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International Financial Contagion and
the IMF: A Theoretical Framework

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Abstract

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We provide a model of contagion where countries borrow or lend for consumption smoothing at the market interest rate or a lower IMF rate. Highly indebted countries hit by large negative shocks to output will default. The resulting reduction in loanable funds raises interest rates, increases the vulnerability of other indebted countries, and can generate further rounds of defaults. In this environment the IMF can limit default and internalize the externality generated by contagion through its lending with conditionality. We characterize the IMF's optimal lending decision in mitigating the loss in world consumption.

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I. Introduction

The International Monetary Fund was set up in response to the disastrous economic conditions of the 1930s. The designers of the Fund viewed the large fluctuations in exchange rates as a major contributor to the world-wide depression in the 1930's. These exchange rate movements were generally viewed as deliberate attempts to achieve an exchange rate depreciation in order to improve the trade balance and thereby boost output and employment. Competitive depreciations by many countries simply spread deflation from one country to another with adverse consequences for all. As pointed out by Chari and Kehoe (1998), to solve this collective action problem the Fund set up rules governing the conditions under which exchange rates could be changed by members. The basis of the Bretton Woods system was that fixed but adjustable exchange rates would avoid the excessive volatility and competitive depreciations which were thought to characterize a floating exchange rate system, while allowing enough flexibility to adjust to fundamental balance of payments disequilibrium.

It was envisioned at that time that private capital flows would play only a limited role in financing payments imbalances and the use of controls was sanctioned to help insulate economies from instability stemming from short-term flows. By contrast, temporary official financing, primarily through the Fund, would enable countries to avoid major adjustments in their domestic economies in the face of adverse balance of payments developments. This key function of the Fund is embodied in Article I (v), which states that one of the purposes of the Fund is "To give confidence to members by making the general resources of the Fund temporarily available to them under adequate safeguards, thus providing them with the opportunity to correct maladjustments in their balance of payments without resorting to measures destructive of national or international prosperity." With clear limits on the ability of private capital flows to finance shifts in external positions, the Fund was to help smooth consumption by providing temporary financing of trade imbalances.

The present international financial system departs sharply from this concept. The exchange rates among the major currencies exhibit significant short-run volatility and substantial fluctuations over the medium term, and a growing number of developing countries have adopted a wide variety of flexible exchange rate arrangements. Private international capital flows have grown tremendously in magnitude and have shown considerable volatility, playing major roles both in financing current account deficits and as autonomous sources of major macroeconomic disturbances transmitted across countries.[2] Finally, industrial countries have by and large abandoned capital controls and developing countries have moved in the direction of liberalizing the capital account.

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[2] For a description of these developments, see Goldstein and Mussa (1993), Mussa and Richards (1999), and IMF (1998).

In this new environment the capital account poses a source of external vulnerability which for many countries may exceed that emanating from the trade account which was initially the rationale for the Fund's operations. This has been particularly evident in the 1990s in the currency crises that have engulfed emerging market economies. A noteworthy feature of these crises has been the spread of financial difficulties from one country to another in the same region and even beyond, in some cases, in a process that has come to be referred to as "contagion."

There has been relatively little analysis of the role of the Fund in providing short-term financing to countries that agree to the conditionality to which the loan is subject. Marchesi and Thomas (1999) develop a theoretical model in which both buybacks and the adoption of an IMF program can be used as a screening device to enable creditors to discriminate between debtors that will repay and those that will not. Miller and Zhang (2000) provide an analysis of sovereign liquidity crises and make a case for a payments standstill as a way of avoiding the moral hazard involved with Fund lending, which they argue causes lenders to cease monitoring the performance of countries they lend to. While both papers offer insights regarding issues dealing with sovereign debt and Fund conditionality, they do not address the problem of how provision of liquidity by the Fund can help reduce the losses that arise from contagion. Kumar, Masson and Miller (2000) point out that while short-term debt provides liquidity and incentives for the sovereign borrowers to undertake adequate effort, at the same time it increases the risk of self-fulfilling liquidity crisis. They suggest that a way out of this dilemma is for the Fund to provide the necessary liquidity together with conditions on its lending to encourage governments to undertake the necessary policy changes. While these papers offer insights regarding issues dealing with sovereign debt and Fund conditionality, they do not address the problem of how provision of liquidity by the Fund can help reduce the losses that arise from contagion in a multi-country setting.

This paper attempts to provide a theoretical framework for such an analysis. It develops a two-period model in which some countries have inherited from the past a given stock of net claims on the rest of the world and others a stock of net foreign liabilities. In the face of shocks to output, countries borrow or lend in order to smooth consumption between the two periods. This can be done both through private capital markets as well as through the Fund. It is assumed that the latter provides short-term financing to members at an interest rate which is below that of private financing. This private borrowing rate is determined endogenously in the world capital market.

We provide a stylized characterization of the operation of the Fund which we believe nevertheless captures certain key features of its functions. We set up a simple theoretical framework in which Fund lending to members compliments that provided by the private capital market and plays a catalytic role for such private financing. This catalytic effect comes about through the policy conditionality associated with Fund lending which is embodied

in a Fund program. When members borrow from the Fund, they are typically obligated to implement changes in monetary and fiscal policies, and in many cases structural policy changes as well, in order to correct balance of payments and other macroeconomic imbalances and thereby enhance prospects for economic growth. In our theoretical framework we assume that these policy measures will increase the future output of the country that borrows from the Fund.[3] As a result, the country's ability to borrow in the private capital market is strengthened. Indeed, we show that stronger IMF conditionality will lead to more capital market borrowing.

If a country is hit by a negative shock to output, it will borrow against next period's output to smooth consumption. To motivate the decision to default in response to such a shock, we simplify the analysis by assuming that there is some minimum level of consumption which will be maintained in the face of a large decline in output. Under these conditions, the country will default on its external liabilities if these are relatively large, implying it has substantial debt service payments, and if output next period is comparatively low, implying that the country's capacity to repay out of future income is limited.

The contagion mechanism in our model is built up mainly from fundamentals and the negative externality effects caused by a default. Compared with the existing literature on financial contagion, including Aghion, Bolton and Dewatripont (1999), Allen and Gale (2000), Goodhart and Huang (2000), Huang and Xu (2000), Kodres and Pritsker (1999), Masson (1999), and others,[4] our mechanism has a number of distinct features. First, it highlights the importance of fundamentals and the structure of the international financial system. When a heavily indebted country is hit by a large negative shock, it chooses to default, which raises the interest rate in the international capital market. If there are other countries with similar levels of international indebtness, and therefore vulnerable to external shocks, this increase in the interest rate on international debt can serve as a trigger for a financial crisis. Second, our contagion mechanism emphasizes the critical role of the negative externality in the international capital markets which links countries through the equilibrium world interest rate. And third, contagion exists in our model of rational expectations. In other words, the propagating contagious mechanism does not rely on the effects of informational asymmetry or coordination failures, as highlighted by Diamond and Dybvig (1983), for example.

Intuitively, the world economy can be seen as an incomplete market in the sense that there is little or no insurance against macroeconomic shocks to indebted developing

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[3] We do not model how the policy conditions in a Fund program affect a country's output. Rather, we simply index the degree of Fund conditionality by a parameter, α, and assume that output in the period following the implementation of the policy conditions is positively related to α. For a recent discussion of Fund conditionality, see IMF (2001).

[4] For additional references to the literature on this topic, see Chapter III, "International Financial Contagion," in *IMF* (1999), and Huang (2000).

economies, and debt contracts between creditor- and debtor-countries are incomplete.[5] The IMF can be viewed as in effect creating a second capital market at a lower and controlled interest rate, and thus providing additional liquidity to the international capital market which addresses in part this market failure. This second market is helpful for both borrowers and lenders. The benefits to borrowers are in the form of additional liquidity from the Fund at below-market interest rates.[6] The benefits to creditors arise from IMF conditionality and monitoring which help to ensure the repayment of loans and thus offset the costs of the subsidized loan rates borne by the creditors. The monitoring provided by the Fund not only compensates for the deficiency of bondholders in monitoring, but also involves economy-of-scale effects in monitoring a la Diamond (1984) in that one monitor can substitute for many monitors. Moreover, the subsidized IMF loan can help prevent an individual debtor country from defaulting on its external debt, which benefits those lending directly to the country, and more generally helps contain international financial contagion, which benefits all lending countries. Even if the IMF loans have moral hazard effects on the borrowing countries, Fund conditionality can serve as a device to control these adverse effects. Indeed, our results show that an increase in the degree of conditionality induces debtor countries to increase their borrowing from the international capital markets, but it may or may not lead them to increase their consumption.[7]

In this setup there is a clear role for the Fund to contain contagion and limit international financial crises by lending at a below-market interest rate to heavily indebted countries and thereby reduce the probability of their defaulting.[8] In our theoretical framework, the Fund can be viewed as varying the interest rate it charges members and/or the degree of conditionality in order to achieve the optimal amount of borrowing from the Fund and the optimal number of defaulting countries. We use this framework to analyze this topic in the last section of the paper.

