WP/99/96

INTERNATIONAL MONETARY FUND

European I Department

Measures of Potential Output: An Application to Israel

Prepared by Fabio Scacciavillani and Phillip Swagel[1]

Authorized for distribution by Thomas Krueger

July 1999

IMF WORKING PAPER

INTERNATIONAL MONETARY FUND

IMF Working Paper

WP/99/96

INTERNATIONAL MONETARY FUND

European I Department

Measures of Potential Output: An Application to Israel

Prepared by Fabio Scacciavillani and Phillip Swagel[1]

Authorized for distribution by Thomas Krueger

July 1999

Abstract

This paper estimates measures of potential output for Israel, with the aim of providing evidence on whether the recent growth slowdown is principally a cyclical slowdown or a structural shift toward a slower growth path after the dramatic developments associated with the years of heavy immigration. Israel poses a challenge because traditional methods of measuring potential output assume relatively stable conditions over an extended period of time. We employ five methodologies to derive estimates and find that four of the measures imply the slowdown stems largely from reduced growth of potential output rather than a cyclical slowdown.

JEL Classification Number: E32

Keywords: potential output, output gap, wavelets analysis

Author's E-Mail Address: pswagel@imf.org

[1]Mr. Scacciavillani is an economist at the European Central Bank, and was an economist at the IMF when this paper was written. The authors thank Eric Clifton, Graham Hacche, Zvi Hercowitz, R. Barry Johnston, Tom Krueger, David W.H. Orsmond, Michel Strawczynski, and Hari Vittas for comments and suggestions, Peter Christoffersen, John McDermott, and Eswar Prasad for helpful conversations and for generously sharing computer programs, and Mandy Hemmati for research assistance.

Contents Page

I. INTRODUCTION

The Israeli economy has changed dramatically over the past decade. The 1985 stabilization program—which saw inflation fall from 185 percent to close to 20 percent within a year—was followed by a massive wave of immigration that increased the population by almost 20 percent. As shown in Figure 1, output grew at an annual rate of nearly 7 percent for sustained periods, expanding on the order of 40 percent over the six years from 1989 to 1995. The unemployment rate rose both before and during the height of the immigration in 1990 and 1991, but then fell back as the new skilled labor force was absorbed (bottom of Figure 1). This process was aided by a shift toward high-technology sectors and an export-oriented investment boom aimed at taking advantage of the increased human capital. However, output growth in Israel slowed to a rate in 1997 of 2.7 percent, and then slowed further to 1.9 percent in 1998, while the unemployment rate rose from 6 percent at the start of 1996 above 9 percent in the middle of 1998 before falling off at the end of the year.

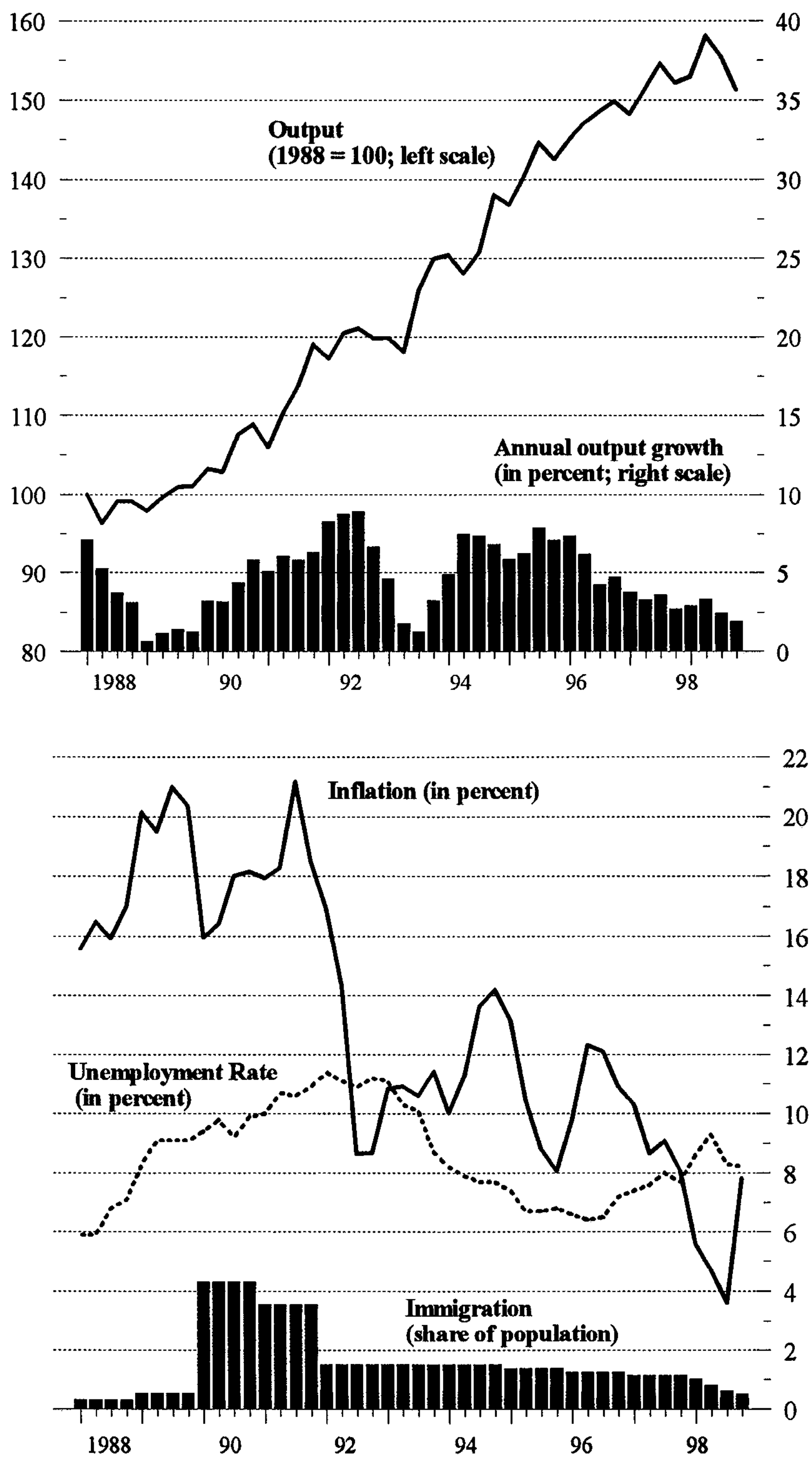
This paper estimates measures of potential output for Israel, with the aim of providing evidence on whether the recent growth slowdown is principally a cyclical shortfall, or instead represents a structural shift toward a slower growth path after the growth spurt associated with the years of heavy immigration. Five methodologies are employed to derive estimates of potential output, including two fairly traditional approaches, and three approaches that feature relatively new or less commonly-used techniques. The dramatic developments and rapidly changing structure of the Israeli economy pose a challenge for efforts to measure potential output, since traditional methods such as the production function and HP filter assume the existence of relatively stable conditions over an extended period of time.[2] In contrast, the more recent statistical approaches are designed so that the estimates of potential output adapt in important ways to changes in economic circumstances. The first two of the new approaches, "running median smoothing" (RMS) and the "wavelets filter," are univariate statistical filters that use a running window on the data. An advantage of this is that disruptive but rare events such as the immigration in Israel do not influence estimates of potential output throughout the sample period but instead only in the period around the event. The last of the nontraditional methodologies examined here is a structural vector autoregression (VAR) that exploits the relationship between output growth and inflation to gauge the extent to which the economy is below or above potential and thus distinguish between permanent changes in potential output and transitory fluctuations in actual output around potential.[3]

---

[2]De Masi (1997) surveys traditional methodologies used to estimate potential output.

[3]A similar methodology is used by DeSerrers, Guay, and St-Amant (1995) and Dupasquier and Guay (1997) for measuring potential output in Mexico and Canada, respectively.

Figure 1. Output, Inflation, and Unemployment



The results from the different measures of potential output uniformly indicate that potential output growth accelerated sharply during the early 1990s, matching the rapid growth of output at the height of the immigration. But in four of five methodologies, potential output growth appears to have slowed markedly in recent years along with actual output, so that relatively modest output gaps are indicated for 1997 and 1998. For 1998, the HP filter and structural VAR give output gaps of 1.3 and 0.5 percent of GDP, respectively, while the running median smoother and wavelets filter indicate essentially no output gap or even a small surplus of output above potential. These results suggest that the recent weakness of activity in Israel largely stems from a slowing of potential output growth rather than business cycle factors. Under this view, rising unemployment in 1997 and 1998 despite the small output gaps can be seen as stemming from the slow absorption of workers displaced from traditional industries in this structural changeover.[4]

In contrast, the production function methodology indicates that growth of potential business sector output has barely slowed the past two years, remaining at a rate around 6 percent in 1997 and 1998. With actual output growth under 2 percent in 1998, business sector output was 6.8 percent below the indicated level of potential in 1998, following a 3.3 percent gap in 1997. The continued expansion of potential under the production function largely reflects strong investment, as the net capital stock grew by nearly 8 percent in 1998. One explanation for the continued strength of investment despite the output slowdown is that the incremental capital reflects adjustment costs incurred as production is redirected away from traditional industries to the high technology sector. Once this adjustment is complete, output growth would be expected to rebound as the new capital is utilized in production.

