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Nonlinearity and Endogeneity in Macro-Asset Pricing

Prepared by Craig Hiemstra 1/ and Charles Kramer

Authorized for distribution by David Folkerts-Landau

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Abstract

We find nonlinear feedback between the stock market and certain macroeconomic factors. This evidence calls into question the adequacy of these factors as a basis for a linear pricing model. It also means that the interaction between the economy and the stock market is more complicated than given by the simple relationship in Chen, Roll and Ross (1986). It also suggests that the univariate evidence for nonlinear dynamics in the stock market may be due to the complicated relationship between the macroeconomy and the stock market.

JEL Classification Numbers:

C22, C52, E44, G14

1/ Mr. Hiemstra is with the Department of Accounting and Finance, University of Strathclyde, Glasgow G4 OLN, Scotland.

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Summary

Linear asset-pricing relations, with macroeconomic factors as state variables, have found wide use in empirical finance. Applications of such relations range from academic studies of market efficiency and market anomalies to practical uses such as risk management and estimation of the cost of capital. These applications make two key assumptions: that the relationship is exclusively linear and that the macroeconomic factors are exogenous to returns. For the set of macro factors commonly used in these applications, both assumptions run counter to economic intuition.

This paper demonstrates that the assumptions are also counter to empirical evidence by testing for linear and nonlinear Granger causality. The tests work as follows. Given two forecasts of a time series--a forecast from its own lags and a forecast from its own lags and the lags of a second series--if the second is more accurate than the first (if the improvement is statistically significant), the second time series is said to Granger cause the first. When two time series Granger cause one another, feedback is said to exist between them.

Linear and nonlinear feedback are found between stock returns and commonly used macroeconomic pricing factors as well as between residuals from linear pricing relations and returns. In addition, there is little evidence to suggest that neglected autoregressive or autoregressive conditionally heteroscedastic dynamics are responsible for these findings, implying that the underlying dynamics are complicated.

The evidence strongly suggests that macroeconomic factors are neither exogenous nor related to stock returns in a solely linear way. Thus, linear asset pricing relations omit interesting and potentially useful aspects of the relationship between stock returns and the macroeconomy. The evidence also sheds light on the literature on univariate nonlinear dynamics in stock returns. It suggests that such dynamics result from a complicated interrelationship between the stock market and the macroeconomy.

I. Introduction 1/

Since the seminal paper of Chen, Roll, and Ross (1986), the empirical finance literature has seen a proliferation of multifactor asset pricing applications that use macroeconomic time series as state variables. 2/ The true state variables in models such as Merton (1973) and Cox, Ingersoll and Ross (1985) are unobservable, so researchers instead use proxies such as output growth, consumption growth, inflation, the term structure, or the bond-market default premium.

Two problems immediately present themselves in this situation. First, returns may not have a linear representation in terms of an arbitrary set of state variables. 3/ Second, unlike the state variables they are supposed to represent, macroeconomic variables are not exogenous to asset returns. Both problems have already been recognized in the empirical finance literature, though little has been done to quantify them. Chen, Roll and Ross (1986) acknowledge the problem of endogenous macrofactors, and Chen (1991) applies two-stage least squares and looks for correlation in higher moments. The problems with estimation and inference in the presence of specification error (e.g., neglected nonlinearity) and endogenous regressors (e.g., the absence of weak exogeneity) are well-known, and indeed may be manifest in the findings of unstable coefficients and the forecasting power of returns for macroeconomic activity. 4/

We take a critical look at the issues of nonlinearity and endogeneity by looking for linear and nonlinear causality between commonly-used macroeconomic factors and stock returns. We find evidence for bi-directional causality between stock returns and macroeconomic factors. We also study the residuals from estimated linear pricing equations, looking for a causal relation between the residuals and returns. Finally, we look for linear and nonlinear temporal dependence in the residuals using a battery of diagnostic tests.

1/ Versions of this paper were presented for the Economics Department at Southern Methodist University, the Time Series Group at the Santa Fe Institute, the Chaos and Nonlinear Dynamics Study Group at the U.S. Bureau of Labor Statistics, and the 1994 North American Summer Meetings of the Econometric Society. We wish to thank the participants at these presentations for helpful comments. We also wish to thank Pedro de Lima, Robert Flood, Ted Jaditz, Jonathan Jones, Francis Longstaff, and anonymous referees for comments. We also thank Janet Shelley for her help in converting this manuscript into WordPerfect.

2/ Examples are Chen (1991), Harvey and Ferson (1991), Chang and Pinegar (1990), and Chan, Chen and Hsieh (1985). More examples are cited in Fama (1991). Due to most readers' familiarity with such models, our discussion of them is intentionally kept brief.

3/ See Bansal, Hsieh and Viswanathan (1992) and Bansal and Viswanathan (1993).

4/ See, *inter alia*, Chen, Roll and Ross (1986) and Chen (1991).

Our evidence can be viewed as critical of the linear macrofactor asset pricing method: it shows that linear models which treat macroeconomic variables as exogenous may be misspecified. On the other hand, it can also be viewed as supportive of the general conclusions drawn from such models. Our evidence for a nonlinear causal relationship between returns and the macroeconomy supports the view that predictable variation in returns is due to changes in macroeconomic fundamentals, while at the same time implying that macroeconomic time series are not good linear proxies for these fundamentals. This evidence is also interesting given the recent surge in interest in nonlinear dynamics in stock returns. 1/ Our approach extends this literature in that we consider joint rather than univariate dynamics, and explore the role of the macroeconomy. Our evidence suggests that some of the complicated behavior of stock returns may stem from their complicated relationship to macroeconomic fundamentals.

Our approach is complementary to the approach of Bansal and Viswanathan (1993), which looks at similar questions in the context of dynamic optimization. Their approach tells us much about the economic significance of nonlinearities in asset pricing, as it imposes economic restrictions. Their tests and the tests in Bansal, Hsieh, and Viswanathan (1993) show support for a nonlinear specification over a linear one. Our approach could be thought of as suggestive of candidate variables for their nonlinear pricing kernel, for example, or as a specification test for such models. For example, our results say something about which factors have nonlinear relations with returns, and which do not, as well as the direction of causality. They also permit us to rule out some factors as having a nonlinear causal relationship with returns, and also some alternative explanations for nonlinear dependence in returns.

Applications of linear macrofactor pricing models range from the construction of measures of systematic risk and assessment of cross-sectional pricing to estimation of systematic causes of time-variation in returns. 2/ Our results have strong implications for cross-sectional studies which use risk measures constructed from linear models, as well as for time-series studies. If the linear model is not correct, neither are risk measures constructed from it. One might argue that the (linear) state space should then be augmented with nonlinear functions of the state variables, but the idea of spanning payoffs with a few factors (relative to the number of securities) is then lost, and with it the empirical usefulness of the theory.

1/ See Hsieh (1991).

2/ For an instance of the practical use of these linear models, see Berry, Burmeister and McElroy (1988).

II. Granger Causality Testing

We next discuss the Granger causality testing methodologies used to generate our findings. Because traditional approaches to testing for Granger causality are well known, we start with a very brief sketch of them. We then discuss the Baek and Brock (1992a) approach to testing for nonlinear Granger causality in more detail, and finish with a discussion of the connections between Granger causality and macrofactor endogeneity.