The remaining part of the paper is organized as follows. The model is set up in Section II. Section III describes the determinants of borrowing and default and the manner in which the world interest rate is affected by a default. The role of the IMF in containing international financial contagion is analyzed in Section IV, and concluding remarks are presented in Section V.

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[5]   See Kletzer and Wright (2000) for an analysis of enforcement of sovereign debt contracts in the sense of complete contracts, and Hart and Holmstrom (1987) for discussions of contractual incompleteness.

[6]   See Holmstrom and Tirole (1998) for further discussion of the public supply of liquidity.

[7]   For discussion and analysis of the effects of conditionality, see Haque and Khan (1998).

[8]   The model presented here can viewed as providing a theoretical basis for the need for an international lender of last resort, as argued by Fischer (2000).

II. The Model

In our model we consider a world economy consisting of N economies, each facing its own budget constraint:

$$A_t^i + B_t^i + Y_t^i = (1 + \bar{r}_{t-1})A_{t-1}^i + (1 + r_{t-1})B_{t-1}^i + C_t^i, \tag{1}$$

where A denotes the value of financial resources borrowed from the IMF, B is the value of an economy's other net foreign borrowing, Y denotes the level of real GNP, C is the level of real consumption, $\bar{r}$ and r are the interest rates on IMF and foreign borrowing, respectively, superscript i indicates country i ($1 \le i \le N$), and subscript t denotes period t ($t \ge 0$). $A^i > 0$ and $B^i > 0$ indicate country i is a debtor, and vice versa if $A^i < 0$ and $B^i < 0$.

$$Y_t^i = Y^i + \delta_t^i,$$

where Y^i is the average long-run level of output in country i, which is assumed to be exogenous, and δ_t^i is an i.i.d. shock with zero mean.

All loans, both from the IMF and other creditors, are assumed to be one period in maturity. The interest rate is determined at the time that borrowing/lending is undertaken. For simplicity, we assume that if a borrower has the ability to repay, it will do so as long as its consumption meets some minimum level. That is, we assume there is no strategic defaulting.

Each economy is assumed to maximize its utility over a two-period time horizon, which is represented as:

$$U_t^i = \sum_{s=t}^{t+2} \beta^s u(C_t^i),$$

where $\beta < 1$ is the discount factor. For simplicity, we also assume that

$$u(C_t^i) = \ln\left(C_t^i - \overline{C}^i\right).$$

We further assume that $\overline{C}^i$ is the minimum level of consumption that country i has to maintain. We do not analyze the determinants of this minimum level of consumption, but simply note that it varies considerably from country to country, depending on their level of economic development, political situation, and a lot of other factors.

It should be noted that our model does not distinguish between private and sovereign debt. Thus our framework is a general one in which agents can be thought of as maximizing consumption of both privately-produced goods and services and those provided by the public sector. By the same token, the minimum level of consumption can be viewed as consisting of public as well as private goods. An obvious extension of the approach provided here would be to model the behavior of the public sector explicitly, perhaps along the line of Alesina

and Tabellini (1990), taking account of the government budget constraint and the impact of a negative output shock on tax revenue.

Without any significant loss of generality, we restrict our attention to a simple two-period model in which M countries, $1 < M < N$, have outstanding net international debts at the beginning of the period, while all other countries are lenders, so that:

$$B_0^1 \geq B_0^2 \geq \dots \geq B_0^M \geq 0 > B_0^{M+1} \geq \dots \geq B_0^N. \tag{2}$$

The interest rate for external debt, r_t, is determined in the international capital market through demand and supply for the one-period instruments.[9] That is, at r_t,

$$\sum_{i=1}^M B_t^i(r_t) = - \sum_{i=M+1}^N B_t^i(r_t),$$

or,

$$\sum_{i=1}^N B_t^i(r_t) = 0.$$

The IMF participates indirectly in the international capital market, as it obtains its loanable funds from its member countries, in particular, those countries that are net creditors in the international financial market. To capture this idea, for simplicity we assume that the credit extended to the IMF by members is proportional to their net foreign lending. That is, for a *lending* country, $i \in [M+1, N]$, such that $B_t^i < 0$, we have

$$A_t^i = \phi_t B_t^i(r_t),$$

where

$$\phi_t = \frac{\sum_{i=1}^M A_t^i(r_t)}{\sum_{i=1}^M B_t^i(r_t)} > 0. \tag{3}$$

For $\bar{r}_t < r_t$, it is obvious that the IMF needs to impose a "no-arbitrage condition" when extending credit to members, i.e., only net debtors are eligible to borrow at the lower

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[9] We make the simplifying assumption that all borrowers face the same interest rate so that private lenders do not differentiate among borrowers in terms of their expected probability of default. It would be possible to make the interest rate charged to each borrower the sum of a common interest rate faced by all borrowers and a country-specific risk premium, i.e., $r_t^i = r_t + \tilde{r}_t^i$, where $\tilde{r}_t^i$ can be a function of its B^i/Y^i ratio, for example. As long as a default raises the common interest rate r_t, due to the nature of incomplete markets and contagious risks which cannot be fully captured by $\tilde{r}_t^i$, our results are not qualitatively altered. We thus would leave the treatment of country-specific risks for a future extension of the model.

rate offered by the IMF. As the IMF's loanable funds come from its member countries, we have:

$$\sum_{i=1}^{M} A_t^i(\bar{r}_t) = - \sum_{i=M+1}^{N} A_t^i(\bar{r}_t),$$

that is,

$$\sum_{i=1}^{N} A_t^i(\bar{r}_t) = 0.$$

Aggregating the budget constraint of the world economy gives:

$$\sum_{i=1}^{N} \left(A_t^i + B_t^i + Y_t^i\right) = \sum_{i=1}^{N} \left[(1 + \bar{r}_{t-1})A_{t-1}^i + (1 + r_{t-1})B_{t-1}^i + C_t^i\right].$$

By using $\sum_{i=1}^{N} A_t^i(\bar{r}_t) = 0$ and $\sum_{i=1}^{N} B_t^i(r_t) = 0$ at t and $t-1$ in the above condition, we have

$$\sum_{i=1}^{N} C_t^i(r_t) = \sum_{i=1}^{N} Y_t^i. \tag{4}$$

This condition has a clear implication in terms of total demand and supply in the real sector. As the goods market has to be cleared each period in this one-good economy, after each economy is hit by an i.i.d. shock and Y_t^i is realized, the good market determines the amount of net borrowing and lending, and thus the equilibrium world interest rate.

To capture the essence of the costs arising from the interest rate charged by the IMF and from the non-financial costs associated with IMF conditionality, the total costs of borrowing from the IMF are represented by the following reduced form:

$$(1 + \bar{r}_t)A_t^i + \alpha_t^i A_t^i, \tag{5}$$

where

$$\alpha_t^i = \alpha A_t^i / 2Y^i > 0.$$

Three features are worth noting of this specification of the conditionality associated with the use of Fund resources by a country. First, it depends on the Fund-wide degree of conditionality, α, which is same across members and therefore consistent with the uniform treatment of members. Second, the degree of conditionality rises with the extent to which a member makes use of Fund resources, A_t^i. This is clear from paragraph five of the Guidelines on Conditionality (*Fund Decision No. 6056 -(79/38)*, March 2, 1979), which stipulates that

phasing and performance clauses are applicable only to the use of Fund resources in stand-by arrangements beyond the first credit tranche. Moreover, recourse to Fund resources is subject to access limits, and greater conditionality is involved in borrowing beyond the access limits. Third, conditionality does not depend on the member's economic size. To control for this, the amount borrowed from the Fund is scaled by the long-run level of output, Y^i, which can be viewed as a proxy for the member's quota.

