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Real and Monetary Factors in the Joint Determination of the Exchange Rate and the Interest Rate

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Abstract

The paper analyzes the joint determination of the exchange rate and the interest rate in terms of the stochastic structure of underlying real and monetary factors. It argues that the information contained in the covariation of the exchange rate and the interest rate, as two variables that respond to the same economic disturbances, can be used to infer the nature of those disturbances underlying exchange rate determination. A statistically significant correlation between the exchange rate and the interest rate is shown to be inconsistent with the stochastic structure of underlying disturbances implied by the strictly random walk property of the exchange rate.

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I. Introduction

This paper investigates the joint determination of the exchange rate and the interest rate in terms of the stochastic structure of underlying real and monetary factors. It will argue that the information contained in the covariation of the exchange rate and the interest rate, as two endogenous variables that respond to the same economic disturbances, can be used to infer the nature of those disturbances underlying exchange rate determination. The paper will show that, while the random walk behavior of empirical exchange rates implies an economic structure in which changes in both the money supply and real variables are permanent, such an economic structure is inconsistent with statistically significant correlations between exchange rates and interest rates that have often been observed. This paradox is made more intuitive by noting that, if both the spot and forward exchange rates follow random walks, covered interest rate parity implies that the interest rate differential also follows a random walk; interest rate differentials, however, have not always followed random walks. This may suggest that random walk tests are not powerful enough to detect a small--yet significant--deviation from a random walk.

The paper is organized as follows. Section I discusses the implications of the empirical behavior of major currency exchange rates in recent years. Section II presents a simple model to analyze the joint determination of the exchange rate and the interest rate. Section III derives and discusses the solution of the model on the assumption of perfect current information; and Section IV, the solution of the model on the assumption of imperfect current information. Section V discusses the covariations of the exchange rate and the interest rate. Section VI makes two inferences from the empirical covariations of exchange rates and interest differentials between major countries. Finally, Section VII presents a summary and a few concluding remarks.

I. Empirical Behavior of Exchange Rates

Under the current regime of floating exchange rates, the exchange rates between major currencies have been found to follow random walks. This observed empirical regularity can be analyzed by using the following equation,

$$e_t = \alpha + \beta e_{t-1} + \eta_t, \tag{1}$$

where e is the log of the spot exchange rate expressed as the domestic currency price of foreign currency (i.e., an increase in e means a depreciation of the domestic currency), η is a serially uncorrelated random error, and t and t-1 are discrete time-subscripts. A random walk means that α=0 and β=1 and that η is serially uncorrelated in equation (1). Table 1 reports two types of random walk tests of the exchange rate movements of four major currencies, using end-of-month data for the period

Table 1. Random Walk Tests of Monthly Exchange Rate Movements, 1975-86. a/

sample period	U.S. dollar/ U.K. pound	U.S. dollar/ Deutsche mark	U.S. dollar/ Japanese yen	U.K. pound/ Deutsche mark	U.K. pound/ Japanese yen	Deutsche mark/ Japanese yen
January 1975- December 1978	2.57 (0.08)	0.50 (0.10)	1.60 (0.04)	2.74 (0.04)	9.39** (0.03)	1.75 (0.15)
January 1979- December 1982	0.82 (0.07)	0.86 (0.06)	2.52 (0.25)	1.30 (0.20)	0.28 (0.17)	0.43 (0.10)
January 1983- August 1986	1.09 (0.16)	0.30 (0.10)	4.27 (0.05)	0.93 (0.20)	2.22 (0.09)	2.49 (0.11)
January 1975- August 1986	1.66 (0.03)	0.71 (0.05)	1.37 (0.05)	2.29 (0.06)	3.86 (0.02)	1.27 (0.04)

Note: ** (*) indicates that the statistic is significant at 1 (5) percent.
a/ The Dickey-Fuller Statistic in the first row; and the Bhargava statistic in parentheses.

1975-86. The first test statistic (Dickey and Fuller, 1981) is given by

$$T_1 = [(n-2)/2][(SSE^*-SSE)/SSE], \tag{2}$$

where SSE^* and SSE are the sums of squared errors with and without the restriction that $(\alpha,\beta)=(0,1)$, and n is the number of observations. The null hypothesis that the log of the exchange rate follows a random walk is rejected if T_1 exceeds a critical value. 1/ The second test statistic (Bhargava, 1986) is given by

$$T_2 = \sum_{t=2}^{n}(e_t-e_{t-1})^2 \Big/ \sum_{t=1}^{n}(e_t-\bar{e})^2 , \tag{3}$$

and the values of T_2's are reported in parentheses in Table 1. The null hypothesis is rejected if T_2 exceeds a critical value. 2/

According to Table 1, the null hypothesis that the logs of the monthly exchange rates among the U.S. dollar, the U.K. pound, the Deutsche mark and the Japanese yen followed random walks could not be rejected for the period of 1975-86, except for the Dickey-Fuller statistic for the pound/yen rate for 1975-78. This random walk property of exchange rate movements is, however, a little puzzling, because the random walk is not a necessary implication of market efficiency. 3/ As Mussa (1979) observed, there is no theoretical reason to expect exchange rates to follow a random walk because there is no a priori reason to think that the anticipated rate of change of the exchange rate, unlike stock or bond prices, is close to being constant. In fact, in order to infer market efficiency from a random walk, the exchange rate must follow a linear trend and the nominal interest rate differential must be constant (Levich, 1979). This may mean that the random walk tests are not powerful enough to detect a small--yet significant--deviation from random walks. Moreover, the random walk property of exchange rate movements, if found indeed valid, must be explained in terms not of market efficiency but of the stochastic properties of the underlying variables that determine the exchange rate (Adams and Boyer, 1986).

It has been said that the random walk property of exchange rate movements reflects the fact that predicted changes account for only a

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1/ This is what Dickey and Fuller (1981) call the Φ_1 statistic. A table of empirical distribution of this statistic is provided on p. 1063.

2/ This is what Bhargava (1986) calls the R_1 statistic. A table of five percent significant points of this statistic is provided on p. 378.

