IMF WORKING PAPER

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WP/90/ 87

INTERNATIONAL MONETARY FUND

Research Department

The Optimal Mix of Inflationary Finance
and Commodity Taxation with Collection Lags

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September 1990

Abstract

When there are collection lags in the tax system, inflation reduces the real revenues. This is often offered as an argument for less reliance on the inflation tax. But the optimal rates of other taxes should also be reconsidered in the light of collection lags. When this is done, the focus shifts from the revenues (which can be recouped by changing the rates of these taxes), to the associated costs of collection. In a benchmark case where the average costs of collection are constant, the optimal inflation tax is independent of the collection lag.

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* Professor of Economics, Princeton University, and Visiting Scholar, International Monetary Fund, June-July 1990. I thank Matthew Canzoneri and Nurun Choudhry for useful conversations, and Pablo Guidotti, Lars Svensson, and Carlos Végh for valuable comments on the first draft. The paper does not necessarily represent any views of the Fund.

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Summary

When there is a lag between the accrual and the payment of taxes, inflation erodes the real value of the government's revenues. This insight was developed, and its importance for less developed countries was stressed, by Tanzi (1977). Subsequent analysis concludes that inflation is even more costly than was previously believed. When there are collection lags, inflationary finance has a twofold harmful effect on the economy. Not only does it drive a wedge between the marginal benefit and the marginal cost of real balances, but it also reduces the revenue from the rest of the tax system. The inference is that inflationary finance should be used less than would be indicated by an analysis that ignored collection lags.

But this analysis is incomplete. This paper therefore expands the analysis to ask how the whole tax system should best respond to the existence of the lags. The question becomes whether less reliance on inflationary finance is indicated even when the rates of the rest of the taxes are reset optimally taking the lags into account.

The analysis is cast in a formal intertemporal model, where the distortion cost of inflation is balanced against the collection cost of consumption taxes to find the optimal mix of these two forms of finance. A collection lag is added to the analysis.

The results do not support the intuition of the earlier incomplete analysis. If a collection lag reduces the real value of the revenue from the consumption tax at its old rate, the rate can be changed to ensure the same real revenue as before. This is formally equivalent to charging interest on the taxes accrued for the period of the collection lag. The question is whether doing so, and leaving the inflation tax at the rate found optimal in the model without collection lags, is the optimal policy.

The answer depends on what happens to the collection cost of the consumption tax as a result of the lag. If the average cost of collection is constant, then the policy of leaving the inflation tax unchanged and raising the rate of consumption tax to restore the real revenue is indeed optimal.

I. Introduction

When there is a lag between the accrual and the payment of taxes, inflation erodes the real value of the government's revenues. This insight was developed, and its importance for less developed countries stressed, by Tanzi (1977). This has since been developed by Tanzi (1978) and many others, including most recently Choudhry (1990). The argument runs as follows. Inflationary finance has a twofold harmful effect on the economy when there are collection lags. Not only does it generate the usual dead-weight loss by driving a wedge between the marginal benefit and cost of real balances, it also reduces the revenue from other kinds of taxes. Therefore, the marginal cost of using inflationary finance is higher when we take collection lags into account. The inference is that inflationary finance should be used less than would be indicated by an analysis that ignores collections lags.

But this is a partial equilibrium insight. It examines the dead-weight burden of inflationary finance at given rates of other kinds of taxes. When we recognize that collection lags exist, we should expand the analysis to consider the effect of these lags on the optimal mix of the inflation tax and other taxes. Since the marginal cost of the inflation tax rises at given rates of other taxes. it becomes desirable to increase the rates of these other taxes. In the process, equilibrium prices in the economy will also change. The question is whether less reliance on inflationary finance is indicated even when all such responses are taken into account and the new optimal mix is established.

The optimal mix of inflation and commodity taxation was examined by Phelps (1973), and his analysis was extended and refined by several others, including Aghevli (1977), Kimbrough (1986), Frenkel (1987), and Végh (1989). A positive inflation tax can be optimal only if other feasible taxes have distortions of their own, and many of the above papers model these as "collection costs." But they do not consider lags, so the Tanzi link between inflation and other taxes is missing. In this paper I shall modify the latest of these models, that of Végh, to incorporate such lags.

The results do not support the partial equilibrium intuition stated above. In a benchmark case where the average cost of collection of commodity taxes is constant, I find that the optimal inflation tax is independent of the length of the lag in the collection of commodity taxes. The optimal rate of commodity taxation takes the full burden of adjustment to offset the revenue loss. This is as if commodity taxation were indexed for inflation, a solution that is indeed adopted in many high-inflation countries. An equivalent procedure is to charge interest for the period of the collection lag on the taxes due.