IMF conditionality aims at helping the borrowing country to improve its economic performance and to restructure its economic system, with the ultimate objective of fostering higher sustainable economic growth.[10] Thus it seems plausible to assume that the output in the second-period, Y_2^i, is a (concave) increasing function of α, that is, $d\left(Y_2^i\left(\alpha\right)\right)/d\alpha > 0$ and $d^2\left(Y_2^i\left(\alpha\right)\right)/d\alpha^2 < 0$. A member therefore faces a trade-off in borrowing from the Fund when solving its dynamic consumption-smoothing problem: it needs to weigh the presumably short-run costs of conditionality against the expected future gains from the enhanced performance of its economy.

Given the implicit cost arising from conditionality, the interest rate the IMF charges, $\bar{r}_t$, is lower than the market rate, r_t. The optimal amount of borrowing from the IMF is determined by the condition that the marginal costs of borrowing from the Fund and the market are the same, i.e.,

$$\frac{\partial}{\partial A_t^i}\left[(1+\bar{r}_t)A_t^i + \frac{\alpha}{2Y^i}\left(A_t^i\right)^2\right] = \frac{\partial}{\partial B_t^i}\left[(1+r_t)B_t^i\right],$$

which leads to

$$1 + \bar{r}_t + \frac{\alpha A_t^i}{Y^i} = 1 + r_t.$$

Thus, as $\alpha/Y^i > 0$, we always have $\bar{r}_t < r_t$.

If country i is a debtor country, it will borrow from the IMF up to the point where:

$$A_t^i = \frac{(r_t - \bar{r}_t)Y^i}{\alpha} = \frac{m_t Y^i}{\alpha}, \tag{6}$$

where $m_t \equiv r_t - \bar{r}_t > 0$ is the margin between the market rate and the IMF interest rate.[11] And

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[10] Achieving these objectives is in the Fund's interests, as it has a fiduciary responsibility for safeguarding its resources over the period during which the country is expected to repay the Fund. In addition, conditionality helps to mitigate moral hazard associated with Fund lending, which, as shown below, has the feature of a public good. For an extensive description of Fund conditionality, see "Conditionality in Fund-Supported Programs – Policy Issues," April 20, 2001, IMF external website.

[11] It should be noted that equation (6) determines the amount of borrowing from the Fund, whereas $A_t^i = \phi_t B_t^i(r_t)$ only holds for countries lending to the Fund.

$$A_t^{i\prime}(\alpha) = \frac{\partial A_t^i(\alpha)}{\partial \alpha} = -\frac{m_t Y^i}{\alpha^2}.$$

As the IMF is concerned about a member's ability to repay the loan, there are limits on the amount a member may borrow from the Fund, namely, after paying off the principle and interest owed to the Fund and to other lenders, the borrowing country must meet its minimum consumption requirement:

$$Y_2^i(\alpha) - (1 + \bar{r}_1)A_1^i = Y_2^i(\alpha) - \frac{m_1}{\alpha}Y^i \geq (1 + r_1)B_1^i + \overline{C}^i.$$

It should be pointed out that r_t in (6) is the equilibrium interest rate on loans in the international capital market and depends on each borrower's B_t^i, and A_t^i. In the next section, it will be clear that both A_t^i and B_t^i are fully endogenized in our model.

III. Borrowing, Defaulting and Interest Rate Dynamics

A. Borrowing and Defaulting

To focus on the issue of international financial contagion, we focus on the problem faced by the debtor countries and start our analysis in a two-period (three-date, $t = \{0, 1, 2\}$) model. At $t = 0$, some countries have outstanding debt (to be repaid at $t = 1$) while others outstanding claims, as shown by equation (2).

Given the nature of two-period model, by the end of the second period there will be no outstanding borrowing or lending. That is, $B_2^1 = B_2^2 = \ldots = B_2^N = 0$. Regarding the IMF lending, we also assume that $A_0^1 = A_0^2 = \ldots = A_0^N = 0$. That is, there is no outstanding IMF lending at $t = 0$. Similarly, by the end of the second period,

$$A_2^1 = A_2^2 = \ldots = A_2^N = 0.$$

Based on the above setup, the budget constraint at $t = 1$ becomes

$$A_1^i + B_1^i + Y_1^i = (1 + r_0^i)B_0^i + C_1^i. \tag{7}$$

Similarly, the budget constraint at $t = 2$ becomes

$$Y_2^i = (1 + \bar{r}_1^i)A_1^i + (1 + r_1^i)B_1^i + C_2^i. \tag{8}$$

Substituting

$$A_1^i = \frac{m_1 Y^i}{\alpha}. \tag{9}$$

into (8), and solving country i's intertemporal utility maximization leads to

$$C_1^{*i} \equiv \Phi^i(r_1) = \frac{(1+r_1)Y_1^i + Y_2^i(\alpha) - (1+r_0)(1+r_1)B_0^i + \frac{(m_1)^2}{\alpha}Y^i}{(1+\beta)(1+r_1)}, \tag{10}$$

$$C_2^{*i} = \frac{\beta\left[(1+r_1)Y_1^i + Y_2^i(\alpha) - (1+r_0)(1+r_1)B_0^i + \frac{(m_1)^2}{\alpha}Y^i\right]}{1+\beta}, \tag{11}$$

as long as $C_t^{*i} \geq \overline{C}^i.$[12] And,

$$B_1^{*i} = -\frac{\beta}{1+\beta}Y_1^i + \frac{1}{(1+\beta)(1+r_1)}Y_2^i(\alpha) + \frac{\beta(1+r_0)}{1+\beta}B_0^i + \left[\frac{1+\bar{r}_1}{1+r_1} - \frac{\beta m_1}{(1+\beta)(1+r_1)}\right]\frac{m_1 Y^i}{\alpha}. \tag{12}$$

It is easy to check that $C_2^{*i}/C_1^{*i} = \beta(1+r_1)$.

From (10), we also have:

$$\frac{\partial C_1^{*i}}{\partial r_1} = -\frac{Y_2^i(\alpha) - \frac{m_1(2+r_1+\bar{r}_1)}{\alpha}Y^i}{(1+\beta)(1+r_1)^2} = \begin{cases} < 0, & \text{if } Y_2^i(\alpha) > \frac{m_1(2+r_1+\bar{r}_1)}{\alpha}Y^i; \\ \geq 0, & \text{if } Y_2^i(\alpha) \leq \frac{m_1(2+r_1+\bar{r}_1)}{\alpha}Y^i. \end{cases}$$

Although $Y_2^i(\alpha) \leq \frac{m_1(2+r_1+\bar{r}_1)}{\alpha}Y^i$ is a theoretical possibility, it is unlikely to be true for reasonably parameter settings, where $\frac{m_1(2+r_1+\bar{r}_1)}{\alpha} \lll 1$. We thus rule it out in the following discussion. Hence:

Lemma 1 *For a borrowing country, $\partial C_1^*/\partial r_1 < 0$ holds.*

Moreover, from (10), it is important to note that consumption in the first period depends on conditionality, α:

$$\frac{\partial C_1^{*i}}{\partial \alpha} = \frac{d\Phi^i(\alpha)}{d\alpha} = \frac{Y_2^{i\prime}(\alpha) + m_1 A_1^{i\prime}(\alpha)}{(1+\beta)(1+r_1)} = \begin{cases} \geq 0, & \text{if } Y_2^{i\prime}(\alpha) \geq -m_1 A_1^{i\prime}(\alpha); \\ < 0, & \text{if } Y_2^{i\prime}(\alpha) < -m_1 A_1^{i\prime}(\alpha). \end{cases}$$

That is:

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[12] Similarly, for lending countries,
$$C_1^{*i} = \frac{(1+r_1)Y_1^i + Y_2^i - (1+r_0)(1+r_1)B_0^i + m_1\phi B_1^i}{(1+\beta)(1+r_1)},$$
$$C_2^{*i} = \frac{\beta\left[(1+r_1)Y_1^i + Y_2^i - (1+r_0)(1+r_1)B_0^i + m_1\phi B_1^i\right]}{1+\beta}.$$

Lemma 2 *For a borrowing country, stronger IMF conditionality will lead it to increase its current consumption if $Y_2^{i'}(\alpha) \geq -m_1 A_1^{i'}(\alpha)$, and vice versa.*

This lemma reflects the fact that conditionality affects consumption through two different channels. First, conditionality can help a country achieve a higher future level of output and thereby enhance its potential to pay back loans. Thus IMF monitoring through its conditionality can make the country a more credible debtor in the eyes of the market and thereby enable it to attract capital inflows to support higher consumption. Second, strong conditionality may lower the consumption level of a borrowing country. This is because the loan from the IMF, at a lower-than-market interest rate, provides a subsidy to a borrowing country. Especially strong conditionality leads the borrowing country to borrow less from the Fund and thus it will receive a smaller subsidy. When the effect of reduction in the subsidy resulting from greater conditionality dominates the effect of increasing future output, which is diminishing with an increasing α, the impact on consumption will be negative. Because of these different effects of conditionality, too much or too little conditionality can be harmful for a borrowing country.