3/ In this particular context, market efficiency means that there are no exploitable profit opportunities from speculation in the spot exchange market. The two other senses in which the word market efficiency has been used in the context of the foreign exchange market are: (1) whether covered interest rate parity holds; and (2) whether the forward rate incorporates all available information. See Kohlhagen (1978) and Levich (1979, 1985).

small fraction of the changes in the exchange rate. In this sense, exchange rate changes have often been characterized as being predominantly determined by "news". At the same time, Dornbusch (1980) and Frenkel (1981) have shown that innovations in the interest rate differentials significantly affect the exchange rate. This comes as no surprise because the exchange rate and the interest rate differential, as two endogenous variables, respond to the same "news". Thus, exploiting the information contained in the covariances between these two endogenous variables may provide additional insight into the forces underlying the actual process of exchange rate determination.

II. The Model

The joint determination of the exchange rate and the interest rate can be analyzed in a simple model with a few economic variables. The model is made as simple as possible without sacrificing the essential features needed for the analysis. First, the economy in question is assumed to be a price taker in the international goods market, such that the log of the domestic price level (p) is exogenously determined by:

$$p_t = e_t + p_t^* + u_t, \tag{4}$$

where p* is the log of the foreign price level (normalized to zero) and u a deviation from purchasing power parity (PPP). The PPP deviation term, u, can be thought of as capturing either the short-run absence of commodity arbitrage or the presence of non-traded goods.

Second, the economy is further assumed to be a price taker in the international asset market, such that the domestic interest rate (R), expressed as the log of one plus the nominal interest rate, is exogenously determined by,

$$R_t = R_t^* + (E_t e_{t+1} - e_t) + \theta_t, \tag{5}$$

where R^* is the foreign nominal interest rate (normalized to zero), E_t a mathematical expectations operator based on the set of information available at t, and θ a serially uncorrelated random deviation from interest rate parity. 1/

Third, turning to the specification of the domestic money market, the log of demand for money (m^d) is assumed to be specified by,

1/ This is not a restrictive assumption because, as it turns out, the only important role of θ_t as an additional disturbance term in the subsequent analysis is to make sure that the economic agent cannot obtain full information about two other disturbance terms from observing two independent signals. This deviation term may reflect a risk premium. For a discussion of the risk premium, see Frankel (1982).

$$m_t^d - p_t = y_t - \gamma R_t + v_t, \tag{6-1}$$

where y is the log of real output, γ the interest elasticity of the demand for money, and v a money demand shock; the income elasticity is assumed to be unitary in this specification. On the supply side, the log of the exogenous supply of money (m^s) is assumed to be determined by

$$m_t^s = m_{t-1}^s + \mu_t, \tag{6-2}$$

where μ, the money supply shock, is assumed to be characterized by the following stochastic process,

$$\mu_t = \pi\mu_{t-1} + \varepsilon_{mt}, \qquad (0 < \pi < 1)$$

with ε_{mt} being a white-noise error. By imposing $m_t^d = m_t^s = m_t$, one obtains from (6-1) and (6-2) the following money market equilibrium condition,

$$m_{t-1} + \mu_t - p_t = y_t - \gamma R_t + v_t. \tag{6-3}$$

Finally, the real side of the model is assumed to be exogenous, such that

$$y_t = \bar{y}_o + w_t, \tag{7}$$

where $\bar{y}_o$ is the log of the natural level of output (normalized to zero) and w a real output shock. This completes the specification of the model economy.

The joint determination of the two endogenous variables, R_t and e_t, can be analyzed in terms of a simple diagram that depicts the equilibrium conditions in the money market and the foreign exchange market. From (4), (6-3) and (7), the money market equilibrium condition that fully takes account of the interaction between the money and the goods markets is given by,

$$R_t = (1/\gamma)e_t - (1/\gamma)m_{t-1} - (1/\gamma)\mu_t + (1/\gamma)\phi_t, \tag{8}$$

where $\phi_t \equiv (w_t + v_t + u_t)$ is an aggregate real disturbance, 1/ the stochastic process of which is assumed to be given by,

$$\phi_t = \lambda\phi_{t-1} + \varepsilon_{rt}, \qquad (0 < \lambda < 1),$$

with ε_{rt} being a white-noise error. An increase in ϕ means an increase in the demand for money (either independently or through an increase in output)

1/ In a monetary model of this kind, a real disturbance affects the endogenous variables through its effect on the demand side of the money market. It is in this sense that a money demand shock (v) can be included in the real disturbance.

or the relative price of the domestic goods. For simplicity, equation (8) is referred to as the money market (MM) equilibrium locus. On the R-e plane, its slope is positive and given by $(1/\gamma)$, as depicted in Figure 1. An intuitive interpretation of the MM locus is as follows: starting at a point on the MM locus, an increase in the value of e_t means a higher price level, and hence a lower real quantity of money, for a given nominal stock of money. Thus, in order to induce the economic agent to hold the lower real quantity of money, the nominal rate of interest must rise.

The foreign exchange market equilibrium condition can be expressed by rewriting equation (5) as,

$$R_t = -e_t + E_t e_{t+1} + \theta_t, \tag{5)'$$

which, for simplicity, is referred to as the exchange market (EM) equilibrium locus. The slope of the locus is minus one on the R-e plane, for a given value of R^*_t, $E_t e_{t+1}$ and θ_t (Figure 1). An intuitive interpretation of the EM locus is as follows: starting at a point on the EM locus, a higher nominal rate of interest means an excess demand for domestic bonds, which requires either a greater expected depreciation or a smaller expected appreciation to clear the market. Given $E_t e_{t+1}$, this means that the domestic currency must appreciate in the current period. 1/

III. Solution under Perfect Current Information

In this section, the solutions for e_t, R_t and $E_t e_{t+1}$ will be obtained, assuming that the economic agent has perfect information about the structure of the model as well as about the past and current values of the exogenous variables. The explicit solution of $E_t e_{t+1}$ will allow the use of the EM-MM diagram for illustrative purposes.