More generally, if the average cost of collecting commodity taxes is a stable and rising function of the real revenue, then a longer collection lag implies a higher optimal inflation tax rate. This runs counter to the intuition outlined above. But the revenue erosion can be made good by

changing the rates of commodity taxes. It is only to the extent that the adjustment of rates changes the cost of collection of those taxes, that the optimal rate of inflation is altered. If the inflation tax is kept unchanged and an indexation or interest scheme adopted for commodity taxes, a longer lag now makes it more costly to collect the commodity tax. There-fore, it becomes optimal to shift some of the tax burden toward the infla-tion tax. Correspondingly, there should be less than full indexation of commodity taxes, or less than full interest charged for the duration of the collection lag.

I should emphasize that the analysis does not negate the importance of collection lags, or the erosion of tax revenues due to inflation when such lags are present. But it does suggest that the optimal policy response to the recognition of these issues is in general not what the earlier partial intuition had led economists to believe.

II. Végh's Model with Collection Lag

Most of Végh's notation will be retained, but the notation for interest and inflation rates will be generalized somewhat. Let π_t denote the infla-tion rate between periods t and (t+1), and $\Pi(t,s)$ the cumulative price multiple between periods t and s, that is

$$\Pi(t,s) = (1+\pi_t)(1+\pi_{t+1}) \ldots (1+\pi_{s-1}).$$

Similarly, i_t is the nominal interest rate and $I(t,s)$ the cumulative nominal interest multiple, and r_t the real rate of interest and $R(t,s)$ the corresponding cumulative real interest multiple.

These satisfy

$$I(t,s) = R(t,s) \, \Pi(t,s) \qquad \text{for all } t,s$$

and

$$I(t_1,t_3) = I(t_1,t_2) \, I(t_2,t_3) \qquad \text{for all } t_1 < t_2 < t_3$$

and similarly for R,Π.

I will assume that the whole profile of real interest multiples $R(t,s)$ is exogenous, and is equal to the reciprocals of the corresponding profile of the consumer's utility discount factors. As in Végh, this is done to eliminate intrinsic dynamics and allow the model to settle down to a steady state at once.

In a steady state, where the real interest rate is constant and equal to r, the nominal interest rate constant and equal to i, and the inflation rate constant and equal to π, we have

$$R(0,t) = (1+r)^t, \quad I(0,t) = (1+i)^t, \quad \Pi(0,t) = (1+\pi)^t.$$

There is one perishable consumption good produced in each period using labor. It takes a unit of labor to produce a unit of the good. Let P_t denote the price of the good relative to labor in period t. Then, taking money in period t. Then, taking money in period 0 as the unit of account, the nominal prices in period t are

$\Pi(0,t)$ for labor, $P_t \Pi(0,t)$ for consumption.

There is a tax on period-t consumption at rate θ_t. If the quantity of consumption at time t is c_t, then the nominal tax accrued at time t is $\theta_t c_t P_t \Pi(0,t)$. This amount is paid by the consumer to the firm at the time of purchase, but turned over by the firm to the government at time (t+k). Thus, k is the collection lag. It is assumed that the lag is fixed and known, as in the case of VAT payments made at fixed dates.

Now the firm's profit from producing and selling one unit of the consumption good at time t is

$$\frac{(1+\theta_t)P_t\Pi(0,t)}{I(0,t)} - \frac{\theta_t P_t \Pi(0,t)}{I(0,t+k)} \cdot \frac{\Pi(0,t)}{I(0,t)}$$

Competition will reduce this to zero. Simplifying the resulting equation, we find

$$(1+\theta_t) P_t - \theta_t P_t/I(t,t+k) - 1 = 0,$$

or

$$P_t = \frac{1}{1+\theta_t - \theta_t/I(t,t+k)} . \tag{1}$$

We can then define an 'effective' tax rate θ'_t by

$$1 + \theta'_t = (1+\theta_t) P_t, \tag{2}$$

or using (1),

$$\theta'_t = \frac{\theta_t/I(t,t+k)}{1+\theta_t - \theta_t/I(t,t+k)} \tag{3}$$

We will see that all the consumer's decisions are made as if, instead of the tax rate θ_t and the price P_t, he faces the tax rate θ'_t and price $P'_t = 1$. When $\theta_t > 0$ and $I(t,t+k) > 1$, we have $\theta'_t < \theta_t$. In other words, the collection lag acts as if consumers face an effective rate of taxation on consumption lower than the announced rate.