Furthermore, from (12) it is clear that

Lemma 3 $B_1^{*i} \geq 0$ *if and only if*

$$\frac{1}{(1+\beta)(1+r_1)} Y_2^i(\alpha) + \frac{\beta(1+r_0)}{1+\beta} B_0^i \geq \frac{\beta}{1+\beta} Y_1^i + \left[1 - \frac{m_1}{(1+\beta)(1+r_1)}\right] \frac{m_1 Y^i}{\alpha}$$

This lemma implies that country i will borrow at $t = 1$ if it has been hit by a severe shock in that period (Y_1^i small) but has a high expected level of output at $t = 2$ (Y_2^i large), or/and has borrowed a large amount of debt at $t = 0$ (B_0 large), and/or it cannot borrow enough from the IMF (high $\bar{r}_1$ or α, or low Y^i) to offset the negative shock.

Moreover,

$$\frac{\partial B_1^{*i}}{\partial \alpha} = \frac{Y_2^{i'}(\alpha)}{(1+\beta)(1+r_1)} + \left[1 - \frac{m_1}{(1+\beta)(1+r_1)}\right] \frac{m_1 Y^i}{\alpha^2} > 0.$$

That is:

Lemma 4 *Stronger IMF conditionality will always induce an indebted country to increase its borrowing from the international capital market.*

The intuition behind this lemma is as follows. Stronger IMF conditionality has two effects on a borrowing country: higher second-period output and lower subsidy at $t = 1$. Both effects lead to higher capital-market borrowing at $t = 1$. Thus the IMF loans to a borrowing

country can substitute for loans by creditor countries in the international capital markets, although it is not a perfect substitution because

$$0 < 1 - \frac{m_1}{(1 + \beta)(1 + r_1)} < 1.$$

Moreover, although a borrowing country always increase its capital-market borrowing if conditionality is stronger, its consumption at $t = 1$ may or may not increase. Its consumption will increase if the gain in the second-period output due to stronger conditionality dominates the loss of the IMF subsidy, and vice versa.

So far the issue of default has not arisen. In this model it arises as a result of a decline in output which is assumed to be exogenous. For simplicity, output in borrowing countries is subject to idiosyncratic shocks which are unrelated to their indebtedness to the financial sector or to the Fund. Thus, defaults are not related directly to specific policies taken by governments. However, high debt stocks (relative to output) inherited from the past do reflect particular policies, such as large fiscal deficits, and therefore there is an indirect relationship between their policies and the likelihood that a country will default. In addition, shocks to output are uncorrelated across countries. An obvious extension of the model would be to introduce such output correlations, which would provide another contagion mechanism distinct from that of through the world interest rate.

If the negative shock to Y_1^i is severe and its outstanding indebtedness is large, the country may not be able to continue its borrowing from the international capital market even if it would very much wish to do so. This arises because country i has to maintain a minimum consumption level, $\overline{C}^i$, in all periods, i.e., $C_t^{*i} \geq \overline{C}^i$, $t = 1, 2$. Notice that the equilibrium condition between C_1^{*i} and C_2^{*i}, i.e., $C_2^{*i}/C_1^{*i} = \beta(1 + r_1)$, together with condition $\beta(1 + r_1) \geq 1$, which we assume to hold, let us to focus on $C_1^{*i} = \Phi^i(r_1)$. That is,

$$C_1^{*i} = \frac{(1 + r_1)Y_1^i + Y_2^i(\alpha) - (1 + r_0)(1 + r_1)B_0^i + \frac{(m_1)^2}{\alpha}Y^i}{(1 + \beta)(1 + r_1)} \geq \overline{C}^i \qquad (13)$$

When a country is hit by a shock at $t = 1$, normally it would borrow against its future output (Y_2) to smooth its consumption. To make our case interesting, we assume that

$$\begin{cases} Y_1^i \geq \overline{C}^i \\ Y_2^i \geq \overline{C}^i \end{cases}.$$

Under this assumption, if a country is hit by a large negative shock in the first period but nonetheless has a bright future, i.e., a high Y_2, and can borrow against Y_2, so that in addition to paying off all outstanding loans, it can further increase current period consumption. In this

case there is no reason for this country to default.[13] The reason is that by choosing not to default, its consumption can exceed its current output:

$$C_1^{*i} > Y_1 > \overline{C}^i.$$

On the other hand, if the country hit by a negative output shock has poor future prospects, i.e., a low Y_2, and thus cannot borrow an amount against Y_2 sufficient to pay off all of its outstanding loans, then it would end up with consumption lower than Y_1, and could end up with consumption lower than $\overline{C}^i$ if the outstanding loans are very high relative to the shock and future output. In this event, the optimal strategy for the country is to default because in doing so it can at least have $\overline{C}^i$ for its first-and-second period consumption.[14]

Thus we have the following proposition.

Proposition 1 *If a debtor country that services its debt obligations is hit by a severe shock such that $Y_1^i \geq \overline{C}^i$, $C_1^{*i} = \Phi^i(r_1) < \overline{C}^i$, and $Y_2^i \geq \overline{C}^i$, then its optimal strategy is to default.*

According to this proposition, a debtor country is more likely to default when it has a low Y_1^i and a low Y_2^i, in addition to its high B_0^i, high $\bar{r}_1$, or high α. That is, if a country has a large stock of outstanding debt, low expected future output, and little or no access to the Fund, it can no longer smooth its consumption according to the classical intertemporal model. Notice that although the country in trouble might still be interested in borrowing from the international capital markets, no one would be willing to lend to it because it would not be able to pay back by the end of the second period. Indeed, it is the limited ability of the country to pay back all the debt that limits its ability to borrow. Thus, if condition (13) is not satisfied, the country will default on its current debt, B_0^i. The consequence of this default, as we show in the next section, can be to trigger an international financial crisis.

B. The Dynamics of Interest Rate Determination with Default

When a country defaults on its debt, the corresponding lender will not be able to get its investment paid back. In order to smooth its own consumption, the lender will have less

---

[13] We assume there are effective mechanisms in place, such as reputational effects and possible sanctions by the international community, which prevent a country from defaulting if it can at least maintain its minimum consumption level while servicing its debt and taking out new loans. With $\overline{C}^i > Y_1 > C_1^{*i}$, however, countries would default, a case we are not particularly interested in this paper; with such a low income level, the country would not be eligible for loans, but for grants.

[14] We assume that if a country defaults, it does so on its liabilities to both the private sector and to the Fund. This is inconsistent with the fact that members have not defaulted on their obligations to the Fund, as pointed out by Jeanne and Zettelmeyer (2001). However, we have made this assumption because it greatly simplifies the analysis below of the Fund's policies to limit the extent of defaults.

to lend or may become a borrower. This will inevitably change the equilibrium condition in the international goods and financial markets. The equilibrium interest rate will be pushed up, and a domino effect may be triggered by even a small default. We provide a set of conditions under which international financial contagion becomes a possibility.

The equilibrium interest rate when there is no default is denoted as r_1^*, which is endogenously determined by equation (4). That is, at r_1^*,

$$\sum_{i=1}^{N} C_1^i(r_1^*) = \sum_{i=1}^{N} Y_1^i. \tag{14}$$

We denote the first country to default to be country 1. As a result of its default, country 1 can no longer borrow from the rest of the world. This implies that its production and consumption would be equal and it would be in a position of autarky. With the country's production and consumption being excluded from the world economy, the equilibrium interest rate after one country has defaulted, $\hat{r}_1(1)$, will be determined by the remaining $N - 1$ economies:

$$\sum_{i=2}^{N} C_1^i(\hat{r}_1(1)) = \sum_{i=2}^{N} Y_1^i. \tag{15}$$

We have the following proposition regarding the new interest rate in equilibrium.

Proposition 2 *The equilibrium interest rate after a default, $\hat{r}_1(1)$, is higher than the rate without the default, r_1^*.*

Proof. When condition (13) for country $i = 1$ is satisfied, it will not default on its borrowing. Its equilibrium level of consumption is simply $C_1^{1*}(r_1^*)$, where r_1^* is determined by the total consumption and production of the N economies with no default, and it can consume above its minimum level of consumption, $\overline{C}^1$. That is, at r_1^*, (14) holds, and $C_1^{1*}(r_1^*) \geq \overline{C}^1$.