1. Granger causality and the Granger direct test

For the case of two scalar-valued, strictly stationary, and ergodic time series, say X and Y, Granger causality is defined in terms of the predictive power of one series for the other. Our focus is on strict Granger causality. 1/ Y is said to strictly Granger cause X if the probability distribution of X conditional on lagged values of X and Y differs from the probability distribution of X conditioned only on lagged values of X. A traditional approach to making the strict Granger causality definition testable in the time domain relies on a VAR specification for the series $\{X_t\}$ and $\{Y_t\}$, t=1,2,.... Let A(L), B(L), C(L), and D(L) denote one-sided lag polynomials of orders a, b, c, and d. Also let $\{U_t\}$ and $\{V_t\}$ denote error terms, assumed to be individually and mutually independent and identically distributed (IIDI) with zero means and constant variances. The VAR specification then can be expressed as,

$$X_t = A(L)X_t + B(L)Y_t + U_t$$

$$Y_t = C(L)X_t + D(L)Y_t + V_t. \tag{1}$$

The null hypothesis that Y does not Granger cause X is rejected if the coefficients on the elements in B(L) are jointly significantly different from zero. Bidirectional Granger causality (or feedback) is said to exist between X and Y if Granger causality runs in both directions. This testing procedure is known as the Granger direct test. In our applications of the Granger direct test we evaluated the hypothesis that Y does not strictly Granger cause X using an F-test for exclusion restrictions for the lagged values of Y. We also used the Akaike (1974) criterion to determine the lag truncation lengths, a and b, on the lag polynomials. 2/

1/ The literature on Granger causality testing is broad. See Geweke (1984), Geweke, Meese, and Dent (1983), and Granger (1990) for more information on the notion of Granger causality and associated statistical tests.

2/ We calculated the Akaike criterion for every combination of a and b where each ran between 1 and 40. The combination of a and b with the smallest value of the Akaike criterion was chosen.

2. Testing for nonlinear Granger causality

Baek and Brock (1992a) propose a method for uncovering nonlinear causal relations that by construction cannot be detected by causality tests which focus on cross-correlations. Through the use of their method, evidence of nonlinear causal relations has been found between money and income (Baek and Brock (1992a)), between the producer and consumer price indices (Jaditz and Jones (1993)), and between aggregate stock returns and volume (Hiemstra and Jones (1994)). In this section we describe their method.

Our discussion of the Baek and Brock approach begins with their testable implication of the strict Granger noncausality notion. Consider two strictly stationary and weakly dependent scalar time series $\{X_t\}$ and $\{Y_t\}$, $t=1,2,\ldots$ 1/ Denote the m-length lead vector of X_t by X_t^m and the Lx- and Ly-length lag vectors of X_t and Y_t by X_{t-Lx}^{Lx} and Y_{t-Ly}^{Ly}. That is,

$$X_t^m \equiv (X_t, X_{t+1}, \ldots, X_{t+m-1}),\ m=1,2,\ldots;\ \ t=1,2\ldots,$$

$$X_{t-Lx}^{Lx} \equiv (X_{t-Lx}, X_{t-Lx+1}, \ldots, X_{t-1}),\ Lx=1,2,\ldots; t=Lx+1, Lx+2\ldots,$$

$$Y_{t-Ly}^{Ly} \equiv (Y_{t-Ly}, Y_{t-Ly+1} \ldots, Y_{t-1}),\ Ly=1,2,\ldots; t=Ly+1, Ly+2,\ldots. \tag{2}$$

The Baek and Brock approach tests for strict Granger causality running from variable Y to X by examining whether the conditional probability that two arbitrarily-selected m-length lead vectors of $\{X_t\}$, say X_t^m and X_m^s, are close to each other given that their corresponding Lx- and Ly-length lag vectors (viz. X_{t-Lx}^{Lx} and X_{s-Ly}^{Ly}, and Y_{t-Ly}^{Ly} and Y_{s-Ly}^{Ly}), are close to each other is the same as the conditional probability that the two lead vectors are close to each other given only that the corresponding lag vectors of X_t and X_m^s are close to each other. More formally, the Baek and Brock approach examines the following implication of strict Granger causality: for given

1/ The Baek and Brock approach to testing for nonlinear Granger causality relies on correlation-integral estimators of certain spatial probabilities corresponding to vector time series. For certain strictly stationary and weakly dependent processes, Denker and Keller (1983) show that estimators such as these (bounded-kernel U-statistics) are consistent estimators. See Denker and Keller (1983, pp. 505-7) for the conditions under which their consistency results hold. Loosely, weakly dependent processes display short-term temporal dependence which decays at a sufficiently fast rate. Formal discussions of weakly dependent processes can be found in Denker and Keller (1983) and their references.

values of m, Lx, and Ly $\geq$ 1 and e > 0, if Y does not strictly Granger cause X then,

$$Pr\left(\|X_t^m - X_s^m\| < e \mid \|X_{t-Lx}^{Lx} - X_{s-Lx}^{Lx}\| < e,\ \|Y_{t-Ly}^{Ly} - Y_{s-Ly}^{Ly}\| < e\right)$$

$$= Pr\left(\|X_t^m - X_s^m\| < e \mid \|X_{t-Lx}^{Lx} - X_{s-Lx}^{LX}\| < e\right) \tag{3}$$

where Pr($\cdot$) denotes probability and $||\cdot||$ denotes a distance norm. We employ the maximum norm in our application. 1/

In implementing a test based on (3) it is useful to express the conditional probabilities in terms of the corresponding ratios of joint probabilities. Let C1(m+Lx,Ly,e)/C2(Lx,Ly,e) and C3(m+Lx,e)/C4(Lx,e) denote the ratios of joint probabilities corresponding to the left- and right-hand sides of (3). They are defined as, 2/

$$C1(m{+}Lx, Ly, e) \equiv Pr\left(\|X_{t-Lx}^{m+Lx} - X_{s-Lx}^{m+Lx}\| < e,\ \|Y_{t-Ly}^{Ly} - Y_{s-Ly}^{Ly}\| < e\right),$$

$$C2(Lx, Ly, e) \equiv Pr\left(\|X_{t-Lx}^{Lx} - X_{s-Lx}^{Lx}\| < e,\ \|Y_{t-Ly}^{Ly} - Y_{s-Ly}^{Ly}\| < e\right),$$

$$C3(m{+}Lx, e) \equiv Pr\left(\|X_{t-Lx}^{m+Lx} - X_{s-Lx}^{m+Lx}\| < e\right),$$

$$C4(Lx, e) \equiv Pr\left(\|X_{t-Lx}^{Lx} - X_{s-Lx}^{Lx}\| < e\right), \tag{4}$$

The Granger-noncausality condition in equation (3) can then be reexpressed as:

1/ The maximum norm for $Z \equiv (Z_1, Z_2, \ldots, Z_K) \in \mathbb{R}^K$ is defined as max (Z_i) i=1,2,...,K. Computational speed in implementing the test is one important reason for using the maximum norm. A more general version of the test can be devised by considering different length scales, e, corresponding to the lead and lag vectors. Also the test can easily be generalized beyond the bivariate case considered here.