II. Concepts of Potential Output

Even abstracting from estimation difficulties, the concept of "potential output" is not well defined, and in various applications the notion of potential output is sometimes used to indicate quite different concepts. Broadly speaking, the literature distinguishes between two definitions. In the first, more along the Keynesian tradition, the business cycle results primarily from movements in aggregate demand in relation to a slowly moving level of aggregate supply. In business cycle downswings, there exist factors of production that are not fully employed; most critically, unemployment remains above its frictional level, and wage and inflation pressures are subdued. A measure of potential output is thus crucial for the setting of demand management policy—both monetary and fiscal—and represents a principal guide for economic policy.

One procedure by which to measure potential output in the Keynesian tradition is through use of an aggregate production function, or more generally a fully specified macroeconomic model that incorporates a production function. In this framework, potential output increases in line with growth of factor inputs, and the output gap measures the shortfall of actual output below

---

[4]See Clifton (1998) for a discussion of factors influencing this process.

this productive potential. Alternately, a measure of potential output can be derived indirectly from the nonaccelerating inflation rate of unemployment (NAIRU), in which inflationary pressures depend on the extent of labor market slack in the economy. Combining the NAIRU with an "Okun's Law" relationship between output and unemployment means that as activity increases, unemployment falls and inflation rises. The output gap is then measured by gauging the extent to which the economy can expand without inflation accelerating. This relationship is the basis for the estimates of potential output derived from the structural VAR.

In the second approach—more along the neoclassical tradition—potential output is driven by exogenous productivity shocks to aggregate supply that determine both the long run growth trend and, to a large extent, short term fluctuations in output over the business cycle. In such a framework, business cycle fluctuations are not necessarily driven by shortfalls or excesses of aggregate demand or by changes in monetary, fiscal, or other policies; instead, to a large extent they result from rational agents reacting to unexpected productivity shocks by writing off old investments and regrouping resources in order to re-coordinate production and thereby adapt to the new conditions.[5] Under the neoclassical approach, output is assumed to fluctuate around potential but generally without any wide or prolonged divergence. Unlike the Keynesian framework where the economy might reach potential only after an extended period, potential output in the neoclassical framework is synonymous with the trend growth rate of actual output. The key measurement problem is thus to distinguish between permanent movements in potential output and transitory movements around potential; in practice, potential output is taken to coincide with a "smoothed" measure of actual output such as the HP filter. Three smoothing filters are employed to measure potential output, including the HP filter in which the path of potential output evolves slowly with changes in output, and the RMS and wavelets filters, in which potential adjusts rapidly to changes in activity.

In practice, of course, use of these methodologies does not necessarily divide neatly into the two intellectual frameworks. For example, a production function is typically used to obtain measures of the technology shocks in the neoclassical model, and similarly, the link between potential output and inflation is not exclusive to the Keynesian framework. In Lucas (1972), for example, inflation affects aggregate supply because, in the face of imperfect information, agents are not able to distinguish between aggregate and sector-specific price movements and thus they increase production in response to inflation on the mistaken belief that the price increase represents higher demand for their output rather than a general increase in the price level. Rather than making artificial divisions in the empirical practice of measuring potential output, the purpose of this paper is to compare the results that come out of the various methods. And again, Israel is a particularly appropriate case for this exercise because its economy has experienced dramatic shocks such as the mass immigration and the structural change of the economy away from traditional industries, both of which would be expected to have important effects on the path of potential output.

---

[5]See Thies (1991). Under this framework, fluctuations in output also occur due to changes in distortions introduced by factors such as the tax regime, protectionist measures, or labor market rigidities. The policy prescription in this case is to remove the distortion.

III. METHODOLOGIES USED TO ESTIMATE POTENTIAL OUTPUT

With these concepts as background, this section describes the methods used to estimate potential output. Results are presented in Section IV.

A. Aggregate Production Function

Estimates of potential output from the production function are derived from a Cobb-Douglas specification, in which output at time t, Y_t, depends on the level of technology (also referred to as the level of total factor productivity), A_t, and the quantities of the factors of production, the stock of physical capital, K_t, and the labor force, L_t:

$$Y_t = A_t K_t^{\alpha} L_t^{\beta}$$

The assumption of constant returns to scale implies that $\alpha + \beta = 1$, while with perfect competition, the coefficient α corresponds to the share of capital income, and thus $\beta = 1 - \alpha$ to the share of labor.

Since the level of technology/productivity cannot be measured directly, values for α and β must be first obtained and then used to calculate the series for A. The empirical results in Section IV employ the simplest way to do this, which is to rely on the assumptions of constant returns and perfect competition, and use the actual share of capital in value-added for α and the share of labor as β. This follows calculations made by the Bank of Israel in its Annual Reports, in which the share of labor is taken to equal 0.68. The level of productivity A, can be calculated as $A_t = Y_t / (K_t^{\alpha} L_t^{\beta})$ where $\alpha = 0.32$ and $\beta = 0.68$. This is the value that transforms the inputs into the actual output.

To calculate potential output, it is further assumed that the growth in the level of productivity is comprised of two parts: an upward trending component representing deterministic productivity growth, and a stochastic component that corresponds to the shortfall or surplus of output around potential—the output gap. A linear trend is used to parameterize deterministic productivity growth, with the growth rate equal to the average growth rate of A_t. Inspection of the calculated values for A for the period after the 1973 Yom Kippur War (not shown) reveals that the growth rate of this productivity level rose markedly after the stabilization from high inflation in 1985, so that separate trends are calculated for the two periods of 1974 to 1985 and 1986 to 1998. Deterministic productivity growth equals 0.69 percent annual before the stabilization in 1985, and then 1.34 percent annually from 1986 to 1998. During this latter interval, productivity growth rose sharply at the onset of the stabilization, stagnated in 1988 and 1989, and then a year into the immigration returned to an annual growth rate of nearly 2 percent, with a 5 percent rise in 1990 alone. This suggests that viewing the immigration as simply an increase in the supply of labor does not fully capture its effects.

Given the values for productivity growth in the two periods, growth of potential output equals the weighted average of the growth rates of capital and labor (weighted by the factor shares), plus the growth rate of deterministic total factor productivity. To compute the output gap, the economy is assumed to have been at potential in 1988, just before the immigration when unemployment was at its recent low of 6 percent, and the growth rates of potential and actual output are used to calculate the gap as a percentage of potential output.

An alternative approach would be to obtain values for α and β from a production function regression and interpret the regression residuals as the level of technology. Unit root tests such as Dickey-Fuller and Phillips-Perron imply that output, capital, and labor are all integrated, as are output and capital per worker (over the sample 1969–98). However, the variables do not appear to be cointegrated: the Engle-Granger test for cointegration does not reject the null of a unit root in the residuals of the cointegrating regression for the production function, and the Johansen-Juselius procedure does not reject the null of zero cointegrating vectors both with and without imposing constant returns to scale.[6] While it is somewhat surprising that output, labor, and capital do not hang together, this absence of cointegration probably reflects the rapid productivity growth in Israel, which gives the result that output grows apart (above) from the growth rates of capital and labor. Fortunately, estimating the production function regression in first differences results in a coefficient of 0.336 for the capital share parameter, α, with a t-statistic of 1.92, and with a standard error of 0.17, this is not statistically different from the actual capital share of 0.32 used in the simulations.

B. Univariate Filters

While the neoclassical paradigm stresses that economic fluctuations are generated by technology shocks, the theory does not provide directions on how to distinguish between permanent movements related to growth of potential output, and temporary fluctuations related to the business cycle. The separation between permanent and transitory movements in a time series has typically been carried out with signal extraction methods such as the filter of Hodrick and Prescott (1997). But there exist many other different filtering techniques that could be appropriate. Indeed, Canova (1998) shows that several detrending methods produce qualitative results and "stylized facts" regarding business cycles in the United States that differ in important ways across techniques. Because of this, we employ the three univariate filtering approaches discussed next to calculate measures of potential output.

---

[6]Cointegration is found only in the specification with a single lag, and this is not the "optimal" lag length suggested by the AIC criterion.