When condition (13) for country 1 is not satisfied, however, it will default on its borrowing so that it can use its entire output to continue to consume above the minimum level of consumption, $\overline{C}^1$. Notice that country 1's decision may also be related to its lower expected output in the second period, Y_2^1. In that event, it may not consume all Y_1^1 at $t = 1$, that is:

$$Y_1^1 \geq C_1^1 > \overline{C}^1.$$

With country 1's production and consumption being excluded from exchange with world economy, the equilibrium interest rate after country 1 has defaulted will be determined by (15).

Subtracting Y_1^1 on both sides of (14), we have

$$\sum_{i=1}^{N} C_1^i(r_1^*) - Y_1^1 = \sum_{i=2}^{N} C_1^i(r_1^*) - \left[Y_1^1 - C_1^1(r_1^*)\right] = \sum_{i=2}^{N} Y_1^i. \tag{16}$$

Clearly at $Y_1^1 - C_1^1(r_1^*) = 0$, we should expect

$$\sum_{i=1}^{N} C_1^i(\widehat{r}_1(1)) = \sum_{i=1}^{N} C_1^i(r_1^*) = \sum_{i=1}^{N} Y_1^i.$$

Thus, country 1 would not default, nor would the remaining economies, and $C_1^i(\widehat{r}_1(1)) = C_1^i(r_1^*)$ for $i = \{1, 2, ...N\}$

As a default by country 1 only takes place when $Y_1^1 \geq \overline{C}^1 \geq C_1^1(r_1^*)$, it follows that:

$$\sum_{i=2}^{N} C_1^i(r_1^*) - \left[Y_1^1 - C_1^1(r_1^*)\right] = \sum_{i=2}^{N} C_1^i(\widehat{r}_1(1)) = \sum_{i=2}^{N} Y_1^i < \sum_{i=2}^{N} C_1^i(r_1^*),$$

and therefore:

$$C_1^i(\widehat{r}_1(1)) < C_1^i(r_1^*) \text{ for } i = \{2, ...N\}.$$

According to Lemma 3.1, this implies that

$$\widehat{r}_1(1) > r_1^*. \tag{17}$$

The intuition underlying this proposition is as follows. After a default, the resource endowment of one or several lenders has been reduced and consequently they will have to adjust their consumption and investment by either reducing their lending or by switching to borrowing. The new equilibrium can only be achieved at a higher interest rate. It is useful to view the default as an additional negative shock to all the affected lenders equal to the amount of the receivable but defaulted credit. As a result of such a shock, the RHS of equation (14) will be reduced and this will require a higher interest rate to bring consumption into alignment with lower output.

Obviously the magnitude of the impact of a default on the world interest rate depends on the size of the defaulting country. The additional resources gained by a small developing

country through defaulting on its foreign loans would have an imperceptible effect on the market interest rate. Thus the model is more applicable to the large emerging market economies, such as Argentina, Brazil, and Russia, as a default on their part would have an impact on interest rates. Moreover, if negative shocks to output are positively correlated across countries (contrary to the i.i.d. assumption in the model), the impact of a default on the world interest rate, and the contagious effect, would be stronger.

However, it should be noted that although renegotiation of loan contracts is not allowed for in the model, and hence defaults are "all or nothing", allowing partial defaults would only weaken our results but would not affect them qualitatively. A partial default occurs through recontracting or providing debt relief which would allow consumption to be maintained at the minimum level, $\overline{C}$, but not above, but there will still be some reduction in resources available in the international capital market, and thus some increase in the equilibrium interest rate. More precisely, a complete default takes away $Y_1^i - C_1^{*i}$ of loanable funds, while a partial default takes away $Y_1^i - \overline{C}^i$ of loanable funds, $C_1^{*i} < \overline{C}^i$ for a defaulting country.

Having established the effects of a default on the equilibrium interest rate, we are ready to present the domino effects triggered by a default and provide a mechanism for international financial contagion.

IV. International Financial Contagion and the Fund

A. International Financial Contagion

After country 1 has defaulted on its debt the new equilibrium interest rate will be higher relative to the rate at which country 1 would not default. At this higher interest rate, another country, call it country 2, may find it will have to default as well. This is because for country 2, although at r_1^*, $C_1^{*2}(r_1^*) = \Phi^2(r_1^*) \geq \overline{C}^2$, but at $\widehat{r}_1(1) > r_1^*$, due to the default of country 1,

$$C_1^{*2}(\widehat{r}_1(1)) = \Phi^2(\widehat{r}_1(1)) < \overline{C}^2,$$

defaulting becomes an optimal decision. As a result of the default by country 2, the new equilibrium interest rate will be pushed up further to $\widehat{r}_1(2) > \widehat{r}_1(1)$. Following the same process, after m countries have defaulted one after another, the new interest rate will be pushed up to a still higher level, $\widehat{r}_1(m)$. This domino-like defaulting process of one country after another is one form of financial contagion which emerges in our model without relying on any information asymmetry, or coordination failure a la Diamond and Dybvig. We have the following proposition to characterize the international financial contagion.

Proposition 3 *After one country has defaulted on its debt payments, this may trigger a domino-like chain default up to m countries, one after another. The crisis stops at country $m + 1$, whose financial situation is so strong that at $\widehat{r}_1(m) > \widehat{r}_1(m - 1)$,*

$$C_1^{*m+1}(\widehat{r}_1(m)) = \Phi^{m+1}(\widehat{r}_1(m)) \geq \overline{C}^{m+1}.$$

For all the remaining $i \in [m + 2, N]$ countries, their financial situations are even stronger, such that at $\widehat{r}_1(m)$, $\Phi^i(\widehat{r}_1(m)) \geq \overline{C}^i$ holds as well.

From this proposition it is easy to see that the key condition whether or not a country is subject to international financial contagion is simply $\Phi^i(\widehat{r}_1(m)) \geq \overline{C}^i$, $i \in [m + 1, N]$. From lemma 2, the lower the production levels in periods 1 and 2, the lower the available IMF loan, the higher the outstanding loans from the international capital market, the higher the minimum level of consumption, and the higher the total number and amount of defaults, the more likely the country will be hit by the financial crisis and whose default worsens international financial conditions and causes others to default. Moreover, $m \leq M$. The intuition behind this result is also quite clear: lenders have no outstanding debt on which to default, and thus the worst situation they face is not having their loans repaid.

It is useful at this point to make the distinction between insolvency and illiquidity.[15] The first country to default faces an insolvency problem in that given its level of international indebtness, a negative shock to its level of income makes it impossible to service its debt and simultaneously achieve its minimum level of consumption. There are not enough foreign assets that could be sold to meet foreign claims coming due. The situation is similar for other borrowers who are adversely affected by the default of the first country, and thus the model here deals basically with the problem of insolvency. However, the consequence of debt default has an aspect of illiquidity, as in Holmstrom and Tirole (1998), in that the higher interest rate generated by defaults raises the price of liquidity. Thus those countries that were relatively liquid at pre-crisis interest rate will find themselves in a less liquid position following the defaults of other countries.

B.   Containment of International Financial Contagion by the IMF

Facing the possibility of international financial contagion and a cascade of defaults by indebted countries, the IMF can play a useful role by containing the contagion and limiting the extent of the defaults. Examining the default condition, i.e., consumption in the first period is below the minimum level:

$$C_1^{*i} = \frac{(1 + r_1)Y_1^i + Y_2^i(\alpha) - (1 + r_0)(1 + r_1)B_0^i + \frac{(m_1)^2}{\alpha}Y^i}{(1 + \beta)(1 + r_1)} < \overline{C}^i,$$

---

[15]   Jeanne (1998) argues that international liquidity mis-match can cause international financial contagion.

one can easily see that there are two channels through which the IMF can help indebted countries from defaulting. The first, and more obvious one, is that the Fund can provide more financing through lowering its interest rate $\bar{r}_1$, so that the above inequality relationship is reversed through an increase in m_1. The second, less obvious one, is that the IMF can increase its conditionality on its financing so that the equilibrium consumption level is raised, as given by Lemma 2 above. This channel operates through raising a country's future production level. The Fund can attempt to avert defaults by relying on one or both instruments in combination.