2/ By definition, the conditional probability Pr(A|B) can be expressed as the ratio Pr(A$\cap$B)/Pr(B). Note that the maximum norm allows us to write $Pr\ (\|X_t^m - X_s^m\| < e, \|X_{t-Lx}^{Lx} - X_{s-Lx}^{Lx}\| < e)$ as $Pr\ (\|X_{t-Lx}^{m+Lx} - X_{s-Lx}^{m+Lx}\| < e)$.

$$\frac{C1(m+Lx,Ly,e)}{C2(Lx,Ly,e)} = \frac{C3(m+Lx,e)}{C4(Lx,e)}. \tag{5}$$

One can test the condition in (5) using correlation-integral estimators of the joint probabilities. For the time series of realizations of X and Y, say $\{x_t\}$ and $\{y_t\}$ (t=1,2,...,T), let $\{x_t^m\}$, $\{x_{t-Lx}^{Lx}\}$, and $\{y_{t-Lx}^{Lx}\}$ (t=max(Lx,Ly)+1,...,T-m+1) denote the time series of m-length lead and Lx-length lag vectors of $\{x_t\}$ and the Ly-length lag vectors of $\{y_t\}$ as defined by (2). Letting $I(Z_1, Z_2, e)$ denote an indicator kernel indicating with 1 whether two conformable vectors Z_1 and Z_2 are within the maximum norm distance e of each other and with 0 otherwise, correlation-integral estimators of these joint probabilities are:

$$C1(m+Lx,Ly,e,n) \equiv \frac{2}{n(n-1)} \sum_{all} \sum_{t<s} I(x_{t-Lx}^{m+Lx},x_{s-Lx}^{m+Lx},e) \cdot I(y_{t-Ly}^{Ly},Y_{s-Ly}^{Ly},e),$$

$$C2(Lx,Ly,e,n) \equiv \frac{2}{n(n-1)} \sum_{all} \sum_{t<s} I(x_{t-Lx}^{Lx},x_{s-Lx}^{Lx},e) \cdot I(y_{t-Ly}^{Ly},Y_{s-Ly}^{Ly},e),$$

$$C3(m+Lx,e,n) \equiv \frac{2}{n(n-1)} \sum_{all} \sum_{t<s} I(x_{t-Lx}^{m+Lx},x_{s-Lx}^{m+Lx},e),$$

$$C4(Lx,e,n) \equiv \frac{2}{n(n-1)} \sum_{all} \sum_{t<s} I(x_{t-Lx}^{Lx},x_{s-Lx}^{Lx},e),$$

$$t,s=\max(Lx,Ly)+1,\ldots,T-m+1$$
$$n=T+1-m-\max(Lx,Ly). \tag{6}$$

The strict Granger noncausality condition in (5) can be tested statistically using the estimators in (6). Specifically, under the assumption that $\{X_t\}$ and $\{Y_t\}$ are strictly stationary, weakly dependent, and satisfy the mixing conditions of Denker and Keller (1983), if for given values of m, Lx, and Ly $\geq$ 1 and e > 0, $\{Y_t\}$ does not strictly Granger cause $\{X_t\}$ then,

$$\sqrt{n}\left(\frac{C1(m+Lx,Ly,e,n)}{C2(Lx,Ly,e,n)} - \frac{C3(m+Lx,e,n)}{C4(Lx,e,n)}\right) \sim AN\left(0,\sigma^2(m,Lx,Ly,e)\right), \tag{7}$$

where σ^2 (m,Lx,Ly,e) and estimators for it are given by expressions described in the Appendix.

One can test for nonlinear Granger causality by applying the test in (7) to the residuals of a VAR in $\{X_t\}$ and $\{Y_t\}$. When one removes linear effects in this way, any rejection of the noncausality null cannot be properly interpreted as indicating a linear causal relation between the series. Instead, when the test in (7) is applied to the VAR residuals series of X and Y, the null hypothesis is that Y does not nonlinearly Granger strictly cause X, and (7) holds for all m, Lx, and Ly ≥ 1 and for all $e > 0$.

To interpret a rejection of the strict Granger noncausality null between two VAR residuals series as indicating a nonlinear causal relation, however, raises a number of issues that we now address. The most important issue concerns the asymptotic distribution of the test when it is applied to residuals rather than disturbances (i.e., in (1), $\{\hat{U}_t\}$ and $\{\hat{V}_t\}$ rather than $\{U_t\}$ and $\{V_t\}$). For linear VAR models such as that in (1) Baek and Brock (1992b) have shown that the asymptotic distribution of their variant of the test in (7) is the same when applied to consistently estimated residuals as when applied to the IIDI errors of the maintained model. Their version of the test (hereafter called the Baek and Brock test) is said to be nuisance-parameter-free (NPF) for such models. The version of the test which we used in this study (hereafter called the modified Baek and Brock test of Hiemstra (1992) and Hiemstra and Jones (1993)), however, applies to the general case in which the IIDI assumption is relaxed. 1/ Such an NPF result has yet to be produced for the modified Baek and Brock test. Nonetheless, there is some Monte Carlo evidence to suggest that the modified test is effectively immune to adverse parameter-estimation effects in linear VAR residuals. Hiemstra and Jones (1993b) have found a close correspondence between the asymptotic and finite-sample properties of the test when applied to consistently estimated VAR residuals in noncausal linear relations.

Another set of issues concerns the finite-sample size and power properties of the test, the selection of the lead and lag truncation lengths (m, Lx, and Ly), and the selection of the length-scale parameter, e. Hiemstra and Jones (1993) have found that for sample sizes of 500 observations, a lead length of m=1, lag lengths of Lx=Ly=1,2,...,5, and length scales of e=1.5, 1.0, and 0.5 corresponding to standardized series of $\{x_t\}$ and $\{y_t\}$ that the modified test displays good finite-sample size and power properties for a variety of relationships (linear and nonlinear, causal and noncausal) in temporally-dependent time series. 2/ However, Baek and Brock (1992a) and Hiemstra and Jones (1993b) have found that the finite-sample size of the Baek and Brock test is considerably larger than its nominal counterpart in some cases where the series to which the test is

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1/ Hiemstra and Jones (1993) modified version of the test holds for the more general assumption that the errors are weakly dependent. The fundamental differences between the two versions of the test are manifested only in estimators of σ^2 (m,Lx,Ly,e) in equation (7).

2/ Hiemstra and Jones (1993b) also find through Monte Carlo simulations that the modified Baek and Brock test is immune to the effects of contemporaneous correlation and neglected nonstationarities due to structural breaks.

applied display temporal dependence. For these reasons we used the modified Baek and Brock test, which corrects this problem.