This is the first of the general equilibrium effects of the collection lag we must consider. As the collection lag reduces the real tax revenues of the government, it gives the firms the benefit of holding on to these revenues for k periods. In equilibrium, competitive firms must pass on this benefit to consumers in the form of lower prices. If we had assumed that consumers directly pay the tax revenue, then they would benefit directly, and again would face a lower effective tax rate.

Next consider the consumer's transaction technology. If he holds an amount of money M_t, and spends a nominal amount E_t, or a real amount $e_t = E_t \Pi(0,t)$, it is assumed that the transaction requires time

$$s_t = \nu(M_t/E_t)\, e_t.$$

Observing that

$$E_t = (1+\theta_t)\, c_t P_t \Pi(0,t),$$

using (2), and defining real balances $m_t = M_t/\Pi(0,t)$, we have

$$s_t = \nu \left[\frac{m_t}{(1+\theta'_t)c_t} \right] (1+\theta'_t)c_t. \tag{4}$$

As in Végh, I shall write the argument of ν as X_t for brevity. The function is defined over the range $[0, X_s]$, where X_s corresponds to the saturation level of real balances. The function satisfies $\nu(X_s) = 0$, and $\nu'(X) < 0$, $\nu''(X) > 0$ for all X in $[0, X_s]$.

Suppose the consumer has an initial endowment of b_0 units of account, and an endowment of a unit of time each period. Let h_t denote his leisure. Define

$$\gamma_t = i_t/(1+i_t) = 1 - I(0,t)/I(0,t+1), \tag{5}$$

the rate of tax on the consumer's real balances, that is, the inflation tax. (Végh's symbol for my γ is I; my I is the cumulative interest multiple.) With all this notation, the consumer's budget constraint is

$$b_0 + \sum_{t=0}^{\infty} R(0,t)^{-1} (1-h_t)$$

$$- \sum_{t=0}^{\infty} R(0,t)^{-1} [(1+\theta_t)c_t P_t + s_t + \gamma_t m_t]$$

$$- \sum_{t=0}^{\infty} R(0,t)^{-1} [(1+\theta_t')c_t + s_t + \gamma_t m_t]. \tag{6}$$

The s_t in (6) is given by (4).

Subject to (6), the consumer chooses the sequence (c_t, h_t, m_t) to maximize

$$\sum_{t=0}^{\infty} R(0,t)^{-1} U(c_t, h_t). \tag{7}$$

This problem is exactly the same as in Végh (1989), except that θ_t is replaced by θ_t'. Therefore, I shall not be given any details, but merely note it defines the equilibrium price functions

$$1 + \theta_t' - y^\theta(c_t, h_t, m_t) = \frac{U_c(c_t, h_t)}{U_h(c_t, h_t)} [1 + \nu(X_t) - X_t \nu'(X_t)^{-1}], \tag{8}$$

and

$$\gamma_t - y^I(c_t, h_t, m_t) = -\nu'(X_t). \tag{9}$$

Now turn to the government's problem. Assume that if it receives T_t units of account at time t, its collection costs are $\phi(T_t/A(0,t))T_t$, where the average cost function is ϕ. The function $A(0,t)$ represents techno-logical progress in collection. Thus $A(0,t+1)/A(0,t)-1$ is the rate of improvement in the collection technology between periods t and (t+1). This is a very general formulation, which seems appropriate since we have no precise theory of the nature of these costs. At one extreme, one might think the cost consists literally of counting nominal cash. Then there is zero technical progress, and $A(0,t)$ is constant. At another extreme, if auditing is costly but the cost is independent of the size of the audit, then the technology improves in pace with the nominal interest rate. More interestingly, Végh defines collection costs as a function of the real revenues. In my notation, this amounts to assuming that the nominal collection technology exactly keeps pace with inflation, or $A(0,t) - \Pi(0,t)$ for all t.