As a first step, from (8) and (5)', e_t can be expressed as a function of its own expectation and the exogenous variables as,

$$\begin{aligned} e_t = {} & [\gamma/(1+\gamma)]E_t e_{t+1} + [1/(1+\gamma)]m_{t-1} + [1/(1+\gamma)]\mu_t \\ & - [1/(1+\gamma)]\phi_t + [\gamma/(1+\gamma)]\theta_t . \end{aligned} \tag{9}$$

Under rational expectations, the following solution for e_t can be obtained by use of the method of undetermined coefficients,

$$e_t = m_{t-1} + [(1+\gamma)/\Delta_m]\mu_t - (1/\Delta_r)\phi_t + [\gamma/(1+\gamma)]\theta_t, \tag{10}$$

where $\Delta m \equiv 1+\gamma(1-\pi)$ and $\Delta r \equiv 1+\gamma(1-\lambda)$. Note that both of these expressions become smaller in value as the coefficients of autoregression (π and λ) approach unity; the money supply follows a random walk for $\pi = 0$ and the real variables follow a random walk for $\lambda = 1$.

1/ This is the predominant mechanism in an overshooting model (Dornbusch, 1986).

FIGURE 1

MONEY MARKET (MM) AND EXCHANGE MARKET (EM) EQUILIBRIUM LOCI

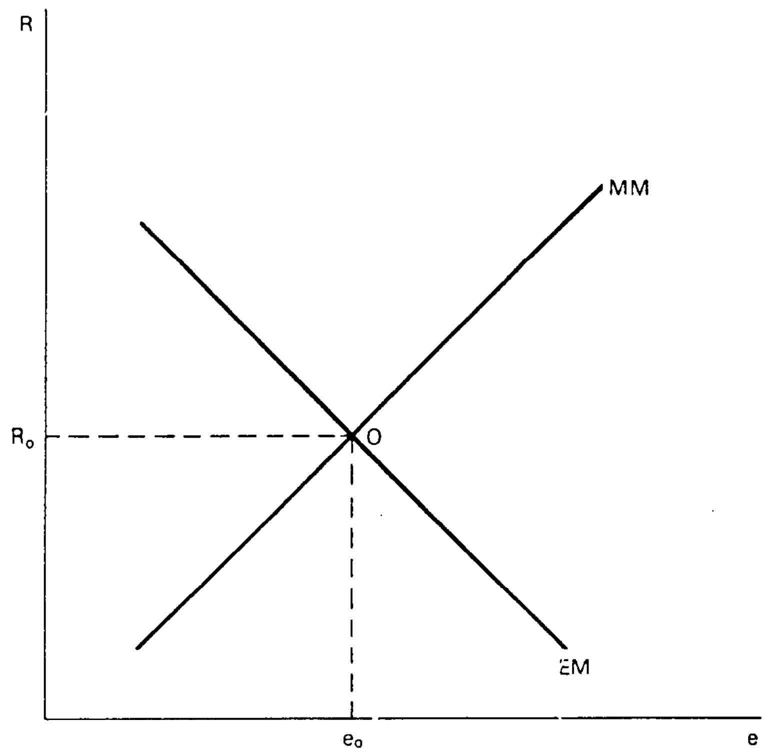
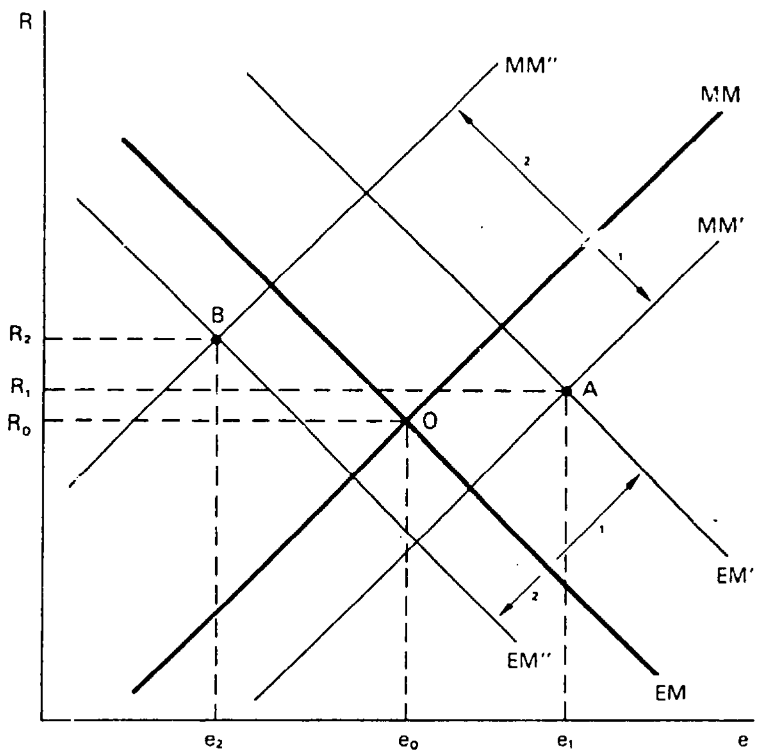


FIGURE 2

EFFECTS OF REAL AND MONETARY SHOCKS UNDER PERFECT CURRENT INFORMATION



1 Indicates the direction of shift caused by a positive monetary shock
2 Indicates the direction of shift caused by a positive real shock

In the solution for e_t, the effect of an increase in m_{t-1} depreciates the spot price of home currency one-to-one, corresponding to the neutrality of money. The same one-to-one change in e_t would result from an increase in μ_t if $\pi=0$; otherwise, the currency would depreciate more than one-to-one because an increase in μ_t would lead the agent to expect an even larger supply of money in the next period. The effect of an increase in ϕ_t would be to appreciate the currency because it implies a temporarily lower domestic price level or a temporarily higher relative price of the domestic goods.

Next, with the explicit solution for e_t, we can solve for R_t by substituting (10) into (8),

$$R_t \;=\; (\pi/\Delta_m)\mu_t + [(1-\lambda)/\Delta_r]\phi_t + [1\ /(1+\gamma)]\theta_t. \tag{11}$$

In the solution for R_t, a change in m_{t-1} would have no effect because it does not alter the future rate of change in the price level. This is a standard result in monetary theory when prices are assumed to be perfectly flexible. The effect of an increase in μ_t is to increase R_t because it implies a higher price level in the next period as long as $\pi>0$. The effect of an increase in ϕ_t is also to increase R_t because an increase in the demand for money requires a higher opportunity cost to induce the agent to hold the existing quantity of money willingly.