The Hodrick-Prescott filter

The Hodrick-Prescott (HP) filter is probably the most widely used method by which to extract a trend from macroeconomic data. The HP filter is a linear filter that constitutes an approximation to a low pass filter, a class of spectral filters that isolates the low frequency components associated with long-run movements such as growth of potential output. Specifically, the HP filter extracts trend GDP, Y*, which is taken to represent potential output, from a raw series on output, Y, by minimizing the size of the actual output fluctuations around its trend, subject to a constraint on the maximum allowable change in the growth of trend output between two periods. Potential output in the HP filter is the series of values that minimizes the expression:

$$\frac{1}{T}\sum_{t=1}^{T}(\ln Y_t - \ln Y_t^*)^2 + \frac{\lambda}{T}\sum_{t=2}^{T-1}\left[(\ln Y_{t+1}^* - \ln Y_t^*) - (\ln Y_t^* - \ln Y_{t-1}^*)\right]^2 \qquad \textbf{(3)}$$

where T is the number of observations and the parameter λ is a weighting factor that determines the smoothness of the trend. In typical use, this is set to λ=1600 for quarterly data and λ=100 for annual data, which has the effect of removing from data on output those cycles with frequencies shorter than eight years. This choice stems from the business cycle work of Burns and Mitchell (1944), who found that the length of the business cycle in the United States varied between two and eight years.

The HP filter has several shortcomings, including the somewhat arbitrary choices of the assumed business cycle frequency and the smoothing parameter λ, the neglect of structural breaks and regime shifts, and the inadequate treatment of nonstationary dynamics.[7] If the structure of the economy is thought to be fairly stable and the growth of potential output relatively smooth, then the HP filter will provide a reasonable estimate of potential output. If, however, there are structural breaks—as has likely been the case in Israel following the immigration—then use of the HP filter could be inappropriate since the filtering procedure may remove from the data shifts that in fact represent a change in the trend level or growth rate of potential output.

---

[7]Indeed, Harvey and Jaeger (1993) uncover a series of spurious "stylized facts" using the HP filter. See Barrell and Sefton (1995) and Coe and McDermott (1997), among others, for further discussions of specific shortcomings. King and Rebelo (1993) illustrate the properties of the HP and low pass filters in general, including the stationarity of series examined. A problem with these critiques is typically the absence of a meaningful alternative for estimating potential output. One exception to this is the filter suggested by Coe and McDermott (1997), which relies on a kernel smoother with a data-dependent bandwidth selection rather than an arbitrary choice of smoothing parameter.

Running median smoothing (RMS)

To investigate the consequences of reducing the "smoothness" imposed by the HP filter, an alternative estimate of potential output is computed using a variant of the "Running Median Smoothing" (RMS) algorithm of Tukey (1977). The simplest form of the RMS filter uses a running window on the data, with the smoothed value of the data in each period being the median of the values inside the window; this has the property of removing outliers that are not "close" to other observations within the window. More complicated versions of the filter involve multiple smoothing passes with different window sizes and weighting of observations. The RMS is a nonlinear operator with the advantage over the HP filter that it "adapts" to structural changes in the data. Moreover, it is not influenced by the particular sample of data—that is, unlike in the HP filter computation, an observation does not influence estimates of potential output in periods outside of the data window.

This paper utilizes a particular version of the RMS called 4(3RSR)2H.[8] This has the property of extreme adaptability; loosely speaking, the criterion used to separate transitory from permanent movements in the data excludes from the permanent components only fairly substantial changes in output, and therefore attributes to movements in potential output part of the fluctuations that would be considered business cycle dynamics in the less adaptable HP filter.

Wavelets filters

The HP filter and the RMS can be viewed as two extremes, one quite rigid, the other quite adaptable. The next measure of potential output uses "wavelets theory" to separate permanent movements in output from transitory fluctuations and can be considered a compromise between rigidity and adaptability. The wavelets methodology is less adaptable than the RMS, but does not suffer from some of the potential difficulties encountered with the use of the HP filter when the shocks experienced by the economy are thought to reflect a changing economic structure. Moreover, unlike the HP filter or other methods based on Fourier analysis, wavelets filters do not hinge on arbitrary assumptions about the regularity of fluctuations. This is because wavelets theory does not assume that an economic variable evolves according to the smooth dynamics conveniently represented by series of sines and cosines, but rather maps the observed data into more general functional spaces the orthogonal bases of which are called "wavelets." This allows the measure of potential output to include time-varying dynamics.

Donoho and Johnstone (1992) prove that if a wavelets basis exists, it provides the optimal method with which to extract a signal from white noise. Donoho (1993, 1994) develops a method, called wavelet denoising or wavelet shrinkage ("waveshrink") to extract the unobserved series of potential output, Y^*, from the regression-like equation in which the observed data for output, Y, equals the sum of potential Y^* and an error component, η_t, that

[8]See Tukey (1977), Chapter 7 and following.

denotes short-term movements in output:

$$Y_t = Y_t^* + \eta_t$$

The waveshrink methodology entails calculating the discrete wavelets transform of output, eliminating the wavelets coefficients that are thought to correspond to business cycle movements, then reconstructing the signal using only the remaining coefficients. The appendix presents an overview of wavelets theory and a brief technical discussion of the waveshrink methodology.

C. Structural Vector Autoregression

The final methodology used to estimate potential output is the structural vector autoregression. This combines aspects of the Keynesian and neoclassical traditions, exploiting the statistical relationship between inflation and growth to distinguish between permanent and transitory movements in output: for example, faster output growth without a pickup in inflation is taken to imply that the economy is at that time operating below potential, while the emergence of inflation in the face of growth would suggest that output is above potential.

The methodology used to estimate the structural VAR follows Blanchard and Quah (1989), with the exception that inflation is used in place of the unemployment rate variable typically used in similar analyses for the United States.[9] This is because the relationship between output and unemployment appears to have been disrupted in Israel as a consequence of the immigration and ongoing structural changes in the economy, so that higher unemployment does not reliably indicate more slack in the economy. Two reduced form regressions are estimated in which inflation and output growth depend on lags of both variables; the regression residuals thus correspond to the effects of contemporaneous developments on the two variables (that is, anything not explained by past data). By imposing identifying restrictions on the relationship between output and inflation, the regression residuals are divided into the effects of supply and demand shocks in each period on output and inflation. The output gap is then defined as the component of the forecast error of output attributed to the demand shock—the shortfall or surplus of output above or below potential due solely to demand-side factors. Potential output equals the sum of actual output and the output gap.

The VAR is structural in the sense that identifying restrictions are imposed on the long-run effects of shocks on output and inflation, while the effects of both shocks are left unconstrained in the short run. This contrasts with the nonstructural VAR methodology in which the effects of shocks on all variables are left unconstrained at all horizons. The restrictions imposed are that demand shocks affect the long-run price level but not the long-run level of output, while supply shocks can have permanent effects on both output and the

---

[9]The Blanchard-Quah estimator is now in widespread use, so that the technical details are not repeated here.

price level.[10] For a sensible measurement of potential output to result from the VAR, it must be found empirically—and it is—that a positive supply shock leads to a permanent increase in the level of GDP and not a decrease, while a positive demand shock leads to higher output in the short run (the estimation is constrained so that there is no permanent effect of demand on output).

As inflation is likely to respond to changes in output growth only with a lag, the variables used in the VAR are the contemporaneous quarterly growth rate of output and the four quarters ahead logarithmic change in the price level (i.e., the fourth lead of the annualized quarterly inflation rate). Examination of Figure 1 shows that there are two distinct periods of inflation, with inflation rates between 15 percent to 25 percent until the beginning of 1991 and then below 15 percent afterwards. Since the results from the VAR depend on the relationship between changes in output and the rate of inflation, this shift must be taken into account; for example, it is important that the inflation of 14 percent in 1994 be seen by the VAR as "high" rather than as "low" since it is below the 18 percent inflation rate in 1990. To account for this change in the inflation regime, the inflation series is "demeaned" or "standardized" by subtracting the average rate of inflation in each of the two periods before and after the second quarter of 1992. The VAR is run with eight lags of each variable, since this should be sufficient to cover any lagged effects of inflation and growth, and likelihood ratio tests do not reject the null hypothesis that additional lags are not statistically significant.

Before estimating the VAR, the time series properties of the data for output and inflation were examined. Standard statistical tests for stationarity such as the Dickey-Fuller and Phillips-Perron tests do not reject the null of a unit root in the levels of prices and output, but do reject the null that a unit root exists in growth rates (the results are not shown here). The Johansen-Juselius test for cointegration does not reject the null of no cointegrating relationship between prices and output for the post-stabilization period, and rejects the null of one cointegrating vector. These results indicate that the VAR should be run on the first differences of output and the price level (that is, output growth and the rate of inflation), but there is no need to augment the VAR with an error correction mechanism.

---

[10]A great deal of attention has focused on the issue of whether "supply" and "demand" are appropriate descriptions of the shocks identified by the Blanchard-Quah procedure. For example, Robertson and Wickens (1997) argue that "real" and "nominal" are a better nomenclature, since aggregate demand shocks do not affect output in the long run only in the unlikely case of a supply curve that is vertical at all time horizons. We do not take a stance on this issue other than to note that the Blanchard-Quah procedure can be viewed as simply an alternative filter with which to distinguish between permanent and transitory movements in the variables under analysis. Indeed, the only precise interpretation of the shocks is in terms of the restrictions by which they are defined, but this is enough for our purposes, since potential output is exactly about permanent movements in output, and thus by definition any deviation from potential output—any "output gap"—is attributable to a transitory shock.

