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On the Sustainability and Optimality of Government Debt

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Abstract

This paper derives the Sustainability and Optimality Rules for government debt within an intertemporal optimizing framework in which both capital and government debt are endogenous, driven by utility and profit maximizing behavior of private agents and tax and expenditures policies of the government. The Rules are expressed purely in terms of familiar economic parameters and their ready applicability in an operational context is illustrated by instructive numerical examples. A discussion of the relationship between the Optimality Rule and the well-known Golden Rule of savings in the literature is also provided.

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Contents

"An increase in government spending must be financed by an increase in taxes. The optimal choice between tax finance and debt finance is really a choice about the timing of those taxes. A permanent increase in government spending must be matched by at least an equally large permanent increase in taxes unless taxes are increased by even more in the short run. There is no way to choose between a permanently higher level of taxes and a permanently higher level of debt." (Feldstein (1985)).

I. Introduction and Summary

In the voluminous literature on economic growth and capital accumulation, no result has richer and more enduring positive and normative implications than the Golden Rule of savings as a guide for the conduct of fiscal policy. In addition to being the rule which governs the welfare-maximizing level of capital intensity in the economy, it has also become the crucial benchmark against which to assess the role government debt plays in the steady state. This latter point was made particularly clear in the exchange between Barro (1976) and Feldstein (1976) on Barro's (1974) earlier re-statement of the Ricardian Equivalence principle, which had assumed a stationary economy. The basic point was that in a growing economy a positive level of government debt in the steady state could be either a net burden on or a net addition to private wealth, depending on whether the steady-state growth rate is, respectively, less than or greater than the interest rate. 1/

While the possibility that government debt could be regarded as net wealth by bond holders in the steady state is intriguing, the prevailing view in the public finance and growth literature is that a steady-state equilibrium that is characterized by a rate of growth which exceeds the interest rate is somehow inconsistent with rational behavior, although it has long been recognized that in a competitive economy there is really no reason , on pure theoretical grounds, to presume that the steady-state level of capital intensity is necessarily below that implied by the Golden Rule. 2/ If one agrees with the prevailing view, then the conclusion reached by Feldstein (1985), quoted at the beginning of the present paper, is inescapable. In a steady state where the level of per capital debt is constant, the total amount of debt must be increasing at the same rate as the labor force. But if the interest rate exceeds this rate, taxes must

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1/ It turns out that, as was shown later by Carmichael (1982), the validity of the Ricardian Equivalence principle, at least within the theoretical framework employed by Barro, is not affected by these complications.

2/ It is worth noting, however, that both Buiter (1980) and Carmichael (1982) have shown that in an overlapping-generations model with operative transfers from children to parents (the gift motive), the steady-state equilibrium is necessarily one in which the growth rate exceeds the interest rate.

be raised in order to service the debt. The choice between debt and taxes as a source of financing the government budget is therefore illusory in the long run under these circumstances.

In writing off the government's ability to reduce taxes in the long run by simply further increasing its borrowing, 1/ the literature has apparently overlooked the fact that steady states with the interest rate being greater than the growth rate could be incompatible with stability, if the level of debt is positive. This observation is superficially obvious under the simplifying assumption that the interest rate can be treated as a constant. For under such circumstances either per capita taxes must be forever increasing to keep the level of per capita debt constant, or with constant taxes per capita debt would rise without limit. Although these consequences do not immediately follow in a more complete model in which the interest rate, or equivalently the stock of capital, is allowed to vary, the dynamic properties of a model whose path of evolution is governed by simultaneous adjustments in debt and capital should be more carefully analyzed than the rather scant attention it has received so far. 2/

The necessary and sufficient conditions for stability are studied in the present paper based on an intertemporal optimizing model in which both capital and government debt are endogenous, driven by utility and profit maximizing behavior of private agents and tax and expenditures policies of the government. In such a model, it turns out that a necessary (but not sufficient) condition for stability in the neighborhood of a steady-state equilibrium with positive debt is that the growth rate of the economy be greater than the interest rate. 3/ Under very general assumptions regarding utility and production functions, an operational rule, expressed purely in terms of familiar economic parameters, for the determination of the critical level of debt above which it cannot be sustained is also derived. Because the model, though simple, is sufficiently rich in economic structure, the Sustainability Rule can be conveniently used to ascertain in a meaningful way the margin between an economy's existing and sustainable debt levels, given unchanged existing economic conditions.

1/ Buiter (1983), for example, likens it to forbidding the government to play Ponzi games forever.

2/ A notable exception are the well-known papers by Blinder and Solow (1973, 1974, 1976), which explicitly recognized stability complications arising from the presence of government debt. Their major focus of attention, however, revolved around the wealth effects of debt in a traditional IS-LM type macoreconomic model, a concern that has certainly faded in importance in the last decade or so.

3/ Masson (1985) also discussed this condition under a differenet model setup and with the utility function restricted to be of the Cobb-Douglas form.

An important implication of the above sustainability analysis is clearly that, if one restricts his attention only to stable equilibria (as is customary), then debt could be considered as a legitimate source of revenue for the government to finance its budget. Assuming lump-sum taxes are not available under realistic economic settings, the choice between debt and (distortionary) tax financing by the public sector therefore has substantive economic content, and provides the motivation for the analysis of the optimal level of government debt.

In this paper, the government's optimization calculus is defined in terms of its objective to maximize the steady-state utility level of the representative individual, subject to its budget constraint as well as the technological and behavioral constraints of the privte sector. Three different policy experiments are analyzed: (i) the determination of the optimal level of expenditures for a given tax rate, (ii) the determination of the optimal tax rate for a given level of expenditures, and (iii) the simultaneous determination of the optimal levels of expenditures and the tax rate. Much of the analysis will be focused on the first experiment, since the government typically has more flexibility in varying expenditure levels than tax rates. In all cases taxation is taken to be of the form of a general income tax at a constant ad valorem rate. Formulating the government's problem in this way implies that the optimal debt level is a derived concept (as it should be) through the government budget constraint.

In each of the policy experiments stated above, the impact of a given government action is transmitted to the private sector by affecting the latter's level of capital intensity. The long-run incidence effects of taxing incomes from productive factors are of course the subject matter of a large body of economic literature (see, for example, Diamond (1970) and Feldstein (1974a, 1974b)), and the fact that variations in the level of debt can similarly produce incidence effects (unless private savings behavior responds to negate them, such as under complete Ricardian Equivalence) is at least as well known. Seen in this light, debt and taxes are essentially fiscal tools by which the government alters the path of private capital accumulation to achieve its objective. 1/

In a decentralized economy in which the government has no direct control over the economy's resources and the optimal conduct of fiscal policy necessitates the use of debt and/or distortionary taxes, the optimal level of capital intensity in the steady state no longer always coincides with that implied by the Golden Rule. 2/ To see the possible

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1/ This, of course, is in striking contrast to the framework used by Barro (1979), where under complete Ricardian Equivalence debt only serves the function for tax smoothing.

2/ If interest income is taxed, as it would be under a general income tax considered here, the proper definition of the Colden Rule must be modified accordingly as one which equates the growth rate of the economy with the net-of-tax interest rate faced by the consumer.

negation of the Golden Rule as an optimality condition, a basic understanding of its implication for fiscal policy is required. When the rate of growth of the economy is exogenously given, the maintenance of a steady-state capital stock of any given size requires the sacrifice of some consumption. Hence, even though a higher capital stock would necessarily produce more output, the amount of resources that is available for consumption is maximized only if the marginal product of capital is equated with the marginal unit of foregone consumption. However, in a life-cycle model such as the one employed in the present paper (see section II), not only the steady-state level of consumption but also its distribution between the young and the old matters. Hence, the optimality of the Golden Rule in this context is implicitly predicated on the ability of the government to peg the interest rate to the rate of population growth and then clear the capital market by varying the level of government debt (no taxes and subsidies are required, however, if there were no autonomous government expenditures). 1/ This is the fundamental reason why the Golden Rule can be characterized, quite paradoxically, entirely by the given rate of growth alone, with other parameters in the production and utility functions playing no role in the optimality condition.