Finally, updating (10), we have for E_te_{t+1},

$$E_te_{t+1} \;=\; m_{t-1} + [(1+\gamma+\pi)/\Delta_m]\mu_t - (\lambda/\Delta_r)\phi_t. \tag{12}$$

With the explicit solution for E_te_{t+1}, the EM-MM diagram provides greater insight into the forces underlying the joint determination of R_t and e_t that can not be gained by a reduced form equation, such as equations (10) and (11).

As an example, the effect of a positive monetary disturbance, μ_t (or ε_{mt}) can be analyzed in the following way (see Figure 2). 1/ First, on the basis of equation (8), this disturbance will shift the MM locus downward to MM' by $-(1/\gamma)$. This is the usual liquidity effect of money that lowers the nominal interest rate. Second, on the basis of equations (5)' and (12), this disturbance will shift the EM locus upward to EM' by $[(1+\gamma+\pi)/\Delta_m]$, which is always larger than $(1/\gamma)$ in numerical terms. This shift is the offsetting expectation effect of money that increases the nominal interest rate through an expected future increase in the price level. Because the expectation effect is larger than the liquidity effect, 2/ the net result of a positive monetary disturbance

1/ Under perfect information, μ_t and ε_{mt} have the identical effect.

2/ This follows from the fact that, for $\pi=0$, a positive monetary disturbance does not alter R_t. This means that, as long as $\pi>0$, R_t must increase.

is to increase both e_t and R_t unambiguously, as implied by equations (10) and (11). This is indicated by the shift of the initial equilibrium (0) to A.

As another example, the effect of a positive real disturbance, ϕ_t (or ε_{rt}) can be analyzed in a similar fashion. 1/ First, on the basis of equation (8), this disturbance will shift the MM locus leftward to MM'' by one. Second, on the basis of equations (5)' and (12), this disturbance will shift the EM locus leftward to EM'' by $-(\lambda/\Delta_r)$, which is always smaller than one in numerical terms. The extent of this leftward shift becomes greater the more persistent the real disturbance becomes, because a greater value of λ means a smaller future increase in the price level or a smaller future deterioration in the terms of trade, hence a smaller expected depreciation. The net result of a positive real disturbance is to raise R_t and to lower e_t unambiguously, as implied by equations (10) and (11) and by the shift of the initial equilibrium (0) to B.

Finally, the time-series property of the exchange rate can be analyzed by finding the expression for the first difference in e_t,

$$\begin{aligned} De_t = {} & (\pi/\Delta_m)\mu_{t-1} + [(1-\lambda)/\Delta_r]\phi_{t-1} \\ & + [\gamma/(1+\gamma)](\theta_t-\theta_{t-1}) + [(1+\gamma)/\Delta_m]\,\varepsilon_{mt} \\ & - (1/\Delta_r)\,\varepsilon_{rt}, \end{aligned} \tag{13}$$

where D is a first difference operator. In order for De_t to approximate a white-noise process, such that e_t follows a random walk, we require that π=0 and λ=1, which guarantees that both the money supply and the real variables follow random walks. 2/ This suggests that the empirical evidence is consistent with an exchange rate generating process in which both monetary and real disturbances are permanent. 3/

IV. Solution under Imperfect Current Information

A further insight can be gained into the workings of the model by relaxing the assumption of perfect current information; thus the joint determination of e_t and R_t is analyzed under the assumption of imperfect current information, i.e., the economic agent does not directly observe the current values of ε_{mt}, ε_{rt} and θ_t, although he has knowledge of their past values. In forming the expectation of e_{t+1}, however, he is assumed to infer the current values of ε_{mt} and ε_{rt} conditional upon the observation of

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1/ Under perfect information, ϕ_t and ε_{rt} have the identical effect.

2/ Similarly, the model can be used to analyze the time-series property of the forward exchange rate (Adams and Boyer, 1986).

3/ Another theoretical possibility is that the interest elasticity of the demand for money (γ) is infinite. Adams and Boyers (1986) discuss its economic interpretation.

two price signals, $\tilde{R}_t \equiv R_t - E_{t-1}R_t$ and $\tilde{e}_t \equiv e_t - E_{t-1}e_t$, which convey information about the three unobservable variables, 1/ according to:

$$\tilde{e}_t = \psi_m \varepsilon_{mt} + \psi_r \varepsilon_{rt} + \psi_\theta \theta_t \quad \text{and} \tag{14}$$

$$\tilde{R}_t \equiv \sigma_m \varepsilon_{mt} + \sigma_r \varepsilon_{rt} + \sigma_\theta \theta_t, \tag{15}$$

where ψ's and σ's are some parameters.

Based on these signals, the conditional expectation of ε_{mt}, for example, is given by,

$$E_t(\varepsilon_{mt}|\tilde{R}_t,\tilde{e}_t) = V_{12}V_{22}^{-1}\begin{vmatrix} \tilde{R}_t \\ \tilde{e}_t \end{vmatrix}, \tag{16}$$

where $V_{12} \equiv |\mathrm{cov}(\varepsilon_{mt},\tilde{R}_t)\mathrm{cov}(\varepsilon_{mt},\tilde{e}_t)|$ and V_{22} is a variance-covariance matrix of $\tilde{R}_t$ and $\tilde{e}_t$. 2/ Expanding (16), we obtain,

$$E_t(\varepsilon_{mt}|\tilde{R}_t,\tilde{e}_t) = [\mathrm{corr}(\varepsilon_{mt},\tilde{R}_t)\mathrm{var}(\varepsilon_{mt})]\tilde{R}_t$$
$$+ [\mathrm{corr}(\varepsilon_{mt},\tilde{e}_t)\mathrm{var}(\varepsilon_{mt})]\tilde{e}_t, \tag{17}$$

where corr means a correlation coefficient and the simplifying assumption of $\mathrm{cov}(\tilde{R}_t,\tilde{e}_t)=0$ is made. What is lost in this assumption is the possible information contained in the co-movements of $\tilde{R}_t$ and $\tilde{e}_t$. 3/ Equation (17) says that the agent forms the conditional expectation of ε_{mt} by simply taking a weighted average of the two pieces of information, $\tilde{R}_t$ and $\tilde{e}_t$. Substituting (14) and (15) into (17), we obtain,

$$E_t(\varepsilon_{mt}|\tilde{R}_t,\tilde{e}_t) = x_1\varepsilon_{mt} + x_2\varepsilon_{rt} + x_3\theta_t, \tag{18}$$

where $x_1 \equiv [\mathrm{corr}(\varepsilon_{mt},\tilde{R}_t)\psi_m + \mathrm{corr}(\varepsilon_{mt},\tilde{e}_t)\sigma_m]\mathrm{var}(\varepsilon_{mt})$,

