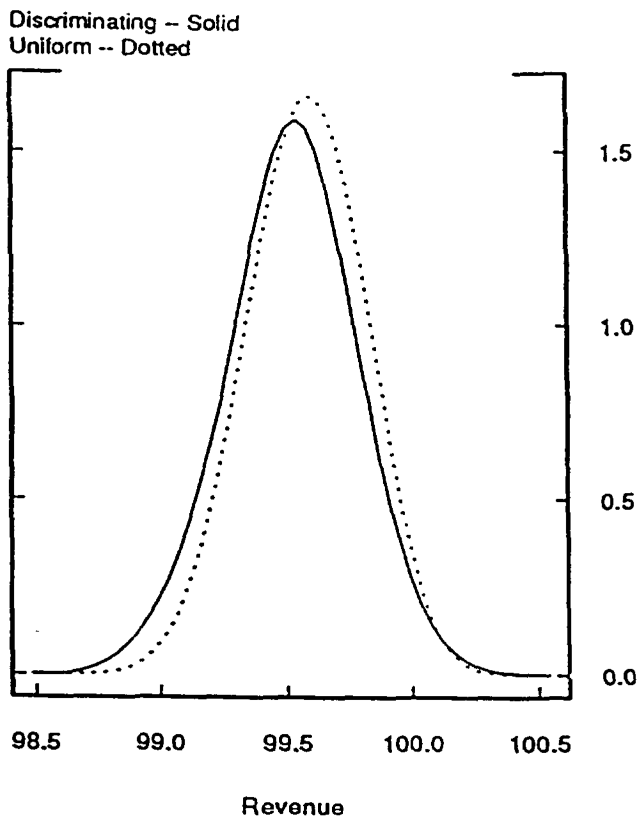


Figure 7

Uniform vs Discriminating Price Revenue
(200 repetitions)



	Discriminating	Uniform
Mean=	99.52	99.58
Median=	99.53	99.59
Variance=	0.05	0.03
Skewness=	-0.21	-0.24
Kurtosis=	0.19	-0.39

	Statistic	P-Value
T-test of Equal Means	-2.91	0
F-test of Equal Variances	1.39	0.02
Wilcoxon Rank Sum	-2.74	0.01
Kolmogorov-Smirnov	0.1	0

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