Now suppose the government can increase or decrease the economy's capital intensity only by changing the level of its outstanding debt through discretionary tax and/or expenditures policies. 2/ In such circumstances, the constraint imposed by the capital market equilibrium condition of a decentralized system becomes binding, and one would expect that the optimal level of capital intensity could either exceed or fall short of that implied by the Golden Rule. Since variations in the capital intensity produce changes in the ratio of the wage to interest rates, which in turn would alter the life-time income profile of the representative individual, the exact location of the optimum relative to the Golden Rule should now depend, among other things, on the nature of both the utility and the production functions, and it does. This paper derives and interprets the optimality condition in the presence of government debt as the Rational Rule of savings. It turns out that, under fairly mild restrictions, the Rational Rule can be characterized in a relatively simple way. Hence, it can conveniently be used as an operational Optimality Rule for the determination of government debt.

Two important implications follow from the above optimality analysis. First, if the level of capital intensity dictated by the Rational Rule is below that corresponding to the Golden Rule, a positive level of government debt (in order to reduce the capital intensity) is required, resulting in an interest rate in the steady state being greater than the rate of growth. The relationship is exactly reversed should the Rational Rule dictate a

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1/ A more detailed discussion on this and other issues relating to the Golden Rule is provided in section V.

2/ In an analysis of optimal fiscal policy, there is, of course, no reason to restrict the level of government debt to be positive.

level of capital intensity which exceeds that implied by the Golden Rule. In either case, a non-zero optimal debt level is an item of outlay in the government's budget constraint in the steady state. This means that when the governments' budget is optimally conceived, debt in itself cannot be a source of revenue, and therefore sufficient positive resources at the disposal of the government are required to attain the optimum. Furthermore, from the earlier sustainability analysis, it can be seen that the optiaml level of government debt, should it be positive, is necessarily incompatible with a stable steady-state equilibrium.

The second important implication is that, if the government has the degree of freedom to choose the optimal level of expenditures and the tax rate simultaneously, then government debt plays no role in its optimization calculus. The reason for this is simply that, since the optimal level of capital intensity is always achievable through an appropriately chosen level of expenditures (and therefore of debt), there is never a reason to use the income tax as a tool for this purpose, as it necessarily entails an excess burden. However, without positive tax revenues, to conduct an expenditures or debt policy by itself is infeasible, unless the government has access to other non-tax revenues. Hence, the third policy experiment stated above is essentially an experiment for the determination of the optimal tax rate, not the opitmal level of government debt. The optimality condition in this case, which simply balances the benefits to be gained by moving a competitive economy towards the Golden Rule with the cost of the excess burden of the tax, is also derived in this paper as the Taxation Rule.

The present analysis of the sustainability and optimality of government debt has been carried out within the context of a closed economy. In view of the large external imbalances experienced by most countries recently, the impact of a change in the level of government debt on an economy's external position is clearly of importance. 1/ The theoretical framework employed in the present paper has already been extended to an open-economy setting by Persson (1985), for example, for a one-good model and by Zee (1987) for a two-good world. The open-economy implications of the issues analyzed here are addressed in a (forthcoming) companion paper.

II. The Model

The basic framework of analysis is the now-familiar overlapping-generations model where each generation lives for two periods, works during the first (when young) and retires in the second (when old). When young, a member of the generation born in period t (henceforth individual t)

1/ A comprehensive study of the international effects of fiscal policies can be found in Frenkel and Razin (1987).

is endowed with one unit of labor. There is no labor-leisure choice. 1/ The size of each generation, i.e., the total labor force in each period, L_t, grows at the rate n:

$$L_t = (1+n)L_{t-1}. \tag{1}$$

Individual t wishes to maximize his utility over his life-cycle given by a strictly quasi-concave, twice-differentiable, increasing real-valued function

$$U_t = u(c_t,\tilde{c}_t) , \tag{2}$$

where c_t and $\tilde{c}_t$ denote, respectively, his consumption when young and when old, subject to the budget constraint

$$\Omega_t = c_t + p_t\tilde{c}_t , \tag{3}$$

where $\Omega_t \equiv w_t\ (1-\tau_t)$ is his net-of-tax wage income (also his life-time net wealth), $p_t \equiv 1/[1 + r_{t+1}(1-\tau_{t+1})]$ is the current price of the next period's consumption, and τ is the ad valorem rate of income tax. r_{t+1} is the gross rate of return he can earn in period t+1 for postponing one unit of consumption in period t. The first-order condition of the above maximization is

$$u_1 = u_2/p_t , \tag{4}$$

where subscripts on u denote its partial derivatives with respect to its corresponding arguments. Together with the budget constraint (3), (4) can be used to solve for the current consumption when young as

$$c_t = c(\Omega_t,p_t) . \tag{5}$$

It is convenient at this juncture to define three useful elasticities relating to consumption. Along any indifference curve in the c-$\tilde{c}$ space, the elasticity of substitution between consumption when young and when old can be defined in the usual manner:

$$\sigma_t \equiv -d\ln(\tilde{c}_t/c_t)/d\ln p_t > 0 . \tag{6}$$

The income elasticity of current consumption, on the other hand, is given by

$$\eta_t \equiv d\ln c_t/d\ln\Omega_t > 0 , \tag{7}$$

1/ Available empirical evidence suggests that the uncompensated elasticity of labor supply is on the whole quite small (see Killingsworth (1983)). Moreover, if leisure is additively separable from consumption, the supply of labor is independent of the rate of a proportional income tax.

where the sign restriction in (7) rules out consumption (in either period) as an inferior good. From the Slutsky equation, the elasticity of savings with respect to the net-of-tax rate of return $[r_{t+1}(1-\tau_{t+1})]$ is related to σ_t and η_t according to

$$\delta_t \equiv d\ln(\Omega_t - c_t)/d\ln[r_{t+1}(1-\tau_{t+1})]$$

$$= (1-s_t)(1-p_t)(\sigma_t - \eta_t) \ , \tag{8}$$

where $s_t \equiv p_t\tilde{c}_t/\Omega_t$ is the share of income devoted to consumption when old, i.e., the rate of savings. (8) gives the well-known result that $\delta_t \gtrless 0$ as $\sigma_t \gtrless \eta_t$.

The production side of the model displays all the standard neoclassical characteristics. Letting x_t to be the per capita output and k_t the capital-labor ratio, the production function

$$x_t = f(k_t) \tag{9}$$

is assumed to satisfy the Inada conditions. Competition in factor markets yields the usual marginal conditions for factor rewards:

$$w_t = f - k_t f' \quad \text{and} \tag{10}$$

$$r_t = f'. \tag{11}$$

It is again convenient at this point to define two useful elasticities relating to production. The first is the elasticity of output:

$$\phi_t \equiv d\ln x_t/d\ln k_t > 0 \ , \tag{12}$$

and the second is the interest elasticity of the demand for capital intensity:

$$\varepsilon_t \equiv d\ln k_t/d\ln r_t < 0 \ . \tag{13}$$

In a model such as this one where the desired capital intensity is always realized without delay, ε_t is equally interpretable as the interest elasticity of gross investment. 1/

The government finances its budget by an ad valorem tax on income (wages and interest) at the rate τ_t and by issuing debt, measured in per capita terms as b_t. These are one-period bonds paying a rate of

1/ It can easily be shown that the elasticity of substitution between capital and labor in the production function, κ_t, is related to ϕ_t and ε_t by the relationship $\kappa_t = -\varepsilon_t(1-\phi_t)$.

return equal to that on capital prevailing in the period of their redemption. Outlays for the government in each period are current expenditures g_t plus the redemption cost (with interest) of debt issued in the previous period. Consequently, its budget constraint can be stated as

$$(1+n)g_t + b_{t-1}/p_{t-1} = (1+n)(\tau_t x_t + b_t) . \tag{14}$$

It should be noted from (14) that taxes are paid on debt interest income to bond holders. This ensures that debt and capital are perfect substitutes in the individuals' decision to save.