We know that $T_{t+k} = \theta_t c_t P_t \Pi(0,t)$. Therefore, the discounted present value of the government's receipts at time (t+k) is

$$T_{t+k}/I(0,t+k) = \theta_t c_t P_t \Pi(0,t)/[\Pi(0,t)R(0,t)I(t,t+k)]$$

$$= \theta_t' c_t/R(0,t).$$

The discounted present value of the collection costs at (t+k) is

$$\phi(T_{t+k}/A(0,t+k))T_{t+k}/(0,t+k) = \phi\left(\frac{\theta_t c_t P_t \Pi(0,t)}{A(0,t+k)}\right)\theta_t c_t P_t \Pi(0,t)/I(0,t+k)$$

$$= \phi\left(\theta_t' c_t \frac{I(t,t+k)\Pi(0,t)}{A(0,t+k)}\right)\theta_t' c_t/R(0,t).$$

For brevity, write

$$B_{t,k} = I(t,t+k)\Pi(0,t)/A(0,t+k). \tag{10}$$

The government purchases exogenously specified quantities (g_t) of the good through time. Then the government's budget constraint becomes

$$\sum_{t=0}^{\infty} R(0,t)^{-1}g_t = \sum_{t=0}^{\infty} R(0,t)^{-1}\{[1-\phi(\theta_t' c_t B_{t,k})]\theta_t' c_t + \gamma_t m_t\}, \tag{11}$$

and the economy's overall resource constraint is

$$b_0 + \sum_{t=0}^{\infty} R(0,t)^{-1} (1-h_t)$$

$$= \sum_{t=0}^{\infty} R(0,t)^{-1}\{[1+\phi(\theta_t' c_t B_{t,k})\theta_t']c_t + g_t + \nu(X_t)c_t(1+\theta_t')\}. \tag{12}$$

The government can be regarded as choosing (c_t, h_t, m_t) subject to these two constraints and the equilibrium price functions, to maximize the utility sum in (7). The policy is assumed to be committed in advance; thus issues of time-inconsistency are not addressed.

III. The Invariance Result

The collection lag k enters this problem only through $B_{t,k}$, which is an argument of the average collection cost function ϕ. Therefore, the simplest benchmark cases to consider are the ones where the value of ϕ is not affected by its argument, and where $B_{t,k}$ does not depend on k. In either of these cases, the solution to the government's optimization problem determines γ_t and θ'_t independently of k. If k changes, the actual tax rates on consumption, θ_t, will be adjusted to maintain the same values of the effective rates θ'_t as before. Inverting (1), we get

$$\theta_t = \frac{\theta'_t I(t,t+k)}{1-\theta'_t[I(t,t+k-1]} \tag{13}$$

This can be seen as a kind of indexation of the tax system.

A better interpretation is that interest is charged on accrued tax payments over the period of the collection lag. A firm that collects one unit of account from the consumer at time t must pay $I(t,t+k)$ units to the government at time $(t+k)$. Let P_t denote the price of the consumer good relative to labor under this system. Now the firm's profit from producing and selling a unit of output at time t is

$$\frac{(1+\theta'_t)F'\Pi(0,t)}{I(0,t)} - \frac{\theta'_t P'_t \Pi(0,t)I(t,t+k)}{I(0,t+k)} - \frac{\Pi(0,t)}{I(0,t)} \cdot$$

Setting this equal to zero for a competitive equilibrium, we get $P'_t = 1$. When I defined the effective rate θ'_t, I said that it could be interpreted as if the consumer faced this rate and a price equal to one. The procedure of collecting interest on the taxes makes that _as if_ into a reality.

Under either system, the same inflation rate as before will remain optimal as the collection lag changes, and the consumption tax will be adjusted to offset the effect on the revenue.

We saw two kinds of circumstances where k does not affect $B_{t,k}$ and therefore the optimal values of θ'_t and r_t: (1) The average cost of collection is constant, that is $\phi(z) = \phi_0$, constant, for all values of the argument z. This is also the benchmark case considered by Végh. (2) The rate of progress in the collection technology always equals the nominal rate of interest, so $I(t,t+k)/A(0,t+k)$ and therefore $B_{t,k}$ is independent of k. This seems an exceptional or 'razor's edge' case, but it helps us develop the intuition behind the result.

As k changes, suppose we hold θ'_t and γ_t at their old optimal values, but charge interest on tax revenues held by firms for the longer collection lag. Then the amount collected goes up at the rate of interest. But if the collection technology improves exactly in step, the cost of collecting each unit of revenue is unchanged, and therefore so is the present value of the collection costs. Then the optimality conditions are unaffected, so it remains optimal to keep θ'_t and γ_t unchanged.