We now analyze the determination of the optimal number of defaulting countries for given resources at the disposal of the Fund. We first analyze the case where the IMF is concerned about the welfare of creditors, and then examine the case where the welfare of borrows is also taken into account. The problem can be formulated as the choice of α and $\bar{r}_1$ to achieve the optimal number of defaulting countries, $k(\alpha, \bar{r}_1)$, which will maximize first- and second-period consumption of the lending countries. This problem is given by:

$$\max_{k(\alpha, \bar{r}_1)} \sum_{t=1}^{2} \sum_{i=M+1}^{N} C_t^i(\widehat{r}_t(k(\alpha, \bar{r}_1))),$$

$$\text{s.t.} \sum_{i=k+1}^{M} A_1^i(\alpha, , \bar{r}_1) \leq \overline{R}. \tag{18}$$

In the above problem, first- and second-period consumption of the lending countries are given by:

$$
\begin{aligned}
C_1^i(\widehat{r}_1(k(\alpha, \bar{r}_1))) &= Y_1^i + (1 + \phi_1) B_1^i(k(\alpha, \bar{r}_1)) + [(1 + \bar{r}_0)\phi_0 + (1 + r_0)](-B_0^i), \\
C_2^i &= Y_2^i + [(1 + \bar{r}_1)\phi_1 + (1 + \widehat{r}_1(k(\alpha, \bar{r}_1)))](-B_1^i(k(\alpha, \bar{r}_1)));
\end{aligned}
$$

$k(\alpha, \bar{r}_1)$ is the cutoff point chosen by the IMF such that for $k \in [0, M]$,

$$
\left\{
\begin{aligned}
C_1^k(\widehat{r}_1(k(\alpha, \bar{r}_1))) &= \Phi^k(\widehat{r}_1(k(\alpha, \bar{r}_1))) < \overline{C}^{k-1}, \\
C_1^{k+1}(\widehat{r}_1(k(\alpha, \bar{r}_1))) &= \Phi^{k+1}(\widehat{r}_1(k(\alpha, \bar{r}_1))) \geq \overline{C}^k;
\end{aligned}
\right. \tag{19}
$$

and $\overline{R}$ is the total amount of available resources the IMF can use.

Substituting $C_1^i(\widehat{r}_1(k(\alpha, \bar{r}_1)))$ and C_2^i into the objective function and noticing that Y_1^i and Y_2^i (of lending countries) do not depend on the choice of k, the problem is equivalent to choosing $k(\alpha, \bar{r}_1)$ to maximize:

$$\sum_{i=M+1}^{N} [\phi_1\bar{r}_1 + \widehat{r}_1(k)][-B_1^i(k)] - \sum_{i=1}^{k} [(1 + \bar{r}_0)\phi_0 + (1 + r_0)] B_0^i, \tag{20}$$

subject to (18).

The first order condition (FOC) of (20) with respect to α is

$$\sum_{i=M+1}^{N} \left[\frac{d\widehat{r}_1}{dk} \frac{\partial k}{\partial \alpha}(-B_1^i) + (\phi_1\bar{r}_1 + \widehat{r}_1)\frac{d(-B_1^i)}{dk} \frac{\partial k}{\partial \alpha} \right]$$

$$= \quad [(1+\bar{r}_0)\phi_0 + (1+r_0)]\, B_0^k \frac{\partial k}{\partial \alpha} - \mu \left[A_1^{k+1} \frac{\partial k}{\partial \alpha} + \frac{\sum_{i=k+1}^{M} m_1 Y^i}{\alpha^2} \right],$$

where $\mu \geq 0$ is the Lagrangian multiplier of the budget constraint.

Dividing both sides by $\partial k/\partial \alpha$, we have

$$\sum_{i=M+1}^{N} \left[(-B_1^i)\frac{d\widehat{r}_1}{dk} + (\phi_1\bar{r}_1 + \widehat{r}_1)\frac{d(-B_1^i)}{dk} \right] - [(1+\bar{r}_0)\phi_0 + (1+r_0)]\, B_0^k + \mu\Delta_k = 0, \quad (21)$$

where

$$\Delta_k \equiv A_1^{k+1} + \frac{\sum_{i=k+1}^{M} m_1 Y^i}{\alpha^2 \frac{\partial k}{\partial \alpha}}.$$

Similarly, the FOC of (20) with respect to $\bar{r}_1$ is:

$$\sum_{i=M+1}^{N} \left[\left(\phi_1 + \frac{\partial \widehat{r}_1}{\partial k}\frac{\partial k}{\partial \bar{r}_1} \right)(-B_1^i) + (\phi_1\bar{r}_1 + \widehat{r}_1)\frac{d(-B_1^i)}{dk}\frac{\partial k}{\partial \bar{r}_1} \right]$$

$$= \quad [(1+\bar{r}_0)\phi_0 + (1+r_0)]\, B_0^k \frac{\partial k}{\partial \bar{r}_1} - \mu \left[A_1^{k+1}\frac{\partial k}{\partial \bar{r}_1} + \frac{\sum_{i=k+1}^{M} Y^i}{\alpha} \right].$$

Dividing both sides by $\partial k/\partial \bar{r}_1$ and using (21) we have

$$\sum_{i=M+1}^{N} (-\phi_1 B_1^i) - \mu\left(\Delta_k + \Delta_{\bar{r}_1} \right)\frac{\partial k}{\partial \bar{r}_1} = 0, \quad (22)$$

where

$$\Delta_{\bar{r}_1} \equiv A_1^{k+1} + \frac{\sum_{i=k+1}^{M} m_1 Y^i}{\alpha \frac{\partial k}{\partial \bar{r}_1}}.$$

Analytically the two FOCs, i.e., (21) and (22), together with the binding constraint of (18), define a set of three equations which give optimal solutions for k^*, α^*, and $\bar{r}_1^*$. The solution can be obtained by taking a two-step approach. First, solving (21) for the optimal solution for k^*. Then, using k^* in (22) and the binding constraint of (18), we further solve for α^*, and $\bar{r}_1^*$.

Intuitively, note first that $\widehat{r}_1(k)$ is an increasing function in k because as the world interest rate rises, there will be a larger the number of defaults. Second, the volume of loans, $-B_1^i(k)$, declines as the number of defaulting countries increases on account of the higher interest rate. Third, the interest and principal on the defaulted loans, $\sum_{i=1}^{k} [(1+\bar{r}_0)\phi_0 + (1+r_0)]\, B_0^i$, is an increasing function of k. Consequently, for any given amount of Fund resources, there is an interior solution for the optimal number of defaults,

k_l^*, which is based on the trade-off between the gains from a higher interest rate, on the one hand, and losses from declining loan volume and increasing non-repayment of existing loans, on the other hand. In addition, as the opportunity cost for creditor countries to provide credit to the IMF is $\hat{r}_1(k)$, while the return of the IMF lending is only $\bar{r}_1$, $\bar{r}_1 < \hat{r}_1(k)$ implies that creditor countries do not wish to provide any additional credit to the IMF beyond that needed to achieve the optimal level of defaults. Thus the resource constraint of the Fund is binding, and $\mu > 0$.

Figure 1 shows the interior solution of k^*. The top curve is the upper panel is the first summation in (20), denoted by F. It initially has a positive slope as the effect of the rising interest rate outweighs the declining loan volume, but at some point the slope becomes negative as the two effects are reversed. The upward-sloping straight line in the upper panel is the second summation of (20), which is denoted by G, and reflects the rising value of interest and principal on defaulted loans. The difference between F and G is the value of (20). The lower panel shows the relationship between k and R, i.e., the value of R required to achieve a given value of k. Obviously, as the number of defaults declines, the corresponding level of resources needed by the Fund increases. The optimum k_l^* is determined where $F - G$ is at its maximum if $R\left(k_l^*\right) \leq \overline{R}$.

We now consider the case where the IMF is concerned about the welfare of the world economy from the perspective of all participants in the international capital market, i.e., both lenders and borrower. This problem can be formulated as follows:

$$\max_{k(\alpha, \bar{r}_1)} \sum_{t=1}^{2} \left[\gamma \sum_{i=M+1}^{N} C_t^i(\hat{r}_t(k)) + (1 - \gamma) \sum_{i=k+1}^{M} C_t^i(\hat{r}_t(k)) \right],$$

subject to (18), where $0 \leq \gamma \leq 1$ and $1 - \gamma$ are the weights of creditor and debtor countries, respectively.