The intertemporal equilibrium for the entire economy is established when the asset market is cleared in each period according to

$$(1+n)k_{t+1} = (\Omega_t - c_t) - b_t. \tag{15}$$

Given initial conditions and the time profiles of g_t and τ_t, (14) and (15) allow one to trace out the complete path along which the economy will evolve over time.

III. Sustainability

Sustainability is a term that has been used with increasing frequency in recent policy discussions on the current world debt situation, but unfortunately with different connotations under different circumstances. There has also been a tendency to inject normative considerations, without being at the same time explicit about the underlying objective function, into what should otherwise be a purely positive concept. To avoid unnecessary ambiguity and confusion, sustainability in the present paper is taken to mean (and only to mean) stability. A sustainable level of public debt is therefore one which allows the economy, in the absence of unanticipated exogenous shocks, to settle eventually onto a steady-state.

The dynamics of the model set out in the previous section are governed by a simultaneous difference equation system where changes in the per capita debt level b_t and the capital labor-ratio k_t (and therefore the rate of interest r_t) from one period to the next are fully described by the government budget constraint (14) and the economy-wide intertemporal equilibrium condition (15). For given and constant levels of g and τ, total differentiation of (14) yields, after leading it by one period,

$$db_{t+1} = db_t/[(1+n)p_t] + \gamma_{1t} dr_{t+1} , \tag{16}$$

where $\gamma_{1t} \equiv [(1-\tau)b_t - (1+n)\tau\varepsilon_{t+1}k_{t+1}]/(1+n) > 0$. Total differentiation of (15), on the other hand, gives

$$\gamma_{2t} dr_{t+1} = -[\gamma_{3t} dr_t + r_{t+1} db_t] , \tag{17}$$

where $\gamma_{2t} \equiv (1+n)k_{t+1}\varepsilon_{t+1} - \Omega_t\delta_t s_t < 0$ and

$$\gamma_{3t} \equiv [1-\eta_t(1-s_t)]k_t r_{t+1}(1-\tau_{t+1}) > 0 .$$

In signing γ_{2t} and γ_{3t}, it has been assumed that $\delta_t > 0$, i.e., the interest elasticity of savings is non-negative, 1/ and that $1 > \eta_t(1-s_t) > 0$, i.e., normality in current and future consumption. 2/ Substituting (17) into (16) to eliminate the dr_{t+1} term from the latter, the dynamic system can be written as

$$\begin{bmatrix} dr_{t+1} \\ db_{t+1} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} dr_t \\ db_t \end{bmatrix} , \tag{18}$$

where

$$a_{11} \equiv - \gamma_{3t}/\gamma_{2t} > 0 ,$$

$$a_{12} \equiv - r_{t+1}/\gamma_{2t} > 0 ,$$

$$a_{21} \equiv - \gamma_{1t}\gamma_{3t}/\gamma_{2t} > 0 , \text{ and}$$

$$a_{22} \equiv 1/[(1+n)p_t] - r_{t+1}\gamma_{1t}/\gamma_{2t} > 0 .$$

With $a_{ij} > 0 \ \forall \ i,j$ in (18), the necessary and sufficient conditions for (local) stability are (i) $(1-a_{11}) > 0$, i.e.,

$$\gamma_{2t} + \gamma_{3t} < 0 , \tag{19}$$

and (ii) $[(1-a_{11})(1-a_{22}) - a_{12}a_{21}] > 0$, i.e.,

$$[n-r_{t+1}(1-\tau)](\gamma_{2t} + \gamma_{3t}) + (1+n)r_{t+1}\gamma_{1t} < 0 . \tag{20}$$

Since $\gamma_{1t} > 0$, by (19) a necessary (but not sufficient) condition for the satisfaction of (20) is clearly

$$n > r_{t+1}(1-\tau), \tag{21}$$

which says that stability requires the growth rate of the labor force (also the real rate of growth of the economy in the steady state) be greater than the net-of-tax interest rate, or, alternatively, the economy

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1/ The stipulated sign for γ_{2t} holds even if $\delta_t<0$, as long as its absolute value is not large enough for the second term in γ_{2t} to overwhelm the first. Available empirical evidence on savings behavior does not support large and negative interest elasticity of savings, however.

2/ This follows from the fact that the weighted average of income elasticities of consumption (current and future) must add to unity, the weights being the relative shares of each period's consumption in total net wealth.

be over-capitalized relative to the level associated with the (tax-modified) Golden Rule of savings. 1/ This necessary condition establishes the important implication that, in the steady state, debt is a source of revenue rather than an item of expenditure for the government.

Expanding (19) and evaluating it at the steady state, this 'low-level' stability condition can be expressed as

$$[1-\eta(1-s)] + (1+n)\varepsilon/[r(1-\tau)] - s\delta(1-\phi)/\phi \equiv \Phi < 0 , \tag{22}$$

which is independent of the level of per capita debt. It also reveals clearly how the inherent dynamic stability capacity of an economy can be enhanced (hampered) by a high (low) interest elasticity of savings. 2/ By similar expansion of (20), again evaluating it at the steady state, one obtains the 'high-level' stability condition as

$$[n-r(1-\tau)]\Phi k - (1+n)\varepsilon k\tau/(1-\tau) + b < 0 . \tag{23}$$

Letting λ_k and λ_b be the capital-output and debt-output ratios respectively, the critical value of λ_b for which (23) can be negated is then

$$\lambda_b^* = \lambda_k\{\varepsilon(1+n)\tau/(1-\tau) - [n-r(1-\tau)]\Phi\} . \tag{24}$$

Hence, a given debt level is sustainable if and only if

$$\lambda_b < \lambda_b^* . \quad \text{(The Sustainability Rule)} \tag{25}$$

Note that the RHS of (24) is comprised only of terms which have transparent economic interpretations and whose numerical magnitudes are usually either readily available or can be easily estimated. Ascertaining the value of λ_b^* is therefore a rather straightforward matter.

Numerical calculations showing the sensitivity of λ_b^* to changes in the values of important economic parameters are provided in Table 1. In the baseline scenario, parameter values are chosen so that they are more or less consistent with either their existing magnitudes or empirical findings in the literature for the U.S. economy. 3/ Alternative scenarios are identical to that of the baseline except for the particular parameter value(s) identified on the left of the table.

1/ The per capita debt level is inconsequential if $n=r(1-\tau)$ in the steady state.

2/ If the utility function (2) takes the (commonly assumed) Cobb-Douglas form, then $\delta=0$ and $\eta=1$, which simplifies (22) to $\varepsilon < -sr(1-\tau)/(1+n)$. For given equilibrium values of the tax, interest, and savings rates, this condition states the minimum required interest responsiveness of investment in order for the economy to be stable, provided that the 'high-level' stability condition is also satisfied (see below).

3/ See Atkinson and Stiglitz (1980) and Auerbach and Kotlikoff (1987) for useful discussions of some of the available empirical evidence.

As can be seen from Table 1, the value of λ_b^* is quite sensitive to η (income elasticity of current consumption), n (the natural growth rate), r (the interest rate), and τ (the proportional income tax rate). A one percentage point decrease in the growth rate or increase in the interest rate, for example, would reduce the sustainable level of debt relative to the baseline by approximately 10 percent of output. A two percent decrease in the income elasticity of consumption would also reduce the sustainable debt level by roughly 5 percent of output. On the other hand, an increase of almost 20 percent of output in the sustainable debt level can be obtained by increasing the taxe rate by 5 percentage points. Changes to the values of the other parameters do not seem to have an appreciable impact on the value of λ_b^*.