3. Implications of strict Granger causality from returns to macrofactors

We motivate our tests of strict Granger causality from returns to macrofactors through references to the hypothesis (maintained by earlier studies) of the exogeneity of the macrofactors. Before proceeding, it is necessary to clearly define exogeneity, and in particular to distinguish various types of exogeneity with respect to their implications for inference. 1/ The practical consequences of exogeneity for our study concern inference on relationships such as:

$$r_t = \alpha + \beta f_t + \epsilon_t, \tag{8}$$

where $\{r_t\}$ denotes an asset-return series and $\{f_t\}$ a vector series of macroeconomic factors. Studies such as Chen, Roll, and Ross (1986) assume that inference can be performed conditionally on the observed outcomes of the macroeconomic factors (e.g., ignoring the process determining macroeconomic factors). In other words, such studies assume that macroeconomic factors are weakly exogenous to stock returns. Loosely, weak exogeneity means that the parameters of the likelihood for $\{r_t\}$ conditional on $\{f_t\}$ are not restricted by the parameters of the marginal likelihood for $\{f_t\}$. 2/ Causality tests can only yield information about strict exogeneity, that is, whether f_t is independent of ε_s for all t and s. 3/ Also, strict exogeneity need not imply nor be implied by weak exogeneity.

If stock returns are rationally determined, as in the models used to justify linear macrofactor regressions, this sheds some light on the implications of our causality tests. Rational expectations implies that the parameters of the driving process (e.g., macrofactors) restrict the parameters of the processes for equilibrium quantities and prices (e.g., returns). 4/ Brock (1982) and Cox, Ingersoll and Ross (1985) are examples of models with this feature; in fact, they are frequently used to motivate linear regressions of stock returns on macrofactors. The cross-equation restrictions of rational expectations mean that the data-generating process assumed for returns must take into account the data-generating process for macrofactors. Otherwise, inference will be affected.

1/ See Engle, Hendry and Richard (1983) for a discussion of the various definitions of exogeneity.

2/ See Engle, Hendry and Richard (1983).

3/ See Geweke (1984). In particular, Granger causality tests can refute (but not establish) a claim of strict exogeneity. That is, finding Granger causality from Y to X implies that Y is not strictly exogenous, while a failure to find Granger causality from Y to X does not necessarily imply that Y is strictly exogenous.

4/ A similar point about rational expectations is made by Geweke (1984), p. 1120.

III. Data

Asset-pricing theory does not identify the relevant state variables, except in special cases such as the CAPM and Breeden's (1979) consumption CAPM. As Chen, Roll, and Ross (1986) note, however, the expected return to equity is a function of expected future cash flows and the discount rate corresponding to those cash flows. This insight has led to a proliferation of macrofactors which are putatively related to cash flows or discount rates, and which are used to explain asset returns. We picked factors which are common to many other empirical studies of asset pricing. The set consists of a default-risk factor, DE, which reflects changes in the return to the pure risk of default on fixed-income securities; a maturity-risk factor, MA, which captures changes in the return to term (the slope of the yield curve); an unexpected inflation factor, UI, which captures the effect of inflation shocks on cash flows; a business-cycle factor, CG, which captures consumption risk; and an industrial-production factor, MP, which captures the risk in aggregate output. Many other factors have been proposed and tested in various empirical applications. These five are typical to multi-factor models, and have been found useful in explaining returns in many contexts.

1. Macrofactor series

The first two factors, DE and MA, capture the effects of debt-market exposure on the expected cash flow of the firm. Following Chen, Roll and Ross (1986) and Burmeister and McElroy (1988), they are constructed from bond returns as follows. The default factor, DE, for the current month is the difference in the return of two bonds with the same maturity but different risks of default. We take the difference in returns on long-term Treasury bonds and long-term corporate bonds. Both returns series come from Ibbotson and Sinquefeld's Stocks, Bonds, Bills and Inflation (SBBI). The maturity or yield-curve factor, MA, for the current month is the difference in return of two bonds with the same risk of default but different maturities. We use the difference in the return on long-term Treasury bonds and the Treasury bill closest to one month in maturity, also from SBBI.

The unexpected-inflation factor measures the effect of inflation shocks on the predicted value of cash flows. The factor used here, UI, is constructed from the seasonally-adjusted Consumer Price Index for all urban consumers. We follow Ferson and Harvey (1991) in using an integrated moving average process IMA(1,1) as a model for expected inflation. The residual from the fitted model is our measure of the unexpected component of inflation. 1/

1/ Estimation of the model yielded an estimated MA parameter of -0.742 with a t-ratio of -21.52.

Formal models of equilibrium asset pricing relate returns to consumption growth. 1/ Consumption growth captures near-term variation in the business cycle as well as consumption-beta pricing. Again following Ferson and Harvey (1991), the consumption-growth factor, CG, is the growth in monthly seasonally-adjusted, real personal consumption of nondurables, i.e., CG(t) = C(t)/C(t-1) where C(t) denotes consumption in month t.

Finally, we construct a macroeconomic-output factor, MP. Following Chen, Roll and Ross (1986) and Chang and Pinegar (1990), we use the growth rate in seasonally-adjusted industrial production. This is constructed as MP(t) = log(IP(t))-log(IP(t-1)), where IP(t) denotes industrial production in month t. As Chen, Roll and Ross (1986) note, since industrial production, a flow variable, is lagged over part of month t, MP(t) should lead returns by one month. We therefore use MP(t+1) as a measure of industrial output in month t.

2. Returns series

We use the return on the Standard & Poor's 500 index, SP, from SBBI, as the dependent variable. The S&P index is a standard benchmark equity returns series for applied asset-pricing work. We examine both raw returns on the S&P, and net-risk-free returns, ST. The net-risk-free returns were constructed by subtracting the one-month T-bill rate (from SBBI) from raw returns. The net-risk-free rate closely approximates the inflation-adjusted rate of return, as well as measuring the equity premium. 2/ Our data span the period February 1959 to December 1989.

3. The linear relationship between returns and the macrofactors

To get a qualitative feel for the usefulness of the five series in explaining returns, we present results in Table 1 of the regression of the two returns series, SP and ST, on the five macrofactors, CG, DE, MA, MP, and UI. The regression coefficients have sensible signs and magnitudes. We expect positive returns to be associated with all risk factors except for inflation, which decreases expected real returns. All factor coefficients are significant at the 5 percent level except for the macroeconomic output factor, MP, in the regression using SP. The coefficients in the two regressions are quite similar. Moreover, the explanatory power (R^2) of the regressions is typical for such financial-markets regressions. 3/

1/ See Breeden (1979).
2/ See Ferson (1990).
3/ See Roll (1988).

Table 1. OLS Regression Results for the Linear Relations 1/

VARIABLE	SP REGRESSION PARAMETER ESTIMATE	t-STAT	ST REGRESSION PARAMETER ESTIMATE	t-STAT
1	0.0067	3.12	0.0014	0.67
DE	0.7238	4.06	0.7318	4.08
MA	0.5705	7.24	0.5725	7.24
UI	-2.3063	-2.59	-2.2947	-2.57
CG	0.4455	1.64	0.4688	1.72
MP	0.3170	1.46	0.3722	1.72
	R^2=0.165		R^2=0.165	

1/ OLS regression results for the linear relations between monthly S&P (SP) and S&P-TBill (ST) series and the macro asset-pricing variables: unexpected inflation, UI; consumption growth, CG; default premium, DE; maturity premium, MA; and industrial production, MP.

IV. Results

In Section IV.1 we present findings from the Granger test of an endogenous relationship between returns and the maturity-risk and industrial-production factors. The modified Baek and Brock test discussed in Section IV.2 indicates feedback between returns and the default-, inflation-, and maturity-risk factors. This evidence for nonlinear interactions between returns and the macroeconomy is corroborated by evidence presented in Section IV.3 of a causal relation between returns and the residuals from a Chen, Roll, and Ross (1986) style returns-generating relation. Evidence of nonlinear temporal dynamics in returns is also presented in Section IV.4.