$x_2 \equiv [\mathrm{corr}(\varepsilon_{mt},\tilde{R}_t)\psi_r + \mathrm{corr}(\varepsilon_{mt},\tilde{e}_t)\sigma_r]\mathrm{var}(\varepsilon_{mt})$,

and $x_3 \equiv [\mathrm{corr}(\varepsilon_{mt},\tilde{R}_t)\psi_\theta + \mathrm{corr}(\varepsilon_{mt},\tilde{e}t)\sigma_\theta]\mathrm{var}(\varepsilon_{mt})$.

Under conditions where the economic agent attributes both signals entirely to a monetary disturbance, x_1 becomes unity and x_2 and x_3 zero, which corresponds to the solution under perfect information. This is so because attributing both signals entirely to a monetary disturbance is the correct thing to do, when the underlying disturbance is monetary.

1/ Based on p_t and R_t, the agent correctly perceives the u_t component of ϕ_t. Once u_t is perceived, however, p_t provides no independent information.

2/ Graybill (1961), p. 64. Barro (1980) uses a similar technique.

3/ This assumption does not seem to be restrictive given the uncertain sign of the empirical covariances of exchange rates and interest differentials (see Table 2).

Similarly, by following the same procedure, we obtain for the conditional expectation of ε_{rt},

$$E_t(\varepsilon_{rt}|\tilde{R}_t,\tilde{e}t) = z_1\varepsilon_{mt} + z_2\varepsilon_{rt} + z_3\theta_t, \tag{19}$$

where z_1, z_2 and z_3 are defined by the expressions analogous to x_1, x_2 and x_3, respectively. Under conditions where the economic agent attributes both signals entirely to a real disturbance, z_2 becomes unity and z_1 and z_3 zero, which corresponds to the solution under perfect information. This is so because attributing both signals entirely to a real disturbance is the correct thing to do when the underlying disturbance is real.

By making use of equations (9), (18) and (19), the solution for e_t can be obtained under imperfect information as,

$$\begin{aligned} e_t = \; & m_{t-1} + [(1+\gamma)/\Delta_m]\pi\mu_{t-1} - [1/\Delta_r]\lambda\phi_{t-1} + c_m\varepsilon_{mt} \\ & + c_r\varepsilon_{rt} + c_\theta\theta_t, \end{aligned} \tag{20}$$

where $c_m \equiv [1/(1+\gamma)]\{[\gamma(1+\gamma+\pi)/\Delta_m]x_1 - (\gamma\lambda/\Delta_r)z_1 + 1\}$,

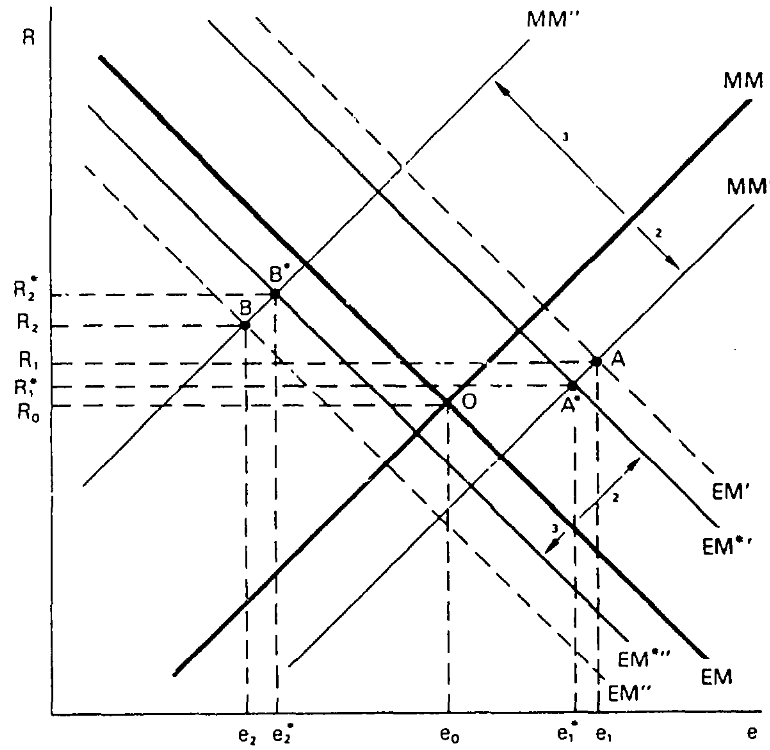
$c_r \equiv [1/(1+\gamma)]\{[\gamma(1+\gamma+\pi)/\Delta_m]x_2 - (\gamma\lambda/\Delta_r)z_2 - 1\}$,

and $c_\theta \equiv [\gamma/(1+\gamma)]\{[(1+\gamma+\pi)/\Delta_m]x_3 - (\lambda/\Delta_r)z_3 + 1\}$.