Table 1. Sustainable Levels of Government Debt

Scenario	λ_b^*
Baseline 1/	0.42
Alternative 2/	
$\delta = 0.2$	0.40
$s = 0.22$	0.42
$\eta = 0.8$	0.36
$\phi = 0.3$, $\varepsilon = -1.43$	0.44
$n = 0.02$ (per annum)	0.32
$r = 0.04$ (per annum)	0.33
$\tau = 0.2$	0.60

1/ Parameter values are $\eta = 1.0$, $s = 0.17$, $\delta = 0.3$, $\phi = 0.25$, $\varepsilon = -1.33$, $\tau = 0.15$, $n = 0.03$ (per annum), and $r = 0.03$ (per annum). Values for both n and r are adjusted in each of the calculations to account for the length of the time period in the model, which corresponds roughly to half a generation (30 years).

2/ Alternative scenarios are identical to that of the baseline except for the parameter value(s) identified below for each calculation.

While the above calculations are only intended to be illustrative, the Sustainability Rule as stated in (24)-(25) is nevertheless useful in providing a first-order approximation of the margin between an economy's existing debt level and the level at which it can be sustained, given unchanged existing economic conditions.

IV. Incidence

The incidence of taxation is a subject of long-standing interest in public finance, undoubtedly stemming from the recognition that the entity upon which a tax is levied is not necessarily the one which bears the burden of the tax. Since the effectiveness and feasibility of any tax is predicated on both the degree and direction by which its burden can be shifted from one entity to another, the determination of the incidence of a tax in a decentralized market economy is probably one of the most important analyses to be performed in the conduct of fiscal policy.

A voluminous literature already exists which investigates the effects of taxing incomes from factors of production. As is well-known, such taxes, apart from their short-run effects (where capital is held fixed), have significant positive and normative implications for the economy in the long run as well through their impact on capital accumulation. Traditional analyses in this area, however, have not properly taken government debt into consideration and have not examined carefully the stability properties of growth models when such debt exists. The model set out in the present paper, on the other hand, allows one to demonstrate precisely the role government debt plays in the determination of whether the capital intensity of an economy in the steady state is increased or reduced by a change in the tax rate (here, the rate of a general income tax).

The impact of debt and taxes on the equilibrium value of the interest rate in the steady state can be obtained by totally differentiating the intertemporal equilibrium condition (15) to get

$$(\gamma_2+\gamma_3)dr = -rw[\delta s + 1-\eta(1-s)]d\tau - rdb \; . \tag{26}$$

For a given level of per capita government expenditures g, changes in taxes cannot be independent from changes in debt. In the steady state, the government budget constraint can be written as

$$(1+n)g = (1+n)\tau x + [n-r(1-\tau)]b. \tag{27}$$

Total differentiation of (27) provides the important relationship between debt and taxes according to

$$[n-r(1-\tau)]db = -[(1+n)\tau\varepsilon k - (1-\tau)b]dr - [(1+n)x + rb]d\tau \; . \tag{28}$$

From (28), it can be seen that, for a constant r, a one unit increase in

τ allows a $[(1+n)x + rb]/[n-r(1-\tau)]$ units decrease in b, a trade-off value which is not independent from the existing debt level. Substituting (28) into (26) to eliminate the term db, one obtains

$$\frac{dr}{d\tau} = \frac{r^2k\{(1+n) - (1-\phi)[n-r(1-\tau)][\delta s + 1-\eta(1-s)]\}/\phi + r^2b}{[n-r(1-\tau)](\gamma_2+\gamma_3) + (1+n)r\gamma_1} . \tag{29}$$

The denominator of (29) is negative by the stability condition (20). The sign of (29) therefore depends on the sign of its numerator. 1/ The critical value of debt for which the numerator is zero is defined by

$$\lambda_b^{**} = \lambda_k\{(1-\phi)[n-r(1-\tau)][\delta s + 1-\eta(1-s)] - (1+n)\}/\phi . \tag{30}$$

Hence, it immediately follows that

$$dr/d\tau \gtreqless 0 \text{ as } \lambda_b \lesseqgtr \lambda_b^{**}, \text{ (The Incidence Rule)} \tag{31}$$

which stipulates the manner by which a change in the tax rate would affect the interest rate (and therefore the capital intensity) in the steady state. The a priori ambiguity of the sign of (29) stems from the fact that, while an increase in τ lowers the net-of-tax income and interest rate (for constant r) and therefore the savings (and also investment) in the economy, it, at the same time, allows a lowering of the debt level in the financing of a constant government budget. The lowered debt level permits a higher level of investment and capital accumulation. Thus, the overall impact on the steady-state capital intensity as a result of any given change in the tax rate would depend on the net outcome of these two opposing forces.

Although from (31) it is seen that an increase in τ would increase the capital intensity only if the existing debt level, expressed as a ratio to output, exceeds the critical value λ_b^{**} calculated from (30), 2/ a little algebraic manipulation reveals that $\lambda_b^{**} < 0$ if the interest elasticity of savings is zero (i.e., $\delta=0$), which then implies $dr/d\tau < 0$ unambiguously. Indeed, it can be shown that an implausibly high value of δ is required in order to produce the opposite outcome. The reason for this is clear. Since the tax impact on savings works through the utility function, but the debt impact on capital accumulation is a direct one-to-one trade-off, only a high δ can lead to a reduction in savings that is sufficient to more than offset the increase in investment afforded by the lowering of the debt level.

1/ In contrast, the incidence of a change in government expenditures, given the tax rate, is unambiguously positive. It can be shown that the derivative dr/dg has the same denominator as that of (29), but its numerator is $[-(1+n)r]$. Hence, an increase in g necessarily reduces the capital intensity of the economy as expected.

2/ Note again that the calculation of this critical value only involves terms whose numerical magnitudes are usually readily obtainable.

V. Optimality

Once it is established that in a stable steady state a given government budget can be financed by either taxation or debt issuance, an interesting and important question immediately arises: how to optimally set the level of expenditure and/or the tax rate so as to maximize the steady-state utility level of a representative individual. 1/ Through the government budget constraint, the solution to this problem implies an optimal level of government debt.

In a decentralized economy in which the government has no direct control over the economy's resources, the individual's budget constraint and first-order condition for utility maximization are constraints in the government's optimization calculus. To take them into account, the government needs to maximize the individual's indirect utility function, obtained by subtituting (3) and (5) into (2), that is,

$$\max u\{c(\Omega,p),[\Omega-c(\Omega,p)]/p\} \ , \tag{32}$$

subject, of course, to its own steady-state budget constraint (27). In achieving (32), three conceptually different policy experiments can be analyzed: (i) the determination of the optimal level of expenditure for a given tax rate, (ii) the determination of the optimal tax rate for a given expenditure level, and (iii) the detemination of the optimal expenditure level and the tax rate simultaneously.