1. Endogeneity and the Granger test

The results of the Granger test are displayed in Table 2. As shown in the table, the Granger test detects causality running from each factor (except consumption growth) to returns at 5 percent nominal significance. The test also detects causality running from returns to each factor (except default and inflation risk). Thus, the Granger test finds evidence of bi-directional causality (feedback) between returns and the maturity-risk

Table 2. Granger Test 1/

	SP<≠CG	ST<≠CG	CG<≠SP	CG<≠ST
Lag lengths: a,b	1 1	1 1	3 30	3 30
Test value	0.697	0.540	1.483*	1.544*
Significance level	0.404	0.462	0.053	0.037
	SP<≠DE	ST<≠DE	DE<≠SP	DE<≠ST
Lag lengths: a,b	1 18	1 18	27 1	27 1
Test Value	1.696*	1.713*	0.204	0.293
Significance Level	0.038	0.035	0.652	0.588
	SP<≠UI	ST<≠UI	UI<≠SP	UI<≠ST
Lag lengths: a,b	2 23	1 4	31 1	31 1
Test Value	1.788*	3.449*	0.139	0.102
Significance Level	0.015	0.008	0.708	0.749
	SP<≠MA	ST<≠MA	MA<≠SP	MA<≠ST
Lag lengths: a,b	2 24	2 7	11 2	11 2
Test value	1.932*	3.878*	4.265*	4.472*
Significance level	0.006	0.000	0.015	0.012
	SP<≠MP	ST<≠MP	MP<≠SP	MP<≠ST
Lag lengths: a,b	1 12	1 12	27 10	27 10
Test value	2.122*	2.023*	3.337*	4.135*
Significance level	0.015	0.021	0.000	0.000

1/ Statistics corresponding to strict Granger causality between monthly S&P (SP) and S&P-Treasury bill (ST) series and the macro asset-pricing variables: unexpected inflation, UI; consumption growth, CG; default premium, DE; maturity premium, MA; industrial production, MP. X<≠Y denotes "Y does not strictly Granger cause X." The lag lengths a and b were determined by the Akaike (1974) criterion. Asterisks denote a rejection of the hypothesis at 5 percent nominal significance.

and industrial-production factors evidence consistent with previous findings of predictability between macroeconomic time series and stock returns. 1/

2. Nonlinear endogeneity and the modified Baek and Brock test

Table 3 displays the results of applying the modified Baek and Brock test to bivariate VAR-residuals of the returns and macrofactor series. The test statistics correspond to the test in equation (7). They are based on a lead value of m=1, lags of Lx=Ly=1,...,5, and a common length scale e of 1.5 standard deviations in the standardized VAR-residuals series. Under the null hypothesis that Y does not nonlinearly strictly Granger cause X, the test statistic (which is studentized) is asymptotically distributed N(0,1).

At 5 percent nominal significance corresponding to a one-sided, right-tailed test, the results in the table indicate the presence of a nonlinear endogenous relationship between returns and the default-, inflation-, and maturity-risk factors. 2/ For the VAR residuals of these factors the modified Baek and Brock test rejects the nonlinear Granger noncausality null running from factors to returns and vice versa in many cases for these series. However, the test provides no evidence of a nonlinear causal relation between returns and the consumption-growth and industrial-production factors.

We interpret these results as evidence of higher-order interactive behavior between returns and these factors. For example, if stocks hedge against inflation, as is commonly believed, they appear to do so in a complicated fashion. Also, given that the default series represents the default risk inherent in corporate bonds versus government bonds of the same term, it is not surprising that feedback appears between default risk and equity returns. A firm financing a risky project and an investor choosing a portfolio consider bonds and stocks to be substitutes. As both stocks and bonds represent claims on the firm's future cash flows, the same business-cycle and production risks determine their ex-ante and ex-post returns. This interpretation is substantiated by similar findings for the maturity risk factor, MA. The results indicate strong evidence for nonlinear feedback between maturity risk and both raw and net-riskless-rate returns. This is consistent with changes in discount rates which affect both the yield curve and stock prices, although not in a simple fashion.

If our two debt-market factors, MA and DE, are bound up in a dynamic relation with equity returns, manifesting their inter-relation with production risk, we would expect the industrial-production factor, MP, to

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1/ See Cochrane (1991), Chen (1991), and references therein.

2/ As can be seen in equation (7), a significant positive test statistic indicates than one series helps to predict another, while a significant negative statistic indicates that one series confounds the prediction of another. Our view is that the Brock and Baek test statistic should be evaluated using the right-tail critical value.

Table 3. Modified Baek and Brock Nonlinear Granger Causality Test 1/

Lx=Ly	CG<≠ST		CG<≠SP		ST<≠CG		SP<≠CG	
	ΔPR	TS	ΔPR	TS	ΔPR	TS	ΔPR	TS
1	−0.0014	−0.222	−0.0009	−0.137	0.0025	0.417	0.0019	0.310
2	0.0009	0.103	0.0017	0.188	−0.0051	−0.575	−0.0056	−0.647
3	0.0006	0.057	0.0029	0.270	−0.0087	−0.793	−0.0091	−0.822
4	−0.0007	−0.050	0.0016	0.119	−0.0160	−1.138	−0.0174	−1.192
5	−0.0045	−0.270	−0.0025	−0.150	−0.0220	−1.202	−0.0227	−1.239

Lx=Ly	DE<≠ST		DE<≠SP		ST<≠DE		SP<≠DE	
	ΔPR	TS	ΔPR	TS	ΔPR	TS	ΔPR	TS
1	0.0107	1.505	0.0107	1.484	0.0156	2.186*	0.0158	2.191*
2	0.0183	1.918*	0.0171	1.830*	0.0101	1.146	0.0101	1.157
3	0.0182	1.798*	0.0159	1.646*	0.0141	1.275	0.0130	1.171
4	0.0223	1.606	0.0195	1.472	0.0289	2.254*	0.0260	2.071*
5	0.0283	1.691*	0.0269	1.659*	0.0225	1.633	0.0208	1.475

Lx=Ly	UI<≠ST		UI<≠SP		ST<≠UI		SP<≠UI	
	ΔPR	TS	ΔPR	TS	ΔPR	TS	ΔPR	TS
1	0.0026	0.484	0.0023	0.437	−0.0001	−0.024	0.0032	0.622
2	0.0041	0.499	0.0054	0.660	0.0081	0.976	0.0060	0.738
3	0.0168	1.859*	0.0154	1.695*	0.0145	1.469	0.0189	1.816*
4	0.0255	2.120*	0.0296	2.637*	0.0366	3.270*	0.0395	3.362*
5	0.0198	1.496	0.0263	2.027*	0.0401	2.532*	0.0428	2.590*