Once again, for $i \in [M + 1, N]$, $C_t^i(\hat{r}_1(k))$ is consumption of the lenders; for $i \in [k + 1, M]$, $C_1^i(\hat{r}_1(k))$ is consumption of borrowers who have not defaulted. Substituting C_t^i into the objective function, the problem is equivalent to choosing $k(\alpha, \bar{r}_1)$ to maximize:

$$\sum_{i=M+1}^{N} [\phi_1 \bar{r}_1 + \hat{r}_1(k)] \left(-B_1^i(k)\right) - \sum_{i=1}^{k} [(1 + \bar{r}_0)\phi_0 + (1 + r_0)] B_0^i \qquad (23)$$

$$+ \frac{1 - \gamma}{\gamma} \left[\sum_{i=k+1}^{M} Y_2^i(\alpha) - \sum_{i=M+1}^{N} [\phi_1 \bar{r}_1 + \hat{r}_1(k)] \left(-B_1^i(k)\right) \right],$$

subject to (18). By taking the same two-step approach, we can solve for the optimal solution for k, α, and $\bar{r}_1$ through using the two FOCs with respect to α and $\bar{r}_1$ respectively, and the budget constraint of (18).

Intuitively, equation (23) is the same as (20) plus an additional term on the second line, which is the difference between of the second-period output of debtor countries (affected by Fund conditionality) and the gain to creditor countries (caused by higher interest rates).[16] This additional item is a decreasing function of k, because $\sum_{i=k+1}^{M} Y_2^i(\alpha)$ is a decreasing function of k which in general dominates the effect of the initial increasing value of $\sum_{i=M+1}^{N} [\phi_1 \bar{r}_1 + \hat{r}_1(k)] (-B_1^i(k))$. For $1 > \gamma > 1/2$, $\frac{1-\gamma}{\gamma} < 1$, and thus the effect of the additional term (its negative slope) shrinks by $\frac{1-\gamma}{\gamma}$. Consequently, an interior solution for k, k_{lb}^*, exists and it is to the left of k_l^*, because the additional term initially only has a moderate negative slope.

Figure 2 shows the interior solution of (23). F and G are the same as in Figure 1. The downward curve in the upper panel is the additional term in (23), which is denoted by H, and reflects the difference between of the second-period output of debtor countries and the gain to creditor countries due to high interest rates. $F - G + H$ is the value of (23), and the optimum k_{lb}^* is determined where $F - G + H$ is at its maximum. Again the lower panel shows the relationship between k and R.

It is interesting to point out that at $\gamma = 1/2$, (23) collapses to

$$\sum_{i=k+1}^{M} Y_2^i(\alpha) - \sum_{i=1}^{k} [(1 + \bar{r}_0)\phi_0 + (1 + r_0)] B_0^i,$$

which is a decreasing function of k, and thus there only exists a corner solution $k_{lb}^* = 0$. The same result may hold for γ close to 1/2, and for $0 < \gamma < 1/2$, whereby the effect of the additional term (its negative slope) is amplified by $\frac{1-\gamma}{\gamma}$ and thus dominates the initial positive slope of $F - G$.

Proposition 4 $0 \leq k_{lb}^* \leq k_l^*$. *That is, the IMF has stronger incentives to contain international financial crises when it is concerned about the interests of both creditor and borrower countries than when it is only concerned about the interest of creditor countries.*

V.   Conclusions and Extensions

In this paper we provide a model of contagion in which countries are linked through the international capital market. All countries face the same market-determined interest rate and borrow or lend to optimally smooth their consumption over time. Borrowing from the Fund also provides a mechanism for countries to smooth consumption intertemporally. A key aspect of the model is that countries which have a high initial levels of debt are vulnerable to negative shocks to output. If these shocks are so large that they make it impossible for a

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[16]   It should be noted when a country defaults, the benefit in terms of higher second-period output generated by Fund conditionality disappears.

country simultaneously to achieve a desired minimum level of consumption and to service its foreign debt, the country will default.

In failing to meet its international financial obligations, the country withdraws net resources from the rest of the world. Rather than repay its debt, it uses the resources involved to maintain its consumption at some minimum level. As a result, there is some upward pressure on world interest rates in order to reestablish balance between total consumption and output in the rest of the world. This higher interest rate then raises the debt service costs of other indebted countries and can generate further rounds of defaults which will increase further the world interest rate. The cascade of defaults and interest rate increases will cease when all countries vulnerable to debt servicing shocks on account of high initial debt stocks have defaulted.

In this environment the Fund has an important systemic function in lending to members to limit the extent of contagion and default. The Fund can be seen as internalizing the externality generated by the contagion that spreads through the channel of the world capital market that links all countries. In the first instance, its role can be viewed as maximizing the welfare of creditors subject to the resources it has available to lend to members. From this perspective and based on the trade-off between the gains from a higher interest rate and losses from declining loan volume and increasing non-repayment of existing loans and a consideration of the opportunity cost of Fund capital, the Fund will be more likely to lend to a country that would do significant damage to the world economy if it defaulted, but it may allow a few indebted countries to default. More generally, its role can be viewed as maximizing the welfare of both creditors and debtors, subject to the resources it has available to lend to members. In this case, the Fund has more incentives to contain the extent of a crisis than when it is only concerned about the welfare of creditors, and it may want to stop a crisis from breaking out.

This analysis of contagion has stressed the role of fundamentals in the form of stocks of international debt, the severity of debt servicing requirements that is affected by both debt levels and the interest rate on the debt, shocks to output, and minimum levels of consumption. This approach is in contrast to many other contributions to the literature which stress the importance of self-fulfilling crises generated by changes in the financial vulnerability of countries unrelated to fundamentals. It also highlights the externality that arises through the linkage of all countries through the international capital market. This externality is in the background in good times when shocks and debt stocks are small and there are no defaults, but it comes to the fore when the fundamentals deteriorate and give rise to defaults that generate a negative externality to other countries in the form or higher borrowing costs. Of course, such negative effects emanating from higher world interest rates can be generated not only by defaults but also by changes in economic conditions and policies in systemically important countries such as the United States.

The model of contagion developed here is based on a number of simplifying assumptions that could be relaxed in further work. One simplification relates to the conditionality associated with access to Fund resources which makes borrowing from the Fund less attractive than obtaining resources from the private market. Such conditionality in the form of tighter budgetary and fiscal policies are designed to reduce the vulnerability of a country to contagion. However, the mechanism through which this operates is not modeled in the paper. In other words, there is no link between the change in policies and the probability of a country going into default. A more complete treatment of the topic would endogenize this probability, which remains fixed here.

We have in addition assumed that all countries can borrow at the same interest rate in the world financial market. This is clearly not a realistic assumption, as the level of international indebtedness, which varies across countries in the model, would clearly be a significant factor affecting the terms on which such debt incurred. However, making this link would considerably complicate the model without necessarily bringing deeper insights into the connection between the process by which contagion spreads in the model and the role of the Fund in containing contagion.

Finally, we have not dealt with the issue of moral hazard. This can arise in two ways. First, private agents may be willing to take on more risk by lending more than they would otherwise if they believe that the Fund will in effect bail out a country and prevent a default from occurring.. Second, countries may adopt policies that result in larger international indebtedness if they consider that the Fund will provide the balance of payments financing resulting from a deteriorating payments position. However, moral hazard would not appear to be a major problem in the context of our model. One reason is that conditionality can be viewed as an instrument utilized by the Fund to limit the ability of a country to pursue policies which increase its indebtedness.[17] An extension of the analysis here could explore the extent to which conditionality can be calibrated to control any incipient tendency for the user of Fund resources to adopt imprudent policies. More fundamentally, in this paper we have not adopted the assumption of asymmetric information which can give rise to moral hazard in most other models in which the public sector provides resources to the private sector when the latter runs into financial difficulties. Here public and private agents are assumed to share the same information set, so that with rational expectations moral hazard would not be expected to arise. However, the model could usefully be extended by relaxing this assumption and exploring the implications of moral hazard.

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[17] For a forceful argument that the Fund is not likely to generate significant moral hazard, see Mussa (1999). Cordella and Dell'Ariccia (2001) and Jeanne and Zettelmeyer (2001) also discuss the role of conditionality in mitigating moral hazard arising from debt relief or international crisis lending by the Fund.

References

Aghion, Philippe, Patrick Bolton and Matias Dewatripont (1999), "Contagious Bank Failures," mimeo, University College London, Princeton University and ECARE, Sept.

Alesina, Alberto and Guido Tabellini (1990), "A Positive Theory of Fiscal Deficits and Government Debt," *Review of Economic Studies*, Vol. 57, No. 3 July: pp. 403-14.