Among the three stated policy experiments, the first is arguably the most relevant from an institutional standpoint, since major tax legislations are both difficult and infrequent. In what follows it will also be shown that it is also theoretically the most important. For this experiment, the solution to (32) satisfies (by using (4))

$$(d\Omega/dg)/\Omega = s(dp/dg)/p \ , \tag{33}$$

which says that at the optimum, the proportionate changes in life-time net wealth and the price of future consumption (the latter being weighted by the savings rate) must be equal. Because a unique relationship exists between the wage rate and the interest rate along the factor-price frontier implied by (10)-(11), (33) can be manipulated to yield

$$(1-\tau)\rho(dr/dg) = 0 \ , \tag{34}$$

where $\rho \equiv [(\Omega-c) - k/p]$ measures how a given change in the interest rate affects the utility level of the individual. Since dr/dg does not

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1/ This is certainly not the only possible form of the objective function of the government. An alternative but much less tractable formulation would be to include in the function the sum of utilities of all generations along the path of transition from one steady-state to another.

equal zero (see Section IV), it is necessary and sufficient to have $\rho = 0$ for (34) to hold. The intuition behind this result is simply that the change in g alters the time profile of the individual's income stream (in the form of a change in the wage relative to interest incomes), at the optimum the change in utility induced by this effect must be zero. By use of the economy-wide equilibrium condition (15) in the steady-state, $\rho = 0$ implies

$$b/k = -[n-r(1-\tau)] \ , \tag{35}$$

which states that the debt-capital ratio in the economy at the optimum is equal to the deviation of the net-of-tax interest rate from the (tax-modified) Golden Rule of savings. 1/ An important implication of (35) is clearly that the optimum can be characterized by either the Golden Rule $[n = r(1-\tau)]$, in which case $b=0$, or an over-capitalization relative to the Golden Rule $[n > r(1-\tau)]$, in which case $b < 0$. A positive optimal debt level is incompatible with the stability requirement, 2/ since under such a circumstance only part of a generation's savings contributes to actual capital stock formation for the next generation (whose size is larger by the rate n), resulting in $[r(1-\tau) > n]$.

The exact location of the optimum relative to the Golden Rule depends on both the nature of the production function and the utility function. By substituting (35) into (15), it can be shown that the optimal net-of-tax interest rate is given by

$$r(1-\tau) = [s(1-\phi)/\phi - 1]^{-1} \ , \tag{36}$$

which can then be compared with n. Since (36) is an implicit equation in k alone, its solution can be obtained in a straightforward manner. It is also of interest to point out that the existence of a meaningful solution to (36), i.e., $r(1-\tau) > 0$, requires the existence of a region along the production function for which the output elasticity (ϕ) is compatible with such a solution. Re-arranging (36), the savings rate at which utility is maximized, or what can be called the Rational Rule of savings, is expressible as

$$s = [1/(1-p)][\phi/(1-\phi)] \ , \text{ (The Rational Rule)} \tag{37}$$

which shows that, ceteris paribus, the higher the elasticity of output or the current price of future consumption, the higher the rational rate of savings.

The optimal debt level (λ_b^{***}) for the economy, given the tax rate τ, can now be stated as

$$\lambda_b^{***} = \lambda_k\{\phi/[s(1-\phi)-\phi] - n\} \ . \text{ (The Optimality Rule)} \tag{38}$$

1/ In a model without government debt, (35) reduces to the Golden Rule.
2/ For $b<0$, the condition $[n>r(1-\tau)]$ is neither necessary nor sufficient for stability.

The substitution of (38) into (27) then solves for the optimal expenditure level as

$$\lambda_g = \tau + [n-r(1-\tau)]\lambda_b^{***}/(1+n) , \tag{39}$$

where λ_g denotes the expenditure-output ratio. (39) can be used for the determination of the optimal response of the level of government expenditure to a given arbitrary change in the tax rate, although the analytical expression for this response is unwieldly, primarily because it involves the second-order (excess burden) effects of taxation.

The characterization of the optimum under the second policy experiment, i.e., the determination of the optimal tax rate for a given expenditure level, is substantially more complicated than the first. This is because any change in the tax rate has a direct revenue effect in the individual's budget that is in <u>addition</u> to its incidence effect (which works only through the interest rate channel). To see this clearly, note first that the first-order condition for this experiment is of the same general form as (33), except that the dg term is now replaced by $d\tau$. Similar manipulations as before yield, however,

$$(1-\tau)\rho(dr/d\tau) - \Theta = 0 , \tag{40}$$

where $\Theta \equiv [r(\Omega-c) + w/p]$ measures the tax revenue effect per unit change in the tax rate. A comparison between (34) and (40) reveals that the two are the same except for the Θ term in the latter. 1/ Because of its presence, the magnitude of the derivative $dr/d\tau$ now has a direct bearing in the solution for the optimum. As given by (29), it is a rather complex analytical expression and therefore the optimum of this experiment cannot in general be characterized in a simple manner. It can also be seen that, apart from the tax revenue effect, the two policy experiments have the same implications for achieving the optimal solution. 2/

In terms of the optimal conduct of fiscal policy, the second experiment considered above is of little relevance, since the government typically has more flexibility in varying expenditure levels than tax rates. Nevertheless, it serves an important intermediate step towards the understanding of the outcome of the third policy experiment, in which

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1/ A possible way to eliminate the Θ term is to postulate that whatever tax revenue received by the government is returned to the individual in a non-distortionary fashion (which is <u>not</u> equivalent to a lump-sum payment to the individual when young). A more detailed discussion on this point is provided below in connection with the third policy experiment.

2/ This does not mean, of course, that the optimal capital intensity and/or utility levels are necessarily identical under the two experiments (that would depend on the <u>levels</u> of g and τ). Rather, their optimality conditions can be characterized in the same way, i.e., $\rho = 0$.

g and τ are chosen simultaneously. The optimal solution in this case requires the concurrent satisfaction of equations (34) and (40). 1/ A quick inspection of these equations immediately reveals that $\rho = 0$ (for (34) to hold) implies $\Theta = 0$ (for (40) to hold). But since this can occur only if (i) the economy is not producing any output (i.e., $x = 0$) with non-zero taxes, 2/ or (ii) at positive output no taxes are levied (i.e., $\tau = 0$), it is therefore optimal for the government to forgo the use of the income tax as an instrument to achieve the optimum. 3/ Following the discussions on the first and second policy experiments, this result should not really come as a surprise. Since the desired incidence effect of varying the tax rate is equally achievable through varying the expenditure level, there is never a reason to use it for this purpose, because a positive distortionary tax necessarily entails an additional excess burden (even if the entire tax revenue is returned). In this sense, the third policy experiment essentially becomes identical to the first, with the given tax rate now set at zero.

This, however, leaves the government in a fundamental dilemma. On one hand, the above analysis shows that it is optimal to have no distortionary taxes. On the other hand, in the course of analysing the first policy experiment, it has already been established that in order to achieve the Rational Rule of savings, positive resources at the disposal of the government are required, unless the optimal capital intensity implied by the Rational Rule coincides with that of the Golden Rule. 4/ A moment's reflection on the nature of this dilemma uncovers a basic fallacy in the use of (27) as the appropriate form of the government budget constraint when g and τ are to be optimally chosen at the same time. Mathematically, (27) implies, with b serving as the residual variable, that the choice of g can be made independent of the choice of τ—an independence which in fact does not exist economically.

1/ Note that account is already taken of the government budget constraint and the economy-wide equilibrium condition in the derivation of (34) and (40).

2/ It can easily be shown that when $\rho = 0$, $\Theta = (rk+w)/p = x/p$, the latter equality follows from the constant-returns-to-scale property of the production function.

3/ In the present model in which distributional issues are completely absent, the validity of this argument hinges purely on efficiency grounds.

4/ Substituting (35) into (27) and setting $\tau = 0$ yields $g = -(n-r)^2k/(1+n) < 0$. A negative g is equivalent to a government subsidy. When the optimal capital intensity is less than that of the Golden Rule, the subsidy is in the form of servicing and maintaining a positive amount of debt. When the optimal capital intensity is greater than that of the Golden Rule, the subsidy takes the form of increasing the private capital stock by the government through a negative amount of debt. In either case, the achievement of the Rational Rule of savings, when it deviates from the Golden Rule, requires sufficient resources at the government's disposal.