IV. Empirical Implementation

Quarterly data on output, inflation, and unemployment are from the *Monthly Bulletin of Statistics* of the Israeli Central Bureau of Statistics (CBS). Owing to the lack of quarterly data on the capital stock, annual data from the Bank of Israel are used for the production function. These cover only the business sector, and include value-added in the business sector from the national accounts, the capital stock net of depreciation, and the numbers of persons in the civilian labor force. The sample used in the structural VAR and HP filter is restricted to the period following the stabilization from high inflation starting with the third quarter of 1986, when the 12 month rate of inflation fell from 74 percent in the second quarter of 1986 to 21 percent in the third quarter.

Figure 1 shows the reductions in inflation that occurred in the early 1990's and again in 1997 and 1998 before the depreciation of the shekel in late 1998 led to an inflationary spike. Because the VAR requires data on inflation four quarters ahead, estimates are used for the last three quarters of 1999 inflation.[11] For the wavelets filters, the sample used is the period from the first quarter of 1982 to the end of 1997 for two technical reasons: these filters are best calculated with a number of observations that is a power of two (in this case, 64 observations), while results for 1998 are not calculated because the wavelets method uses a window of data past the end of the sample period and gives unreliable results at the endpoints. As discussed below, however, the output gaps from the wavelets methodology are uniformly quite small, so the likely results for 1998 are not difficult to surmise.

Table 1 summarizes the results for the output gap from the production function and the other methodologies starting in 1988. In the four approaches that use quarterly data, annual potential output is calculated as the sum of potential output in the four quarters of each year, and then the output gap is calculated as the deviation of this annual output from the sum of the corresponding four quarters of actual output.

Figure 2 depicts data on output and factor inputs starting in 1988 and the results for potential output and the output gap. The estimates of potential output from the production function suggest that following a slowdown 1989 and 1990, potential output growth in the business sector jumped to around 7 percent from 1990 to 1995. This was spurred initially by the 7.3 percent rise in the labor force in 1991, and then maintained by the surge in investment, as the net capital stock grew by double-digit rates from 1995 to 1997. Even with the strong growth of potential output in the early 1990's, actual GDP was either above or fairly close to potential until 1997. Investment remained strong in 1997 and fell off only partially in 1998 despite the growth slowdown, so that growth of potential remained strong and the output gap

---

[11]The actual value of inflation is used for the first quarter. Inflation in 1999 is expected to fall back to 4 percent by the end of the year after the effects of the 1998 shekel devaluation are passed through. This gives values of year on year inflation of 7.0, 7.0, 7.6, and then 4.0 percent for the four quarters of 1999, where the final quarter is low because this is more than a year past the fall of the shekel.

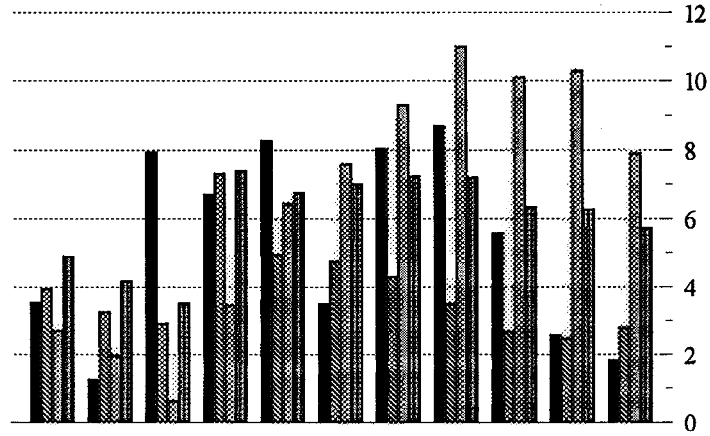
Table 1. Estimates of Potential Output and the Output Gap

	Actual Output		Potential Output				
Year	GDP	Business GDP	Production Function	Hodrick-Prescott Filter	RMS Smoothing	Wavelets Filter	Structural VAR
			(Rates of growth of output and potential output; in percent)				
1988	3.1	3.5	4.9	2.3	3.1	2.7	3.1
1989	1.3	1.2	4.2	4.2	1.9	1.8	1.4
1990	5.8	7.9	3.5	4.7	4.7	4.8	5.4
1991	6.2	6.7	7.4	5.3	7.1	7.2	7.6
1992	6.6	8.3	6.8	5.5	6.1	6.0	5.0
1993	3.2	3.5	7.0	5.6	3.7	3.5	2.9
1994	6.8	8.0	7.2	5.5	6.5	6.7	7.6
1995	7.1	8.7	7.2	5.1	7.0	7.1	6.5
1996	4.7	5.6	6.3	4.5	4.3	4.5	5.2
1997	2.7	2.6	6.3	3.9	2.9	2.8	3.4
1998	1.9	1.8	5.7	3.5	1.4	...	1.7
			(Output gap under different methodologies; in percent of GDP)				
1988			0.0	0.7	0.2	0.2	0.0
1989			-2.8	-2.1	-0.5	-0.3	-0.1
1990			1.3	-1.2	0.5	0.6	0.3
1991			0.7	-0.3	-0.3	-0.3	-1.0
1992			2.1	0.7	0.1	0.2	0.4
1993			-1.2	-1.5	-0.4	-0.1	0.7
1994			-0.5	-0.4	-0.2	0.0	-0.1
1995			0.9	1.4	-0.1	0.0	0.4
1996			0.2	1.5	0.2	0.1	0.0
1997			-3.3	0.3	0.0	0.0	-0.8
1998			-6.8	-1.3	0.5	...	-0.5

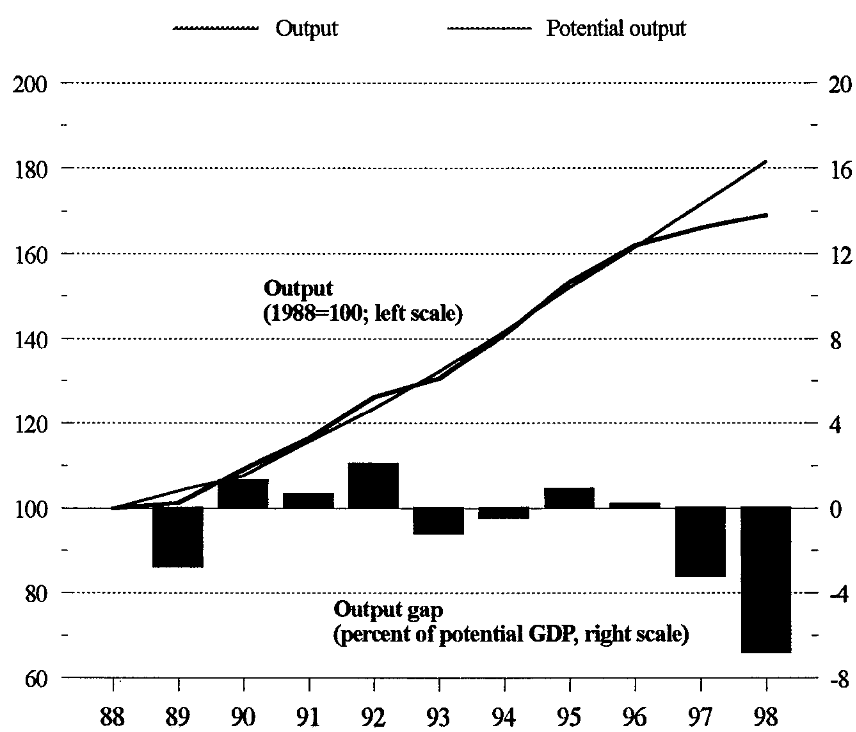
Note: a positive output gap indicates that output is above potential, a negative gap indicates that output is below potential.

Figure 2. Production Function: Business Sector GDP

(Growth rates of output, potential output, and factor inputs; in percent)



	88	89	90	91	92	93	94	95	96	97	98
Output	3.5	1.2	7.9	6.7	8.3	3.5	8.0	8.7	5.6	2.6	1.8
Labor	3.9	3.2	2.9	7.3	4.9	4.8	4.3	3.5	2.7	2.5	2.8
Capital	2.7	2.0	0.6	3.5	6.4	7.6	9.3	11.0	10.1	10.3	7.9
Potential	4.9	4.2	3.5	7.4	6.8	7.0	7.2	7.2	6.3	6.3	5.7



widened to 3.3 percent in 1997 and then 6.8 percent in 1998. The continued strength of business sector investment in Israel despite slowing growth could be tied to costs associated with the restructuring of the economy from traditional industries to the high technology sector. If this is correct, this would then be expected to lead to an increase in the capital share of output and thus an increased response of potential output to investment.