IV. The General Case

When the average cost of collection ϕ is an increasing function, and k affects $B_{t,k}$, the length of the collection lag does alter the optimal inflation tax rate. To examine this in more detail, I specialize the analysis to a steady state, as does Végh at the corresponding point in his paper. Now the time subscripts can be dropped from all the variables, and a constant capitalization factor $(1+r)/r$ dropped from the present value constraints. I follow Végh in setting $b_0 = 0$ and choosing the utility function

$$U(c,h) = \ln c + \ln h; \tag{14}$$

both are matters of algebraic convenience that do not affect the basic results.

Now the consumer's optimization yields $h = \frac{1}{2}$, and then the equilibrium price functions can be written

$$c(1+\theta') = \frac{1}{2} \frac{1}{1+\nu(X)-X\nu'(X)}, \tag{15}$$

and

$$\gamma = -\nu'(X), \tag{16}$$

where

$$X = m/(c(1+\theta')). \tag{17}$$

If a steady state is to exist, $B_{t,k}$ must be independent of t. In a steady state, (10) becomes

$$B_{t,k} = (1+i)^k (1+\pi)^t/(1+a)^{t+k},$$

where a is the constant rate of improvement of the collection technology. For this to be independent of t, we need $a = \pi$, and then

$$B_{t,k} = (1+i)^k/(1+\pi)^k = (1+r)^k.$$

This is exactly the implicit assumption about technical progress in Végh. He defines the real average costs of collection as a stable function of the real revenues. In my notation, this amounts to assuming that the collection technology improves in step with inflation, that is, $A(0,t) = \Pi(0,t)$, or with constant rates in a steady state, $a = \pi$. Therefore, I make this assumption to proceed with the steady state analysis.

Write $B_{t,k}$ as B for short. Since the real rate of interest must be positive to ensure convergence of the consumer's and the government's objective functions, b is increasing in k.

Next write $z = \theta'c$, so $c(1+\theta') = (c+z)$. Then the government's budget constraint (11) becomes

$$[1-\phi(zB)]\ z - (c+z)\ X\nu'(X) = g. \tag{18}$$

The overall resource constraint (12) reduces to (15). The objective function is simply $\ln(c)$.

We can regard c, z, and X as the choice variables. Since these are subject to the two constraints (15) and (18), there is one degree of freedom, and one independent first-order condition emerges. After some algebra, it can be written as

$$\phi(zB) + zB\phi'(zB) = \frac{X[(\nu'(X))^2 - \nu(X)\nu''(X)] - \nu'(X)[1+\nu(X)]}{X\nu''(X)}. \tag{19}$$

The solution can be shown diagrammatically, following Végh's method. It is most convenient to do this in terms of two variables, a compound variable $\zeta = zB$, and the inflation tax rate γ, which is negatively related to X by (16).

The two equations in terms of ζ and X are the Fiscal Equilibrium condition (FE)

$$\frac{1}{B}\ [1-\phi(\zeta)]\zeta - \frac{1}{2}\frac{X(\nu'(X)}{1+\nu(X)-X(\nu'(X))} = g, \tag{20}$$

and the Optimality Condition (OC)

$$\phi(\zeta) + \zeta\phi'(\zeta) = \frac{X[\nu'(X))^2 - \nu(X)\nu''(X)] - \nu'(X)[1+\nu(X)]}{X\nu''(X)}. \tag{21}$$

These are generalizations of the correspondingly numbered and labelled equations of Végh, incorporating collection lags.

Figure 1 shows the two curves in (γ, ζ) space. Following Végh's argument, the curve FE is downward-sloping and the curve OC is upward-sloping. Their intersection determines the optimal values of γ and ζ. The former is just the inflation tax; the consumption tax rate θ' can be computed from ζ.

At the optimum, the left-hand side of (20) is an increasing function of ζ and a decreasing function of X; this is because the tax revenue is increasing in the tax rates. Now an increase k raises B, which lowers the first term in (20). To restore equality, we must increase ζ or reduce X (increase γ). Thus the increase in k shifts the FE curve upward and to the right to the new position FE'. The optimal γ increases.

The intuition is as follows. If θ' and γ are kept unchanged, an increase in B raises ζ equiproportionately, which increases the average cost of collection of taxes on consumption. This disturbs the balance of benefits and costs of the two kinds of taxes. The consumption tax now has a higher marginal cost per unit of revenue than the inflation tax, making it desirable to switch some of the burden of taxation away from consumption and toward inflation. At the new optimum, the inflation tax is levied at a higher rate, and the consumption tax is less than fully indexed, or less than full interest is charged for the tax revenues held by the firms for the length of the collection lag. Note that the conventional argument based on the loss of revenue is not the driving force; instead, the general equilibrium reasoning rests on the 'secondary' effects on collection costs.