Instead, the correct government budget constraint to be used is $g = x\tau$. Thus, if the government is not constrained to an existing positive tax rate and has no access to other non-tax revenues sufficiently large to finance an optimally determined expenditure level, then government debt has no role to play in the optimal conduct of fiscal policy. 1/

Under ordinary circumstances, the government seldom has the degree of freedom to choose both g and τ optimally at the same time. As stated earlier, changes in τ are infrequent, and even when they do occur, they are usually more arbitrary than optimally conceived. For this reason, the first policy experiment and its associated results remain the most relevant and important under realistic economic settings to guide the conduct of fiscal policy.

As an illustrative example, consider the particularly simple case of a Cobb-Douglas production function

$$x = \psi k^{\alpha}\ , \quad \psi > 0,\ 1 > \alpha > 0 \tag{41}$$

and a CES utility function

$$u = (c^{1-1/\sigma} + \beta\tilde{c}^{1-1/\sigma})/(1-1/\sigma)\ , \quad 1 \neq \sigma > 0 \tag{42}$$

$$= \ln c + \beta \ln \tilde{c}\ , \quad \sigma = 1$$

where $\beta > 0$ is a measure of the individual's preference for future relative to current consumption and subsumes his pure rate of time

1/ In a model in which g is not intrinsically valued (i.e., it does not enter directly into either the utility or production function), the optimal choice of τ (and therefore g) is purely a matter of balancing the benefit of moving the economy towards the Golden Rule of savings with the cost of the excess burden of the tax, assuming tax revenues are returned as lump-sum transfers to the individual distributed over the two periods of his life in exactly the same proportions as the shares of the wage and interest income taxes in total revenues raised. More formally, let T and $\tilde{T}$ be the transfer payments the individual receives when young and when old, repectively, where $T = w\tau$ and $\tilde{T} = r(\Omega-c)\tau$. The maximization of the individual's indirect utility function, now written as

$$u[c,\ (\Omega+T-c)/p + \tilde{T}],$$

yields the first-order condition

$$\rho + \tau(1+n)k\varepsilon = 0.$$

The term $[\tau(1+n)k\varepsilon]$ measures the excess burden of the tax, and vanishes at $\tau = 0$, a well-known result in the taxation literature. Re-writing the above condition slightly as

$$[n-r(1-\tau)] + \tau(1+n)\varepsilon = 0,$$

one can immediately see that if the optimum is at a level of capital intensity less than that implied by the Golden Rule, a subsidy ($\tau < 0$) would be required to achieve it, and the government would run into the same type of problem discussed earlier. Note also that the optimal tax rate can be solved explicitly as $\tau = (r-n)/[r+(1+n)\varepsilon]$. (The Taxation Rule) preference. For the above production and utility functions, it is easily verified that $\phi = \alpha$, $\varepsilon = 1/(\alpha-1)$, $\eta = 1$, and the rate of savings implied by (42) is

$$s = 1 - 1/(p^{1-\sigma}\beta^{\sigma} + 1) \; . \tag{43}$$

To solve for the Rational Rule of savings, equate (43) with (37) and search for an economically meaningful root to the following implicit equation in p:

$$1 - 1/(p^{1-\sigma}\beta^{\sigma} + 1) - [1/(1-p)][\phi/(1-\phi)] = 0 \; . \tag{44}$$

Note that when $\sigma = 1$, s becomes a constant and p (and therefore r) can be solved explicity from (44).

Table 2. Optimal Levels of Government Debt
(in percent)

Scenario	λ_b^{***}
Baseline 1/	5.8
Alternative 2/	
$\sigma = 0.8$	12.9
$\sigma = 2.0$	-3.9
$\beta = 0.6$	12.9
$\beta = 1.5$	-2.0
$\phi = 0.3$	16.1
$\phi = 0.2$	-3.1
$\tau = 0.05$	6.5
$\tau = 0.25$	5.2

1/ Parameter values are $\sigma = 1.0$, $\beta = 0.9$, $\phi = 0.25$, $\tau = 0.15$, and $n = 0.03$ (per annum). Values for both n and (solved) r are adjusted in each of the calculations to account for the length of the time period in the model, which corresponds roughly to half a generation (30 years).

2/ Alternative scenarios are identical to that of the baseline except for the parameter value identified below for each calculation.

Calculations for the optimal levels of government debt under different sets of parameter values are provided in Table 2. As with Table 1, alternative scenarios are identical to the baseline scenario except for the particular parameter value identified on the left of the table.

Employing plausible parameter values, Table 2 strikingly illustrates the fundamental theoretical result derived earlier that the optimal level of government debt can be either positive or negative, which is equivalent to saying that the optimal capital intensity of the economy can be either smaller or greater than that implied by the Golden Rule. The numerical calculations also clearly show the influence of various parameters in the utility and production functions on the optimal debt level. Ceteris paribus, a high debt level would be opitmal for the economy, for example, if the individual's intertemporal elasticity of substitution in consumption (σ) is low, or that his preference for future relative to current consumption (β) is low, since either tends to lead to a low savings rate. A low elasticity of output in the production function, on the other hand, would lead to a low level of optimal debt.

Table 2 also provides some interesting results on the trade-off between debt and taxes when the level of government expenditures is optimally determined. In general, because of the interest rate impact of debt, a change of one percentage point in the tax rate leads to a less than proportionate change in the optimal debt level in the opposite direction.

For the graphically inclined, the relationship between the Golden Rule and the Rational Rule of savings is illustrated in Figures 1-3. In Figure 1, the production space and the consumption space are represented respectively by the RHS and LHS quadrants. Consider the arbitrarily chosen capital-labor ratio k^* and the associated level of per capita output y^*. Since the slope at point I on the production function equals r^*, income to capital (r^*k^*) is measured by the distance By^* on the vertical axis. The distance OB then measures the wage rate w^*. Suppose there are no debt and taxes. To maximize utility, the representative individual would choose a point on his budget constraint BJ where it is tangent to one of his indifference curves. 1/ Not all points on BJ are compatible, however, with a long-run equilibrium for the economy as a whole. A feasible steady state must be such that the amount of savings in the economy, such as measured by the distance Bc^*, just equals $(1+n)k^*$. Applying the same procedure for alternative budget constraints, one can therefore ascertain a different feasible steady state for every point on the production function. The curve OUHGZ traces out the locus of all such feasible steady states (LFSS). The underlying capital-labor ratio increases as one moves along it from the origin O towards Z.

1/ The slope of the individual's budget constraint is $-(1+r^*)^{-1}$ with respect to the horizontal axis. Hence, in general it will be different from that of the line IBJ. To reduce clutter in the diagram, it has been assumed, without loss of generality, that the two lines coincide at the interest rate r^*. This would occur, of course, only if $r^* \approx 0.62$.

FIGURE 1.