Allen, Franklin and Douglas Gale (2000), "Financial Contagion," *Journal of Political Economy*, Vol. 108, no. 1, February.

Chari, V.V., and Patrick J. Kehoe, 1998, "Asking the Right Questions About the IMF," *Federal Reserve Bank of Minneapolis 1998 Annual Report*.

Cordella, Tito and Giovanni Dell'Ariccia (2001), "Conditionality and Debt Relief," mimeo, International Monetary Fund, Washington, D.C.

Diamond, Douglas (1984), "Financial Intermediation and Delegated Monitoring," *Review of Economic Studies*, 51(July): 393-414.

Diamond, Douglas, and Phillip Dybvig (1983), "Bank Runs, Liquidity, and Deposit Insurance," *Journal of Political Economy*, 91:401-419.

Fischer, Stanley, 2000, "On the Need for an International Lender of Last Resort," *Journal of Economic Perspectives*, Fall 1999, pp.85-104.

Goldstein, Morris, and Michael Mussa, 1993, "The Integration of World Capital Markets," IMF Working Paper 93/95.

Goodhart, Charles and Haizhou Huang (2000), "A Simple Model of an International Lender of Last Resort," Working Paper 00/75, International Monetary Fund, Washington, D.C.

Haque, Nadeem and Mohsin Khan (1998), "Do IMF-Supported Programs Work? A Survey of the Cross-Country Empirical Evidence," Working Paper 98/169, International Monetary Fund, Washington, D.C.

Hart, Oliver and Bengt Holmstrom (1987), "The Theory of Contracts," in T.F. Bewley (ed.), *Advances in Economic Theory*. Cambridge: Cambridge University Press, 71-155.

Holmstrom, Bengt and Jean Tirole (1998), "Private and Public Supply of Liquidity," *Journal of Political Economy*, 106(1): 1-40.

Huang, Haizhou (2000), "Financial Contagion: ABC Channels," *IMF Research Bulletin*, 1(2).

Huang, Haizhou and Chenggang Xu, "Financial Institutions, Financial Contagion, and Financial Crises," Working Paper 00/92, International Monetary Fund, Washington, D.C.

International Monetary Fund (1998), *International Capital Markets: Developments, Prospects, and Key Issues*, Washington, D.C.

International Monetary Fund (2001), "Conditionality in Fund-Supported Programs – Policy Issues," available from the IMF's external website.

Jeanne, Olivier (1998), "The International Liquidity Mismatch and the "New Architecture"," unpublished, International Monetary Fund, Washington, D.C.

Jeanne, Olivier and Jeromin Zettelmeyer(2001), "International Bailout, Moral Hazard and Conditionality," unpublished, International Monetary Fund, Washington, D.C.

Kletzer, Kenneth M. and Brian D. Wright (2000), "Sovereign Debt as Intertemporal Barter," *American Economic Review* Vol.90, n.3 (June): 621-39.

Kodres, Laura and Mathew Pritsker (1999), "A Rational Expectations Model of Financial Contagion," mimeo, IMF and the Board of Governors of the Federal Reserve System.

Kumar, Manmohan S., Paul R. Masson and Marcus Miller, (2000), "Global Financial Crises - Institutions and Incentives," Working Paper 00/105, International Monetary Fund, Washington, D.C.

Marchese, Silivia, and Jonathan P. Thomas, 1999, "IMF Conditionality as a Screening Device," *The Economic Journal*, Vol. 109, pp. 111 - 125.

Masson, Paul R., (1999), "Multiple Equilibria, Contagion, and the Emerging Markets Crises," Working Paper 99/164, International Monetary Fund, forthcoming in Glick, Moreno and Spiegel, ed., *Financial Crises in Emerging Markets*, Cambridge University Press.

Miller, Marcus, and Lei Zhang (2000), "Sovereign Liquidity Crises: The Strategic Case for a Payments Standstill," *The Economic Journal*,Vol.110, n460: 335-362.

Mussa, Michael (1999), "Reforming the International Financial Architecture: Limiting Moral Hazard and Containing Real Hazard," in the Proceedings of a Conference held at the H.C. Coombs for Financial Studies, Kirribili, 9-10 August 1999, *Capital Flows and the International Financial System*, edited by David Gruen and Luke Gower (Australia: J.S. McMillan Printing Group), pp. 216-236.

Mussa, Michael, and Anthony Richards, 1999, "Capital Flows in the 1999s Before and After the Asian Crisis," paper presented at the World Bank/International Monetary Fund/World Trade Organization Conference on Capital Flows, Financial Crises, and Policies, The World Bank, Washington, D.C., April 15-16, 1999.

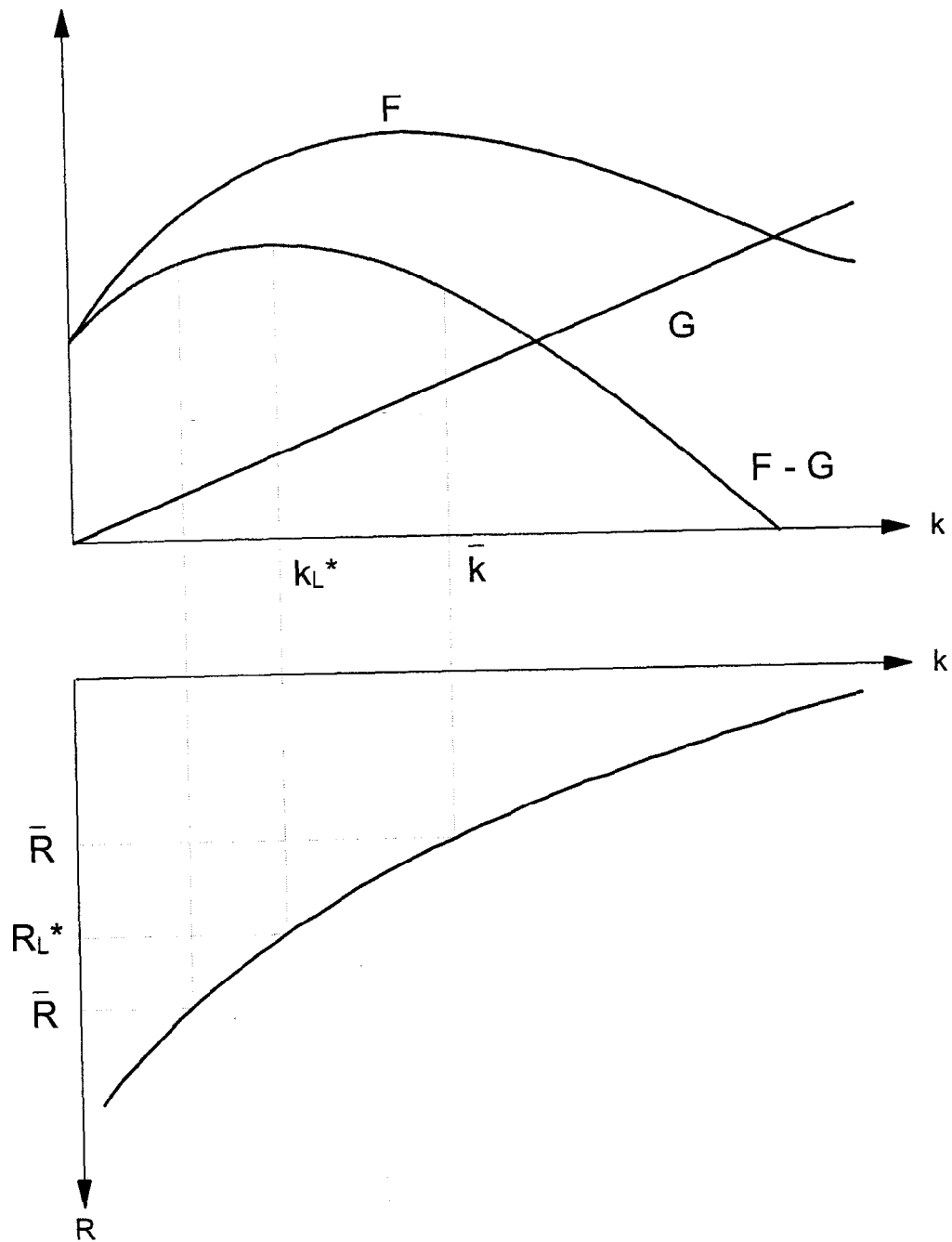# Figure 1. Determination of Optimal K:
Welfare of Lenders Only

Figure 2. Determination of Optimal K:
Welfare of Both Lenders and Borrowers