In the solution for e_t under imperfect information, we note that the coefficients of the anticipated variables, m_{t-1}, $\pi\mu_{t-1}$ and $\lambda\phi_{t-1}$ are identical to the coefficients of m_{t-1}, μ_t and ϕ_t under perfect information, respectively. The coefficient of an unanticipated monetary disturbance, ε_{mt}, converges to the full information solution, as x_1 approaches unity and z_1 approaches zero. As long as x_1 is less than unity and z_1 greater than zero, however, the coefficient of the unanticipated monetary disturbance is algebraically smaller under imperfect information than under perfect information. This follows from the fact that the future change in the money supply (when $\pi\neq0$) would be underestimated by the agent who does not perceive part of ε_{mt}. The coefficient of an unanticipated real disturbance, ε_{rt}, converges to the full information solution, as x_2 approaches zero and z_2 approaches unity. As long as x_2 is greater than zero and z_2 is less than unity, however, the coefficient of the unanticipated real disturbance is algebraically greater under imperfect information. This follows from the fact that the smaller future depreciation that would result from an increase in ϕ_t (when $\lambda\neq0$) is discounted by the agent who does not perceive part of ϕ_t. Finally, the coefficient of the deviation from interest rate parity converges to the full information solution as both x_3 and z_3 approach zero. As a rough measure, the coefficient under imperfect information is greater if the deviation is perceived more as a monetary disturbance, and smaller if it is perceived more as a real disturbance.

FIGURE 3

EFFECTS OF REAL AND MONETARY SHOCKS UNDER IMPERFECT CURRENT INFORMATION[1]



[1] The variables with asterisks refer to the solutions under imperfect information, the shifts of MM are identical under perfect and imperfect information for both types of shocks.
[2] Indicates the direction of shift caused by a positive monetary shock.
[3] Indicates the direction of shift caused by a positive real shock

Substituting (20) into (8), we have for R_t,

$$R_t = (\pi/\Delta_m)\pi\mu_{t-1} + [(1-\lambda)/\Delta_r]\lambda\phi_{t-1} + q_m\varepsilon_{mt} + q_r\varepsilon_{rt} + q_\theta\theta_t, \tag{21}$$

where $q_m \equiv [1/(1+\gamma)]\{[(1+\gamma+\pi)/\Delta_m]x_1 - (\lambda/\Delta_r)z_1 - 1\}$,

$q_r \equiv [1/(1+\gamma)]\{[(1+\gamma+\pi)/\Delta_m]x_2 - (\lambda/\Delta_r)z_2 + 1\}$,

and $q_\theta \equiv [1/(1+\gamma)]\{[(1+\gamma+\pi)/\Delta_m]x_3 - (\lambda/\Delta_r)z_3 + 1\}$.

In the solution for R_t under imperfect information, we note that the coefficients of the anticipated variables are identical to those under perfect information. The coefficient of the unanticipated monetary disturbance, which converges to the full information solution as x_1 approaches unity and z_1 zero, is algebraically smaller under imperfect information. The coefficient of the unanticipated real disturbance, which converges to the full information solution as x_2 approaches zero and z_2 unity, is algebraically larger under imperfect information. Finally, as a rough measure, the coefficient of the deviation from interest parity, which converges to the full information solution as x_3 and z_3 approach zero, will be greater under imperfect information if the deviation is perceived more as a monetary disturbance; the coefficient will be smaller if it is perceived more as a real disturbance.

Finally, updating (20), we have for E_te_{t+1},

$$E_te_{t+1} = m_{t-1} + [(1+\gamma+\pi)/\Delta_m]\pi\mu_{t-1} - (\lambda/\Delta_r)\lambda\phi_{t-1} + s_m\varepsilon_{mt} + s_r\varepsilon_{rt} + s_\theta\theta_t, \tag{22}$$

where $s_m \equiv \{[(1+\gamma+\pi)/\Delta_m]x_1 - (\lambda/\Delta_r)z_1\}$,

$s_r \equiv \{[(1+\gamma+\pi)/\Delta_m]x_2 - (\lambda/\Delta_r)z_2\}$,

and $s_\theta \equiv \{[(1+\gamma+\pi)/\Delta_m]x_3 - (\lambda/\Delta_r)z_3\}$.

With the explicit solution for E_te_{t+1}, the EM-MM diagram provides greater insight into the forces underlying the joint determination of e_t and R_t under imperfect information.

As an example, the effect of a positive monetary disturbance, ε_{mt}, can be analyzed in the following way (see Figure 3). 1/ First, on the basis of equation (8), this disturbance will shift the MM locus downward

1/ Under imperfect information, μ_t can be divided into the anticipated component ($\pi\mu_{t-1}$) and the unanticipated component (ε_{mt}). The effect of a change in the anticipated component is the same as the effect of a change in μ_t under perfect information.

to MM' by $-(1/\gamma)$. The magnitude of the shift of the MM locus is the same as under perfect information. Second, on the basis of equations (5)' and (22), this disturbance will shift the EM locus upward to EM*' by s_m, which is always smaller than the magnitude of the vertical shift (to EM') under perfect information given by $[(1+\gamma+\pi)/\Delta_m]$. Therefore, the effect of imperfect information works only through the economic agent's expectation of the future exchange rate. Unlike the perfect information case, the net result of a positive monetary disturbance on R_t under imperfect information becomes ambiguous, because the effect of a monetary disturbance on the future price level is underestimated. It is even possible for a depreciation of the exchange rate, which is caused by an increase in ε_{mt}, to be entirely perceived to be caused by a fall in ε_{rt}, such that a positive monetary disturbance results in an expected fall in the price level. The new equilibrium A^* in Figure 3 is drawn for the case where the monetary disturbance is sufficiently perceived as monetary, so that the interest rate rises.

As another example, the effect of a positive real disturbance, ε_{rt} can be analyzed in a similar fashion. 1/ First, on the basis of equation (8), this disturbance will shift the MM locus leftward to MM' by one as in the case of perfect information. Second, on the basis of equations (5)' and (22), this disturbance will shift the EM locus by s_r (to EM*''), which is always algebraically larger than the magnitude of the shift under perfect information given by $-(\lambda/\Delta_r)$; the shift of the EM locus may or may not be leftward. The shift of EM in Figure 3 is drawn for the case where a large enough portion of the real disturbance is correctly perceived, such that the locus shifts to the left. It can be verified that, as long as the exchange rate appreciates in response to a real disturbance, 2/ its net effect on R_t is unambiguously positive.