Lx=Ly	MA<≠ST		MA<≠SP		ST<≠MA		SP<≠MA	
	ΔPR	TS	ΔPR	TS	ΔPR	TS	ΔPR	TS
1	−0.0017	−0.361	0.0027	0.551	0.0143	2.483*	0.0153	2.416*
2	0.0071	0.915	0.0073	0.907	0.0117	1.452	0.0135	1.561
3	0.0152	1.884*	0.0161	1.793*	0.0202	2.057*	0.0224	2.072*
4	0.0164	1.854*	0.0182	1.892*	0.0339	2.973*	0.0351	2.830*
5	0.0196	1.918*	0.0221	1.954*	0.0356	2.862*	0.0373	2.705*

Lx=Ly	MP<≠ST		MP<≠SP		ST<≠MP		SP<≠MP	
	ΔPR	TS	ΔPR	TS	ΔPR	TS	ΔPR	TS
1	0.0029	0.419	0.0035	0.518	0.0029	0.454	0.0021	0.331
2	0.0015	0.190	0.0029	0.363	−0.0081	−0.993	−0.0082	−1.018
3	0.0017	0.167	0.0012	0.116	−0.0193	−2.061	−0.0209	−2.246
4	0.0031	0.228	0.0021	0.153	−0.0088	−0.680	−0.0060	−0.450
5	0.0089	0.526	0.0080	0.456	−0.0102	−0.620	−0.0100	−0.576

1/ Modified Baek and Brock nonlinear Granger causality test statistics corresponding to strict Granger causality between VAR residuals of monthly S&P (SP) and S&P-Treasury bill (ST) series and the macro asset-pricing variables: unexpected inflation, UI; consumption growth, CG; default premium, DE; maturity premium, MA; and industrial production, MP. X<≠Y denotes "Y does not nonlinearly Granger cause X." ΔPR denotes the difference in the conditional probabilities corresponding to the test in equation (3.7). TS denotes the studentized test statistic which under the null of Granger strict noncausality is asymptotically distributed N(0,1). Asterisks denote nominally significant test statistics at the 5 percent level for a one-sided, right-tailed test.

also display feedback. However, while we found evidence for such feedback by the Granger test, there is no evidence from the Baek and Brock tests for nonlinear causality from production growth to returns or vice versa. For consumption growth, CG, there is also no evidence from the Baek and Brock statistics of nonlinear Granger causality in either direction. This outcome is only moderately surprising. In the linear returns-generating relation shown in Table 1, consumption growth has marginal explanatory power for returns. The Granger test also failed to find feedback between consumption growth and returns. Moreover, consumption-risk pricing tends to be subsumed by market-risk pricing, so that the information in lagged consumption may already be captured by lagged stock-market returns. 1/ Finally, seasonal adjustment of these series may confound their relationship to stock returns, which are not seasonally adjusted.

3. Nonlinearity and the Granger causality tests

We next use the Granger and modified Baek and Brock causality tests to look for nonlinearity in the relationship of stock returns to the macroeconomy in a different way. Since the residual of a linear regression of stock returns on macroeconomic factors captures any neglected components of a nonlinear relationship between returns and the macroeconomy, any evidence of a causal relationship between returns and returns residuals would corroborate the findings of the modified Baek and Brock test of a nonlinear relation between returns and the macroeconomy. Also, one can think of these tests as controlling for any nonlinearities in returns that are due to a contemporaneous, linear relationship to the set of macrofactors (which may themselves display nonlinear behavior).

Table 4 displays Granger and modified Baek and Brock test statistics corresponding to the returns series and the residuals of the Chen, Roll, and Ross style returns-generating relation (shown in Table 1). Once again focusing on rejections of the strict Granger noncausality null at 5 percent nominal significance, note in the table that the Granger tests indicate the presence of feedback between the residuals and the returns series. Moreover, the modified Baek and Brock test, when applied to the bivariate VAR-residuals between returns and the residuals series, detects nonlinear feedback between the returns and the residuals. These results tests yield more evidence against the linearity of the macrofactor returns-generating function.

4. Nonlinear temporal dependence and residual diagnostics

Temporal dependence in raw stock returns has been reported by a number of authors. 2/ We use a battery of tests to examine the residuals from the macrofactor pricing model for such dependence. This evidence gives us a clue as to whether a contemporaneous relationship with the macroeconomy

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1/ See Mankiw and Shapiro (1986) and Giovannini and Weil (1989).
2/ See Hsieh (1991) and references therein.

Table 4. Granger and Modified Baek and Brock Causality Tests 1/

Granger Tests

	SP<≠SPR	ST<≠STR	SPR<≠ SP	STR<≠ ST
Lag lengths: a,b	1 1	1 1	1 1	1 1
Test value	12.273*	13.141*	5.381*	6.361*
Significance level	0.000	0.000	0.020	0.012

Modified Baek And Brock Tests

Lx=Ly	STR<≠ST		SPR<≠SP		ST<≠STR		SP<≠SPR	
	ΔPR	TS	ΔPR	TS	ΔPR	TS	ΔPR	TS
1	-0.0016	-0.792	-0.0019	-0.972	0.0050	1.616	0.0043	1.423
2	0.0006	0.200	-0.0004	-0.127	0.0062	1.701*	0.0061	1.778*
3	0.0090	1.977*	0.0074	1.703*	0.0140	3.379*	0.0133	3.313*
4	0.0111	2.055*	0.0092	1.802*	0.0167	3.273*	0.0154	3.253*
5	0.0139	2.356*	0.0117	2.069*	0.0170	3.326*	0.0164	3.235*

1/ Granger and modified Baek and Brock causality test statistics corresponding to strict Granger causality between monthly S&P (SP) and S&P-Treasury bill (ST) series and the linear pricing residuals corresponding to the regressions shown in Table 1. SPR and STR denote SP- and ST-residuals series. For the Granger test X<≠Y denotes "Y does not strictly Granger cause X." The lag lengths a and b were determined by the Akaike (1974) criterion. The modified Baek and Brock tests are applied to to VAR-residuals series. For them, X<≠Y denotes "Y does not nonlinearly Granger cause X," ΔPR denotes the difference in the conditional probabilities corresponding to the test in equation (3.7), and TS denotes the studentized test statistic of the modified Baek and Brock test which under the null of Granger strict noncausality is asymptotically distributed N(0,1). Askerisks denote nominally significant test statistics at the 5 percent level for a one-sided, right-tailed test.

might account for these results alone, or whether more complicated dynamics might also be at work. Evidence that the residual has complicated temporal dynamics would imply that the complicated dynamics of stock returns are not due to a contemporaneous, linear relationship with the macrofactors.

Table 5 displays tests for temporal dependence in the residuals of the Chen, Roll, and Ross style returns-generating relation shown in Table 1. We subjected the residuals series to the Durbin-Watson test, Diebold's (1988) adjusted-for-ARCH Ljung-Box test, Mcleod and Li's (1983) and Engle's (1982) tests, Lee, White, and Granger's (1993) RESET and neural net tests, and to Brock, Dechert, and Scheinkman's (1987) BDS test. 1/ Note in the table that only the RESET and BDS tests (which have power against nonlinear dependence) indicate the presence of temporal dependence in the residuals series at 5 percent nominal significance. 2/ Given that the Durbin-Watson and the adjusted Ljung-Box tests are optimal tests against linear AR dependence and that the Engle and McLeod and Li tests are optimal against ARCH dependence, the rejections by the RESET and BDS tests seem to indicate the presence of nonlinear temporal dependence in the residuals that is unlikely to be attributable to ARCH dynamics. These results suggest that the dynamics of the asset-pricing process are indeed complicated, and their complexity seems to stem from nonlinear temporal dependence.