In contrast to the production function, the HP filter imposes smoothness on the measure of potential, so that potential output growth in the economy as a whole estimated to have increased steadily from 2.3 percent annually in 1988 to 5.6 percent in 1993, then slowed gradually to 3.5 percent in 1998. As seen in Figure 3, the economy was mostly below potential from 1988 until the middle of 1991, reaching a trough in which output was nearly 4 percent below potential in the first quarter of 1991. The subsequent growth spurt quickly lifted output above potential in the second half of 1991 and 1992, reaching a surplus of nearly 4 percent in the third quarter of 1991, and averaging 1.8 percent above potential from the third quarter of 1991 to the third quarter of 1992. Following a slowdown in 1993 that featured a quarterly output gap of 5 percent of potential GDP in the second quarter of 1993 and a smaller dip in early 1994, growth resumed at an annual pace over 7 percent in the second quarter of 1994, with output above potential until the present slowdown began in the second half of 1996. With growth of actual output only 1.9 percent for all of 1998 but potential growing at a 3.5 percent annual rate, the HP filter indicates an output gap of 1.3 percent for 1998, a figure driven largely by output declines in the second half of the year.

Figure 4 shows the estimates of potential output and the implied output gap from the running median smoother. The RMS filter provides estimates of potential that evolve smoothly, but track GDP more closely than the HP filter. This results in output gaps around 1 percent or less in most quarters and only very small annualized deviations from potential in Table 1. Indeed, the gap from the RMS exceeds 2 percent of GDP in either direction only in periods when output changes rapidly, and then for only one or two quarters in a row. The RMS filter indicates that potential output growth jumped sharply with actual output growth in the early 1990s but then slowed to 2.9 percent in 1997 and only 1.4 percent in 1998. This exceeds the slowdown in actual output, so that the RMS filter suggests no output gap in 1997 and actually a small surplus in 1998. Again, this result is obtained because the filter tracks output closely and thus interprets the 1997–98 slowdown as representing a change in potential rather than a cyclical episode.