Finally, the time-series property of the exchange rate can be analyzed by finding the expression for De_t,

$$De_t = (\pi^2/\Delta_m)\ \mu_{t-2} + [\lambda(1-\lambda)/\Delta_r]\ \phi_{t-2} + \{[(1+\gamma+\pi)/\Delta_m]-c_m\}\ \varepsilon_{mt-1}$$
$$- [(\lambda/\Delta_r)+c_r]\ \varepsilon_{rt-1} + c_m\ \varepsilon_{mt} + c_r\ \varepsilon_{rt} + c_\theta\ (\theta_t-\theta_{t-1}). \quad (23)$$

As in the case of perfect information, a random walk of the exchange rate requires that both monetary and real disturbances be permanent. Thus, the introduction of imperfect information does not change the fundamental implication of the empirical regularity for the stochastic nature of the underlying disturbances.

1/ Under imperfect information, ϕ_t can be divided into the anticipated component $(\lambda\phi_{t-1})$ and the unanticipated component (ε_{rt}). The effect of a change in the anticipated component is the same as the effect of a change in ϕ_t under perfect information.

2/ This is a sufficient, but not a necessary, condition for this result to hold.

V. The Covariation of the Exchange Rate and the Interest Rate

If it is indeed the case that $\pi=0$ and $\lambda=1$, the covariance of e_t and R_t will become simply $[\gamma/(1+\gamma)^2](X_3-Z_3+1)^2 var(\theta)>0$. If the variance of θ is small, no statistically significant relationship between e_t and R_t is likely to be found. In fact, in calculating the covariances of the exchange rates and the interest differentials among the four major countries during 1978-86, more than half (13 out of 24) of the calculated covariances were found to be statistically insignificant in terms of the implied correlation coefficients, consistent with the random walk nature of the underlying exchange rates. At the same time, the presence of statistically significant covariances in the others is somewhat inconsistent with the strictly random walk nature of the exchange rates. This may mean either that the random walk tests may not be powerful enough to detect the presence of a small--yet statistically significant--deviation from a random walk, or that a monetary model of the kind presented in the paper is not appropriate.

Assuming $\pi\neq0$ and $\lambda\neq1$, the covariance of e_t and R_t will change in sign depending on the strength of the underlying real or monetary disturbances. On the one hand, under perfect current information, e_t and R_t will unambiguously move in opposite directions for a real disturbance, while they will move in the same direction for a monetary disturbance. 1/ On the other hand, under imperfect current information, the signs of the covariances that result from real and monetary disturbances will become ambiguous, although they are likely to remain the same as under perfect current information. This ambiguity can be demonstrated by expressing the solutions for e_t and R_t in the following general forms,

$$e_t = m_{t-1} + b_m\pi\mu_{t-1} + c_m\varepsilon_{mt} + b_r\lambda\phi_{t-1} + c_r\varepsilon_{rt} + c_\theta\theta_t, \tag{24}$$

$$R_t = d_m\pi\mu_{t-1} + q_m\varepsilon_{mt} + d_r\lambda\phi_{t-1} + q_r\varepsilon_{rt} + q_\theta\theta_t, \tag{25}$$

where, under perfect information, $c_m=b_m\equiv[(1+\gamma)/\Delta_m]>0$, $c_r=b_r\equiv-(1/\Delta_r)<0$, $q_m=d_m\equiv(\pi/\Delta_m)>0$, and $q_r=d_r\equiv[(1-\lambda)/\Delta_r]>0$; and, under imperfect information, $c_m<b_m$, $c_r>b_r$, $q_m<d_m$ and $q_r>d_r$. The covariance of e_t and R_t can be expressed as,

$$\begin{aligned} Cov(e_t,R_t) = {} & c_\theta q_\theta\ var(\theta_t) + \pi^2 b_m d_m\ var(\mu_{t-1}) + c_m q_m\ var(\varepsilon_{mt}) \\ & + \lambda^2 b_r d_r\ var(\phi_{t-1}) + c_r q_r\ var(\varepsilon_{rt}) \\ & + \lambda\pi(b_m d_r + b_r d_m)\ cov(\pi_{t-1}, \phi_{t-1}) \\ & + (c_m q_r + c_r q_m)\ cov(\varepsilon_{mt}, \varepsilon_{rt}), \end{aligned} \tag{26}$$

1/ Engel (1986) has argued that the covariance of e_t and R_t can become negative even in a model with only monetary disturbances. However, his result is based on the covariance in first difference and does not follow when the covariance in level is considered. It can be verified that, for monetary disturbances, $cov(e_t, R_t)>cov(De_t,DR_t)$.

where the assumption that ε_m and ε_r are uncorrelated is relaxed in the light of the empirically observed cyclical co-movement of monetary and real variables, and the two covariance terms are assumed to be non-negative. It can be immediately verified that the algebraic value of the covariance decreases as the autoregressive coefficient of the monetary disturbance becomes smaller and the autoregressive coefficient of the real disturbance becomes greater.

In the second row in equation (26), which represents the covariance resulting from monetary disturbances, the algebraic value of $c_m q_m$ is unambiguously smaller under imperfect information; it can even become negative if $z_1 > x_1$ (i.e., a monetary disturbance is perceived more as real than as monetary) and the interest elasticity of the demand for money (γ) is sufficiently small. In the third row in (26), which represents the covariance resulting from real disturbances, the algebraic value of $c_r q_r$ is unambiguously larger under perfect information; it can even become positive if $x_2 > z_2$ (i.e., a real disturbance is perceived more as monetary than as real) and γ is sufficiently large. Thus, the effect of imperfect information is to decrease the algebraic value of the covariance of e_t and R_t resulting from monetary disturbances and to increase the value resulting from real disturbances.

Thus, in general, a positive covariance of e_t and R_t suggests an economic environment in which monetary disturbances are relatively more dominant in the sense that the variances of monetary disturbances are large and a relatively large portion of both real and monetary disturbances is perceived as monetary under imperfect information. On the other hand, a negative covariance suggests an economic environment in which real disturbances are relatively more dominant in the sense that the variances of real disturbances are large and a relatively large portion of both real and monetary disturbances is perceived as real under imperfect information. These results would not be substantively altered if the effects of the covariance terms and the deviation from interest rate parity on the right hand side of equation (26) were taken into account: in the first covariance term, the effects of $b_m d_r$ and $b_r d_m$ are offsetting, so that the net effect is likely to be small; and, in the second covariance term and the variance of θ_t, the algebraic values of $(c_m q_r + c_r q_m)$ and $c_\theta q_\theta$ decrease as z_1, z_2 and z_3 increase relative to x_1, x_2 and x_3.