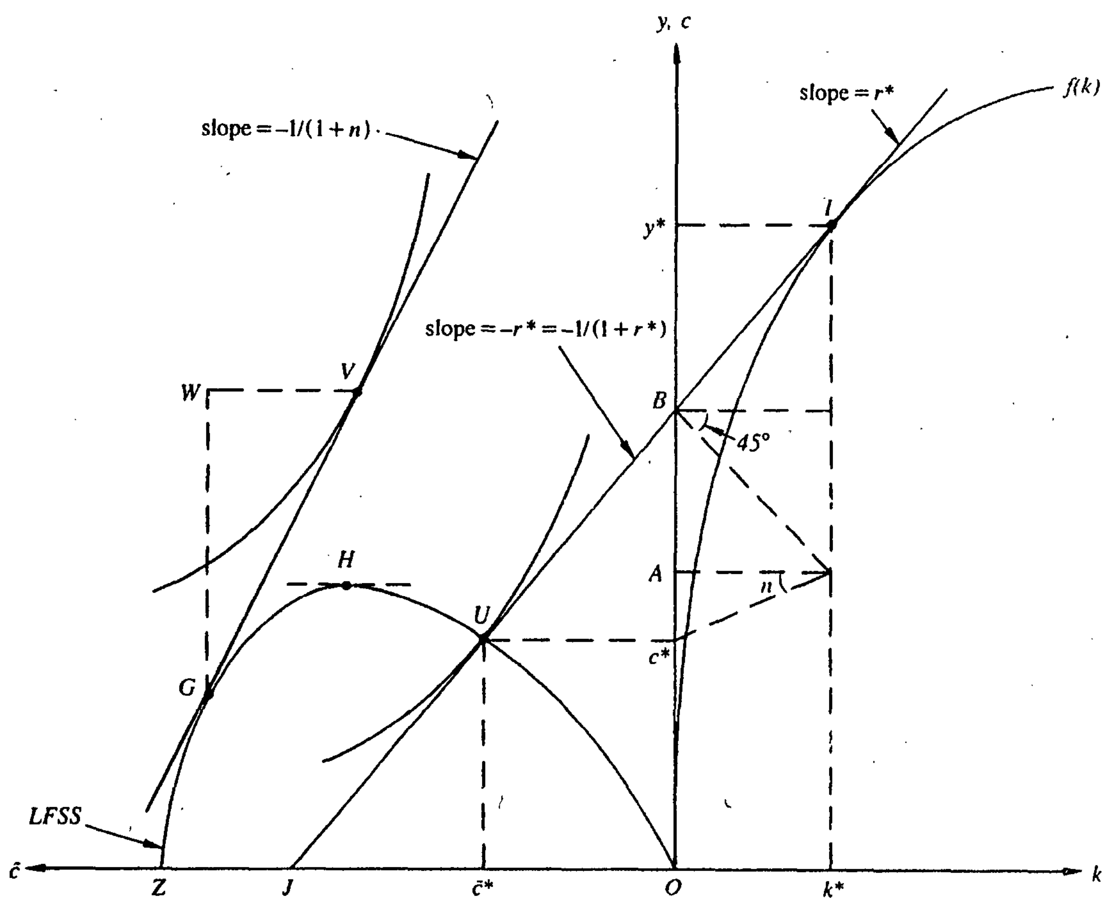
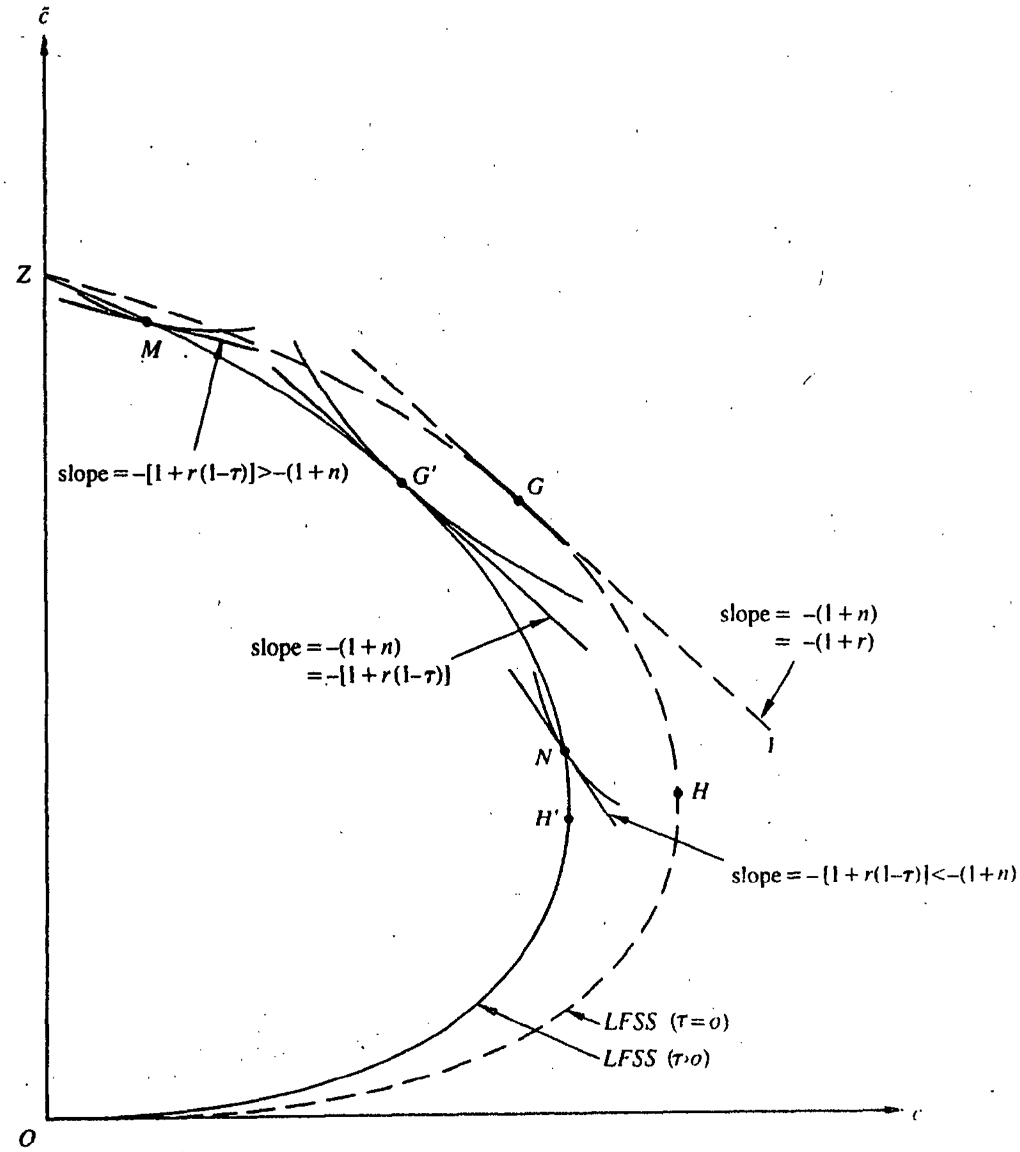
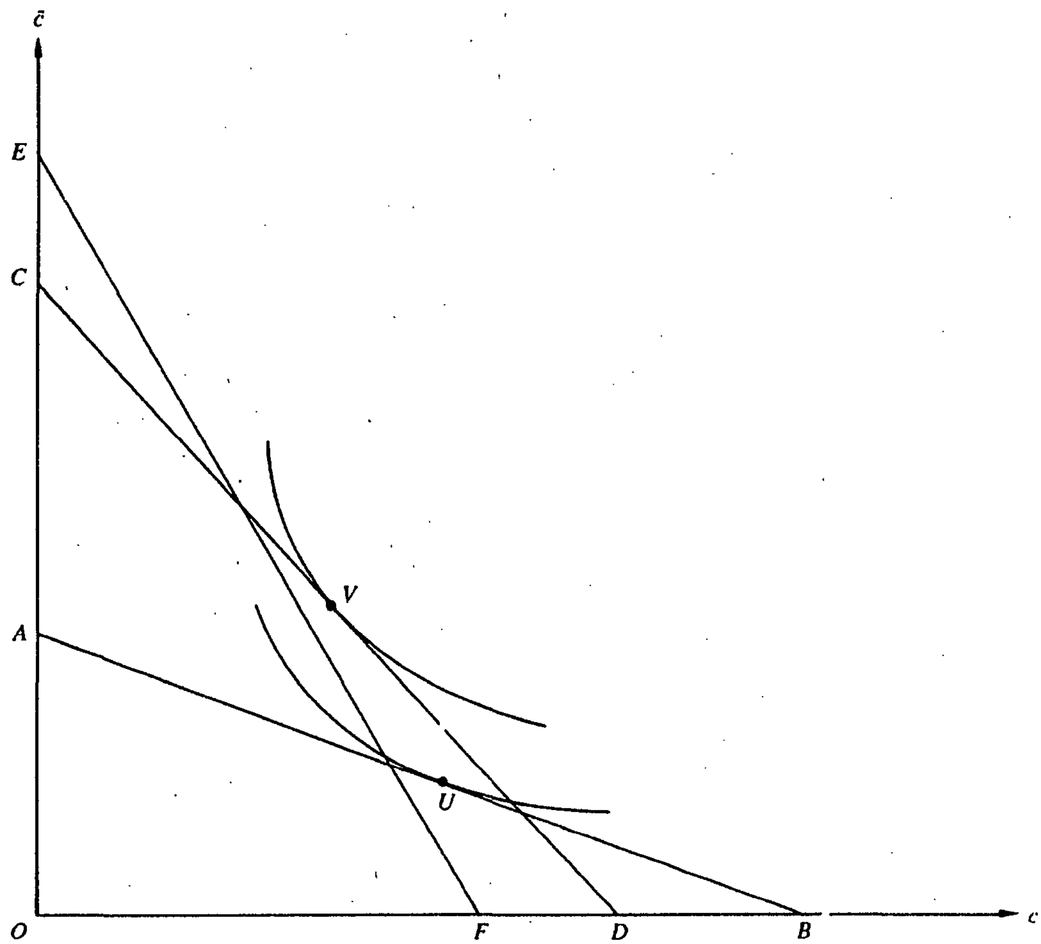