1/ We used 24 autocorrelation and autocovariance terms to implement the adjusted-for-ARCH Ljung-Box test. We also used 24 autocovariance terms in implementing the Engle and McLeod and Li tests. Under the IID null these tests are asymptotically distributed χ^2 (24). The RESET test employed here is based on the residuals of an AR(p) model fit to the residuals series. We used the Akaike (1974) criterion using a maximum lag length of 10 periods to fit the series. The test is also based on the 2nd through 4th principal components of the raised-to-the-2nd through 6th AR(p) forecasts of the series. Under the null of no nonlinear temporal dependence in the conditional mean of the series, the test statistic is asymptotically distributed χ^2 (3). Our implementation of the neural net test relies on N(0,1) realizations to generate so-called hidden factors. In all other respects it conforms to Lee, White, and Granger's NEURRAL1 test. Under the assumption of no nonlinear dependence in the mean of a series, the neural net test is asymptotically distributed χ^2 (3). In implementing the BDS test we considered length scales equal to 1.5, 1.0, and 0.5 standard deviations in the series and embedding dimensions (or maximum lag lengths) ranging from 1 to 4. Under the IID null the BDS test is asymptotically distributed N(0,1).

2/ The test statistics corresponding to an embedding dimension of 4 are also significant at the appropriate 5 percent finite-sample level of significance for the IID N(0,1) case (see Brock, Hsieh, and LeBaron (1991)).

Table 5. Tests for Temporal Dependence 1/

		SP			SP-TB		
	DW	1.95			1.95		
	LB-ADJ	17.97			19.55		
	ENGLE	22.73			22.43		
	ML	23.24			23.15		
	RESET	8.40*			0.73		
	NEURAL	0.76			0.44		
	BDS	length scale			length scale		
		1.5σ	1.0σ	0.5σ	1.5σ	1.0σ	0.5σ
	2	1.79*	1.12	1.56	1.87*	1.11	1.46
embedding dimension	3	1.82*	1.39	1.33	1.85*	1.28	1.23
	4	2.46*	2.63*	3.22*	2.49*	2.49*	2.72*

1/ Tests for temporal dependence in the residuals of the Chen, Roll, and Ross style relation between monthly S&P (SP) and S&P-Treasury bill (ST) series and the macro asset-pricing variables: unexpected inflation, consumption growth, default premium, maturity premium, and industrial production. Regression results of the relation are displayed in Table 1. DW, LB-ADJ, ENGLE, ML, RESET, NEURAL, and BDS denote the Durbin-Watson, adjusted-for-ARCH Ljung-Box, Engle, McLeod and Li, RESET, neural net, and BDS tests mentioned in Section 4.4. Asterisks denote a rejection of the IID null for the BDS test and a rejection of no nonlinear dependence in the conditional mean of the residuals series for the RESET test at 5 percent nominal significance.

V. Conclusion

We have explored a number of hypotheses relating to endogeneity and nonlinearity in the relationship of stock returns to the macroeconomy, using a typical linear macrofactor asset pricing model. We find evidence, summarized in Table 6, which is consistent with both phenomena.

Table 6. Summary of the 5 Percent Nominal Significance Rejections 1/

Test	GD	MBB	Test	GD	MBB	Test	GD	MBB
CG≠>ST	A	A	DE≠>ST	R	R	UI≠>ST	R	R
CG≠>SP	A	A	DE≠>SP	R	R	UI≠>SP	R	R
ST≠>CG	R	A	ST≠>DE	A	R	ST≠>UI	A	R
SP≠>CG	R	A	SP≠>DE	A	R	SP≠>UI	A	R
MA≠>ST	R	R	MP≠>ST	R	A	STR≠>ST	R	R
MA≠>SP	R	R	MP≠>SP	R	A	SPR≠>SP	R	R
ST≠>MA	R	R	ST≠>MP	R	A	ST ≠>STR	R	R
SP≠>MA	R	R	SP≠>MP	R	A	SP ≠>SPR	R	R

1/ Summary of the 5 percent nominally significance rejections, R, and acceptances, A, for the Granger tests G, and the Modified Baek and Brock Nonlinear Granger causality tests, MBB, shown in Tables 2-4 of the null hypothesis that series Y does not strictly Granger cause X (Y≠>X).

Note: For the Lx=Ly and e values considered in the MBB tests, a rejection of the null hypothesis was considered to occur if any of the test statistics took on a value in excess of the critical value of the 95 percent tail region of the normal distribution (1.65). SP, ST, UI, CG, DE, MA, and MP correspond to the monthly return on the S&P, the SP return less the TBILL rate, and the macro-asset pricing variables unexpected inflation, consumption growth, default premium, maturity premium, and industrial production. SPR and STR denote the linear pricing residuals corresponding to the regressions shown in Table 1. Here, SPR and STR denote SP- and ST-residuals series.

Three conclusions can be drawn from this evidence. First, the linear model using these factors is probably misspecified. There is feedback between returns and macrofactors, and at least some of this feedback is nonlinear. While this evidence implies the absence of strict rather than weak exogeneity, we argue that rational expectations would make it likely that our tests will uncover the absence of weak exogeneity. Second, there is more evidence for a relationship between the macroeconomy and the stock market than can be adduced from a linear model. While our evidence is critical of the linear specification, it is supportive of the goal to which that specification is applied: at least some of the predictability in stock returns may be due to a relationship to the macroeconomy, albeit a complicated one. Third, our evidence adds to the body of work on univariate nonlinear dynamics in stock returns. Our evidence for a nonlinear causal relationship between the macroeconomy and the stock market implies that nonlinear dynamics in returns may stem from a complicated economic process.

These results suggest several avenues for further research. One possibility is a further econometric exploration of the relationship between macroeconomic fundamentals and the stock market. For example, it is possible that causality in variance, perhaps relating to Factor-ARCH dynamics, underlies our findings for nonlinear causality. 1/ It may be that changes in the variances of the factors cause changes in the mean or variance of returns. Such an exercise could help in determining the source of the causality results while employing some of the insights of asset-pricing theory. One attempt by us to explain the nonlinear univariate dynamics in returns using a macrofactor-ARCH specification for expected returns has met with some success. 2/

Another interesting exercise would be to extend the linear forecasting results in the finance literature (both from stock returns to macrofactors and vice versa) to a nonlinear setting, perhaps using a neural net model. 3/ Our results imply that the linear model omits some interesting and potentially useful relationships between the stock market and the macroeconomy. A forecasting exercise could help us judge the practical significance of this implication.

Another possibility is an exploration of the possible microeconomic sources for the nonlinearities we find. Given the nature of our results, some nontrivial economic modeling would likely be required to generate them. One possibility is that the factors and stock returns are bound up in a nonlinear interactive system. One way to describe such an interactive system is using Brock's (1993) emergent noise economy, in which interacting agents can generate time-series behavior similar to what is observed in returns data. Development of this line of inquiry awaits future research efforts.

1/ Factor-ARCH models of asset markets include Engle, Ng, and Rothschild (1990) and Ng, Engle and Rothschild (1992).