A restatement of the intuition further clarifies the issue. Suppose we proceed as suggested by the invariance result, keeping θ'_t and γ_t unchanged, and charging nominal interest on the taxes due for the period of the collection lag. Since the real interest rate is positive, when the revenues are finally received, they are larger in real terms. Under Végh's specification, the average cost of collection is a stable function of the real revenues. Therefore, the average cost of collecting the larger real revenues is now higher.
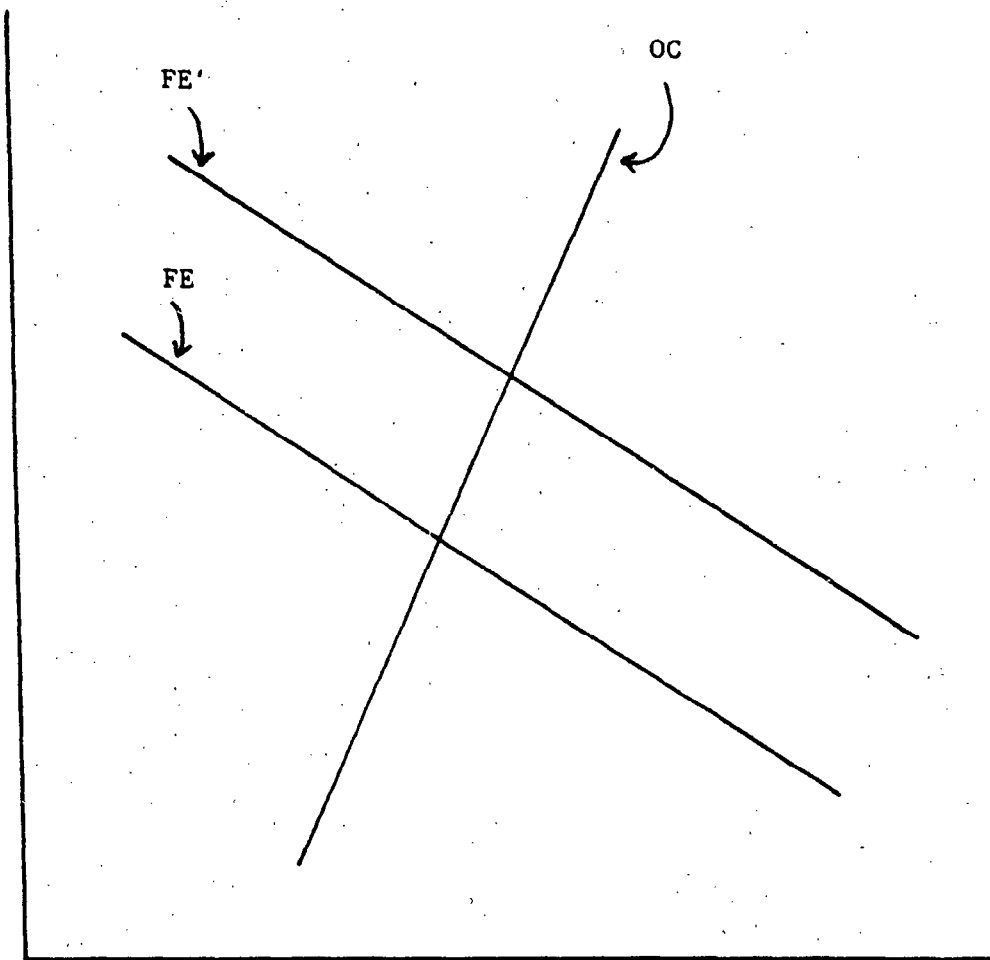
V. Concluding Comments

The particular model studied in this paper embodies many specific assumptions. The 'regular' tax is levied on consumption, it is collected by the firms in advance and held for the duration of the collection lag, and so on. But similar analysis comparing other kinds of taxes (for example, on labor income) and the inflation tax can be constructed. The specific results will depend on the nature of the collection or distortion costs associated with the tax system. For example, the costs of the regular tax system are a function of the effective rate θ', then the invariance result will prevail.

Figure 1

But as a general point, the analysis emphasizes the need for looking at the issue in a wider framework than has been common. When we recognize the existence of a collection lag, our response should be to re-calculate the whole tax structure, and not merely the inflation tax. When this is done, the important channel of interaction is the effect of the inflation tax on the collection (or distortion) costs associated with the other taxes, not on the revenues from those taxes.

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