y, c
slope = r*
f(k)
slope = –1/(1 + n)
y*
I
slope = –r* = –1/(1 + r*)
V
W
B
45°
H
A
n
U
c*
G
LFSS
c̃
Z
J
c̃*
O
k*
k

FIGURE 2.



$\tilde{c}$
Z
M
slope = –[1 + r(1–τ)] > –(1 + n)
G'
G
slope = –(1 + n)
= –(1 + r)
slope = –(1 + n)
= –[1 + r(1–τ)]
I
N
H
H'
slope = – [1 + r(1–τ)] < –(1 + n)
LFSS (τ = o)
LFSS (τ > o)
O
c

FIGURE 3.



For a general constant-returns-to-scale production function, the shape of the LFSS is not easy to characterize. From the individual's budget constraint (3) and the economy-wide equilibrium condition (15), one can obtain the slope of the LFSS as

$$dc/d\tilde{c} = -[(1+n) + r/\varepsilon]/\{(1+n)[(1+r) + r/\varepsilon]\} \ . \tag{45}$$

As ε is a function of k in general, the LFSS may contain more than one inflection points. However, for a production function with a constant ε (such as the Cobb-Douglas), the LFSS is well-behaved and everywhere concave towards the horizontal axis. This can be seen by noting that

$$\begin{aligned} dc/d\tilde{c} &= -[(1+n)(1+\varepsilon)]^{-1} > 0 && \text{as } r = \infty && \text{(point 0)} \\ &= 0 && \text{as } r = -\varepsilon(1+n) && \text{(point H)} \\ &= -1 < 0 && \text{as } r = 0 && \text{(point Z).} \end{aligned} \tag{46}$$

In Figure 1, the LFSS is drawn on the assumption that ε is a constant. An important implication of (45) is clearly that, at the Golden Rule where n=r, the slope of the LFSS equals $-(1+n)^{-1}$ (point G). Hence, all points on the OHG and GZ portions of the curve are associated with, respectively, r > n and n > r.

A <u>competitive</u> steady state that is feasible is one in which the chosen consumption vector, such as $(c^*,\tilde{c}^*)$, happens to lie on the LFSS. This can occur on either side of point G, depending on the utility function. For illustrative purposes, it is shown to be at point U in Figure 1, which corresponds to a level of capital intensity below that corresponding to point G. If the government has direct control over the economy's resources and its objective is to maximize the level of steady-state utility, then it clearly pays for it to move the economy to point G, i.e., the Golden Rule. However, should the tangency between the indifference curve and the individual's budget constraint associated with point G not coincide with the latter (such as point V), to clear the capital market the government would need to provide an instrument (such as debt) to satisfy the desired intertemporal consumption pattern of the individual. The validity of the foregoing analysis is clearly predicated on the government's ability to choose G directly, bypassing a decentralized market mechanism altogether. Once there, the injection of government debt into the economy would have no real consequence, since at n=r debt of any size is self-financing. It is for this reason that the Golden Rule is optimal, and can be characterized even in a life-cycle model without reference to the nature of the utility and production functions.

The analysis becomes substantially different if the government must respect the market mechanism and can change the economy's level of capital intensity only indirectly through debt and taxes. The LFSS in Figure 1 is reproduced in Figure 2 (with the axes rotated clockwise by 90 degrees) as the broken curve labeled LFSS(τ=0). For τ > 0, the feasible consumption vector associated with any given output level must lie to the

South-west of the vector for $\tau = 0$, except at the end-points (points 0 and Z). Hence, for positive taxes the LFSS(τ>0) shrinks relative to that with no taxes. 1/ The curvature properties of the LFSS(τ>0) continue to be characterizable by those stated in (46) if the term r is replaced by the term $r(1-\tau)$, with the implication that at the Golden Rule (point G'), $n = r(1-\tau)$.

If the individual's indifference curve is tangent to the LFSS(τ>0) at G', then the Golden Rule is optimal and no government debt would be required. But such an outcome is purely coincidental. As discussed earlier, in a competitive economy the steady-state level of capital intensity could either exceed (point M) or fall short (point N) of the level that corresponds to the Golden Rule. In such circumstances, steady-state welfare could be improved with the injection of government debt (positive or negative), the optimal amount of which is to be determined by the nature of both the production and utility function.

Consider the competitive feasible steady state M in Figure 2. At point M, $n > r(1-\tau)$, and the government could increase the individual's steady-state utility level by introducing a positive amount of debt in order to lower the economy's level of capital intensity, 2/ which in turn increases the wage and decreases the interest rates. Figure 3 illustrates the impact of such an action on the individual's budget constraint and his level of utility. At the initial equilibrium, the consumption vector is point U (corresponding to point M in Figure 2) on the budget constraint AB. A positive amount of government debt shifts the budget constraint to CD, resulting in the new consumption vector V, which is on a higher indifference curve. A further increase in the debt level could lower welfare, however, if it were to shift the budget constraint to a line such as EF. At the optimum, therefore, a marginal change in the slope and location of the budget constraint, induced by a marginal change in the debt level, must leave the individual's level of utility unchanged. The same analysis applies should the initial competitive feasible steady state be at point N in Figure 2, only in this case the amount of government debt to be introduced would be negative. The optimal amount of debt determined in this manner is in accordance with the Rational Rule of savings.

VI. Concluding Remarks

The applicability and robustness of the various theoretical results and operational rules developed in the present paper would be enhanced if the model is extended in two important directions. First, allowance should be made for government expenditures to have intrinsic value to the private economy, either through public-good benefits in the utility function of

1/ It is possible for LFSS(τ>0) to intersect LFSS(τ=0) along the positively-sloped portion of the latter.

2/ Alternatively, this could be interpreted as a way to return part of the tax revenues to the private sector.

the representative individual, or through productive benefits (say, the provision and maintenance of the private economy's infrastructure) in the production function. This extension would render a more complete characterization of the government's optimization problem, expanding the role of fiscal policy to more than its customary incidence effects on capital accumulation.

Secondly, the response of private savings to changes in the level of government debt should be taken into account. The basic challenge of this extension is to incorporate such behavioral response without producing the degenerate case of complete Ricardian Equivalence. 1/ If successfully developed, these two extensions would allow one to differentiate in a meaningful way between the effects of changes in government debt arising from changes in expenditures and those stemming from changes in taxes.

1/ Blanchard's (1985) uncertain lifetime approach, though conceptually intriguing, remains somewhat ad hoc in spirit.

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