2/ See Hiemstra and Kramer (1993).

3/ Examples of neural net forecasting in finance are Weigend, Huberman, and Rumelhart (1992) and Kuan and Liu (1994).

The Variance of the Modified Baek And Brock Test

To describe the variance of the modified Baek and Brock test and a consistent estimator for it, we begin by defining the joint probabilities

$hl_{C1}(x_{t-Lx}^{Lx+m}, y_{t-Ly}^{Ly}, e)$, $hl_{C2}(x_{t-Lx}^{Lx}, y_{t-Ly}^{Ly}, e)$, $hl_{C3}(x_{t-Lx}^{m+Lx}, e)$, and

$hl_{C4}(x_{t-Lx}^{Lx}, e)$, which are conditioned on combinations of the realizations

x_t^m, x_{t-Lx}^{Lx}, and Y_{t-Ly}^{Ly}, as 1/

$$hl_{C1}(x_{t-Lx}^{m+Lx}, y_{t-Ly}^{Ly}, e) = Pr\left(\| x_{t-Lx}^{m+Lx} - X_{s-Lx}^{m+Lx}\| < e, \| y_{t-Ly}^{Ly} - Y_{s-Ly}^{Ly}\| < e\right), \tag{A.1}$$

$$hl_{C2}(x_{t-Lx}^{Lx}, y_{t-Ly}^{Ly}, e) = Pr\left(\| x_{t-Lx}^{Lx} - X_{s-Lx}^{Lx}\| < e, \| y_{t-Ly}^{Ly} - Y_{s-Ly}^{Ly}\| < e\right),$$

$$hl_{C3}(x_{t-Lx}^{m+Lx}, e) = Pr\left(\| x_{t-Lx}^{m+Lx} - X_{s-Lx}^{m+Lx}\| < e\right),$$

$$hl_{C4}(x_{t-Lx}^{Lx}, e) = Pr\left(\| x_{t-Lx}^{Lx} - X_{s-Lx}^{Lx}\| < e\right).$$

Using (A.1) and the delta method (see Serfling (1980), pp. 122-125), under the assumption that the underlying series are strictly stationary, weakly dependent, and satisfy the mixing conditions of Denker and Keller (1983), an expression for the variance of the Baek and Brock test in (7) is given by

$$\sigma^2 \ (\mathrm{m, Lx, Ly, e}) = \mathbf{d}\ \Sigma\ \mathbf{d}^T, \tag{A.2}$$

where

$$\mathbf{d} = [d_i], \ i=1,..,4 \tag{A.3}$$

$$= [1/C2(Lx,Ly,e), \ -C1(m+Lx,Ly,e)/C2^2(Lx,Ly,e),$$

$$-1/C4(Lx,e), \ C3(m+Lx,e)/C4^2(Lx,e)],$$

1/ As previously noted, we follow the standard convention of denoting random variables in the upper case and their realizations in the lower case. These joint probabilities relate to the probability that arbitrarily selected triplets $(X_s^m, X_{s-Lx}^{Lx}, Y_{s-Ly}^{Ly})$ defined in (2) are close to the realized triplets $(x_t^m, x_{t-Lx}^{Lx}, y_{t-Ly}^{Ly})$.

$$\Sigma = [\Sigma_{ij}],\ i,j=1,\ldots,4$$
$$= 4 \sum_{k\geq 1} \omega(k) E(A_{i,t} A_{j,t+k-1}),$$

$$\omega_k = \begin{cases} 1, & if\ k=1 \\ 2, & otherwise \end{cases}$$

$$A_{1,t} = h1_{C1}(x^{m+Lx}_{t-Lx}, y^{Ly}_{t-Ly}, e) - C1(m+Lx, Ly, e)$$

$$A_{2,t} = h1_{C2}(x^{Lx}_{t-Lx}, y^{Ly}_{t-Ly}, e) - C2(Lx, Ly, e)$$

$$A_{3,t} = h1_{C3}(x^{m+Lx}_{t-Lx}, e) - C3(m+Lx, e)$$

$$A_{4,t} = h1_{C4}(x^{Lx}_{t-Lx}, e) - C4(Lx, e),$$

and where E(·) denotes expected value and the Ci(·) terms are defined in (4).

Using the results of Denker and Keller (1983) and Newey and West (1987), a consistent estimator of the Σ_{ij} elements in (A.2) is given by

$$\hat{\Sigma}_{i,j}(n) = 4\sum_{k=1}^{K(n)} \omega_k(n)\Big[\frac{1}{2(n-k+1)}\sum_t \Big(\hat{A}_{i,t}(n)\hat{A}_{j,t-k+1}(n) + \hat{A}_{i,t-k+1}(n)\hat{A}_{j,t}(n)\Big)\Big],$$

$$t = \max(Lx, Ly) + k, \ldots, T-m+1,$$
$$n = T+1-m-\max(Lx, Ly),$$
$$K(n) = int(n^{1/4}),$$
$$\omega_k(n) = \begin{cases} 1, & if\ k=1 \\ 2(1-[(k-1)/K(n)]), & otherwise \end{cases}$$

where

$$\hat{A}_{1,t}(n) = \frac{1}{n-1}\sum_{s\neq t} I(x^{m+Lx}_{t-Lx}, x^{m+Lx}_{s-Lx}, e)\cdot I(y^{Ly}_{t-Ly}, Y^{Ly}_{s-Ly}, e) - C1(m+Lx, Ly, e, n)$$

$$\hat{A}_{2,t}(n) = \frac{1}{n-1}\sum_{s\neq t} I(x^{Lx}_{t-Lx}, x^{Lx}_{s-Lx}, e)\cdot I(y^{Ly}_{t-Ly}, Y^{Ly}_{s-Ly}, e) - C2(Lx, Ly, e, n),$$

$$\hat{A}_{3,t}(n) = \frac{1}{n-1} \sum_{s \neq t} I(x_{t-Lx}^{m+Lx}, x_{s-Lx}^{m+Lx}, e) - C3(m+Lx, e, n)$$

$$\hat{A}_{4,t}(n) = \frac{1}{n-1} \sum_{s \neq t} I(x_{t-Lx}^{Lx}, x_{s-Lx}^{Lx}, e) - C4(Lx, e, n),$$

$$t, s = \max(Lx, Ly)+1, \ldots, T-m+1,$$

and where the Ci(·,n) correlation integrals are defined in (6) and the I(·) indicators are described in Section III.2. The Ci(·,n) correlation integrals provide a consistent estimator of **d** in equation (A3), namely,

$$\hat{\mathbf{d}}(n) = [1/C2(Lx,Ly,e,n),\ -C1(m+Lx,Ly,e,n)/C2^2(Lx,Ly,e,n),\ -1/C4(Lx,e,n),\ C3(m+Lx,e,n)/C4^2(Lx,e,n)].$$

A consistent estimator for $\sigma^2(m,Lx,Ly,e)$ in (7) can then be expressed as

$$\hat{\sigma}^2(m,Lx,Ly,e,n) = \hat{\mathbf{d}}(n)\ \hat{\Sigma}(n)\ \hat{\mathbf{d}}(n)^T.$$

The test statistics reported in Tables 2-4 use this variance estimator.

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