VI. Empirical Covariations of Exchange Rates and Interest Differentials

As an illustrative application, the model can be used to see what kind of inferences can be made from the empirical covariances of exchange rates and interest differentials about the strength of the underlying real and monetary variables. However, a few words of caution are in order. First, the relationship based on the small country assumption of the model may not be strictly applicable to the relationship involving two large countries. To the extent that the key behavioral parameters are similar across two countries, however, the basic qualitative implications of the model for the domestic variables are expected to hold for the same variables expressed as a difference between the foreign and domestic values. Second, the model has not allowed for the possibility of sticky prices and hence real interest rate changes. In the models of Dornbusch (1976) and Frankel (1979), for example, the assumption of sticky prices places the entire burden of short-run money market adjustment on the real interest rate. 1/ Moreover, Meyer and Startz (1982) have recently presented some evidence that real interest rate disturbances are an important component of exchange rate determination. Third, the model has not explicitly taken into consideration real trade variables that may be important in determining the real exchange rate (Dornbusch, 1980; Hooper and Morton, 1982). In a model where a current account is incorporated into analysis, for example, a current account disturbance can influence not only the real exchange rate through the goods market clearing condition but also the risk premium through portfolio adjustments. 2/ Finally, what the model strictly implies is the signs of conditional covariances and not of unconditional covariances that can be observed empirically.

Against this background, the covariances of exchange rates and interest differentials among the four major countries were calculated for the period 1978-86 (Table 2). While Table 2 indicates that the signs of the covariances can be negative, positive or insignificant, depending on the sample period and the country pair, two interesting patterns become apparent. First, the consistently positive covariances observed for the U.K. pound/deutsche mark rate suggest the predominance of monetary factors in the determination of the exchange rate throughout the sample period. This may be indicative of the fact that Germany had a substantially lower rate of monetary expansion than the United Kingdom throughout the period and that this difference in monetary growth between the two countries was reflected in a large variance of monetary variables.

Second, the consistently negative covariances observed for the bilateral exchange rates of the U.S. dollar against the other currencies during the most recent sample period suggest the predominance of real factors. This is consistent with the large movements of the U.S. dollar exchange rates that were correlated with the differences in real economic activity as well as with the large PPP deviations that were observed

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1/ The adjustment path implied by such a model (i.e., the so-called overshooting phenomenon), along which the initial exchange rate depreciation and interest rate decline are followed by a subsequent exchange rate appreciation and interest rate rise, is also consistent with a negative covariance of the exchange rate and the interest rate.

2/ Frankel (1982) has presented evidence, however, that the effect of changes in the relative supplies of foreign and domestic bonds on the risk premium may be insignificant.

Table 2. Covariances of Exchange Rates and Nominal Interest Differentials, 1978-86.

sample period	U.S. dollar/ U.K. pound	U.S. dollar/ Deutsche mark	U.S. dollar/ Japanese yen	U.K. pound/ Deutsche mark	U.K. pound/ Japanese yen	Deutsche mark/ Japanese yen
January 1978- December 1980	-7.04**	-2.65	2.05	1.50	-2.95	-5.34*
January 1981- December 1983	14.87**	0.77	6.65*	7.46**	-7.61**	-3.77
January 1984- May 1986	-0.65	-0.91	-6.53*	2.87	5.08*	0.15
January 1978- May 1986	2.80	3.25	-1.32	8.85**	-5.74	-2.78

(Scale: 1/10,000)

Note: ** (*) indicates that the implied correlation coefficient is significant at 1 (5) percent.

between the U.S. and the other countries. Moreover, it suggests an economic environment characterized by uncertainty in which, because the predominance of real factors had been incorporated into the expectations formation process, even some of monetary factors were misperceived as real.

VII. A Summary and Concluding Remarks

The simple model presented in this paper has suggested that the near random walk property of the observed exchange rate behavior is not necessarily an implication of market efficiency but rather a reflection of a particular stochastic structure of the economic environment under which the world economy has operated during recent years. In particular, the model has shown that the random walk of the exchange rate is consistent with an economic structure in which innovations in both the money supply and real variables are permanent, such that both real and monetary variables follow random walks.

Moreover, the paper has argued that the information contained in the covariance of the exchange rate and the interest rate, as two endogenous variables that respond to the same disturbances, can be used to infer the strength of real and monetary factors underlying the process of exchange rate determination. As illustrations, the model was used to make inferences that monetary factors dominated the determination of the U.K. pound/deutsche mark exchange rate and that real factors dominated the determination of the U.S. dollar exchange rates during the most recent period. Moreover, the model suggested that imperfect information might be an important factor characterizing the environment in which the market's expectations of the future exchange rate are formed.

There remains, however, an unresolved paradox: the random walk behavior of exchange rates is inconsistent with the presence of statistically significant correlations between exchange rates and interest rates. This paradox is made more intuitive by noting that, if both the spot and forward exchange rates follow random walks, covered interest rate parity implies that the interest rate diffential follows a random walk--an empirical regularity that has not been consistently observed. This may mean that the random walk tests are not powerful enough to detect a small--yet significant--deviation from random walks.

Alternatively, it is possible that a simple monetary model of the kind presented in the paper may not be an adequate representaion of the exchange rate determination process. While the strength of the model lies in its quite general structure and its ability to allow inferences without requiring the arbitrary choice of a price index, the limitation of the model lies in its focus on the relationship between the nominal exchange rate and the nominal interest rate to the exclusion of the relationship between the real exchange rate and the real interest rate (Meese and Rogoff, 1985). Thus, it would be of interest to extend the analysis by incorporating the mechanism by which the real exchange rate and the real interest rate are determined through current accounts, sticky prices or other factors that were not explicitly considered in the paper.

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