Figure 3. Potential Output: HP Filter

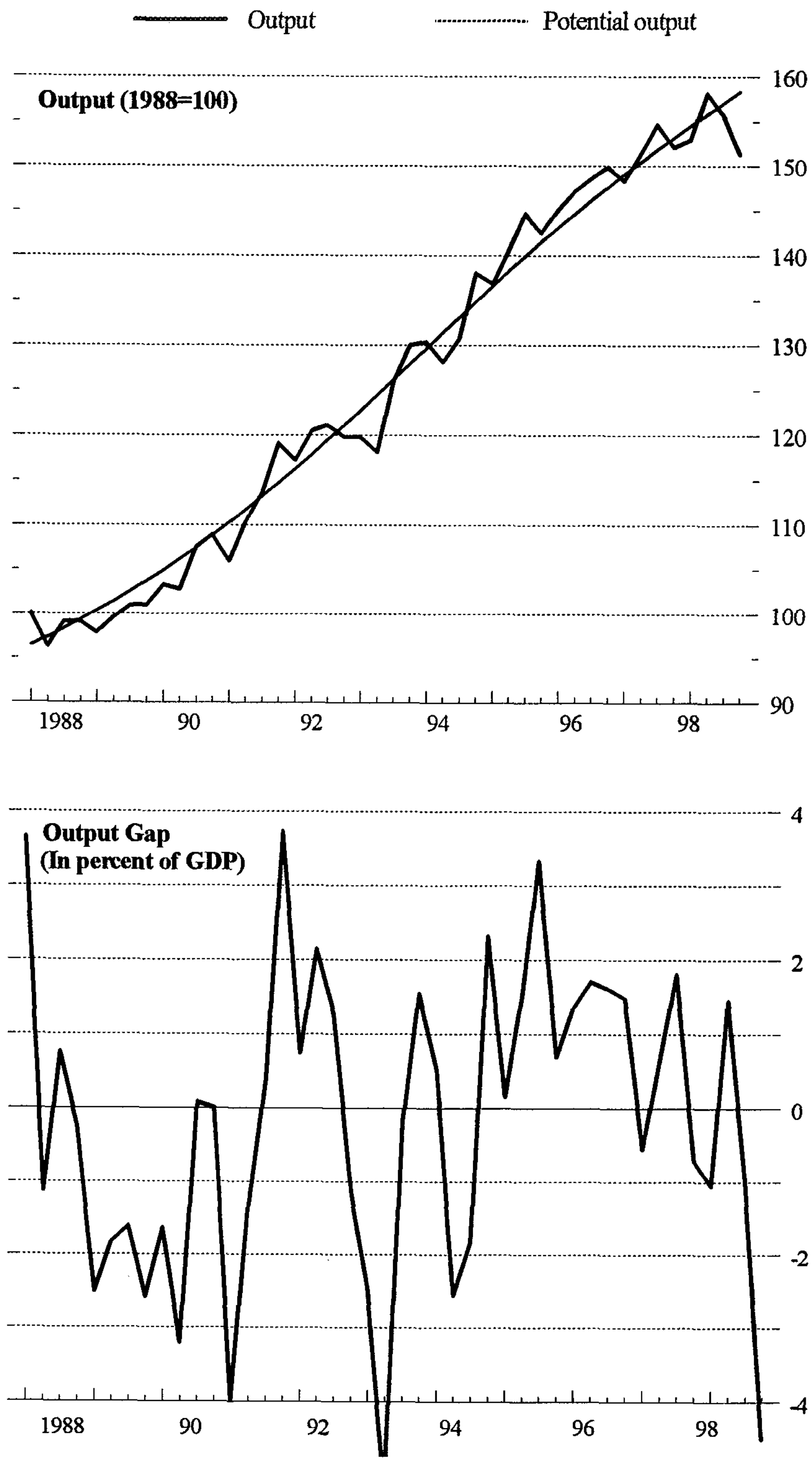


Figure 4. Potential Output: Running Median Smoothing

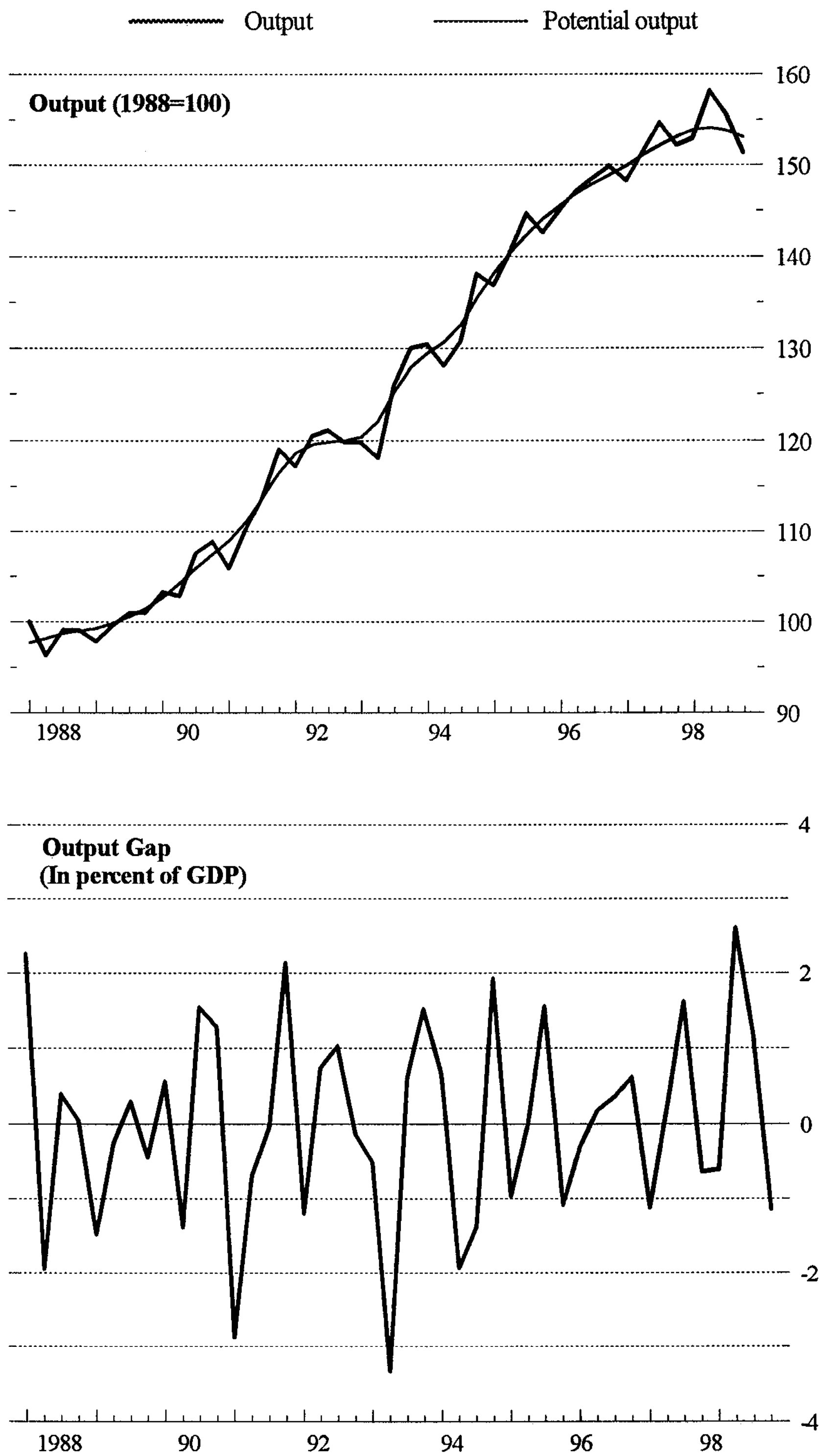


Figure 5 shows estimates of potential output and the output gap derived from the S4 wavelet, a particular type of wavelet that is not as adaptable as the RMS filter but less rigid than the HP filter. Given this, it is not surprising that the wavelets results imply generally only small deviations of output from potential, falling in between those of the RMS and HP filters. Growth of potential is calculated to have reached a low of just 1.8 percent before the immigration at the start of 1989, then accelerated rapidly in the early 1990s, reaching an annual rate of 8 percent in the second quarter of 1992. Output was nearly 2 percent above potential in 1993, though this was offset by the subsequent slowing at the end of the year so that the gaps for all of 1992 and 1993 are close to zero. Growth of this measure of potential output slowed to 2.8 percent in 1997, giving essentially zero output gap throughout 1997. It is likely that potential will be seen as having slowed again in 1998 in line with actual output growth.

Figure 6 shows that the VAR produces sensible results for the effects of supply and demand shocks: a positive supply shock leads to a permanent increase in the level of output (and thus a smaller output gap) and a fall in the price level, while a positive demand shock leads to higher prices and an immediate increase in output that dissipates within three years.

The VAR results for potential output and the output gap in Figure 7 are based on the assumption that there was no output gap in the second quarter of 1988—this is just prior to the first immigration wave when unemployment was at its recent low of 6 percent, and is generally consistent with the small output gap in the HP filter in the middle of 1988 (the gaps are -1.1 and +0.8 for the second and third quarters of the year). The timing of many of the larger movements in the output gap correspond to those from the HP filter, including the slowdowns in 1990–91, 1994, and 1996–97. In contrast to the results from the HP filter, however, the structural VAR implies that the slowdown in late 1993 and early 1993 was entirely a structural slowdown caused by slow growth of potential output. This is because inflation was essentially flat over this period and then rose somewhat over the next year, indicating to the VAR that there was not slack in the economy as would be the case in a cyclical slowdown. The growth spurts in 1994 and 1995 are similarly found to result principally from supply factors; because potential is estimated to have increased with output, this leads to no surplus of output above potential in 1994 despite the high rate of actual growth. The implied output gap from the VAR widens to 1 percent in late 1996 before recovering in 1997. While the output gaps for 1997 and 1998 are consistent with the severely deflationary stance of monetary policy in this period, the VAR suggests an important role for slowing growth of potential output beyond the effects of tight policy on aggregate demand: actual output growth slowed by 2 percentage points from 1996 to 1997, but much of this is accounted for by the 1.8 percentage point decline in the rate of potential growth (from 5.2 to 3.4 percent), so that the output gap widens only to 0.8 percent for the year as a whole. And actual growth in 1998 is slightly above potential growth implied by the VAR, leading to a narrowing of the output gap.

Figure 5. Potential Output: S4 Wavelet

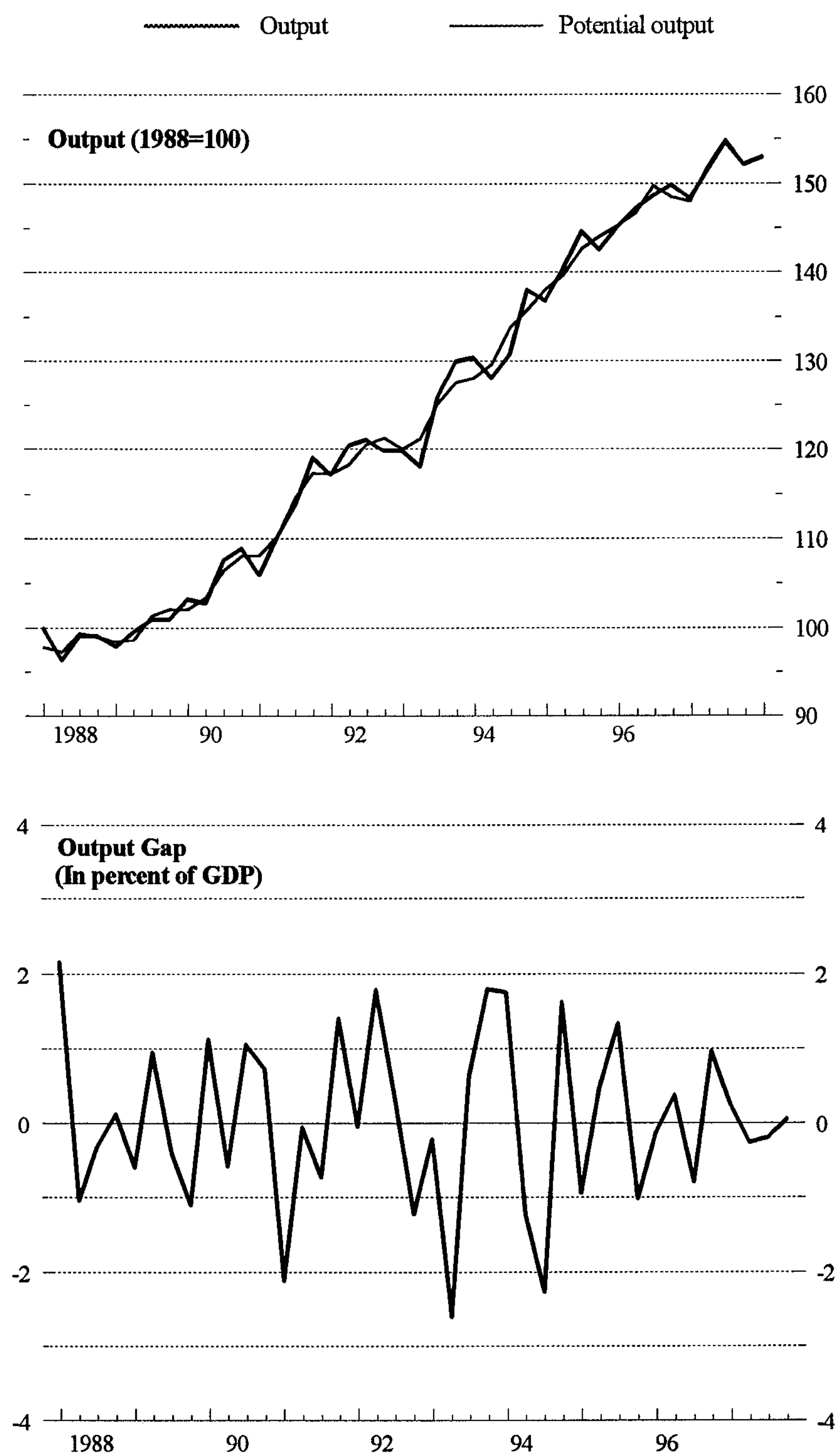
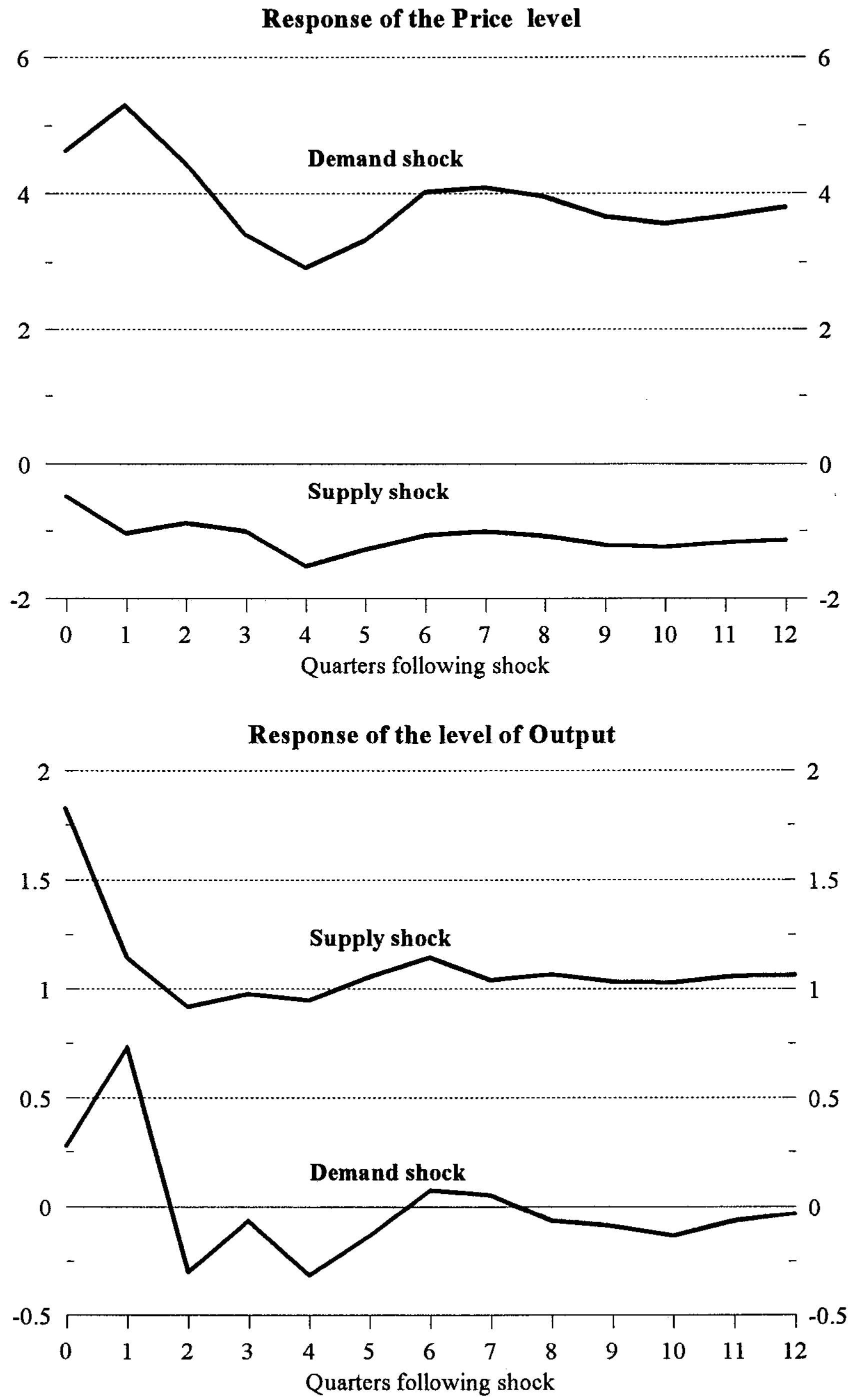
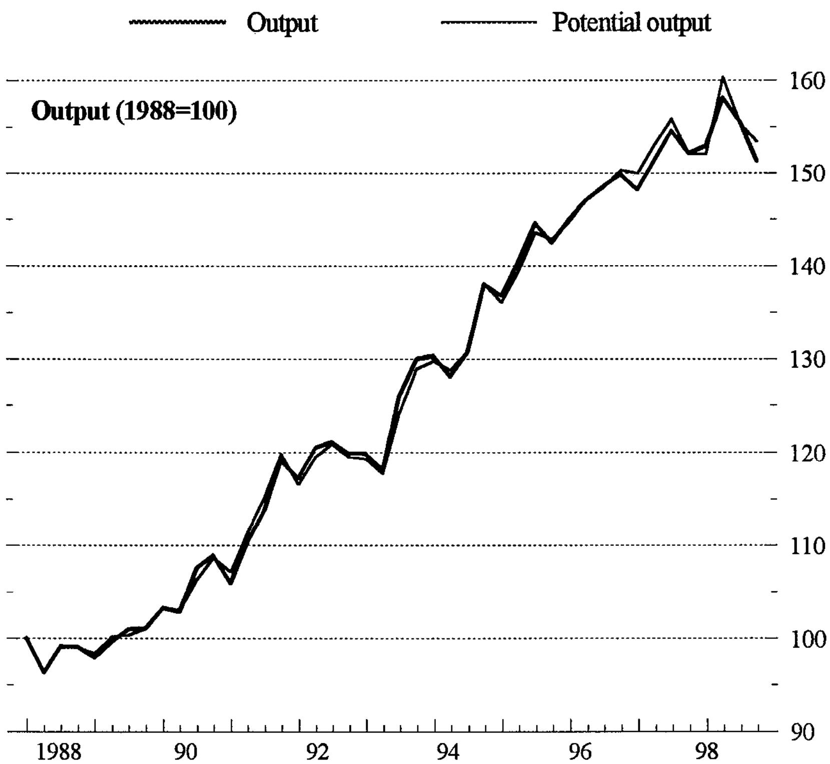


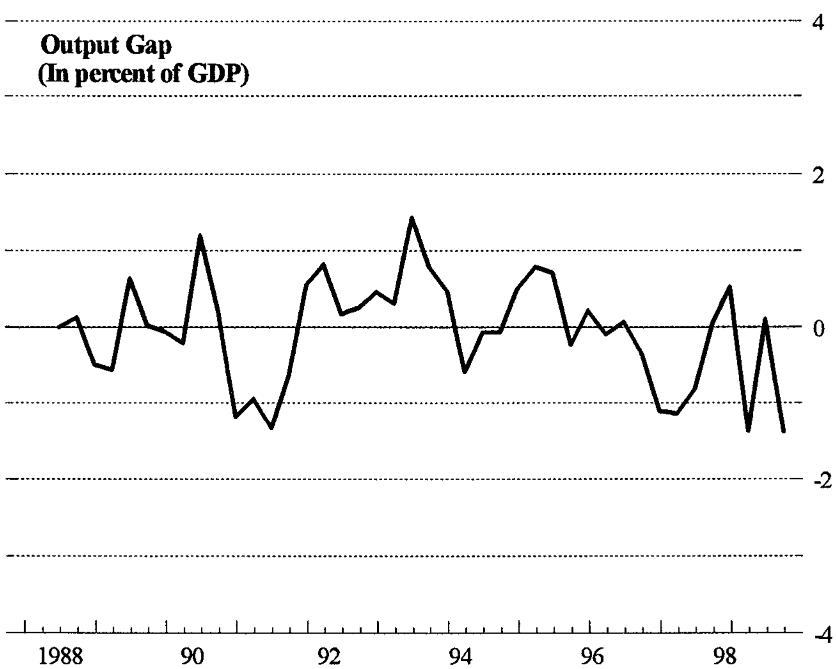
Figure 6. Structural VAR: Response to Shocks



Note: The charts shows the percentage response of output and inflation to "typical" supply and demand shocks (shocks one standard deviation in magnitude). The response of output growth and inflation to the shocks have been cumulated to show the effects of supply and demand on the levels of output and prices.

Figure 7. Potential Output: Structural VAR





One caution with regard to these results is that the inflationary spike at the end of 1998 is interpreted by the VAR as suggesting a lack of slack at the end of 1997, even though the sharp increase in inflation was related to the decline in the value of the shekel rather than to strong activity. To examine the importance of this effect, the VAR was estimated with inflation starting in the third quarter of 1998 staying at 4 percent for the subsequent period rather than spiking upwards and then declining by the end of 1999. While on the one hand this lower inflation in 1998 would be expected to imply a wider output gap in 1997, the revised "data" change the estimated coefficients in the VAR that quantify the relationship between output and inflation. The overall results of the VAR with the modified data are actually smaller output gaps of -0.3 percent of potential in 1997 and -0.2 percent in 1998. The results of the VAR thus appear to be fairly robust in finding only moderate output gaps in 1997 and 1998 despite slow growth.

V. Conclusion

The results presented here are consistent in the finding of a high growth rate of potential in the period of heavy immigration. Using five quite different approaches to measure potential output in Israel, the annual estimates vary somewhat from year to year, but each methodology indicates that annual potential output growth accelerated during the 1990s to reach around 7 percent by 1995. The output gaps likewise vary by methodology, but most imply that output was above potential for a lengthy period in the early or mid 1990's. For 1997 and 1998, however, the results are more eclectic. In the face of continued strong investment, the production function suggests an output gap that reaches nearly 7 percent of potential output in 1998 and thus implies a substantial amount of underutilized resources in the economy. In contrast, the other methodologies cast the weak activity in 1997 and 1998 as more of a structural phenomenon, with the differences in their estimates depending primarily on how sensitive the methodology is to the slowdown in growth. Because it imposes the most smoothness on potential output, the HP filter implies the largest gap of the statistical techniques for 1998, while the two filters that adapt rapidly find little deviation from potential. Deciding which of these measures is most appropriate in the case of Israel requires a judgement to be made as to the importance and pace of the structural change occurring in the Israeli economy. Nonetheless, all four measures of potential output aside from the production function imply that an important part of the slowdown in output stems from slower growth of potential output rather than solely a cyclical retrenchment. These supply influences presumably include the end of the immigration, adjustment costs incurred in the transition to a high-tech economy, and the slowing of foreign investment associated with delays in the peace process.

The Wavelets Transform[12]

The wavelets decomposition represents data as a superimposition of basic functions, called wavelets, with precise frequency and position in time. This differs from Fourier analysis, which represents a data series as a superimposition of sines and cosines, and thus imposes a specific form of regularity on the cyclical movements. There are numerous varieties of wavelets, and some care must be taken to select the variety that is most appropriate for the data to be analyzed.

Wavelets have a "gender": the mother wavelet $\psi(t)$ integrates to zero, while the father wavelet $\phi(t)$ integrates to 1. Given a discrete signal f(t) of finite length, the orthogonal wavelets series approximation can be written as a linear combination of functions of the two genders:

$$f(t) = \sum_{k=1}^{N} s_{J,k}\phi_{J,k}(t) + \sum_{k=1}^{N} d_{J,k}\psi_{J,k}(t) + \sum_{k=1}^{N} d_{J-1,k}\psi_{J-1,k}(t) + \ldots + \sum_{k=1}^{N} d_{1,k}\psi_{1,k}(t) \quad (1)$$

where J indicates the number of components of the equation and N denotes the number of coefficients in the specified component. The coefficients $s_{J,k}$ and $d_{J,k}$ $d_{J-1,k}$ ... $d_{1,k}$ are called the wavelets transform coefficients; these represent the coordinates in the functional space spanned by the approximating wavelets functions and are generated by combining equation (1) with:

$$\phi_{j,k}(t) = 2^{-j/2}\phi(2^{-j}t-k) = 2^{-j/2}\phi\left(\frac{t-2^j k}{2^j}\right) \quad (2)$$

and

$$\psi_{j,k}(t) = 2^{-j/2}\psi(2^{-j}t-k) = 2^{-j/2}\psi\left(\frac{t-2^j k}{2^j}\right) \quad (3)$$

Equations (2) and (3) show that the bases of the functional space are generated by scaling and translating both the mother and the father wavelets. The scale (or dilation) factors are the elements of the dyadic series 2^j, while the translation parameters (or location) are the elements of the dyadic series multiplied by the number of coefficients, that is, $2^j k$. The level index j is associated with scale 2^j, and the shift index k is associated with translation $2^j k$.

For the wavelets to form a basis in a functional space they must be orthogonal, that is, they must satisfy the following properties:

---

[12]This summarizes results from Donoho (1993, 1994) and Donoho and Johnstone (1992).

$$\int \phi_{J,k}(t)\phi_{J,k'}(t)dt = \delta_{k,k'}$$

$$\int \psi_{j,k}(t)\phi_{J,k'}(t)dt = 0$$

$$\int \psi_{j,k}(t)\psi_{j',k'}(t)dt = \delta_{j,j'}\delta_{k,k'}$$

where $\delta_{i,j} = 1$ for I=j and 0 otherwise. The crucial aspect of wavelets analysis is finding the appropriate basis—the functional form for equations (2) and (3).

To calculate the wavelets approximation to a time series, a discrete wavelets transform is used to map the signal vector $f = (f_1, f_2, \ldots f_n)'$ into a set of wavelets coefficients $w = (w_1, w_2, \ldots, w_n)'$ through a matrix operator W:

$$w = Wf \tag{4}$$

The elements of vector w are the coefficients $s_{J,k}$ and $d_{j,k}$ from equation (1). The former, called the smooth coefficients, represent the long-term behavior of the series at the coarse scale 2^J, while the latter, called the detail coefficients, represent the deviation from the long-term trend at progressively fine scale as the coefficients move from $d_{J-1,k}$ toward $d_{1,k}$. The vector w is therefore composed of:

$$\boldsymbol{w} \equiv \begin{pmatrix} \boldsymbol{s}_J \\ \boldsymbol{d}_J \\ \boldsymbol{d}_{J-1} \\ \cdot \\ \cdot \\ \cdot \\ \boldsymbol{d}_1 \end{pmatrix}$$

where each component is defined as:

$\mathbf{s}_j = (s_{J,1}, s_{J,2}, s_{J,3}, \ldots, s_{J,n/2^J})'$

$\mathbf{d}_J = (d_{J,1}, d_{J,2}, d_{J,3}, \ldots, d_{J,n/2^J})'$

$\mathbf{d}_{J-1} = (d_{J-1,1}, d_{J-1,2}, d_{J-1,3}, \ldots, d_{J-1,n/2^J})'$

$\mathbf{d}_1 = (d_{1,1}, d_{1,2}, d_{1,3}, \ldots, d_{1,n/2^J})'$

The coefficients S_T represent potential output, while the $d_{J,K}$ coefficients can be interpreted as business cycle movements.

The wavelets shrinkage developed by Donoho and Johnstone entails estimating the elements of w, then shrinking the detail coefficients at the j finest scale to obtain a new set of coefficients:

$$\tilde{d}_1 = \delta_{\lambda_1\sigma_1}(d_1),\ \ \tilde{d}_2 = \delta_{\lambda_2\sigma_2}(d_2),\ \ldots\ ,\ \ \tilde{d}_j = \delta_{\lambda_j\sigma_j}(d_j)$$

where δ_c is calculated using the "hard shrinkage" function:

$$\delta_c(x) = \begin{cases} 0 & if\ |x| \leq c \\ x & if\ |x| > c \end{cases}$$

The last element to define is $c_j = \lambda_j\sigma_j$, the product of the shrinkage threshold and the scale of noise, respectively. For both of these coefficients, there exist a number of rules to select the optimal level. The scale of noise, σ_j, is derived from the details coefficients, while the rule used to calculate the shrinkage threshold yields the largest threshold and as a consequence the smoothest signal: $\lambda_j = \sqrt{2\log(n)}$.

Once all of the coefficients $\tilde{d}_j$ have been calculated, the inverse wavelets transform can be applied to reconstruct a signal where the noise has been filtered out.

REFERENCES

Barrell, Ray and James Sefton, 1995, "Output Gaps: Some Evidence from the United Kingdom, France, and Germany," *National Institute Economic Review*, No. 151 (February), pp. 65–73, .

Blanchard, Olivier, and D. Quah, 1989, "The Dynamic Effects of Aggregate Demand and Supply Disturbances," *American Economic Review*, Vol. 79, pp. 655–73.

Burns, Arthur F. and Wesley C. Mitchell, 1944, *Measuring Business Cycles*, (New York: National Bureau of Economic Research).

Canova, Fabio, 1998, "Detrending and Business Cycle Facts," *Journal of Monetary Economics*, Vol. 41, pp. 475–512.

Clifton, Eric, 1998, "The Decline of Traditional Sectors in Israel–The Role of the Exchange Rate and the Minimum Wage," IMF Working Paper 98/167 (Washington: International Monetary Fund).

Coe, David T. and C. John McDermott, 1997, "Does the Gap Model Work in Asia?" *Staff Papers*, (Washington: International Monetary Fund) Vol. 44 (March), pp. 59–80.

De Masi, Paula, 1997, "IMF Estimates of Potential Output: Theory and Practice," *Staff Studies for the World Economic Outlook*, (Washington: International Monetary Fund).

DeSerrers, Alain, Alain Guay, and Pierre St-Amant, 1995, "Estimating and Projecting Potential Output Using Structural VAR Methodology: The Case of the Mexican Economy," Bank of Canada Working Paper 95-2 (March).

Donoho, D. L., 1993, "Nonlinear Wavelets Methods for Recovery of Signals, Densities and Spectra from Indirect and Noisy Data" in Daubechies I. (ed.) Proceedings of the Symposia in Applied Mathematics. American Mathematical Society.

________, 1994, "De-noising by Soft Thresholding," *IEEE Transactions on Signal Processing*.

________ and I. Johnstone, 1992, "Ideal Spatial Adaptation via Wavelets Shrinkage" Technical Report No. 400, Department of Statistics, Stanford University.

Dupasquier, Chantal, and Alain Guay, 1997, "A Comparison of Alternative Methodologies for Estimating Potential Output and the Output Gap," Bank of Canada Working Paper 97-5 (February).

Harvey, A.C. and A. Jaeger, 1993, "Detrending, Stylized Facts and the Business Cycle," *Journal of Applied Econometrics*, Vol. 8.

Hodrick, Robert J. and Edward C. Prescott, 1997, "Postwar U.S. Business Cycles: An Empirical Investigation," *Journal of Money, Credit, and Banking*, Vol. 29 (February), pp. 1–16.

King, Robert G. and Sergio T. Rebelo, 1993, "Low Frequency Filtering and Real Business Cycles," *Journal of Economic Dynamics and Control*, Vol. 17 (January–March), pp. 207–31 .

Lucas, Robert, 1972, "Expectations and the Neutrality of Money," *Journal of Economic Theory*, Vol. 4, (April) pp. 103–124.

Robertson, Donald, and M.R. Wickens, 1997, "Measuring Real and Nominal Macroeconomic Shocks and the International Transmission Under Different Monetary Systems," *Oxford Bulletin of Economics and Statistics*, Vol. 59 No. 1, pp. 5–27.

Thies, Clifford F., 1991, "Business Cycles and Black Holes," *Critical Review*, Vol. 5, No. 2.

Tukey, John W., 1977, *Exploratory Data Analysis*, (Reading, Massachusetts: Addison-Wesley).